Exercises 2

Here we only consider the underlying topology space of the spectrum.

1. (a) Show that the maximal spectrum of $\operatorname{Spec}\mathbb{C}[x]$ is the complex plane $\mathbb{C}$ with finite complement topology.

   (b) Show that the maximal spectrum of $\operatorname{Spec}\mathbb{R}[x]$ is $\mathbb{C}$ quotiented by a $\mathbb{Z}/2\mathbb{Z}$ action.

   (c) Show that the closure of (x^2+y^2+1) in $\operatorname{Spec}\mathbb{C}[x,y]$ is isomorphic to $\operatorname{Spec}\mathbb{C}[x,y]/(x^2+y^2+1)$, and whose maximal spectrum are
   $$\{(s,t)\in\mathbb{C}\times\mathbb{C}|s^2+t^2+1=0\}$$
   with finite complement topology. Then show that there is an isomorphism $i:\mathbb{C}^*(i.e.\mathbb{C}-\{0\})\to\operatorname{Spec}\mathbb{C}[x,y]/(x^2+y^2+1)$

2. Let k be a field, find out what are the prime spectrum of the following rings, and the open sets of Zariski topology.

   (a) $\operatorname{Spec} k[x]/(x^2)$

   (b) $\operatorname{Spec} k[[x]](:=\operatorname{Spec}\lim_n k[x]/(x^n))$

   (c) $\operatorname{Spec} k[x]_{(x)}$

   (d) $\operatorname{Spec}\mathbb{Z}[x]$

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[1]find also the exercises on https://github.com/iamcxds/AG-exercise