Asymmetric Generalized Gaussian Mixture Models and EM algorithm for Image Segmentation

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***Abstract*–In this paper, a parametric and unsupervised histogram-based image segmentation method is presented. The histogram is assumed to be a mixture of asymmetric generalized Gaussian distributions. The mixture parameters are estimated by using the Expectation Maximization algorithm. Histogram fitting and region uniformity measures on synthetic and real images reveal the effectiveness of the proposed model compared to the generalized Gaussian mixture model.**

Keywords- AGGMM; EM algorithm; histogram fitting; image segmentation

I. INTRODUCTION

In an image analysis system, the segmentation constitutes one of the most significant problems because the result, obtained at the end of this step, governs strongly the final quality of interpretation [1].

Many segmentation methods are quoted in the literature among which some are based on parametric approaches where the classes defining the various image partitions are, separated by:

(1) considering a statistical model to approach the class probability density functions *pdf* or by

(2) determining the threshold optimal values as functions of the class parameters (case of multi-thresholding) or as estimates of the weighted class *pdf*s computed according to the considered model parameters (case of the mixture models).

The major part of the parametric approaches considers mixture models using the Gaussian distribution (GMM) for their simplicity of computation. However, a normalized histogram or *pdf* of a real image may be inherently non-Gaussian or even a mix of several distributions, so using a GMM with a finite number of modes cannot portray the data accurately. In [2,3,4,5], the authors use the generalized Gaussian mixture models (GGMM) for the image segmentation task where, in fact, the results are improved compared to GMM but remains insufficient because the asymmetry aspect often included in the image histogram is not taken into account. To this end and to overcome the limitations presented by such models, we propose a mixture model based on the asymmetric generalized Gaussian distribution [6] (AGGMM) as an alternative model so that the resulted image partitions match as best as possible with the various semantic components contained in the real image. Indeed, this model can describe in the best possible way the class statistical behaviour in the image because the asymmetric generalized Gaussian distribution (AGGD) not only can approximate a large class of statistical distributions (e.g. impulsive, Laplacian, Gaussian and uniform distributions) but also include the asymmetry. The modelling capacity confers to this model more flexibility compared to GGMM. In this paper, the Expectation Maximization (EM) algorithm is used to estimate the AGGMM parameters. In experiments, region uniformity [7] and histogram fitting measures are used to compare the proposed model results with those of GGMM.

II. MIXTURE MODEL AND EM ALGORITHM

Let $X = \{x_i\}$, $i = 1,\ldots, n$ be a set of n realizations of a random d-dimensional vector χ with a *pdf* $f(x_i)$. Thus the parameterized *pdf* can be written as a combination of *pdf*s of the M components C_m ($m = 1,\ldots,M$) characterizing the finite mixture model :

$$f(x_i \mid \Theta) = \sum_{m=1}^{M} \pi_m f_m(x_i \mid \theta_m) \tag{1}$$

where $\Theta=(\pi_m,\theta_m)$ is the vector of parameters to estimate with π_m the prior probability of the m^{th} component which satisfies : $\pi_m \geq 0$ and $\sum_{m=1}^{M}\pi_m = 1$. In presence of iid observations, the likelihood can be expressed as:

$$L(\Theta) = \prod_{i=1}^{n} f(x_i \mid \Theta) = \prod_{i=1}^{n}\sum_{m=1}^{M} \pi_m f_m(x_i \mid \theta_m) \tag{2}$$

Consider the vector X as a partial observation of the considered phenomenon, then the maximization of $L(\Theta)$ is difficult to perform directly. We introduce a random variable Z corresponding to the incomplete data such as $z_i =$

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$(z_{i,1},\ldots,z_{i,M})$ where $z_{i,M} = 1$ if $x_i \in C_m$ and $z_{i,M} = 0$ elsewhere. Let $Y = (X,Z)$ the complete data. Thus, EM algorithm consists of maximizing likelihood law in presence of incomplete data by maximizing iteratively the expectation of the complete log-likelihood given by:

$$\log L_c(\Theta/Y) = \sum_{i=1}^{n}\sum_{m=1}^{M} z_{i,m} \log[\pi_m f_m(x_i/\theta_m)] \tag{3}$$

