Практика

№ 7.3.11

1) $$\lim_{x\to 0} \frac{\ln \sin 3x}{\ln x} = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{x\to 0} \frac{(\ln \sin 3x)'}{(\ln x)'} = \lim_{x\to 0} \frac{3x \cdot \cos 3x}{\sin 3x} =$$

$$= 3 \lim_{x\to 0} \cos 3x \cdot \lim_{x\to 0} \frac{1}{\left(\frac{\sin 3x}{x}\right)} =$$

$$= 3 \cancel{\lim} \frac{1}{\lim\limits_{x\to 0} \left(\frac{\sin 3x}{\cancel{x}}\right)} = 1$$

2) $$\cancel{\lim_{x\to 0} x^3 = \lim_{x\to 0}}$$

$$\lim_{x\to 0} \frac{x^3}{x - \sin x} = \left[\frac{0}{0}\right]$$

$$\lim_{x\to 0} \frac{x^3}{x - \sin x} = \lim_{x\to 0} \frac{(x^3)'}{(x - \sin x)'} =$$

$$= \lim_{x\to 0} \frac{3x^2}{1 - \cos x} = \left[\frac{0}{0}\right] = \lim_{x\to 0} \frac{(3x^2)'}{(1 - \cos x)'} =$$

$$= \lim_{x\to 0} \frac{6x}{\sin x} = 6 \cancel{\lim} \frac{1}{\lim\limits_{x\to 0} \frac{\sin x}{x}} = 6$$

w7.3.12

$$\lim_{x\to 2}\frac{x^3+x-10}{x^3-3x-2}=\left[\frac{2^3+2-10}{2^3-3\cdot 6-2}=\frac{0}{0}\right]=$$

$$=\lim_{x\to 2}\frac{(x^3+x-10)'}{(x^3-3x-2)'}=\lim_{x\to 2}\frac{3x^2+1}{3x^2-3}=$$

$$=\left[\frac{3\cdot 2^2+1}{3\cdot 2^2-3}=\frac{13}{9}\right]=1\frac{4}{9}$$

w7.3.13

$$\lim_{x\to 1}\frac{\ln x}{x-1}=\left[\frac{0}{0}\right]=\lim_{x\to 1}\frac{(\ln x)'}{(x-1)'}=$$

$$=\lim_{x\to 1}\frac{\frac{1}{x}}{1}=\lim_{x\to 1}\frac{1}{x}=1$$

w7.3.14

$$\lim_{x\to 0}\frac{e^x-1}{\sin x}=\left[\frac{0}{0}\right]=\lim_{x\to 0}\frac{(e^x-1)'}{(\sin x)'}=$$

$$=\lim_{x\to 0}\frac{e^x}{\cos x}=\frac{1}{1}=1$$

w7.3.15

$$\lim_{x\to +\infty}\frac{\ln x}{x}=\left[\frac{\infty}{\infty}\right]=\lim_{x\to +\infty}\frac{(\ln x)'}{(x)'}=$$

$$= \lim_{x\to+\infty} \frac{1}{x} = 0$$

w7.3.16

$$\lim_{x\to+\infty}\frac{e^x}{x^3} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to+\infty}\frac{e^x}{3x^2} = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{x\to\infty}\frac{e^x}{6x} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to+\infty}\frac{e^x}{6} =$$

$$= \frac{+\infty}{6} = +\infty$$

w7.3.17

$$\lim_{x\to0}\frac{\ln x}{ctg\,2x} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to0}\frac{\frac{1}{x}}{-\frac{1}{\sin^2 2x\cdot 2}} =$$

$$= \lim_{x\to0} -\frac{2\sin^2 2x}{x} = \left[\frac{0}{0}\right] =$$

$$= -2\lim_{x\to0}\frac{(\sin^2 2x)'}{(x)'} = -2\lim_{x\to0}\big(2\sin 2x\cdot$$

$$\cdot\cos 2x\cdot 2\big) = -2\lim_{x\to0} 4\sin 2x\cos 2x =$$

$$= -4\lim_{x\to0}\sin 4x = -4\cdot 0 = 0$$

№7.3.18

1) $$\lim_{x\to 0+0} x\ln x = [0\cdot\infty] =$$

$$= \lim_{x\to 0+0} \frac{\ln x}{\left(\frac{1}{x}\right)} = \left[\frac{\infty}{\infty}\right] = \lim_{x\to 0+0} \frac{(\ln x)'}{\left(\frac{1}{x}\right)'} =$$

$$= \lim_{x\to 0+0} \frac{\left(\frac{1}{x}\right)}{\left(-\frac{1}{x^2}\right)} = -\lim_{x\to 0+0} x = 0$$

