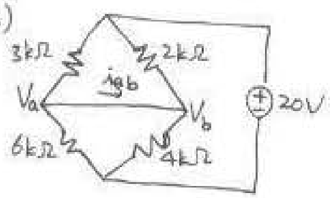
[1] (a) $V_a = 20 \cdot \frac{6}{3+6} = \frac{40}{3}\,V$

$V_b = 20 \cdot \frac{4}{2+4} = \frac{40}{3}\,V$ $\quad \therefore \underline{V_{ab} = 0\,V}$

(b)



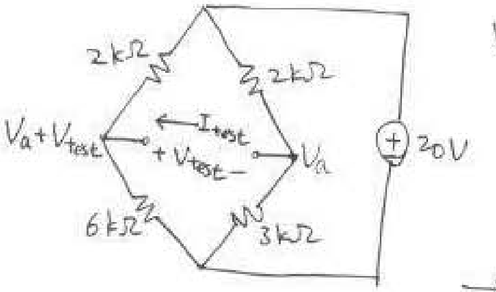
$V_a = V_b$

$\frac{V_a - 20}{3k\Omega} + \frac{V_a}{6k\Omega} + i_{ab} = 0$

$\frac{V_b - 20}{2k\Omega} + \frac{V_b}{4k\Omega} - i_{ab} = 0$

$\rightarrow \underline{i_{ab} = 0\,A}$

(c) ~ (f) : $\Delta R_1 = 1k\Omega$, $\Delta R_2 = 0\,k\Omega$, Thévenin equivalent circuit is,



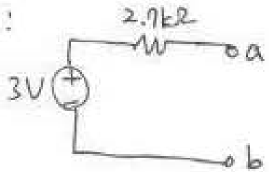
$\frac{V_a + V_{test} - 20}{2k\Omega} + \frac{V_a + V_{test}}{6k\Omega} - I_{test} = 0$

$\frac{V_a - 20}{2k\Omega} + \frac{V_a}{3k\Omega} + I_{test} = 0$

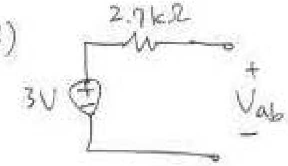
$\rightarrow I_{test} = \frac{V_{test} - 3}{2.7\,k\Omega}$

$\rightarrow V_t = 3V,\ R_t = 2.7k\Omega$

Thévenin equivalent circuit :

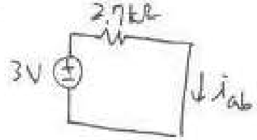


(c)



$\underline{V_{ab} = 3V}$

(d)



$i_{ab} = \frac{3V}{2.7k\Omega} = \frac{10}{9}\,mA$

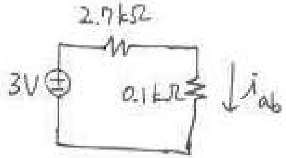
$\rightarrow i_{ab} = \frac{10}{9}\,mA \simeq \underline{1.111\,mA}$

(e)

$V_{ab} = 3V \cdot \frac{0.06M\Omega}{2.7k\Omega + 0.06M\Omega}$

$= \frac{600}{209} \simeq \underline{2.871\,V}$

(f)



$i_{ab} = \frac{3V}{2.7k\Omega + 0.1k\Omega}$

$= \frac{15}{14}\,mA \simeq \underline{1.071\,mA}$

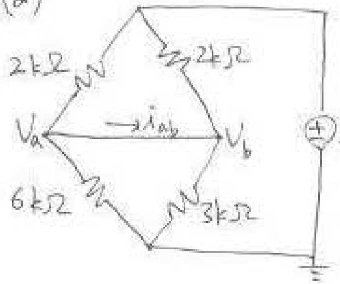
(c) ~ (f) : another solution

(c) $V_a = 20 \cdot \frac{6}{2+6} = 15V$

$V_b = 20 \cdot \frac{3}{2+3} = 12V$

$\therefore V_{ab} = 3V$

(d)



Applying KCL,

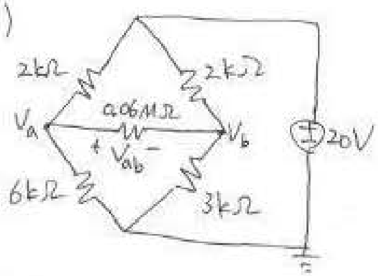
$$\frac{V_a-20}{2k\Omega} + i_{ab} + \frac{V_a}{6k\Omega} = 0$$

$$\frac{V_b-20}{2k\Omega} - i_{ab} + \frac{V_b}{3k\Omega} = 0$$

$$V_a = V_b$$

$$\rightarrow i_{ab} = \frac{10}{9}mA \simeq 1.111\ mA$$

(e)



Applying KCL,

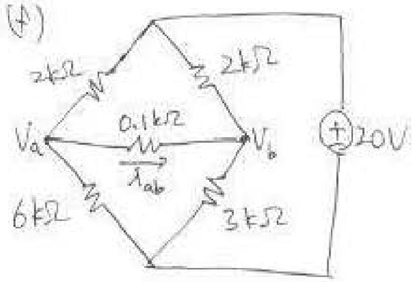
$$\frac{V_a-20}{2k\Omega} + \frac{V_a}{6k\Omega} + \frac{V_a-V_b}{0.06M\Omega} = 0$$

$$\frac{V_b-20}{2k\Omega} + \frac{V_b}{3k\Omega} + \frac{V_b-V_a}{0.06M\Omega} = 0$$

$$\rightarrow V_a = \frac{127920}{8569}V, \quad V_b = \frac{2520}{209}V$$

$$\rightarrow V_{ab} = \frac{600}{209}V \simeq 2.871\ V$$

(f)



applying KCL,

$$\frac{V_a-20}{2k\Omega} + \frac{V_a}{6k\Omega} + \frac{V_a-V_b}{0.1k\Omega} = 0$$

