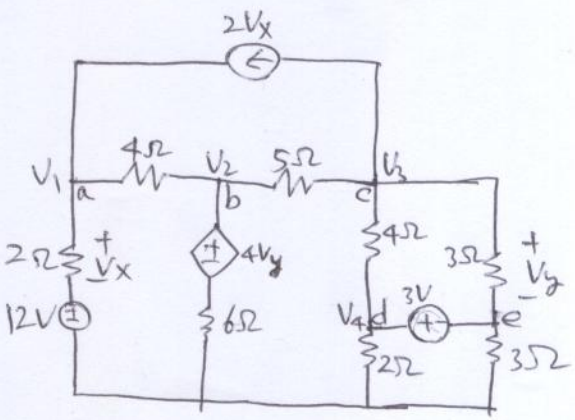
2012년 1학기 중간고사 풀이

[1]



$V_x = V_1 - 12$

$V_y = V_3 - (V_4 - 3)$

Applying KCL,

node a: $\frac{V_1-12}{2} + \frac{V_1-V_2}{4} - 2V_x = 0$

node b: $\frac{V_2-V_1}{4} + \frac{V_2-V_3}{5} + \frac{V_2-4V_y}{6} = 0$

node c: $2V_x + \frac{V_3-V_2}{5} + \frac{V_3-V_4}{4} + \frac{V_y}{3} = 0$

supernode d-e: $\frac{V_4-V_3}{4} + \frac{V_4}{2} + \frac{V_4-3}{3} + \frac{V_4-3-V_3}{3} = 0$

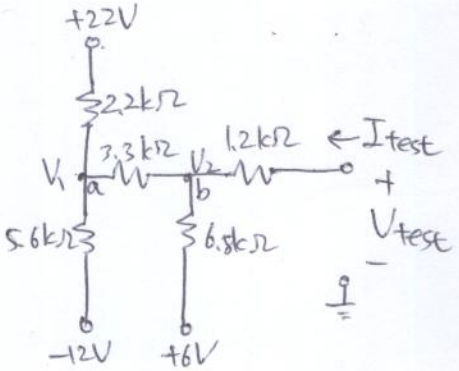
$\Rightarrow V_1 = \frac{693}{26}\ V$

$V_2 = -\frac{1593}{26}\ V$

$V_3 = -\frac{999}{13}\ V$

$V_4 = -\frac{393}{13}\ V$

[2] (a) Applying KCL,



node a: $\frac{V_1-22}{2.2}+\frac{V_1+12}{5.6}+\frac{V_1-V_2}{3.3}=0$

node b: $\frac{V_2-6}{6.8}+\frac{V_2-V_1}{3.3}+\frac{V_2-V_{test}}{1.2}=0$

$$I_{test}=\frac{V_{test}-V_2}{1.2}$$

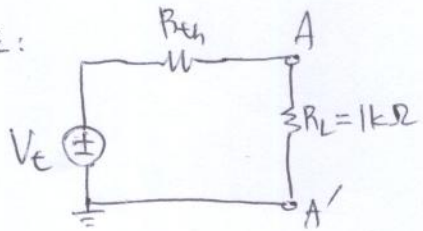
V_1, V_2를 소거하여 정리하면,

$$I_{test}=\frac{V_{test}-\frac{3510936}{360756}}{\frac{18222336}{4509450}}$$

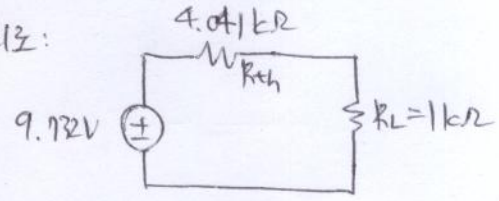
$\therefore$ $V_t=\frac{3510936}{360756}\simeq 9.732\,V$

$R_{th}=\frac{18222336}{4509450}\simeq 4.041\,k\Omega$

등가회로:



(b) 등가회로:



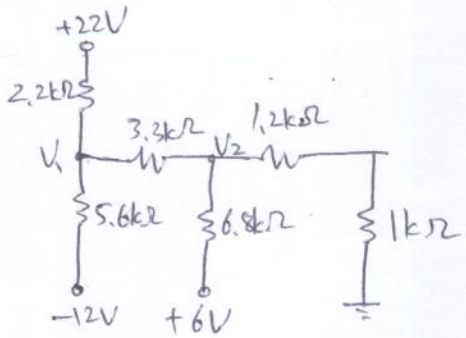
R_{th}가 소모하는 전력

$$P_{R_{th}}=i^2R_{th}$$

$$=\left(\frac{9.732}{(4.041+1)\times 1000}\right)^2\cdot 4.041\times 1000\ [W]$$

$$\doteq 15.06\ mW$$

(c) Applying KCL,



$$\frac{V_1-22}{2.2}+\frac{V_1+12}{5.6}+\frac{V_1-V_2}{3.3}=0.$$

$$\frac{V_2-V_1}{3.3}+\frac{V_2-6}{6.8}+\frac{V_2}{2.2}=0.$$

$$\Rightarrow V_1=\frac{1758145848}{179791443?}\simeq 9.771\ V$$

$$V_2=\frac{64367160}{15116464}\simeq 4.258V.$$

$$P=P_{2.2k\Omega}+P_{3.3k\Omega}+P_{5.6k\Omega}+P_{6.8k\Omega}+P_{1.2k\Omega}$$

$$=\frac{(9.771-22)^2}{2.2}+\frac{(9.771-4.258)^2}{3.3}+\frac{(9.771+12)^2}{5.6}+\frac{(4.258-6)^2}{6.8}+\frac{\left(\frac{1.2}{2.2}\cdot 4.258\right)^2}{1.2}$$

$$\doteqdot 67.98+9.21+84.64+0.45+4.49$$

$$\doteqdot 166.76\ [mW]$$

[3] (a) 회로망의 방정식 세우기 $V_b = V_x$

① $\frac{V_a - V_b}{3} + \frac{V_a - V_c}{2} + \frac{V_a - 3V_x}{2} + \frac{V_a - 3V_x - (V_c - 10)}{4} = 0$] → supernode 사용

② $\frac{V_c - V_a}{2} + \frac{V_c - V_b}{1} + \frac{V_c - 10 - (V_a - 3V_b)}{4} - 6 = 0$

③ $\frac{V_b - V_a}{3} + \frac{V_b - V_c}{1} + \frac{V_b}{1} = 0$

각 식당 2점 x3 = 6점

(b) 전압 V_a, V_b, V_c

① x 12 → $19V_a - 31V_b - 9V_c = -30$

② x 4 → $-3V_a - V_b + 7V_c = 34$

③ x 3 → $-V_a + 7V_b - 3V_c = 0$

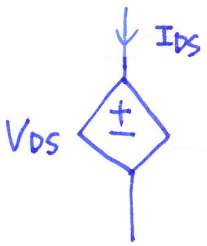
$$\begin{bmatrix} 19 & -31 & -9 \\ -3 & -1 & 7 \\ -1 & 7 & -3 \end{bmatrix} \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} -30 \\ 34 \\ 0 \end{bmatrix}$$

$V_a = 21.8V$ $V_b = 9.8V$ $V_c = 15.6V$

각 전압당 3점, 단위 안쓰면 1점씩 감점.

(a)의 회로망의 방정식을 잘못 세워서 전압값이 틀리면 1점

(c) V_a에서 아래로 향하는 전류: I_{DS}



$\frac{V_a - V_c}{2} + I_{DS} + \frac{V_a - V_b}{3} = 0$

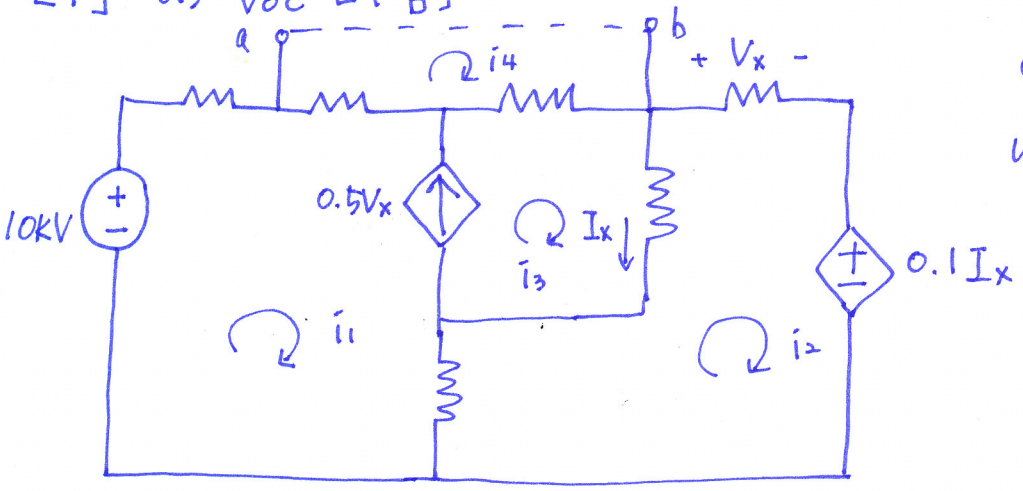
$I_{DS} = -\frac{V_a - V_c}{2} - \frac{V_a - V_b}{3} = -7.1A$ → 2점 (식은 맞는데 계산이 답이 틀리면 1점 감점)

