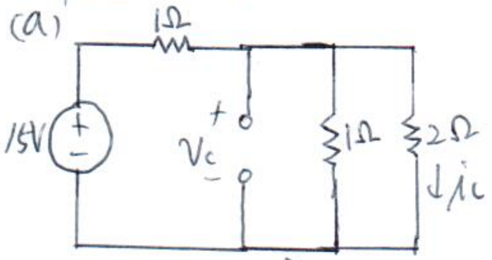
# #1 Solution.

초기조건 $t=0^-$

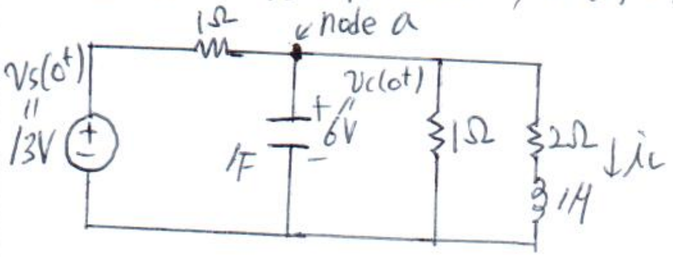
(a)




$v_c(0^-) = \frac{\frac{2}{3}}{\frac{2}{3}+1} \times 15 = 6V \qquad v_c(0^-) = v_c(0^+) = 6V$

$i_L(0^-) = \frac{6}{2} = 3A \qquad i_L(0^-) = i_L(0^+) = 3A$

$t = 0^+ \quad v_s = 13\cos 2t \;, \; v_s(0) = 13V$




node a 에서 KCL

$$\frac{v_c(0^+) - v_s(0^+)}{1} + i_L + C\frac{dv_c}{dt}\bigg|_{0^+} + \frac{v_c(0^+)}{1} = 0$$

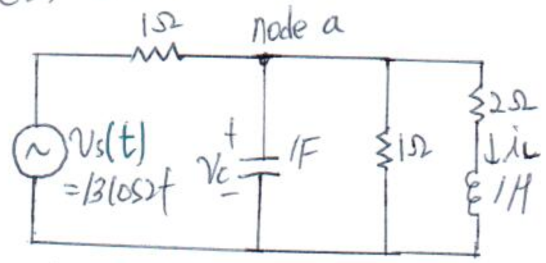
$$\frac{6-13}{1} + 3 + 1\frac{dv_c}{dt}\bigg|_{0^+} + 6 = 0$$

$$\therefore \frac{dv_c}{dt}\bigg|_{0^+} = -2$$

$v_c(0^+) = L\frac{di_L}{dt}\Big|_{0^+} + 2i_L(0^+)$ 이므로

$$\therefore \frac{di_L}{dt}\bigg|_{0^+} = 0$$

(b) $t > 0^+$




다시 node a 에서 KCL

$$\frac{v_c - v_s}{1} + i_L + C\frac{dv_c}{dt} + \frac{v_c}{1} = 0 \quad \cdots\cdots (1)$$

$$v_c = L\frac{di_L}{dt} + 2i_L \quad \cdots\cdots (2)$$

(1)식에 (2) 대입

$$\left(\frac{di_L}{dt} + 2i_L - v_s\right) + i_L + \frac{d}{dt}\left(\frac{di_L}{dt} + 2i_L\right) + \frac{di_L}{dt} + 2i_L = 0$$

$$\therefore \frac{d^2 i_L}{dt} + 4\frac{di_L}{dt} + 5i_L = 13\cos 2t$$

(c) $i_L(t)$

$i_L(t) = i_{Lh} + i_{Lp}$로 구성

$i_{Lh} = ke^{st}$라 하자

$$\frac{d^2 i_{Lh}}{dt^2} + 4\frac{di_{Lh}}{dt} + 5i_{Lh} = 0 \rightarrow s^2 + 4s + 5 = 0$$