2) $$\lim_{x\to 1}\left(\frac{1}{\ln x} - \frac{1}{x-1}\right) = [\infty - \infty] =$$

$$= \lim_{x\to 1} \frac{x-1-\ln x}{(x-1)\ln x} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to 1} \frac{(x-1-\ln x)'}{((x-1)\ln x)'} = \lim_{x\to 1} \frac{1-\frac{1}{x}}{\ln x + \frac{x-1}{x}} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to 1} \frac{\left(1-\frac{1}{x}\right)'}{\left(\ln x + \frac{x-1}{x}\right)'} = \lim_{x\to 1} \frac{\frac{1}{x^2}}{\frac{1}{x} + \frac{1}{x^2}} = \frac{1}{2}$$

№7.3.19

$$\lim_{x\to +\infty} x^2\cdot e^{-x} = \cancel{[\infty\cdot 0]} = \lim_{x\to +\infty} \frac{x^2}{e^x} =$$

$$= \left[\frac{\infty}{\infty}\right] = \lim_{x\to +\infty} \frac{(x^2)'}{(e^x)'} = \lim_{x\to +\infty} \frac{2x}{e^x} =$$

$$= \left[\frac{\infty}{\infty}\right] = \lim_{x\to +\infty} \frac{(x\cdot 2)'}{(e^x)'} = \lim_{x\to +\infty} \frac{2}{e^x} = 0$$

№7.3.20

$$\lim_{x\to 0}\left(\frac{1}{x}-\frac{1}{\sin x}\right) = [\infty-\infty] =$$

$$= \lim_{x\to 0}\frac{\sin x - x}{x\sin x} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to 0}\frac{(\sin x - x)'}{(x\sin x)'} = \lim_{x\to 0}\frac{\cos x - 1}{\sin x + x\cos x} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to 0}\frac{(\cos x - 1)'}{(\sin x + x\cos x)'} = \cancel{\lim_{x\to 0}\frac{-\sin x}{\cos x - \cos x + x\sin x}} =$$

$$\cancel{= \lim_{x\to 0}\frac{1}{x} = \infty}$$

$$= \lim_{x\to 0}\frac{-\sin x}{2\cos x - x\sin x} = \frac{0}{2\cdot 1 - 0} = \frac{0}{2} = 0$$

№7.3.21

$$\lim_{x\to\infty} x\left(e^{\frac{1}{x}}-1\right) = \lim_{x\to\infty} xe^{\frac{1}{x}} - x =$$

$$= [\infty-\infty] = \lim_{x\to\infty}\frac{x^2\left(e^{\frac{1}{x}}-1\right)}{x} = \left[\frac{\infty\cdot\infty}{\infty}\right] =$$

$$= \left[\frac{\infty}{\infty}\right] = \lim_{x\to\infty}\frac{\left(x^2\left(e^{\frac{1}{x}}-1\right)\right)'}{x'} =$$

$$= \lim_{x\to\infty}\frac{2x\left(e^{\frac{1}{x}}-1\right) + x^2\cdot e^{\frac{1}{x}}\cdot\left(-\frac{1}{x^2}\right)}{1} =$$

$$= \lim_{x\to\infty} 2x\left(e^{\frac{1}{x}}-1\right) - e^{\frac{1}{x}} = [\infty-\infty].$$

w7.3.22

$$\lim_{x\to 1}\left(\frac{1}{1-x^3}-\frac{1}{1-x^2}\right)=[\infty-\infty]=$$

$$=\lim_{x\to 1}\frac{1-x^2-1+x^3}{(1-x^3)(1-x^2)}=\lim_{x\to 1}\frac{x^3-x^2}{(1-x^3)(1-x^2)}=\left[\frac{0}{0}\right]=$$

$$=\lim_{x\to 1}\frac{(x^3-x^2)'}{((1-x^3)(1-x^2))'}=\lim_{x\to 1}\frac{3x^2-2x}{-3x^2(1-x^2)-2x(1-x^3)}=$$

$$=\left[\frac{3\cdot 1-2\cdot 1}{-3\cdot 1\cdot(1-1)-2\cdot 1\cdot(1-1)}=\frac{1}{0}\right]=\infty$$

