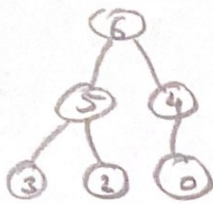


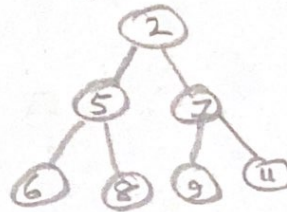
HEAP TREE

- It is a complete binary tree



Max heap tree

$$A[\text{parent}] \geq A[\text{child}]$$

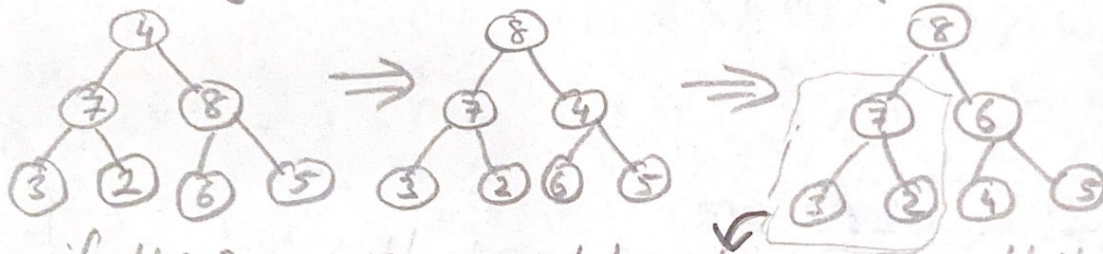


Min heap tree

$$A[\text{parent}] \leq A[\text{child}]$$

- Left child may be greater than right child

Heapify: If the heap tree is wrong, switch the parent with max child (for max heap). And if we do this ^{as} "recursive" "switched side subtree" eventually will be okay for heap order.



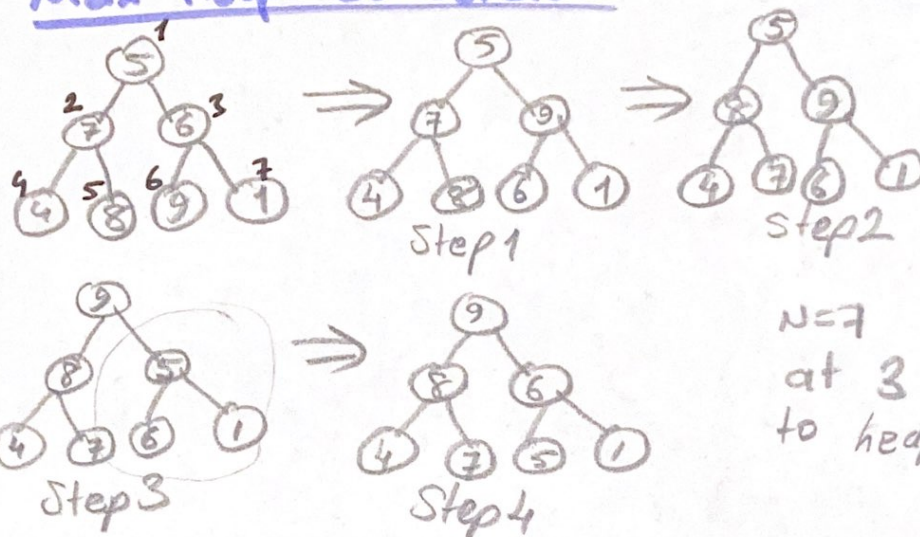
if there was a mistake here, we couldn't fix it only with heapify. Because it looks only switched subtree

Complexity of heapify $\rightarrow O(\log n)$!!

- If we wanted to control whether it is heap tree or not, it would be enough $N/2$ to 1 index since more than $N/2$ are leaf nodes. leaf nodes cannot be swapped.

- To find the min element in max-heap tree, we should check all leafs. It means $N/2$ to N

Max Heap Conversion

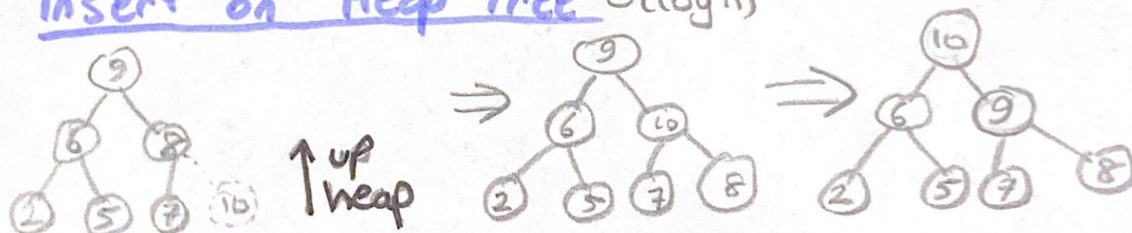


$N=7$ so we start at 3 (last parent) to heapify

```
void build_max_heap (int array[], int N) {
    int i;
    for (i = N/2; i >= 1; i--) {
        max_heapify(array, i, N);
    }
}
```