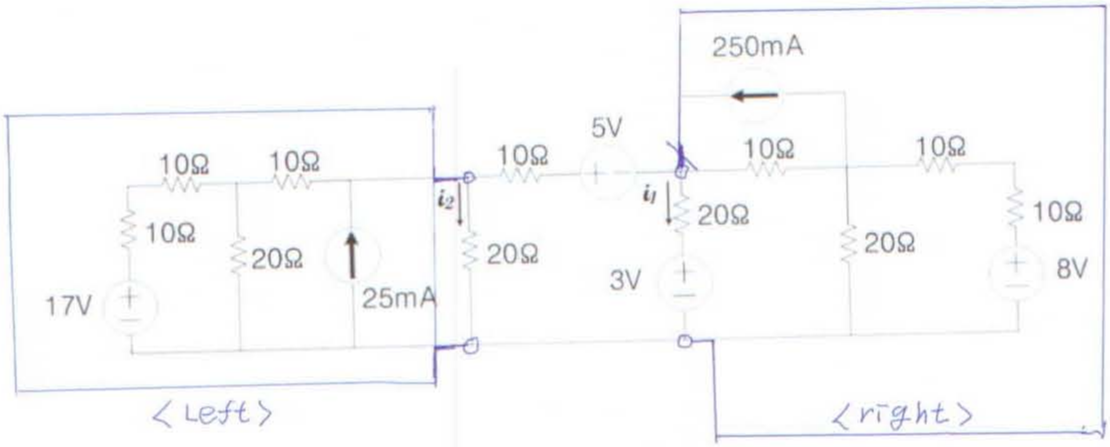
$$\frac{V_b-20}{2k\Omega} + \frac{V_b}{3k\Omega} + \frac{V_b-V_a}{0.1k\Omega} = 0$$

$$i_{ab} = \frac{V_a-V_b}{0.1k\Omega}$$

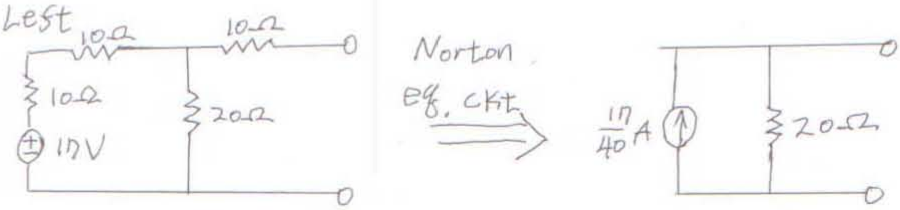
$$\rightarrow i_{ab} = \frac{15}{14}mA \simeq 1.071\ mA$$

2.

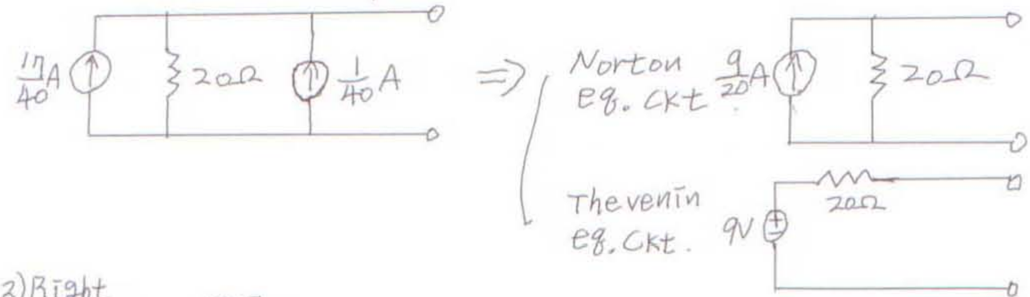
Sol 1)



1) Left

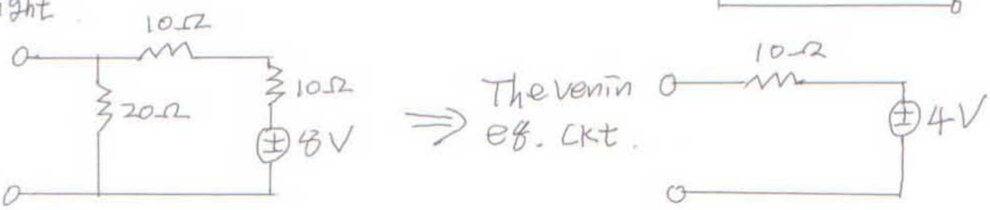


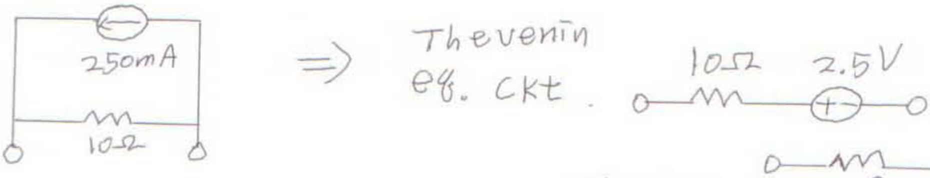
With 25 mA $(=\frac{1}{40}A)$ current source,

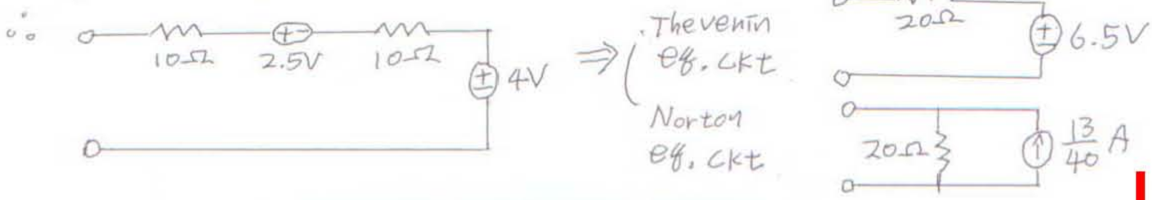


5pts

2) Right

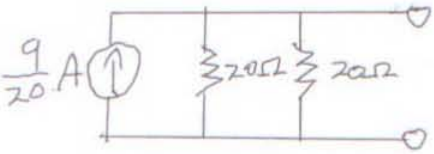




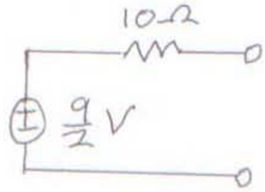


5pts

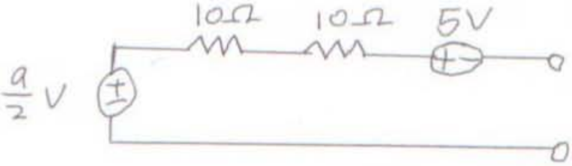
i_1) left with 20Ω resistor.