Hereafter $L_c(\Theta)$ indicates $\log L_c(\Theta/Y)$. The parameters Θ are estimated by using expectation and maximization steps. E-step computes the $L_c(\Theta)$ expectation $Q(\Theta/\Theta^{(t)}) = \mathrm{E}[L_c(\Theta),\Theta^{(t)}]$ by using the posterior probability $z_{i,m}^{(t)}$ which depends on the current parameters $\Theta^{(t)}$. After, M-step comes to maximize the Q function in relation to Θ to estimate the new model parameter values $\Theta^{(t+1)}$. These steps are repeated iteratively until a convergence criterion is reached.

III. AGGMM FOR SEGMENTATION

Let h_g ($g\in[0,L-1]$ where L is the number of gray levels) the normalized histogram of an image X which can be seen as an estimate of the true *pdf* $f(g)$ of the image. In this work, the gray level histogram will be estimated by a mixture of univariate AGGDs which have as analytical expression

$$f_m(g/\theta_m) = \begin{cases} \dfrac{\beta_m}{(\alpha_{1,m}+\alpha_{2,m})\Gamma(1/\beta_m)} e^{-[(-g+\mu_m)/\alpha_{1,m}]^{\beta_m}} & \text{if } g < \mu_m \\ \dfrac{\beta_m}{(\alpha_{1,m}+\alpha_{2,m})\Gamma(1/\beta_m)} e^{-[(g-\mu_m)/\alpha_{2,m}]^{\beta_m}} & \text{if } g \geq \mu_m \end{cases} \tag{4}$$

where $\theta_m = (\mu_m,\alpha_{m,1},\alpha_{m,2},\beta_m)$ represents the vector where the components are: pseudo-mean, left and right scale parameters and shape parameter, respectively; Γ is the gamma function defined by $\Gamma(\xi) = \int_0^{\infty} e^{-t} t^{\xi-1} dt$.

Fig. 1 shows an example of mixture of three AGGDs.

The scale parameters which express the width of the distribution are linked to standard deviations by

$$\alpha_{i,m} = \sigma_{i,m}\sqrt{\Gamma(1/\beta_m)/\Gamma(3/\beta_m)}\,,\; i = 1,2 \tag{5}$$

For AGGMM, the complete log-likelihood is given by

$$L_c(\Theta) = \sum_{g=0}^{L-1}\sum_{m=1}^{M} z_{g,m} h_g \log[\pi_m f_m(g/\theta_m)] \tag{6}$$

where $\Theta = (\pi_m,\mu_m,\alpha_{1,m},\alpha_{2,m},\beta_m)\,;\, m = 1,\cdots,M$.

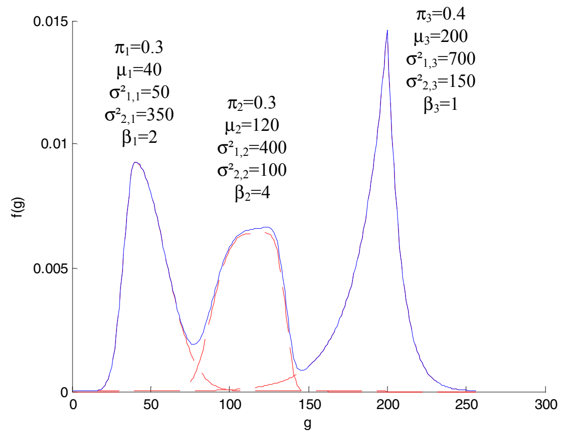

Figure 1. Mixture of AGGDs with different parameters.

