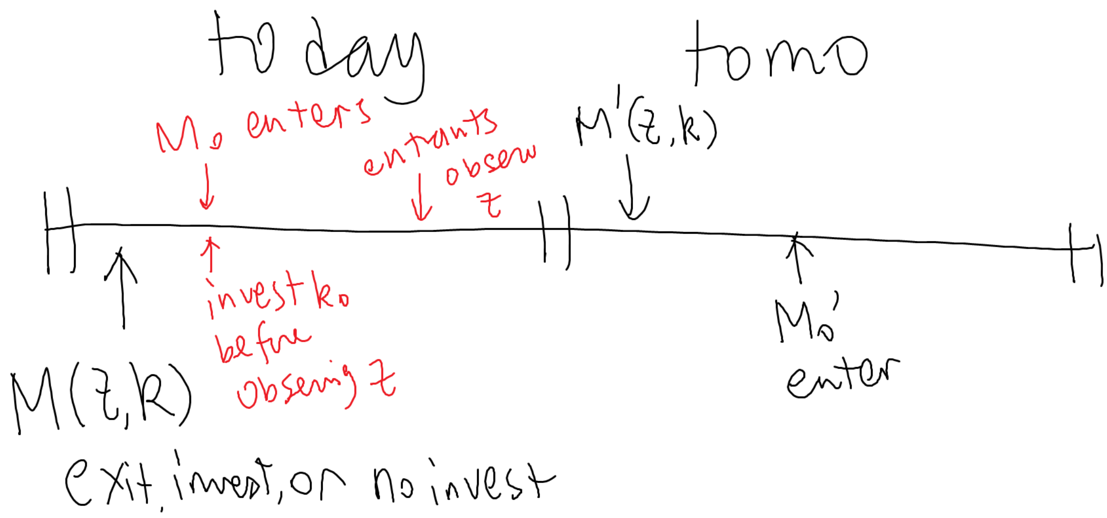
1 dFixed z over time

Timing:



Entrants:

- Firms can pay a fixed labor cost $c_E w$ to enter
- Before drawing z, they decide how much initial capital k_0 to invest
  - Cost of investing k_0 capital is $\frac{k_0}{A}$ units of final/investment/consumption goods today
- No operation or anything today
- End of today:
  - they draw z from initial distribution $f(z)$, cannot change investment decision further.
- Tomorrow, they are effectively an existing firm with (z, k_0) (for example, they could exit right away)

The problem for the entry firm conditional on choosing to enter is:

$$V^E \equiv \max_{k_0} -c_E w - c_0 w - \frac{k_0}{A} + \beta (1-\psi) E_z [V(z, k_0)]$$

Existing firms:

- To fix the idea:
  - c, i: are potatoes (final goods)
  - Each potato turns into A units of capital k' (potato factories) tomorrow
- Existing firms have state variable (z, k)
- Each period, they could
  - Exit: sell capital k at a loss
  - Operate without investment: don't pay investment cost, next period end up with $(z, (1-d)\, k)$
  - Operate with investment: pay investment cost, decide how much to invest, operate with k today, get k' tomorrow

Investment technology:

- For firm (z, k) that operates without investing

$$V^{\text{no invest}}(z, k) = \max_{n} z\left(k^{\alpha} n^{1-\alpha}\right)^{\xi} - wn - c_f w + \beta(1-\psi)\, V(z, (1-\delta)\, k)$$

  - c_f: cost of operating

- For firm (z, k) that invests or de-invests, value is

$$V^{\text{invest}}(z, k) = \max_{n, i, k'} z\left(k^{\alpha} n^{1-\alpha}\right)^{\xi} - wn - c_f w - c_0 w - i + \beta(1-\psi)\, V(z, (1-\delta)\, k + Ai)$$

  - $c_0 w$: fixed cost of investment or de-investment (capital adjustment)
  - A: exogenously increasing over time, capturing cheaper investment cost over time (similar to Simone)
  - Note: investment i has the same cost as consumption, but the return is Ai, which adds to capital tomorrow

- For firm (z, k) that **exits**,
  - They smash their capital k back into $\frac{\omega k}{A}$ units of final goods/consumption/investment
  - They sell their capital k for a total price of $\frac{\omega k}{A}$
  - $\omega < 1$ captures a loss of sale

