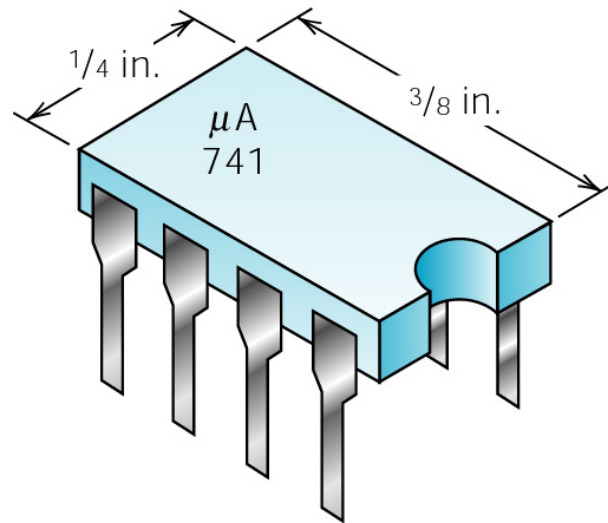


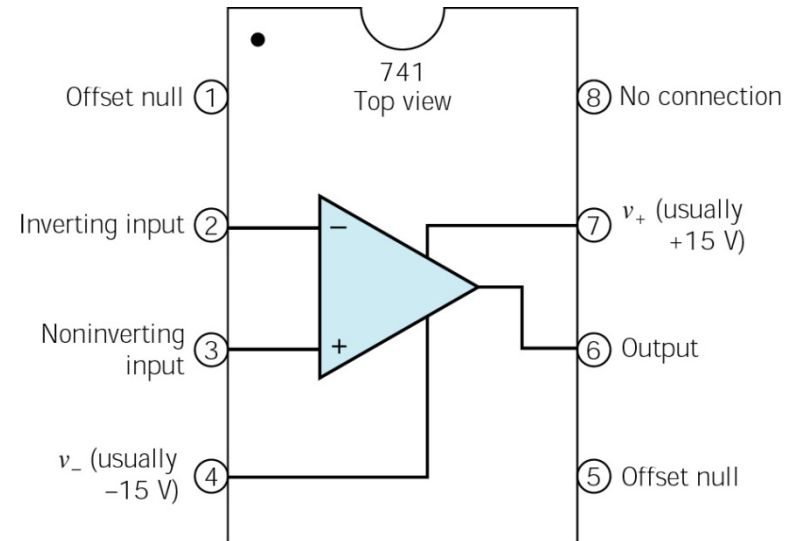
Operational Amplifier



(a) A $\mu A741$ integrated circuit has eight connecting pins

주요한 단자

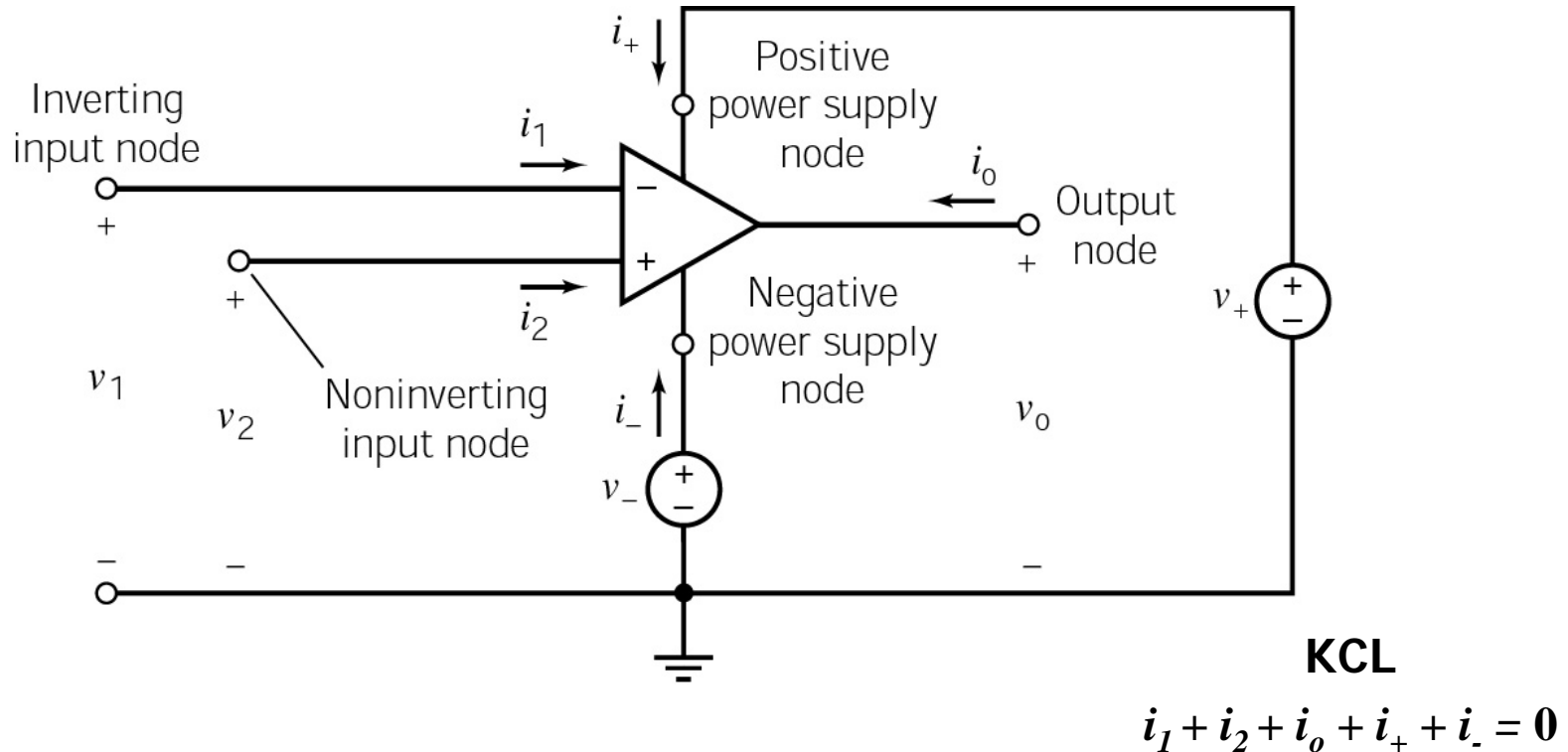
1. inverting input
2. noninverting input
3. output
4. positive power supply (v^+)
5. negative power supply (v^-)



(b) The correspondence between the circled pin numbers of the integrated circuit and the nodes of the operational amplifier.

- NC : no connection
- Balance(offset null) : compensate for a degradation

Symbol and Circuits



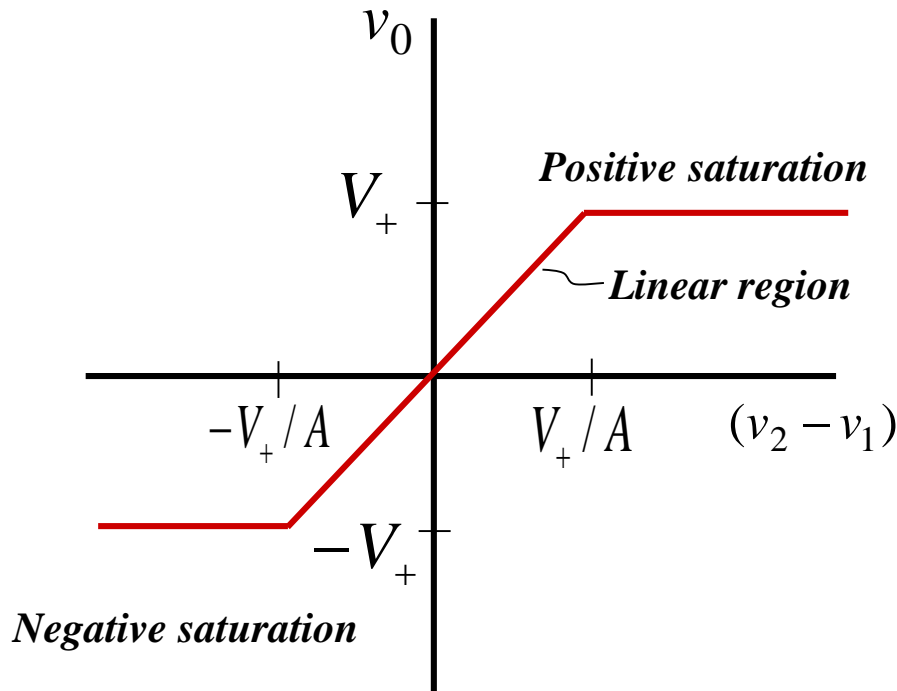
An op amp, including power supplies v^+ and v^- .

Common node : reference

- All voltages rise from the reference node.
- All currents come into the amplifier.

Richard C. Dorf and James A. Svoboda, Introduction to Electric Circuits, 8th edition, John Wiley and Sons, 2010, p. 209

Ideal Operational Amplifier



- Op amp가 선형이기 위해서는
다음의 조건을 만족해야 한다.

$$|v_o| \leq v_{sat}$$

$$|i_o| \leq i_{sat}$$

$$\left| \frac{dv_o(t)}{dt} \right| \leq SR \quad (SR, \text{ slew rate})$$

- For $\mu A 741$,

$$v_{sat} = 14 \text{ V}, \quad i_{sat} = 2 \text{ mA}, \quad SR = 500,000 \text{ V/s}$$

Ideal Operational Amplifier

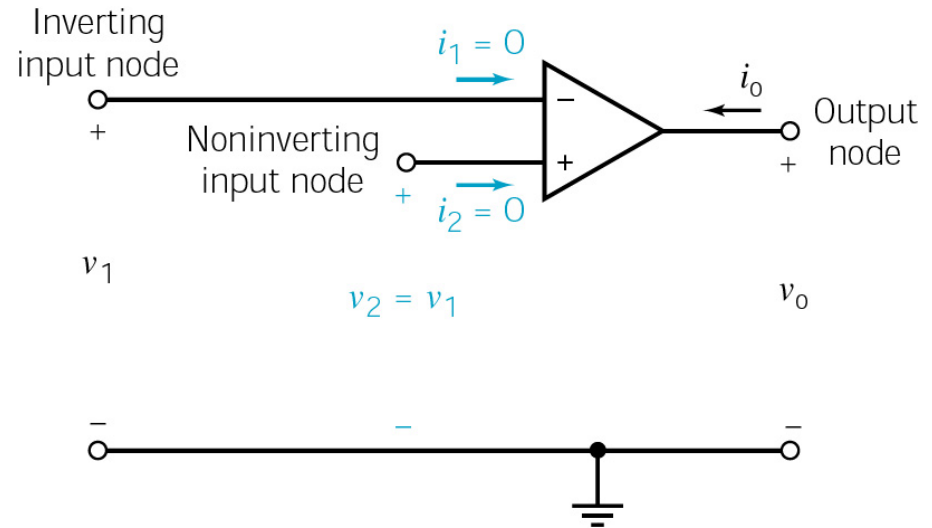


Table. Operating Condition for an Ideal Operational Amplifier

Variable	Ideal Condition
Inverting node input current	$i_1 = 0 \leftarrow R_i \rightarrow \infty$
Noninverting node input current	$i_2 = 0$
Voltage difference between inverting node voltage v_1 and noninverting node voltage v_2	$v_2 - v_1 = 0 \leftarrow A \rightarrow \infty$

The ideal operational amplifier

– Ideal operational amplifier
Op amp input current는 영이다.

$$i_1 = 0, \quad i_2 = 0$$

Input node voltage는 같다.

