

# Интегралы, часть 1.

## Практика

№ 8.1.2

$$\int x^{10} dx = \frac{x^{10+1}}{10+1} + C = \frac{x^{11}}{11} + C$$

№ 8.1.3

$$\int \frac{dx}{x^7} = \int x^{-7} dx = \frac{x^{-6}}{-6} + C = -\frac{1}{6x^6} + C$$

№ 8.1.4

$$\int \sqrt[4]{x} dx = \int x^{\frac{1}{4}} dx = \frac{x^{\frac{5}{4}}}{\frac{5}{4}} + C =$$

$$= \frac{4}{5} x^{5/4} + C$$

№ 8.1.5

$$\int \frac{dx}{x^2+9} = \frac{1}{3} \operatorname{arctg} \frac{x}{3} + C$$

№ 8.1.6

$$\int \frac{dx}{x^2 - \frac{1}{2}} = \frac{1}{x \cdot \frac{\sqrt{2}}{2}} \ln \left| \frac{x - \frac{\sqrt{2}}{2}}{x + \frac{\sqrt{2}}{2}} \right| + C =$$

$$= \frac{\sqrt{2}}{2} \ln \left| \frac{x - \frac{\sqrt{2}}{2}}{x + \frac{\sqrt{2}}{2}} \right| + C$$

W 8.1.7

$$\int \frac{dx}{\sqrt{x^2+3}} = \ln |x + \sqrt{x^2+3}| + C$$

W 8.1.8

$$\int \frac{x^4 + x^2 - 6x}{x^3} dx = \int \frac{x^4}{x^3} dx +$$

$$+ \int \frac{x^2}{x^3} dx - 6 \int \frac{x}{x^3} dx =$$

$$= \int x dx + \int \frac{1}{x} dx - 6 \int \frac{1}{x^2} dx =$$

$$= \frac{x^2}{2} + \ln |x| - 6 \cdot \left(-\frac{1}{x}\right) + C =$$

$$= \frac{x^2}{2} + \ln |x| + \frac{6}{x} + C$$

W 8.1.10

$$\int \left( \frac{5}{x} - \frac{10}{\sqrt[4]{x^3}} - \frac{3}{x^2+7} \right) dx =$$

$$= 5 \int \frac{dx}{x} - 10 \int x^{-3/4} dx - 3 \int \frac{dx}{x^2+7} =$$

$$= 5 \ln |x| - 10 \frac{x^{7/4}}{\frac{7}{4}} - 3 \frac{1}{\sqrt{7}} \operatorname{arctg} \frac{x}{\sqrt{7}} + C$$

$$= 5 \ln|x| - \frac{30x^{7/3}}{7} - \frac{3}{\sqrt{x}} \operatorname{arctg} \frac{x}{\sqrt{x}} + C$$

~ 8.1.11

$$\begin{aligned} \int \sqrt{x} (x^2 + 1) dx &= \int (x^2 \sqrt{x} + \sqrt{x}) dx = \\ &= \int x^{5/2} dx + \int \sqrt{x} dx = \frac{2x^{7/2}}{7} + \frac{2x^{3/2}}{3} + C \end{aligned}$$

~ 8.1.12

$$\begin{aligned} \int \frac{3 + \sqrt{4-x^2}}{\sqrt{4-x^2}} dx &= 3 \int \frac{1}{\sqrt{4-x^2}} dx + \int dx = \\ &= 3 \arcsin \frac{x}{2} + x + C \end{aligned}$$

~ 8.1.13

$$\begin{aligned} \int \frac{(x^3 + 2)^2}{\sqrt{x}} dx &= \int \frac{x^6 + 4x^3 + 4}{\sqrt{x}} dx = \\ &= \int x^{11/2} dx + 4 \int x^{5/2} dx + 4 \int \frac{1}{\sqrt{x}} dx = \\ &= \frac{2x^{13/2}}{13} + \frac{8x^{7/2}}{7} + 8\sqrt{x} + C \end{aligned}$$

