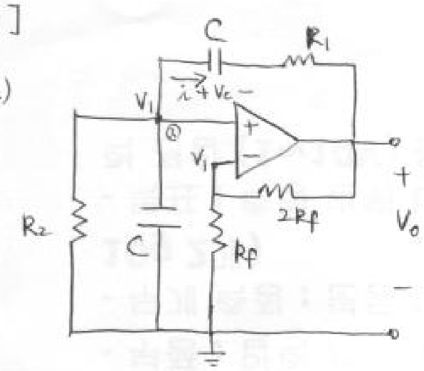
[1]

(a)



$$V_1 = \frac{R_f}{R_f + 2R_f} V_o = \frac{1}{3} V_o$$

$$V_1 - V_o = V_c + R_1 i$$

$$-\frac{2}{3} V_o = V_c + R_1 C \frac{dV_c}{dt} \quad \cdots ①$$

a node에서 KCL을 이용하면

$$\frac{\frac{1}{3}V_o}{R_2} + C\frac{d(\frac{V_o}{3})}{dt} + i = 0 \quad \cdots ②$$

$$i = \frac{\frac{1}{3}V_o - V_c - V_o}{R_1} = \frac{-\frac{2}{3}V_o - V_c}{R_1} \quad \cdots ③$$

③을 ② 식에 대입하면

$$\frac{V_o}{3R_2} + \frac{C}{3}\frac{dV_o}{dt} - \frac{\frac{2}{3}V_o + V_c}{R_1} = 0$$

$$V_c = \frac{R_1}{3R_2}V_o - \frac{2}{3}V_o + \frac{R_1 C}{3}\frac{dV_o}{dt} \quad \cdots ④$$

④ 식에 $-\frac{3}{2} \times$ ①을 대입하면

$$-\frac{2}{3}V_o = \frac{R_1}{3R_2}V_o - \frac{2}{3}V_o + \frac{R_1 C}{3}\frac{dV_o}{dt} + R_1 C\left(\frac{R_1}{3R_2}\frac{dV_o}{dt} - \frac{2}{3}\frac{dV_o}{dt} + \frac{R_1 C}{3}\frac{d^2V_o}{dt^2}\right)$$

정리하면

$$R_1 R_2 C^2 \frac{d^2V_o}{dt^2} + (R_1 C - R_2 C)\frac{dV_o}{dt} + V_o = 0$$

단진동할 조건은 $R_1 C - R_2 C = 0$

그러므로 $R_1 = R_2$

(b) $R_1 = R_2$ 이므로

$$R_1^2 C^2 \frac{d^2V_o}{dt^2} + V_o = 0$$

$$-R_1^2 C^2 w^2 + 1 = 0$$

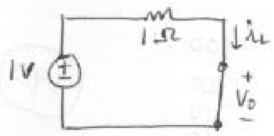
$$w^2 = \frac{1}{R_1^2 C^2} \qquad w = \frac{1}{RC} = \frac{1}{1\times10^3 \cdot \frac{1}{6.28}\times10^{-6}} = 6.28\times10^3$$

$$f = \frac{w}{2\pi} = 10^3 (Hz)$$

[2]

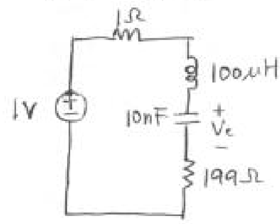
(a)