- Whoever buys k can only use it tomorrow on. This will save the buyer some investment i today of $\frac{\omega k}{A}$ units

$$V^{\text{exit}}(z,k) = \frac{\omega k}{A}$$

- Firm value is

$$V(z,k) = \max\left\{V^{\text{exit}}(z,k), V^{\text{no invest}}(z,k), V^{\text{invest}}(z,k)\right\}$$

- Let's call policy function:

$$I_t(z,k) = \begin{cases} 0 & \text{firm chooses to exit at } t \\ 1 & \text{firm chooses not to invest at } t \\ 2 & \text{firm chooses to invest at } t \end{cases}$$

Optimal capital if invest:

$$k^*_{t+1}(z,k) = (1-\delta)k + A_t \times i_t(k,z)$$

Note: in our model, k^*_{t+1} doesn't depend on k.

- Next period capital if invest or no invest:

$$k'_{t+1}(z,k) = \begin{cases} -\inf \text{ (this is just arbitrary number)} & \text{firm chooses to exit at } t \\ (1-\delta)k & \text{firm chooses not to invest at } t \\ k^*_{t+1}(z,k) & \text{firm chooses to invest at } t \end{cases}$$

Market Clearing Condition:

- Goods market in terms of final goods

$$\int_{\text{no invest} \mid \text{invest}} y(z,k)\,dM + \int_{\text{exit}} \frac{\omega k}{A} dM = C + \int i(z,k)\,dM + M_0 \frac{k_0}{A}$$

- Labor market

$$\int_{\text{no invest} \mid \text{invest}} n(z,k)\,dM + c_f \int_{\text{no invest} \mid \text{invest}} dM + c_0 \int_{\text{invest}} dM + M_0(c_E + c_0) = N$$

- HH FOC

$$w = \frac{v'(N)}{u'(C)}$$

Measures:

- Suppose in period t, measure of firms are given by $M_t(z,k)$ and M_{0t} new firms enter
- Then measure of firms in period $t+1$ is $M_{t+1}(z,k)$

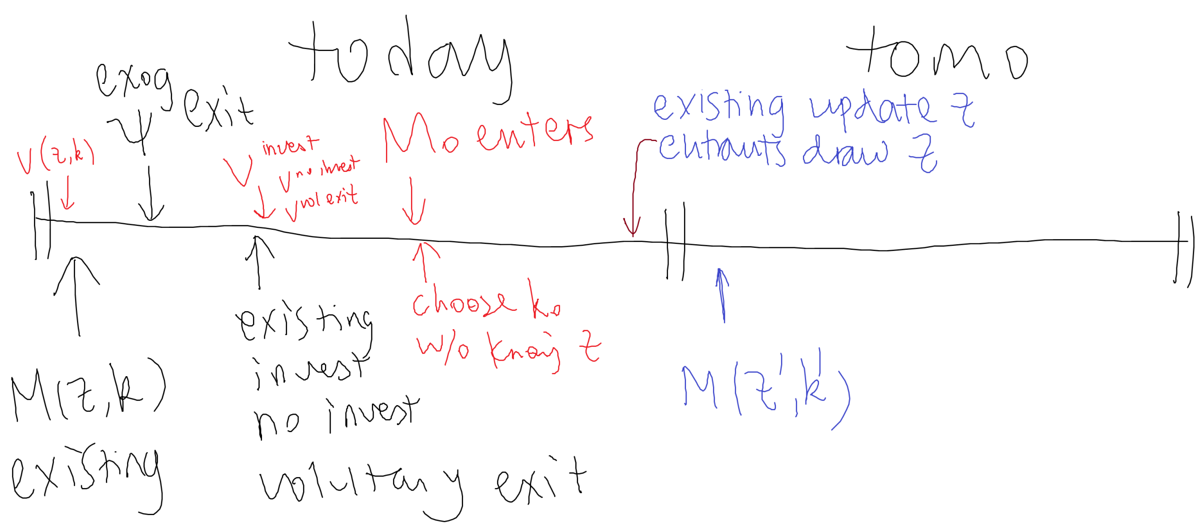
$$M_{t+1}(z,k') = M_{0t}\times 1(k_{0t}=k') + (1-\psi)\times 1\left(I_t\left(\frac{k'}{1-\delta},z\right)=1\right)\times M_t\left(z,\frac{k'}{1-\delta}\right) + \int_{I_t(z,k)=2,k^*_{t+1}(z,k)=k'}$$

Strategy

- What we have: $V_t(z,k)$;w_t; We found this using the free entry condition.
- $M_{0,t=1}$; $M_{t=1}(z,k)$ (We assume the world is in some stationary state before the beginning of time to find these.) (This might be irrational since we're going to start along a trajectory of changes in A)
- What we need is $M_{0,t}$ and $M_t(z,k)$
- Idea: We can solve for measure variables sequentially in time. $M_t(z,k)$ can be derived easily from $M_{0,t}$ & $M_{t-1}(z,k)$ and the policy function.
  - Either use a solver or guess on $M_{0,t}$ and see if $w = \frac{v'(N)}{u'(C)}$ is satisfied.