특성방정식의 해는 $s = -2 \pm i$

$$i_{Lh} = e^{-2t}(k_1\cos t + k_2 \sin t)$$

$i_{Lp}$

$$\frac{d^2 i_{Lp}}{dt^2} + 4\frac{di_{Lp}}{dt} + 5i_{Lp} = 13\cos 2t$$

$$\left.\begin{aligned} &i_{Lp}\text{의 꼴은 } i_{Lp} = A\cos 2t + B\sin 2t \\ &i_{Lp}' = -2A\sin 2t + 2B\cos 2t \\ &i_{Lp}'' = -4A\cos 2t - 4B\sin 2t \end{aligned}\right\} \text{를 위식에 대입}$$

$$(-4A\cos 2t - 4B\sin 2t) + 4(-2A\sin 2t + 2B\cos 2t) + 5(A\cos 2t + B\sin 2t) = 13\cos 2t$$

$$(A\cos 2t + 8B\cos 2t) + (B\sin 2t - 8A\sin 2t) = 13\cos 2t$$

$$A + 8B = 13, \quad B - 8A = 0$$

$$\therefore A = \frac{1}{5}, \quad B = \frac{8}{5}$$

$$i_{Lp} = \frac{1}{5}\cos 2t + \frac{8}{5}\sin 2t$$

$$i_L(t) = i_{Lh} + i_{Lp} = e^{-2t}(k_1 \cos t + k_2 \sin t) + \frac{1}{5}\cos 2t + \frac{8}{5}\sin 2t$$

$$i_L(0) = 3 = k_1 + \frac{1}{5}$$

$$\therefore k_1 = \frac{14}{5}$$

$$i_L'(t) = -2k_1 \cos t \cdot e^{-2t} - k_1 \sin t \cdot e^{-2t} - 2k_2 \sin t \cdot e^{-2t} + k_2 \cos t \cdot e^{-2t} - \frac{2}{5}\sin 2t + \frac{16}{5}\cos 2t$$

$$i_L'(0) = 0 = -2k_1 + k_2 + \frac{16}{5}$$

$$\therefore k_2 = \frac{12}{5}$$

$$\therefore i_L(t) = e^{-2t}\left(\frac{14}{5}\cos t + \frac{12}{5}\sin t\right) + \frac{1}{5}\cos 2t + \frac{8}{5}\sin 2t$$

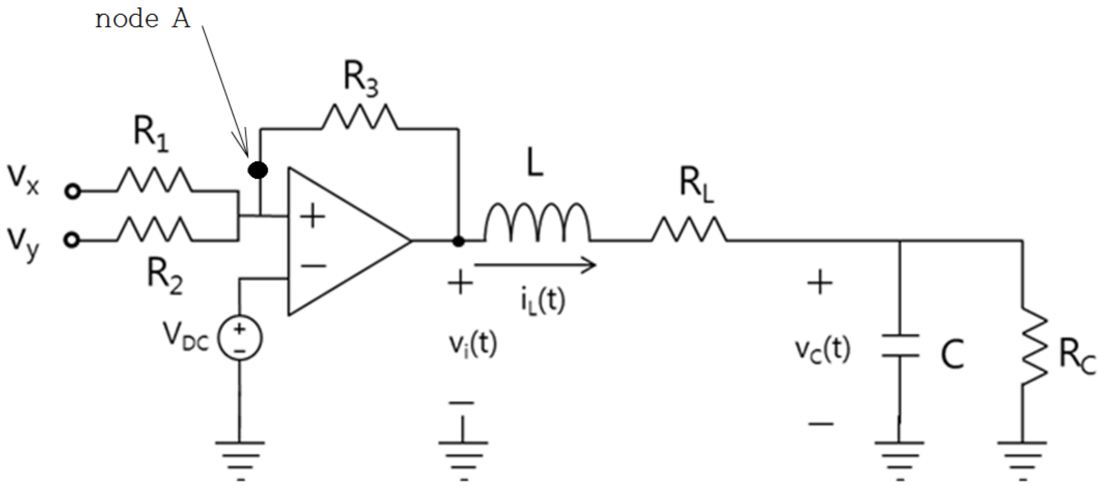
(d) $v_C(t)$

$$v_C = L\frac{di_L}{dt} + 2i_L$$

$$v_C = \frac{d}{dt}\left(e^{-2t}\left(\frac{14}{5}\cos t + \frac{12}{5}\sin t\right) + \frac{1}{5}\cos 2t + \frac{8}{5}\sin 2t\right) + 2\left(e^{-2t}\left(\frac{14}{5}\cos t + \frac{12}{5}\sin t\right) + \frac{1}{5}\cos 2t + \frac{8}{5}\sin 2t\right)$$

$$= e^{-2t}\left(\frac{12}{5}\cos t - \frac{14}{5}\sin t\right) + \frac{14}{5}\sin 2t + \frac{18}{5}\cos 2t$$

