Q2 (a): Cournot Duopoly Model

Cournot model involves two firms producing a homogeneous product.

They simultaneously choose quantities. Market price: $P = a - b(Q1 + Q2)$

Firm 1's Profit: $pi1 = aQ1 - bQ1^2 - bQ1Q2$

Maximizing: $dpi1/dQ1 = a - 2bQ1 - bQ2 = 0 \Rightarrow Q1 = (a - bQ2)/2b$

Similarly: $Q2 = (a - bQ1)/2b$

Solving:

$Q1 = Q2 = a/3b \Rightarrow Q = 2a/3b \Rightarrow P = a - bQ = a/3$

Profit = $PQ1 = (a/3)(a/3b) = a^2/9b$

Q2 (b): Bayesian Game

Bayesian games involve incomplete information.

Each player has types and beliefs over others types.

Example: Market Entry Game

- Player 1 (Entrant) chooses Enter or Stay Out.

- Player 2 (Incumbent) is Strong (60%) or Weak (40%).

Payoffs show Strong fights, Weak accommodates.

Expected payoff if Enter = 0.6(0) + 0.4(3) = 1.2

Stay Out = 1 => Entrant chooses Enter.

Bayesian Equilibrium: Entrant Enters, Strong Fights, Weak Accommodates.

Q3 (a): Hill Climbing Limitations

Limitations:

1. Local Maxima

2. Plateaus

3. Ridges

4. Greedy search

Solution: Simulated Annealing

- Accepts worse moves temporarily.

- Helps escape local optima.

P(accept) = exp(-DeltaE/T)

Q3 (b): Heuristic & TSP

Heuristic = estimated cost from node to goal, h(n)

TSP Heuristic:

h(n) = MST(unvisited) + min(current to unvisited) + min(unvisited to start)

Admissible and useful for A* search.

Q5 (a): Logistic Regression

Given: X = [3,2,1,3,0,4.19], W = [2.5,-5,-1.2,0.5,2,0.7], b = 0.1

z = W.X + b = 0.833

Sigmoid = 1 / (1 + e^-z) approx 0.697

=> Review is Positive (since > 0.5)

Q5 (b): Least Squares

x_bar = 7, y_bar = 72.5

m = (Sum xy - n*x_bar*y_bar) / (Sum x^2 - n*x_bar^2) = 4.5

c = y_bar - m*x_bar = 41

Line: y = 4.5x + 41

For x = 7 => y = 72.5