$t=0^-$ 일때



$i_L(0^-)=1A \quad V_0(0^-)=0V$

$t=0^+$ 일때



$i_L(0^+)=1A$

$V_0(0^+)=0V$

$V_0(0^+)=V_c(0^+)+199\cdot i_L(0^+)$

$0=V_c(0^+)+199$

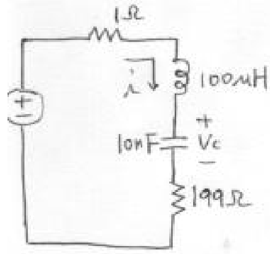
$V_c(0^+)=-199V$

$i_L(0^+)=C\cdot \frac{dV_c}{dt}\Big|_{0^+}$

$1=C\cdot\frac{dV_c}{dt}\Big|_{0^+}$

$\frac{dV_c}{dt}\Big|_{0^+}=\frac{1}{C}=\frac{1}{10\times10^{-9}}=10^8 (V/s)$

(b) $t>0$



$1=1\cdot i+L\frac{di}{dt}+V_c+199i$

$i=C\frac{dV_c}{dt}$ 를 대입.

$1=200\cdot C\frac{dV_c}{dt}+LC\frac{d^2V_c}{dt^2}+V_c$

$LC\frac{d^2V_c}{dt^2}+200\cdot C\frac{dV_c}{dt}+V_c=1$

$\frac{d^2V_c}{dt^2}+\frac{200}{L}\frac{dV_c}{dt}+\frac{1}{LC}V_c=\frac{1}{LC}$

$\frac{d^2V_c}{dt^2}+2\times10^6\frac{dV_c}{dt}+10^{12}V_c=10^{12}$

$s^2+2\times10^6 s+10^{12}=0$

$s=-10^6 \qquad V_{ch}=k_1e^{-10^6t}+k_2te^{-10^6t}, \quad V_{cp}=1 \qquad V_c=V_{ch}+V_{cp}$

$V_c=k_1e^{-10^6t}+k_2te^{-10^6t}+1$

초기조건 $V_c(0^+)=-199V, \ i_L(0^+)=1A$

$-199=k_1+1$

$k_1=-200$

$1=C\frac{dV_c}{dt}=C\left(k_1(-10^6)e^{-10^6t}+k_2e^{-10^6t}+k_2t(-10^6)e^{-10^6t}\right]$

$=C[k_1(-10^6)+k_2]$

$k_2=3\times10^8$

$\therefore V_c(t)=-200e^{-10^6t}+3\times10^8te^{-10^6t}+1 \ (V)$

(C) $V_o(t) = V_c(t) + 199 i$

$$= V_c(t) + 199 \cdot C \frac{dV_c}{dt}$$

$$V_o(t) = -200 e^{-10^6 t} + 3\times10^8 t\, e^{-10^6 t} + 1$$

$$+ 199 \cdot 10^{-8} (200 \times 10^6 e^{-10^6 t} + 3\times10^8 e^{-10^6 t} + 3\times10^8 \cdot t (-10^6) e^{-10^6 t})$$

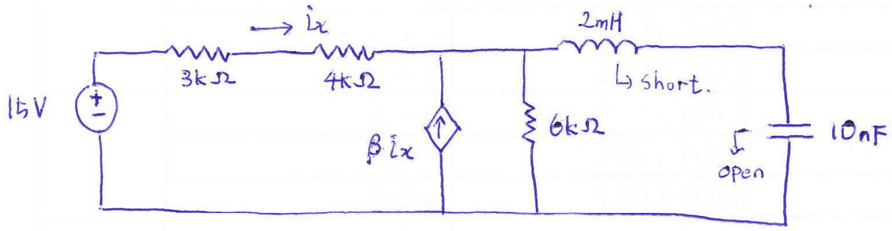
$$= -200 e^{-10^6 t} + 3\times10^8 t\, e^{-10^6 t} + 1 + e^{-10^6 t} \cdot 199 \cdot 10^{-8} (5\times10^8 - 3\times10^{14} t)$$

$$= -200 e^{-10^6 t} + 3\times10^8 t\, e^{-10^6 t} + 1 + 995 e^{-10^6 t} - 597\times10^6 \cdot t\, e^{-10^6 t}$$

$$= 795 e^{-10^6 t} - 297\times10^6 \cdot e^{-10^6 t} \cdot t + 1 \quad (V)$$

[3]

(1) for $t < 0$,



$$\Rightarrow 15 = 7000\, i_x + (1+\beta)\, 6000\, i_x \quad \Rightarrow i_x = \frac{15}{7000 + (1+\beta)\,6000}$$

$$\therefore \quad v_c(0) = \frac{15}{7000 + (1+\beta)6000} \cdot 6000 \ \text{[V]} .$$

$$i_L(0) = 0\,\text{[A]} .$$

* 총 3점.

