Производные (часть 6)

Домашняя работа

№ 11.5.41

$z = \cos(ax + e^y)$

Найти: $\dfrac{\partial^3 z}{\partial x \partial y^2}$

$\dfrac{\partial z}{\partial x} = (\cos(ax+e^y))'_x = -\sin(ax+e^y) \cdot$

$\cdot a = -a\sin(ax+e^y)$

$\dfrac{\partial^2 z}{\partial x \partial y} = (-a\sin(ax+e^y))'_y =$

$= -a\cos(ax+e^y) \cdot e^y = -ae^y\cos(ax+e^y)$

$\dfrac{\partial^3 z}{\partial x \partial y^2} = (-ae^y\cos(ax+e^y))'_y =$

$= -ae^y\cos(ax+e^y) + ae^y\sin(ax+e^y) \cdot$

$\cdot e^y = ae^y(e^y\sin(ax+e^y) - \cos(ax+e^y))$

№ 11.5.42

$z = \dfrac{x^4 - 8xy^3}{x - 2y}$; Найти: $\dfrac{\partial^3 z}{\partial x^2 \partial y}$

$$z = \frac{x^4 - 8xy^3}{x-2y} = \frac{x(x^3 - (2y)^3)}{x-2y} =$$

$$= \frac{x(x-2y)(x^2+2xy+4y^2)}{x-2y} =$$

$$= x^3 + 2x^2y + 4xy^2$$

$$\frac{\partial z}{\partial x} = 3x^2 + 4xy + \cancel{8} 4y^2$$

$$\frac{\partial^2 z}{\partial x^2} = 6x + 4y$$

$$\frac{\partial^3 z}{\partial x^2 \partial y} = 4$$

<u>w 11.5.43</u>

$$u = x\ln(xy), \quad \frac{\partial^3 u}{\partial x^2 \partial y} = ?$$

$$u = x\ln(xy) = x\ln x + x\ln y$$

$$\frac{\partial u}{\partial x} = \ln x + \frac{x}{x} + \ln y = \ln x + \ln y + 1$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{x}$$

$$z = \frac{x^4 - 8xy^3}{x-2y} = \frac{x\left(x^3 - (2y)^3\right)}{x-2y} =$$

$$= \frac{x(x-2y)(x^2+2xy+4y^2)}{x-2y} =$$

$$= x^3 + 2x^2y + 4xy^2$$

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№ 11.5.43

$$u = x\ln(xy), \quad \frac{\partial^3 u}{\partial x^2 \partial y} = ?$$

$$u = x\ln(xy) = x\ln x + x\ln y$$

$$\frac{\partial u}{\partial x} = \ln x + \frac{x}{x} + \ln y = \ln x + \ln y + 1$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{1}{x}$$

$$\frac{\partial^2 u}{\partial x^2 \partial y} = 0$$

№ 11.5.44

$u = x^3 \sin y + y^3 \sin x$, $\frac{\partial^3 u}{\partial x \partial y \partial z} - ?$

$\frac{\partial^3 u}{\partial x \partial y \partial z} = 0$, т.к. в ур-ии нет z

№ 11.5.45

$u = e^{xyz}$, $\frac{\partial^3 u}{\partial x \partial y \partial z}$

$$\frac{\partial u}{\partial x} = e^{xyz} \cdot yz = yze^{xyz}$$

$$\frac{\partial^2 u}{\partial x \partial y} = ze^{xyz} + xze^{xyz} \cdot yz =$$

$$= ze^{xyz}(xyz + 1)$$

$$\frac{\partial^3 u}{\partial x \partial y \partial z} = e^{xyz}(xyz+1) + xyze^{xyz}(xyz+1) + xyze^{xyz} =$$

$$= e^{xyz}(xyz + 1 + xyz(xyz+1) + xyz) =$$

$$= e^{xyz}(2xyz + 1 + x^2y^2z^2 + xyz) =$$

$= e^{xyz}\left(x^2y^2z^2 + 3xyz + 1\right)$

№11.5.46

$$z = \ln \frac{1}{\sqrt{(x-u)^2+(y-v)^2}}, \quad \frac{\partial^4 z}{\partial x \partial y \partial u \partial v}$$