$$v_2 = v_1$$

*** Virtual short condition.**

Richard C. Dorf and James A. Svoboda, Introduction to Electric Circuits, 8th edition, John Wiley and Sons, 2010, p. 210

Ideal Operational Amplifier

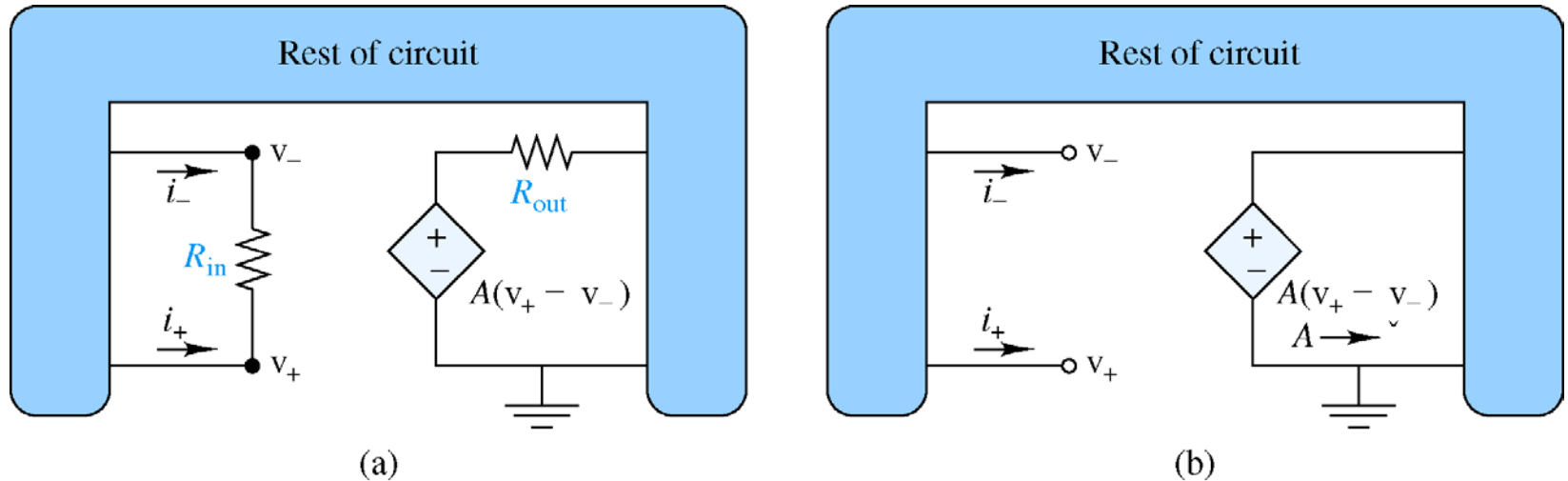


Figure 4.4 (a) Op amp model. (b) Idealized model.

Raymond A. DeCarlo and Pen-Min Lin, Linear Circuit Analysis, 7th edition, Oxford University Press, New York, 2001, p. 136

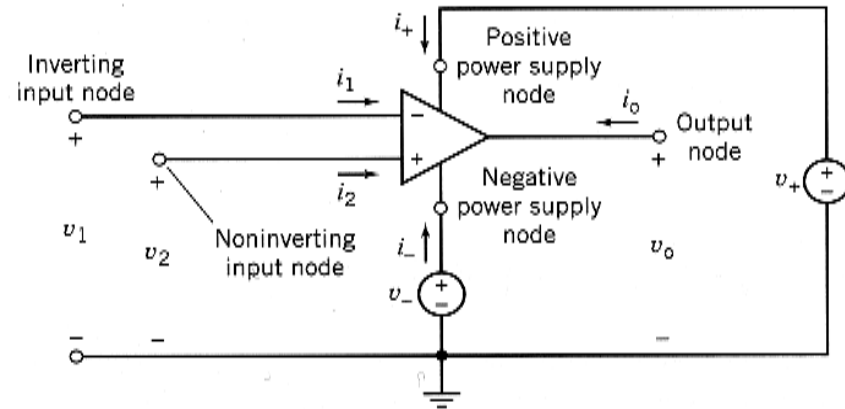
-Ideal OP Amp

(1) Infinite gain, (2) infinite input resistance, (3) zero output resistance

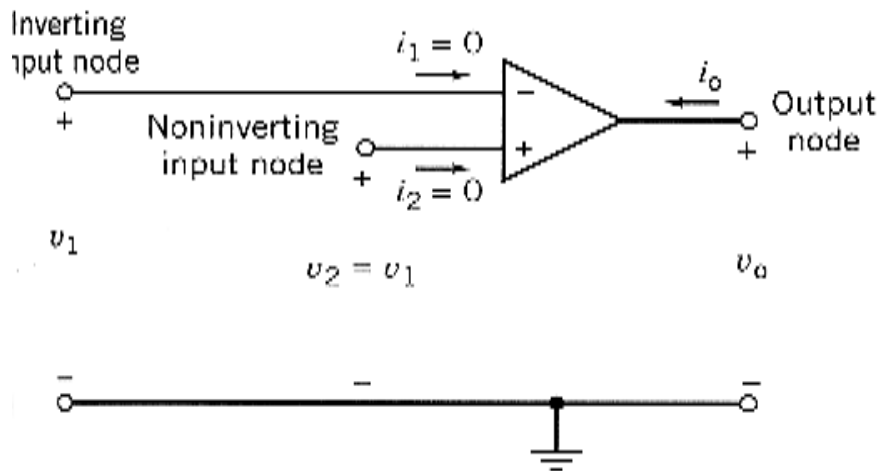
$$R_i \rightarrow \infty, \quad R_o \rightarrow 0, \quad A \rightarrow \infty$$

- 실제 소자 거동(e.g. saturation)을 정확히 묘사하지 못하나, 해석을 단순화.

Op amp 회로의 간략화



An op amp, including power supplies v^+ and v^- .



The ideal operational amplifier

Richard C. Dorf and James A. Svoboda, Introduction to Electric Circuits, 8th edition, John Wiley and Sons, 2010, p. 209-210

KCL

$$i_2 + i_1 + i_o + i_+ + i_- = 0$$

여기서 i_2, i_1 은 매우 작으므로

$$i_2 = i_1 \approx 0$$

따라서, $i_o = -(i_+ + i_-)$

입력전류는 영이지만 출력전류는 상당히 흐른다.

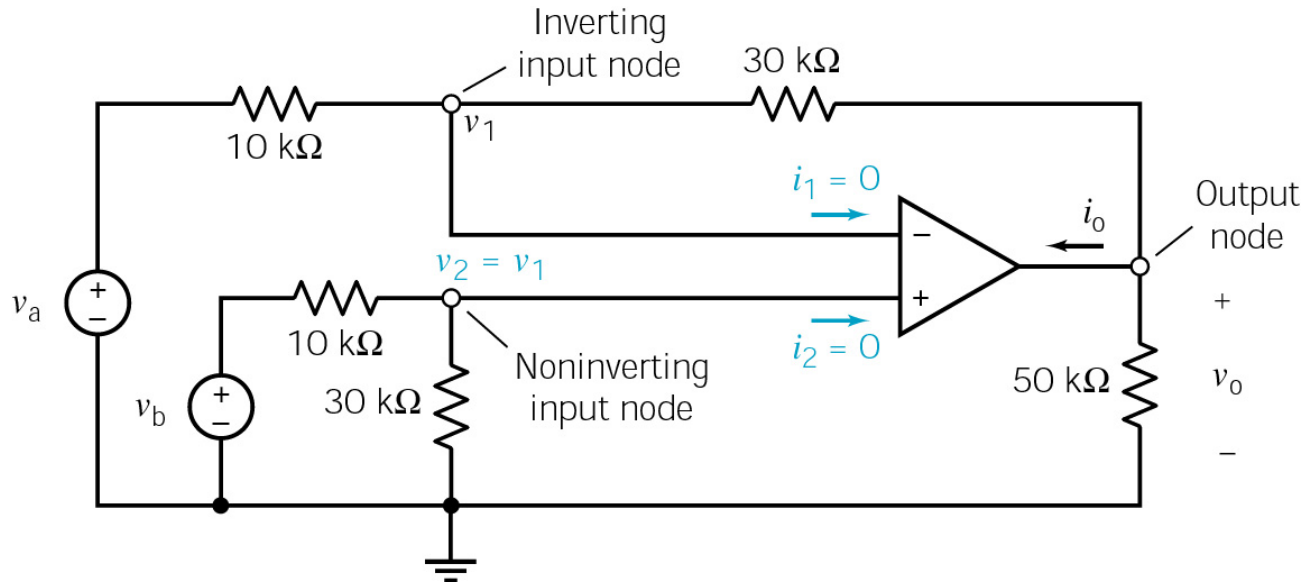
Op amp 회로는 선형 구간에서 그림과 같이 간략화 할 수 있다.

여기서 $|v_o| < V_+$ 이고

$i_2 + i_1 + i_o = 0$ 은 성립하지 않는다.

왜냐하면 이 회로는 간략화한 회로이기 때문이다.

Nodal Analysis of Op Amp Circuits



Op amp : virtual short condition

$v_2 = v_1, i_1 = i_2 = 0$
Input 단자에서 KCL 적용.

Node 1

$$\frac{v_1 - v_a}{10k\Omega} + \frac{v_1 - v_o}{30k\Omega} + 0 = 0 \quad (1)$$

Node 2 $\frac{v_1 - v_b}{10k\Omega} + \frac{v_1 - 0}{30k\Omega} + 0 = 0 \quad (2)$

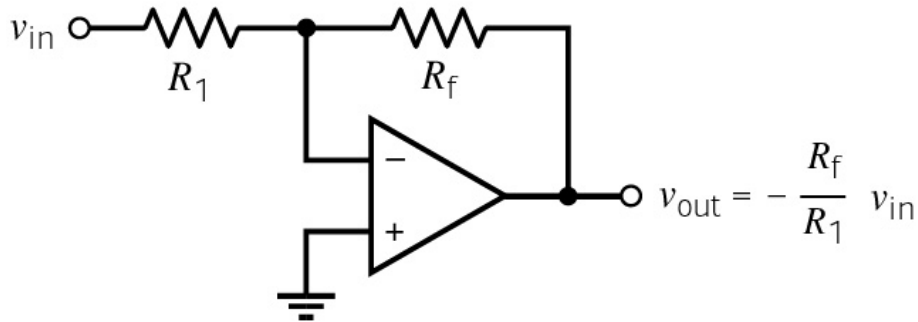
v_1, v_o 가 미지수 $\frac{4}{3}v_1 - v_a - \frac{v_o}{3} = 0 \quad (1')$

$$\frac{4}{3}v_1 - v_b = 0 \quad (2')$$

따라서, $v_o = -3(v_b - v_a)$

Richard C. Dorf and James A. Svoboda, Introduction to Electric Circuits, 8th edition, John Wiley and Sons, 2010, p. 211

Inverting Amplifier



(a) Inverting amplifier

$v_2 = 0$ 이고 $v_2 = v_1$ 이므로 $v_1 = 0, i_1 = 0$

KCL에서 $i_{R1} + i_{Rf} = 0$.

$$\frac{0 - v_{in}}{R_1} + \frac{0 - v_o}{R_f} = 0$$

$$v_o = -\frac{R_f}{R_1} v_{in} \quad \left(\frac{R_f}{R_1} : \text{scaling factor}\right)$$

$|v_o| < V_+$ 이어야 하므로

$$\left| \frac{R_f}{R_1} v_{in} \right| < V_+ \rightarrow |v_{in}| < \frac{V_+}{R_f / R_1}$$

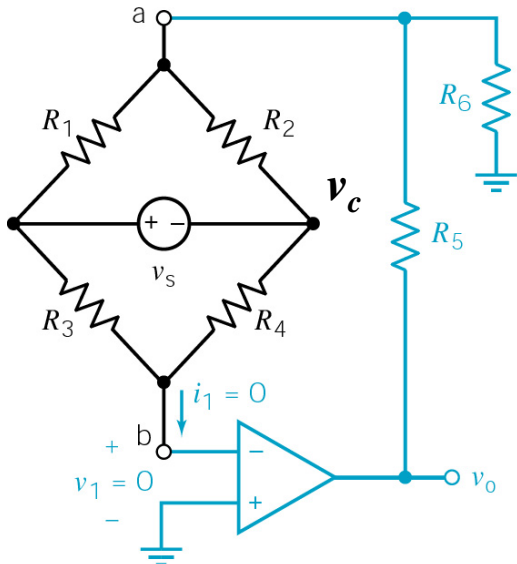
R_f 가 없는 **open loop**인 경우
 $v_o = -Av_1$ 이 되고 $i_1 \approx 0$ 이므로
 $v_1 \approx v_{in}$ 가 된다.

