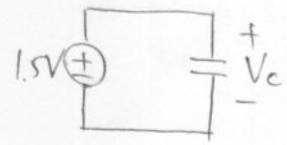
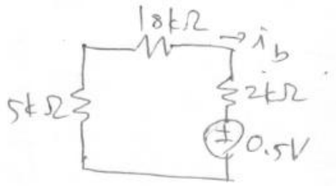
[1] (a) $t<0$:

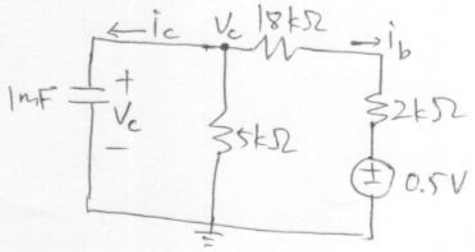




$V_c(0^-) = 1.5\,V \cdots ①$

$i_b(t) = \dfrac{-0.5V}{25k\Omega} = -\dfrac{2}{100}\,mA$

$t \geq 0$:



KVL: $i_c + \dfrac{V_c}{5k\Omega} + \dfrac{V_c - 0.5}{20k\Omega} = 0$

$i_c = \dfrac{1}{1000}\dfrac{dV_c}{dt}$

$\Rightarrow 4\dfrac{dV_c}{dt} + V_c = \dfrac{1}{10}$

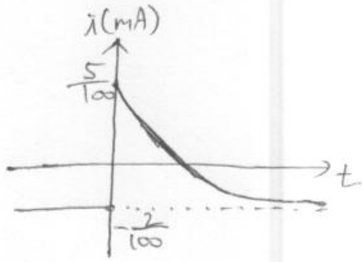
$\Rightarrow V_c = Ae^{-\frac{t}{4}} + \dfrac{1}{10}$

by ①, $V_c(0) = A + \frac{1}{10} = 1.5 \quad \Rightarrow A = 1.4$

$V_c(t) = 1.4e^{-\frac{t}{4}} + 0.1$

$i_b(t) = \dfrac{V_c - 0.5}{20k\Omega} = \dfrac{7e^{-\frac{t}{4}} - 2}{100}\ (mA)$

$\therefore i_b(t) = \begin{cases} \dfrac{7e^{-\frac{t}{4}} - 2}{100}\ (mA) & \text{at } t \geq 0 \\ -\dfrac{2}{100}\ (mA) & \text{at } t < 0 \end{cases}$



(b) Relay current $= 50\,i_b = \dfrac{7}{2}e^{-\frac{t}{4}} - 1\ (mA)$

The lamp turn off at 0.5mA

$\dfrac{7}{2}e^{-\frac{t}{4}} - 1 = 0.5$

$t \doteqdot 3.39$ sec.

[2] (a)

KVL at node ①, ②:

$$\frac{V_1 - V_s}{\alpha R_1} + \frac{V_1 - V_2}{\alpha R_2} = 0$$

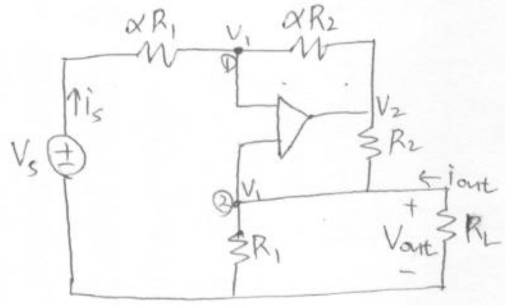
$$\frac{V_1}{R_1} + \frac{V_1 - V_2}{R_2} + \frac{V_1}{R_L} = 0$$

$$\Rightarrow V_1 = -V_s \frac{R_L}{R_1}$$

$$V_2 = -V_s \frac{R_2 R_L}{R_1} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_L} \right)$$

$$i_{out} = -\frac{V_1}{R_L}$$

$$i_{out} = V_s \cdot \frac{1}{R_1}$$



(b)

$$i_{out} = V_s \cdot \frac{1}{R_1}$$

$\Rightarrow$ $R_1 = 500\Omega$. (from condition (ii))

condition (i) : $i_s = \frac{V_s - V_1}{\alpha R_1} = V_s \cdot \frac{R_1 + R_L}{\alpha R_1^2} \leq 0.5\,mA$