$$z = \ln \frac{1}{\sqrt{(x-u)^2+(y-v)^2}} =$$

$$= -\ln \left((x-u)^2+(y-v)^2\right)^{\frac{1}{2}} =$$

$$= -\frac{1}{2} \ln \left((x-u)^2+(y-v)^2\right)$$

$$\frac{\partial z}{\partial x} = -\frac{1}{2} \cdot \frac{1}{(x-u)^2+(y-v)^2} \cdot 2(x-u) =$$

$$= -\frac{x-u}{(x-u)^2+(y-v)^2} = \frac{u-x}{(x-u)^2+(y-v)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \left(\frac{u-x}{(x-u)^2+(y-v)^2}\right)'_y =$$

$$= \frac{x-u}{\left((x-u)^2+(y-v)^2\right)^2} \cdot 2(y-v) =$$

$$= \frac{2(x-u)(y-v)}{\left((x-u)^2+(y-v)^2\right)^2}$$

$$\frac{\partial^3 z}{\partial x \partial y \partial u} = \Big[\left(2(x-u)(y-v)\right)'_u =$$

$$= -2(y-v) = 2(v-y);$$

$$\left(\left((x-u)^2+(y-v)^2\right)^2\right)'_u =$$

$$= 2\left((x-u)^2+(y-v)^2\right)\cdot 2(x-u)\cdot(-1) =$$

$$= -4(x-u)\left((x-u)^2+(y-v)^2\right) \Big] =$$

$$= \frac{2(v-y)\left((x-u)^2+(y-v)^2\right)^2}{\left((x-u)^2+(y-v)^2\right)^4} -$$

$$- \frac{-8(x-u)^2(y-v)\left((x-u)^2+(y-u)^2\right)}{\left((x-u)^2+(y-v)^2\right)^4} =$$

$$= \frac{-2(y-v)}{\left((x-u)^2+(y-v)^2\right)^3}\left(-4(x-u)^2+(x-u)^2+(y-v)^2\right)$$

$$= \frac{-2(y-v)}{\left((x-u)^2+(y-v)^2\right)^3}\left((y-v)^2-3(x-u)^2\right) =$$

$$= \frac{2(y-v)\left(3(x-u)^2-(y-v)^2\right)}{\left((x-u)^2+(y-v)^2\right)^3}$$

$$\frac{\partial^4 z}{\partial x\partial y\partial u\partial v} = \left[\left(2(y-v)\left(3(x-u)^2-(y-v)^2\right)\right)'_v = \right.$$

$$= -6\left((x-u)^2-(y-v)^2\right);$$

$$\left(\left((x-u)^2+(y-v)^2\right)^3\right)'_v =$$

$$\left. = -6(y-v)\left((\cancel{x}-u)^2+(y-v)^2\right)^2 \right] =$$

$$= \frac{-6\left((x-u)^2-(y-v)^2\right)\left((x-u)^2+(y-v)^2\right)^3}{\left((x-u)^2+(y-v)^2\right)^6} -$$

$$- \frac{2(y-v)\left(3(x-u)^2-(y-v)^2\right)(-6)(y-v)\left((x-u)^2+(y-v)^2\right)^2}{\left((x-u)^2+(y-v)^2\right)^6} =$$

$$= \frac{-6}{\left((x-u)^2+(y-v)^2\right)^3}\left((x-u)^2-(y-v)^2-2(y-v)^2\cdot\right.$$

$$\left.\cdot\left(3(x-u)^2-(y-v)^2\right)\left((x-u)^2+(y-v)^2\right)^2\right)$$

