$$= 2\,\frac{(y^2-x^2)dx^2 - 4xy\,dx\,dy + (x^2-y^2)dy^2}{(x^2+y^2)^2}$$

№ 11.5.23

$[y(x)=]\; x^3y^2 - xy^5 + 5x - y = 0$

$y'''(0)$?

$y' \Rightarrow$

$(x^3y^2)' - (xy^5)' + (5x)' - (y)' = 0$

- $3x^2y^2 + 2x^3yy' - y^5 - 5xy^4y' + 5 - y' = 0$

$y'' \Rightarrow$

$6xy^2 + 6x^2yy' + 6x^2yy' + 2x^3(y')^2 + 2x^3yy'' - 5y^4y' - 5y^4y' - 20xy^3(y')^2 - 5xy^4y'' - y'' = 0$

- $6xy^2 + 12x^2yy' + 2x^3(y')^2 + 2x^3yy'' - 10y^4y' - 20xy^3(y')^2 - 5xy^4y'' - y'' = 0$

$y''' \Rightarrow$

$6y^2 + 12xyy' + 12xyy' + 12x^2(y')^2 + 12x^2yy'' + 6x^2(y')^2 + 6x^3y'y'' +$

$+ 6x^2yy'' + 2x^3y'y'' + 2x^3yy''' - 40y^3(y')^2 -$
$- 10y^4y'' - 20y^3(y')^2 - 60xy^2(y')^3 -$
$- 40xy^3y'y'' - 5y^4y'' - 20xy^3y'' -$
$- 5xy^4y''' - y''' = 0$

$6y^2 + 36xyy' + 18x^2(y')^2 + 18x^2yy'' +$
$+ 6x^3y'y'' + 2x^3yy''' - 60y^3(y')^2 -$
$- 15y^4y'' - 60xy^2(y')^3 - 40xy^3y'y'' -$
$- 20xy^3y'' - 5xy^4y''' - y''' = 0$

1) $x = 0, -y = 0 \Rightarrow y = 0$

2) $x = 0, y = 0$ ~~$\Rightarrow$~~
$5 - y' = 0 \Rightarrow y' = 5$

3) $x = 0, y = 0, y' = 5 \Rightarrow$
$y'' = 0$

4) $x = 0, y = 0, y' = 5, y'' = 0 \Rightarrow$
$y''' = 0$

№ 11.5.24

$ye^x + e^y = 0 \quad y'' - ?$

1) $y'e^x + ye^x + e^y y' = 0$

$y''e^x + y'e^x + y'e^x + ye^x + e^y(y')^2 + e^y y'' = 0$

2) $y''e^x + e^y y'' = 2y'e^x - ye^x - e^y(y')^2$

$y'' = -\frac{-2y'e^x + ye^x + e^y(y')^2}{e^x + e^y}$

3) $y'e^x + y'e^y = -ye^x$

$y' = -\frac{ye^x}{e^x + e^y}$

4) $y'' = -\frac{2e^x\left(-\frac{ye^x}{e^x+e^y}\right) + ye^x + e^y\left(-\frac{ye^x}{e^x+e^y}\right)^2}{e^x + e^y} =$

$= \frac{2e^{2x}y}{(e^x+e^y)^2} - \frac{ye^x}{e^x+e^y} + \frac{e^y y^2 e^{2x}}{(e^x+e^y)^3} =$

$= \frac{2e^{2x}y(e^x+e^y) - ye^x(e^x+e^y)^2 + e^{2x}e^y y^2}{(e^x+e^y)^3} =$

$= \frac{2e^{3x}y + 2e^{2x+y}y - ye^{2x}(e^x + 2e^{x+y} + e^{2y}) + e^{2x+y}y^2}{(e^x+e^y)^3} =$

$= \frac{2e^{3x}y + 2e^{2x+y}y - ye^{3x} - 2e^{2x+y}y - ye^{2y+x} + e^{2x+y}y^2}{(e^x+e^y)^3} =$

$= \boxed{\frac{ye^{3x} - ye^{2y+x} + y^2e^{2x+y}}{(e^x+e^y)^3}}$

№ 11.5.31

$y^2 - 3x^2 + 2x + 3y - 9 = 0$

$y', y'' - ?$

1) $2yy' - 6x + 2 + 3y' = 0$

$2yy' + 3y' = 6x - 2$

$y' = \frac{6x-2}{2y+3} = \boxed{\frac{2(3x-1)}{2y+3}}$

2) $2(y')^2 + 2yy'' - 6 + 3y'' = 0$

$3y'' + 2yy'' = 6 - 2(y')^2$

$y'' = \frac{6 - 2(y')^2}{3 + 2y}$

3) $y'' = \frac{6 - 2\left(2 \cdot \frac{3x-1}{2y+3}\right)}{3+2y} =$

$= \frac{6}{3+2y} - 4\,\frac{3x-1}{(3+2y)^2} =$

$= \frac{18 + 12y - 12x + 4}{(3+2y)^2} = \boxed{\frac{12y - 12x + 22}{(2y+3)^2}}$