$100\Omega \leq R_L \leq 500\Omega$, so $V_s \frac{R_1 + R_L}{\alpha R_1^2} \leq V_s \cdot \frac{R_1 + 500}{\alpha R_1^2}$

$\therefore V_s \cdot \frac{R_1 + 500}{\alpha R_1^2} \leq 0.5$ to satisfy condition (i)

$$5 \cdot \frac{500 + 500}{\alpha \cdot 500^2} \leq 0.5.$$

$$\alpha \geq 40.$$

condition (iii): $|V_2| = V_s \cdot \frac{R_2 R_L}{R_1} \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_L} \right) \leq 20.$

$|V_2|$ is maximum at $R_L = 500\Omega$.

$$\therefore 5 \cdot \frac{R_2 \cdot 500}{500} \left(\frac{1}{500} + \frac{1}{R_2} + \frac{1}{500} \right) \leq 20$$

$$R_2 \leq 750\Omega$$

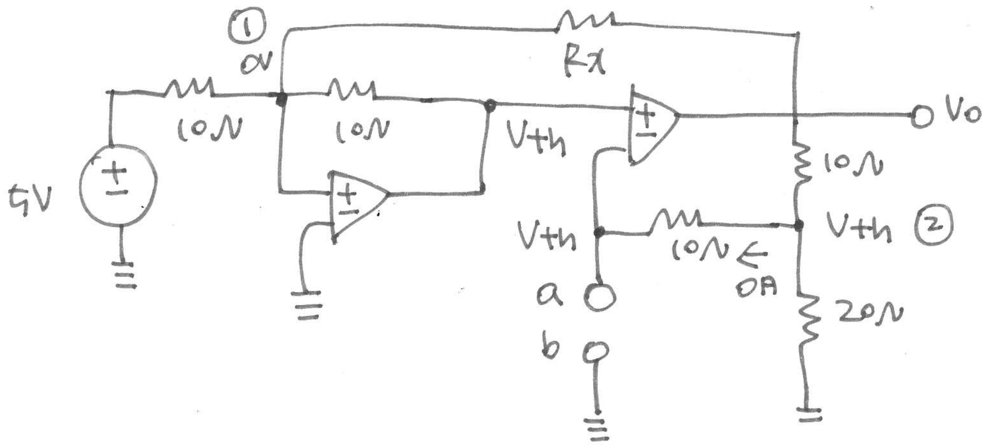
$\therefore$ $R_1 = 500\Omega$

$R_2 \leq 750\Omega$

$\alpha \geq 40$

3

1. (a) 단자 a와 b 사이의 Thevenin 등가회로를 구하시오.



i) Vth 구하기

① $\dfrac{0-5}{10} + \dfrac{0-V_o}{Rx} + \dfrac{0-V_{th}}{10} = 0 \quad \times 10Rx$