w7.3.21

② $\left[t=\frac{1}{x}\Rightarrow t\to 0;\ x=\frac{1}{t}\right]=$

$$=\lim_{t\to 0}\frac{1}{t}(e^t-1)=\lim_{t\to 0}\frac{e^t-1}{t}=\left[\frac{0}{0}\right]=$$

$$=\lim_{t\to 0}\frac{(e^t-1)'}{t'}=\lim_{t\to 0}\frac{e^t}{1}=1$$

w7.3.23

1) $\lim_{x\to 0}x^x=[0^0]$ [также 1^∞; ∞^0] $=$

$$=[y=x^x]=\lim_{x\to 0}\ln y=\lim_{x\to 0}\ln(x^x)=$$

$$=\lim_{x\to 0}x\ln x=0\ (\text{по w7.3.18})\Rightarrow$$

$$\Rightarrow \ln\lim_{x\to 0}y=\lim_{x\to 0}\ln y=0\Rightarrow$$

$\Rightarrow \lim\limits_{x\to 0} y = 1$, т.е. $\lim\limits_{x\to 0} x^x = 1$

2) $\lim\limits_{x\to 0} (\cos x)^{\frac{1}{x}} = [1^\infty] =$

$= [y = (\cos x)^{\frac{1}{x}}] = \lim\limits_{x\to 0} \ln y =$

$= \lim\limits_{x\to 0} \ln (\cos x)^{\frac{1}{x}} = \lim\limits_{x\to 0} \frac{\ln(\cos x)}{x} = \left[\frac{0}{0}\right] =$

$= \lim\limits_{x\to 0} \frac{(\ln(\cos x))'}{x'} = \lim\limits_{x\to 0} (-\operatorname{tg} x) = 0$

$\Rightarrow \ln \lim\limits_{x\to 0} y = \lim\limits_{x\to 0} \ln y = 0 \Rightarrow$

$\Rightarrow \lim\limits_{x\to 0} y = \lim\limits_{x\to 0} (\cos x)^{\frac{1}{x}} = 1$

<u>№7.3.24</u>

$\lim\limits_{x\to 0} x^{\operatorname{tg} x} = [0^0] \Rightarrow$

$\Rightarrow \ln \lim\limits_{x\to 0} x^{\operatorname{tg} x} = [y = x^{\operatorname{tg} x}] =$

$= \ln \lim\limits_{x\to 0} y = \lim\limits_{x\to 0} \ln y = \lim\limits_{x\to 0} \ln(x^{\operatorname{tg} x}) =$

$= \lim\limits_{x\to 0} \operatorname{tg} x \ln x = [0 \cdot \infty] = [$

$= \lim\limits_{x\to 0} \frac{\operatorname{tg} x}{\frac{1}{\ln x}} = \left[\frac{0}{0}\right] =$

$$= \lim_{x\to 0} \frac{(tg x)'}{\left(\frac{1}{\ln x}\right)'} = \lim_{x\to 0} \frac{\frac{1}{\cos^2 x}}{-\frac{1}{\ln^2 x}\cdot\frac{1}{x}} =$$

$$= \lim_{x\to 0}\left(-\frac{x\ln^2 x}{\cos^2 x}\right) = \left[-\frac{0\cdot\infty}{1}\right] = \boxed{\circledcirc}$$

$$(2)\ \lim_{x\to 0} \frac{\ln x}{\frac{1}{tg x}} = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{x\to 0} \frac{(\ln x)'}{\left(\frac{1}{tg x}\right)'} = \lim_{x\to 0} \frac{\frac{1}{x}}{\cos^2 x} = \lim_{x\to 0} \frac{1}{x\cos^2 x} =$$

$$= \left[\frac{1}{0\cdot 1} = \infty\right] = \infty \Rightarrow$$

$$\Rightarrow \ln \lim_{x\to 0} x^{tg x} = \infty$$

№7.3.25

$$\lim_{x\to 0} (\cos 2x)^{\frac{1}{x^2}} = [1^\infty] =$$

$$= \lim_{x\to 0} \ln\left((\cos 2x)^{\frac{1}{x^2}}\right) = \lim_{x\to 0} \frac{1}{x^2}\ln(\cos 2x) =$$

$$= \lim_{x\to 0} \frac{\ln(\cos 2x)}{x^2} = \left[\frac{0}{0}\right] =$$

$$= \lim_{x\to 0} \frac{(\ln(\cos 2x))'}{(x^2)'} = \lim_{x\to 0} \frac{\frac{1}{\cos 2x}\cdot(-\sin 2x)\cdot 2}{2x} =$$