2 Persistent z over time

Timing:



Entrants:

- Firms can pay a fixed labor cost $c_E w$ to enter
- Before drawing z, they decide how much initial capital k_0 to invest
  - Cost of investing k_0 capital is $\frac{k_0}{A}$ units of final/investment/consumption goods today
  - Maybe: include a a minimum amount of initial investment $k_0 \geq \underline{k}$ (not necessary)
- No operation or anything today
- End of today: they draw z from initial distribution $f(z)$, cannot change investment decision further.
- Tomorrow, they are effectively an existing firm with (z, k_0) (for example, they could exit right away)

The problem for the entryfirm conditional on choosing to enter is:

$$V^E \equiv \max_{k_0} -c_E w - c_0 w - \frac{k_0}{A} + \beta E_z \left[V(z, k_0) \right]$$

Existing firms:

- To fix the idea:
  - c, i: are potatoes (final goods)
  - Each potato turns into A units of capital k' (potato factories) tomorrow
- Existing firms have state variable (z, k):
  - z is productivity today, k is capital they enter the period with
- Each period, **existing** firm (z, k)
  1. First they receive a shock of **exogenous exit with prob** ψ, if they get the exog exit shock, they cannot operate/invest, just smashing their capital back today to

     $$\frac{\omega k}{A}$$

     - Alternatively, if we assume their capital is destroyed instead of smashed back, then this will encourage more volunatary exits (because operating has the risk of hitting ψ, which leads to loss of all capital)
  2. If they don't receive the shock of exog exit, then they could

  - Voluntary Exit: pay fixed investment adjustment cost to exit, sell capital k at no loss
  - Operate without investment: don't pay investment cost, next period end up with $(z', (1-d)\,k)$
  - Operate with investment or de-investment: pay investment fixed cost, decide how much to invest, operate with k today, get k' tomorrow

Investment technology:

- $V^{\text{operate}}, V^{\text{voluntary exit}}$: are values contigent on no exoge exit (which happens at beginning of period)
- For firm (z,k) that **operates (invest, no invest, or de-invest)**, value is

$$V^{\text{operate}}(z,k) = \max_{n,i,k'} z\left(k^{\alpha}n^{1-\alpha}\right)^{\xi} - wn - c_f w - c_0 w \times 1\{i \neq 0\} - i + \beta E_{z'|z}\left[V\left(z', (1-d)\,k + Ai\right)\right]$$

  - **Problem**: if de-invest, should also have the option to smashing and not paying fixed investment cost
    * Solution??? set $\omega = 0$ and no worry about it!!
  - $c_f w$: cost of operating
  - $c_0 w$: fixed cost of investment or de-investment. If firm chooses inaction, then don't pay this cost
    * This is in spirit of Bachmann, Caballero, Engel (2013)
  - A: exogenously increasing over time, capturing cheaper investment cost over time (similar to Simone)
  - Note: investment i has the same cost as consumption, but the return is Ai, which adds to capital tomorrow

- For firm (z,k) that **voluntarily exits**

$$V^{\textbf{voluntary exit}}(z,k) = \max\left\{\frac{\omega\,(1-\delta)\,k}{A}, -c_0 w + \frac{(1-\delta)\,k}{A}\right\}$$

  - Assume: even if firms voluntarily exit, their capital still depreciate. This way, there's no jump discontinuity between voluntary exits and de-invest.