$-5Rx - 10V_o - RxV_{th} = 0$

② $\dfrac{V_{th}}{20} + \dfrac{V_{th}-V_o}{10} = 0 \quad \times 20$

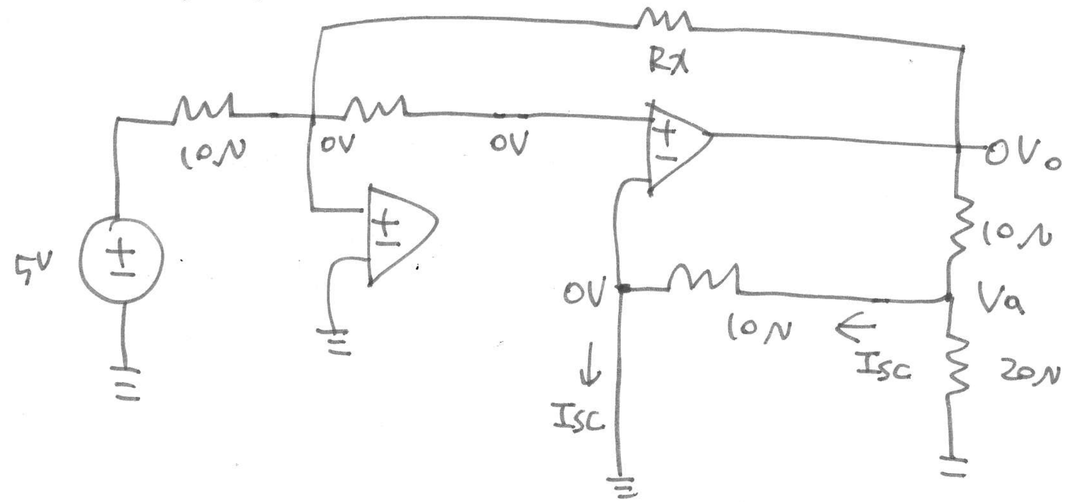
$3V_{th} - 2V_o = 0 \qquad V_o = \dfrac{3}{2}V_{th}$

① & ②

$-5Rx - 15V_{th} - RxV_{th} = 0$

$$V_{th} = \frac{-5Rx}{Rx+15}$$

(T) Isc 구하기



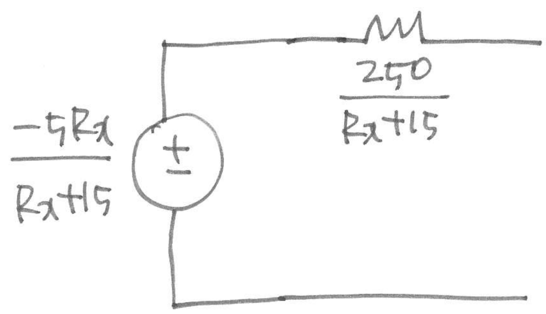
$$\frac{0-5}{10} + \frac{0-V_o}{Rx} = 0 \qquad V_o = -\frac{1}{2}Rx$$

$$\frac{Va}{10} + \frac{Va}{20} + \frac{Va-V_o}{10} = 0 \qquad 2Va + Va + 2Va - 2V_o = 0$$

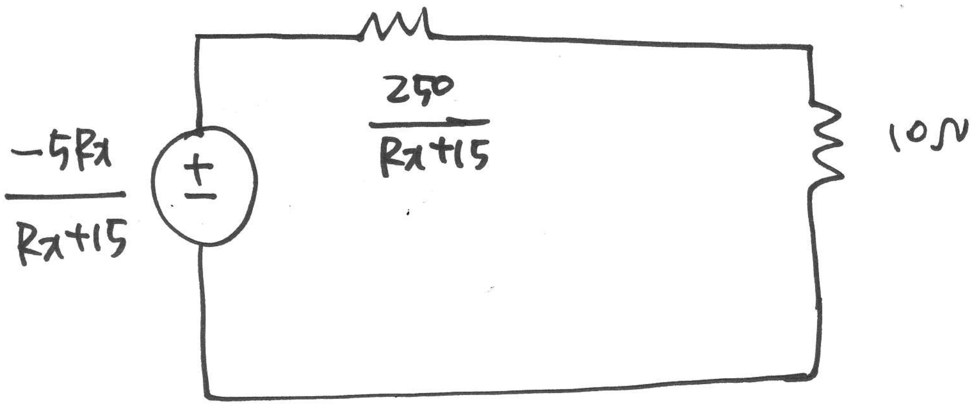
$$5Va = 2V_o$$

$$Isc = \frac{Va}{10} = \frac{2V_o/5}{10} = \frac{1}{25}V_o = -\frac{1}{50}Rx$$

$$R_{th} = \frac{V_{th}}{Isc} = \frac{\frac{-5Rx}{Rx+15}}{-\frac{1}{50}Rx} = \frac{250}{Rx+15}$$



