Лабораторная работа №1 Чупель, 581073

Свойства дискретного преобразования Фурье.

Вып. работы: 1. Вычислить спектр ДПФ посл. Построить график амплитуд. спектра этой посл.
2. Вычислить обратное дискрет. преобраз.
3. Теорема свертки. Вычислить свертку (автосвертку) исходной посл. по опред. (построить график свертки)
4. Теорема о корреляции.
5 Подтвердить теорему Парсеваля

Посл.:
$x(n) = \{-1 \quad 1 \quad 3 \quad \underset{\uparrow}{0} \quad -1 \quad 1 \quad 3 \quad 0\}$

1.

Дискретное преобразование Фурье:

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}} = \sum_{n=0}^{n-1} x(n) W^{kn} \; ; \; k = 0; 1; \ldots; N-1$$

$$x(n) = \{ -1 \quad 1 \quad 3 \quad 0 \quad -1 \quad 1 \quad 3 \quad 0 \}$$

$$\begin{bmatrix}
W^0 & W^0 & W^0 & W^0 & W^0 & W^0 & W^0 & W^0 \\
W^0 & W^1 & W^2 & W^3 & W^4 & W^5 & W^6 & W^7 \\
W^0 & W^2 & W^4 & W^6 & W^0 & W^2 & W^4 & W^6 \\
W^0 & W^3 & W^6 & W^1 & W^4 & W^7 & W^2 & W^5 \\
W^0 & W^4 & W^0 & W^4 & W^0 & W^4 & W^0 & W^4 \\
W^0 & W^5 & W^2 & W^7 & W^4 & W^1 & W^6 & W^3 \\
W^0 & W^6 & W^4 & W^2 & W^0 & W^6 & W^4 & W^2 \\
W^0 & W^7 & W^6 & W^5 & W^4 & W^3 & W^2 & W^1
\end{bmatrix}$$

$$W^{kn} = e^{-j\frac{2\pi kn}{N}} = \cos\frac{2\pi kn}{N} - j\sin\frac{2\pi kn}{N}$$

$$W^0 = \exp(0) = \cos 0 - j\sin 0 = 1 - 0 = 1$$

$w^1 = \exp(-j\frac{\pi}{4}) = \cos 45 - j\sin 45 = \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$

$w^2 = \exp(-j\frac{\pi}{2}) = \cos 90 - j\sin 90 = 0 - j = -j$

$w^3 = \exp(-j\frac{3\pi}{4}) = \cos 135 - j\sin 135 = -\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}$

$w^4 = \exp(-j\pi) = \cos 180 - j\sin 180 = -1 + 0 = -1$

$w^5 = \exp(j\frac{3\pi}{4}) = \cos 225 - j\sin 225 = -\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$

$w^6 = \exp(-j\frac{3\pi}{2}) = \cos 270 - j\sin 270 = 0 + j = j$

$w^7 = \exp(-j\frac{7\pi}{4}) = \cos 315 - j\sin 315 = \frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}$

$$\begin{bmatrix}
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & \frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} & -j & -\frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} & -1 & -\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2} & j & \frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2} \\
1 & -j & -1 & j & 1 & -j & -1 & j \\
1 & -\frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} & j & \frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2} & -1 & \frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2} & -j & \frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} \\
1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\
1 & -\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2} & -j & \frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} & -1 & \frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} & j & -\frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} \\
1 & j & -1 & -j & 1 & j & -1 & -j \\
1 & \frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2} & j & -\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2} & -1 & -\frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2} & -j & \frac{\sqrt{2}}{2}-j\frac{\sqrt{2}}{2}
\end{bmatrix}
\begin{bmatrix} -1 \\ 1 \\ 3 \\ 0 \\ -1 \\ 1 \\ 3 \\ 0 \end{bmatrix}$$