따라서, $v_o = -Av_{in}$ 이고

$|v_s| < V_+ / A$ 이어야 하므로

v_{in} 는 매우 작아야 **Op amp**가 선형 동작한다.

Bridge Amplifier Circuits (I)



(a) A bridge amplifier, including the bridge circuit

Virtual short condition

$$v_2 = v_1 = v_b = 0, \quad i_1 = 0$$

v_c , $v_c + v_s$ 로 Node voltage 정의

Node b 의 KCL

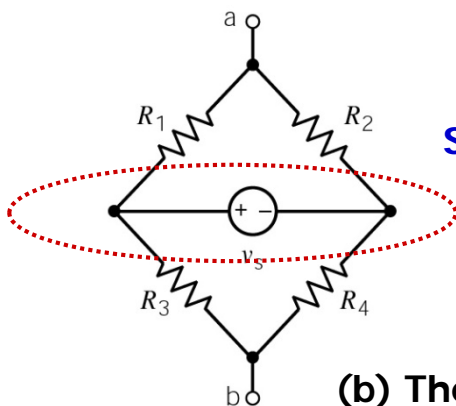
$$\frac{0 - (v_c + v_s)}{R_3} + \frac{0 - v_c}{R_4} = 0 \quad (1)$$

Node a 의 KCL

$$\frac{v_a - (v_c + v_s)}{R_1} + \frac{v_a - v_c}{R_2} + \frac{v_a - v_o}{R_5} + \frac{v_a}{R_6} = 0 \quad (2)$$

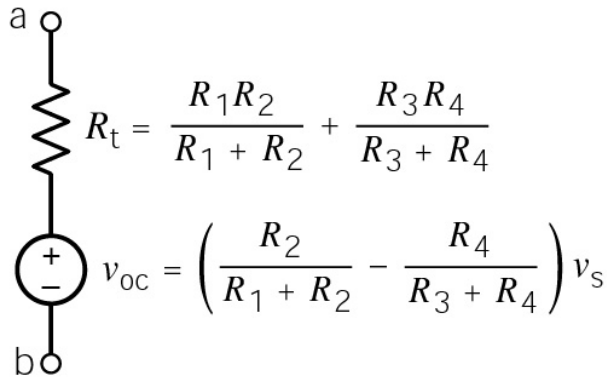
Supernode c, d 의 KCL

$$\frac{v_c - v_a}{R_2} + \frac{v_c - 0}{R_4} + \frac{v_c + v_s - v_a}{R_1} + \frac{v_c + v_s - 0}{R_3} = 0 \quad (3)$$



(b) The bridge circuit

Bridge Amplifier Circuits (II)



v_c , v_o , v_o 가 미지수.

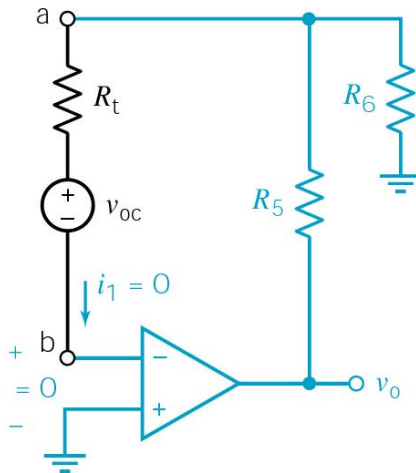
$$v_c \left(\frac{1}{R_3} + \frac{1}{R_4} \right) = -\frac{v_s}{R_3} \quad (1')$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_5} + \frac{1}{R_6} \right) v_a - \left(\frac{1}{R_1} + \frac{1}{R_2} \right) v_c - \frac{1}{R_5} v_o = \frac{v_s}{R_1} \quad (2')$$

$$-\left(\frac{1}{R_1} + \frac{1}{R_2} \right) v_a + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4} \right) v_c = -\left(\frac{1}{R_1} + \frac{1}{R_3} \right) v_s \quad (3')$$

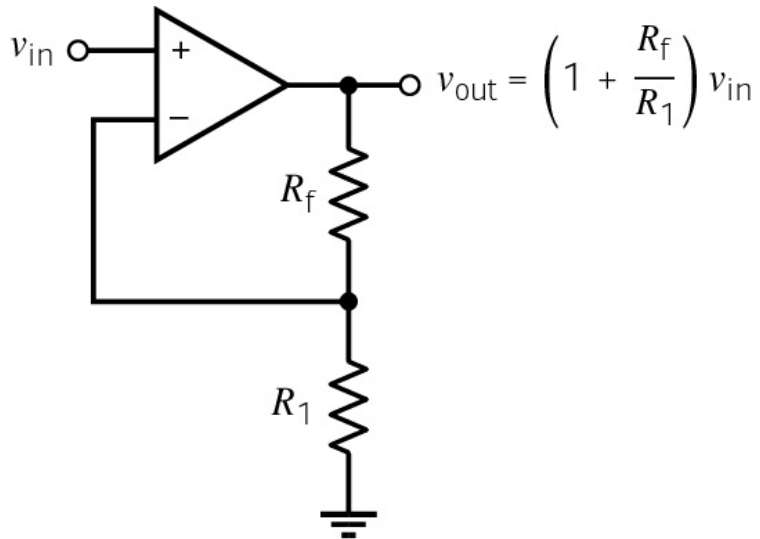
v_o , v_c 를 소거하면 v_o 를 구할 수 있다.

(c) Its Thévenin equivalent circuit



(d) The bridge amplifier, including the Thévenin equivalent of the bridge

Noninverting Amplifier



(b) Noninverting amplifier

$i_2 \approx 0$ 이므로 R_g 에서의 전압강하 = 0.

따라서, $v_2 \approx v_{in}$ 이고 $v_1 \approx v_2$ 이므로

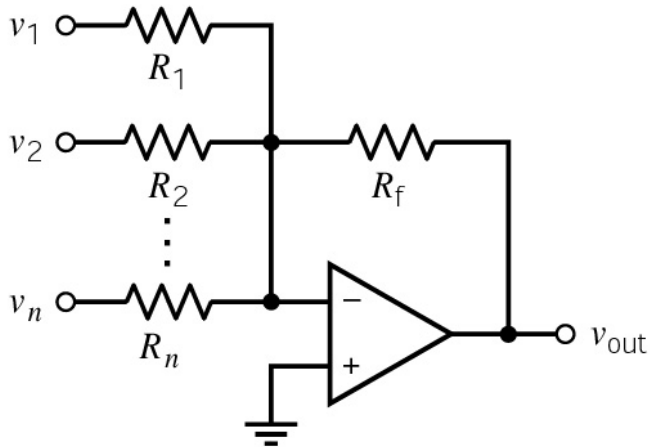
$$v_1 = v_{in}$$

KCL에서
$$\frac{v_{in} - 0}{R_1} + \frac{v_{in} - v_o}{R_f} = 0$$

$$\frac{v_o}{R_f} = \left(\frac{1}{R_1} + \frac{1}{R_f} \right) v_{in}$$

$$v_o = \left(\frac{R_f}{R_1} + 1 \right) v_{in}$$

Summing Amplifier



$v_p = v_n = 0$ 이고 KCL을 적용.

$$v_{out} = - \left(\frac{R_f}{R_1} v_1 + \frac{R_f}{R_2} v_2 + \dots + \frac{R_f}{R_n} v_n \right)$$

(d) Summing amplifier

$$\frac{v_n - v_o}{R_f} + \frac{v_n - v_1}{R_1} + \frac{v_n - v_2}{R_2} + \frac{v_n - v_3}{R_3} = 0$$

$$v_o = \left(-\frac{R_f}{R_1} \right) v_1 + \left(-\frac{R_f}{R_2} \right) v_2 + \left(-\frac{R_f}{R_3} \right) v_3$$

따라서, v_o 는 **scale**된 **n**개의 입력 전압의 합이고 부호는 역전되어 있다.

Noninverting Summing Amplifier

$v_p = v_n$ 이고 KCL을 적용.

$$\frac{v_n - 0}{R_b} + \frac{v_n - v_0}{(K_4 - 1)R_b} = 0 \Rightarrow$$

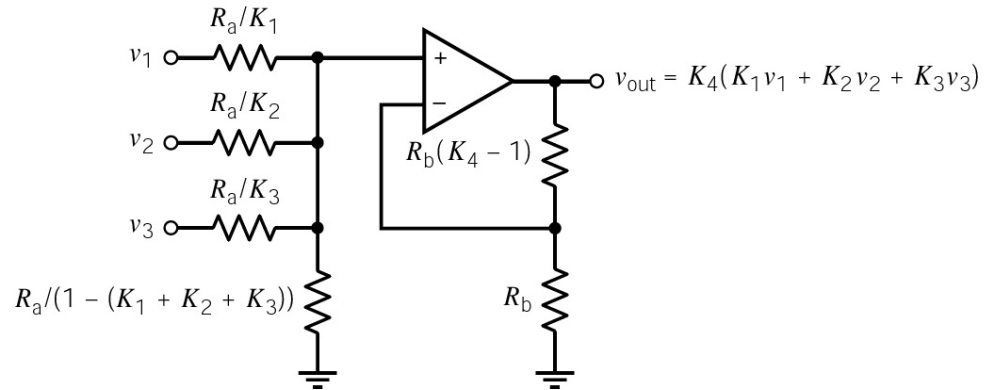
$$\frac{K_4 v_n}{(K_4 - 1)R_b} = \frac{v_0}{(K_4 - 1)R_b} \Rightarrow v_n = \frac{v_0}{K_4}$$

$$\frac{v_n - v_1}{R_a / K_1} + \frac{v_n - v_2}{R_a / K_2} + \frac{v_n - v_3}{R_a / K_3} + \frac{v_n - 0}{R_a / (1 - (K_1 + K_2 + K_3))} = 0 \Rightarrow$$

$$\frac{K_1 v_n}{R_a} + \frac{K_2 v_n}{R_a} + \frac{K_3 v_n}{R_a} + \frac{(1 - (K_1 + K_2 + K_3))v_n}{R_a} = \frac{K_1 v_1 + K_2 v_2 + K_3 v_3}{R_a}$$