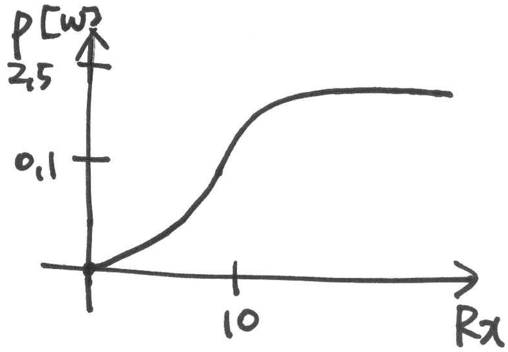
(b)



$$P = \left(\frac{\frac{-5R_x}{R_x+15}}{\frac{250}{R_x+15} + 10} \right)^2 \cdot 10$$

$$= \left(\frac{-R_x}{2R_x+80} \right)^2 \cdot 10$$

$$\frac{dP}{dR_x} = \frac{1600R_x}{(2R_x+80)^3}$$

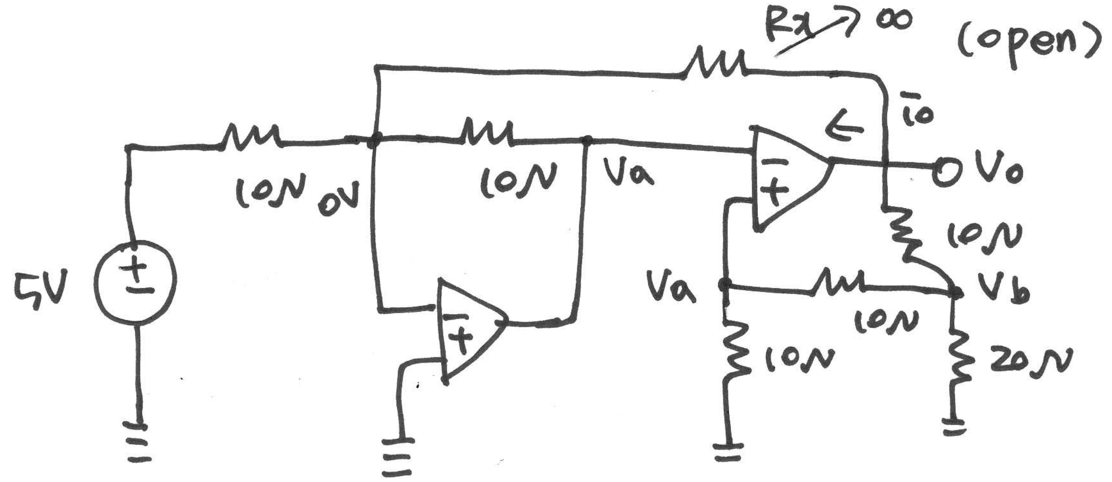


$R_x \to \infty$ $\quad P = 2.5W$

3점 1점

이 외에 답을 구하고자 한 노력이 보이는 경우

1점 / 4점



$$\frac{0-5}{10} + \frac{0-V_a}{10} = 0 \qquad V_a = -5V$$

$$\frac{V_a}{10} + \frac{V_a - V_b}{10} = 0 \qquad V_b = 2V_a = -10V$$

$$i_o + \frac{V_o - V_b}{10} = 0$$

$$\frac{V_b - V_a}{10} + \frac{V_b}{20} + \frac{V_b - V_o}{10} = 0$$

$$2V_b - 2V_a + V_b + 2V_b - 2V_o = 0$$

$$5V_b - 2V_a = 2V_o \qquad V_o = -20V \rightarrow 3점$$

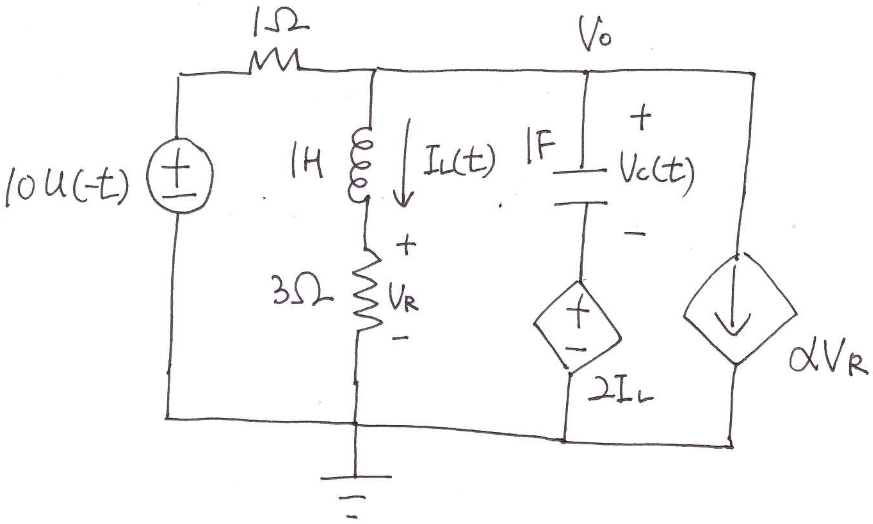
$$i_o = \frac{V_b - V_o}{10} = \frac{10}{10} = 1A \rightarrow 3점$$

답이 틀렸으나 구하고자 하는 방법이 옳은 경우 — i_o, V_o 각 2점

계산을 하고자 하였으나 방정식을 잘못 세운 경우 — i_o, V_o 각 1점

[4]