$X(0) = x(0)w^0 + x(1)w^0 + x(2)w^0 + x(3)w^0 + x(4)w^0 +$
$+ x(5)w^0 + x(6)w^0 + x(7)w^0 = (-1)\cdot 1 + 1\cdot 1 + 3\cdot 1 + 0\cdot 1 +$
$+ (-1)\cdot 1 + 1\cdot 1 + 3\cdot 1 + 0\cdot 1 = -1 + 1 + 3 + 0 - 1 + 1 +$
$+ 3 + 0 = 6$

$X(1) = x(1)w^0 + x(1)w^1 + x(2)w^2 + x(3)w^3 + x(4)w^4 +$
$+ x(1)w^5 + x(1)w^6 + x(1)w^7 = 1(-1) + \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2} +$
$+ (-j)3 + 0 + (-1)(-1) = -\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2} + 3j + 0 =$
$= -1 + 1 = 0$

$X(2) = (-1)^{-1}\,1 + (-j)^{-3}\,1 + (-1)^{-5}\,3 + j^{0}\,0 + (-1)^{-1}\,1 + (-j)^{-3}\,1 +$
$+ (-1)^{-3}\,3 + j^{0}\,0 =$

$X(3) = -1 + \left(-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) + 3j + 0 + 1 - \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2} - 3j + 0 =$

$= 0$

$X(4) = -1 + (-1) + 3 + 0 + (-1) + (-1) + 3 + 0 = -4 + 6 = 2$

$X(5) = (-1) + \left(-\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) - 3j + 0 + 1 + \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2} +$
$+ 3j + 0 = 0$

$X(6) = \sqrt{(-8)^2 + (2\cdot(-1))^2} = \sqrt{68} = 8.24$

$X(6) = -1 - j - 3 + 0 + 1 + j - 3 + 0 = -8 + 2j$

$X(7) = -1 + \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2} + 3j + 0 + 1 - \frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2} - 3j + 0 =$

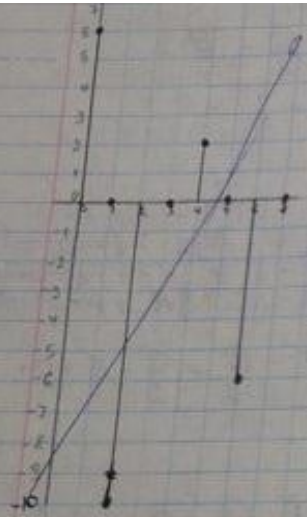
$= 0$

$X(k) = \{6, 0, -8+2j, 0, 2, 0, -8-2j, 0\}$

$X(k) = \{6, 0, 8.24, 0, 2, 0, 8.24, 0\}$

$X(2) = -1 - j - 3 - 1 - j - 3 = -8 - 2j$

$\sqrt{(-8)^2 + (-2\cdot(-1))^2} = \sqrt{68} = 8.24$



2) обратное ДПФ

$$x(n) = N^{-1}\sum_{k=0}^{N-1} X(k) e^{j\frac{2\pi kn}{N}} = N^{-1}\sum_{k=0}^{N-1} X(k) W^{-kn}, \quad n = 0, 1, \dots N-1$$

$$V_2^{-1} \begin{bmatrix} W^0 & W^0 & W^0 & W^0 & W^0 & W^0 & W^0 & W^0 \\ W^0 & W^7 & W^6 & W^5 & W^4 & W^3 & W^2 & W^1 \\ W^0 & W^6 & W^4 & W^2 & W^0 & W^6 & W^4 & W^2 \\ W^0 & W^5 & W^2 & W^7 & W^4 & W^1 & W^6 & W^3 \\ W^0 & W^4 & W^0 & W^4 & W^0 & W^4 & W^0 & W^4 \\ W^0 & W^3 & W^6 & W^1 & W^4 & W^7 & W^2 & W^5 \\ W^0 & W^2 & W^4 & W^6 & W^0 & W^2 & W^4 & W^6 \\ W^0 & W^1 & W^2 & W^3 & W^4 & W^5 & W^6 & W^7 \end{bmatrix}$$