- Let's call voluntary exit policy function

$$VolExit_t(z,k) = \begin{cases} 1 & \text{voluntary exit by smashing and get } \frac{\omega k}{A} \\ 2 & \text{voluntary exit by paying fixed cost and get } -c_0 w + \frac{k}{A} \\ 0 & \text{operate (invest, de-invest, or inaction)} \end{cases}$$

  - Could pay fixed cost $c_0 w$ to recycle all capital with no loss
  - Or could take all capital to junk yard and suffer a loss

- Firm value is

$$V(z,k) = (1-\psi)\max\left\{V^{\text{exit}}(z,k), V^{\text{operate}}(z,k)\right\} + \psi\frac{\omega(1-d)k}{A}$$

  - Exogenous exit shock ψ: capital partially obselete (Nokia)
  - Voluntary exit or de-invest: capital still usable, not subject to ω depreciation

- Optimal capital if invest:

$$k^*_{t+1}(z,k) = (1-d)k + A_t \times i_t(k,z)$$

  Note: in our model, k^*_{t+1} doesn't depend on k.

- Next period capital if invest or no invest:

$$k'_{t+1}(z,k) = \begin{cases} \text{this firm no longer exits} & \text{firm chooses to exit at } t \\ (1-d)k & \text{firm chooses not to invest at } t \\ k^*_{t+1}(z,k) & \text{firm chooses to invest or de-invest at } t \end{cases}$$

  - In the code, if firm exits, we set k' to $-\inf$ to avoid accidental errors

Market Clearing Condition:

- Goods market in terms of final goods

$$
\begin{aligned}
(1-\psi)\int_{VolExit=0} y(z,k)\,M(z,k) \\
(1-\psi)\int_{VolExit=1} \frac{\omega(1-\delta)k}{A}M(z,k) \\
(1-\psi)\int_{VolExit=2} \frac{(1-\delta)k}{A}M(z,k) \\
+\frac{\omega(1-\delta)k}{A}\psi\int M(z,k) \\
= \\
(1-\psi)\int_{VolExit=0} i(z,k)\,M(z,k) \\
+C+M_0\frac{k_0}{A}
\end{aligned}
$$

- Labor market

$$
\begin{aligned}
(1-\psi)\int_{VolExit=0}\left[n(z,k)+c_f\right]M(z,k) \\
+(1-\psi)c_0\int_{VolExit=0 \text{ and } i(z,k)\neq 0} M(z,k) \\
+(1-\psi)c_0\int_{VolExit=2} M(z,k) \\
+M_0(c_E+c_0) \\
=N
\end{aligned}
$$

- HH FOC

$$
w=\frac{v'(N)}{u'(C)}
$$

Measures:

- Define $M(z,k)$ to be entering measure
- Define $O(z,k)$ to be production measure
- Suppose in period t, measure of firms are given by $M_t(z,k)$ and M_{0t} new firms enter
- Then measure of firms in period $t+1$ is $M_{t+1}(z,k)$:
  - When z is fixed over time:

$$
M_{t+1}(z,k')=M_{0t}\times 1(k_{0t}=k')\times f(z)+(1-\psi)\times 1\left(I_t\left(z,\frac{k'}{1-\delta}\right)=1\right)\times M_t\left(z,\frac{k'}{1-\delta}\right)+\int_{I_t(z,k)=
$$

- When z is persistent over time:
  - Measure at beginning of period

$$
\begin{aligned}
M_{t+1}\left(z', k'\right) = M_{0t} \times 1\left(k_{0t} = k'\right) \times f\left(z'\right) \\
+ (1-\psi) \int_{z: E\left(z, k=\frac{k'}{1-d}\right)=0, i\left(z, k=\frac{k'}{1-d}\right)=0} M_t\left(z, \frac{k'}{1-d}\right) \times f\left(z' | z\right) dz \\
+ (1-\psi) \iint_{z, k: E(z,k)=0, i(z,k)\neq 0} 1\left\{k_{t+1}^{*}(z) = k'\right\} \times M_t(z,k) \times f\left(z'|z\right) dk dz
\end{aligned}
$$

  - Measure of firms at production stage

$$
O_t(z,k) = (1-\psi) M_t(z,k) 1\{E(z,k)=0\}
$$

Numerical Strategy

- What we have: $V_t(z,k)$;w_t; We found this using the free entry condition.
- $M_{0,t=1}$; $M_{t=1}(z,k)$ (We assume the world is in some stationary state before the beginning of time to find these.) (This might be irrational since we're going to start along a trajectory of changes in A)
- What we need is $M_{0,t}$ and $M_t(z,k)$
- Idea: We can solve for measure variables sequentially in time. $M_t(z,k)$ can be derived easily from $M_{0,t}$ & $M_{t-1}(z,k)$ and the policy function.
  - Either use a solver or guess on $M_{0,t}$ and see if $w = \frac{v'(N)}{u'(C)}$ is satisfied.