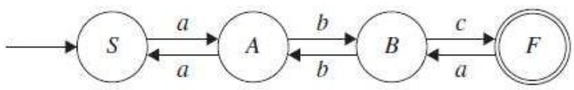
Exercise No : 1

1. Prove by giving suitable example that if AUB = A UC, then it is not necessary that B = C.
2. For the given sets A and B, there is a possibility that A-B = B-A, if yes, When?
3. For the two finite sets A and B, is it possible that
   A-B = B? if yes, when?
   A-B = A? if yes, when?

4. Let A = {1,2,3} and S = A X A, define a relation R on S such that (a,b)R(a',b') if and only, if ab = a'b'. show that R is an equilvalence relation.
5. Prove the following using PMI: $1 + r + r^2 + r^3 + \ldots\ldots + r^n = (r^n - 1)/(r - 1)$

6. Find the path for the strings abb, abca, aa,abb, abbc in the finite automaton shown in the following figure.



7. Design a finite automaton M over {0,1} to accept all strings satisfying the following conditions:
   a. Ending with 111 or 000
   b. Starting with 111 or 000
   c. Containing the substring 000 or 111

8. Design a finite automaton M over {0,1} to accept all strings satisfying the following conditions:
   a. Containg exactly two 0's
   b. Containing at least two 0's
   c. Containing at the most two 0's

9. Design the DFA equivalent for the NFA given in the following table:Starting state is q_0 and ending state is q_3.

Current state	Input Symbol	
	a	b
q_0	q_0, q_1	q_0, q_2
q_1	-	q_3
q_2	q_0, q_3	q_1
q_3	q_2	-

10. For the Mealy machine in the following table, find the equivalent Moore Machine. Starting state is q_0.

Current state	Input symbol			
	a		b	
	Next State	Output	Next State	Output
q_0	Q_1	1	q_3	1
q_1	q_1	0	q_0	1
q_2	q_0	1	q_2	0
q_3	q_3	0	q_1	1

11. For the Moore Machine given in the following table. Find the equivalent Mealy Machine, Start state in q0.

Current state	Input Symbol		output
	a	b	
q_0	q_1	q_2	1
q_1	q_3	q_4	1
q_2	q_4	q_0	0
q_3	q_1	q_2	0
q_4	q_3	q_0	1

Exercise - 3

Regular Grammar & Regular Sets

Q 1: List all the strings of length up to five corresponding to the following regular expressions over {a,b}.

(1) a(a+b)*
(2) a(aa)*
(3) (a+b)*c
(4) (aa+bb)c(ab+ba)
(5) (aa+bb)*c(ab+ba)

Q 2: For the following regular expressions, draw the corresponding finite automata:

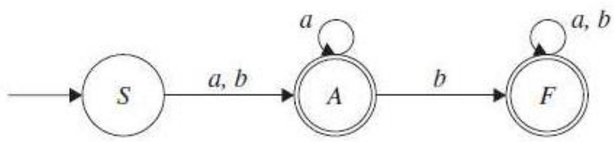
(1) (111+000)*1
(2) (0+1)*0(0+11)*
(3) 0+10*+001*00
(4) (0+1)*(01+1110)

Q 3: Draw a finite automaton M accepting the grammar S -> bS | aA, A -> bA | a. Find the regular expression corresponding to M.

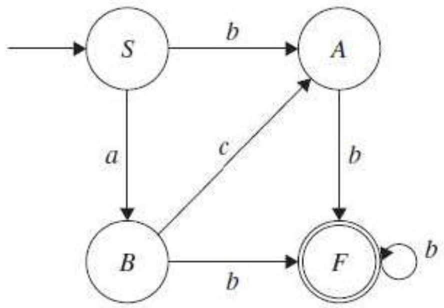
Q 4: For the following language over {a,b}, find the corresponding regular expression R.

(1) Every word in the language contains exactly three a's.
(2) Every word in the language contains minimum three a's.
(3) Every word contains alternate 00s and 11s.
(4) L={$a^m b^n$ | m,n>1}
(5) Every word begins and ends with 00.