$$= \lim_{x\to 0}\left(\frac{-\sin 2x}{\cos 2x}\cdot\frac{1}{x}\right) = \cancel{\lim_{x\to 0} \frac{tg 2x}{tg x}}$$

$$= \lim_{x\to 0}\left(\frac{\sin(-2x)}{x}\cdot\frac{1}{\cos 2x}\right) = \left[-2\cdot\frac{1}{1}\right] =$$

$$= -2$$

$$\lim_{x\to 0} \ln\left((\cos 2x)^{\frac{1}{x^2}}\right) = -2 \Rightarrow$$

$$\Rightarrow \lim_{x\to 0} (\cos 2x)^{\frac{1}{x^2}} = e^{-2}$$

w7.3.26

$$\lim_{x\to 0}\left(\frac{1}{x}\right)^{x^2} = [\infty^0] = \lim_{x\to 0} \ln\left(\left(\frac{1}{x}\right)^{x^2}\right) =$$

$$= \lim_{x\to 0} x^2 \ln\frac{1}{x} = [0\cdot\infty] =$$

$$= \lim_{x\to 0} \frac{\ln\frac{1}{x}}{\frac{1}{x^2}} = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{x\to 0} \frac{\left(\ln\frac{1}{x}\right)'}{\left(\frac{1}{x^2}\right)'} = \lim_{x\to 0}\frac{-\frac{1}{x}}{-\frac{2}{x^3}} =$$

$$= \lim_{x\to 0} \cancel{\frac{2}{x^2}}\ \frac{x^2}{2} = 0 \Rightarrow$$

$$\Rightarrow \lim_{x\to 0} \ln\left(\left(\frac{1}{x}\right)^{x^2}\right) = 0 \Rightarrow$$

$$\Rightarrow \lim_{x\to 0}\left(\frac{1}{x}\right)^{x^2} = 1$$

№7.3.27

$$\lim_{x\to 0} x^{\frac{1}{1+\ln x}} = [0^0] =$$

$$= \lim_{x\to 0} \frac{1}{1+\ln x}\ln x = \lim_{x\to 0} \frac{\ln x}{1+\ln x} = \left[\frac{\infty}{\infty}\right] =$$

$$= \lim_{x\to 0} \frac{(\ln x)'}{(1+\ln x)'} \lim_{x\to 0} \frac{\frac{1}{x}}{\frac{1}{x}} = 1$$

$$\lim_{x\to 0} \ln\left(x^{\frac{1}{1+\ln x}}\right) = 1 \Rightarrow \lim_{x\to 0} x^{\frac{1}{1+\ln x}} = e$$

~~№7.3.28~~

Фор-ла Тейлора:

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \ldots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n$$

№7.3.29

$P(x) = x^3 + 4x^2 - 6x - 8$, $x_0 = -1$,

$n \leq 3$

$$P(x) = P(-1) + \frac{P'(-1)}{1!}(x+1) + \frac{P''(-1)}{2!}(x+1)^2 +$$

$$+ \frac{P'''(-1)}{3!}(x+1)^3 + o((x+1)^3)$$

$$P(-1) = (-1)^3 + 4\cdot(-1)^2 - 6\cdot(-1) - 8 =$$

$$= -1 + 4 + 6 - 8 = 1$$

$P'(x) = 3x^2 + 8x - 6$

$P'(-1) = 3\cdot(-1)^2 + 8\cdot(-1) - 6 = 3-8-6 =$

$= -11$

$P''(x) = 6x + 8$

$P''(-1) = 6\cdot(-1) + 8 = -6+8 = 2$

$P'''(x) = 6$

$P'''(-1) = 6$

$$P(x) = 1 + \frac{-11}{1}(x+1) + \frac{2}{2}(x+1)^2 +$$

$$+ \frac{6}{6}(x+1)^3 + o(x^3)$$

$o(x^3) = 0$

$\cancel{P(x) = 1 - 11x - 11 + x^2 + 2x + 1}$

$P(x) = 1 - 11(x+1) + (x+1)^2 + (x+1)^3 + \cancel{o(x)}$

<u>№ 7.3.30</u>

$P(x) = x^5 - 3x^4 + 7x + 2$, $x_0 = 2$,

$n \le 5$

$$P(x) = P(2) - \frac{P'(2)}{1!}(x-2) + \frac{P''(2)}{2!}(x-2)^2 +$$

$$+ \frac{P'''(2)}{3!}(x-2)^3 + \frac{P^{(IV)}(2)}{4!}(x-2)^4 +$$

$$+ \frac{P^{(V)}(2)}{5!}(x-2)^5$$

$P(2) = 2^5 - 3 \cdot 2^4 + 7 \cdot 2 + 2 = 32 - 3 \cdot 16 +$
$+ 14 + 2 = 48 - 48 = 0$