№ 11.5.47

$$u = (x-x_0)^p(y-y_0)^q; \quad \frac{\partial^{p+q}u}{\partial x^p\partial y^q} = ?$$

$$\frac{\partial u}{\partial x} = (y-y_0)^q \, p \, (x-x_0)^{p-1}$$

$$\frac{\partial^p u}{\partial x^p} = p!\,(x-x_0)(y-y_0)^q$$

$$\frac{\partial^{p+1} u}{\partial x^p \partial y} = p!\,(x-x_0)\, q\, (y-y_0)^{q-1}$$

$$\frac{\partial^{p+q} u}{\partial x^p \partial y^q} = p!\,q!\,(x-x_0)(y-y_0)$$

№ 11.5.48

$$u = \frac{x+y}{x-y}, \quad \frac{\partial^{m+n} u}{\partial x^m \partial y^n} = ?$$

$$m=1, \quad \frac{\partial u}{\partial x} = \frac{-2y}{(x-y)^2} = (-1)^m \frac{2y \cdot 1}{(x-y)^{m+1}};$$

$$m=2, \quad \frac{\partial^2 u}{\partial x^2} = \frac{4y}{(x-y)^3} = (-1)^m \frac{2y \cdot 1 \cdot 2}{(x-y)^{m+1}};$$

$$m=3, \quad \frac{\partial^3 u}{\partial x^3} = \frac{-12y}{(x-y)^4} = (-1)^m \frac{2y \cdot 1 \cdot 2 \cdot 3}{(x-y)^{m+1}};$$

$$\frac{\partial^m u}{\partial x^m} = (-1)^m \frac{m!\, 2y}{(x-y)^{m+1}}$$

$$n=1,\ \frac{\partial^{m+1}u}{\partial x^m \partial y} = (-1)^m 2m! \cdot$$

$$\cdot \frac{(x-y)^{m+1} - y(m+1)(x-y)^m}{(x-y)^{2m+2}} =$$

$$= (-1)^m 2m! \cdot \frac{(x-y)^m}{(x-y)^{2m+2}} \left(x-y-y(m+1)\right) =$$

$$= (-1)^m 2m! \frac{(-1)^1}{(x-y)^{m+1+1}} \left(y(m+1+1)-x\right);$$

$$n=2,\ \frac{\partial^{m+2}u}{\partial x^m \partial y^2} = (-1)^m 2m! (-1)^1 \cdot$$

$$\cdot \frac{(m+1+1)(x-y)^{m+1+1} - \left(y(m+2)-x\right)(m+2)(x-y)^{m+1}}{(x-y)^{2m+4}} =$$

$$= (-1)^m 2m! (-1)^1 \cdot \frac{(m+1+1)(x-y)^{m+1}}{(x-y)^{2m+4}} \left(x-y-y(m+2)+x\right) =$$

$$= (-1)^m 2m! (-1)^2 \cdot \frac{(m+1+1)}{(x-y)^{m+1+2}} \left(y(m+1+2)-2x\right);$$

$$n=3,\ \frac{\partial^{m+3}u}{\partial x^m \partial y^3} = (-1)^m 2m! (-1)^2 (m+1+1) \cdot$$

$$\cdot \frac{(m+1+2)(x-y)^{m+1+2} - \left(y(m+1+2)-2x\right)(m+1+2)(x-y)^{m+2}}{(x-y)^{2m+2+4}} =$$

$$= (-1)^m 2m! (-1)^2 (m+1+1) \cdot$$

$$\cdot \frac{(m+1+2)(x-y)^{m+2}}{(x-y)^{2m+2+4}}(x-y-y(m+1+2)+2x)$$

$$= (-1)^m 2m!(-1)^3 (m+1+1)\cdot$$

$$\cdot \frac{m+1+2}{(x-y)^{m+1+3}}(y(m+1+3)-3x)$$

$$\frac{\partial^{m+n}u}{\partial x^m \partial y^n} = \frac{(-1)^{m+n}2(m+n)!}{(m+1)(x-y)^{m+n+1}}(y(m+n+1)-nx)$$

