A single quantum cannot be cloned

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If a photon of definite polarization encounters an excited atom, there is typically some nonvanishing probability that the atom will emit a second photon by stimulated emission. Such a photon is guaranteed to have the same polarization as the original photon. But is it possible by this or any other process to amplify a quantum state, that is, to produce several copies of a quantum system (the polarized photon in the present case) each having the same state as the original? If it were, the amplifying process could be used to ascertain the exact state of a quantum system: in the case of a photon, one could determine its polarization by first producing a beam of identically polarized copies and then measuring the Stokes parameters[1]. We show here that the linearity of quantum mechanics forbids such replication and that this conclusion holds for all quantum systems.

Note that if photons could be cloned, a plausible argument could be made for the possibility of faster-than-light communication[2]. It is well known that for certain non-separably correlated Einstein–Podolsky–Rosen pairs of photons, once an observer has made a polarization measurement (say, vertical versus horizontal) on one member of the pair, the other one, which may be far away, can be for all purposes of prediction regarded as having the same polarization[3]. If this second photon could be replicated and its precise polarization measured as above, it would be possible to ascertain whether, for example, the first photon had been subjected to a measurement of linear or circular polarization. In this way the first observer would be able to transmit information faster than light by encoding his message into his choice of measurement. The actual impossibility of cloning photons, shown below, thus prohibits superluminal communication by this scheme. That such a scheme must fail for some reason despite the well-established existence of long-range quantum correlations[6-8], is a general consequence of quantum mechanics[9].

A perfect amplifying device would have the following effect on an incoming photon with polarization state $|s\rangle$:

$$|A_0\rangle|s\rangle \rightarrow |A_s\rangle|ss\rangle \qquad (1)$$

Here $|A_0\rangle$ is the 'ready' state of the apparatus, and $|A_s\rangle$ is its final state, which may or may not depend on the polarization of the original photon. The symbol $|ss\rangle$ refers to the state of the radiation field in which there are two photons each having the polarization $|s\rangle$. Let us suppose that such an amplification can in fact be accomplished for the vertical polarization $|\updownarrow\rangle$ and for the horizontal polarization $|\leftrightarrow\rangle$. That is,

$$|A_0\rangle|\updownarrow\rangle \rightarrow |A_{\text{vert}}\rangle|\updownarrow\updownarrow\rangle \qquad (2)$$

and

$$|A_0\rangle|\leftrightarrow\rangle \rightarrow |A_{\text{hor}}\rangle|\leftrightarrow\leftrightarrow\rangle \qquad (3)$$

According to quantum mechanics this transformation should be representable by a linear (in fact unitary) operator. It therefore follows that if the incoming photon has the polarization given by the linear combination $\alpha|\updownarrow\rangle + \beta|\leftrightarrow\rangle$—for example, it could be linearly polarized in a direction 45° from the vertical,

so that $\alpha = \beta = 2^{-1/2}$—the result of its interaction with the apparatus will be the superposition of equations (2) and (3):

$$|A_0\rangle(\alpha|\updownarrow\rangle + \beta|\leftrightarrow\rangle) \rightarrow \alpha|A_{\text{vert}}\rangle|\updownarrow\updownarrow\rangle + \beta|A_{\text{hor}}\rangle|\leftrightarrow\leftrightarrow\rangle \qquad (4)$$

If the apparatus states $|A_{\text{vert}}\rangle$ and $|A_{\text{hor}}\rangle$ are not identical, then the two photons emerging from the apparatus are in a mixed state of polarization. If these apparatus states are identical, then the two photons are in the pure state

$$\alpha|\updownarrow\updownarrow\rangle + \beta|\leftrightarrow\leftrightarrow\rangle \qquad (5)$$

In neither of these cases is the final state the same as the state with two photons both having the polarization $\alpha|\updownarrow\rangle + \beta|\leftrightarrow\rangle$. That state, the one which would be required if the apparatus were to be a perfect amplifier, can be written as

$$2^{-1/2}(\alpha a^+_{\text{vert}} + \beta a^+_{\text{hor}})^2|0\rangle = \alpha^2|\updownarrow\updownarrow\rangle + 2^{1/2}\alpha\beta|\updownarrow\leftrightarrow\rangle + \beta^2|\leftrightarrow\leftrightarrow\rangle$$

which is a pure state different from the one obtained above by superposition [equation (5)].