2016 1학기 기초회로이론 및 실험 중간고사 2차, 2번문제 해설




**$v_x$ = 1.5u(t) [V], $v_y$ = 3u(t) [V], $V_{DC}$ = 3 [V]**

**$R_1$ = 2kΩ, $R_2$ = 4kΩ, $R_3$ = 8kΩ, $R_L$ = 4 Ω, $R_C$ = 2 Ω**

**C = 0.125 F, L = 4 H**

한글 해설지 (영문 해설지는 뒤에 있습니다/ English solution is provided on the last pages)

(a) $V_i(t)$를 구하시오

op-amp의 특성에 따라 node A의 전위는 $V_{DC}$와 같다.

R1에 흐르는 전류 : $I_{R_1} = \dfrac{V_x - V_{DC}}{R_1}$

R2에 흐르는 전류 : $I_{R_2} = \dfrac{V_y - V_{DC}}{R_2}$

R3에 흐르는 전류 : $I_{R_3} = \dfrac{V_x - V_{DC}}{R_1} + \dfrac{V_y - V_{DC}}{R_2}$

$$
\begin{aligned}
V_i(t) &= V_{DC} - R_3 I_{R_3} \\
&= 3 - 4(1.5u(t) - 3) - 2(3u(t) - 3) \\
&= 21 - 12u(t)\,[V]
\end{aligned}
$$

(2점)

(b) $i_L(0^+)$, $v_c(0^+)$를 구하시오

전류와 전압은 시간에 대해 연속이어야 하므로 $t=0^+$일 때 $i_L$은 $t=0^-$일 때 $i_L$과 같다

$t=0^-$일 때, L과 C가 평형상태에 도달하여 있으므로 L은 도선처럼 작용하고, C는 끊어진 회로로 작용한다.

$$i_L(0^+) = i_L(0^-) = \frac{21}{6} = \frac{7}{2}[A]$$ (2점)

$$v_c(0^+) = v_c(0^-) = \frac{7}{2} \times 2 = 7\,[V]$$ (2점)

미지수: $i_L$, $V_c$

$$9-4\frac{di_L}{dt}-4i_L-V_c=0 \quad \text{——①}$$

$$i_L=\frac{1}{8}\frac{dv_c}{dt}+\frac{1}{2}v_c \quad \text{——②}$$

미분

$$\frac{di_L}{dt}=\frac{1}{8}\frac{d^2v_c}{dt^2}+\frac{1}{2}\frac{dv_c}{dt} \rightarrow \text{① 에 대입}$$

$$9-4\left(\frac{1}{8}\frac{d^2V_c}{dt^2}+\frac{1}{2}\frac{dV_c}{dt}\right)-4\left(\frac{1}{8}\frac{dv_c}{dt}+\frac{1}{2}v_c\right)-V_c=0$$

$$9-\frac{1}{2}\frac{d^2V_c}{dt^2}-2\frac{dV_c}{dt}-\frac{1}{2}\frac{dv_c}{dt}-2V_c-V_c=0$$

$$-\frac{1}{2}\frac{d^2V_c}{dt^2}-\frac{5}{2}\frac{dv_c}{dt}-3V_c=-9$$

$$\frac{d^2V_c}{dt^2}+5\frac{dv_c}{dt}+6V_c=18$$

let $(V_c-3)=P$

$$\frac{d^2V_c}{dt^2}=\frac{d^2P}{dt^2}, \quad \frac{dV_c}{dt}=\frac{dP}{dt}$$

$$\frac{d^2P}{dt^2}+5\frac{dP}{dt}+6P=0$$

trial solution: $P(t)=e^{\lambda t}$

$$\lambda^2e^{\lambda t}+5\lambda e^{\lambda t}+6e^{\lambda t}=0$$

$$\lambda^2+5\lambda+6=0 \rightarrow \lambda=-2,-3$$

$$P(t)=Ae^{-2t}+Be^{-3t}$$

$$V_c(t) = 3 + Ae^{-2t} + Be^{-3t}$$

초기조건 : $V_c(0^+) = 7$ 이므로

$$7 = 3 + A + B \longrightarrow A+B=4. \quad \text{— ⓐ}$$

$$i_L(t) = \frac{1}{8}\frac{dV_c}{dt} + \frac{1}{2}V_c$$

$$= \frac{1}{8}(-2Ae^{-2t} - 3Be^{-3t}) + \frac{1}{2}(3 + Ae^{-2t} + Be^{-3t})$$

$$= \frac{3}{2} + \left(-\frac{A}{4} + \frac{A}{2}\right)e^{-2t} + \left(-\frac{3}{8}B + \frac{B}{2}\right)e^{-3t}$$