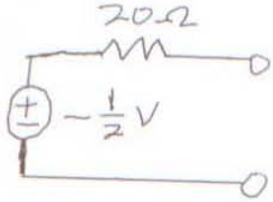
$\Rightarrow$ Thevenin eq. ckt.



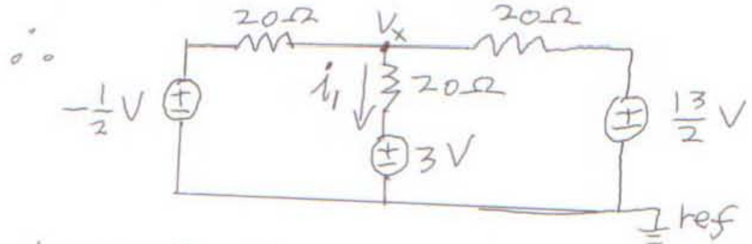
With 10Ω resistor and 5V voltage source,



$\Rightarrow$



$\therefore$



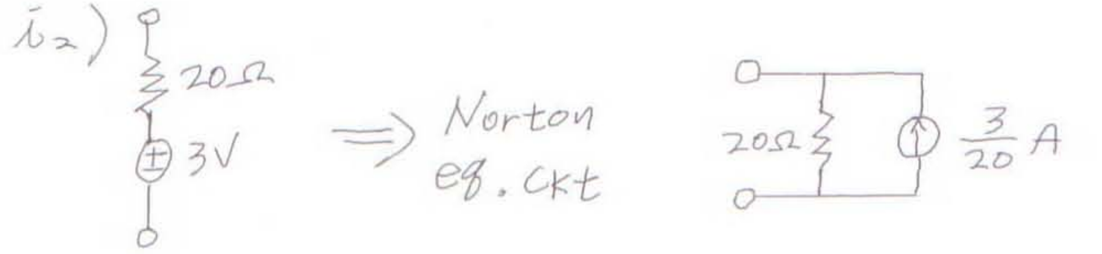
KCL @ node X:

$$\frac{V_x - 3}{20} + \frac{V_x - \frac{13}{2}}{20} + \frac{V_x + \frac{1}{2}}{20} = 0$$

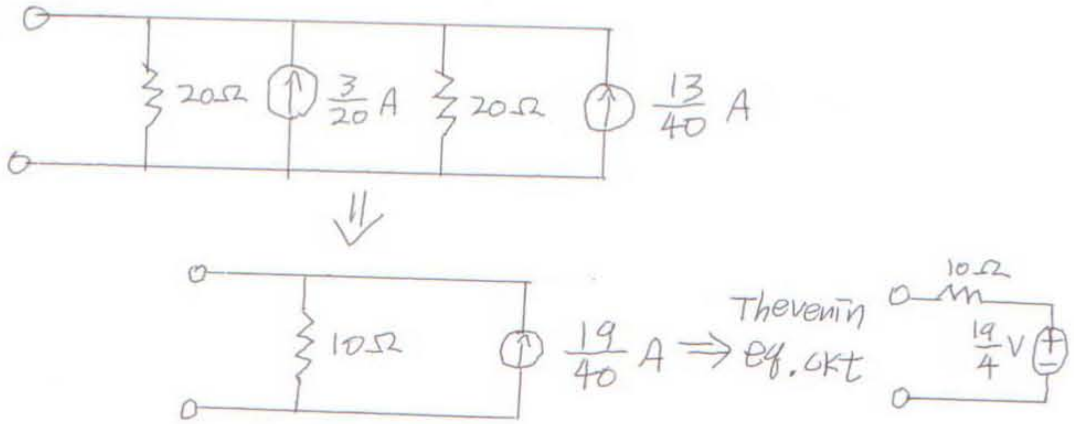
$$V_x = 3$$

$$\therefore \underline{\underline{i_1 = 0\,A}}$$

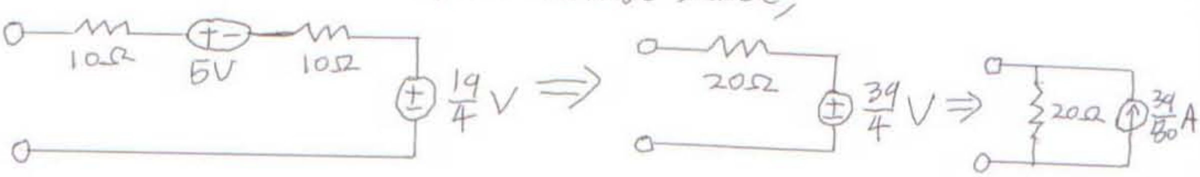
5pts



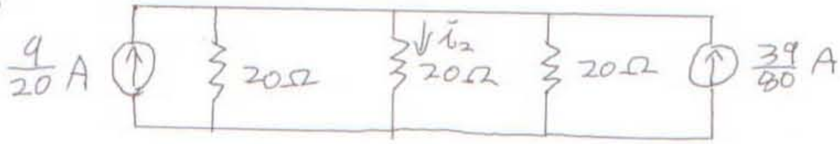
With previous work,



With 10Ω resistor & 5V voltage source,



∴



$$i_2 = \frac{\frac{9}{20} + \frac{39}{80}}{3} = \frac{25}{80}\ \text{A} = 0.3125\ \text{A}$$