In E-Step, the posterior probabilities are given by

$$z_{g,m}(t) = \pi_m f_m(g/\theta_m(t)) \Big/ \sum_{l=1}^{M} \pi_l f_l(g/\theta_l(t)) \tag{7}$$

In M-Step, the estimation of Θ implies:

$$\pi_m(t+1) = \sum_g z_{g,m}(t) h_g \tag{8}$$

$$\begin{aligned} &\frac{1}{\alpha_{1,m}^{\beta_m(t)}(t)} \sum_{g<\mu_m(t)} z_{g,m}(t) h_g (-g+\mu_m(t+1))^{\beta_m(t)-1} \\ &-\frac{1}{\alpha_{2,m}^{\beta_m(t)}(t)} \sum_{g\geq\mu_m(t)} z_{g,m}(t) h_g (g-\mu_m(t+1))^{\beta_m(t)-1} = 0 \end{aligned} \tag{9}$$

$$\alpha_{1,m}^{\beta_m(t)+1}(t+1) - K_1\alpha_{1,m}(t+1) - K_1\alpha_{1,m}(t+1) = 0 \tag{10}$$

where $K_1 = \dfrac{\beta_m(t) \sum_{g<\mu_m(t)} z_{g,m}(t) h_g (-g+\mu_m(t))^{\beta_m(t)}}{\sum_g z_{g,m}(t) h_g}$

$$\alpha_{2,m}^{\beta_m(t)+1}(t+1) - K_2\alpha_{2,m}(t+1) - K_2\alpha_{2,m}(t+1) = 0 \tag{11}$$

where $K_2 = \dfrac{\beta_m(t) \sum_{g\geq\mu_m(t)} z_{g,m}(t) h_g (g-\mu_m(t))^{\beta_m(t)}}{\sum_g z_{g,m}(t) h_g}$

$$\sum_{g} z_{g,m}(t)h_g\left[\frac{1}{\beta_m(t+1)}+\frac{\Psi(1/\beta_m(t+1))}{\beta_m^2(t+1)}\right]$$
$$-\sum_{g<\mu_m(t)} z_{g,m}(t)h_g\left(\frac{(-g+\mu_m(t))}{\alpha_{1,m}(t)}\right)^{\beta_m(t+1)}\log\left(\frac{(-g+\mu_m(t))}{\alpha_{1,m}(t)}\right) \quad (12)$$
$$-\sum_{g\geq\mu_m(t)} z_{g,m}(t)h_g\left(\frac{(g-\mu_m(t))}{\alpha_{2,m}(t)}\right)^{\beta_m(t+1)}\log\left(\frac{(g-\mu_m(t))}{\alpha_{2,m}(t)}\right)=0$$

where $\Psi(\xi)=\partial\log\Gamma(\xi)/\partial\xi$. We remark that the equations from (9) to (12) related to all AGGD parameters are non linear. They can be solved by numeric methods such as Newton-Raphson method.

To map the segmented image, the optimal realization $S(u,v)$ for each pixel with (u,v) coordinates is assigned according to the Bayes decision rule

$$S(u,v)=\begin{cases}\omega_1(\mathrm{t_f}) \text{ if } p_1(t_f)=\max_m\{p_1(t_f):m=1,\cdots,M\}\\ \vdots \\ \omega_M(\mathrm{t_f}) \text{ if } p_M(t_f)=\max_m\{p_M(t_f):m=1,\cdots,M\}\end{cases} \quad (13)$$

where $p_m(t_f)=\pi_m f_m(g\mid\theta_m(t_f))$ and $\omega_m(t_f)$ is the final estimated real mean parameter which can be given by

$$\omega_m(t+1)=\sum_g z_{g,m}(t)h_g g\Big/\sum_g z_{g,m}(t)h_g \quad (14)$$

IV. Experiments

In order to assess the effectiveness of the proposed model in histogram fitting and image segmentation, one simulated image (S) and two real images (R1, R2) are used as shown in Fig. 2. S consists of three image classes representing two concentric rings and one background. All classes are corrupted by asymmetric generalized Gaussian noise. For R1, the number of mixture modes is fixed to M = 4 which corresponds to the number of segmented regions expected by the user; whilst R2 represents a magnetic resonance image (MRI) of a brain consisting, after background removal, of three modes (white matter (WM), gray matter (GM) and cerebrospinal fluid (CSF)). For EM initialization, all initial prior probabilities $\pi_m(0)$ are taken equal to $1/M$; the pseudo-means $\mu_m(0)$ are chosen so that $\mu_m(0)=H^{-1}(0.05)+m\times\left(H^{-1}(0.95)-H^{-1}(0.05)\right)/(M+1)$ where H denotes the cumulated histogram; whilst $\sigma_{1,m}^2(0)=\sigma_{2,m}^2(0)=(\mu_2(0)-\mu_1(0))^2/4$ and all initial shape parameters $\beta_m(0)$ are taken equal to 1.2.