(1) 회로망의 방정식



① $\frac{dI_L}{dt} + 3I_L = V_c + 2I_L = V_o$

$$V_c = \frac{dI_L}{dt} + I_L \quad — ⓐ$$

V_o에서 KCL

② $\frac{V_o}{1} + I_L + 3\alpha \cdot I_L + \frac{dV_c}{dt} = 0$

①의 식 대입

$$\left(3I_L + \frac{dI_L}{dt}\right) + I_L + 3\alpha I_L + \frac{dV_c}{dt} = 0$$

$$(4+3\alpha) I_L + \frac{dI_L}{dt} + \frac{dV_c}{dt} = 0$$

특성방정식 : $s^2 + 2s + (4+3\alpha) = 0$

(2)

① Overdamped

$\alpha < -1$ and $-1 + \sqrt{-3(\alpha+1)} < 0$

$-\frac{4}{3} < \alpha < -1$

② Critically damped

$-3(\alpha+1) = 0 \qquad \alpha = -1$

③ Underdamped

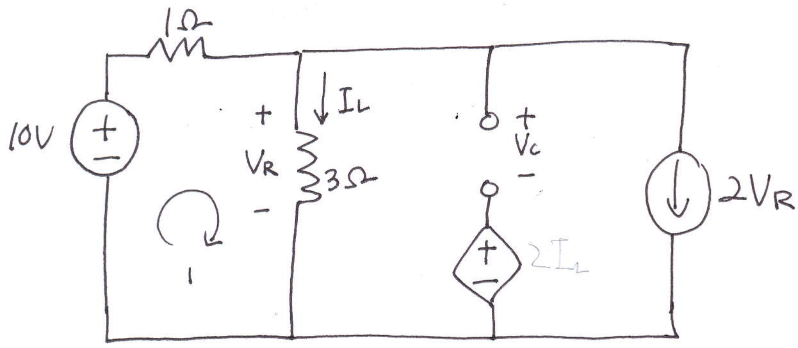
$-3(\alpha+1) < 0$

$\alpha + 1 > 0$

$\alpha > -1$

(3) $\alpha = 2$ 일때 초기조건

$V(0-)$, $I(0-)$ 계산



1에서 KVL

$10 = 1(I_L + 2V_R) + 3I_L \quad V_R = 3I_L \qquad I_L(0^-) = 1A$

$= 10I_L \qquad V_c(0^-) = 1V$

$V_c + 2I_L = 3$

$I_L(t) = A_1 e^{-t}\cos 3t + A_2 e^{-t}\sin 3t = e^{-t}(A_1\cos 3t + A_2 \sin 3t)$

