

EXPERIMENTAL DESIGN AND REGRESSION ANALYSIS

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OBJECTIVE AND VARIABLES

- Examining how brewing tea under different conditions affects its taste.

FACTORS	FACTOR LEVEL 1	FACTOR LEVEL 2
Type of Tea	Black	Green
Water Temperature	80°C	100°C
Steeping Time	3 Minutes	5 Minutes

- **Output Variable:** After each experiment, the tea is tasted and assigned a “Taste Score” out of 10.

EXPERIMENTS

Observation Number	Conditions	Date	Observation Value
1	Black Tea, 80°C, 3 min	19.05.2024	5
2	Black Tea, 80°C, 3 min	19.05.2024	6
3	Black Tea, 80°C, 3 min	19.05.2024	5
4	Black Tea, 80°C, 5 min	19.05.2024	4
5	Black Tea, 80°C, 5 min	19.05.2024	5
6	Black Tea, 80°C, 5 min	19.05.2024	4
7	Green Tea, 80°C, 3 min	20.05.2024	8
8	Green Tea, 80°C, 3 min	20.05.2024	7
9	Green Tea, 80°C, 3 min	20.05.2024	7
10	Green Tea, 80°C, 5 min	20.05.2024	6
11	Green Tea, 80°C, 5 min	20.05.2024	7
12	Green Tea, 80°C, 5 min	20.05.2024	8
13	Black Tea, 100°C, 3 min	21.05.2024	7
14	Black Tea, 100°C, 3 min	21.05.2024	8
15	Black Tea, 100°C, 3 min	21.05.2024	8
16	Black Tea, 100°C, 5 min	21.05.2024	6
17	Black Tea, 100°C, 5 min	21.05.2024	7
18	Black Tea, 100°C, 5 min	21.05.2024	6
19	Green Tea, 100°C, 3 min	22.05.2024	8
20	Green Tea, 100°C, 3 min	22.05.2024	9
21	Green Tea, 100°C, 3 min	22.05.2024	9
22	Green Tea, 100°C, 5 min	22.05.2024	7
23	Green Tea, 100°C, 5 min	22.05.2024	8
24	Green Tea, 100°C, 5 min	22.05.2024	7

Water Temperature (B) and Steeping Time (C)

Type of Tea (A)	80°C 3 min	80°C 5 min	100°C 3 min	100°C 5 min
Black Tea	5	8	7	8
	6	7	8	9
	5	7	8	9
Green Tea	4	6	6	7
	5	7	7	8
	4	8	6	7

HYPOTHESES

DECISION RULE

❖ If $F_0 > F_{\text{crit}}$, then H_0 is rejected.

❖ $F_{\text{crit}} = F_{0.05, 1, 16} = 4.49$

TOTALS

Water Temperature (B) and Steeping Time (C)					
Type of Tea (A)	80°C 3 min	80°C 5 min	100°C 3 min	100°C 5 min	Row Sum
Black Tea	5	8	7	8	90
	6	7	8	9	
	5	7	8	9	
Green Tea	4	6	6	7	80
	5	7	7	8	
	4	8	6	7	
Column Sum	35	45	41	49	170
	80		90		

- ✓ $(y_{..}) = 170$
- ✓ $(y_{i..}) = (90,80)$
- ✓ $(y_{.j.}) = (80,90)$
- ✓ $(y_{.k.}) = (76,94) = ((35 + 41), (45 + 49))$
- ✓ $(y_{ij.}) = (42,48,38,42)$
- ✓ $(y_{i.k.}) = (41,49,35,45)$
- ✓ $(y_{.jk.}) = (35,45,41,49)$
- ✓ $(y_{ijk.}) = (19,23,22,26,16,22,19,23) = ((6 + 7 + 6), (8 + 7 + 8), ..., (6 + 7 + 6), (8 + 7 + 8))$

A×B Total		
	$y_{ij..}$	
	Water Temperature (B)	
Type of Tea (A)	80°C	100°C
Black Tea	42	48
Green Tea	38	42

A×C Total		
	$y_{i.k.}$	
	Steeping Time (C)	
Type of Tea (A)	3 min	5 min
Black Tea	41	49
Green Tea	35	45

