Chapter 4

Recursion

Principle of Recursion

- A recursion is a process of defining a problem (or solution to a problem) in terms of (a simple version of) itself.
- A **recursive algorithm** is one expressed in terms of itself. In other words, at least one step of a recursive algorithm is a “call” to itself.
- In programming, a **recursive function** is one that calls itself. We use recursive function to implement recursive algorithms in programming languages
- A recursive algorithm is more elegant and easier to understand but it is less efficient (extra calls consume time and space).

Principle of Recursion

- Given a recursive algorithm, how can we sure that it terminates?
- The algorithm must have:
  - one or more “easy” cases or stopping conditions
  - one or more “hard” cases or recursive calls
- In a stopping condition, the algorithm must give a direct answer without calling itself.
- In a recursive call, the algorithm may call itself, but only to deal with an “easier” case.

Example: Factorial

- **Algorithm:**

n! = 1 if n = 0

n! = n * (n – 1)! If n > 0

- **C-Function:**

```
int factorial(int n)
{
        if(n == 0)
                return 1;
        else
                return n * factorial(n - 1);
}
```

Example: Fibonacci Number

- **Algorithm:**

fib(n) = n if n = 0 or n = 1

fib(n) = fib(n – 1) + fib(n – 2) If n > 1

- **C-Function:**

```
int fibo(int n)
{
        if(n <= 1)
                return n;
        else
                return fibo(n - 1) + fibo(n - 2);
}
```

Example: Greatest Common Divisor

- **Algorithm:**

gcd(p, q) = q if q exactly divides p

gcd(p, q) = gcd(q, p mod q) if q does not exactly divides p

- **C-Function:**

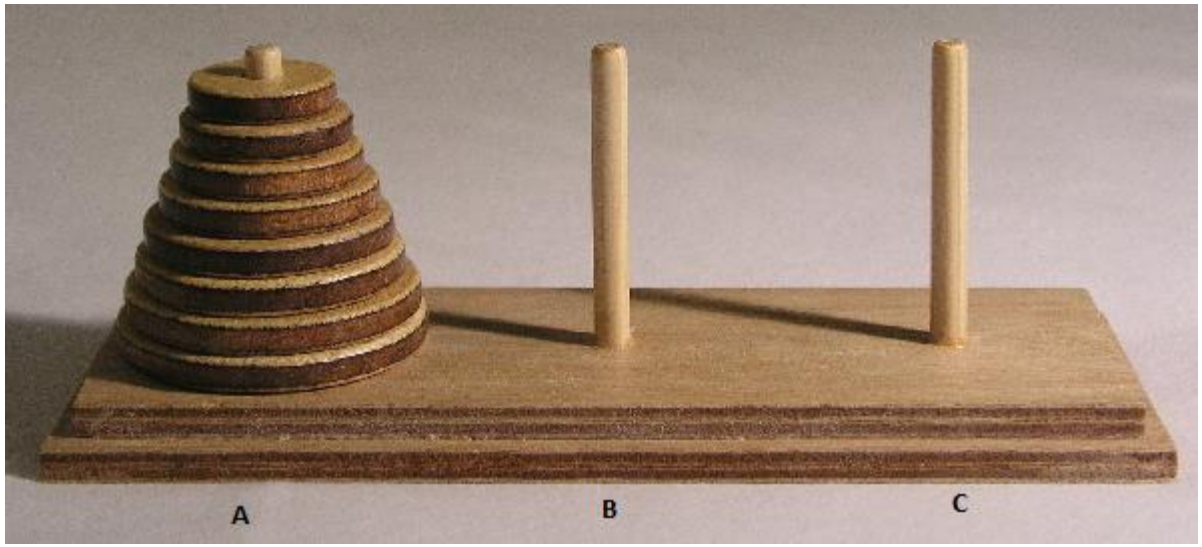
```
int gcd(int p, int q)
{
        if(p % q == 0)
                return q;
        else
                return gcd(q, p % q);
}
```

Example: Tower of Hanoi

- Three vertical poles (A, B, C) are mounted on a platform.
- A number of differently-sized disks are threaded on to pole A, forming a tower with the largest disk at the bottom and the smallest disk at the top.
- We may move **one** disk at a time, from any pole to any other pole, but we must never place a larger disk on top of a smaller disk.
- Problem: Move the tower of disks from pole A to pole B.

Example: Tower of Hanoi

- **Algorithm:** To move n disks from A to C, using B as auxiliary
  1. If n = 1, move the single disk from A to C and stop.
  2. Move the top n - 1 disks from A to B using C as auxiliary.
  3. Move the remaining disk from A to C.
  4. Move then n - 1 disks from B to C using A as auxiliary.



Example: Tower of Hanoi

- **C-Function:**

```
void towers(int n, char frompeg, char topeg, char auxpeg)
{
        if(n == 1)
        {
                printf("\nMove disk 1 from peg %c to peg %c", frompeg, topeg);
                return;
        }
        towers(n - 1, frompeg, auxpeg, topeg);
        printf("\nMove disk %d from peg %c to peg %c", n, frompeg, topeg);
        towers(n - 1, auxpeg, topeg, frompeg);
}
```

Recursion vs. Iteration

- **Recursion:**
  - Recursion uses selection structure.
  - Infinite recursion occurs if the recursive step does not reduce the problem in a manner that converges on some condition (base case) and Infinite recursion can crash the system.
  - Recursion terminates when a base case is recognized.
  - Recursion is usually slower than iteration due to the overhead of maintaining the stack.
  - Recursion uses more memory than iteration.
  - Recursion makes the code smaller.

Recursion vs. Iteration

- **Iteration:**
  - Iteration uses repetition structure.
  - An infinite loop occurs with iteration if the loop condition test never becomes false and Infinite looping uses CPU cycles repeatedly.
  - An iteration terminates when the loop condition fails.
  - An iteration does not use the stack so it's faster than recursion.
  - Iteration consumes less memory.
  - Iteration makes the code longer.

Efficiency of Recursion

- Nonrecursive version of a program executes more efficiently in terms of time and space than a recursive version because extra calls in recursive programs consume time and space
- Overhead involved in entering and exiting a block is avoided in the nonrecursive version
- In a nonrecursive program stacking activity can be eliminated
- Sometimes a recursive solution is the most natural way of solving a problem because recursive solution flows directly from the recursive definitions