Thus no apparatus exists which will amplify an arbitrary polarization. The above argument does not rule out the possibility of a device which can amplify two special polarizations, such as vertical and horizontal. Indeed, any measuring device which distinguishes between these two polarizations, a Nicol prism for example, could be used to trigger such an amplification.

The same argument can be applied to any other kind of quantum system. As in the case of photons, linearity does not forbid the amplification of any given state by a device designed especially for that state, but it does rule out the existence of a device capable of amplifying an arbitrary state.

Milonni (unpublished work) has shown that the process of stimulated emission does not lead to quantum amplification, because if there is stimulated emission there must also be—with equal probability in the case of one incoming photon—spontaneous emission, and the polarization of a spontaneously emitted photon is entirely independent of the polarization of the original.

It is conceivable that a more sophisticated amplifying apparatus could get around Milonni's argument. We have therefore presented the above simple argument, based on the linearity of quantum mechanics, to show that no apparatus, however complicated, can amplify an arbitrary polarization.

We stress that the question of replicating individual photons is of practical interest. It is obviously closely related to the quantum limits on the noise in amplifiers[10,11]. Moreover, an experiment devised to establish the extent to which polarization of single photons can be replicated through the process of stimulated emission is under way (A. Gozzini, personal communication; and see ref. 12). The quantum mechanical prediction is quite definite; for each perfect clone there is also one randomly polarized, spontaneously emitted, photon.

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On replicating photons

WOOTTERS AND ZUREK[1] have recently considered whether it is possible to build a quantum mechanical device which will simply duplicate an arbitrarily polarized incoming photon. They consider two possible situations. In the first, the final state of the device depends on the polarization of the photon. In this case, a photon beam of arbitrary polarization will give rise to a mixed, rather than a pure, final state and will therefore not be properly replicated.

In the second situation the final state of the replicator is considered to be independent of the photon polarization. The authors (as also in a subsequent paper by Dieks[2]) demonstrate an inconsistency in the quantum mechanical description of this situation which leads them to conclude that in it, too, photon replication is impossible to achieve. However, this second situation is unphysical for a rather serious reason: if the final state of the replicator is independent of the photon polarization, then angular momentum conservation is violated. Photons of different polarization are in different spin states (or different linear combinations of spin states). Thus the polarization of the emitted photon must affect the final angular momentum state of the replicator which emits it.

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WOOTTERS AND ZUREK REPLY— Bussey points out that if the amplifier's final state were independent of polarization, then angular momentum would not be conserved. The question of angular momentum conservation is, however, more subtle than it may seem at first, as the following example shows. (This example is related to work of Wigner, Araki and Yanase on the limitations imposed by conservation laws on the accuracy of measurements[1-4].)

Let $|l\rangle$ be a certain state of the amplifier which is an eigenstate of L_z with eigenvalue l, L_z being the component of angular momentum along the direction of motion of the photon. Assume that when a right- or left-handed circularly polarized photon interacts with the amplifier in this state, the following angular-momentum-conserving transformation occurs:

|one right-handed photon⟩ $\otimes |l\rangle \rightarrow$

  |two right-handed photons⟩ $\otimes |l-1\rangle$

|one left-handed photon⟩ $\otimes |l\rangle \rightarrow$

  |two left-handed photons⟩ $\otimes |l+1\rangle$

Now suppose we take as our 'ready' state of the amplifier the state

$$|A_0\rangle \propto \sum_{l=-\infty}^{\infty} e^{-\alpha l^2}|l\rangle$$

Then it is a simple matter to show that for small α, the final state of the amplifier is practically independent of the polarization of the incoming photon, even though angular momentum is conserved. Indeed, the inner product between the two possible pure final states of the amplifier is $\exp(-2\alpha)$, which can be made arbitrarily close to unity. Thus, if angular momentum conservation were the only constraint, then the second situation we consider (that is, cloning with the final state independent of polarization) could in principle still be achieved to an arbitrarily good approximation. This is why it was necessary to give the argument based on the linearity of quantum mechanics, which clearly rules out this sort of cloning. Furthermore, the argument we gave can be generalized to apply to other quantum systems, for which angular momentum conservation may not be a relevant factor.

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