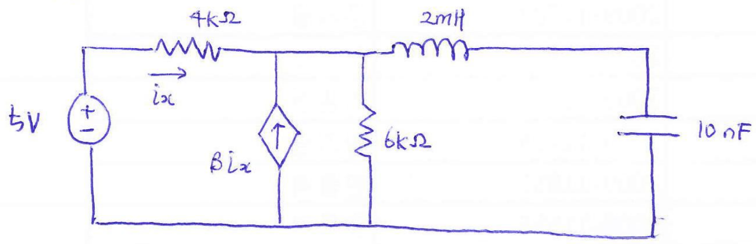
풀이과정 있고 답 없으면 1점,

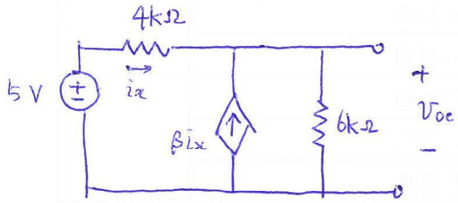
" " 답 틀리면 2점,

" " 답 맞으면 3점.

단위 안 쓰면 -1점.

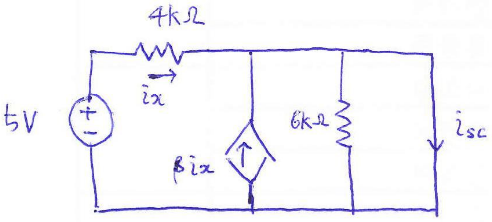
(2) for $t \geq 0$





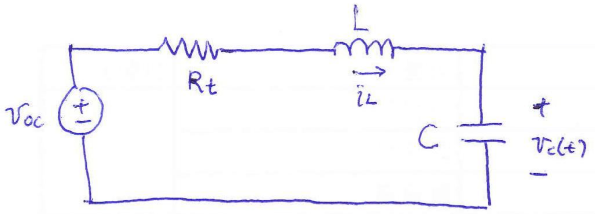
$$\Rightarrow \quad i_x = \frac{5 - V_{oc}}{4000}, \quad i_x + \beta i_x = \frac{V_{oc}}{6000}$$

$$\Rightarrow V_{oc} = \frac{5 \times 6000(1+\beta)}{4000 + 6000(1+\beta)}$$



$$\Rightarrow i_{sc} = (1+\beta)\, i_x = \frac{5}{4000}(1+\beta) .$$

$$\Rightarrow R_t = \frac{V_{oc}}{i_{sc}} = \frac{4000 \times 6000}{4000 + 6000(1+\beta)} .$$



$$\Rightarrow \begin{cases} v_{oc} = R_t i_L(t) + L\dfrac{di_L(t)}{dt} + v_c(t) \\ i_L(t) = C\dfrac{dv_c(t)}{dt}. \end{cases}$$

$$\Rightarrow \frac{d^2 v_c(t)}{dt^2} + \frac{R_t}{L}\cdot\frac{dv_c(t)}{dt} + \left(\frac{1}{CL}\right)v_c(t) = \frac{v_{oc}}{CL}.$$

$(L = 2\text{mH},\ C = 10\text{nF}).$

* 총 6점

풀이과정만 있으면 2점.

풀이과정이 틀려서 답이 틀리면 3점.

풀이과정은 맞지만 실수로 〃 4점.

