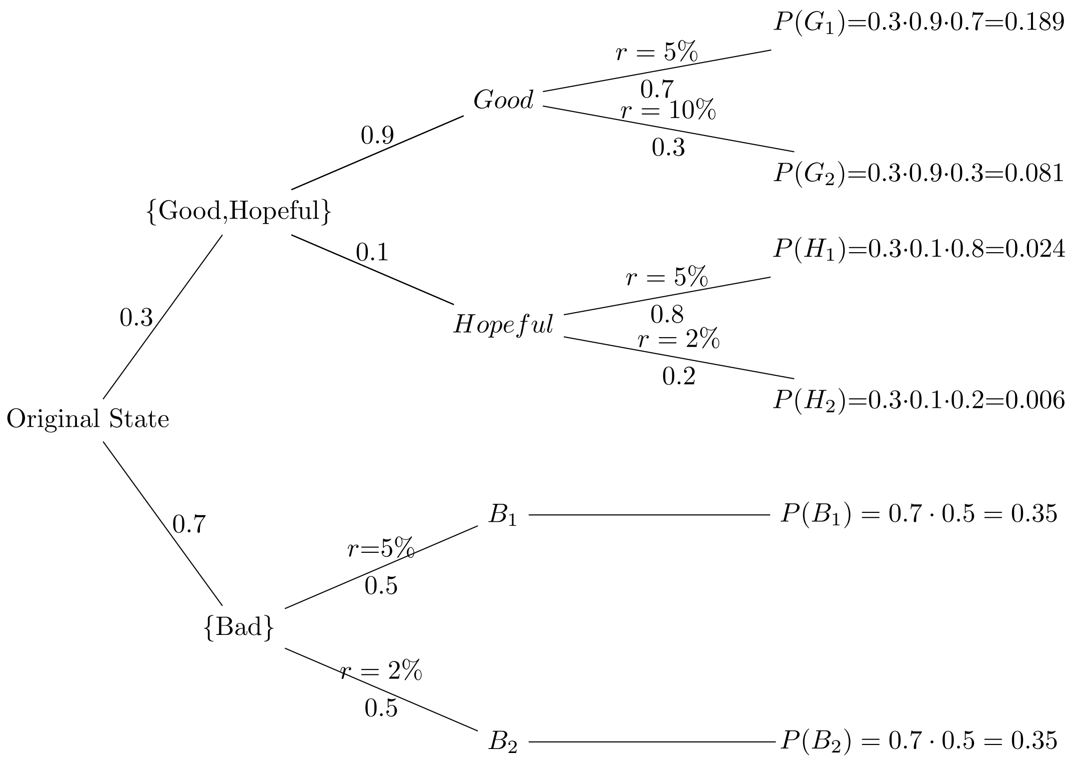
Information: Part One

Suppose we have an asset that has a dividend, d, that pays \$20 per period, d=\$20. In period 2 we learn if we are in a "bad" world state with probability of 0.7, in which the dividend falls to d_3^b=\$5. There is a 0.3 probability we are in the $\{Good, Hopeful\}$ state where the dividend remains d=\$20. If we are in the **bad** world then the dividend payments continues forever. We learn in period three whether we are in the **Good** state with probability 0.9 or we could be in the **Hopeful** with a probability of 0.1. The **Good** world pays a dividend,d_4^g=\$40, and it continues this dividend forever. The **Hopeful** world has dividend payment that is the same as the initial dividend, and the payment continues forever.

Information: Part Two

Assume that Federal Reserve sets the interest rates in the economy. In period 4 we learn that the Federal Reserve could react to each world state by changing the interest rates or leaving them unchanged. In the **Good** state, the Fed believes that the rise in asset prices is due to a bubble with a probability 0.3, and because of this they raise the interest rate to r=10%. If we are in the **Hopeful** state then the Fed sees the fall in asset prices harmful to the economy with a probability 0.2, and as such they lower the interest rate to r=4%. Finally, if we are in the **Bad** state, the Fed believes that we are in a recession with a probability of 0.5 and cuts the interest rate to r=2%

Probability



$$Pr\{Good, Hopeful\} = Pr\{G_1\} + Pr\{G_2\} + Pr\{H_1\} + Pr\{H_1\} = 0.3 \tag{1}$$
$$Pr\{Bad\} = Pr\{B_1\} + Pr\{B_2\} = 0.7 \tag{2}$$

