Homework Assignment 1

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1. **Find the L_1, L_2, and L_∞ norms of the following vectors.**
   a. **(5, 2)**

Solution:

A norm is a function that measures the size of things. L_1, L_2 and L_∞ are most popular norms. Which are defined as follow.

L_1: $||x||_1 = \sum_i^n \left|xi^1\right| = |x_1| + |x_2| \dots + |x_n|$

L_2: $||x||_2 = \left(\sum_i^n |xi^2|\right)^{1/2}$

$= |x_1^2 + x_2^2 + \dots + x_n^2|$

L_∞: $||x||_\infty = \max_{i \in (1,2,..,n)} |xi|$

$= \max\{ |x_1| , | x_2|, \dots , |x_n|$

Now, Here given point is $(5, 2) \in \mathbf{R^2}$, norms L_1, L_2 and L_∞ are

define respectively as,

L_1 : $||x||_1 = |5| + |2| = 7$

L_2: $||x||_2 = (25 + 4)^{1/2} = 5.2$

L_∞: $||x||_\infty = \max\{2,5\} = 5$

From above we can see that $||x||_\infty < ||x||_2 < ||x||_1$

b. **(-4 , 2, 3)**

The point $(-4 , 2 , 3) \in \mathbf{R^3}$ so, corresponding norms L_1, L_2 and L_∞ are define as

L_1 : $||x||_1 = |-4| + |2| + |3| = 9$

L_2: $||x||_2 = (16 + 4 + 9)^{1/2} = 5.2$

L_∞: $||x||_\infty = \max\{ -4, 2, 3\} = 3$

From above we can say $||x||_\infty < ||x||_2 < ||x||_1$.

c. (1,2,3,4)

The point (1, 2, 3, 4) ∈ $\mathbf{R^4}$ so, norms L_1, L_2 and L_∞ are define as

L_1 : $||x||_1$ = |1| + |2| + |3| + |4| = 10

L_2: $||x||_2$ = $(1 + 4 + 9 + 16)^{1/2}$ = 5.21

L_∞: $||x||_\infty$ = max{1,2,3,4} = 4

Hence, we saw $||x||_\infty < ||x||_2 < ||x||_1$.

d. (4,-1,1,3)

The point (4, -1, 1, 3) ∈ $\mathbf{R^4}$ norms L_1, L_2 and L_∞ are define as

L_1 : $||x||_1$ = |4| + |-1| +|1| + |3| = 9

L_2: $||x||_2$: $(16 + 1 + 1 + 9)^{1/2}$ = 5.44

L_∞: $||x||_\infty$: max{ 4,-1,1,3} = 3

Here also we found, $||x||_\infty < ||x||_2 < ||x||_1$.

e. (0,0,0,7,0,0)

The point (0, 0, 0, 7, 0, 0) ∈ $\mathbf{R^6}$ so, corresponding norms L_1, L_2 and L_∞ are define as

L_1 : $||x||_1$: |0| + |0| + |0| +|7| + |0| + |0| = 7

L_2: $||x||_2$: $(49)^{1/2}$ = 7

L_∞: $||x||_\infty$: max{ 0,0,0,7,0,0} = 7

In this point we found $||x||_\infty = ||x||_2 = ||x||_1$.

Overall, we conclude that $||x||_\infty \le ||x||_2 \le ||x||_1$.