1 toy growth model

This is a simple neoclassic growth model. Households solve the following problem:

$$\max \sum_{t=0}^{\infty} \beta^t u\left(c_t\right)$$

subject to

$$c_t + k_{t+1} = f\left(k_t\right) + \left(1-\delta\right) k_t$$

The associated Bellman equation is

$$V\left(k\right) = \max u\left(c\right) + \beta V\left(k'\right)$$

subject to

$$c + k' = f\left(k\right) + \left(1-\delta\right) k$$

The Bellman equation is solved using a simple value function iteration.

2 toy growth model with transition

Assume there is a sudden (unexpected) increase in productivity:

- Old $f\left(k\right) = k^{\alpha}$
- New $zf\left(k\right) = zk^{\alpha}$ where $z > 1$

This code calculates the transition from the old stationary equilibrium to the new stationary equilibrium.

Algorithm:

1. Solve for old and new stationary equilibrium capital levels k^*_{old} and k^*_{new}. For this growth model, this can be done by hand:

$$\beta\left(1-\delta+f'\left(k^*_{old}\right)\right) = 1$$

$$\beta\left(1-\delta+zf'\left(k^*_{new}\right)\right) = 1$$

2. Assume transition is completed in T periods, where T is a safely large number.

3. Calculate a transition path of $\{k_t\}_{t=1}^{T}$ using fsolve

   (a) Initial guess can be linear interpolation between k^*_{old} and k^*_{new}.

   (b) Based on any guessed path, calculate the associated level of consumption c_t from

   $$c_t + k_{t+1} = zf\left(k_t\right) + \left(1-\delta\right) k_t$$

   (c) Calculate how much we are off from the Euler equations:

   $$u'\left(c_t\right) = \beta\left(1-\delta+zf'\left(k_t\right)\right) u'\left(c_{t+1}\right)$$

   (d) k_1 and k_T will be fixed at k^*_{old}, k^*_{new}, leaving $T-2$ free variables. We can use Euler equation in period $t = 1, 2, 3, ..., T-2$