<u>№11.5.49</u>

$$u = (x^2+y^2)e^{x+y}, \quad \frac{\partial^{m+n}u}{\partial x^m \partial y^n} = ?$$

$$\frac{\partial u}{\partial x} = e^{x+y}(x^2+y^2)+e^{x+y}\cdot 2x =$$

$$= e^{x+y}(x^2+2x+y^2); \quad m=1$$

$$\frac{\partial^2 u}{\partial x^2} = e^{x+y}(x^2+2x+y^2)+e^{x+y}(2x+2) =$$

$$= e^{x+y}(x^2+4x+2+y^2); \quad m=2$$

$$\frac{\partial^3 u}{\partial x^3} = e^{x+y}(x^2+4x+2+y^2)+e^{x+y}(2x+4) =$$

$$= e^{x+y}(x^2+6x+6+y^2); \quad m=3$$

$$\frac{\partial^m u}{\partial x^m} = e^{x+y}(x^2+2mx+(m-1)m+y^2)$$

$$\frac{\partial^{m+1} u}{\partial x^m \partial y} = e^{x+y}(x^2+2mx+m(m-1)+y^2)+$$
$$+e^{x+y}\cdot 2y = e^{x+y}(x^2+2mx+m(m-1)+y^2+2y)$$

$$\frac{\partial^{m+2} u}{\partial x^m \partial y^2} = e^{x+y}(x^2+2mx+m(m-1)+y^2+2y)+$$
$$+e^{x+y}(2y+2) =$$
$$= e^{x+y}(x^2+2mx+m(m-1)+y^2+4y+2); \quad n=2$$

$$\frac{\partial^{m+3} u}{\partial x^m \partial y^3} = e^{x+y}(x^2+2mx+m(m-1)+y^2+4y+$$
$$+2)+e^{x+y}(2y+4) =$$
$$= e^{x+y}(x^2+2mx+m(m-1)+y^2+6y+6); \quad n=3$$

$$\frac{\partial^{m+n} u}{\partial x^m \partial y^n} = e^{x+y}(x^2+2mx+m(m-1)+$$
$$+y^2+2ny+n(n-1))$$

~~№11.5.50~~

~~$u = \operatorname{arctg} \frac{y}{x}$~~

№11.5.51

$d^{10}u = ?$, если $u = \ln(x+y)$

$$d^{10}u = \frac{\partial^{10}u}{\partial x^{10}}dx^{10} + \frac{\partial^{10}u}{\partial x^{9}\partial y}dx^{9}dy + \ldots + \frac{\partial^{10}u}{\partial x\partial y^{9}}dx\,dy^{9} + \frac{\partial^{10}u}{\partial y^{10}}dy^{10}$$

$$\frac{\partial u}{\partial x} = \frac{1}{x+y}; \quad \frac{\partial^{2}u}{\partial x^{2}} = -\frac{1}{(x+y)^{2}}$$

$$\frac{\partial^{3}u}{\partial x^{3}} = \frac{2}{(x+y)^{3}} \Rightarrow \frac{\partial^{10}u}{\partial x^{10}} = -\frac{9!}{(x+y)^{10}}$$

$$\frac{\partial^{10}u}{\partial x^{9}\partial y} = \left(\frac{8!}{(x+y)^{9}}\right)'_y = -\frac{9!}{(x+y)^{10}}$$

$$\frac{\partial^{10}u}{\partial x^{8}\partial y} = \left(-\frac{7!}{(x+y)^{8}}\right)'_y = \frac{8!}{(x+y)^{9}}$$

$$\Rightarrow \frac{\partial^{10}u}{\partial x^{n}\partial y^{m}} = \frac{\partial^{10}u}{\partial x^{n+1}} = \frac{(-1)^{n}n!}{(x+y)^{n+1}}$$

$$d^{10}u = -\frac{9!}{(x+y)^{10}}dx^{10} - \frac{9!}{(x+y)^{10}}dx^9dy -$$

$$-\frac{9!}{(x+y)^{10}}dx^8dy^2 - \ldots - \frac{9!}{(x+y)^{10}}dx^2dy^8 -$$

$$-\frac{9!}{(x+y)^{10}}dxdy^9 - \frac{9!}{(x+y)^{10}}dy^{10} =$$

$$= -\frac{9!}{(x+y)^{10}}(dx^{10} + dx^9dy + \ldots + dxdy^9 + dy^{10})$$

№ 11.5.52

$d^4u = ?$, если $u = \ln(x^x y^y z^z)$

$$u = \ln(x^x y^y z^z) = \ln x^x + \ln y^y + \ln z^z =$$

$$= x\ln x + y\ln y + z\ln z$$

$$\frac{\partial u}{\partial x} = \ln x + 1; \quad \frac{\partial^2 u}{\partial x^2} = \frac{1}{x};$$