$$= \frac{3}{2} + \frac{A}{4}e^{-2t} + \frac{1}{8}Be^{-3t}$$

$i_L(0^+) = \frac{7}{2}$ 이므로

$$\frac{7}{2} = \frac{3}{2} + \frac{A}{4} + \frac{1}{8}B \rightarrow \frac{A}{4} + \frac{1}{8}B = 2$$

$$2A + B = 16 \quad \text{— ⓑ}$$

ⓐ, ⓑ 로부터 $A=12$, $B=-8$

$$\boxed{\begin{aligned} \therefore\ V_c(t) &= 3 + 12e^{-2t} - 8e^{-3t} \\ i_L(t) &= \frac{3}{2} + 3e^{-2t} - e^{-3t} \end{aligned}}$$

Check

$V_c(0^+) = 3 + 12 - 8 = 7$ ✓

$V_c(\infty) = 3$ ✓

$i_L(0^+) = \frac{3}{2} + 3 - 1 = \frac{7}{2}$ ✓

$i_L(\infty) = \frac{3}{2}$ ✓

$f' \xrightarrow{\mathcal{L}} sF - f(0)$

*라플라스 변환으로 푸는 방법

$$9-4\frac{di_L}{dt}-4i_L-V_c=0 \quad \cdots ①$$

$$i_L=\frac{1}{8}\frac{dV_c}{dt}+\frac{1}{2}V_c \quad \cdots ②$$

$$i_L(0^+)=\frac{7}{2}, \quad V_c(0^+)=7$$

① 번식 라플라스변환 : $\frac{9}{s}-4\left(sI_L-\frac{7}{2}\right)-4I_L-V_c=0$

② 번식 라플라스변환 : $I_L=\frac{1}{8}(sV_c-7)+\frac{1}{2}V_c$.

$$(4s+4)I_L+V_c=\frac{9}{s}+14$$

$$-I_L+\left(\frac{s}{8}+\frac{1}{2}\right)V_c=\frac{7}{8}$$

$$\begin{pmatrix} 4s+4 & 1 \\ -1 & \frac{s}{8}+\frac{1}{2} \end{pmatrix}\begin{pmatrix} I_L \\ V_c \end{pmatrix}=\begin{pmatrix} \frac{9}{s}+14 \\ \frac{7}{8} \end{pmatrix}$$

$$\begin{pmatrix} I_L \\ V_c \end{pmatrix}=\frac{1}{(4s+4)\left(\frac{s}{8}+\frac{1}{2}\right)+1}\begin{pmatrix} \frac{s}{8}+\frac{1}{2} & -1 \\ +1 & 4s+4 \end{pmatrix}\begin{pmatrix} \frac{9}{s}+14 \\ \frac{7}{8} \end{pmatrix}$$

$$\left(\begin{array}{l} I_L = \dfrac{2}{s^2+5s+6} \times \dfrac{7s^2+29s+18}{4s} = \dfrac{\frac{7}{2}s^2+\frac{29}{2}s+9}{s(s+2)(s+3)} \\[2em] V_c = \dfrac{2}{s^2+5s+6} \times \dfrac{7s^2+35s+18}{2s} = \dfrac{7s^2+35s+18}{s(s+2)(s+3)} \end{array}\right.$$

i) $$I_L = \frac{\frac{7}{2}s^2+\frac{29}{2}s+9}{s(s+2)(s+3)} = \frac{A}{s} + \frac{B}{s+2} + \frac{C}{s+3}$$

$$= \frac{(A+B+C)s^2 + (5A+3B+2C)s + 6A}{s(s+2)(s+3)}$$

$$\left(\begin{array}{l} A+B+C = \frac{7}{2} \\ 5A+3B+2C = 29/2 \\ 6A = 9 \end{array}\right) \longrightarrow \left(\begin{array}{l} A = 3/2 \\ B = 3 \\ C = -1 \end{array}\right)$$

