ENM 326 – Network Optimization and Algorithms Project Assignment Presentation

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What Is Our Problem?

6.2. Nurse staff scheduling (Khan and Lewis [1987]). To provide adequate medical service to its constituents at a reasonable cost, hospital administrators must constantly seek ways to hold staff levels as low as possible while maintaining sufficient staffing to provide satisfactory levels of health care. An urban hospital has three departments: the emergency room (department 1), the neonatal intensive care nursery (department 2), and the orthopedics (department 3). The hospital has three work shifts, each with different levels of necessary staffing for nurses. The hospital would like to identify the minimum number of nurses required to meet the following three constraints: (1) the hospital must allocate at least 13, 32, and 22 nurses to the three departments (over all shifts); (2) the hospital must assign at least 26, 24, and 19 nurses to the three shifts (over all departments); and (3) the minimum and maximum number of nurses allocated to each department in a specific shift must satisfy the following limits:

Department and Shift Requirements

Dept 1 ≥ 13 nurses

Dept 2 ≥ 32 nurses

Dept 3 ≥ 22 nurses

Shift 1 ≥ 26 nurses

Shift 2 ≥ 24 nurses

Shift 3 ≥ 19 nurses

		Department 1	Department 2	Department 3
Shift	1	(6, 8)	(11, 12)	(7, 12)
	2	(4, 6)	(11, 12)	(7, 12)
	3	(2, 4)	(10, 12)	(5, 7)

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Problem Data

```
!pip install gurobipy networkx matplotlib
```

```
import gurobipy as gp
from gurobipy import GRB
import numpy as np
import networkx as nx
import matplotlib.pyplot as plt
```

```
params = {
    "WLSACCESSID": '5e515f08-0e42-48bd-9fc8-cac321d9fc57',
    "WLSSECRET": '2e0bcb85-f43c-4c6f-9421-fcdaf69ac602',
    "LICENSEID": 2631488,
}


env = gp.Env(params=params)
m = gp.Model("MaximumFlow_NurseScheduling", env=env)
```

```
shifts = [1, 2, 3]
departments = [1, 2, 3]
```

```
shift_reqs = {1: 26, 2: 24, 3: 19}
dept_reqs = {1: 13, 2: 32, 3: 22}
```

```
bounds = {
    (1, 1): (6, 8), (1, 2): (11, 12), (1, 3): (7, 12),
    (2, 1): (4, 6), (2, 2): (11, 12), (2, 3): (7, 12),
    (3, 1): (2, 4), (3, 2): (10, 12), (3, 3): (5, 7)
}
```

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Decision Variables (Flow Variables)

```
f_s_shift = m.addVars(shifts, lb=0.0, vtype=GRB.CONTINUOUS, name="f_s_shift")
```

```
f_shift_sd = m.addVars(shifts, departments, lb=0.0, vtype=GRB.CONTINUOUS, name="f_shift_sd")
```

```
f_sd_dept = m.addVars(shifts, departments, lb=0.0, vtype=GRB.CONTINUOUS, name="f_sd_dept")
```

```
f_dept_t = m.addVars(departments, lb=0.0, vtype=GRB.CONTINUOUS, name="f_dept_t")
```

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Constraints

```
m.addConstrs((f_sd_dept[s, d] <= bounds[(s, d)][1] for s in shifts for d in departments), name="Capacity_Upper")
m.addConstrs((f_sd_dept[s, d] >= bounds[(s, d)][0] for s in shifts for d in departments), name="Capacity_Lower")
```

```
m.addConstrs((f_s_shift[s] == gp.quicksum(f_shift_sd[s, d] for d in departments) for s in shifts), name="Flow_Shift")
```

```
m.addConstrs((f_shift_sd[s, d] == f_sd_dept[s, d] for s in shifts for d in departments), name="Flow_SD")
```

```
m.addConstrs((gp.quicksum(f_sd_dept[s, d] for s in shifts) == f_dept_t[d] for d in departments), name="Flow_Dept")
```

```
m.addConstrs((f_dept_t[d] >= dept_reqs[d] for d in departments), name="Dept_Demand")
```

```
m.addConstrs((f_s_shift[s] >= shift_reqs[s] for s in shifts), name="Shift_Demand")
```

Objective Function

```
m.setObjective(gp.quicksum(f_s_shift[s] for s in shifts), GRB.MINIMIZE)
```