$P = -V_{DS} \cdot I_{DS} = -(3V_b)(-7.1) = 21.3V_b = 208.74W$ → 5점

부호 틀리면 2점 감점

$V_{DS} = 3V_b$ 가 위에서 틀린경우 1점 감점

[4] a) V_{oc} [기점]



equivalent circuit using source transformation (all of R : 1kΩ)

$I_x = i_3 - i_2$

$V_x = 1000\, i_2$

loop 1 (super mesh)

$$2000\, i_1 + 1000\, i_3 + 1000(i_3 - i_2) + 1000(i_1 - i_2) = 10^4$$

$$\Rightarrow\ 3i_1 - 2i_2 + 2i_3 = 10 \quad \cdots ①$$

loop 2

$$1000(i_2 - i_1) + 1000(i_2 - i_3) + 1000\, i_2 + 0.1(i_3 - i_2) = 0$$

$$\Rightarrow\ 10000\, i_1 - 29999\, i_2 + 9999\, i_3 = 0 \quad \cdots ②$$

$$i_3 - i_1 = 0.5\, V_x = 500\, i_2 \ \Rightarrow\ i_1 + 500\, i_2 - i_3 = 0 \quad \cdots ③$$

solve (①②③) $i_1 = 10.1657\,A$ $i_2 = -0.0409\,A$ $i_3 = -10.2894\,A$

$$V_{oc} = 1000(i_1 + i_3) = -123.75\,[V]$$

b) I_{sc} [계산]

$$3i_1 - 2i_2 + 2i_3 - 2i_4 = 10$$

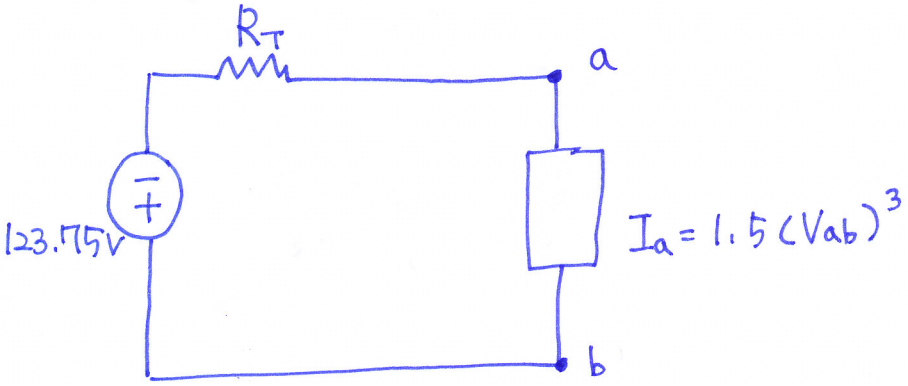
$$10000\, i_1 - 29999\, i_2 + 9999\, i_3 = 0$$

$$i_1 + 500\, i_2 - i_3 = 0$$

$$i_1 + i_3 - 2i_4 = 0$$

solve $I_{sc} = -61$ [mA]

c) [계산]



$$R_T = \frac{V_{oc}}{I_{sc}} = 2.0248\,k\Omega$$

KCL on a

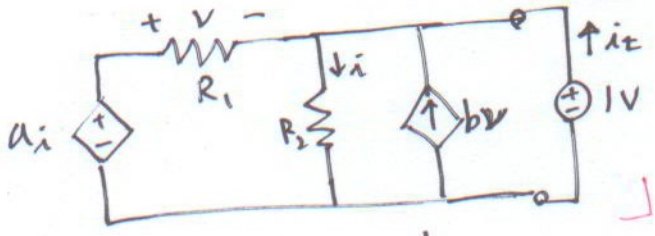
$$\frac{V_{ab} + 123.75}{2.0248\,k\Omega} + 1.5(V_{ab})^3 = 0$$

$$V_{ab} = -334.18\ [mV]$$

$$I_a = -55.98\ [mA]$$

[5] -(1) (total 8pt)