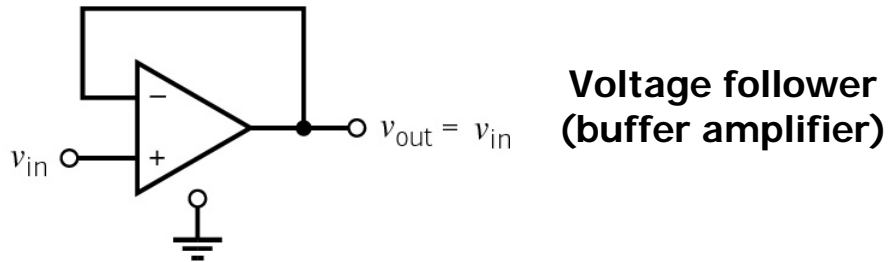
$$v_n = K_1 v_1 + K_2 v_2 + K_3 v_3$$

$$\text{따라서, } v_o = K_4 (K_1 v_1 + K_2 v_2 + K_3 v_3)$$



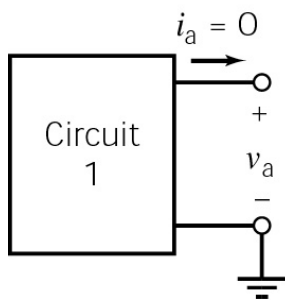
(e) Noninverting summing amplifier

Voltage Follower and Loading Effect

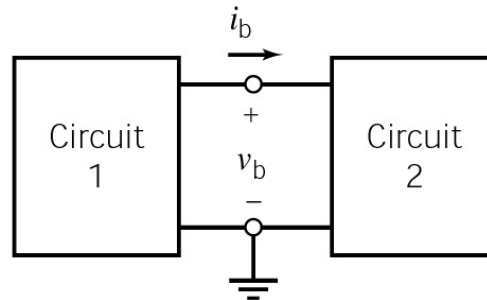


$$v_- = v_{in} = v_{out}$$

Circuit #1의 출력은 Circuit #2를 연결하는 순간 변하고 만다. 이를 **Loading effect**라고 한다.

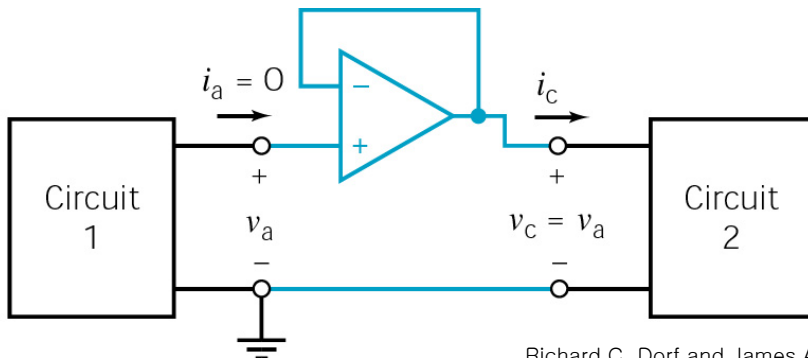


(a) Circuit#1 before



(b) After Circuit#2 is connected

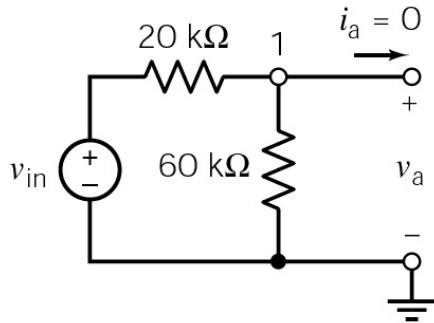
그림(b)와 같은 전압은 바뀌게 된다. Op amp의 **voltage follower**를 이용하면 출력전압을 그대로 유지할 수 있다.



(c) Preventing loading
using a voltage follower

Richard C. Dorf and James A. Svoboda, Introduction to Electric Circuits, 8th edition, John Wiley and Sons, 2010, p. 217-218

Voltage Follower (Buffer or Isolation Amplifier)



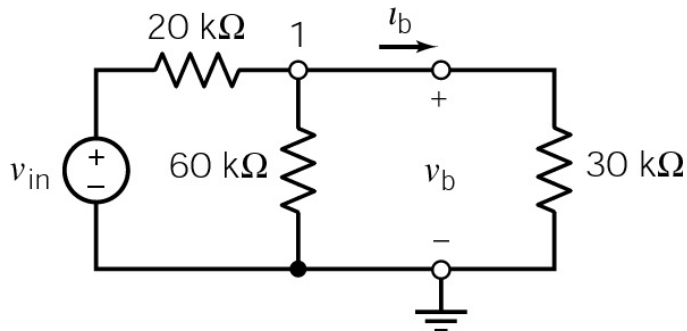
(a) A voltage divider before a 30 kΩ resistor is added

그림 (a)의 경우
$$v_a = \frac{60}{20 + 60} v_{in} = \frac{3}{4} v_{in}$$

그림 (b)의 경우.

30 kΩ 의 저항을 연결했으므로

$$v_b = \frac{60 // 30}{20 + 60 // 30} v_{in} = \frac{1}{2} v_{in}$$



(b) A voltage divider after a 30-kΩ resistor is added

그림 (c)와 같이 voltage follower를 삽입.

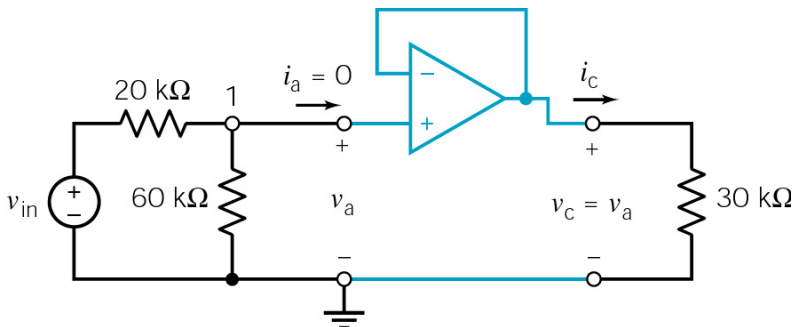
Node a의 KCL

$$\frac{v_a - v_{in}}{20k\Omega} + \frac{v_a - 0}{60k\Omega} = 0 \Rightarrow \left(\frac{1}{20k\Omega} + \frac{1}{60k\Omega} \right) v_a = \frac{v_{in}}{20k\Omega}$$

$$v_a = \frac{3}{4} v_{in}$$

$$v_{out} = v_a \text{ 이므로 } v_{out} = \frac{3}{4} v_{in}$$

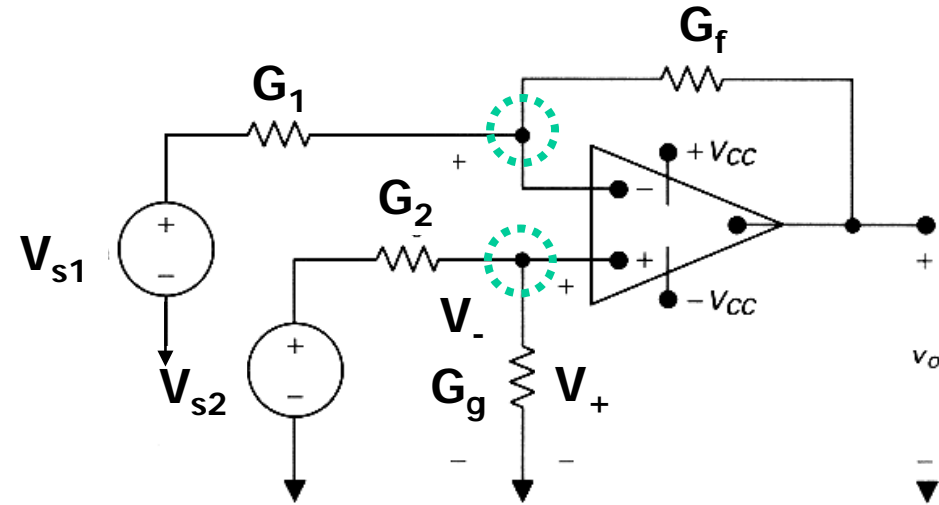
$$i_c = \frac{3}{4} v_{in} / 30k\Omega = v_{in} / 40k\Omega$$



(c) A voltage follower is added to prevent loading

Richard C. Dorf and James A. Svoboda, Introduction to Electric Circuits, 8th edition, John Wiley and Sons, 2010, p. 219

Difference Amplifier



$i_+ = 0$ 이므로

$$G_2(v_+ - v_{s2}) + G_g v_+ = 0$$

$i_- = 0$ 이므로

$$G_1(v_+ - v_{s1}) + G_f(v_+ - v_o) = 0$$

두 식에서

$$v_o = -\frac{G_1}{G_f} v_{s1} + \left(1 + \frac{G_1}{G_f}\right) \left(\frac{G_2}{G_2 + G_g}\right) v_{s2}$$

만약 $G_1 = G_f$ 이고 $G_2 = G_g$ 이면 $v_o = v_{s2} - v_{s1}$

만약 $G_1 = kG_f$ 이고 $G_2 = kG_g$ 이면 $v_o = k(v_{s2} - v_{s1})$

Saturation & the Active Mode

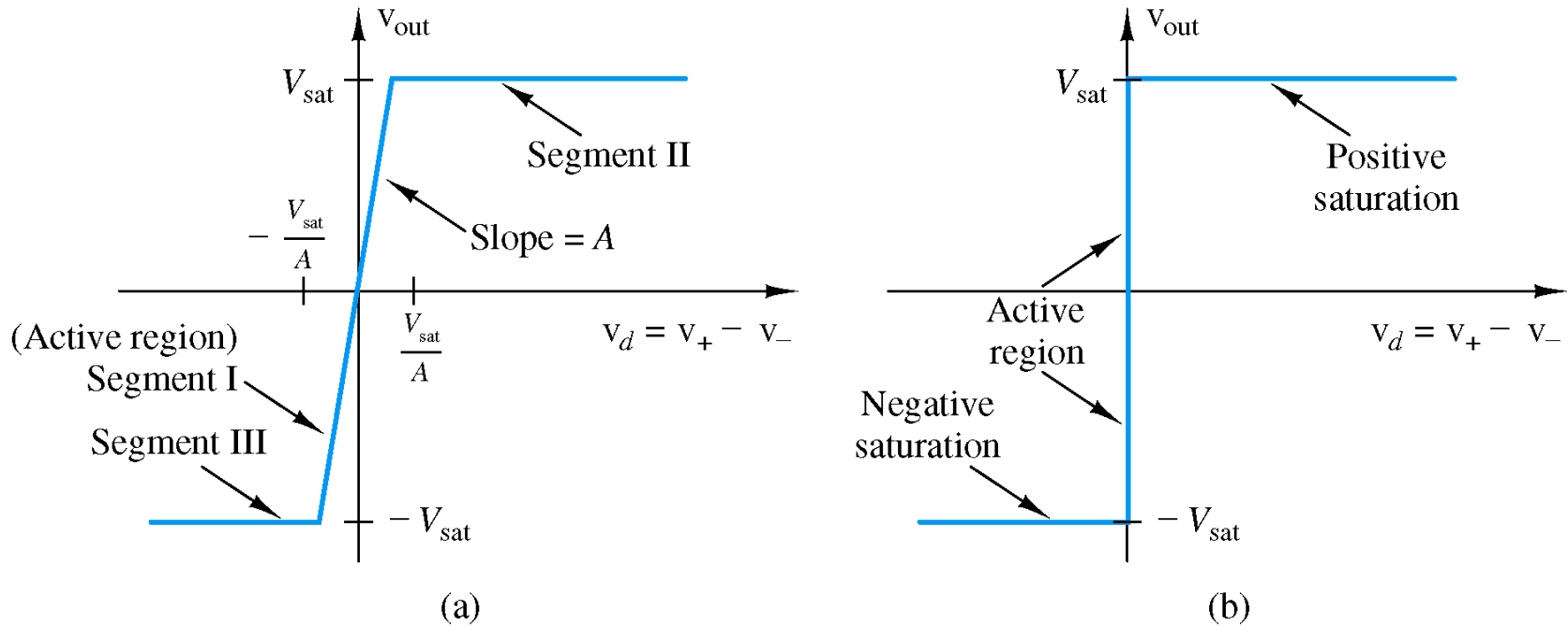


Figure 4.17 Piecewise linear (three-segment) curve for op amp, specifying the active and the positive and negative saturation regions of operation. (a) Finite gain A . (b) (Ideal) infinite gain A .