$P'(x) = 5x^4 - 12x^3 + 7$

$P'(2) = 5 \cdot 2^4 - 12 \cdot 2^3 + 7 = 5 \cdot 16 - 12 \cdot 8 + 7 =$
$= 80 - 96 + 7 = -9$

$P''(x) = 20x^3 - 36x^2$

$P''(2) = 20 \cdot 2^3 - 36 \cdot 2^2 = 20 \cdot 8 - 36 \cdot 4 =$
$= 160 - 144 = 16$

$P'''(x) = 60x^2 - 72x$

$P'''(2) = 60 \cdot 2^2 - 72 \cdot 2 = 240 - 144 =$
$= 96$

$P^{(IV)}(x) = 120x - 72$

$P^{(IV)}(2) = 120 \cdot 2 - 72 = 240 - 72 =$
$= 168$

$P^{(V)}(x) = P^{(V)}(2) = 120$

$P(x) = -9(x-2) + 8(x-2)^2 + 16(x-2)^3 +$
$+ 7(x-2)^4 + (x-2)^5$

w7.3.32

$f(x) = 2^x$, $x_0 = \log_2 3$

$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \dots + \frac{f^{(n)}(x_0)}{n!}(x-x_0)^n + o((x-x_0)^n)$

$f(x_0) = 2^{\log_2 3} = 3$

$\cancel{f'(x_0) = 2^{\log_2 3} \ln 2 \cdot (\log_2 3)'}$

$f'(x) = 2^x \ln 2$

$f'(x_0) = 2^{\log_2 3} \ln 2 = 3\ln 2$

$f''(x) = 2^x \ln^2 2$

$f''(x_0) = 2^{\log_2 3} \ln^2 2 = 3\ln^2 2$

$f^{(n)}(x) = 2^x \ln^n 2$

$f^{(n)}(x_0) = 3\ln^n 2$

$f(x) = 3 + 3\ln 2 (x - \log_2 3) +$
$+ 3\ln^2 2 (x-\log_2 3)^2 + \dots +$
$+ 3\ln^n 2 (x - \log_2 3)^n + o((x-\log_2 3)^n)$, $x \to \log_2 3$

w7.3.33

$f(x) = \frac{x^2 \ln x}{2}$, $x_0 = 1$

$f(x_0) = \frac{1^2 \ln 1}{2} = 0$

$$f'(x) = \frac{1}{2}(x^2 \ln x)' = \frac{1}{2}\left((x^2)' \ln x + x^2 (\ln x)'\right) =$$

$$= \frac{1}{2}\left(2x \ln x + \frac{x^2}{x}\right) = x \ln x + \frac{x}{2}$$

$$f'(x_0) = 1 \ln 1 + \frac{1}{2} = \frac{1}{2}$$

$$f''(x) = (x \ln x)' + \frac{1}{2} = x' \ln x + x(\ln x)' + \frac{1}{2} =$$

$$= \ln x + \frac{x}{x} + \frac{1}{2} = \ln x + \frac{3}{2}$$

$$f''(x_0) = \ln 1 + \frac{3}{2} = \frac{3}{2}$$

$$f'''(x) = (\ln x)' = \frac{1}{x}$$

$$f'''(x_0) = \frac{1}{1} = 1 \qquad 3 - 3 = 0$$

$$f^{(IV)}(x) = \left(\frac{1}{x}\right)' = -\frac{1}{x^2}$$

$$f^{(IV)}(x_0) = -\frac{1}{1^2} = -1$$

$$f^{(V)}(x) = \left(-\frac{1}{x^2}\right)' = \frac{2}{x^3}$$

$$f^{(V)}(x_0) = \frac{2}{1} = 2$$

$$f^{(VI)}(x) = \left(\frac{2}{x^3}\right)' = -\frac{6}{x^4}$$

$$f^{(VI)}(x_0) = -\frac{6}{1^4} = -6$$

$$f^{(VII)}(x) = \frac{24}{x^5}$$

$f^{(VIII)}(x_0) = \frac{24}{1} = 24 \qquad n(n-1)(n-2)(n-3)!$

при $n \geq 3$

$$\frac{f^{(n)}(x_0)}{n!}(x-x_0)^n = \frac{(-1)^{n-3}\cdot(n-3)!\,(x-x_0)}{n!} =$$