$I_L(0^-) = 1 \quad \Rightarrow A_1 = 1$

$I'_L(t) = -e^{-t}(A_1\cos 3t + A_2\sin 3t) + e^{-t}(-3A_1\sin 3t + 3A_2\cos 3t)$

$I'_L(0) = -A_1 + 3A_2 = 0 \quad A_2 = \frac{1}{3}$

앞의 ⓐ식에서 (회로망 방정식) $V_c(0) = \frac{dI_L(0)}{dt} + I_L(0)$ $\quad I_L = e^{-t}$

$I_L'(0) = 0$

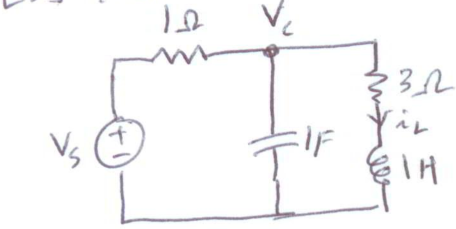
$I_L(t) = e^{-t}\cos 3t + \frac{1}{3}e^{-t}\sin 3t$

$V_c(t) = \frac{dI_L}{dt} + I_L$

$= -\frac{10}{3}e^{-t}\sin 3t + e^{-t}\cos 3t + \frac{1}{3}e^{-t}\sin 3t$

$= -3e^{-t}\sin 3t + e^{-t}\cos 3t$

[5] Method I; Direct Method.



KCL at node V_c.

$$\frac{V_s - V_c}{1} = \frac{dV_c}{dt} + i_L, \quad V_c = 3i_L + \frac{di_L}{dt}$$

$$\Rightarrow V_s = 4i_L + 4\frac{di_L}{dt} + \frac{d^2 i_L}{dt^2}$$

Homogeneous Solution, $i_{L,H}(t) = Ae^{-2t} + Bte^{-2t}$

Particular solution, $i_{L,p}(t) = Ct^2 e^{-2t}$

Total solution, $i_L(t) = i_{L,H}(t) + i_{L,p}(t) = (A + Bt + Ct^2)e^{-2t}$

If $t<0$; $i_L(0^-) = 1A$ $\left(\because V_s(0^-) = 4V,\ i_L(0^-) = \frac{4V}{4\Omega} = 1A\right)$

$$\begin{cases} i_L(0^-) = i_L(0^+) \rightarrow \text{continuity for inductor} \\ V_c(0^-) = V_c(0^+) \Leftrightarrow 3i_L(0^-) + \left.\frac{di_L}{dt}\right|_{t=0^-} = 3i_L(0^+) + \left.\frac{di_L}{dt}\right|_{t=0^+} \end{cases}$$

$$\therefore \left.\frac{di_L}{dt}\right|_{t=0^-} = \left.\frac{di_L}{dt}\right|_{t=0^+} = 0 \quad \left(\because t<0,\ \frac{di_L}{dt} = 0\right)$$

∴ Boundary condition for $i_L(t)$

① $i_L(0) = 1A$

② $i_L'(0) = 0$

⇒ we can find A and B in equation of $i_L(t)$

$A = 1, \quad -2A + B = 0 \quad \therefore B = 2$

In addition, we can also find C by Particular Solution,

$$\frac{d^2}{dt^2}\left(Ct^2e^{-2t}\right) + 4\frac{d}{dt}\left(Ct^2e^{-2t}\right) + 4Ct^2e^{-2t} = e^{-2t} = V_s(t)$$

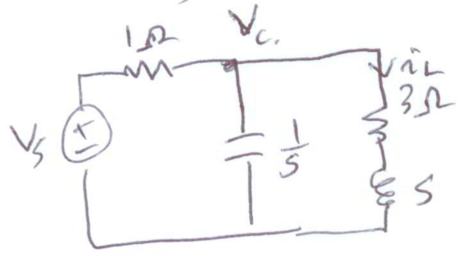
$$\Leftrightarrow \left(2Ce^{-2t} - 8Cte^{-2t} + 4Ct^2e^{-2t}\right) + 4\left(2Cte^{-2t} - 2Ct^2e^{-2t}\right) + 4Ct^2e^{-2t} = e^{-2t}$$

$$\Leftrightarrow 2Ce^{-2t} = e^{-2t} \quad \therefore C = 0.5$$

$$\boxed{\therefore i_L(t) = e^{-2t} + 2te^{-2t} + 0.5t^2e^{-2t} \quad (A)}$$

[5]