5pts

Sol 2) using KVL, KCL without equivalent circuit,

equations : 10 pts

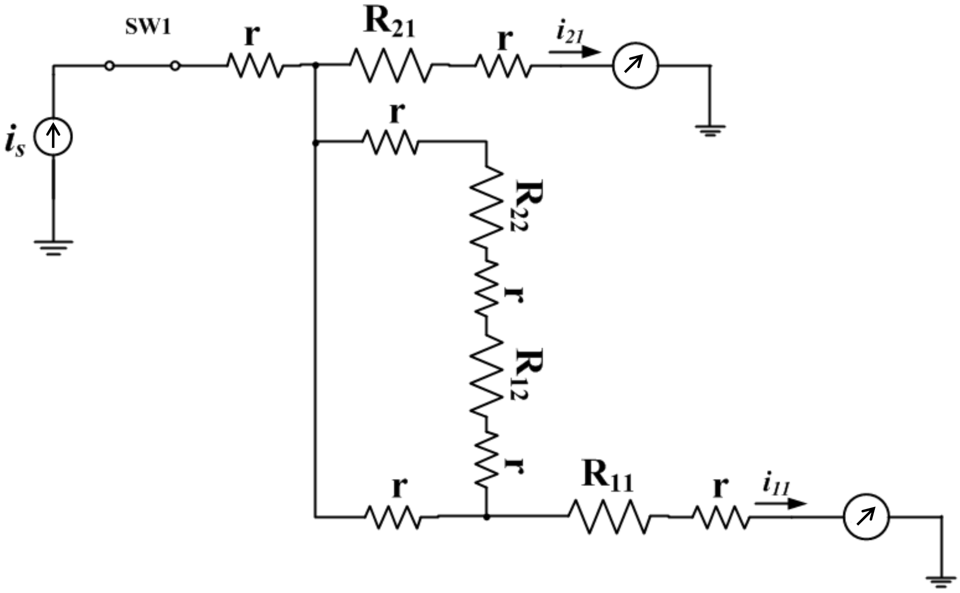
i_1=0A: 5 pts

$i_2=\frac{25}{80}$A=0.3125A: 5 pts

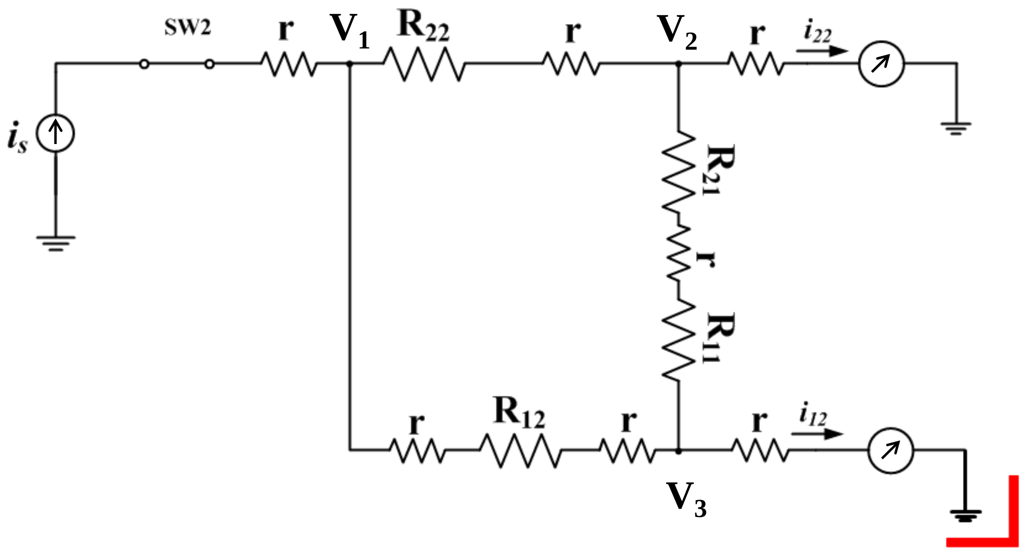
3.

Below figure shows the equivalent circuit of the 2x2 pressure sensor array.

SW1 Connected



SW2 Connected



8 points

Resistivity of conductive line,

$$r = \rho \cdot \frac{l}{s} = \left(10 \times 10^{-6}\ \mu\Omega \cdot cm\right) \times \frac{1\ cm}{10^{-2}\ cm \times 10^{-5}\ cm} = 100\ \Omega$$

1 points

Resistivity of the sensor

Unpressed, $R_{11}=R_{12}=R_{21}=R_{22}=R= 10\ k\Omega$

Pressed, $R_{11}'=R_{12}'=R_{21}'=R_{22}'=0.8R= 8\ k\Omega$

2 points

Case I. All sensors are unpressed

1. SW1 connected

$$\left[\begin{array}{l} i_{21} = \dfrac{(R_{22}+R_{12}+3r)//r+R_{11}+r}{(R_{21}+r)+((R_{22}+R_{12}+3r)//r+R_{11}+r)} \times i_s \\ \quad = \dfrac{(2R+3r)//r+R+r}{(R+r)+((2R+3r)//r+R+r)} \times i_s < 1\,mA \;\Rightarrow\; i_s < 1.99\ mA \\ i_{11} = \dfrac{R_{21}+r}{(R_{21}+r)+((R_{22}+R_{12}+3r)//r+R_{11}+r)} \times i_s \\ \quad = \dfrac{R+r}{(R+r)+((2R+3r)//r+R+r)} \times i_s < 1\,mA \;\Rightarrow\; i_s < 2.01\ mA \end{array}\right] \Rightarrow i_s < 1.99\ mA$$

2. SW2 connected

Using below equations with KCL, we can solve i_s.