(3) 미분방정식을 $\dfrac{d^2 v_c(t)}{dt^2} + 2\alpha\dfrac{dv_c(t)}{dt} + \omega_0^2 v_c(t) = f(t)$ 로 두면,

$$\Rightarrow 2\alpha = \frac{R_t}{L},\quad \omega_0^2 = \frac{1}{CL},\quad f(t) = \frac{v_{oc}}{CL}.$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

$\Rightarrow$ $v_c(t)$ 가 stable 하려면, $s_{1,2}$ 가 복소평면의 left half plane 에 있어야 하므로, $\alpha < 0$ 이 되어야 한다.

$$\alpha = \frac{R_t}{2L} = \frac{4000 \times 6000}{2 \times 0.002 \times (4000 + 6000(1+\beta))} < 0$$

$$\Rightarrow \frac{2 \times 0.002 \times (4000 + 6000(1+\beta))}{4000 \times 6000} > 0$$

$$\Rightarrow 4000 + 6000(1+\beta) > 0$$

$$\Rightarrow \beta > -\frac{5}{3}.$$

* 총 6점.

기준 (2)와 같음.

(4) $\beta = 5$ 일 경우,

$$\alpha = \frac{4000 \times 6000}{2 \times 0.002\,(4000 + 6000\,(1+5))} = 150000.$$

$$\omega_0^2 = \frac{1}{CL} = \frac{1}{10 \times 10^{-9} \times 0.002} = 5 \times 10^{10}$$

$$S_{1,2} = -150000 \pm j165831.$$

$\therefore$ underdamped.

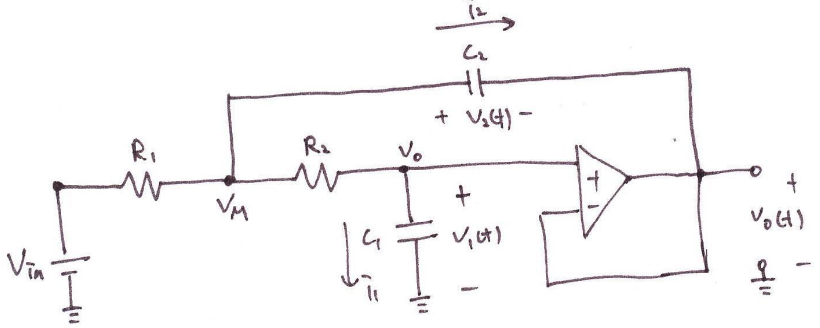
*총 5점.

풀이 과정만 있으면 1점.

" " 답 틀리면 2점.

[4] (1)

8



+1 $i_1 + \dfrac{V_0 - V_M}{R_2} = 0$, $V_M = V_2 + V_0 \rightsquigarrow V_2 = R_2 i_1$

대입

+1 $\dfrac{V_M - V_{in}}{R_1} + i_2 + \dfrac{V_2}{R_2} = 0 \rightsquigarrow \dfrac{V_2 + V_0 - V_{in}}{R_1} + i_2 + \dfrac{V_2}{R_2} = 0$

$\therefore R_2(R_2 i_1 + V_0 - V_{in}) + R_1 R_2 i_2 + R_1 R_2 i_1 = 0$

여기서 $i_1 = C_1 \dfrac{dV_0(t)}{dt}$, $i_2 = C_2 \dfrac{dV_2(t)}{dt} = R_2 C_1 C_2 \dfrac{d^2 V_0(t)}{dt^2}$

$\therefore R_1 R_2 R_2 C_1 C_2 \dfrac{d^2 V_0(t)}{dt^2} + R_2 C_1 (R_1 + R_2) \dfrac{dV_0(t)}{dt} + R_2 V_0(t) = R_2 V_{in}$

$\hookrightarrow \dfrac{d^2 V_0(t)}{dt^2} + \dfrac{R_1 + R_2}{R_1 R_2 C_2} \dfrac{dV_0(t)}{dt} + \dfrac{1}{R_1 R_2 C_1 C_2} V_0(t) = \dfrac{1}{R_1 R_2 C_1 C_2} V_{in}$ +2