$$\frac{\partial^3 u}{\partial x^3} = -\frac{1}{x^2}; \quad \frac{\partial^4 u}{\partial x^4} = \frac{2}{x^3};$$

$$\frac{\partial^4 u}{\partial x^3 \partial y} = \left(-\frac{1}{x^2}\right)'_y = 0 = \frac{\partial^4 u}{\partial x^3 \partial z}$$

$$\frac{\partial^4 u}{\partial x^2 \partial y^2} = \left(\frac{1}{x}\right)''_y = 0'_y = 0 = \frac{\partial^4 u}{\partial x^2 \partial z^2}$$

$$\frac{\partial^4 u}{\partial x \partial y^3} = (\ln x + 1)'''_y = 0''_y = 0'_y = 0 =$$

$$= \frac{\partial^4 u}{\partial x \partial z^3}$$

$$\frac{\partial u}{\partial y} = \ln y + 1 \Rightarrow \frac{\partial^4 u}{\partial y^4} = \frac{2}{y^3}$$

$$\frac{\partial u}{\partial z} = \ln z + 1 \Rightarrow \frac{\partial^4 u}{\partial z^4} = \frac{2}{z^3}$$

$$\frac{\partial^4 u}{\partial y^3 \partial z} = \left(-\frac{1}{y^2}\right)'_z = 0$$

$$\frac{\partial^4 u}{\partial y^2 \partial z^2} = \left(\frac{1}{y}\right)''_z = 0'_z = 0$$

$$\frac{\partial^4 u}{\partial y \partial z^3} = (\ln y + 1)'''_z = 0''_z = 0'_z = 0$$

$$\frac{\partial^4 u}{\partial x^2 \partial y \partial z} = \left(\left(\frac{1}{x}\right)'_y\right)'_z = 0 = \frac{\partial^4 u}{\partial x \partial y^2 \partial z} =$$

$$= \frac{\partial^4 u}{\partial x \partial y \partial z^2}$$

$$d^4 u = \frac{2}{x^3} dx^4 + \frac{2}{y^3} dy^4 + \frac{2}{z^3} dz^4$$

№ 11.5.53

$d^m u$, если $u = e^{ax+by}$

$$\frac{\partial u}{\partial x} = (ax+by)'_x e^{ax+by} = a e^{ax+by}$$

$$\frac{\partial^2 u}{\partial x^2} = a (ax+by)'_x e^{ax+by} = a^2 e^{ax+by}$$

$$\frac{\partial^m u}{\partial x^m} = a^m e^{ax+by}$$

$$\frac{\partial u}{\partial y} = (ax+by)'_y e^{ax+by} = b e^{ax+by}$$

$$\frac{\partial^2 u}{\partial y^2} = b (ax+by)'_y e^{ax+by} = b^2 e^{ax+by}$$

$$\frac{\partial^m u}{\partial y^m} = b^m e^{ax+by}$$

$$\frac{\partial^m u}{\partial x^{m-1} \partial y} = (a^{m-1} e^{ax+by})'_y = a^{m-1} (ax+by)'_y \cdot$$