$W^{kn} = e^{j\frac{2\pi kn}{N}} = \cos\frac{2\pi kn}{N} + j\sin\frac{2\pi kn}{N}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{\sqrt2}{2}+j\frac{\sqrt2}{2} & j & -\frac{\sqrt2}{2}+j\frac{\sqrt2}{2} & -1 & -\frac{\sqrt2}{2}-j\frac{\sqrt2}{2} & -j & \frac{\sqrt2}{2}-j\frac{\sqrt2}{2} \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & -\frac{\sqrt2}{2}+j\frac{\sqrt2}{2} & -j & \frac{\sqrt2}{2}-j\frac{\sqrt2}{2} & -1 & -\frac{\sqrt2}{2}+j\frac{\sqrt2}{2} & j & -\frac{\sqrt2}{2}-j\frac{\sqrt2}{2} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\frac{\sqrt2}{2}-j\frac{\sqrt2}{2} & j & \frac{\sqrt2}{2}-j\frac{\sqrt2}{2} & -1 & \frac{\sqrt2}{2}+j\frac{\sqrt2}{2} & -j & -\frac{\sqrt2}{2}+j\frac{\sqrt2}{2} \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & \frac{\sqrt2}{2}-j\frac{\sqrt2}{2} & -j & -\frac{\sqrt2}{2}-j\frac{\sqrt2}{2} & -1 & -\frac{\sqrt2}{2}+j\frac{\sqrt2}{2} & j & \frac{\sqrt2}{2}+j\frac{\sqrt2}{2} \end{bmatrix}$$

$W^0 = 1$
$W^1 = \frac{\sqrt2}{2} + j\frac{\sqrt2}{2}$
$W^2 = j$
$W^3 = -\frac{\sqrt2}{2} + j\frac{\sqrt2}{2}$
$W^4 = -1$
$W^5 = -\frac{\sqrt2}{2} - j\frac{\sqrt2}{2}$
$W^6 = -j$
$W^7 = \frac{\sqrt2}{2} - j\frac{\sqrt2}{2}$

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & \frac{\sqrt2}{2}+j\frac{\sqrt2}{2} & j & -\frac{\sqrt2}{2}+j\frac{\sqrt2}{2} & -1 & -\frac{\sqrt2}{2}-j\frac{\sqrt2}{2} & -j & \frac{\sqrt2}{2}-j\frac{\sqrt2}{2} \\ 1 & j & -1 & -j & 1 & j & -1 & -j \\ 1 & -\frac{\sqrt2}{2}+j\frac{\sqrt2}{2} & -j & \frac{\sqrt2}{2}-j\frac{\sqrt2}{2} & -1 & \frac{\sqrt2}{2}+j\frac{\sqrt2}{2} & j & -\frac{\sqrt2}{2}-j\frac{\sqrt2}{2} \\ 1 & -1 & 1 & -1 & 1 & -1 & 1 & -1 \\ 1 & -\frac{\sqrt2}{2}-j\frac{\sqrt2}{2} & j & \frac{\sqrt2}{2}-j\frac{\sqrt2}{2} & -1 & \frac{\sqrt2}{2}+j\frac{\sqrt2}{2} & -j & -\frac{\sqrt2}{2}+j\frac{\sqrt2}{2} \\ 1 & -j & -1 & j & 1 & -j & -1 & j \\ 1 & \frac{\sqrt2}{2}-j\frac{\sqrt2}{2} & -j & -\frac{\sqrt2}{2}-j\frac{\sqrt2}{2} & -1 & -\frac{\sqrt2}{2}+j\frac{\sqrt2}{2} & j & \frac{\sqrt2}{2}+j\frac{\sqrt2}{2} \end{bmatrix} \begin{bmatrix} 8 \\ 0 \\ -8-2j \\ 0 \\ 2 \\ 0 \\ -8+2j \\ 0 \end{bmatrix} = \begin{bmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}$$