$\therefore \alpha = \dfrac{1}{2} \cdot \dfrac{R_1 + R_2}{R_1 R_2 C_2}$, $\omega_0 = \sqrt{\dfrac{1}{R_1 R_2 C_1 C_2}}$

+2 +2

- 회로에 관련된 식을 쓰면 1점씩
- α, ω_0 는 각각 2점씩
- 미분 방정식 제대로 쓰면 2점

(2) 4

$$\frac{R_1+R_2}{R_1R_2C_2} = \frac{5000}{6\times10^6 \times \frac{1}{6}\times10^{-6}} = 5000$$

· 단위 없으면 -1점

$$\frac{1}{R_1R_2C_1C_2} = \frac{1}{C_1}$$

$$\therefore\ 5000^2 - \frac{4}{C_1} < 0 \ \sim\ C_1 < \frac{4}{25}\times10^{-6} \qquad \therefore\ \underline{C_1 < \frac{4}{25}\mu F}$$ +4

(3) 8

$$\frac{d^2V_o(t)}{dt^2} + 5000\frac{dV_o(t)}{dt} + \frac{25}{2}\times10^6\, V_o(t) = \frac{25}{2}\times10^8$$

① natural response

$$s^2 + 5000s + \frac{25}{2}\times10^6 = 0 \ \sim\ s = \frac{-5000\pm5000i}{2} = -2500\pm2500i$$

$$\therefore\ \underline{V_o^n(t) = B_1e^{-2500t}\cos2500t + B_2e^{-2500t}\sin2500t\ (V)}$$ +2

② forced response

$V_o^f = K$ 라 하면,

$$\cancel{\frac{25}{2}\times10^6}\cdot K = \cancel{\frac{25}{2}\times10^6}\times V_{in} \qquad \therefore\ \underline{V_o^f = V_{in} = 100V}$$ +1

$$\therefore\ V_o(t) = V_o^n(t) + V_o^f$$

$$\underline{= B_1e^{-2500t}\cos2500t + B_2e^{-2500t}\sin2500t + 100\ (V)}$$ +1

초기조건 ① $V_o(0) = 0$

② $V_2(0) = R_2 i_1(0) = R_2 L_1 \frac{dV_o(0)}{dt} = 2$

$\therefore 2000 \times \frac{2}{25} \times 10^{-6} \times V_o'(0) = 2 \rightsquigarrow V_o'(0) = \frac{25}{2} \times 10^3$ (V) +1

① $\Rightarrow B_1 + 100 = 0 \rightsquigarrow B_1 = -100$ +1

② $\Rightarrow V_o'(t) = B_1(-2500e^{-2500t}\cos 2500t - 2500e^{-2500t}\sin 2500t)$

$+ B_2(-2500e^{-2500t}\sin 2500t + 2500e^{-2500t}\cos 2500t)$

$V_o'(0) = -2500B_1 + 2500B_2 = \frac{25}{2} \times 1000$

$\therefore B_2 = B_1 + 5 = -95$ +1

$\therefore V_o(t) = -100e^{-2500t}\cos 2500t - 95e^{-2500t}\sin 2500t + 100$ (V) +1

- natural response 잘 구했으면 2점
- forced response 구했으면 1점

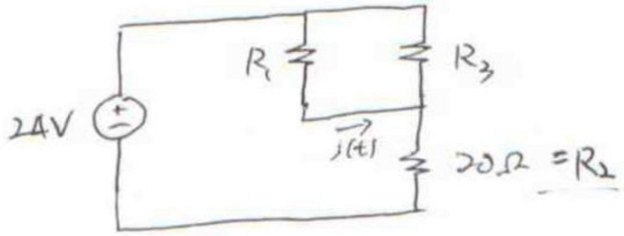
⇒ 잘 합쳐 total $V_o(t)$ 식 적었으면 1점

- $V_o(0) = 0$ 이용해서 B_1 구했으면 1점
- $V_o'(0) = \frac{25}{2} \times 10^3$ (V) 구하면 1점, B_2 구하면 1점
- $V_o(t)$ 구하면 1점 단위없으면 -1점