Q 5: For the finite automaton in the following figure find the corresponding regular expression.



Q 6: For the finite automaton in the following figure find the corresponding regular expression.



<u>Exercise 2</u>

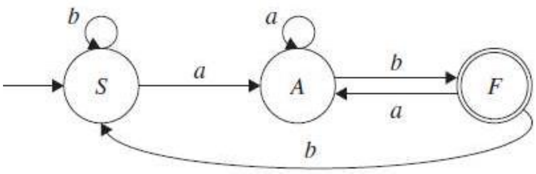
Regular Grammar & Regular Sets

Q 1: Construct the Non Deterministic Finite automata for the following regular expressions.

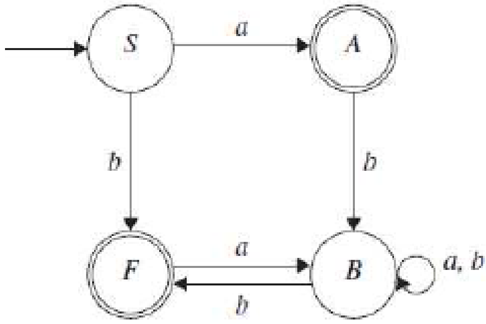
(1) (a+b+c)*
(2) (ab+bc)d
(3) (ab+bc)*k*(d+e)
(4) a+bb+cc

Q 2: Let M_1 and M_2 be two finite automata accepting the language L_1 and L_2 respectively as shown in following figure. Construct the finite automata to accept the language.

(1) $L_1 \cup L_2$
(2) $L_1 \cap L_2$
(3) $L_1 - L_2$
(4) $L_2 - L_1$

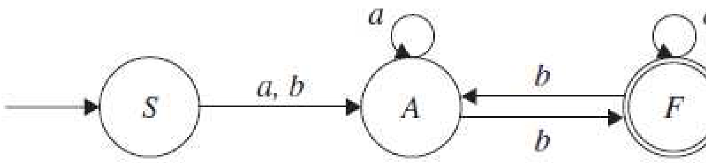


Q 3: For the finite automaton given in the following figure, find the corresponding regular expression.



Q 4: Design a finite automaton for the regular expression 10+(0+11)0*1. If it is an NFA, then convert it into its equivalent DFA.

Q 5: For the finite automaton in the following figure find the corresponding regular expression.



Q 6: For the following regular expressions, draw an ϵ-NFA and convert them into their equivalent DFA.

(1) (a+b)*(abb+ababab)(a+b)*
(2) (a+b)(ba)*(abb)*

Exercise 4

Context-free Grammars & Languages

Q 1: Identify the nonterminals and terminals in the following grammars.

(1) S -> Aba | b   A -> BB | aa   B -> bB | c   C -> cC | d
(2) S -> XY1 | 0   X -> 00X | 1   Y -> 1X1
(3) S -> XY   X -> YSY   X -> YY | a   Y -> aXb | b
(4) S -> XY   X -> YSY   X -> YY | 1   Y -> 0X1 | 1

Q 2: Convert the following CFG to CNF:

(1) S -> aAC   A -> aB | bAB   B -> b   C -> c
(2) S -> 0X1Y   X -> 0X | 0   Y -> 1Y | 1
(3) S -> abSab | a |aAAb   A -> bS | aAAb | c

Q 3: Identify and remove the nonreachable nonterminals from the following grammars:

(1) S - > XY1 | 0   X -> 00X | 1   Y -> 1X1   Z -> 00
(2) S -> XZ | 0   X -> YA | 1   Y -> Z1 | A2   A -> 01   B-> X | 2

Q 4: Identify Language

(1) L={ $a^ib^ic^i$ | i>=1 }
    a. Regular Language
    b. CFL
    c. Both CFL & Regular
    d. Neither CFL nor Regular

(2) L={ $a^ib^ic^j$ | I,j>=1 }
    a. Regular Language
    b. CFL
    c. Both CFL & Regular
    d. Neither CFL nor Regular

(3) L={ $a^nb^nc^md^m$| n,m>=1 }
    a. Regular Language
    b. CFL
    c. Both CFL & Regular
    d. Neither CFL nor Regular