№ 11.5.35

$$x^2 + 2y^2 + 3z^2 + xy - z - 9 = 0$$

$$\frac{\partial^2 z}{\partial x^2}, \frac{\partial^2 z}{\partial x \partial y}, \frac{\partial^2 z}{\partial y^2} = ?$$

1) $2x + 6z\frac{\partial z}{\partial x} + y - \frac{\partial z}{\partial x} = 0$

$$\frac{\partial z}{\partial x} = \frac{-2x - y}{6z - 1} = -\frac{2x + y}{6z - 1} = \frac{2x + y}{1 - 6z}$$

2) $4y + 6z\frac{\partial z}{\partial y} + x - \frac{\partial z}{\partial y} = 0$

$$\frac{\partial z}{\partial y} = -\frac{4y + x}{6z - 1} = \frac{4y + x}{1 - 6z}$$

3) $\frac{\partial^2 z}{\partial x^2} = \left(\frac{\partial z}{\partial x}\right)' = \left(\frac{2x + y}{1 - 6z}\right)'$

$$2 + 6\left(\frac{\partial z}{\partial x}\right)^2 + 6z\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x^2} = 0$$

$$\frac{\partial^2 z}{\partial x^2} = -\frac{6\left(\frac{\partial z}{\partial x}\right)^2 + 2}{6z - 1} = \frac{6\left(\frac{\partial z}{\partial x}\right)^2 + 2}{1 - 6z}$$

$$\frac{\partial^2 z}{\partial x^2} = \frac{6\left(\frac{2x+y}{1-6z}\right)^2 + 2}{1-6z} =$$

$$= \frac{6(2x+y)^2 + 2(1-6z)^2}{(1-6z)^3} = \boxed{\frac{12x+6y+2-12z}{(1-6z)^2}}$$

4) $$6z\frac{\partial^2 z}{\partial x \partial y} + 6\cdot\frac{\partial z}{\partial y}\cdot\frac{\partial z}{\partial x} + 1 - \frac{\partial^2 z}{\partial x \partial y} = 0$$

$$\frac{\partial^2 z}{\partial x \partial y} = -\frac{6\frac{\partial z}{\partial y}\frac{\partial z}{\partial x} + 1}{6z-1} = \frac{6\frac{\partial z}{\partial x}\frac{\partial z}{\partial y} + 1}{1-6z}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{6\frac{2x+y}{1-6z}\cdot\frac{4y+x}{1-6z} + 1}{1-6z} =$$

$$= \frac{6(2x+y)(4y+x) + 1(1-6z)^2}{(1-6z)^3} =$$

$$= \frac{48xy + 24y^2 + 12x^2 + 6yx + 1 - 12z + 36z^2}{(1-6z)^3} =$$

$$= \boxed{\frac{12x^2 + 24y^2 + 36z^2 + 54xy - 12z + 1}{(1-6z)^3}}$$

5) $4 + 6\left(\frac{\partial z}{\partial y}\right)^2 + 6z\frac{\partial^2 z}{\partial y^2} - \frac{\partial^2 z}{\partial y^2} = 0$

$$\frac{\partial^2 z}{\partial y^2} = \frac{4 + 6\left(\frac{\partial z}{\partial y}\right)^2}{1-6z}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{4 + 6\left(\frac{4y+x}{1-6z}\right)^2}{1-6z} =$$

$$= \frac{4(1-6z)^2 + 6(4y+x)^2}{(1-6z)^3} =$$

$$= \frac{4 - 48z + 144z^2 + 96y^2 + 48xy + 6x^2}{(1-6z)^3} =$$

$$= \frac{6x^2 + 96y^2 + 144z^2 + 48xy - 48z - 4}{(1-6z)^3}$$

№ 11.5.36

по номеру 11.5.35:

$$\frac{\partial^2 z}{\partial x^2} = \frac{12x + 6y + 2 - 12z}{(1-6z)^2} \cdot \frac{6(2x+y)^2 + 2(1-6z)}{(1-6z)^3}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{6(2x+y)(4y+x) + (1-6z)^2}{(1-6z)^3}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{4(1-6z)^2 + 6(4y+x)^2}{(1-6z)^3}$$

Тогда при $x=1$, $y=-2$, $z=1$:

$$\frac{\partial^2 z}{\partial x^2} = \frac{6(2\cdot 1-2)^2 + 2(1-6\cdot 1)^2}{(1-6\cdot 1)^3} =$$

$$= \frac{2\cdot 25}{-125} = -\frac{50}{125} = -\frac{2}{5}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{6(2\cdot 1-2)(4\cdot(-2)+1)+(1-6\cdot 1)^2}{(1-6\cdot 1)^3} =$$

$$= \frac{25}{-125} = -\frac{1}{5}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{4(1-6\cdot 1)^2 + 6(4\cdot(-2)+1)^2}{(1-6\cdot 1)^3} =$$

$$= \frac{100+294}{-125} = -\frac{394}{125}$$