5. (a)

$t < 0$, switch closed.

at steady state

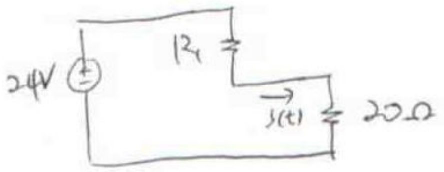


$$i(\ ) = \frac{24}{(R_1 // R_3) + 20} = 0.24 + 0.193 \cos(-100^\circ)\ (A) = 0.2 (A)$$

$$(R_1 // R_3) = \frac{24}{0.2} - 20$$

$t > 0$, switch open

at steady state



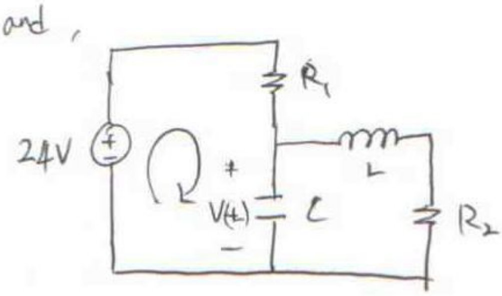
$$i(\infty) = \frac{24}{R_1 + 20}$$

$$\lim_{t\to\infty}\left(0.24 + 0.193 e^{-6.25t} \cos(4.3t - 100^\circ)\right) = 0.24$$

$\therefore\ R_1 = 80\,\Omega$

$80 // R_3 = 100 \qquad \Rightarrow\ R_3 = 80\,\Omega$

and,



$$24 = R_1\left(i(t) + C\frac{dV(t)}{dt}\right) + V(t)$$

and $$V(t) = L\frac{di(t)}{dt} + R_2\, i(t)$$

$$\Rightarrow R_1 C L \frac{d^2 i(t)}{dt^2} + (R_1 C R_2 + L)\frac{d}{dt} i(t) + (R_1 + R_2)\, i(t) = 24$$

$$\frac{d^2 i(t)}{dt^2} + \left(\frac{R_1 C R_2 + L}{R_1 C L}\right)\frac{d}{dt} i(t) + \left(\frac{R_1 + R_2}{R_1 C L}\right) i(t) = \frac{24}{R_1 C L}$$

$$\Rightarrow \frac{d^2 i(t)}{dt^2} + 2\alpha \frac{d}{dt} i(t) + w_0^2 i(t) = f(t)$$

$$2\alpha = \frac{R_1 C R_2 + L}{R_1 C L}, \quad w_0^2 = \frac{R_1 + R_2}{R_1 C L} \quad \text{and} \quad f(t) = \frac{24}{R_1 C L}$$

where $i(t) = 0.24 + 0.193 e^{-6.25t} \cos(9.29t - 102°)$ (A)

$$\alpha = 6.25 \quad w_d = 9.29 \text{ rad/s}$$

$$\Rightarrow w_0 = \sqrt{w_d^2 + \alpha^2} = 11.18 \text{ rad/s}$$

$$\therefore \left.\begin{aligned} 2(6.25) &= \frac{R_1 C R_2 + L}{R_1 C L} = \frac{20}{L} + \frac{1}{80C} \\ (11.18)^2 \simeq 125 &= \frac{R_1 + R_2}{R_1 C L} = \frac{1.25}{CL} \end{aligned}\right\}$$

$$\Rightarrow \begin{cases} C = 1.25 \text{ mF} \\ L = 8 \text{ H} \end{cases} \quad \begin{cases} C = 5 \text{ mF} \\ L = 2 \text{ H}. \end{cases}$$

$$\therefore R_1 = R_2 = 80\,\Omega, \quad C = 5 \text{ mF}, \quad L = 2 \text{ H}.$$

we have initial condition $i(0) = 0.2$ A, $V(0) = 8$.

