

1. Limits and Continuity

- $\lim(x \rightarrow a) f(x) = L$
 - $\lim(x \rightarrow \infty) 1/x^n = 0$, for $n > 0$
 - $\lim(x \rightarrow 0) \sin(x)/x = 1$
 - $\lim(x \rightarrow 0) (e^x - 1)/x = 1$
 - L'Hôpital's Rule: If limit is $0/0$ or ∞/∞ , then
 $\lim f(x)/g(x) = \lim f'(x)/g'(x)$
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2. Derivatives

- $d/dx (x^n) = n \cdot x^{n-1}$
- $d/dx (e^x) = e^x$
- $d/dx (\ln x) = 1/x$
- $d/dx (\sin x) = \cos x$
- $d/dx (\cos x) = -\sin x$
- $d/dx (\tan x) = \sec^2 x$
- $d/dx (\arcsin x) = 1/\sqrt{1 - x^2}$
- $d/dx (\arctan x) = 1/(1 + x^2)$

Rules

- Product Rule: $(uv)' = u'v + uv'$

- Quotient Rule: $(u/v)' = (u'v - uv') / v^2$
 - Chain Rule: $d/dx [f(g(x))] = f'(g(x)) \cdot g'(x)$
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3. Integration

- $\int x^n dx = x^{n+1} / (n + 1) + C$, for $n \neq -1$
- $\int 1/x dx = \ln|x| + C$
- $\int e^x dx = e^x + C$
- $\int \sin x dx = -\cos x + C$
- $\int \cos x dx = \sin x + C$
- $\int \sec^2 x dx = \tan x + C$
- $\int 1/(1 + x^2) dx = \arctan x + C$
- $\int 1/\sqrt{1 - x^2} dx = \arcsin x + C$

Techniques

- Substitution: $\int f(g(x))g'(x) dx = \int f(u) du$
 - Integration by Parts: $\int u dv = uv - \int v du$
 - Partial Fractions: Break rational expressions before integrating
 - Trig Substitution: Use trig identities for expressions with square roots
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4. Definite Integrals and Applications

- $\int_a^b f(x) dx$: Area under the curve
 - Average value of $f(x)$: $(1/(b - a)) \int_a^b f(x) dx$
 - Net change: $\int_a^b f'(x) dx = f(b) - f(a)$
 - Area between curves: $\int_a^b [f(x) - g(x)] dx$
 - Volume by Disk Method: $V = \pi \int_a^b [f(x)]^2 dx$
 - Volume by Shell Method: $V = 2\pi \int_a^b x \cdot f(x) dx$
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5. Series and Sequences

- Geometric Series: $\sum ar^n = a / (1 - r)$, for $|r| < 1$
 - Taylor Series:

$$f(x) = \sum [f^n(a) / n!] \cdot (x - a)^n$$
 - Maclaurin Series (Taylor series at $a = 0$):

$$e^x = \sum x^n / n!$$

$$\sin x = \sum (-1)^n x^{2n+1} / (2n + 1)!$$

$$\cos x = \sum (-1)^n x^{2n} / (2n)!$$

$$\ln(1 + x) = \sum (-1)^{n+1} x^n / n$$
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6. Multivariable Calculus

- Partial Derivatives: $\partial f / \partial x$, $\partial f / \partial y$
- Gradient: $\nabla f = \langle \partial f / \partial x, \partial f / \partial y, \dots \rangle$
- Double Integrals: $\iint f(x, y) dA$

- Jacobian: Used for change of variables in multiple integrals
 - Lagrange Multipliers: $\nabla f = \lambda \nabla g$ (for constrained optimization)
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7. Differential Equations

- Separable Equations: $dy/dx = g(x) \cdot h(y)$
 $\Rightarrow \int (1/h(y)) dy = \int g(x) dx$
 - First-Order Linear: $dy/dx + P(x)y = Q(x)$
 $\Rightarrow y = e^{\int P(x) dx} \cdot [\int Q(x) \cdot e^{\int P(x) dx} dx + C]$
 - Exponential Growth/Decay: $y = y_0 \cdot e^{kt}$
 - Newton's Law of Cooling: $dT/dt = -k(T - T_{\text{env}})$
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8. Constants and Identities

- $e \approx 2.71828$
- $\pi \approx 3.14159$
- $\ln(e) = 1$
- $\ln(1) = 0$
- $d/dx (\ln|x|) = 1/x$
- $d/dx (a^x) = a^x \ln(a)$