~ 8.1.14

$$\int \left( 4 \sin x + 8x^3 - \frac{11}{\cos^2 x} \right) dx =$$

$$= 4 \int \sin x dx + 8 \int x^3 dx - 11 \int \frac{1}{\cos^2 x} dx =$$

$$= -4 \cos x + \frac{8x^4}{4} - 11 \tan x + C =$$

$$= -4 \cos x + 2x^4 - 11 \tan x + C$$

W 8.1.16

$$\int \cos 2x dx = \int \frac{\cos 2x \cdot 2 dx}{2} =$$

$$= \frac{1}{2} \int \cos 2x d(2x) = \frac{1}{2} \sin 2x + C$$

W 8.1.17

$$\int (9x+2)^{17} dx = \frac{1}{9} \int \frac{d(9x+2)}{1} (9x+2)^{17} =$$

$$= \frac{1}{9} \frac{(9x+2)^{18}}{18} + C = \frac{(9x+2)^{18}}{162} + C$$

W 8.1.18

$$\int \frac{dx}{8x-1} = \frac{1}{8} \int \frac{d(8x-1)}{8x-1} = \frac{1}{8} \ln |8x-1| + C$$

W 8.1.19

$$\int 4^{3-5x} dx = -\frac{1}{5} \int 4^{3-5x} d(3-5x) =$$

$$= -\frac{1}{5} \frac{4^{3-5x}}{\ln 4} + C = -\frac{4^{3-5x}}{5 \ln 4} + C$$

W8.1.20

$$\int \sqrt{3x+4} dx = \frac{1}{3} \int \sqrt{3x+4} d(3x+4) \\ = \frac{1}{3} \frac{(3x+4)^{3/2}}{\frac{3}{2}} + C = \frac{2(3x+4)^{3/2}}{9} + C$$

W8.1.21

$$\int \frac{dx}{3x^2-25} = \frac{1}{\sqrt{3}} \int \frac{d(\sqrt{3}x)}{3x^2-25} = \\ = \frac{1}{\sqrt{3}} \cdot \frac{1}{2 \cdot 5} \ln \left| \frac{x\sqrt{3}-5}{x\sqrt{3}+5} \right| + C = \\ = \frac{1}{10\sqrt{3}} \ln \left| \frac{x\sqrt{3}-5}{x\sqrt{3}+5} \right| + C$$

W8.1.23

$$\int \cos^2 x dx = \int \frac{1+\cos 2x}{2} dx = \\ = \frac{1}{2} \int dx + \frac{1}{2} \int \cos 2x dx = \\ = \frac{1}{2} \int dx + \frac{1}{2} \cdot \frac{1}{2} \int \cos 2x d(2x) = \\ = \frac{x}{2} + \frac{\sin 2x}{4} + C$$



w 8.1.24

$$\int \frac{x-2}{x+3} dx = \int \frac{x}{x+3} d\cancel{x} -$$

$$- 2 \int \frac{1}{x+3} d(x+3) = \int \frac{(x+3)-3}{x+3} d(x+3) -$$

$$- 2 \int \frac{1}{x+3} d(x+3) = \int d\cancel{x} - 3 \int \frac{d(x+3)}{x+3} -$$

$$- 2 \int \frac{d(x+3)}{x+3} = \int d\cancel{x} - 5 \int \frac{d(x+3)}{x+3} =$$

$$= x\cancel{-} - 5 \ln|x+3| + C$$

w 8.1.25

$$\int \frac{x^2 dx}{x^2-9} = \int \frac{(x^2-9)+9}{x^2-9} dx =$$

$$= \int dx + 9 \int \frac{dx}{x^2-9} = x + 9 \cdot \frac{1}{2 \cdot 3} \cdot$$

$$\cdot \ln \left| \frac{x-3}{x+3} \right| + C = x + \frac{3}{2} \ln \left| \frac{x-3}{x+3} \right| + C$$

w 8.1.26

$$\int \frac{5 + \sin^3 x}{\sin^2 x} dx = 5 \int \frac{dx}{\sin^2 x} + \int \sin x dx =$$

$$= -5 \cot x - \cos x + C$$