$$V(t) = L\frac{di(t)}{dt} + R_2 i(t)$$

$$\Rightarrow \frac{di(0)}{dt} = \frac{V(0)}{L} - \frac{R_2 i(0)}{L} = \frac{8}{L} - \frac{4}{L} = \frac{4}{L}$$

$$\left.\frac{di(t)}{dt}\right|_{t=0} = 0.193(-6.25)e^{-6.25t}\{\cos 102° \cdot \cos(9.29t) + \sin 102° \cdot \sin(9.29t)\}$$
$$+ 0.193 e^{-6.25t}\{(-9.29)\cos 102° \cdot \sin(9.29t) + (9.29)\sin 102° \cdot \cos(9.29t)\}$$

$$= (0.193)(-6.25)\cos 102° + (0.193)(9.29)\sin 102° \simeq 2 = \frac{4}{L}$$

$\therefore\ L = 2 \quad , \quad C = 5\,mF.$

and $\quad R_1 = R_3 = 80\,\Omega.$

---

5. (b)

$$V(t) = L\frac{di(t)}{dt} + R_3\, i(t)$$

$$= 2\Big[(0.193)(-6.25)e^{-6.25t}\left\{\cos 102^\circ \cos(9.29t) + \sin 102^\circ \cdot \sin(9.29t)\right\}$$

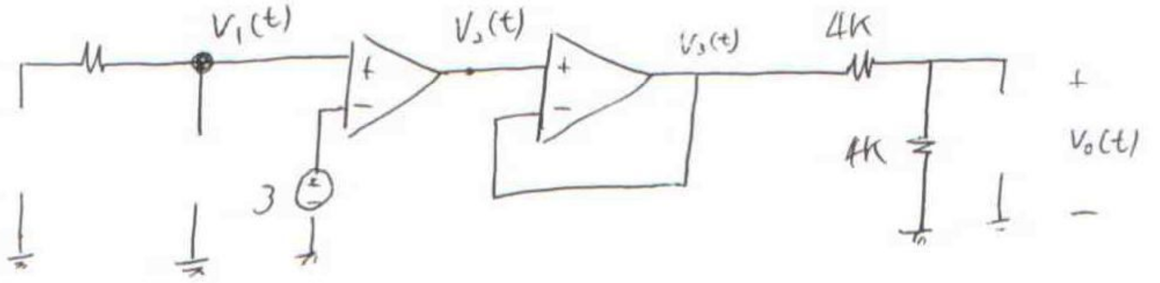
$$+ (0.193)\, e^{-6.25t}\left\{(-9.29)\cos 102^\circ \sin(9.29t) + (9.29)\sin(102^\circ)\cos(9.29t)\right\}\Big]$$

$$+ 20\Big[0.24 + 0.193e^{-6.25t}\left\{\cos 102^\circ \cos(9.29t) + \sin 102^\circ \sin(9.29t)\right\}\Big]$$

$$\simeq 4.8 + 3.2\cos(9.29t) + 2.16\sin(9.29t)$$

6.

$t < 0$, steady state.