$$I_L = \frac{3/2}{s} + \frac{3}{s+2} + \frac{-1}{s+3}$$

$$\boxed{i_L(t) = \frac{3}{2} + 3e^{-2t} - e^{-3t} \ (A)}$$

ii) $$V_c = \frac{7s^2+35s+18}{s(s+2)(s+3)} = \frac{D}{s} + \frac{E}{s+2} + \frac{F}{s+3}$$

$$= \frac{(D+E+F)s^2 + (5D+3E+2F)s + 6D}{s(s+2)(s+3)}$$

$\cancel{D+E+F=7}$

$\Rightarrow$

$$\begin{pmatrix} D+E+F = 7 \\ 5D+3E+2F = 35 \\ 6D = 18 \end{pmatrix} \longrightarrow \begin{pmatrix} D=3 \\ E=12 \\ F=-8 \end{pmatrix}$$

$$V_C = \frac{3}{s} + \frac{12}{s+2} + \frac{-8}{s+3}$$

$$\boxed{v_c(t) = 3 + 12e^{-2t} - 8e^{-3t} \ (V)}$$

(d) $i_L(\infty)$, $v_c(\infty)$를 구하시오
시간이 많이 흐르면 회로는 saturated되어 유도기는 도선처럼, 축전기는 끊어진 회로처럼 작용한다.

$$i_L(\infty) = \frac{9\,(V)}{6\,(\Omega)} = \frac{3}{2}\,[A]$$ (2점)

$$v_c(\infty) = \frac{3}{2} \times 2 = 3\,[V]$$ (2점)

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English Version

(a) find Vi(t)

Due to characteristics of ideal op-amp, voltage of node A is same as $V_{DC}$.

voltage of node A : 3V

current through R1 : $I_{R_1} = \dfrac{V_x - V_{DC}}{R_1}$

current through R2 : $I_{R_2} = \dfrac{V_y - V_{DC}}{R_2}$

current through R3 : $I_{R_3} = \dfrac{V_x - V_{DC}}{R_1} + \dfrac{V_y - V_{DC}}{R_2}$

$$\begin{aligned} V_i(t) &= V_{DC} - R_3 I_{R_3} \\ &= 3 - 4(1.5u(t) - 3) - 2(3u(t) - 3) \\ &= 21 - 12u(t)\,[V] \end{aligned}$$

(2points)

(b) find $i_L(0^+)$ and $v_c(0^+)$

Since current and voltage have to be continuous, $i_L(0^+)$ is equal to $i_L(0^-)$ and $v_c(0^+)$ is equal to $v_c(0^-)$.

At $t=0^-$, the circuit is saturated and L can be substituted with a short line and C can substituted with a open circuit.

$i_L(0^+) = i_L(0^-) = \dfrac{21}{6} = \dfrac{7}{2}[A]$ (2points)

$v_c(0^+) = v_c(0^-) = \dfrac{7}{2} \times 2 = 7\,[V]$ (2points)

(c) find $i_L(t)$, $v_c(t)$

Please check the Korean version

(d) find $i_L(\infty)$, $v_c(\infty)$

At $t=\infty$, the circuit is saturated. So L can be substituted with a short line and C can substituted with an open circuit.

$$i_L(\infty) = \frac{9\,(V)}{6\,(\Omega)} = \frac{3}{2}\,[A] \text{ (2points)}$$

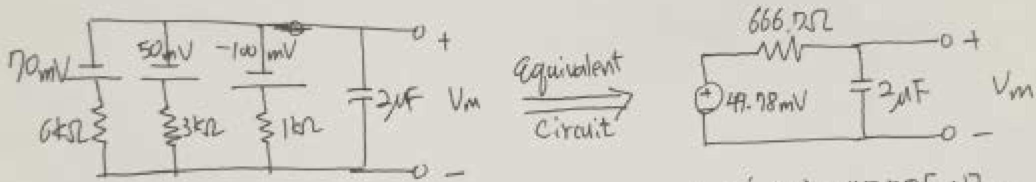
$$v_c(\infty) = \frac{3}{2}\times 2 = 3\,[V] \text{ (2points)}$$

Solution - Mid Term #2

[3]

(a) At steady-state, capacitance becomes open. $\therefore V_m = -70.00$ [mV] ... 2 pts