TEST STATISTICS

- $SS_A = \frac{1}{12}(90^2 + 80^2) - \frac{170^2}{24} = 4,166666667$
- $SS_B = \frac{1}{12}(80^2 + 90^2) - \frac{170^2}{24} = 4,166666667$
- $SS_C = \frac{1}{12}(76^2 + 94^2) - \frac{170^2}{24} = 13,5$
- $SS_{AB} = \frac{1}{6}(42^2 + 48^2 + 38^2 + 42^2) - 4,166666667 - 4,166666667 = 0,166666667$
- $SS_{AC} = \frac{1}{6}(41^2 + 49^2 + 35^2 + 45^2) - 4,166666667 - 13,5 = 0,166666667$
- $SS_{BC} = \frac{1}{6}(35^2 + 45^2 + 41^2 + 49^2) - 4,166666667 - 13,5 = 0,166666667$
- $SS_{ABC} = \frac{1}{3}(19^2 + 23^2 + 22^2 + \dots + 19^2 + 23^2) - 4,166666667 - 4,166666667 - 13,5 - 0,166666667 - 0,166666667 - 0,166666667 = 0,166666667$
- $SS_T = (6^2 + 8^2 + 7^2 + 9^2 + 7^2 + \dots + 7^2 + 6^2 + 8^2) = 27,83333333$
- $SS_E = SS_T - SS_{Subtotals} = 27,83333333 - (4,166666667 + 4,166666667 + 13,5 + 0,166666667 + 0,166666667 + 0,166666667) = 5,333333333$

Analysis of Variance Table (ANOVA)