(4) L={ $0^n1^m2^{m+n}$ | n,m>=1 }
    a. Regular Language
    b. CFL
    c. Both CFL & Regular
    d. Neither CFL nor Regular

Q 5: Define Property

(1) CFLs are closed under
    a. Union
    b. Complementation
    c. Intersection
    d. All the above
(2) The CFLs and regular languages are both closed over
    a. Union
    b. Complementation

c. Intersection
d. None of the above

(3) The CFLs and regular languages are both closed over
a. Difference
b. Intersection
c. Complement
d. Concatenation

(4) CFLs are not closed under
a. Union
b. Concatenation
c. Intersection
d. Homomorphism

Q 6:

(1) The regular expression corresponding to the CFG S -> aS | bS | a | b is
a. a+b
b. (a+b)*
c. (a+b)*(a+b)
d. None of the above

(2) The CFG corresponding to the language L={0^k1^k | k>=1} is
a. S -> 0S1 | 01
b. S -> 0S1 | 01 | ϵ
c. S -> 0A1, A -> 01
d. All the above

(3) The CFL L={a^nb^n | n>0} can be generated by the following CFG:
a. S -> ϵ | ab | aSb
b. S -> ab | aSb
c. S -> ϵ | aSb
d. All of the above

Exercise 5

Context-free Grammars & Languages

Q 1: Remove unit productions from the following grammars and generate equivalent grammar.

(1) S -> ABC | 0, A -> 1, B -> C | 0, C -> D, D -> E, E -> 2
(2) S -> ABCD | 0, A -> BC | 1, B -> C, C -> D, D -> d

Q 2: Convert the following CFGs to GNF:

(1) S -> XY1 | 0	A -> 00X | 1	Y -> 1X1	
(2) S -> XY	X -> YSY	X -> YY | 1	Y -> 0X1 | 1
(3) S -> Xa	X -> aY	Y -> Xa | b	

Q 3: Identify the nonterminals from the following grammers, which fail to generate terminal

(1) S - > XY1 | 0	X -> 00X	Y -> 1X1 | 2	
(2) S -> XZ | 0	X -> YA | 1	Y -> Z1 | A2 | 3	Z -> 3Z

Q 4: Consider the following grammar:

S -> ASA | BSB | ASB | BSA | 1   A -> 0   B -> 1

Derive the strings 010, 111, 00101, 11100 using both left and right derivation

Q 5: Define Property

(1) A CFL is accepted by a
    a. Pushdown Automata
    b. Finite Automata
    c. Turing Machine
    d. None of the above
(2) A Pumping lemma is used for proving that
    a. A language is context free
    b. A language is not context free
    c. Two CFLs are the same
    d. Two CFLs are different
(3) A CFG is a
    a. Type 0 Grammar
    b. Type 1 Grammar

c. Type 2 Grammar
d. Type 3 Grammar

(4) The intersection of a CFL and regular language is a
a. Regular language
b. CFL
c. Neither CFL nor regualr
d. Cannoy say

Q 6:

(1) Which of the following CFGs can also be recognized by a FSM?
a. S -> aS| Bs | aa | b
b. S -> aS | bS | a | b
c. S -> aS | bSb | a | b
d. All of the above

(2) Let G be a CFG in GNF and L be the corresponding CFL. Let there be string $z \in L$. The number of productions used in deriving z is
a. $|z|$
b. $2|z|$
c. $|z+1|$
d. $|z|+1$

(3) Let $L_1=\{0^n1^n2^m|n,m>=1\}$ and $L_2=\{0^m1^n2^n|n,m>=1\}$; given m>n, the intersection of L_1 and L_2 is the language
a. $L=\{0^m1^n2^m|n,m>=1\}$
b. $L=\{0^m1^m2^m|m>=1\}$
c. $L=\{0^n1^n2^n|n>=1\}$
d. None of the above

Exercise 6

Pushdown Automata

Q 1: Design a PDA to accept the language L over Σ = {a,b} consisting of all the string with equal number of a's and b's.