Dividends

State	*sym*	Prob.	r_{future}	t	1	2	3	4	5	6	...
G_1	d_g	0.189	r_{future}=5%		\$20	\$20	\$20	\$40	\$40	\$40	\$40
G_2	d_g	0.081	r_{future}=10%		\$20	\$20	\$20	\$40	\$40	\$40	\$40
H_1	d_h	0.024	r_{future}= 5%		\$20	\$20	\$20	\$20	\$20	\$20	\$20
H_2	d_h	0.006	r_{future}=4%		\$20	\$20	\$20	\$20	\$20	\$20	\$20
B_1	d_b	0.35	r_{future}=5%		\$20	\$20	\$5	\$5	\$5	\$5	\$5
B_2	d_b	0.35	r_{future}=2%		\$20	\$20	\$5	\$5	\$5	\$5	\$5

Prices of Asset

The prices of the asset can be represented by the equation:

$$P_t = \frac{E_t d_{t+1} + E_t P_{t+1}}{1 + r_t} \tag{3}$$

State	*sym*	Prob.	r_{future}	t	1	2	3	4	5	6	...
G_1	P^{G_1}	0.189	r_{future}=5%		\$314.99	\$310.74	\$663.86	\$685.71	\$800	\$800	\$800
G_2	P^{G_2}	0.081	r_{future}=10%		\$314.99	\$310.74	\$663.86	\$685.71	\$800	\$400	\$400
H_1	P^{H_1}	0.024	r_{future}= 5%		\$314.99	\$310.74	\$663.86	\$419.05	\$400	\$400	\$400
H_2	P^{H_2}	0.006	r_{future}=4%		\$314.99	\$310.74	\$663.86	\$419.05	\$500	\$500	\$500
B_1	P^{B_1}	0.35	r_{future}=5%		\$314.99	\$310.74	\$168.03	\$171.43	\$100	\$100	\$100
B_2	P^{B_2}	0.35	r_{future}=2%		\$314.99	\$310.74	\$168.03	\$171.43	\$250	\$250	\$250

Calculations

Period 4 Prices

$$\begin{aligned} E_4 P_5^G &= (0.8)P_5^{G_1} + (0.2)P_5^{G_2} \\ &= (0.8)800 + (0.2)400 \\ &= 560 + 120 \\ &= 680 \end{aligned}$$

$$\begin{aligned} P_4^G = \frac{d_5 + E_4 P_5}{1 + r_4} &= \frac{\$40 + \$680}{1 + .05} \\ &= \frac{\$720}{1.05} \\ &= \$685.71 \end{aligned}$$

$$\begin{aligned} E_4 P_5^H &= (0.8)P_5^{H_1} + (0.2)P_5^{H_2} = (0.8)\$400 + (0.2)\$500 \\ &= \$320 + \$100 \\ &= \$420 \end{aligned}$$

$$\begin{aligned} P_4^H &= \frac{d_5 + E_4 P_5}{1 + r_4} = \frac{\$20 + \$420}{1 + 0.05} \\ &= \frac{\$440}{1.05} \\ &= \$419.05 \end{aligned}$$

$$\begin{aligned} E_4 P_5^B &= (0.5) P_5^{B_1} + (0.5) P_5^{B_2} = (0.5)\$100 + (0.5)\$250 \\ &= \$50 + \$125 \\ &= \$175 \end{aligned}$$

$$\begin{aligned} P_4^B &= \frac{d_5 + E_4 P_5}{1 + r_4} = \frac{\$5 + \$175}{1 + 0.05} \\ &= \frac{\$180}{1.05} \\ &= \$171.43 \end{aligned}$$

Period 3 Prices

$$\begin{aligned} E[d_4 \mid S \in \{Good, Hopeful\}] &= (0.9) d_4^G + (0.1) d_5^H \\ &= (0.9)\$40 + (0.1)\$20 \\ &= \$36 + \$2 \\ &= \$38 \end{aligned}$$

$$\begin{aligned} E[P_4 \mid S \in \{Good, Hopeful\}] &= (0.9) P_4^G + (0.1) P_4^H \\ &= (0.9)\$685.71 + (0.1)\$419.05 \\ &= \$617.14 + \$41.91 \\ &= \$659.05 \end{aligned}$$

$$\begin{aligned} P_3 \mid \{Good, Hopeful\} &= \frac{E_3 d_4 + E_3 P_4}{1 + r_3} = \frac{\$38 + \$659.05}{1 + 0.05} \\ &= \frac{\$697.05}{1.05} \\ &= \$663.86 \end{aligned}$$

$$\begin{aligned} P_3 \mid \{Bad\} &= \frac{d_4 + E_3 P_4}{1 + r_3} = \frac{\$5 + \$171.43}{1 + 0.05} \\ &= \frac{\$176.43}{1.05} \\ &= \$168.03 \end{aligned}$$