$\cdot e^{ax+by} = a^{m-1} b e^{ax+by}$

Следовательно:

$$\frac{\partial^m u}{\partial x^i \partial y^j} = a^i b^j e^{ax+by}$$

$$d^m u = \sum_{j=0}^{m} \sum_{i=m}^{0} a^i b^j e^{ax+by} dx^i dy^j =$$

$$= a^{10} e^{ax+by} dx^{10} + a^9 b e^{ax+by} dx^9 dy + \ldots +$$

$$+ a b^9 e^{ax+by} dx dy^9 + b^{10} e^{ax+by} dy^{10}$$

№ 11.5.54

$d^n u$, если $u = e^{ax+by+cz}$

По аналогии с ~~...~~ № 11.5.53

$$d^n u = \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{k=0}^{n} a^i b^j c^k e^{ax+by+cy} \cdot$$

$$\cdot dx^i dy^j dz^k$$

№ 11,5.56

$$z = \left(\frac{y}{x}\right)^x$$

$$d^2 z = \frac{\partial^2 z}{\partial x^2} dx^2 + 2\frac{\partial^2 z}{\partial x \partial y} dx dy +$$

$$+ \frac{\partial^2 z}{\partial y^2} dy^2$$

$$\frac{\partial z}{\partial x} = \left(\left(\frac{y}{x}\right)^x\right)'_x = \left(\frac{y}{x}\right)^x \ln\frac{y}{x} \cdot \left(-\frac{y}{x^2}\right) =$$

$$= -\frac{y^{x+1}}{x^{x+2}}(\ln y - \ln x) = -\left(\frac{y}{x}\right)^{x+1} \cdot \frac{1}{x} \cdot (\ln y - \ln x)$$

$$\frac{\partial^2 z}{\partial x^2} = \left(-\left(\frac{y}{x}\right)^{x+1} \cdot \frac{1}{x} \cdot (\ln y - \ln x)\right)'_x =$$

$$= -\left(\frac{y}{x}\right)^{x+1}(\ln y - \ln x)^2 \cdot \frac{1}{x} +$$

$$+ \left(\frac{y}{x}\right)^{x+1} \cdot \frac{1}{x^2}(\ln y - \ln x) -$$

$$- \left(\frac{y}{x}\right)^{x+1} \cdot \frac{1}{x}\left(-\frac{1}{x}\right) =$$

$$= -\left(\frac{y}{x}\right)^{x+1} \frac{1}{x}\left((\ln y - \ln x)^2 - \frac{1}{x}(\ln y - \ln x) - \frac{1}{x}\right)$$

$$\frac{\partial z}{\partial x} = \left(\left(\frac{y}{x}\right)^x\right)'_x = \left(x^{-x} y^x\right)'_x =$$

$$= -x^{-x} \ln x \, y^x + y^x \ln y \, x^{-x} =$$

$$= x^{-x} y^x (\ln y - \ln x)$$

$$\frac{\partial^2 z}{\partial x^2} = -x^{-x} \ln x \, y^x (\ln y - \ln x) +$$

$$+ x^{-x} y^x \ln y (\ln y - \ln x) +$$

$+x^{-x}y^{x}\cdot\left(-\frac{1}{x}\right)=x^{-x}y^{x}\cdot$

$\cdot\left(\cancel{\ln x\ln y}-\ln^2 x+\ln^2 y-\cancel{\ln x\ln y}-\frac{1}{x}\right)=$

$=x^{-x}y^{x}\left(\ln^2 y-\ln^2 x-\frac{1}{x}\right)$

$$\frac{\partial z}{\partial x\partial y}=x^{-x+1}y^{x-1}(\ln y-\ln x)+$$
$$+x^{-x}y^{x}\cdot\frac{1}{y}=$$
$$=x^{-x}y^{x-1}(x\ln y-x\ln x+1)$$

$$\frac{\partial z}{\partial y}=\left(x^{-x}y^{x}\right)'_y=x^{-x+1}y^{x-1}$$

$$\frac{\partial^2 z}{\partial y^2}=\left(x^{-x+1}y^{x-1}\right)'_y=x^{-x+2}y^{x-2}$$

$$d^2z=x^{-x}y^{x}\left(\ln^2 y-\ln^2 x-\frac{1}{x}\right)dx^2+$$
$$+2x^{-x}y^{x-1}(x\ln x-x\ln y+1)\,dxdy+$$
$$+x^{-x+2}y^{x-2}\,dy^2$$