$$x(0) = \frac{1}{8}(1\cdot 6 + 0 + (-8-j)\cdot 1 + 0 + 2 - 8 + 2j + 0) =$$

$$= \frac{1}{8}\left(1\cdot 6 + 1\cdot 0 + 1\cdot(-8-j) + 1\cdot 0 + 1\cdot 2 + 1\cdot 0 + 1\cdot(-8+2j) + 0\right) = \frac{1}{8}(-8) =$$

$$= \frac{-8}{8} + \frac{j}{8} = -1$$

$$x(1) = \frac{1}{8}\left(1\cdot 6 + \frac{\sqrt2}{2} + j\frac{\sqrt2}{2} + 0 + j\cdot(-8-2j) + 0 + (-2) + 0 + (-j)(-8+2j)\right) =$$

$$= \frac{1}{8}(6 + 0 + \dots) = \frac{1+\dots}{8}$$

$$x(1) = \frac{1}{8}(6\cdot 1 + 0 + j\cdot(-8-2j) + 0 - 2 + 0 + (-j)\cdot(-8+2j) + 0) =$$

$$= \frac{1}{8}(6 + (-8j - 2j^2) + 0 - 2 + 8j - 2j^2) = \frac{1}{8}(4 - 4j^2) =$$

$$= 1$$

$\cdot(-8+2j)+0) = 3$

$x(3) = \frac{1}{8}(6+0+(-j)(-8-2j)+(-1)2+(j)(8+2j)-0) =$

$= 0$

$x(4) = -1$

$x(5) = 1$

$x(6) = 3$

$x(7) = 0$

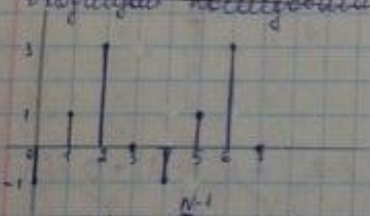
$x(n) = \{-1, 1, 3, 0, -1, 1, 3, 0\}$

№3

Свёртка

$x(n) = \{-1; 1; 3; 0; -1; 1; 3; 0\}$ $n(n) = \{-1, 1, 3, 0, -1, 1, 3\}$

~~порядок последовательности~~



$$y(n) = \sum_{m=0}^{N-1} x(m)\, n(n-m), \quad n = 0, \ldots, N-1$$

~~$y(0)$~~

Автосвёртка – это взаимная свёртка 2х одинаковых фун

$n=0$ $y(0) = x(0) \cdot n(0) = 1$

$n=1$ $y(1) = x(0) \cdot n(1) + x(1) \cdot n(0) = (-1) \cdot 1 + 1 \cdot (-1) = 0-2$

$= -3 + 1 - 3 = -5$

$y(3) = x(0)\, n(3) + x(1) \cdot n(2) + x(2) \cdot n(1) + x(3) \cdot n(0) =$ $n=3$

$= 0 + 3 + 3 + 0 = 6$

$y(4) = x(0) \cdot n(4) + x(1)\, n(3) + x(2) \cdot n(2) + x(3) \cdot n(1) +$ $n=4$

$+ x(4)\, n(0) = (-1) \cdot (-1) + 1 \cdot 0 + 3 \cdot 3 + 0 \cdot 1 + (-1)(-1) =$

$= 1 + 9 + 1 = 11$

$y(5) = x(0)\, n(5) + x(1) \cdot n(4) + x(2)\, n(3) + x(3) \cdot n(2) +$ $n=5$

$+ x(4) \cdot n(1) + x(5) \cdot n(0) = -1 + (-1) + 0 + 0 + -1 + -1 =$

$= -4$

$y(6) = x(0)\, n(6) + x(1) \cdot n(5) + x(2) \cdot n(4) + x(3) \cdot n(3) +$ $n=6$

