Exercises 2

Here we only consider the underlying topological space of the spectrum. And the maximal spectrum is the subspace of maximal ideals.

1. (a) Show that the maximal spectrum of $\operatorname{Spec}\mathbb{C}[x]$ is the complex plane $\mathbb{C}$ with cofinite topology.

   (b) Show that the maximal spectrum of $\operatorname{Spec}\mathbb{R}[x]$ is $\mathbb{C}$ quotiented by a $\mathbb{Z}/2\mathbb{Z}$ action with cofinite topology.

   (c) Show that the maximal spectrum of $\operatorname{Spec}\mathbb{C}[x,y]/(x^2+y^2+1)$ is

   $$\{(s,t)\in\mathbb{C}^2 | s^2+t^2+1=0\}$$

   with cofinite topology. Then show that there is an isomorphism $i : \operatorname{Spec}\mathbb{C}[x,x^{-1}] \to \operatorname{Spec}\mathbb{C}[x,y]/(x^2+y^2+1)$

   (d) * Show that the maximal spectrum of $\operatorname{Spec}\mathbb{R}[x,y]/(x^2+y^2-1)$ consists of

   $\{(s,t)\in\mathbb{R}^2 | s^2+t^2=1\}$ and $\{l \in lines(\mathbb{R}^2) | l \text{ not intersect with } s^2+t^2=1\}$

   with cofinite topology.

2. Let GL_n be $\operatorname{Spec}\mathbb{Z}[x_{11},\dots,x_{1n},\dots,x_{nn},\iota]/(\iota\det(x)-1)$ where $\det(x)$ is the polynomial of determine. Show that for ring R

   $\mathrm{GL}_n(R) := \mathrm{Hom}_{Sch}(\operatorname{Spec}R, \mathrm{GL}_n) \cong \{\text{invertible matrices with coefficient } R\}$

3. Let k be a field, find out what are the prime spectra of the following rings, and the open sets of Zariski topology.

   (a) $\operatorname{Spec} k[x]/(x^2)$

   (b) $\operatorname{Spec} k[[x]](:= \operatorname{Spec}\lim_n k[x]/(x^n))$

   (c) $\operatorname{Spec} k[x]_{(x)}$

   (d) * $\operatorname{Spec}\mathbb{Z}[x]$

[1]

[1]find also the exercises on https://github.com/iamcxds/AG-exercise, you can skip the question with * if it is difficult.