Q 2: Design a PDA corresponding to the following CFGs.

a. S -> 0S0 | 1S1 | A	A -> 2B3	B -> 23|31
b. S -> bX|aY	A -> bXX|aS|a	Y -> aYY |bS|b
c. S -> 0Y|1X	X -> 0S|1XX|0	Y -> 1S|0YY|1

Classify these PDA into deterministic and nondeterministic categories.

Q 3: Why cannot the following language be implemented on PDA?

L={$a^m b^m$|m>=1} U {$a^m b^{2m}$|m>=1}

Q 4: Design a top-down parser to implement the following CFG and parse the string 0102313010

S -> 0S0|1S1|A A -> 2B3 B -> 23|31

Q 5: Convert the following grammar to LL(a) type.

S -> S+A S -> A A -> A/B A -> B B -> a1|a2|a3

Where {a,1,2,3,+,/} is the set of terminals.

Q 6:

(1) The production of the type A -> Aα involves
  a. Left recursion
  b. Right recursion
  c. Left factoring
  d. Right factoring

(2) The production of the type A -> αβ|αg|αδ involves
  e. Left recursion
  f. Right recursion
  g. Left factoring
  h. Right factoring

(3) Shift process in shift-reduce parsing involves
  a. Popping of a terminal from pushdown store
  b. Popping of a nonterminal from pushdown store
  c. Pushing of a nonterminal from pushdown store

d. Pushing of a terminal from pushdown store

(4) The language $L=\{a^n b^n | n>=1\}$

a. Cannot be accepted by a PDA
b. Can be accepted by a PDA of null store type only
c. Can be accepted by a PDA of final state type only
d. Can be accepted by a PDA of both types, null store and final state type.

Exercise 7

Turing Machines

Q 1: Design a Turing Machine M over {0,1} such that L(M)={w|w contains equal numbers 0s and 1s}.

Q 2: Design a Turing Machine M over {0,1} such that L(M)=$\{0^n 1^{2n} | n>=1\}$

Q 3: Design a Turing Machine M over {0,1,2} such that L(M)=$\{0^n 1^{2n} 2^n | n>=1\}$

Q 4: Design a Post Machine M over {a,b} such that L(M)=$\{0^n 1^n 0^n | n>=1\}$

Q 5: Design a Turing Machine M to find the predecessor of a positive integer.

Q 6: Design a Turing Machine M over {a,b} such that L(M)={x| length of x is odd}

Exercise 9

Context-free Grammars & Languages

Q 1: Identify the nonterminals and terminals in the following grammars.

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(2)	S -> XY1 | 0	X -> 00X | 1	Y -> 1X1	
(3)	S -> XY	X -> YSY	X -> YY | a	Y -> aXb | b
(4)	S -> XY	X -> YSY	X -> YY | 1	Y -> 0X1 | 1

Q 2: Convert the following CFG to CNF:

(1)	S -> aAC	A -> aB | bAB	B -> b	C -> c
(2)	S -> 0X1Y	X -> 0X | 0	Y -> 1Y | 1	
(3)	S -> abSab | a |aAAb	A -> bS | aAAb | c		

Q 3: Identify and remove the nonreachable nonterminals from the following grammars:

(1)	S - > XY1 | 0	X -> 00X | 1	Y -> 1X1	Z -> 00	
(2)	S -> XZ | 0	X -> YA | 1	Y -> Z1 | A2	A -> 01	B-> X | 2

Q 4: Identify Language

(1) L={ $a^ib^ic^i$ | i>=1 }
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  b. CFL
  c. Both CFL & Regular
  d. Neither CFL nor Regular

(3) L={ $a^nb^nc^md^m$| n,m>=1 }
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  b. CFL
  c. Both CFL & Regular
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(4) L={ $0^n1^m2^{m+n}$ | n,m>=1 }
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   b. Intersection
   c. Complement
   d. Concatenation
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   b. Concatenation
   c. Intersection
   d. Homomorphism

Q 6:

(1) The regular expression corresponding to the CFG S -> aS | bS | a | b is
   a. a+b
   b. (a+b)*
   c. (a+b)*(a+b)
   d. None of the above