$+ x(4) \cdot n(2) + x(5) \cdot n(1) + x(6) \cdot n(0) = (-3) + 1 + (-3) +$

$+ 0 + (-3) + 1 \cdot (-3) = -12 + 2 = -10$

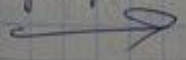
$y(7) = x(0) \cdot n(7) + x(1) \cdot n(6) + x(2)\, n(5) + x(3) \cdot n(4) +$ $n=7$

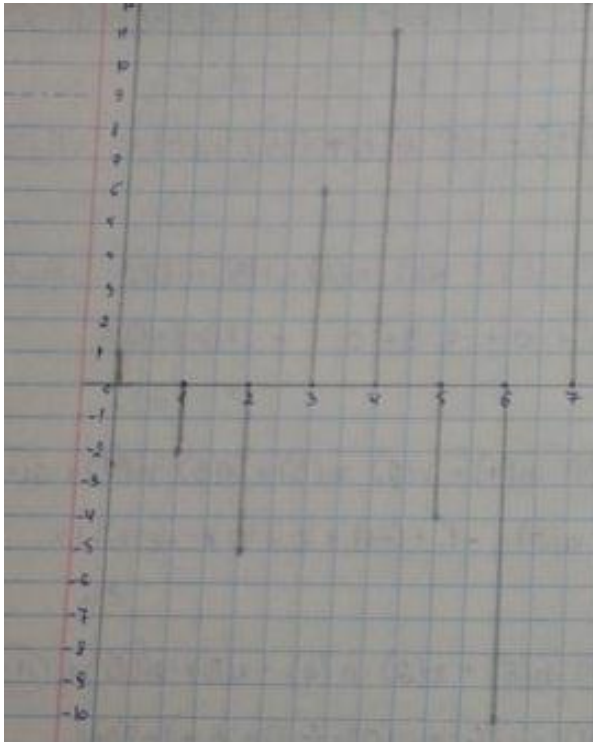
$+ x(4) \cdot n(3) + x(5) \cdot n(2) + x(6) \cdot n(1) + x(7) \cdot n(0) =$

$= 0 + 3 + 3 + 0 + 0 + 3 + 3 + 0 = 12$

$y(n) = \{1, -2, -5, 6, 11, -4, -10, 12\}$

график





4) Корреляция

$x(n) = \{-1; 1; 3; 0; -1; 1; 3; 0\}$

$r(n) = \sum_{m=0}^{N-1} x(m)\, h(n+m)$, $n = 0, N-1$

$r(0) = x(0)\, h(0) + x(1) \cdot h(1) + x(2)\, h(2) + x(3)\, h(3) +$

$+ x(4) \cdot h(4) + x(5) \cdot h(5) + x(6)\, h(6) + x(7)\, h(7) =$

$= -1 + 1 + 9 + 0 + 1 + 1 + 9 + 0 = 22$

$r(1) = x(0) \cdot h(1) + x(1)\, h(2) + x(2) \cdot h(3) + x(3) \cdot h(4) +$

$+ x(4) \cdot h(5) + x(5) \cdot h(6) + x(6) \cdot h(7) = -1 + 3 + 0 +$

$0 + (-1) + 3 + 0 = -2 + 6 = 4$

$r(2) = x(0)\, h(2) + x(1)\, h(3) + x(2) \cdot h(4) + x(3) \cdot h(5) + x(4) \cdot h(6) +$