A sum of squared error (SSE) is used to measure the histogram curve fitting

$$SSE=\sum_g\left(h(g)-f\left(g,\hat{\Theta}\right)\right)^2 \quad (15)$$

A region uniformity measure U is used to evaluate the segmentation performance method. It is given by

$$U=1-\sum_{m=1}^{M}\pi_m\sigma_m^2/\sigma_T^2 \quad (16)$$

where π_m is the area ratio of the m^{th} segmented region and σ_m^2 its variance, whilst σ_T^2 is the total image variance. The highest (near to 1) is the value of U, the highest is the segmentation quality.

The synthetic image, the original and segmented real images and their fitted histograms based GGMM and AGGMM are illustrated in Fig. 2. The estimated parameters, the uniformity measures and the histogram fitting errors are reported in Table 1. From the simulated image, we try to show the limitation of GGMM to fit an AGGD noised image histogram. The asymmetry characterizing the histogram of this latter can occur in many real image histograms. We can visually note that the estimated histogram given by GGMM is not able to match the original mixture *pdf* especially at the third mode. From Table 1, we can see that the SSE for AGGMM is the least and the estimated parameters such as the means are the nearest to those of the designed model. The recovering of the best mode mean value for a model is important in this work because it is used to generate the segmented image as it is explained at the end of Section 3 and as it will be established in the following. The tested real images do not include prior information on the histogram shape. Since the reference segmented map is not provided, a comparison between the used models will be performed by an unsupervised segmentation evaluation U and the histogram fitting error SSE. As it can be seen, the considered measures for AGGMM give the best results that can be reinforced by visual examination. The limited capabilities of GGMM to recover accurately the *pdf* modes for both images can also be noted. For the "polyhydric" image, the right side of the AGGMM-segmented image of the middle polyhedron is well separated from the right polyhedron, conversely to the GGMM-segmented one. Also, for the brain MRI the WR, GM and CSF proportions are in reasonable agreement with the original image in the AGGMM-segmented image compared to the GGMM-segmented one.

V. Conclusion

Due to the flexibility of the AGGD to model large class of statistical behaviours, we try to show the strength of the proposed mixture model to overcome the problem of asymmetry often encountered in real image histograms.