(2) The CFG corresponding to the language L={0^k1^k | k>=1} is
   a. S -> 0S1 | 01
   b. S -> 0S1 | 01 | ϵ
   c. S -> 0A1, A -> 01
   d. All the above

(3) The CFL L={anbn | n>0} can be generated by the following CFG:

a. S -> ϵ | ab | aSb
b. S -> ab | aSb
c. S -> ϵ | aSb
d. All of the above

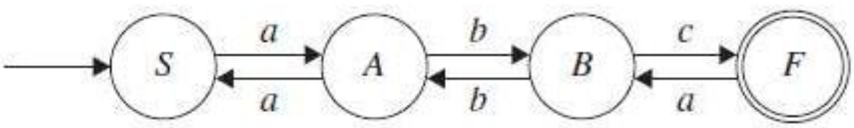
Exercise No : 1

1. Prove by giving suitable example that if AUB = A UC, then it is not necessary that B = C.
2. For the given sets A and B, there is a possibility that A-B = B-A, if yes, When?
3. For the two finite sets A and B, is it possible that
   A-B = B? if yes, when?
   A-B = A? if yes, when?

4. Let A = {1,2,3} and S = A X A, define a relation R on S such that (a,b)R(a',b') if and only, if ab = a'b'. show that R is an equilvalence relation.
5. Prove the following using PMI: $1 + r + r^2 + r^3 + \ldots\ldots + r^n = (r^n - 1)/(r - 1)$

Exercise No : 2

1. Find the path for the strings abb, abca, aa,abb, abbc in the finite automaton shown in the following figure.



2. Design a finite automaton M over {0,1} to accept all strings satisfying the following conditions:
   (a) Ending with 111 or 000
   (b) Starting with 111 or 000
   (c) Containing the substring 000 or 111

3. Design a finite automaton M over {0,1} to accept all strings satisfying the following conditions:
   (a) Containg exactly two 0's
   (b) Containing at least two 0's
   (c) Containing at the most two 0's

4. Design the DFA equivalent for the NFA given in the following table:Starting state is q_0 and ending state is q_3.

Current state	Input Symbol	
	a	b
q_0	q_0, q_1	q_0, q_2
q_1	-	q_3
q_2	q_0, q_3	q_1
q_3	q_2	-

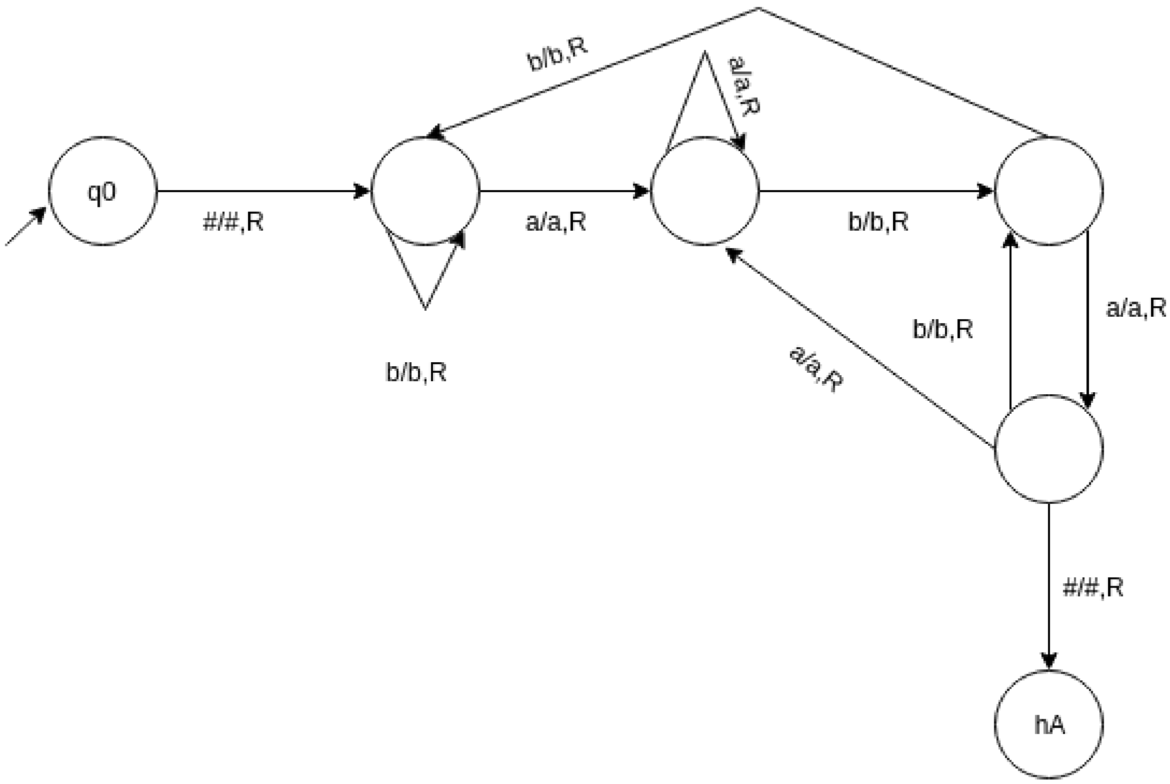
5. For the Mealy machine in the following table, find the equivalent Moore Machine. Starting state is q_0.