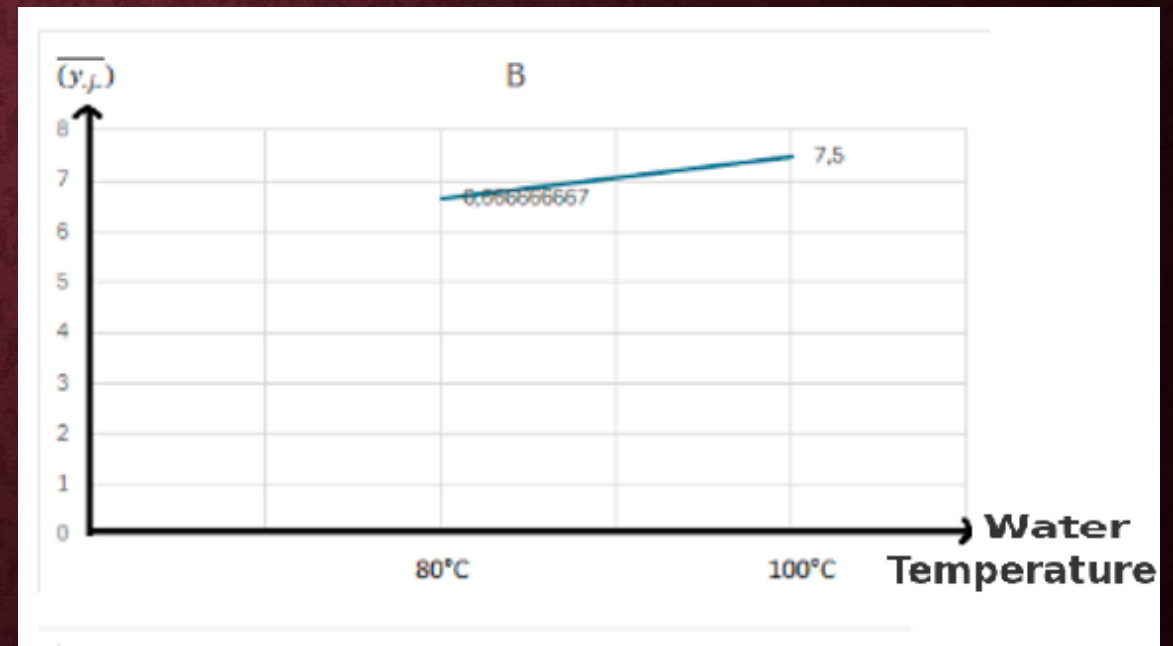
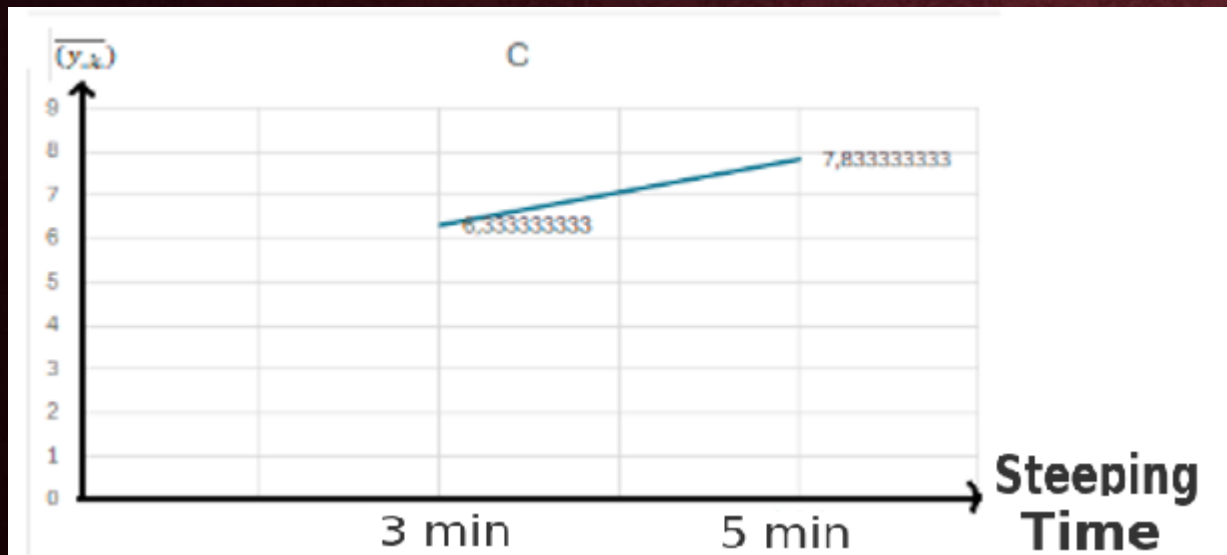
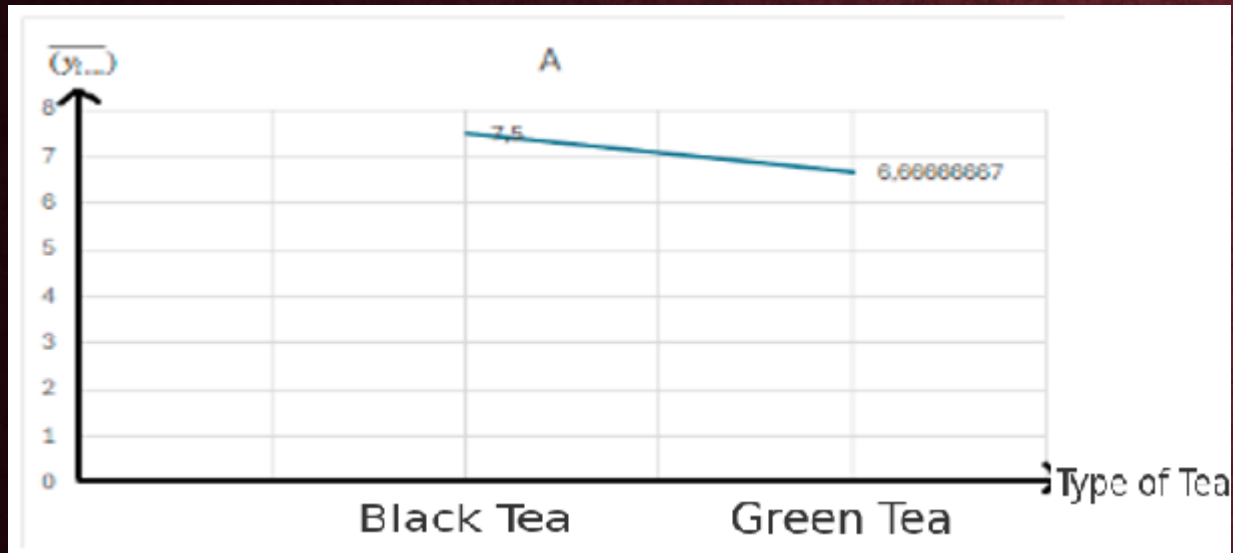
Source of Variation	Degrees of Freedom (df)	Sum of Squares (SS)	Mean Square (MS)	F ₀
Tea Type (A)	1	4.1667	4.1667	12.5
Water Temp. (B)	1	4.1667	4.1667	12.5
Steep. Time (C)	1	13.5	13.5	40.5
AxB	1	0.1667	0.1667	0.5
AxC	1	0.1667	0.1667	0.5
BxC	1	0.1667	0.1667	0.5
AxBxC	1	0.1667	0.1667	0.5
Error	16	5.3333	0.3333	
Total	23	27.8333		

CONCLUSION

Source of Variation	F_0	Decision
Tea Type (A)	12.5	Reject H_0
Water Temperature (B)	12.5	Reject H_0
Steeping Time (C)	40.5	Reject H_0
A×B (Tea × Temp)	0.5	Fail to Reject H_0
A×C (Tea × Steep)	0.5	Fail to Reject H_0
B×C (Temp × Steep)	0.5	Fail to Reject H_0
A×B×C (Tea × Temp × Steep)	0.5	Fail to Reject H_0