$V_{oc} = V_{th} = 0V$ (∵ No independent source) ⌋ 1 pt



⌋ 2pt

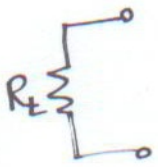
$iR_2 = 1V \Leftrightarrow i = \frac{1}{R_2}$

$V = ai - 1 = \frac{a}{R_2} - 1$

KCL; $\frac{V}{R_1} + i_t + bv = i$

$\therefore i_t = \frac{1}{R_2} - b\left(\frac{a}{R_2} - 1\right) - \frac{1}{R_1}\left(\frac{a}{R_2} - 1\right)$

$\therefore R_t = \frac{1}{i_t} = \frac{1}{1/R_2 - \frac{b(a-R_2)}{R_2} - \frac{(a/R_2 - 1)}{R_1}}$ ⌋ 1Pt ⌋ 3pt.



⌋ 1 pt

(Same result when 1A current source is used instead of 1V voltage source)

[5]-(2) (total 12pt)

i)

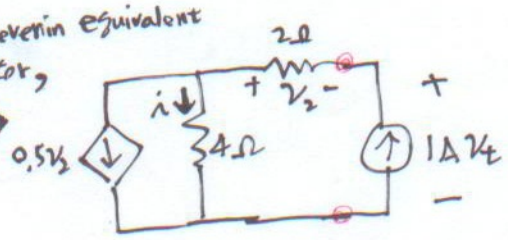
by using the result of [5]-(1),

$$R_t = \frac{1}{\frac{1}{20} - \frac{-10(20/20-1)}{1} - \frac{(20/20-1)}{10}} = 20\,\Omega$$

→ 2pt

ii)

to find Thevenin equivalent resistor,



$v_2 = -2V$

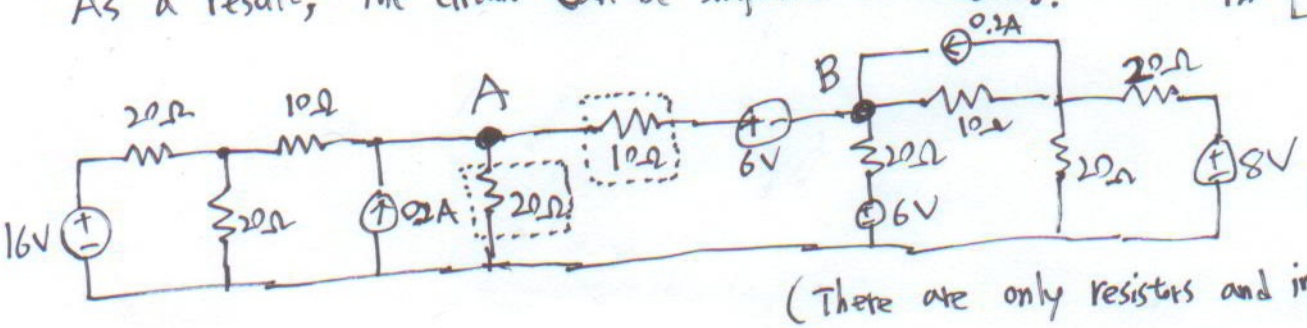
$i = 1A + 1A = 2A$

$\therefore v_t = 8V + 2V = 10V$

$\therefore R_t = \frac{V_t}{1A} = \boxed{10\,\Omega}$

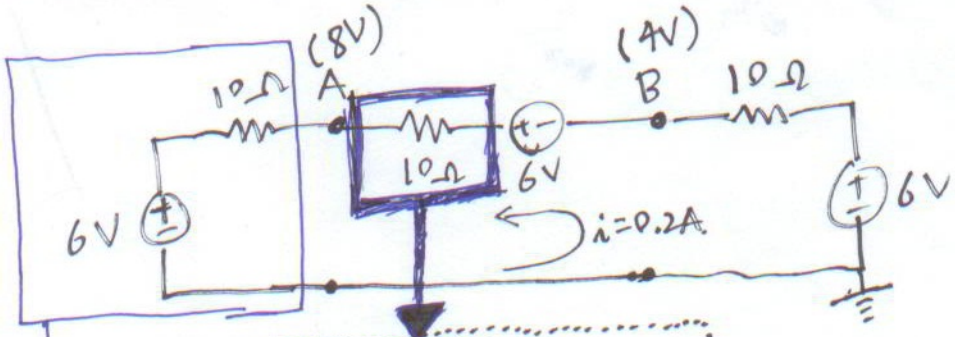
→ 2pt

As a result, The circuit can be simplified as following.