Method II; Impedance.



$$i_L = \frac{V_s}{\text{total impedance}} \times \frac{\frac{1}{s}}{3+s+\frac{1}{s}}$$

$$\text{total impedance} = \frac{1}{s} \| (3+s) + 1$$

$$= \frac{\frac{1}{s}(3+s)}{\frac{1}{s}+3+s} + 1 = \frac{s+3}{s^2+3s+1} + 1$$

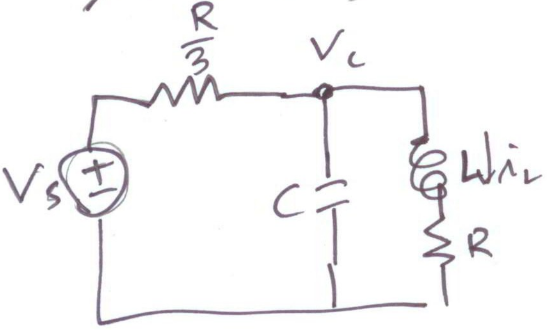
$$= \frac{s^2+4s+4}{s^2+3s+1}$$

$$\therefore i_L = V_s \cdot \frac{\cancel{s^2+3s+1}}{s^2+4s+4} \times \frac{1}{\cancel{s^2+3s+1}}$$

$$\therefore \underline{V_s = i_L(s^2+4s+4)}$$

$\Rightarrow$ Same as method I.

[5]. $1\mu F$으로 풀경우,



$$\frac{V_s - v_c}{\frac{R}{3}} = C\frac{dv_c}{dt} + i_L$$

$$v_c = L\frac{di_L}{dt} + Ri_L$$

$$\Rightarrow V_s = \frac{RC}{3}L\frac{d^2 i_L}{dt^2} + \left(L + \frac{R^2 C}{3}\right)\frac{di_L}{dt} + \frac{4}{3}Ri_L$$

$$\Rightarrow e^{-2t} = 1\mu\frac{d^2 i_L}{dt^2} + (1+3\mu)\frac{di_L}{dt} + 4i_L$$

일반해, $i_{LH} = Ae^{+\alpha t} + Be^{+\beta t}$,

특수해 $i_{L,P} = Ce^{-2t}$

$\therefore i_L(t) = Ae^{+\alpha t} + Be^{+\beta t} + Ce^{-2t}$

$$\alpha = \frac{-(1+3\mu) + \sqrt{(1+3\mu)^2 - 16\mu}}{2\mu} \cong -4$$

$$\beta = \frac{-(1+3\mu) - \sqrt{(1+3\mu)^2 - 16\mu}}{2\mu} \cong -1000000$$

★ 상수 A, B, C 구하기!

Boundary condition; $i_L(0) = 1A$, $\left.\frac{di_L}{dt}\right|_{t=0} = 0$

• 상수 C 구하기; $i_L(t) = Ce^{-2t}$ 대입;

$$(4\mu + (-2-6\mu) + 4)\,C = 1.$$

$\therefore C = \frac{1}{2-2\mu}$, $C \cong 0.5$ (근사)

• 상수 A, B 구하기; Boundary condition 이용.