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Solve and Print Results

```
m.optimize()
```

```
if m.status == GRB.OPTIMAL:
    print("Optimal objective value (Minimum Nurse Count):", m.objVal)
    for s in shifts:
        for d in departments:
            val = f_sd_dept[s, d].x
            if val > 0:
                print(f"Flow from Shift {s} to Department {d}: {val}")
else:
    print("Optimization was not successful. Status:", m.status)
```

```
m.write('Nurse_MaxFlow.lp')
m.write('Nurse_MaxFlow.sol')
```

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Network Graph Visualization

```
G = nx.DiGraph()

edge_labels = {}
```

```
for s in shifts:
    val = f_s_shift[s].x
    G.add_edge("S", f"Shift{s}", weight=val)
    edge_labels[("S", f"Shift{s}")] = f"{val:.0f}"
```

```
for s in shifts:
    for d in departments:
        val1 = f_shift_sd[s, d].x
        val2 = f_sd_dept[s, d].x
        G.add_edge(f"Shift{s}", f"S{s}D{d}", weight=val1)
        edge_labels[(f"Shift{s}", f"S{s}D{d}")] = f"{val1:.0f}"
        G.add_edge(f"S{s}D{d}", f"Dept{d}", weight=val2)
        edge_labels[(f"S{s}D{d}", f"Dept{d}")] = f"{val2:.0f}"
```

```
for d in departments:
    val = f_dept_t[d].x
    G.add_edge(f"Dept{d}", "T", weight=val)
    edge_labels[(f"Dept{d}", "T")] = f"{val:.0f}"
```

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Manual Node Positioning

```
pos = {}
pos["S"] = (-1, 0)
for i, s in enumerate(shifts):
    pos[f"Shift{s}"] = (0, 1 - i)
for i, s in enumerate(shifts):
    for j, d in enumerate(departments):
        pos[f"S{s}D{d}"] = (1, 1 - i - j * 0.3)
for i, d in enumerate(departments):
    pos[f"Dept{d}"] = (2, 0.5 - i)
pos["T"] = (3, 0)
```

Drawing the Graph

```
plt.figure(figsize=(16, 8))
nx.draw_networkx_nodes(G, pos, node_size=1800, node_color="lightsteelblue")
nx.draw_networkx_edges(G, pos, arrows=True, arrowstyle="->", arrowsize=20, edge_color="gray")
nx.draw_networkx_labels(G, pos, font_size=10, font_weight="bold")
nx.draw_networkx_edge_labels(G, pos, edge_labels=edge_labels, font_color="darkred", font_size=9)
```

```
plt.title("Nurse Scheduling Max Flow Network (Katmanlı Görsel)", fontsize=14)
plt.axis("off")
plt.tight_layout()
plt.show()
```

Results and Allocation

```
Academic license 2631488 - for non-commercial use only - registered to hu___@ogr.eskisehir.edu.tr
Optimize a model with 39 rows, 24 columns and 66 nonzeros
Model fingerprint: 0xad8139ac
Coefficient statistics:
  Matrix range     [1e+00, 1e+00]
  Objective range  [1e+00, 1e+00]
  Bounds range     [0e+00, 0e+00]
  RHS range        [2e+00, 3e+01]
Presolve removed 34 rows and 12 columns
Presolve time: 0.01s
Presolved: 5 rows, 12 columns, 18 nonzeros

Iteration    Objective       Primal Inf.    Dual Inf.      Time
       0    6.9000000e+01   1.500000e+00   0.000000e+00      0s
       4    6.9000000e+01   0.000000e+00   0.000000e+00      0s

Solved in 4 iterations and 0.01 seconds (0.00 work units)
Optimal objective  6.900000000e+01
Optimal objective value (Minimum Nurse Count): 69.0
Flow from Shift 1 to Department 1: 6.0
Flow from Shift 1 to Department 2: 12.0
Flow from Shift 1 to Department 3: 8.0
Flow from Shift 2 to Department 1: 5.0
Flow from Shift 2 to Department 2: 12.0
Flow from Shift 2 to Department 3: 7.0
Flow from Shift 3 to Department 1: 2.0
Flow from Shift 3 to Department 2: 10.0
Flow from Shift 3 to Department 3: 7.0
```

Objective: Minimize the total number of nurses while meeting all department and shift requirements.