Period 2 Prices

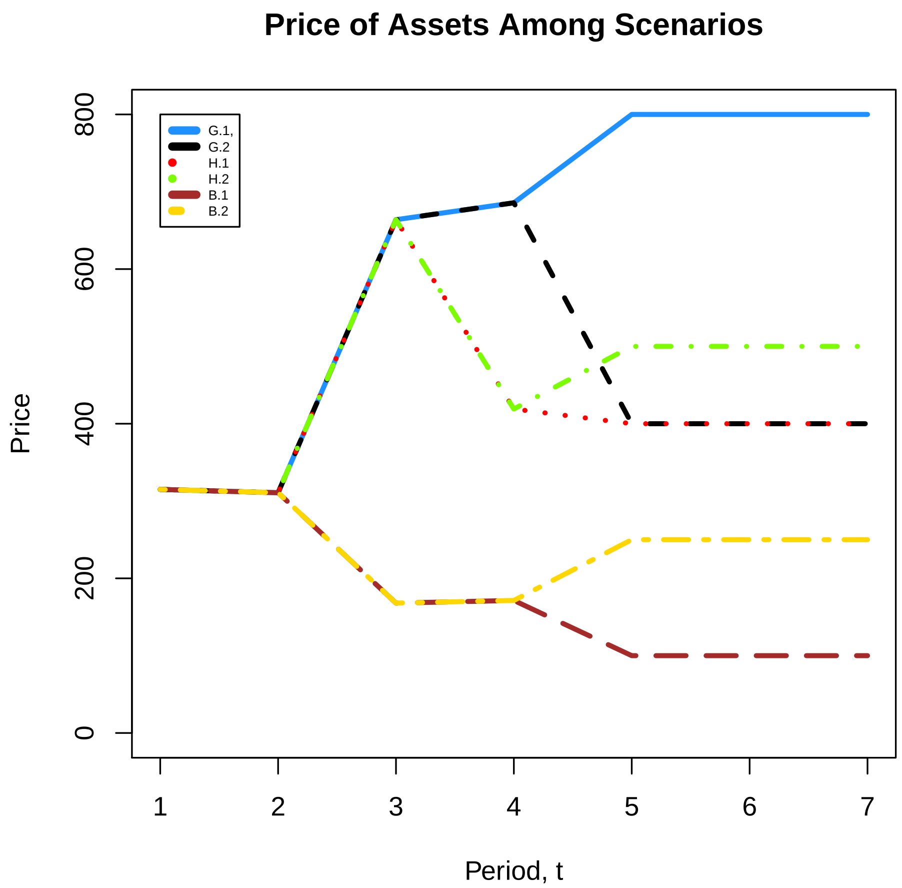
$$\begin{aligned} E_2 d_3 &= (0.3)d_3^{\{H,G\}} + (0.7)d_3^B = (0.3)\$20 + (0.7)\$5 \\ &= \$6 + \$3.5 \\ &= \$9.5 \end{aligned}$$

$$\begin{aligned} E_2 P_3 &= (0.3)P_3^{\{H,G\}} + (0.7)P_3^B = (0.3)\$663.86 + (0.7)\$234.55 \\ &= \$199.16 + \$117.62 \\ &= \$316.78 \end{aligned}$$

$$\begin{aligned} P_2 &= \frac{E2d_3 + E_2 P_3}{1 + r_2} = \frac{\$9.5 + \$316.78}{1 + 0.05} \\ &= \frac{\$326.28}{1.05} \\ &= \$310.74 \end{aligned}$$

Period 1 Prices

$$\begin{aligned} P_1 &= \frac{d_2 + P_2}{1 + r_1} = \frac{\$20 + \$310.74}{1 + 0.05} \\ &= \frac{\$330.74}{1.05} \\ &= \$314.99 \end{aligned}$$



I used LaTeX, a document processing and markuo language, and the R programming language to complete this homework. I would have attached the coding but it is pretty long. If you really want to see it, I could print it out. However, I have added a repository on Github that contains the coding for each of the homeworks where I have used computer codes. It can be found at
`https://github.com/iamcolonelreb/Financial_Crises_and_Bubbles`