$$v_d = v_+ - v_- \neq 0$$

- (1) **Finite gain** : typically 10^4 to 10^6 .
- (2) **Saturation** : Output voltage cannot exceed the saturation voltage

Typical Op-Amp

Parameter	Units	OPA101				
		$\mu A741$	LF351	TL051C	AM	OP-07E
Saturation voltage, v_{sat}	V	13	13.5	13.2	13	13
Saturation current, i_{sat}	mA	2	15	6	30	6
Slew rate, SR	V/ μs	0.5	13	23.7	6.5	0.17
Bias current, i_b	nA	80	0.05	0.03	0.012	1.2
Offset current, i_{os}	nA	20	0.025	0.025	0.003	0.5
Input offset voltage, v_{os}	mV	1	5	0.59	0.1	0.03
Input resistance, R_i	M Ω	2	10^6	10^6	10^6	50
Output resistance, R_o	Ω	75	1000	250	500	60
Differential gain, A	V/mV	200	100	105	178	5000
Common mode rejection ratio, $CMRR$	V/mv	31.6	100	44	178	1413
Gain bandwidth product, B	MHz	1	4	3.1	20	0.6

Equivalent Circuit of a 741 Op Amp

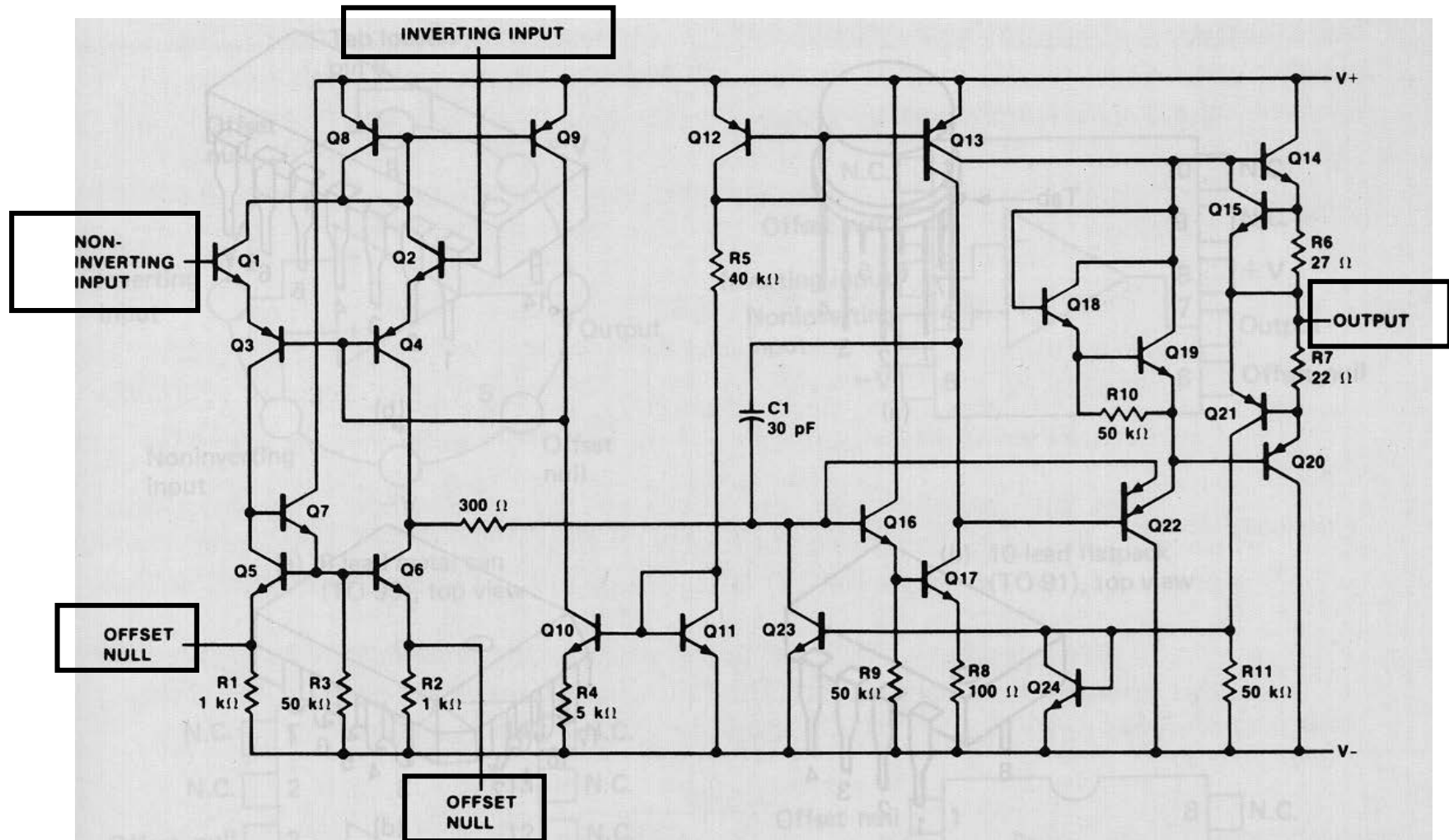
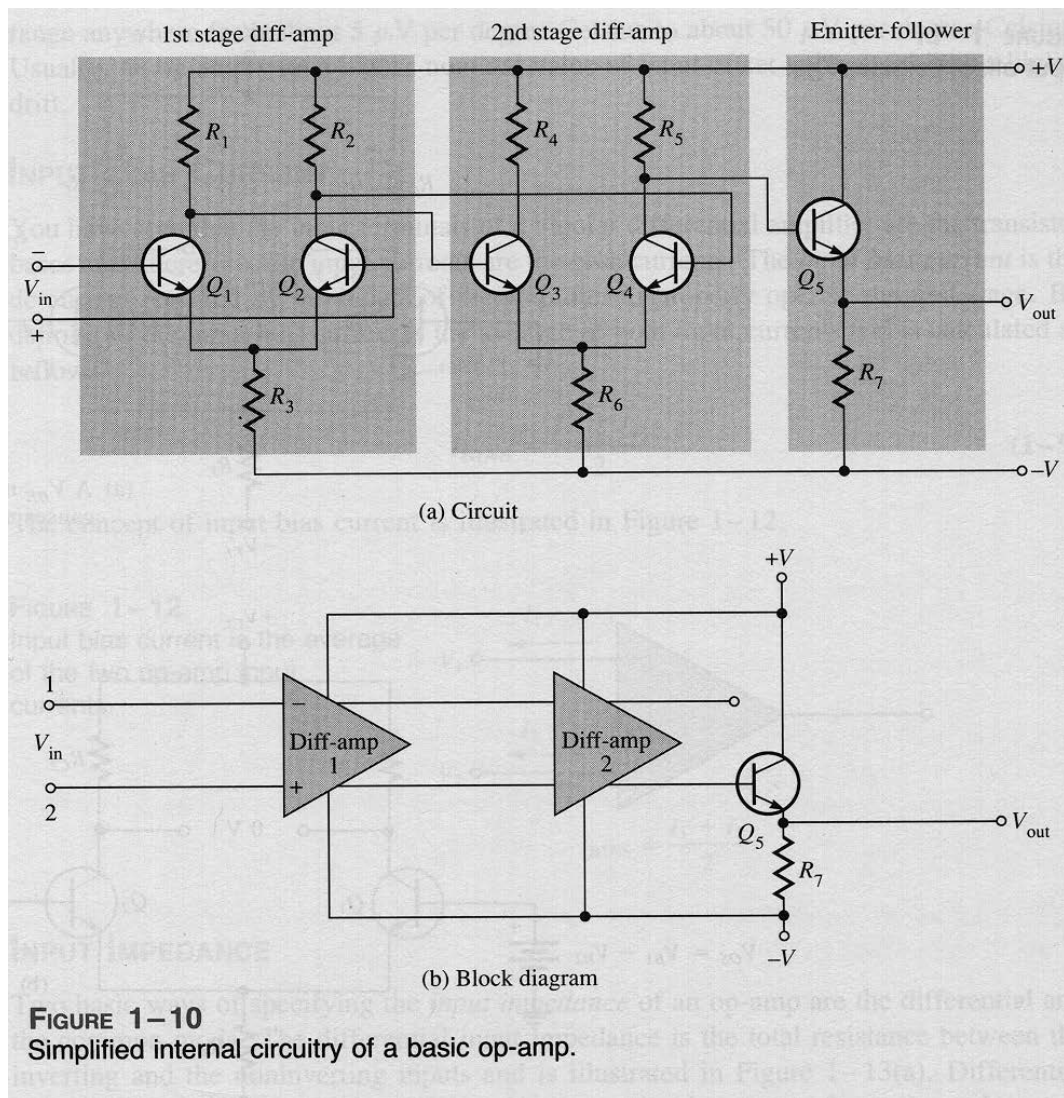


FIGURE 1-2 Equivalent circuit of a 741 op amp. (Courtesy of Fairchild Semiconductor, a Division of Fairchild Camera and Instrument Corporation.)

Simplified Internal Circuitry of a Basic Op Amp



Practical Op-Amp

Ideal op amp.

$$i_1 = 0, \quad i_2 = 0, \quad v_1 - v_2 = 0$$

Practical op amp.

- **nonzero bias currents** (i_{b1} , i_{b2})
- **nonzero input offset voltage** (v_{os})
- **finite input resistance** (R_i)
- **nonzero output resistance** (R_o)
- **finite voltage gain** (A)

$$i_1 = i_{b1}, \quad i_2 = i_{b2}, \quad v_1 - v_2 = v_{os}$$

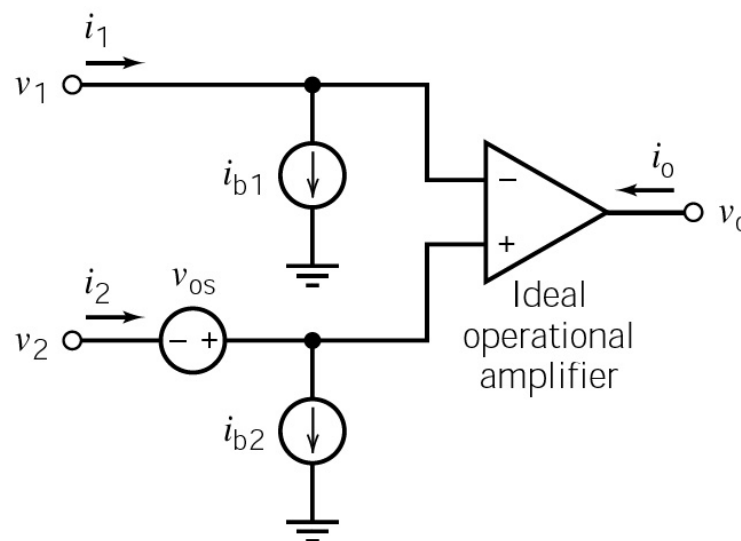
$$i_{os} = i_{b1} - i_{b2}$$

For μA 741,

$$|i_{b1}| \leq 500 \text{ nA}, \quad |i_{b2}| \leq 500 \text{ nA}$$

$$|i_{b1} - i_{b2}| \leq 200 \text{ nA}$$

$$|v_{os}| \leq 5 \text{ mV}$$



(b) The offsets model of an operational amplifier

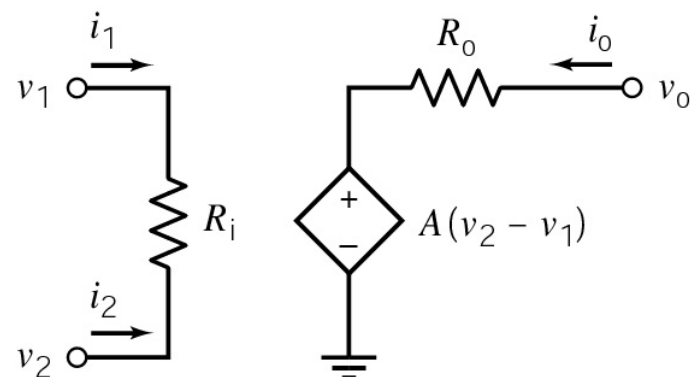
Practical Op-Amp

Ideal op amp.