№ 11.5.57

$z=\ln\operatorname{tg}\frac{y}{x},\quad d^2z=?$

$$\frac{\partial z}{\partial x}=\frac{1}{\operatorname{tg}\frac{y}{x}}\cdot\frac{1}{\cos^2\frac{y}{x}}\cdot-\frac{y}{x^2}=$$
$$=-\frac{y}{\sin\frac{y}{x}\cos\frac{y}{x}\,x^2}=-\frac{2y}{\sin\frac{2y}{x}\,x^2}$$

$$\frac{\partial^2 z}{\partial x^2} = \left(-\frac{2y}{x^2\sin\frac{2y}{x}}\right)'_x =$$

$$= \frac{4y}{x^3}\cdot\left(-\frac{1}{\sin^2\frac{2y}{x}}\right)\cdot\cos\frac{2y}{x}\cdot\left(-\frac{2y}{x^2}\right) =$$

$$= \frac{8y}{x^5}\left(\operatorname{ctg}^2\frac{2y}{x}+1\right)\cdot\cos\frac{2y}{x}$$

$$\frac{\partial^2 z}{\partial x\partial y} = \left(-\frac{2y}{x^2\sin\frac{2y}{x}}\right)'_y =$$

$$= -\frac{2x^2\sin\frac{2y}{x} - 2y\,x^2\cos\frac{2y}{x}\cdot\frac{2}{x}}{x^4\sin^2\frac{2y}{x}} =$$

$$= \frac{2x\left(y\cos\frac{2y}{x} - x\sin\frac{2y}{x}\right)}{x^4\sin^2\frac{2y}{x}} =$$

$$= \frac{2\left(y\cos\frac{2y}{x} - x\sin\frac{2y}{x}\right)}{x^3\sin^2\frac{2y}{x}}$$

$$\frac{\partial z}{\partial y} = \frac{1}{\operatorname{tg}\frac{y}{x}}\cdot\frac{1}{\cos^2\frac{y}{x}}\cdot\frac{1}{x} =$$

$$= \frac{1}{x\sin\frac{y}{x}\cos\frac{y}{x}} = \frac{1}{x\sin\frac{2y}{x}}$$

$$\frac{\partial^2 z}{\partial y^2} = \left(\frac{1}{x\sin\frac{2y}{x}}\right)'_y =$$

$$= -\frac{1}{x\sin^2\frac{2y}{x}} \cdot \cos\frac{2y}{x} \cdot \frac{2}{x} =$$

$$= -\frac{2\cos\frac{2y}{x}}{x^2\sin^2\frac{2y}{x}}$$

$$d^2z = \frac{8y\cos\frac{2y}{x}}{x^5\sin^2\frac{2y}{x}}dx^2 +$$

$$+\frac{4\left(y\cos\frac{2y}{x} - x\sin\frac{2y}{x}\right)}{x^3\sin^2\frac{2y}{x}}dx\,dy -$$

$$-\frac{2\cos\frac{2y}{x}}{x^2\sin^2\frac{2y}{x}}dy^2 =$$

№ 11.5.59

$$z = x^4 + 3x^3y - 4x^2y^2 + 5xy^3 - y^4$$

$$\frac{\partial z}{\partial x} = 4x^3 + 9x^2y - 8xy^2 + 5y^3$$

$$\frac{\partial^2 z}{\partial x^2} = 12x^2 + 18xy - 8y^2$$

$$\frac{\partial^3 z}{\partial x^3} = 24x + 18y$$

$$\frac{\partial z}{\partial y} = 3x^3 - 8x^2y + 15xy^2 - 4y^3$$

$$\frac{\partial^2 z}{\partial y^2} = -8x^2 + 30xy - 12y^2$$

$$\frac{\partial^3 z}{\partial y^3} = 30x - 24y$$

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$z = e^{xy}$

$$\frac{\partial z}{\partial x} = e^{xy} \cdot y = ye^{xy}$$

$$\frac{\partial^2 z}{\partial x^2} = y^2 e^{xy}$$

$$\frac{\partial z}{\partial y} = xe^{xy}$$

$$\frac{\partial^2 z}{\partial y^2} = x^2 e^{xy}$$

Номера 11.5.61 – 11.5.68 были логически выполнены