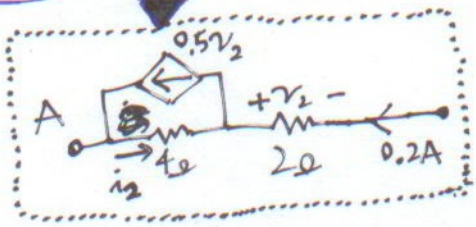


(There are only resistors and independant sources)

Using source transformation, we can simplify the circuit into the following circuit.



The current from node B to node A is 0.2A

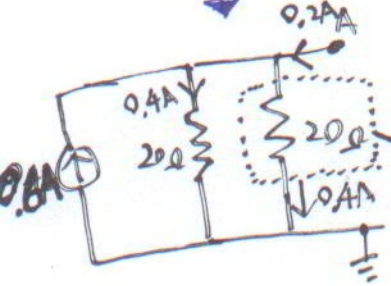


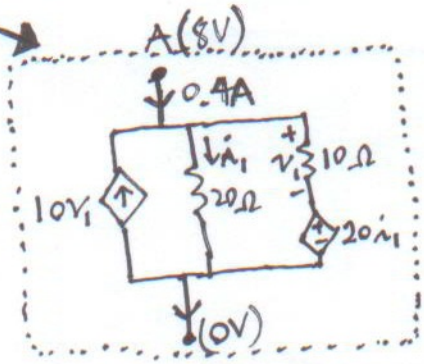
$v_2 = -0.4V$

$i_2 + 0.2 = 0.5v_2 = -0.2$

$\therefore i_2 = -0.4A$

→ 4pt





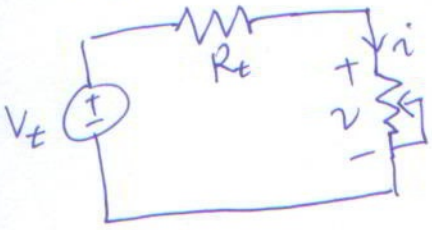
$20i_1 = 8V$

$\therefore i_1 = 0.4A$

→ 4pt

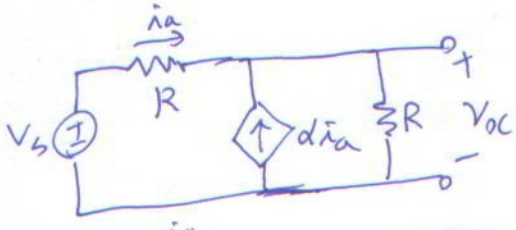
[6]-(a) (total 12pt)

Using Thevenin equivalent ckt,



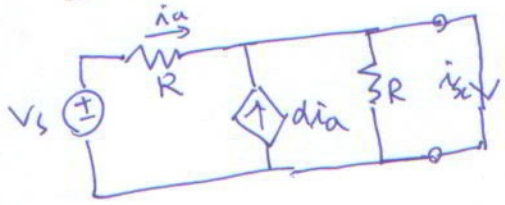
$v = V_t - iR_t$

$\therefore V_t = 5\,V$, $R_t = 625\,\Omega$ — 1pt, 1pt



$V_{oc} = V_t = R\,i_a(1+\alpha) = R(1+\alpha)\dfrac{(V_s - V_t)}{R} = (1+\alpha)(V_s - V_t)$ ····· ①

$\left(i_a = \dfrac{V_s - V_t}{R}\right)$ — 2pt



$i_{sc} = (\alpha+1)i_a = \dfrac{(\alpha+1)V_s}{R}$

$i_a = \dfrac{V_s}{R}$

$R_t = \dfrac{V_t}{i_{sc}} = \dfrac{R\,V_t}{(\alpha+1)V_s}$ ····· ② — 2pt

Combining ① and ②;

$$\begin{cases} V_s = \dfrac{V_t R}{R - R_t} = \dfrac{5R}{R-625} & \text{1pt} \\ \alpha = \dfrac{5}{V_s - 5} - 1 & \text{1pt} \end{cases}$$

