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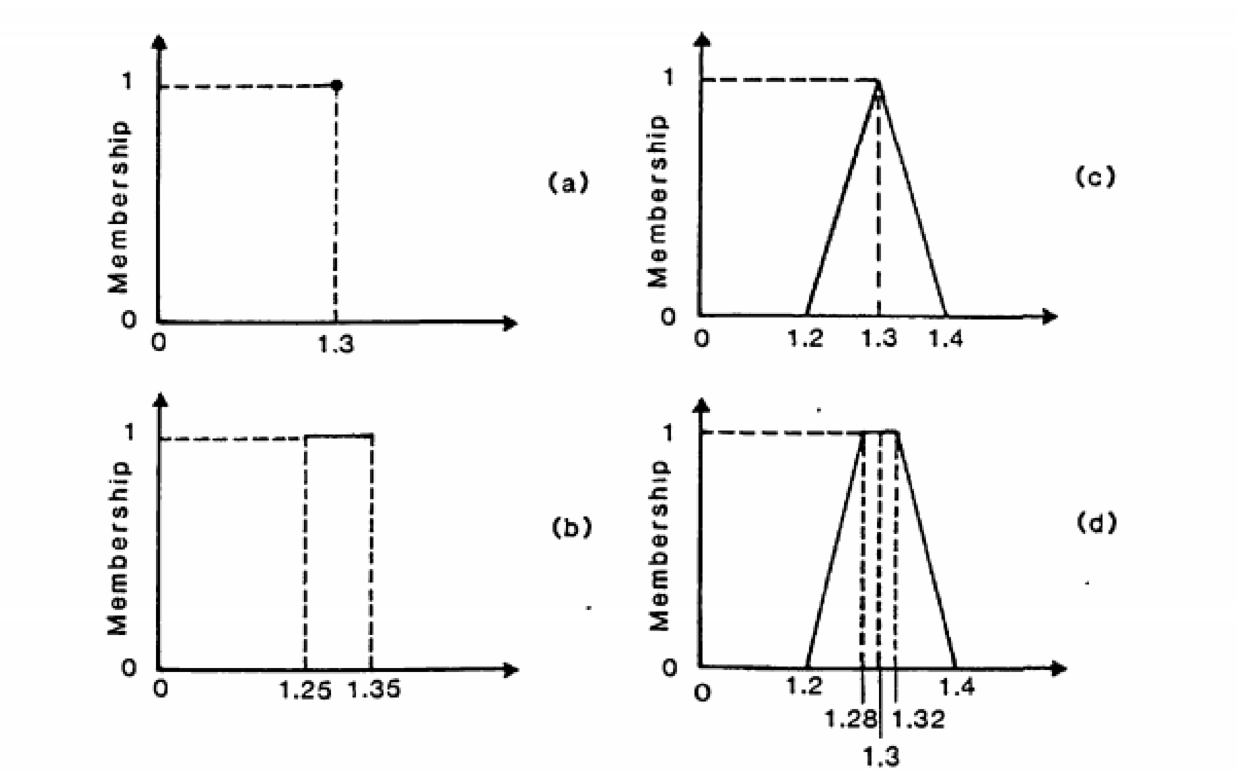
Chapter Review

FUZZY NUMBERS

Among the various types of fuzzy sets, of special significance are fuzzy sets that are defined on the set R of real numbers. Membership functions of these sets, which have the form

A : R -> (0, 1]

To qualify as a fuzzy number, a fuzzy set A on IR must possess at least the following three properties:
(i) A must be a normal fuzzy set
(ii) °A must be a closed interval for every a E (0, 1]
(iii) the support of A, °+A, must be bounded



Special cases of fuzzy numbers include ordinary real numbers and intervals of real numbers, as

illustrated
(a) an ordinary real number 1.3
(b) an ordinary (crisp) closed interval [1.25, 1.35]
(c) a fuzzy number expressing the proposition "close to 1.3"
(d) a fuzzy number with a flat region (a fuzzy interval)

LINGUISTIC VARIABLES

The concept of a fuzzy number plays a fundamental role in formulating quantitative fuzzy variables. These are variables whose states are fuzzy numbers. When, in addition, the fuzzy numbers represent linguistic concepts, such as very small, small, medium, and so on, as interpreted in a particular context, the resulting constructs are usually called linguistic variables.

- Each linguistic variable the states of which are expressed by linguistic terms interpreted as specific fuzzy numbers is defined in terms of a base variable, the values of which are real numbers within a specific range.
- Each linguistic variable is fully characterized by a quintuple (v, T, X, g, m)

ARITHMETIC OPERATIONS ON INTERVALS

Fuzzy arithmetic is based on two properties of fuzzy numbers:
- each fuzzy set, and thus also each fuzzy number, can fully and uniquely be represented by its a-cuts
- a-cuts of each fuzzy number are closed intervals of real numbers for all a E (0, 1].

These properties enable us to define arithmetic operations on fuzzy numbers in terms of arithmetic operations on their a-cuts (i.e., arithmetic operations on closed intervals). The latter operations are a subject of interval analysis, a well-established area of classical mathematics.

Let * denote any of the four arithmetic operations on closed intervals:
addition + , subtraction - , multiplication . ,division / ,Then,

$$\mathbf{[a,b] * [d,e] = \{ f * g | a < f < b , d < g < e \}}$$

is a genetal property of all arithmetic operations on closed intervals, except that [a, b]/[d, e] is not defined when 0 E [d, e]. That is, the result of an arithmetic operation on closed intervals is again a closed interval. The four arithmetic operations on closed intervals are defined as follows:

[a, b] + [d, e] = [a + d, b + e]

[a,b]-[d,e] = [a-e,b-d]

[a, b] [d, e] = [min(ad, ae, bd, be), max(ad, ae, bd, be)]

provided that 0 ¢ [d, e],

[a, b]/[d, e] = [a, b] [11e, l/d] = [min(a/d, ale, b/d, b/e), max(a/d, ale, b/d, b/e)].

Arithmetic operations on closed intervals satisfy some useful properties. To overview them, let A = [at, a2], B = (bt, b2], C = [cl, c2], 0 = [0, 0],1 = [1, 1]. Using these symbols, the properties are formulated as follows:

1. A+B=.B+A, A B = B . A (commutativity).
2. (A+B)+C=A+(B+C) (A B) C = A (B C) (associativity).
3. A=O+A=A+O A = 1 A = A -1 (identity).
4. A (B + C) C A - B + A - C (sub distributivity).
5. If b . c >_ 0 for every b E B and C E C, then A - (B +C) = A - B +A - C (distributivity).
   Furthermore, if A = [a, a], then a (B + C) = a - B + a - C.

6.OEA-Aand1EA/A. 7. If A C E and B C F, then: A+B C E+F, A-B C E-F, C E-F, A/B c E/F (inclusion monotonicity)