Current state	Input symbol			
	a		b	
	Next State	Output	Next State	Output
q_0	Q_1	1	q_3	1
q_1	q_1	0	q_0	1
q_2	q_0	1	q_2	0
q_3	q_3	0	q_1	1

6. For the Moore Machine given in the following table. Find the equivalent Mealy Machine, Start state in q0.

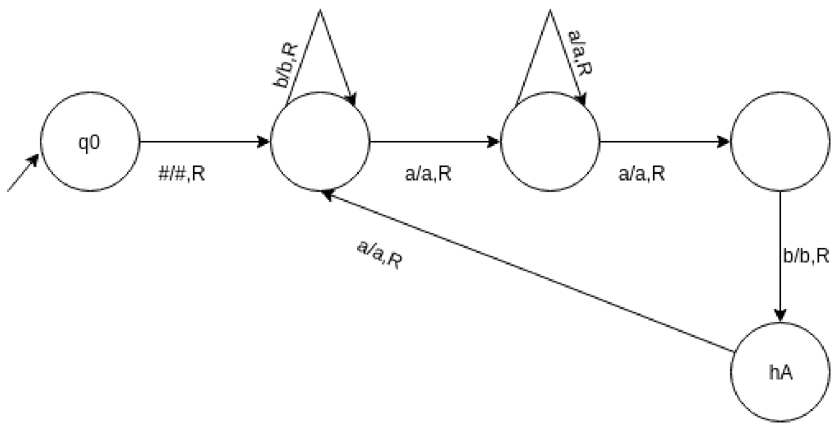
Current state	Input Symbol		output
	a	b	
q_0	q_1	q_2	1
q_1	q_3	q_4	1
q_2	q_4	q_0	0
q_3	q_1	q_2	0
q_4	q_3	q_0	1