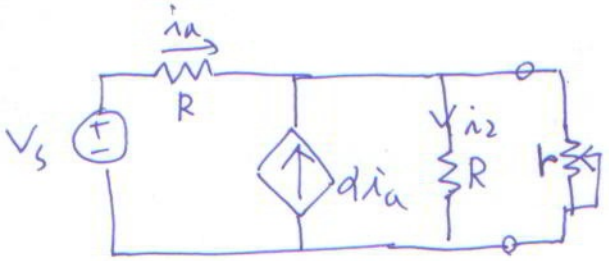
for student A, $R = 500\,\Omega$,

$$\therefore \begin{cases} V_s = -20\,V & \leftarrow 1\,pt \\ \alpha = -1.2\ A/A & \leftarrow 1\,pt \end{cases}$$

for student B, $R = 1k\Omega$,

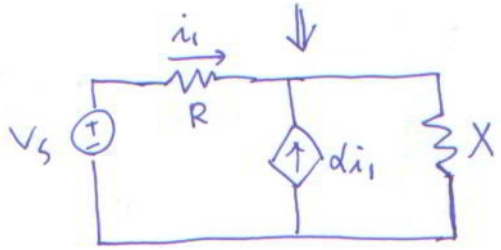
$$\therefore \begin{cases} V_s = 40/3\ V & \leftarrow 1\,pt \\ \alpha = -0.4\ A/A & \leftarrow 1\,pt \end{cases}$$

[6]-(b) (total 8pt)



r is variable resistor. $(0 \le r < \infty)$

Let $R \| r = X$ $\quad (0 \le X < R)$



$$i_2 = (\alpha+1) i_1 \frac{r}{R+r} = \frac{(\alpha+1) i_1 X}{R}$$

i_1 and i_2 must be located in the range of $-15mA \sim 15mA$

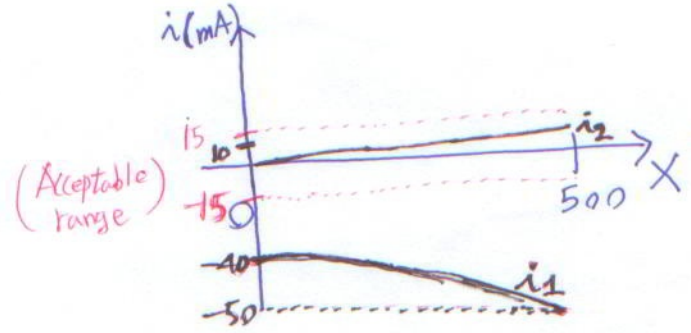
✡ (i.e; $-15mA < i_1 < 15mA$, $-15mA < i_2 < 15mA$)

KVL; $-V_s + i_1 R + (\alpha+1) i_1 X = 0$

$$\therefore i_1 = \frac{V_s}{(\alpha+1)X+R}, \quad i_2 = \frac{(\alpha+1) V_s X}{R[(\alpha+1)X+R]} \quad (0 \le X < R)$$

for student A, $R = 500\Omega$, $V_s = -20V$, $\alpha = -1.2$

$$\therefore i_1 = \frac{-20}{-0.2X+500} = \frac{100}{X-2500}, \quad i_2 = \frac{-0.04X}{X-2500} \quad (0 \le X < 500)$$



⇒ Student A's circuit is not working properly. because the current in the resistor exceeded acceptable range.

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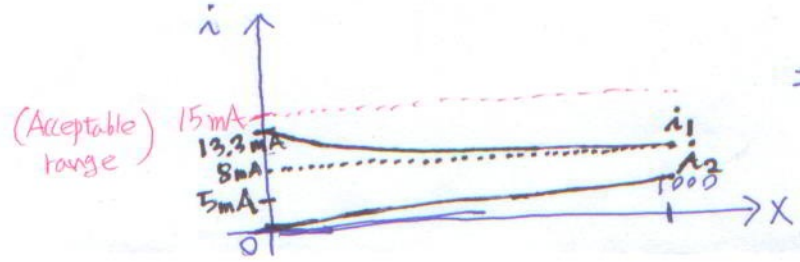
for student B, $R = 1k\Omega$, $V_s = 40/3\ V$, $\alpha = -0.4$

$$\therefore i_1 = \frac{40/3}{0.6X+1000}, \quad i_2 = \frac{8X}{1000(0.6X+1000)} \quad (0 \le X < 1000)$$



⇒ Student B's circuit is working normally because the currents in the resistor do not exceed acceptable range.