(b) $V_m(0^+) = V_m(0^-) = -70.00$ [mV] ... 1pt




$\therefore V_m(\infty) = 47.78$ [mV] ... 1pt

$\tau = RC = 1.334 \times 10^{-3}$ [sec] ... 1pt

$\therefore V_m(t) = 47.78 - 117.8\, e^{-\frac{t}{1.334 \cdot 10^{-3}}}$ [mV]

... 1pt

(c) $V_m(3\,\text{msec}) \fallingdotseq 35.35$ mV ... 1pt

$V_m(\infty) = -96.29$ mV, $R_t = 96.77\Omega \rightarrow \tau = RC = 0.1935$ ms ... 1pt

... 1pt

$\therefore V_m(t) = -96.29 + 131.6\, e^{\frac{-(t - 3.000 \cdot 10^{-3})}{0.1935 \cdot 10^{-3}}}$ [mV] ... 1pt

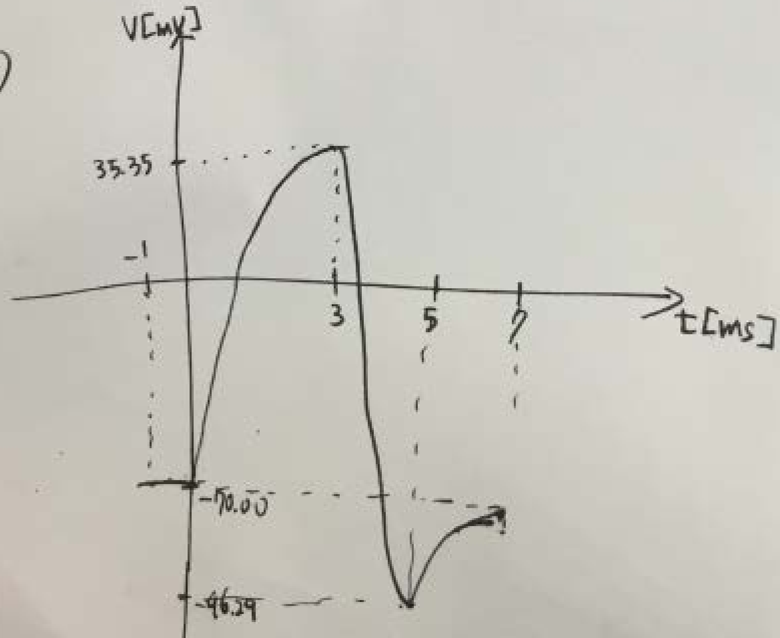
(d) $V_m(5\,\text{msec}) = -96.29$ mV ... 1pt

$V_m(\infty) = -70.00$ mV ... 1pt

$\tau = RC = 0.2000$ ms ... 1pt

$\therefore V_m(t) = -70.00 - 26.29\, e^{-\frac{t - 5.000 \cdot 10^{-3}}{0.2000 \cdot 10^{-3}}}$ [mV] ... 1pt

(e)

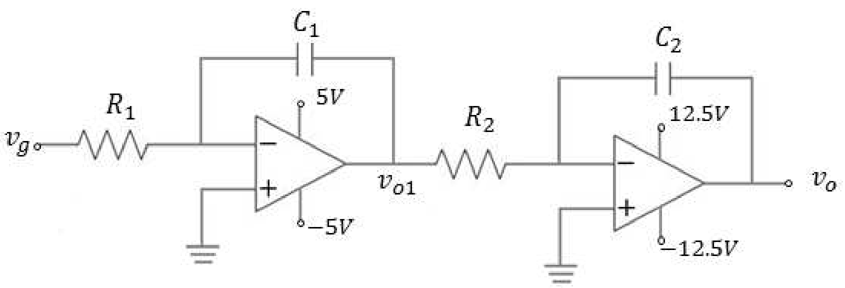



Graph 6점.

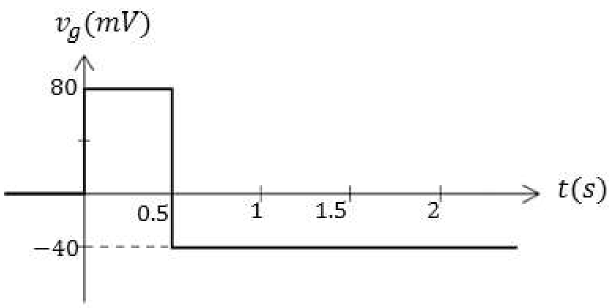
주요 point가 틀리거나 안쓰면 -1점.