$$\left[\begin{array}{l} i_s = \dfrac{V_1-V_2}{R_{22}+r} + \dfrac{V_1-V_3}{R_{12}+2r} \\ \dfrac{V_2-V_1}{R_{22}+r} + \dfrac{V_2-V_3}{R_{11}+R_{21}+r} + i_{22} = 0 \\ \dfrac{V_3-V_1}{R_{12}+2r} + \dfrac{V_3-V_2}{R_{11}+R_{21}+r} + i_{12} = 0 \\ i_{12} = \dfrac{V_3}{r}, i_{22} = \dfrac{V_2}{r}, i_s = i_{12}+i_{22} \end{array}\right] \Rightarrow i_s < 1.99\ mA$$

$\Rightarrow\; i_s < 1.99\ mA \quad \cdots$ ① **2 points**

Case II. Only one cell is pressed

R of the pressed cell is changed. R→ 0.8R

1. SW1 connected

S$_{21}$ pressed,

$i_{21} > 1\,mA \,\&\, i_{11} < 1\,mA \;\Rightarrow\; 1.80\,mA < i_s < 2.26\,mA \;\cdots$ ②   **1 point**

S$_{11}$ pressed,

$i_{11} > 1\,mA \,\&\, i_{21} < 1\,mA \;\Rightarrow\; 1.81\,mA < i_s < 2.23\,mA \;\cdots$ ③   **1 point**

S$_{22}$ or S$_{12}$ pressed,

$i_{11} \,\&\, i_{21} < 1\,mA \;\Rightarrow\; i_s < 1.99\,mA \;\cdots$ ④   **0.5 point**

2. SW2 connected

S$_{21}$ or S$_{11}$ pressed,

$i_{22} \,\&\, i_{12} < 1\,mA \;\Rightarrow\; i_s < 1.99\,mA \;\cdots$ ⑤   **0.5 point**

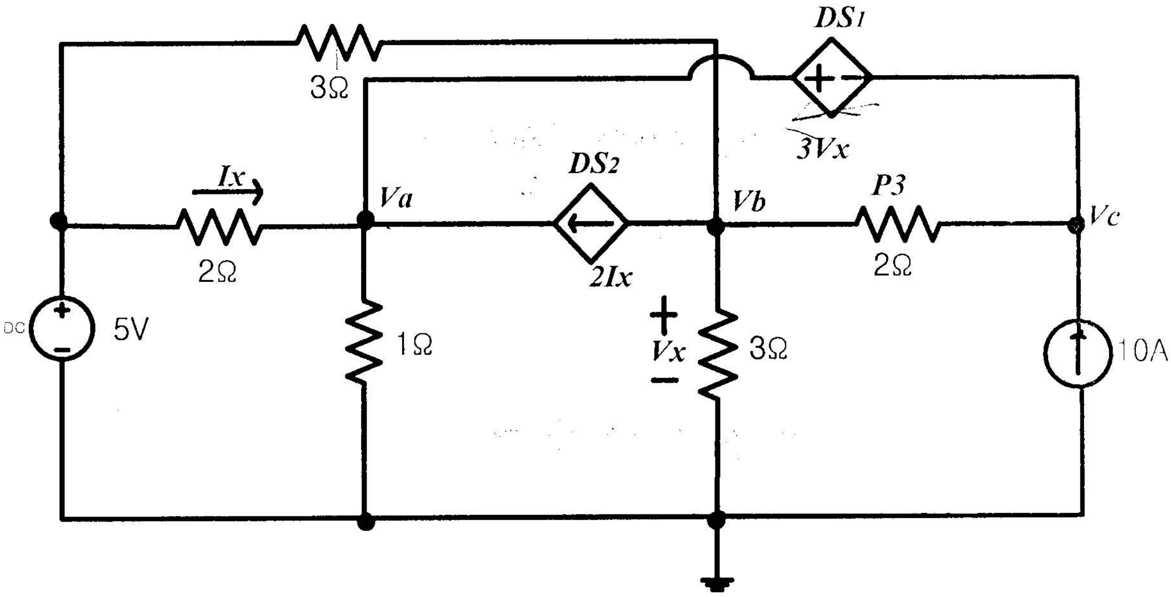
S$_{22}$ pressed,

$i_{22} > 1\,mA \,\&\, i_{12} < 1\,mA \;\Rightarrow\; 1.80\,mA < i_s < 2.25\,mA \;\cdots$ ⑥   **1 point**

S$_{12}$ pressed,

$i_{12} > 1\,mA \,\&\, i_{22} < 1\,mA \;\Rightarrow\; 1.82\,mA < i_s < 2.23\,mA \;\cdots$ ⑦   **1 point**

By ①~⑦, **1.82 mA < i_s < 1.99 mA**   **2 points**



4번

(a) 최소의 변수를 사용하여 회로망 세우기 (8점)

Node voltage Analysis 사용

o supernode 사용

$$-2I_x + \frac{V_a-5}{2} + V_a - 10 + \frac{V_c-V_b}{2} = 0 \quad \cdots ①$$

$I_x = \frac{5-V_a}{2}$ ⋯ ②　①, ② 결합하여 두배 $5V_a - V_b + V_c = 35$ ⋯ ③ ← 3점

o $V_a - V_c = 3V_x$ ⋯ ④　$V_x = V_b$ ⋯ ⑤

④, ⑤식 결합　$V_a - 3V_b - V_c = 0$ ⋯ ⑥ ← 2점

o $$2I_x + \frac{V_b}{3} + \frac{V_b-V_c}{2} + \frac{V_b-5}{3} = 0 \quad \cdots ⑦$$

②, ⑦ 결합　$6V_a - 7V_b + 3V_c = 20$ ⋯ ⑧ ← 3점

식을 세우는 과정 맞으면 만점 (8점)

