Exercises 2

[1] Here we only discuss the underlying topological space of the spectrum. And the maximal spectrum is the subspace of maximal ideals.

1. (a) Show that the maximal spectrum of $\operatorname{Spec}\mathbb{C}[x]$ is the complex plane $\mathbb{C}$ with cofinite topology.
   (b) Show that the maximal spectrum of $\operatorname{Spec}\mathbb{R}[x]$ is $\mathbb{C}$ quotiented by a $\mathbb{Z}/2\mathbb{Z}$ action with cofinite topology.
   (c) Show that the maximal spectrum of $\operatorname{Spec}\mathbb{C}[x,y]/(x^2+y^2+1)$ is
   $$\{(s,t)\in\mathbb{C}^2|s^2+t^2+1=0\}$$
   with cofinite topology. Then show that there is an isomorphism $i:\operatorname{Spec}\mathbb{C}[x,x^{-1}]\to\operatorname{Spec}\mathbb{C}[x,y]/(x^2+y^2+1)$
   (d) * Show that the maximal spectrum of $\operatorname{Spec}\mathbb{R}[x,y]/(x^2+y^2-1)$ consists of
   $$\{(s,t)\in\mathbb{R}^2|s^2+t^2=1\} \text{ and } \{l\in lines(\mathbb{R}^2)|l \text{ not intersect with } s^2+t^2=1\}$$
   with cofinite topology.

2. Let GL_n be $\operatorname{Spec}\mathbb{Z}[x_{11},\ldots,x_{1n},\ldots,x_{nn},\iota]/(\iota\det(x)-1)$ where $\det(x)$ is the polynomial of determine. Show that for ring R
   $$\mathrm{GL}_n(R):=\mathrm{Hom}_{Sch}(\operatorname{Spec}R,\mathrm{GL}_n)\cong\{\text{invertible matrices with coefficient } R\}$$

3. Let k be a field, find out what are the prime spectra of the following rings, and the open(or closed) sets of Zariski topology.
   (a) $\operatorname{Spec}k[x]/(x^2)$
   (b) $\operatorname{Spec}k[[x]]$(where $k[[x]]:=\operatorname{Spec}\lim_n k[x]/(x^n)$ is the formal polynomial ring)
   (c) $\operatorname{Spec}k[x]_{(x)}$
   (d) * $\operatorname{Spec}\mathbb{Z}[x]$

[1]find also the exercises on https://github.com/iamcxds/AG-exercise, you can skip the question with * if it is difficult.