Formula Booklet - Essentials

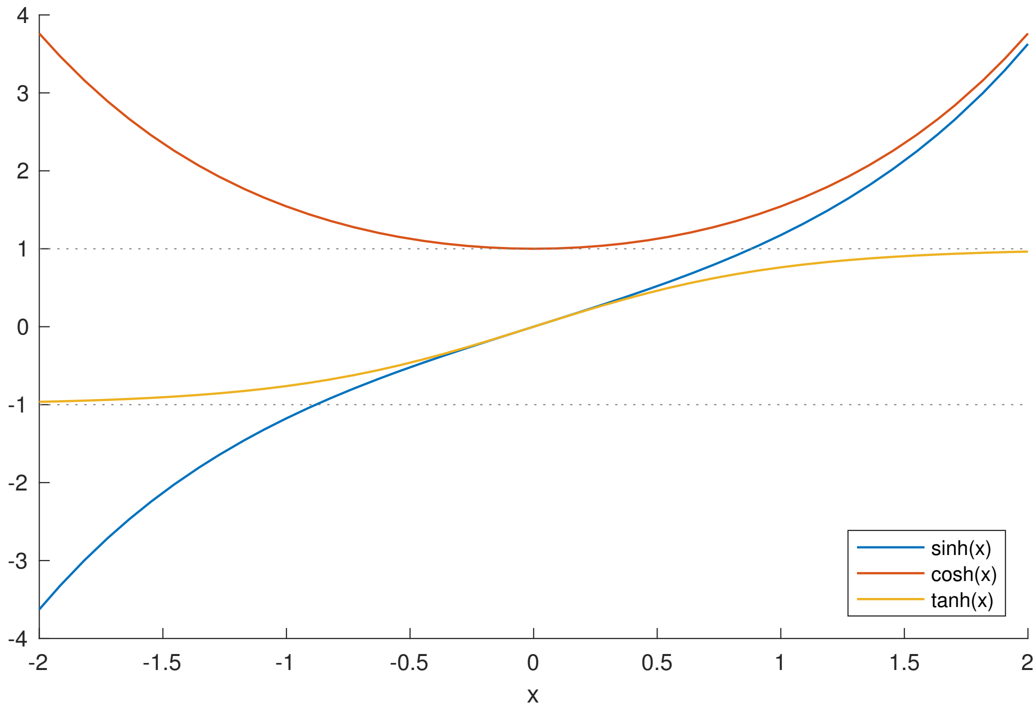
Trigonometric

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$e^{ix} = \cos(x) + i\sin(x)$$
$$\cos x = \frac{e^{ix} + e^{-ix}}{2}, \qquad \sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

Hyperbolic

$$\sinh(A \pm B) = \sinh A \cosh B \pm \cosh A \sinh B$$
$$\cosh(A \pm B) = \cosh A \cosh B \pm \sinh A \sinh B$$
$$\cosh x = \frac{e^{x} + e^{-x}}{2}, \qquad \sinh x = \frac{e^{x} - e^{-x}}{2}$$



Quotient rule

$$f(x) = \frac{u(x)}{v(x)} \quad \Rightarrow \quad f'(x) = \frac{vu' - uv'}{v^2}$$

Integrating factor

$$\frac{dy}{dx} + P(x)y = R(x,y) \quad \Rightarrow \quad I(x) = \exp\left(\int P(x)dx\right)$$

Taylor Series

$$\begin{aligned} f(x) &= \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \\ &= f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \cdots \end{aligned}$$

Residue at n-th order pole at $k = k_0$

$$Res(f, k_0) = \frac{1}{(n-1)!} \lim_{k \to k_0} \frac{d^{n-1}}{dk^{n-1}} \Big((k-k_0)^n f(k) \Big)$$

so for a simple-pole (n=1), we have

$$Res(f, k_0) = \lim_{k \to k_0} (k-k_0) f(k)$$

Contour integration (with n-th order pole at $k = k_0$)

$$\int_C f(x)dx = \left\{ \begin{array}{ll} 2\pi i \times Res(f, k_0) & \text{if } C \text{ counter-clockwise} \\ -2\pi i \times Res(f, k_0) & \text{if } C \text{ clockwise} \end{array} \right.$$

Integrals

$$\begin{aligned} \int \frac{a}{a^2+x^2} dx &= \tan^{-1}\left(\frac{x}{a}\right) \\ \int \frac{1}{\sin x} dx &= \log \left| \tan\left(\frac{x}{2}\right) \right| \\ \int \frac{1}{\cos x} dx &= \log \left| \sec(x) + \tan(x) \right| \end{aligned}$$

Heaviside function

$$H(x) = \left\{ \begin{array}{ll} 1 & \text{if } x > 0, \\ 0 & \text{if } x < 0. \end{array} \right.$$

Dirac delta function

$$\delta(x) = \left\{ \begin{array}{ll} \infty & \text{if } x = 0, \\ 0 & \text{if } x \neq 0. \end{array} \right. \qquad \text{where} \qquad \int_{-\infty}^{\infty} \delta(x)dx = 1$$