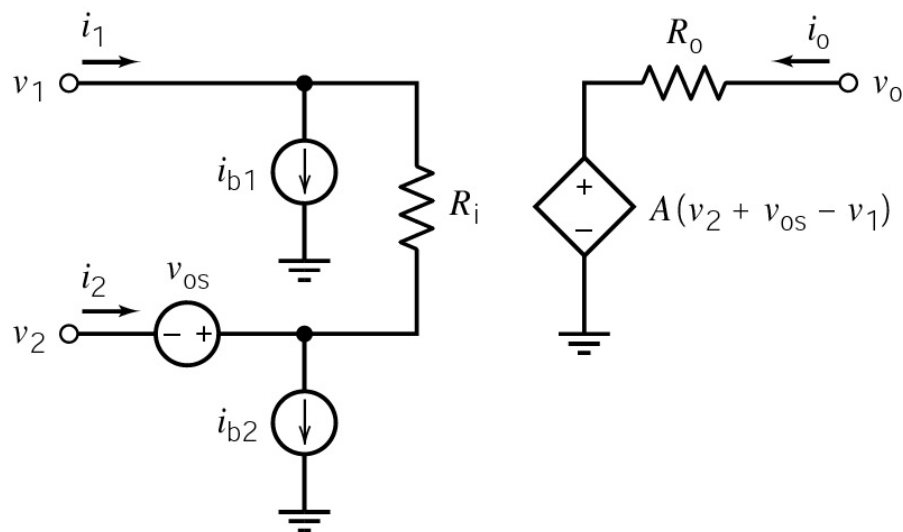
$$i_1 = 0, \quad i_2 = 0, \quad v_1 - v_2 = 0$$

Practical op amp.

- nonzero bias currents (i_{b1} , i_{b2})
- nonzero input offset voltage (v_{os})
- finite input resistance (R_i)
- nonzero output resistance (R_o)
- finite voltage gain (A)

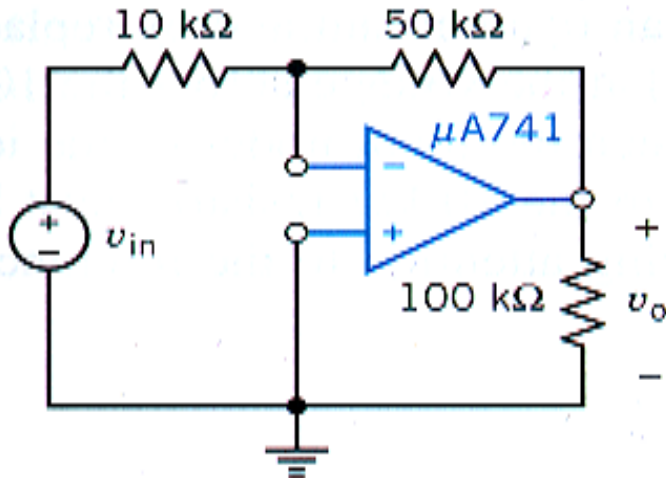


(c) The finite gain model of an operational amplifier



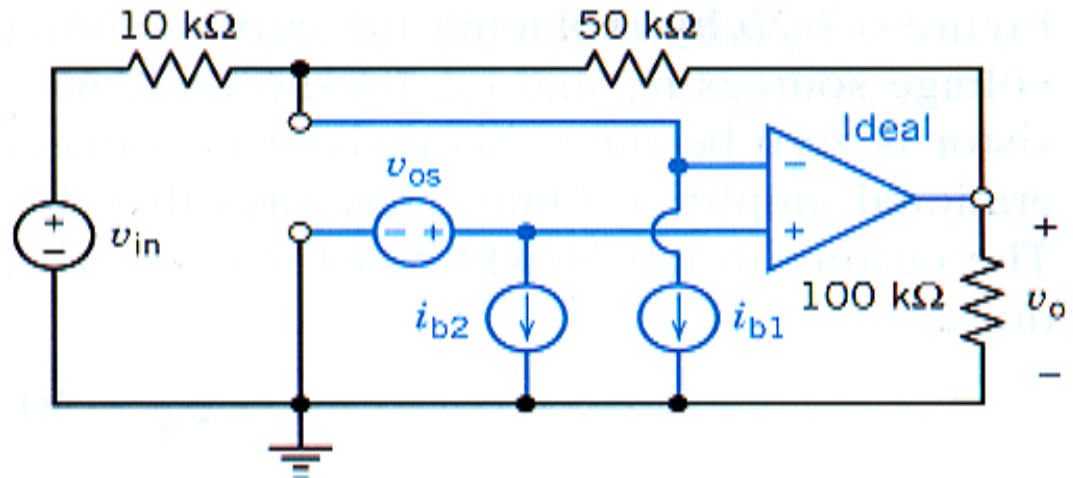
(d) The offsets and finite gain model of an operational amplifier

Realistic Model - Inverting Amp(I)



(a) An inverting amplifier

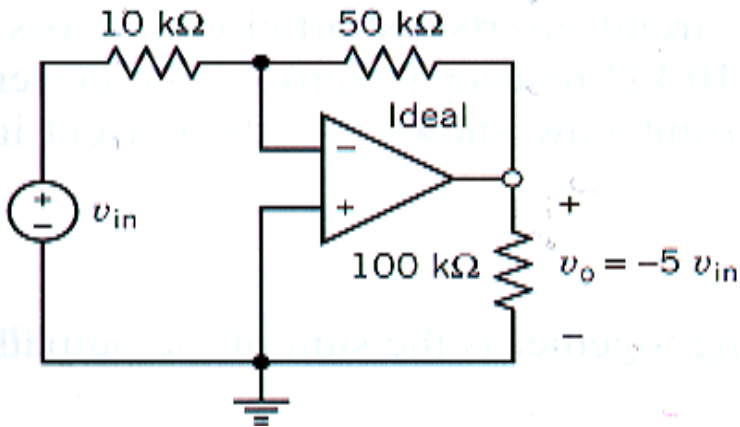
- Op amp는 μA 741임.



(b) An equivalent circuit that accounts for the input offset voltage and bias currents of the operational amplifier

- 실제 Op amp는 bias current source 두 개와 offset voltage source 한 개가 ideal Op amp에 더해져 있는 것으로 간주 (그림 (b)).

Realistic Model - Inverting Amp (II)

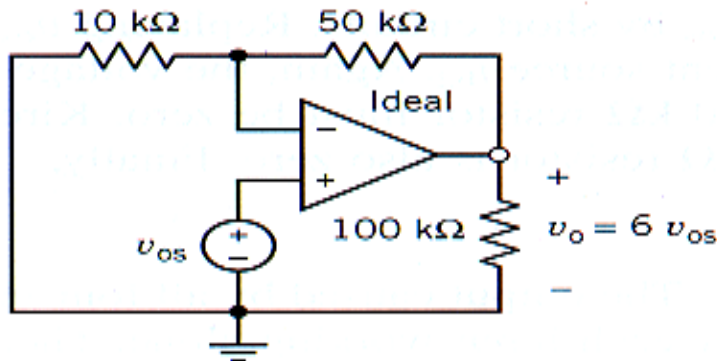


- 그림 (c)는 ideal Op amp.

$$\frac{0 - v_{in}}{10 \text{ k}\Omega} + \frac{0 - v_o}{50 \text{ k}\Omega} = 0$$

$$v_o = -5v_{in}$$

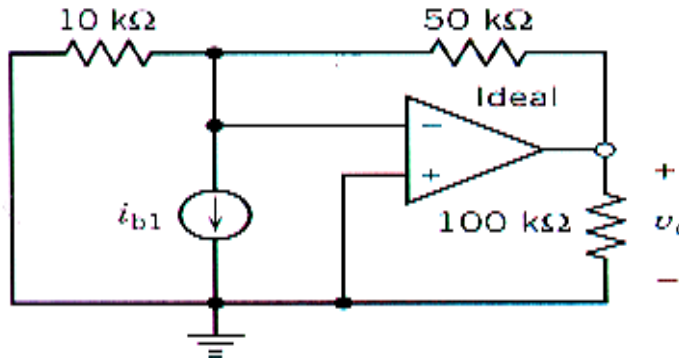
(c) Analysis using superposition



- 그림 (d) : Offset voltage source

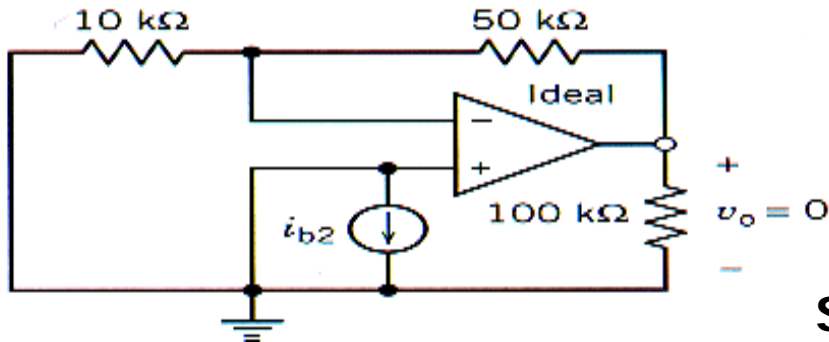
$$\frac{v_{os} - 0}{10 \text{ k}\Omega} + \frac{v_{os} - v_o}{50 \text{ k}\Omega} = 0 \Rightarrow v_o = 6v_{os}$$

Realistic Model - Inverting Amp (III)



- 그림 (e) : Bias current source, i_{b1}

$$i_{b1} + \frac{0-0}{10 \text{ k}\Omega} + \frac{0-v_o}{50 \text{ k}\Omega} = 0 \Rightarrow v_o = 50 \text{ k}\Omega \cdot i_{b1}$$



- 그림 (f) : Bias current source, i_{b2}

$i_n = 0$, $v_p = v_n = 0$ 이므로
10 kΩ 에 흐르는 전류=0 이고
50 kΩ 에 흐르는 전류=0. $v_o = 0$

Superposition에 의해서

$$v_o = -5v_{in} + \underbrace{6v_{os} + 50\text{k}\Omega \cdot i_{b1}}_{\text{output offset voltage}}$$