$6V_a - 4V_b = 35$

$9V_a - 16V_b = 20$

(b) (a)의 식을 연립방정식으로 풀면 각 2점 6점 만점

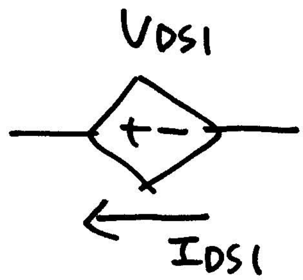
(계산 틀리면 각 1점)

$V_a = 8\,V$ $\quad V_b = \frac{13}{4}V = 3.25V$ $\quad V_c = -\frac{7}{4}V = -1.75V$

(a)의 식이 틀리면 6점 만점중 2점

(c) DS1이 만들어 내는 전력

$V_{DS1} = 3V_x = \frac{39}{4}V$ ... 1점



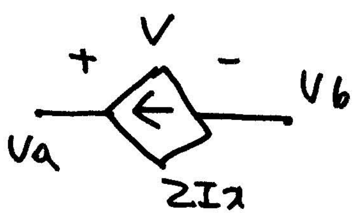
$I_{DS1} = 10 - \frac{V_c - V_b}{2}$ ... 1점

$= 10 - \frac{-\frac{20}{4}}{2} = 12.5A = \frac{25}{2}A$

DS1의 생산전력 $= \frac{39}{4} \times \frac{25}{2} = \frac{975}{8}\,W$

$= 121.875\,W$ ... 1점

DS2이 만들어 내는 전력



$V = V_a - V_b$

$= 8 - \frac{13}{4} = \frac{19}{4}V$ ... 1점

$2I_x = 5 - V_a = -3A$ ... 1점

DS2의 생산 전력 $= \frac{19}{4} \times -3 = -\frac{57}{4}\,W$

$= -14.25\,W$ ... 1점

부호 틀리면 1점씩 감점

과정은 맞으나 (a), (b)에서 구한 값이 이미 틀렸으면 2점/6점

Solution채점 기준

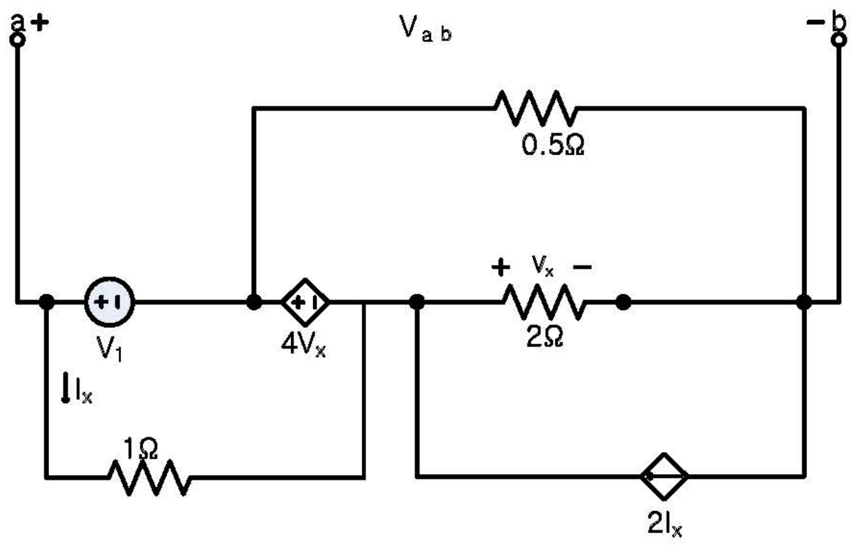
숫자나 답이 틀릴 경우에는 반점만 드렸습니다.

(a)

Superposition 이용한 경우

전원 별로 superposition을 사용하여 구한다.

i) V_1만 고려



Ground에서 KCL 적용

$$\left(\frac{4+1}{0.5}\right)V_x + \frac{V_x}{2} = 2I_x$$

$$\therefore I_x = 5.25V_x$$

(+1점)

위 식을 아래 식에 대입

$$I_x = \frac{V_1 + 4V_x}{1}$$

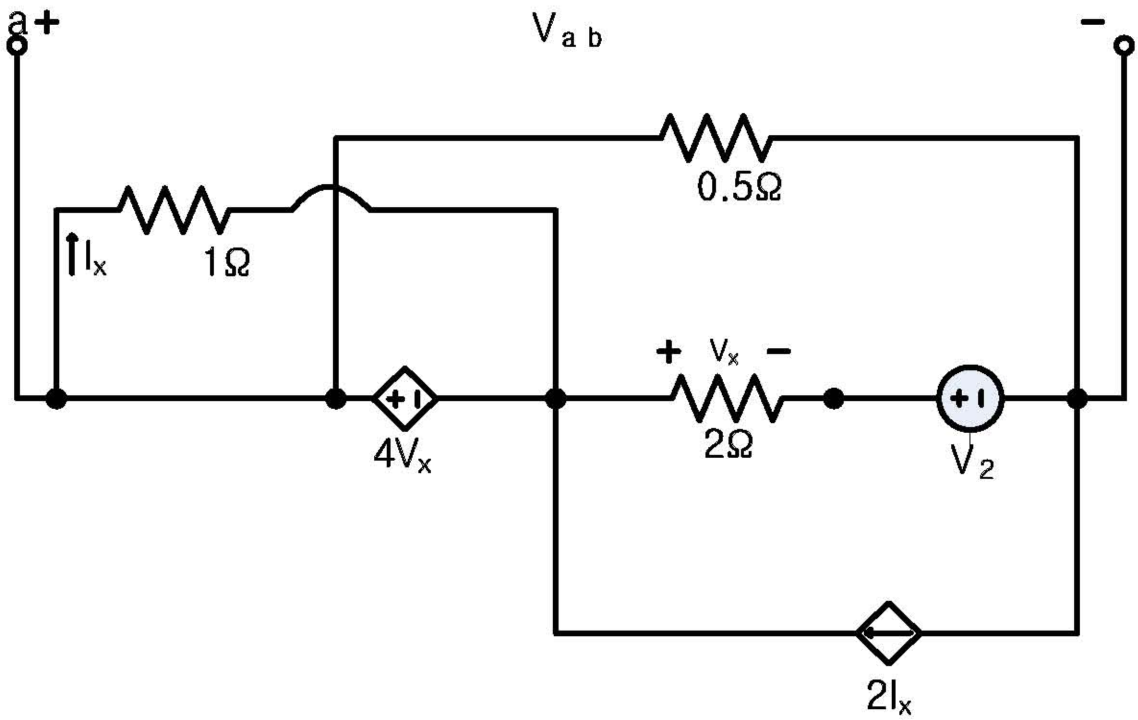
$$\therefore V_x = 0.8V_1$$

(+1점)

$$\therefore V_{ab} = V_1 + 5V_x = 5V_1$$

(+1점)

ii) V_2만 고려



$$I_x = \frac{4V_x}{1}$$

(+1점)