* (a) ~ (e) 답에서 유효숫자 틀릴 때마다 -1점

**[4]** For the cascaded integrating amplifier below, answer the following questions. (Suppose the op-amp is ideal) **(20pts)**




(figure 1) Cascaded integrating amplifier




(figure 2) $v_g - t$ graph

(a) Find the differential equations expressing the relations between $v_g$ and $v_{o1}$, $v_g$ and $v_o$. (5pts)

By ideal op-amp & KCL,

$\frac{v_g}{R_1} = -C_1\frac{dv_{o1}}{dt}$ …① (+2pts)

$\frac{v_{o1}}{R_2} = -C_2\frac{dv_o}{dt}$ …②

②를 ①에 대입하면 $v_g = -R_1C_1\frac{d}{dt}\left(-R_2C_2\frac{dv_o}{dt}\right)$

=> $\frac{d^2v_o}{dt^2} = \frac{1}{R_1C_1}\frac{1}{R_2C_2}v_g$ (+3pts, 사소한 실수 -1)

(b) For the given $v_g$ as in figure 2, find each $v_{o1}(t)$, $v_o(t)$ for $0 \le t \le 0.5(s)$ and $0.5 \le t \le t_{sat}$. Suppose the $R_1C_1 = 50ms$, $R_2C_2 = 80ms$, and initial charge of capacitor is zero.($t_{sat}$ is a saturation time when $v_o$ is saturated to $v_{ss}$ or $-v_{ss}$) (10pts)

capacitor의 initial charge 0이므로,
$v_{C_1}(0^-) = v_{C_1}(0^+) = 0$, $v_{C_2}(0^-) = v_{C_2}(0^+) = 0$이고, 따라서 $v_{o1}(0) = 0$, $v_o(0) = 0$
**(초기조건 둘 다 맞은 경우 +1pt)**

① $0 \le t \le 0.5(s)$에서
$\frac{dv_{o1}}{dt} = -\frac{v_g(t)}{R_1C_1} = -1.6$ 이므로 $v_{o1} = -1.6t[V]$ $(v_{o1}(0) = 0)$ **(+2pts)**
$\frac{d^2v_o}{dt^2} = \frac{1}{R_1C_1}\frac{1}{R_2C_2}v_g(t) = 20$ 이므로 $v_o(t) = 10t^2[V]$ $\left(\frac{dv_o}{dt_{t=0}} = -\frac{1}{R_2C_2}v_{o1}(0) = 0,\ v_o(0) = 0\right)$
**(+2pts)**

$t = 0.5s$에서 $v_{o1}(0.5) = -0.8V$, $v_o(0.5) = 2.5V$ **(경계조건 둘 다 맞은 경우 +1pt)**

② $0.5 \le t \le t_{sat}$에서
$\frac{dv_{o1}}{dt} = -\frac{v_g(t)}{R_1C_1} = 0.8$ 이므로 $v_{o1} = 0.8t + A_1$
$v_{o1}(0.5) = 0.4 + A_1 = -0.8$ => $A_1 = -1.2$
$v_{o1}(t) = 0.8t - 1.2[V]$ **(+2pts)**

$\frac{d^2v_o}{dt^2} = \frac{1}{R_1C_1}\frac{1}{R_2C_2}v_g(t) = -10$ 이므로 $v_o(t) = -5t^2 + A_2t + A_3$
$\frac{dv_o(t)}{dt_{t=0.5}} = -\frac{1}{R_2C_2}v_{o1}(0.5) = 10 = -5 + A_2$ => $A_2 = 15$
$v_o(0.5) = -1.25 + 0.5A_2 + A_3 = 2.5$ => $A_3 = -3.75$
$v_o(t) = -5t^2 + 15t - 3.75[V]$ **(+2pts)**

(c) Calculate the $t_{sat}$. (5pts)
$v_o(t) = -5(t-1.5)^2 + 7.5$ 이므로 $t_{sat}$은 $v_o = -V_{ss} = -12.5$에 도달하는 시간 **(+3pts, 이유 없으면 -2pt)**
$v_o(t_{sat}) = -5t_{sat}^2 + 15t_{sat} - 3.75 = -12.5$
=> $t_{sat}^2 - 3t_{sat} - 1.75 = 0$
=> $t_{sat} = 3.5s$ $(t > 0)$ **(+2pts)**