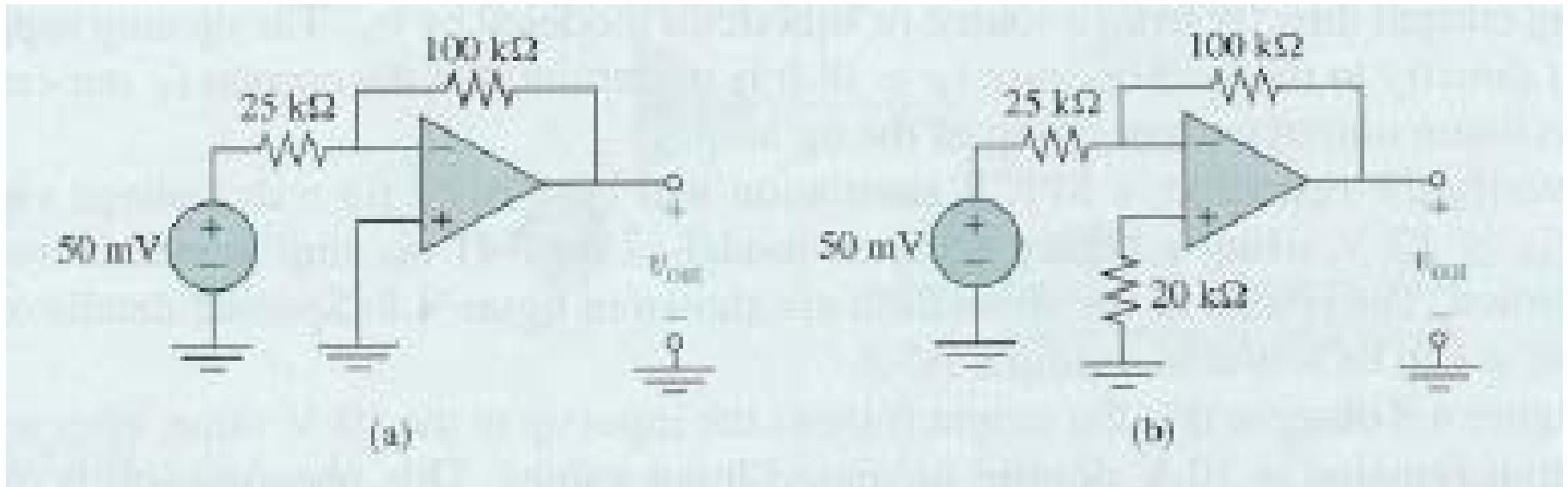
Output offset voltage for $\mu\text{A} 741$

$$= \underbrace{6 \times 5 \text{ mV}}_{\text{최대}} + 50 \text{ k}\Omega \cdot \underbrace{500 \text{ nA}}_{\text{최대}} = 55 \text{ mV}$$

$5v_{in} > 500 \text{ mV}$ 인 영역에서 offset voltage를 무시.

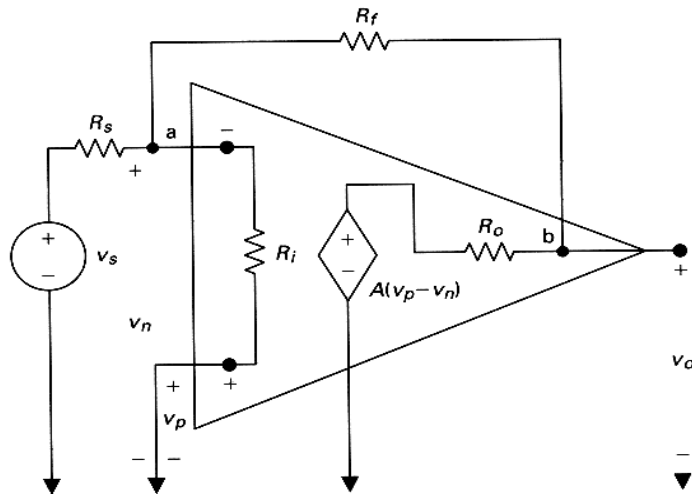
Offset Voltage - Inverting Amp

20 k Ω 의 역할



- (a) Bias current 에 의해 offset 전압이 발생.
- (b) Offset current에 의해 offset 전압이 발생.
- (c) 대개 offset current는 bias current 의 1/4 정도.

Real Inverting Op-Amp Circuit



$$\text{node } a: \frac{v_n - 0}{R_i} + \frac{v_n - v_s}{R_s} + \frac{v_n - v_o}{R_f} = 0$$

$$\text{node } b: \frac{v_o - v_n}{R_f} + \frac{v_o - A(0 - v_n)}{R_o} = 0$$

$$\frac{1}{R_i} = G_i, \frac{1}{R_s} = G_s, \frac{1}{R_f} = G_f, \frac{1}{R_o} = G_o \text{ 라 하면}$$

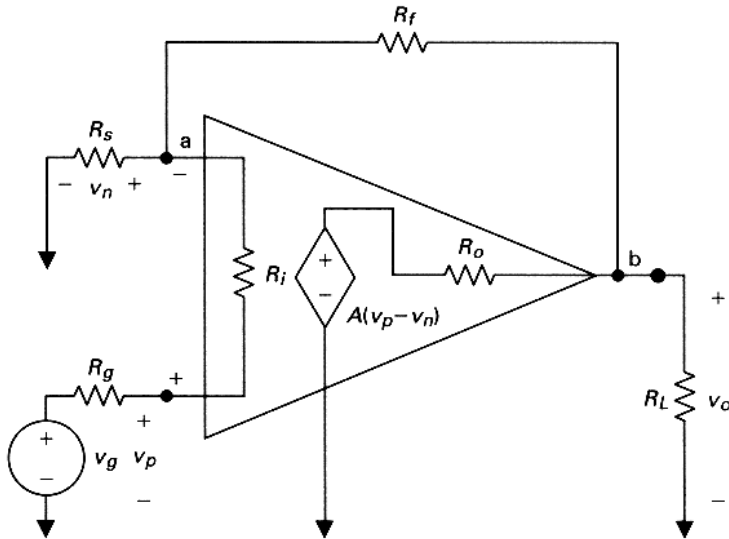
$$(G_i + G_s + G_f)v_n - G_f v_o = G_s v_s$$

$$(AG_o - G_f)v_n + (G_f + G_o)v_o = 0$$

$$v_o = \frac{D_2}{D} = \frac{-G_s(AG_o - G_f)v_s}{(G_i + G_s + G_f)(G_f + G_o) + G_f(AG_o - G_f)}$$

Ideal op amp의 경우, $A \rightarrow \infty$, $G_i \rightarrow 0$, $G_o \rightarrow \infty$ 이므로 이를 대입하면 앞의 예와 같다. 출력 단에 부하저항 R_L 을 연결하면 v_o 가 바뀌며 이 값도 **KCL**에 의해서 구할 수 있다.

Real Noninverting Op-Amp Circuit



$$\text{node } a : G_s v_n + \frac{v_n - v_g}{R_i + R_g} + G_f (v_n - v_o) = 0$$

$$\text{node } b : G_f (v_o - v_n) + G_o (v_o - A(v_p - v_n)) + G_L v_o = 0$$

또한 R_i 와 R_g 에 흐르는 전류가 같으므로

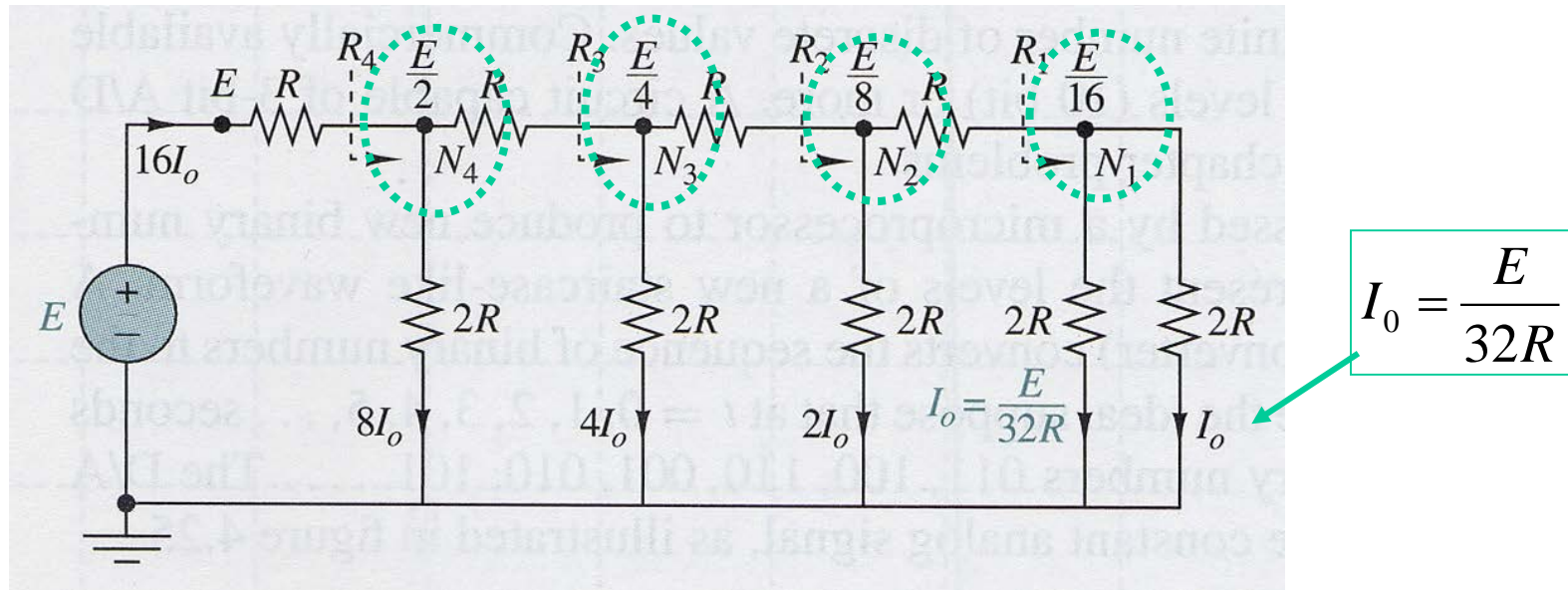
$$\frac{v_n - v_g}{R_i + R_g} = \frac{v_p - v_g}{R_g}$$

여기서, v_p , v_n , v_o 가 미지수이고 식이 세 개이므로 v_o 를 구할 수 있다.