1. Which of the following regular expression resembles the given diagram?

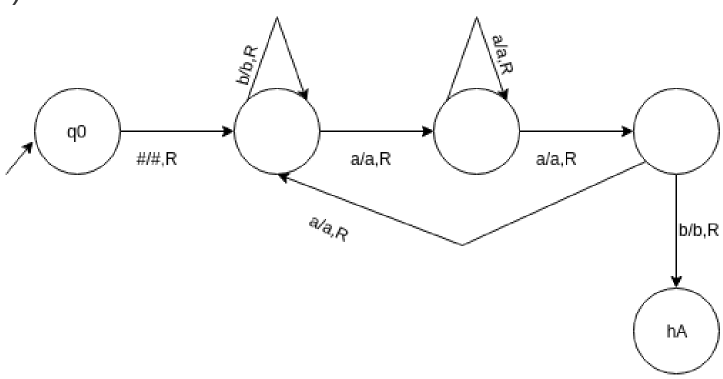


a) {a}*{b}*{a,b}
b) {a,b}*{aba}
c) {a,b}*{bab}
d) {a,b}*{a}*{b}*

2. Construct a turing machine which accepts a string with ‘aba’ as its substring.

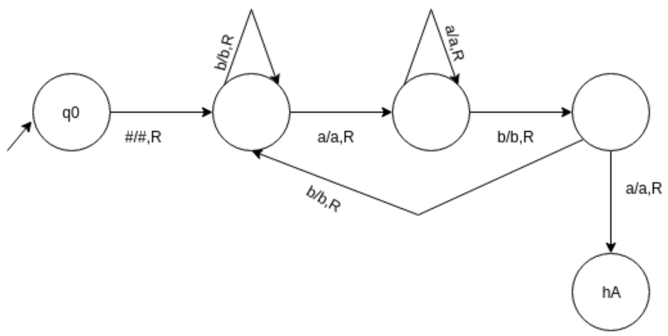


a)

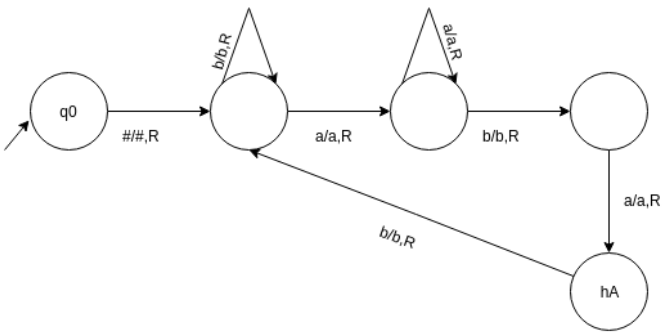


b)

C)



d)



3. The number of states required to automate the last question i.e. {a,b}*{aba}{a,b}* using finite automata:

a) 4

b) 3

c) 5

d) 6

4. The machine accept the string by entering into hA or it can:

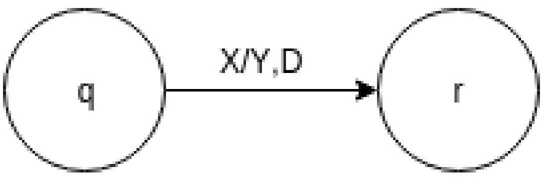
a) explicitly reject x by entering into hR

b) enter into an infinte loop

c)Both (a) and (b)

d) None of the mentioned

5.d(q,X)=(r,Y,D) where D cannot be:



a) L

b) R

c) S

d) None of the mentioned

6. Which of the following can accept even palindrome over {a,b}

a) Push down Automata

b) Turing machine

c) NDFA

d) All of the mentioned

7. Which of the functions can a turing machine not perform?

a) Copying a string

b) Deleting a symbol

c) Accepting a pal

d) Inserting a symbol

8. If T1 and T2 are two turing machines. The composite can be represented using the expression:

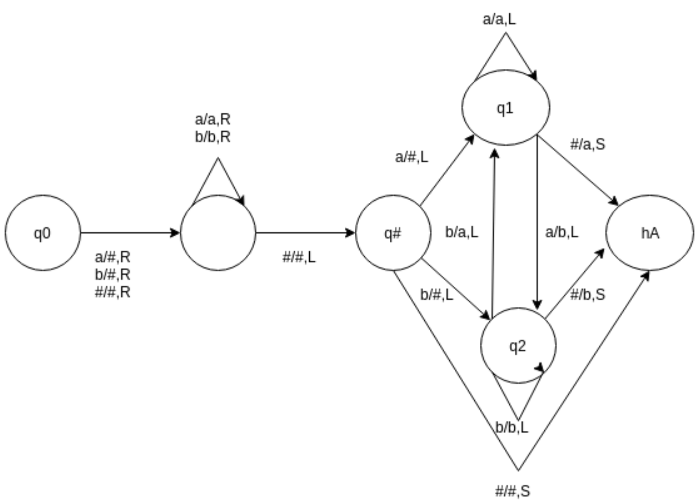
a) T1T2

b) T1 U T2

c) T1 X T2

d) None of the mentioned

9. The following turing machine acts like:



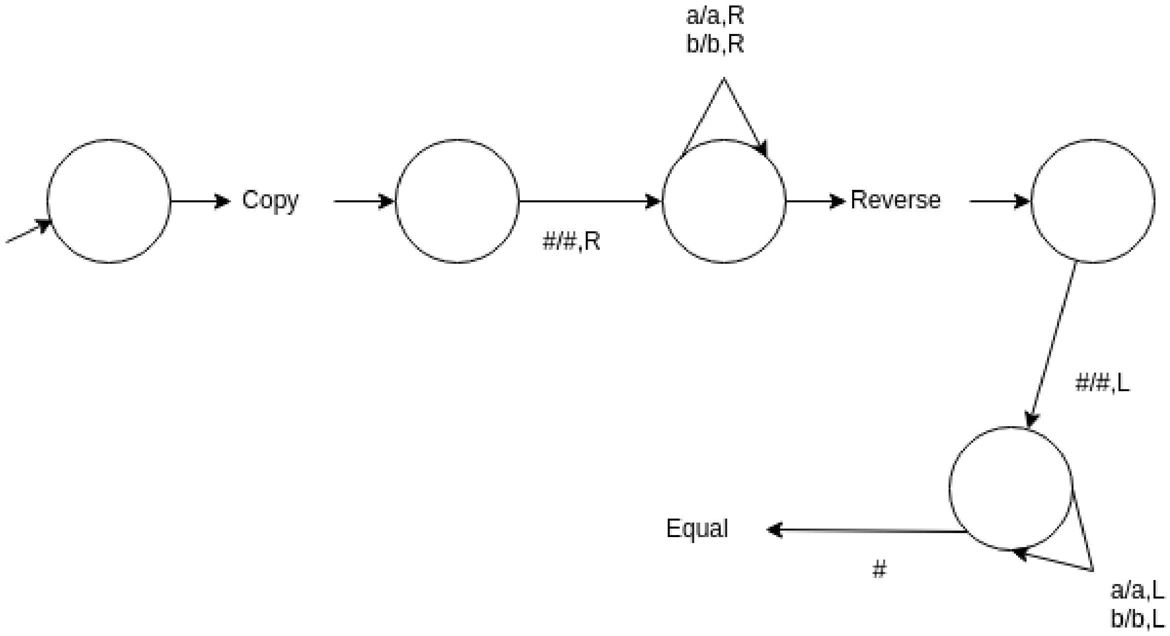
a) Copies a string

b) Delete a symbol

c) Insert a symbol

d) None of the mentioned

10. What does the following transition graph shows:



a) Copies a symbol

b) Reverses a string

c) Accepts a pal

d) None of the mentioned

7.1 Design a Turing machine M over $\{0, 1\}$ such that $L(M) = \{w \mid w \text{ contains equal numbers 0s and 1s}\}$.

7.2 Design a Turing machine M to compute $\sum_{k=1}^{n} k$ for a given positive integer n.

7.3 Design a Turing machine M over $\{0, 1\}$ such that $L(M) = \{0^n 1^{2n} \mid n \geq 1\}$.

7.4 Design a Turing machine M over $\{0, 1\}$ such that $L(M) = \{0^{2n} 1^n \mid n \geq 1\}$.

7.5 Design a Turing machine M over $\{0, 1, 2\}$ such that $L(M) = \{0^n 1^{2n} 2^n \mid n \geq 1\}$.

7.6 Design a Turing machine M over $\{0, 1, 2, 3\}$ such that $L(M) = \{0^{2n} 1^n 2^n 3^{2n} \mid n \geq 1\}$.

7.7 Design a Post machine M over $\{a, b\}$ such that $L(M) = \{0^n 1^n 0^n \mid n \geq 1\}$.

7.8 Design a two-track Turing machine M to compute $\sum_{k=1}^{n} k$ for a given positive integer n.

7.9 Design a Turing machine M to find the predecessor of a positive integer.

7.10 Design a Turing machine M to find the successor of a positive integer.

7.11 Design a Turing machine M over $\{a, b\}$ such that $L(M) = \{ x \mid \text{length of } x \text{ is odd}\}$.