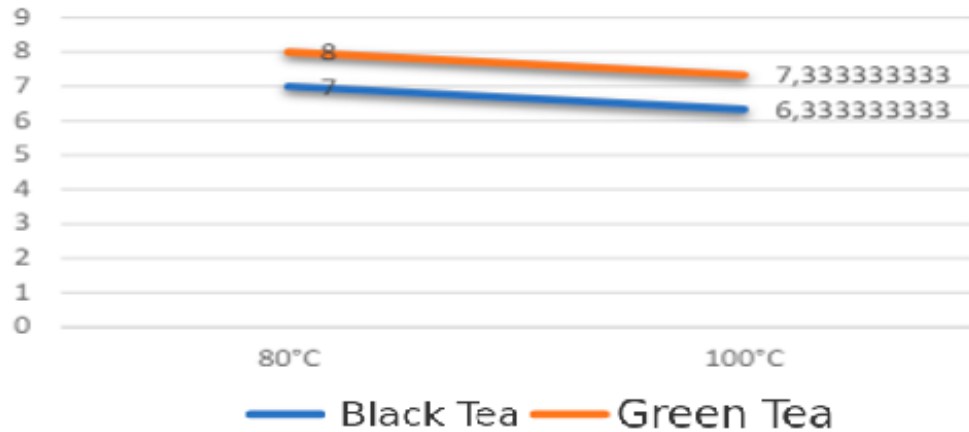
According to the obtained analysis of variance table, all three factors affect the taste of tea at the 5% significance level. However, the interactions among these factors are not statistically significant.