Applications - D/A Converter

Building-Weighted Summing Circuit (I)

$$[b_{n-1}, \dots, b_0] \longrightarrow \boxed{\text{D/A converter}} \longrightarrow v_{out} = [b_{n-1}2^{n-1} + \dots + b_12^1 + b_0]E_0$$



Node 1 voltage : $E/16$

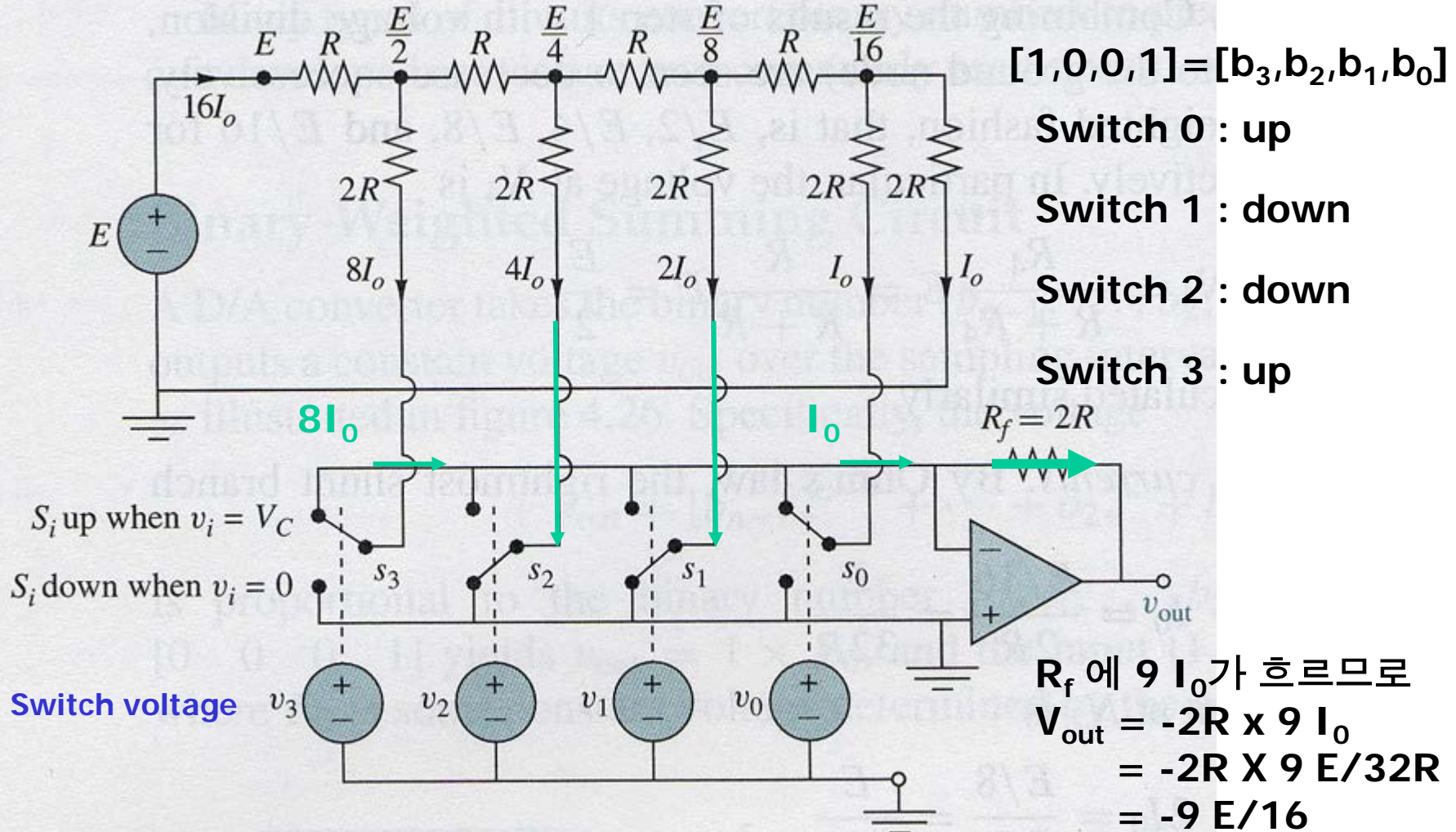
Node 2 voltage : $E/8$

Node 3 voltage : $E/4$

Node 4 voltage : $E/2$

Applications - D/A Converter

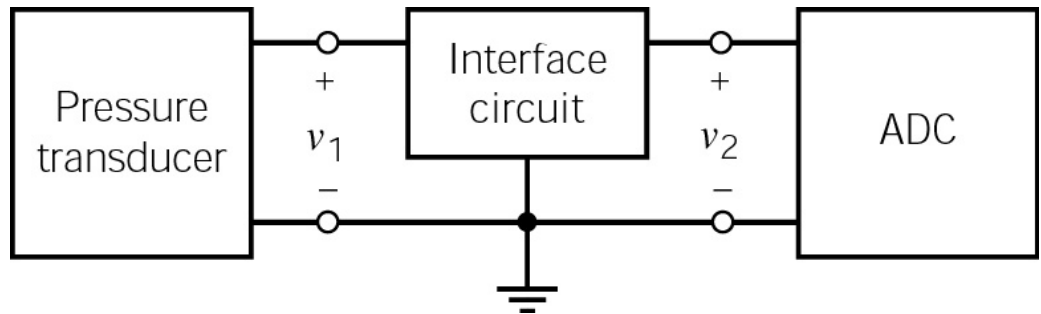
Building-Weighted Summing Circuit (II)



Raymond A. DeCarlo and Pen-Min Lin, Linear Circuit Analysis, 7th edition, Oxford University Press, New York, 2001, p. 154

Transducer Interface Circuit (I)

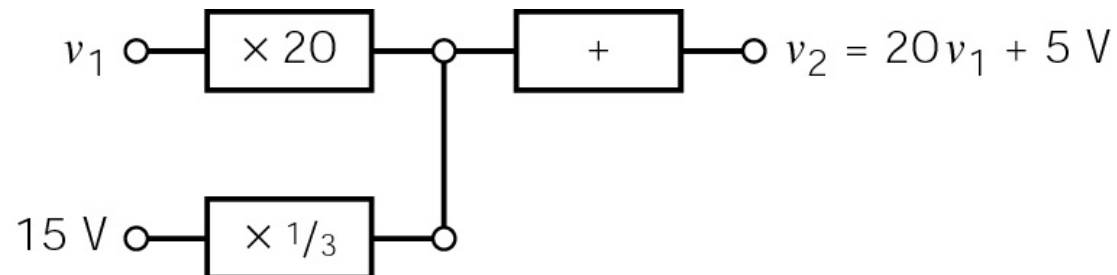
- Pressure sensor 의 출력을 PC에 입력을 하려면 ADC (analog-digital converter)를 이용해야 한다.
- ADC 는 0 ~ 10 V 의 입력을 필요로 하는데 pressure sensor의 출력은 - 250 mV ~ 250 mV 이다.
- 이것을 증폭시켜야 한다.



$$-250 \text{ mV} \leq v_1 \leq 250 \text{ mV}$$

$$0 \text{ V} \leq v_2 \leq 10 \text{ V}$$

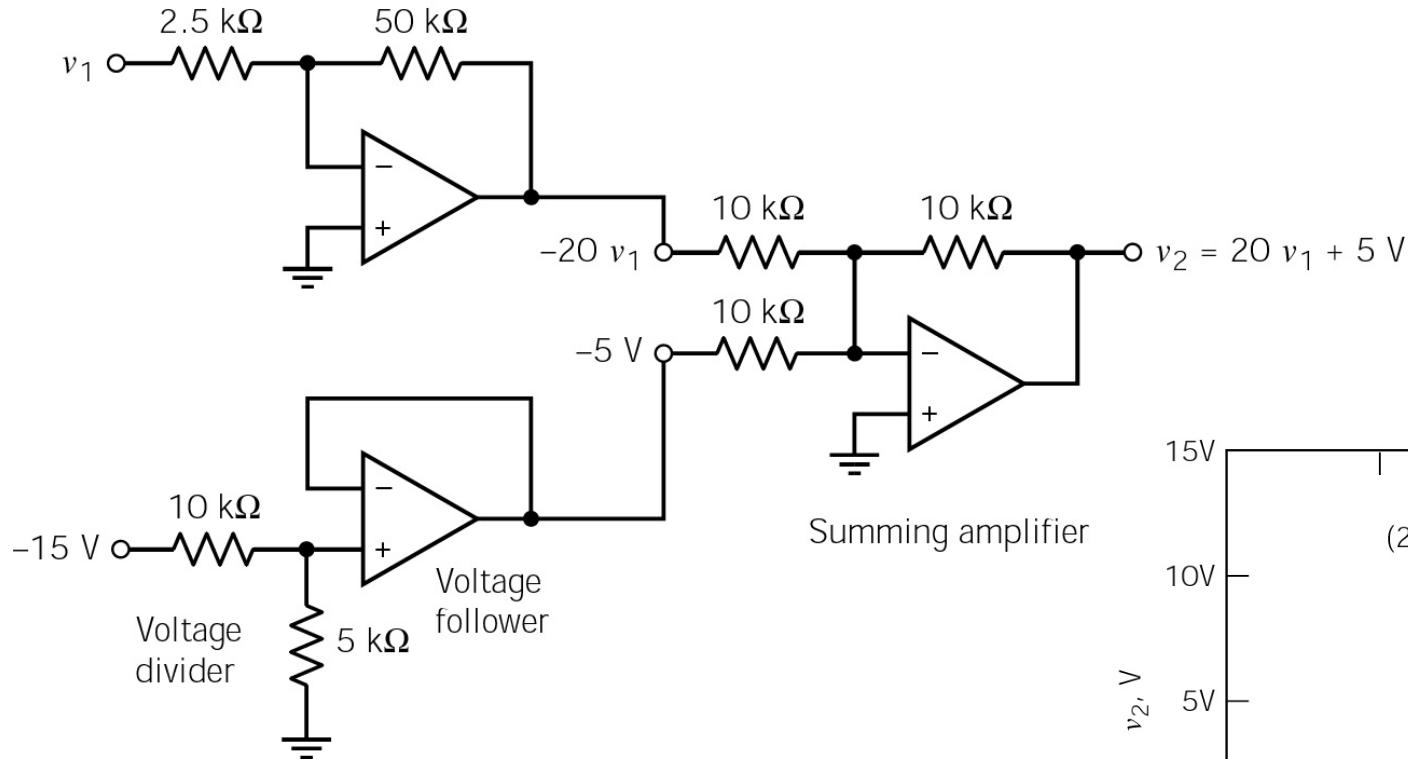
$$v_2 = a \cdot v_1 + b \Rightarrow v_2 = 20v_1 + 5 \text{ V}$$



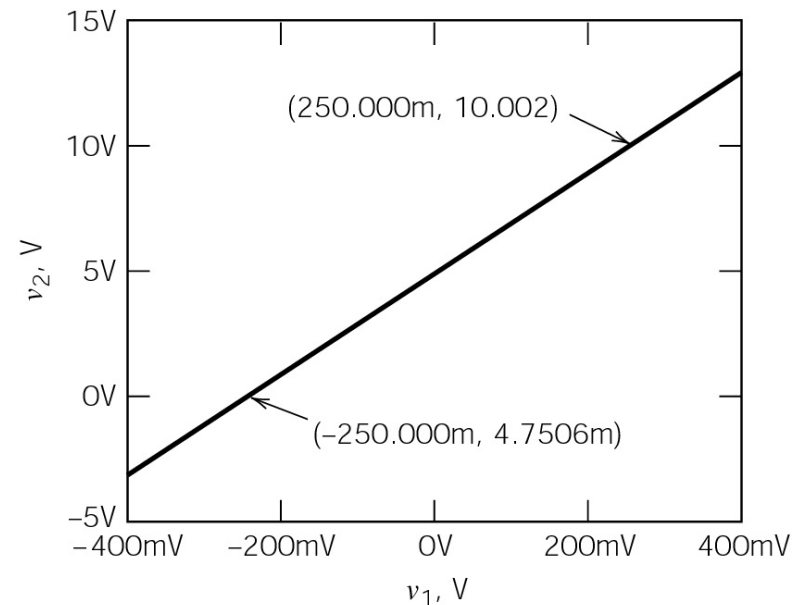
Richard C. Dorf and James A. Svoboda, Introduction to Electric Circuits, 8th edition, John Wiley and Sons, 2010, p. 239-240

Transducer Interface Circuit (II)

Inverting amplifier



- Inverting amplifier, voltage follower, summing amplifier를 이용하여 회로를 완성한다.



Richard C. Dorf and James A. Svoboda, Introduction to Electric Circuits, 8th edition, John Wiley and Sons, 2010, p. 240-241