$$= (-1)^{n-3}\frac{1}{n(n-1)(n-2)}$$

$$f(x) = \frac{\frac{1}{2}}{1!}(x-1) + \frac{\frac{3}{2}}{2!}(x-1)^2 +$$

$$+ \frac{1}{3!}(x-1)^3 - \frac{1}{4!}(x-1)^4 + \frac{2}{5!}(x-1)^5 -$$

$$- \frac{6}{6!}(x-1)^6 + \dots + (-1)^{n-3}\frac{(x-1)^n}{n(n-1)(n-2)} + o((x-1)^n),$$
$x \to 1$

№ 7.3.34

$f(x) = e^{2-x}$ до $o(x^4)$

$f'(x) = e^{2-x}\cdot(-1) = -e^{2-x}$

$f''(x) = (-e^{2-x})' = e^{2-x}$

$f'''(x) = -e^{2-x}$

$f^{(IV)}(x) = e^{2-x}$

$$f(x) = e^{2-x_0} - \frac{e^{2-x_0}}{1}(x-x_0) +$$

$$+\frac{e^{2-x_0}}{2}(x-x_0)-\frac{e^{2-x_0}}{6}(x-x_0)+$$

$$+\frac{e^{2-x_0}}{24}(x-x_0)+o((x-x_0)^4)$$

$$f(x)=e^{2-x_0}-e^{2-x_0}(x-x_0)+\frac{1}{2}e^{2-x_0}(x-x_0)^2-$$

$$-\frac{1}{6}e^{2-x_0}(x-x_0)^3+\frac{1}{24}e^{2-x_0}(x-x_0)^4+$$

$$+o((x-x_0)^4)$$

при $x_0=0$

$$f(0)=e^2-e^2x+\frac{e^2x^2}{2}-\frac{e^2x^3}{6}+$$

$$+\frac{e^2x^4}{24}+o(x^4)$$

№ 7.3.35

$f(x)=\arcsin x$ до $o(x^3)$

$$\cancel{f(x)}\ f(0)=\arcsin 0=0$$

$$f'(x)=\frac{1}{\sqrt{1-x^2}}$$

$$f'(0)=\frac{1}{\sqrt{1-0^2}}=\frac{1}{1}=1$$

$$\cancel{f''(x)=\left(\frac{1}{\sqrt{1-x^2}}\right)'=-\frac{1}{2x\sqrt{x}}\cdot 2x=-\frac{1}{\sqrt{x}}}$$

$\cancel{f'''(0) = -\frac{1}{\sqrt{0}} = -\infty}$ ~~$\Rightarrow$ не входит в ряд M.~~

$\cancel{f'''(x) = \left(-\frac{1}{\sqrt{x}}\right)' = \frac{1}{2x\sqrt{x}}}$

$$f''(x) = \left(\frac{1}{\sqrt{1-x^2}}\right)' = -\frac{1}{2(1-x^2)\sqrt{1-x^2}}\cdot 2x =$$

$$= -\frac{x}{(1-x^2)\sqrt{1-x^2}} = -\frac{x}{\sqrt{(1-x^2)^3}}$$

$$f''(0) = -\frac{0}{1} = 0$$

$$f'''(x) = \left(-\frac{x}{\sqrt{(1-x^2)^3}}\right)' = -\frac{x'\sqrt{(1-x^2)^3} - x\left(\sqrt{(1-x^2)^3}\right)'}{(1-x^2)^3} =$$

$$= -\frac{\sqrt{(1-x^2)^3} - \frac{x}{2\sqrt{(1-x^2)^3}}\cdot 3(1-x^2)^2\cdot 2x}{(1-x^2)^3} =$$

$$= -\frac{(1-x^2)^3 - 3(1-x^2)^2\cdot x^2}{(1-x^2)^3\sqrt{(1-x^2)^3}} =$$

$$= -\frac{\cancel{(1-x^2)^2}(1-x^2-3x^2)}{(1-x^2)^{4}\sqrt{1-x^2}} =$$

$$= -\frac{1-4x^2}{\sqrt{1-x^2}\,(1-x^2)^2}$$

$$f'''(0) = -\frac{1-4\cdot 0^2}{\sqrt{1-0^2}\,(1-0^2)^2} = -1$$

$$f(x) = x - \frac{x^3}{6} + o(x^3)$$