Ground에 KCL을 적용한 뒤 위 식을 대입

$$\frac{5V_x + V_2}{0.5} + \frac{V_x}{2} = 2I_x$$

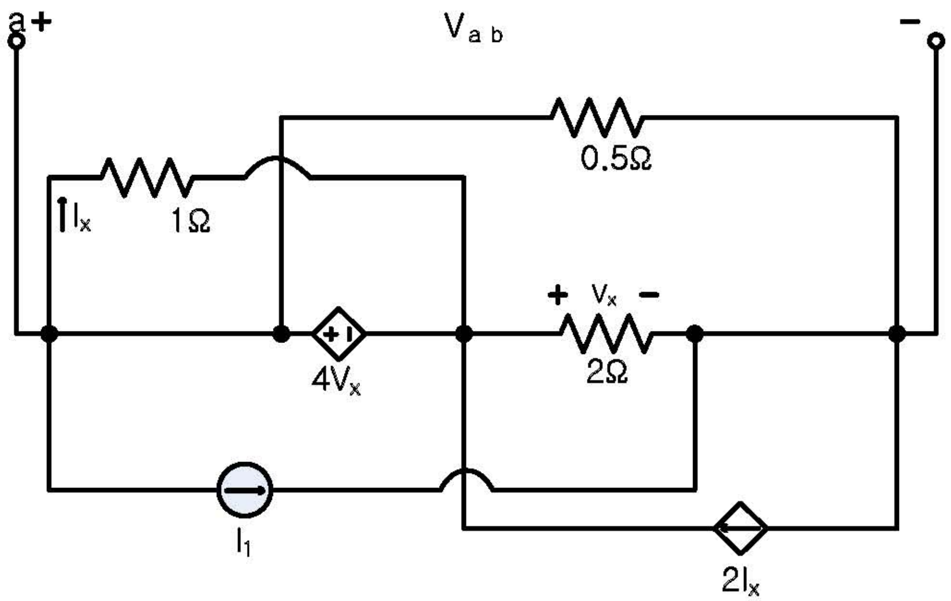
$$\therefore V_x = -0.8V_2$$

(+1점)

$$\therefore V_{ab} = V_2 + 5V_x = -3V_2$$

(+1점)

iii) I_1만 고려



$$I_x = \frac{4V_x}{1}$$

(+1점)

Ground에 KCL을 적용한 뒤 위 식을 대입

$$\frac{5V_x}{0.5} + \frac{V_x}{2} + I_1 = 2I_x$$

$$\therefore V_x = -0.4I_1$$

(+1점)

$$\therefore V_{ab} = 5V_x = -2I_1$$

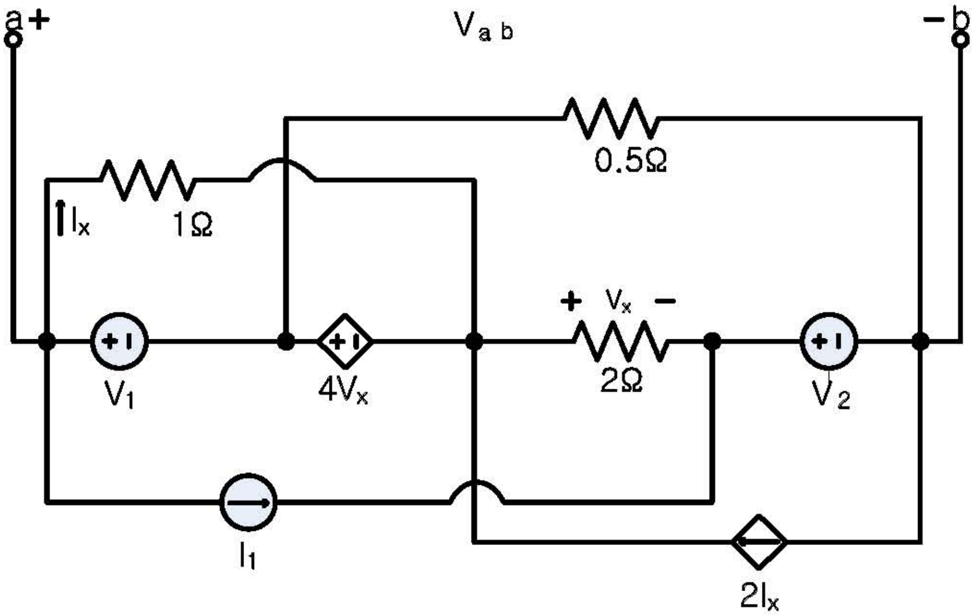
(+1점)

위에서 구한 V_1, V_2, i_1에 대해 superposition을 활용하면 다음과 같이 나온다.

$$V_{ab} = 5V_1 - 3V_2 - 2I_1$$

(+1점)

Superposition 을 이용하지 않은 경우



$$I_x = \frac{V_1 + 4V_x}{1}$$

(+2 점)

B node 에서 supernode 를 활용하여 KCL 적용

$$I_1 + \frac{V_x}{2} + \frac{5V_x + V_2}{0.5} - 2I_x = 0$$

(+2 점)