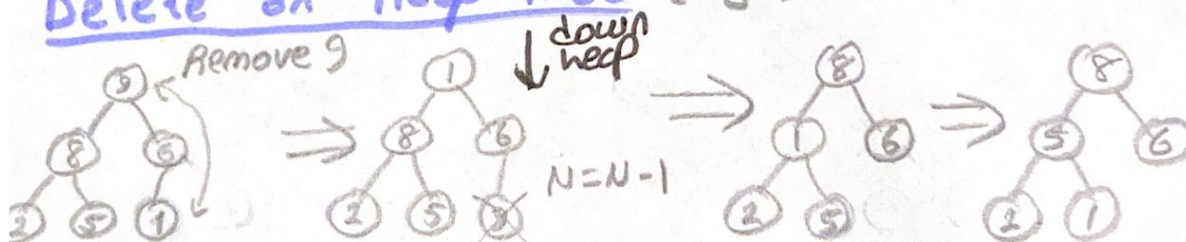
complexity of build_max_heap $\rightarrow O(n)$

Insert on Heap Tree $O(\log n)$



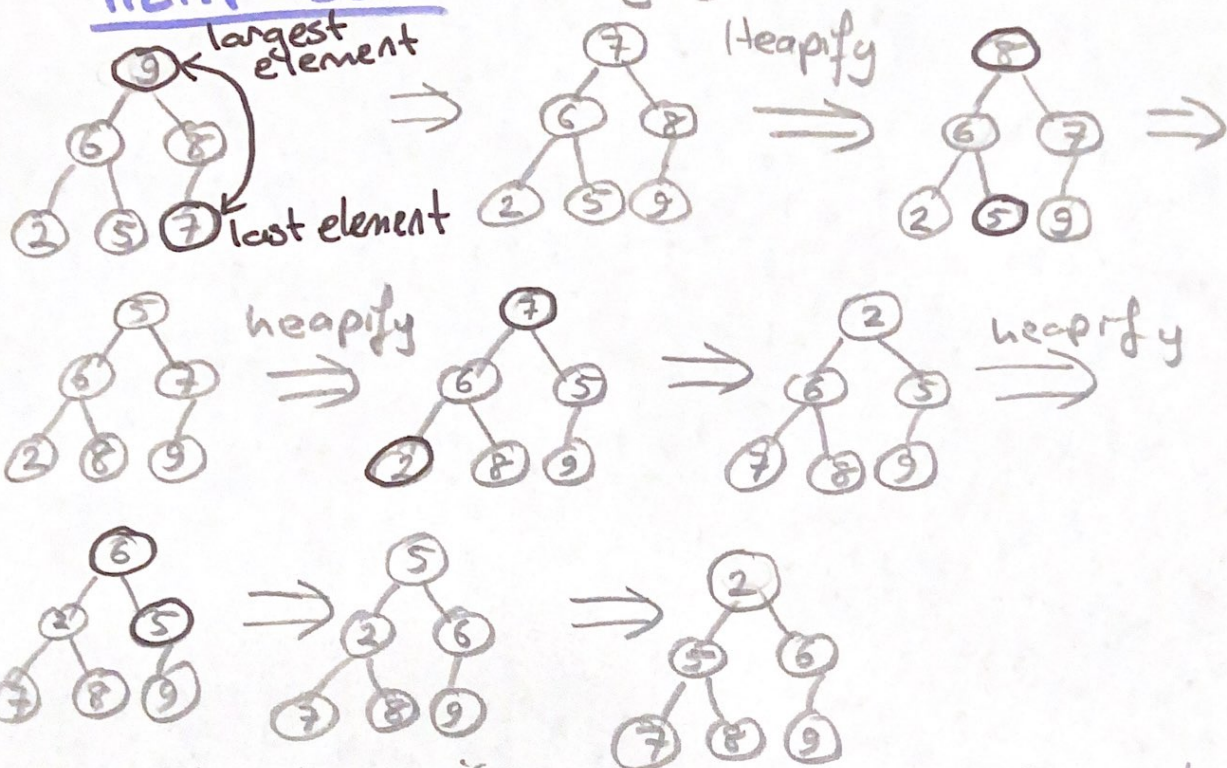
we add the value to the end and heapify...
bottom \rightarrow up: we only check whether it is greater than parent

Delete on Heap Tree $O(\log n)$



top \rightarrow down: we check both child and max one is switched with parent
we switched last element with it and we reduce the size by one

HEAP SORT $O(N \log N)$



α Switched element is firstly N (last element) and every step it decreases. We always switch it with largest element

```
void maxHeapify (int array[], int i, int N) {
    left = 2*i; // since index starts with 1
    right = 2*i + 1;
    if (left ≤ N && array[left] > array[i])
        largest = left;
    else
        largest = i;
    if (right ≤ N && array[right] > array[largest])
        largest = right;
    if (largest != i) {
        swap(array[i], array[largest]);
        maxHeapify (array, largest, N);
    }
}
```