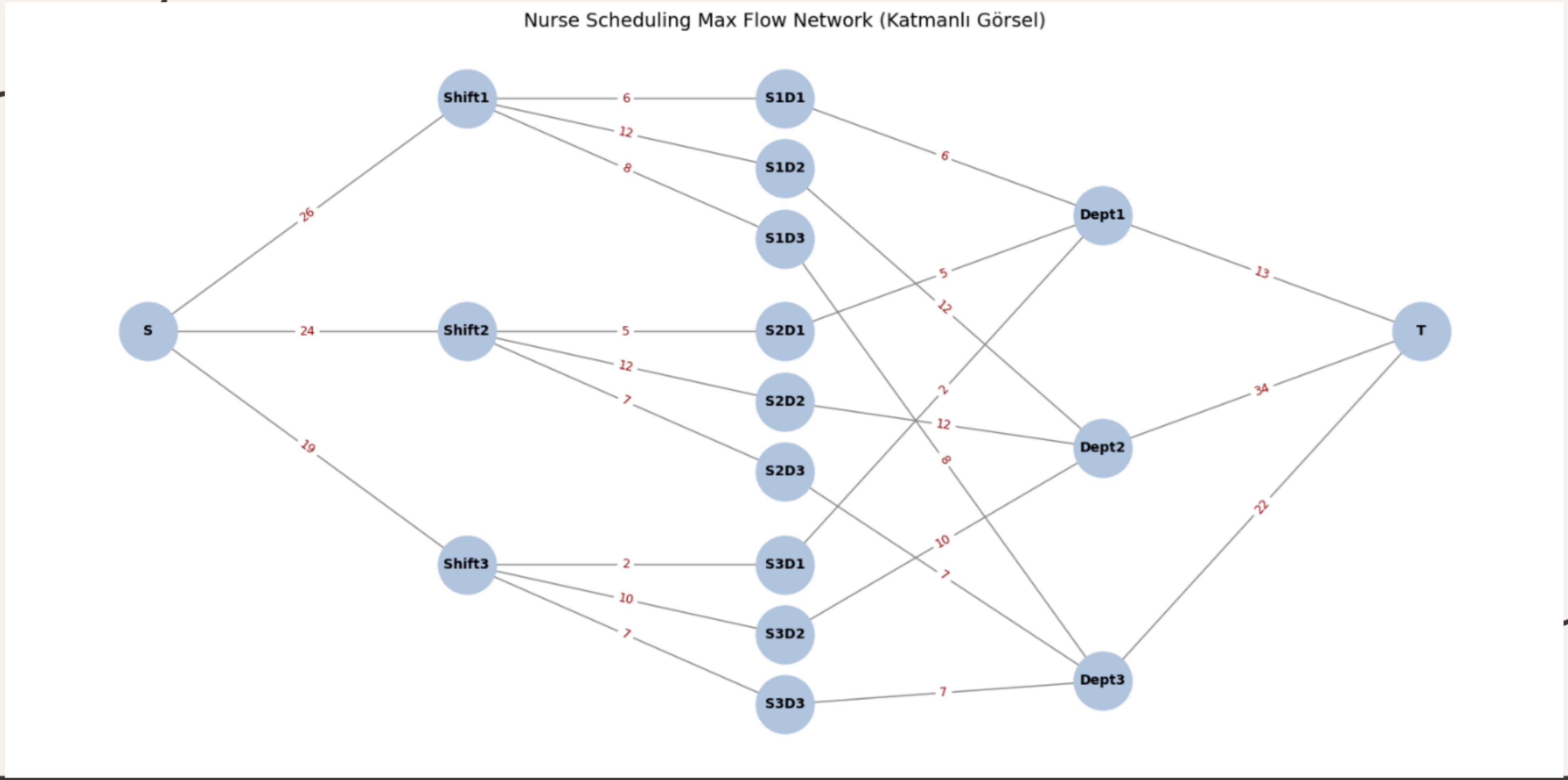
Total Nurses Required: 69 nurses in the optimal solution.

Shift-wise Nurse Allocation:
- Shift 1: 26 nurses
- Shift 2: 24 nurses
- Shift 3: 19 nurses

Shift-to-Department Distribution:
- Shift 1: 6 → Dept 1, 12 → Dept 2, 8 → Dept 3
- Shift 2: 5 → Dept 1, 12 → Dept 2, 7 → Dept 3
- Shift 3: 2 → Dept 1, 10 → Dept 2, 7 → Dept 3

Department-wise Totals:
- Dept 1: 13 nurses (requirement met exactly)
- Dept 2: 34 nurses (exceeds minimum of 32)
- Dept 3: 22 nurses (requirement met exactly)

MAX FLOW NETWORK OF THE SOLUTION



THANKS

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