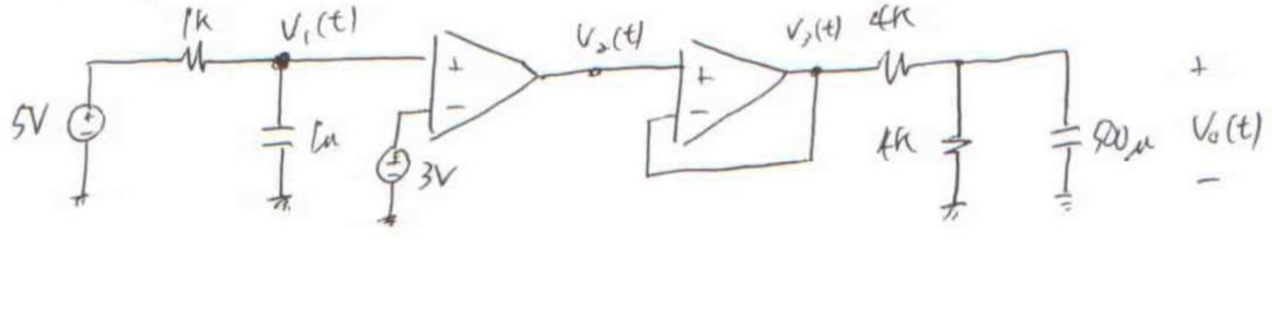
$V_1(0^-) = 0$,

$V_2(0^-) = 2\ V$ — where $V_2(t) = \begin{cases} 6\ V, & V_1(t) > 3 \\ 2\ V, & V_1(t) < 3 \end{cases}$

$V_2(t) = V_3(t)$

$\therefore\ V_0(0^-) = \frac{4k}{4k + 4k} \cdot 2 = 1V.$

$t > 0$,



$$\frac{V_1(t) - 5}{1k} + 1\mu \frac{dV_1(t)}{dt} = 0.$$

$$\Rightarrow \frac{dV_1(t)}{dt} + 1000\ V_1(t) = 5000$$

natural response $\quad V_{1n}(t) = Ae^{-1000t}$

forced response $\quad V_{1f}(t) = 5.$

$\therefore$ Complete response of $V_1(t) = 5 + Ae^{-1000t}$

and $V_1(0^-) = V_1(0^+) = 5 + Ae^0 = 0$

$\therefore A = -5.$

$\therefore V_1(t) = 5(1 - e^{-1000t})$

$$v_2(t) \begin{cases} 6V, & V_1(t) > 3 \quad \equiv \quad t > \frac{1}{1000}\ln(5/2) \\ 2V, & V_1(t) < 3 \quad \equiv \quad t < \frac{1}{1500}\ln(5/2) \end{cases}$$

$$\left(\begin{array}{l} \therefore 5(1 - e^{-1000t}) > 3 \\ \therefore t > \frac{1}{1000}\ln(5/2) \end{array} \right)$$

op amp's output voltage $V_3(t) = V_2(t)$

node eq. at output node

$$\frac{V_0(t) - V_2(t)}{4k} + \frac{V_0(t)}{4k} + 500\mu\frac{dV_0(t)}{dt} = 0.$$

$$\Rightarrow \frac{dV_0(t)}{dt} + V_0(t) = \frac{1}{2}V_2(t)$$

$t < \frac{1}{1000}\ln(5/2)$

$$\frac{dV_0(t)}{dt} + V_0(t) = 1.$$

complete response $V_0(t) = 1 + Be^{-t}$

initial condition $V_0(0) = 1 + B = 1$ $\therefore B = 0.$

$\therefore \quad V_o(t) = 1\,V \qquad$ when $\quad t < \frac{1}{7000}\ln(5/2)$

$t > \frac{1}{7000}\ln(5/2)$

$$\frac{dV_o(t)}{dt} + V_o(t) = 3$$

complete response $\qquad V_o(t) = \underbrace{3}_{\text{forced}} + \underbrace{Ce^{-t}}_{\text{natural}}$

boundary condition $\qquad V_o\left(\frac{1}{7000}\ln(5/2)\right) = 3 + Ce^{-\frac{1}{7000}\ln 5/2} = 1$

$$C \doteq -2.00\ldots$$

$\therefore \quad V_o(t) = 3 - 2e^{-t} \qquad$ when $\quad t > \frac{1}{7000}\ln(5/2)$

$$\therefore \quad V_o(t) \begin{cases} 1\ V, & t < \frac{1}{7000}\ln 5/2 \\ 3 - 2e^{-t}, & t > \frac{1}{7000}\ln 5/2 \end{cases}$$