MAIN EFFECTS GRAPHS

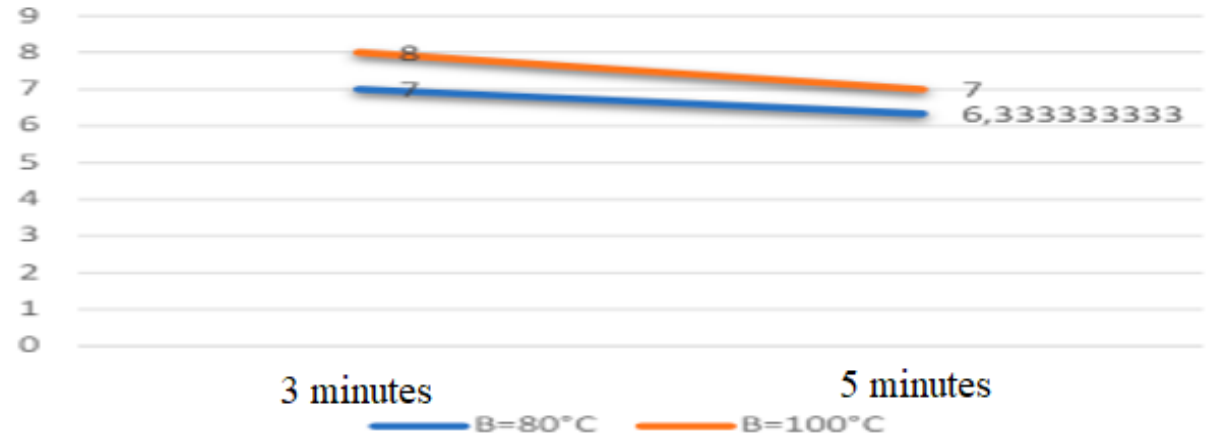


INTERACTION CHARTS

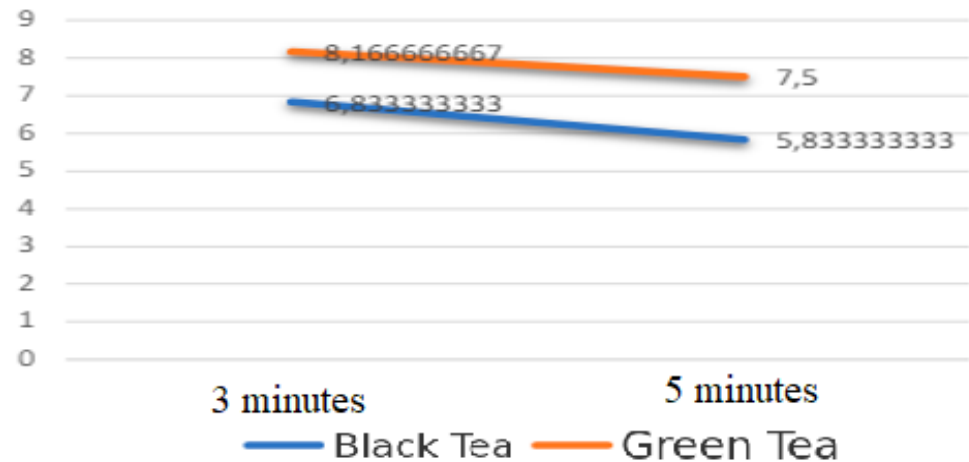
AxB Interaction Chart



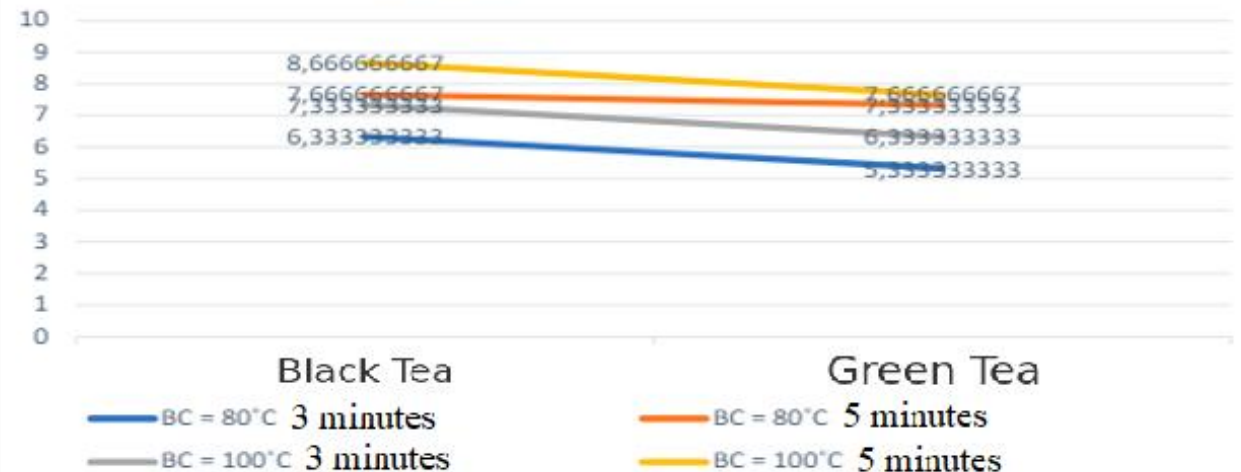
BxC Interaction Chart



AxC Interaction Chart



AxBxC Interaction Chart



TUKEY TEST

$$\diamond \text{ Tukey value: } T_{\alpha} = q_{0.05}(2, 16) \sqrt{\frac{0.33333333}{12}} = 0.4997$$

❖ Tukey's Test for Factor A:

- Black Tea: $\bar{y}_{1..} = \frac{90}{12} = 7.5$
- Green Tea: $\bar{y}_{2..} = \frac{80}{12} = 6.6667$
- $|\bar{y}_{1..} - \bar{y}_{2..}| = 0.833$

Since $0.833 > 0.4997$, the difference between the mean scores of Black Tea and Green Tea is statistically significant.

❖ Tukey's Test for Factor B:

- 80°C: $\bar{y}_{1.} = \frac{80}{12} = 6.6667$
- 100°C: $\bar{y}_{2.} = \frac{90}{12} = 7.5$
- $|\bar{y}_{1.} - \bar{y}_{2.}| = 0.833$

Since $0.833 > 0.4997$, the difference between the mean scores at 80 °C and 100 °C is statistically significant.

❖ Tukey's Test for Factor C:

- 3 min: $\bar{y}_{.1.} = \frac{76}{12} = 6.3333$
- 5 min: $\bar{y}_{.2.} = \frac{94}{12} = 7.8333$
- $|\bar{y}_{.1.} - \bar{y}_{.2.}| = 1.5$

Since $1.5 > 0.4997$, the difference between the mean scores at 3 min and 5 min steeping times is statistically significant.

CONCLUSIONS AND RECOMMENDATIONS

- Since Tea Type (A), Water Temperature (B) and Steeping Time (C) are important main effects, it is recommended to carefully select the levels of these factors.
- Specific levels should be chosen according to the desired outcome (e.g., taste, strength, aroma).
- Generally, more information about the preferred levels of these factors is required or previous experience may guide these selections.
- Since interactions between factors do not have a significant effect on the outcome, focusing individually on the optimal levels of A, B and C is sufficient, and these interactions do not have statistically significant effects on the taste score.
- For more detailed recommendations regarding factor levels, more detailed information about the levels tested for each factor is needed.