$+ x(5) \cdot h(7) = -3 + 0 + (-3) + 0 + (-3) + 0 = -9$

$r(3) = x(0)\, h(3) + x(1)\, h(4) + x(2) \cdot h(5) + x(3)\, h(6) +$

$+ x(4)\, h(7) = 0 + (-1) + 3 + 0 + 0 = 2$

$r(4) = x(0)\, h(4) + x(1) \cdot h(5) + x(2) \cdot h(6) + x(3) \cdot h(7) =$

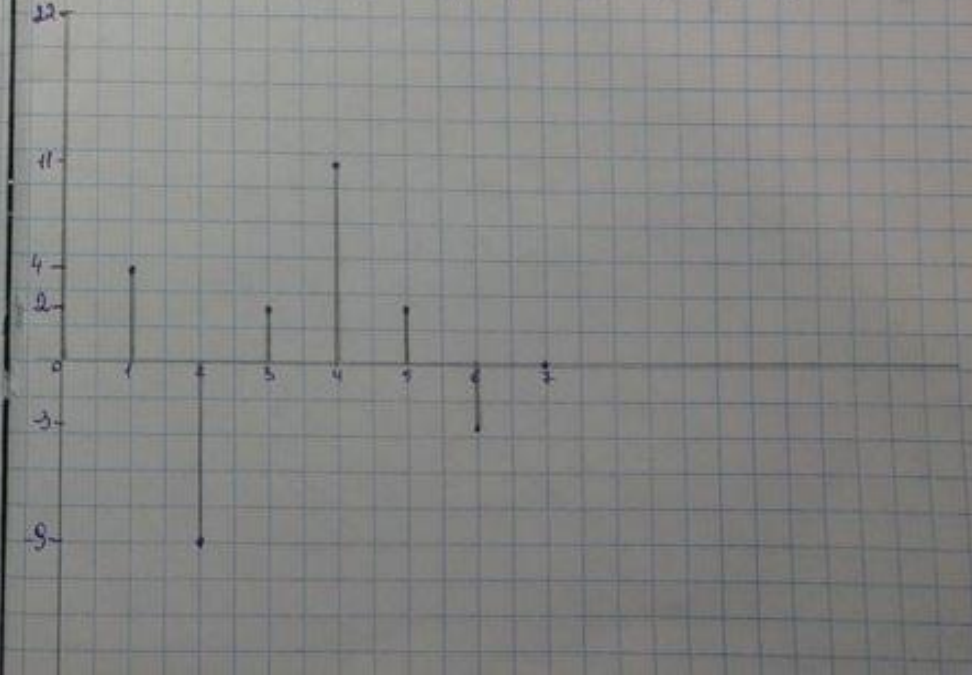
$= 1 + 1 + 9 + 0 = 11$

$r(5) = x(0) \cdot h(5) + x(1)\, h(6) + x(2)\, h(7) = -1 + 3 + 0 = 2$

$r(6) = x(0) \cdot h(6) + x(1) \cdot h(7) = -3 + 0 = -3$

$r(7) = x(0) \cdot h(7) = 0$

$r(n) = \{22; 4; -9; 2; 11; 2; -3; 0\}$



5) $x(n) = \{-1; 1; 3; 0; -1; 1; 3; 0\}$

$$N^{-1} \sum_{k=0}^{N-1} |X(k)|^2 = \sum_{n=0}^{N-1} |x(n)|^2$$

$$\sum_{n=0}^{N-1} |x(n)|^2 = |x(0)|^2 + |x(1)|^2 + |x(2)|^2 + |x(3)|^2 + |x(4)|^2 + |x(5)|^2 + |x(6)|^2 + |x(7)|^2 = (-1)^2 + 1^2 + 3^2 + 0^2 + (-1)^2 + 1^2 + 3^2 + 0^2 =$$

$$= 1 + 1 + 9 + 0 + 1 + 1 + 9 + 0 = 22$$

$$\sum_{n=0}^{N-1} |X(n)|^2 = |X(0)|^2 + |X(1)|^2 + |X(2)|^2 + |X(3)|^2 + |X(4)|^2 + |X(5)|^2 + |X(6)|^2 + |X(7)|^2 = 6^2 + 0^2 + 8{,}24^2 + 0^2 + 2^2 + 8{,}24^2 + 0^2 =$$

$$= 36 + 0 + 67{,}89 + 4 + 67{,}9 = 175{,}8$$

$$N^{-1} \sum_{k=0}^{N-1} |X(k)|^2 = \frac{1}{8} \cdot 175{,}8 \approx 22$$