두 식으로부터 다음과 같이 정리

$$5V_x = 4V_1 - 4V_2 - 2I_1$$

(+2 점)

$$V_{ab} = V_1 + V_2 + 5V_x$$

(+2 점)

$$V_{ab} = 5V_1 - 3V_2 - 2I_1$$

(+2 점)

(b)

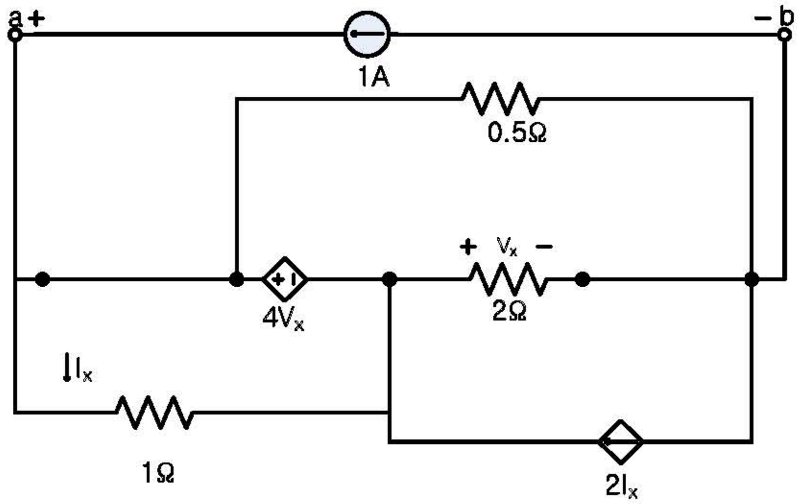
(a)에서 구한 관계식으로 V_{th}를 구할 수 있다.

$$V_{th} = 5V_1 - 3V_2 - 2I_1 = 30 - 12 - 4 = 14V$$

(+2점)

1A 방법 사용

아래 그림과 같이 a-b양단에 1A 전류원을 달아서 양단 전압을 구해서 R_{th}를 구한다.



$$I_x = \frac{4V_x}{1}$$

(+1점)

Node b에서 KCL 적용

$$1 + 2I_x = \frac{V_x}{2} + \frac{5}{0.5}V_x$$

$$\therefore V_x = 0.4V$$

$$V_{ab} = 5V_x = 2V$$

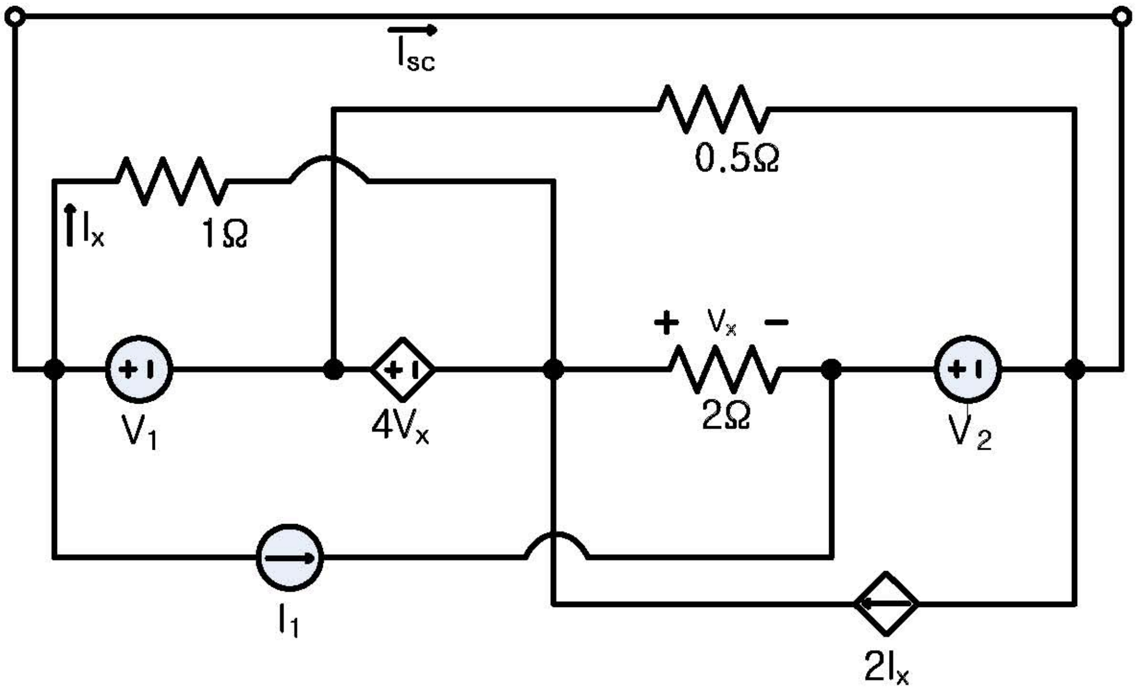
(+1점)

따라서 R_{th}는 다음과 같다.

$$R_{th} = \frac{2V}{1A} = 2\Omega$$

(+2점)

Isc를 구하는 방법 사용



$$I_x = \frac{V_1 + 4V_x}{1}$$

$$5V_x + V_1 + V_2 = 0$$

$$V_x = -2V, I_x = -2A$$

(+1점)

B node에서 KCL적용

$$I_{sc} = 2I_x - I_1 - \frac{V_x}{2} - \frac{5V_x + V_2}{0.5}$$

$$\mathrm{I}_{sc} = 7A$$

(+1점)

$$R_{th} = \frac{14V}{7A} = 2\Omega$$

(+2점)

(c)

$R_{th}=R_L$일 때 최대 전력이 전달된다.(+1점)

$R_L=2\Omega$(+1점)

$$I_{R_L}=\frac{14}{4}=\frac{7}{2}\mathrm{A}$$

$$P_{max}=I_{R_L}^2R_L=\frac{49}{2}W$$

(+2점)