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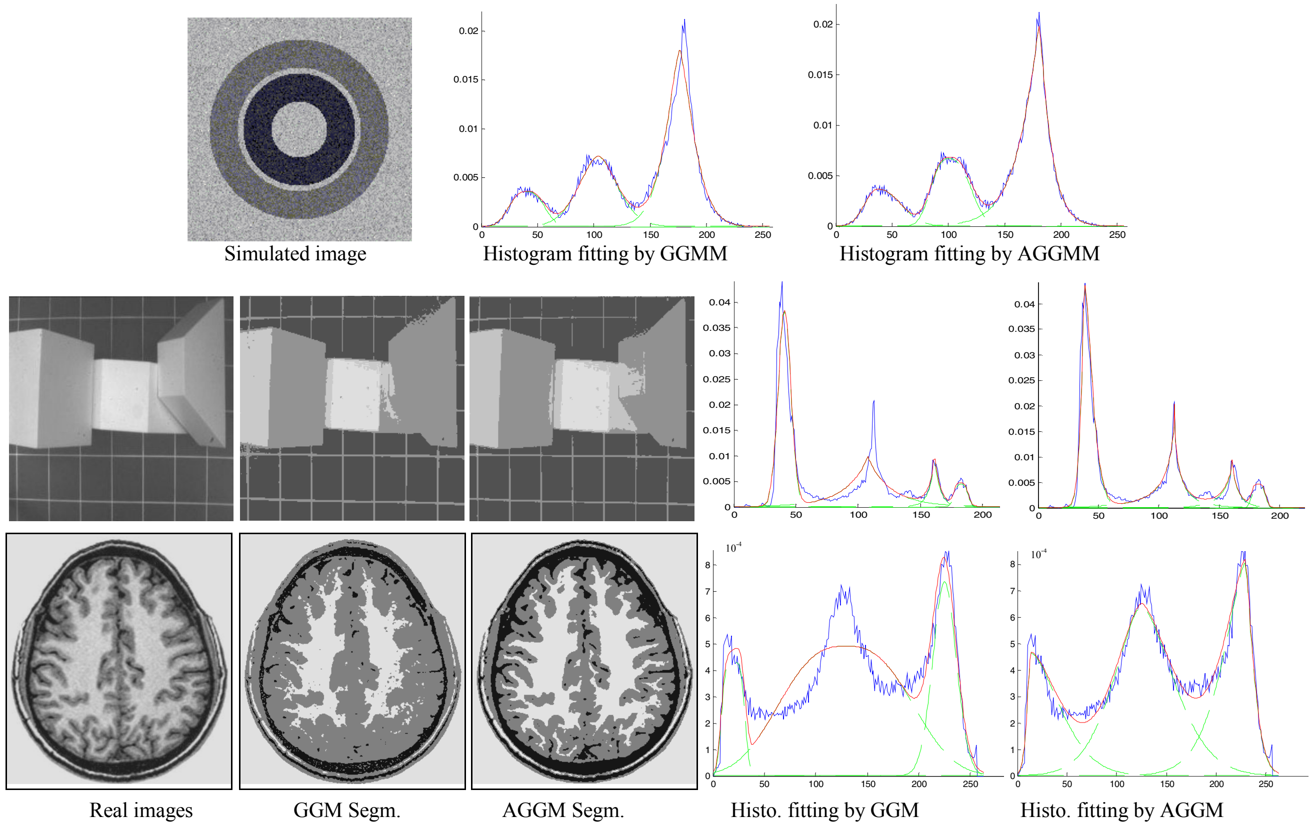

Figure 2. Histogrm curve fitting and image segmentation results

TABLE I. ESTIMATED PARAMETERS AND SEGMENTATION EVALUATION SCORES FOR SIMULATED AND REAL IMAGES

		π	μ	ω	α or α_1	α_2	β	SSE ($\times 10^{-4}$)	U
S	Design. Model	{0.15 0.26 0.59}	{35 100 180}	{43.9 102.7 173.6}	{15 20 18}	{30 25 10}	{2 3 1}	—	—
	GGM	{0.13 0.31 0.56}	{39.5 102.9 175.3}	{39.5 102.9 175.3}	{19.7 24.2 **17.1**}	—	{2.6 1.7 1.4}	2.14	—
	AGGM	**{0.14 0.26 0.60}**	**{36.7 99.1 180.1}**	**{42.8 102.0 172.7}**	**{16.3 19.2** 19.9}	{27.6 24.7 11.2}	**{2.1 2.6 1.1}**	**0.28**	—
R1	GGM	{0.49 0.37 0.09 0.05}	{80.1 147.0 200.6 221.9}	{80.1 147.0 200.6 221.9}	{7.3 18.3 4.3 6.8}	—	{2 0.9 0.9 3.2}	15	0.914
	AGGM	{0.51 0.31 0.13 0.05}	{78.2 152.1 200.4 222.6}	{80.7 145.4 196.1 221.9}	{4.3 6.3 6.9 7.1}	{8.7 4.0 4.5 6.6}	{1.5 0.6 0.8 3.2}	**6.27**	**0.943**
R2	GGM	{0.10 0.69 0.21}	{18.9 128.3 224.1}	{18.9 128.3 224.1}	{12.9 78.9 16.2}	—	{3.9 2.7 1.9}	1.88	0.836
	AGGM	{0.19 0.49 0.32}	{13.1 124.3 228.2}	{34.7 128.0 214.3}	{5.5 39.4 31.9}	{40.9 45.6 11.6}	{1.6 1.6 1.4}	**0.51**	**0.907**