(i) $i_L(0) = 1 \Leftrightarrow A + B + C = 1 \Rightarrow A + B = 1 - \frac{1}{2-2\mu}$ ....①

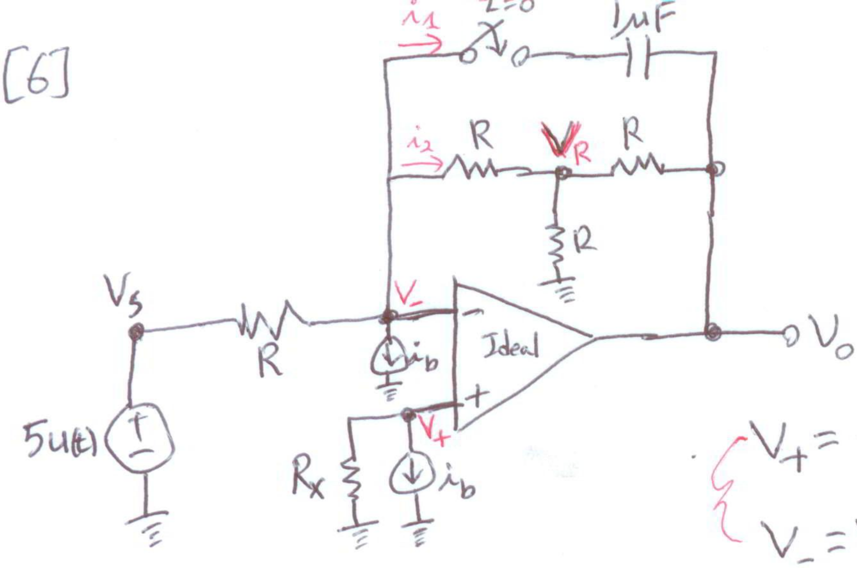
(ii) $i_L'(0) = 0 \Leftrightarrow +\alpha A + \beta B - 2C = 0 \Rightarrow \alpha A + \beta B = 2C = +\frac{1}{1-\mu}$ ....②

①, ② 연립하면, $A = \frac{\beta C + 2C - \beta}{\alpha - \beta}$, $B = \frac{-\alpha C - 2C + \alpha}{\alpha - \beta}$

$A \cong 0.5$, $B \cong -3\mu$

$$\therefore i_L(t) \cong 0.5e^{-\alpha t} - 3\mu e^{-\beta t} + 0.5e^{-2t} \text{ [A]}$$

[6]



$R = 500\Omega$

$i_b = 80nA$

$V_+ = -R_x i_b$

$V_- = V_+ = -R_x i_b$

KCL at node V_- ; $\boxed{\dfrac{V_s - V_-}{R} = i_b + i_1 + i_2}$ ...... 식 (1)

■ $i_1 = C\dfrac{d(V_- - V_o)}{dt} = -C\dfrac{d}{dt}(R_x i_b - V_o) = \boxed{-C\dfrac{dV_o}{dt}}$ ($\because R_x i_b$ is constant)

■ i_2 구하기; $\dfrac{V_- - V_R}{R} = \dfrac{V_R}{R} + \dfrac{V_R - V_o}{R} = i_2$
(KCL at node V_R)

$\Rightarrow V_R = V_- - i_2 R$

$\Rightarrow V_o = 2V_R - Ri_2 = 2V_- - 3i_2 R = -2R_x i_b - 3i_2 R$

$\boxed{\therefore\ i_2 = -\dfrac{2R_x}{3R} i_b - \dfrac{V_o}{3R}}$

$\therefore$ i_1, i_2를 식 (1)에 대입하면,

$V_s - V_- = Ri_b - RC\dfrac{dV_o}{dt} - \dfrac{2}{3}R_x i_b - \dfrac{V_o}{3}$

$\therefore V_s = -RC\dfrac{dV_o}{dt} - \dfrac{V_o}{3} + Ri_b - \dfrac{5}{3}R_x i_b$ → Should be zero

~~[illegible]~~ $V_o(t) = f(t)u(t)$ 꼴을 만족시키기 위해서,

$Ri_b - \dfrac{5}{3}R_x i_b = 0$

$\boxed{\therefore R_x = \dfrac{3}{5}R = 300\Omega}$

한편, $V_s = 5u(t) = -RC\dfrac{dV_o}{dt} - \dfrac{V_o}{3}$, $V_o(0) = 0\,V$ 이용하면,

$\Rightarrow V_o(t) = 15(e^{-\frac{t}{3RC}} - 1)u(t)$ $\boxed{\therefore f(t) = 15(e^{-\frac{2000t}{3}} - 1)}$