№ 11.53

$d^2z = ?, \quad z = \operatorname{arctg}\frac{y}{x}$

1) $dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$

$= -\frac{y}{x^2+y^2}dx + \frac{x}{x^2+y^2}dy$

2) $\frac{\partial^2 z}{\partial x^2} = \frac{\partial}{\partial x}\left(-\frac{y}{x^2+y^2}\right) =$

$= -y \cdot \left(-\frac{1}{(x^2+y^2)^2}\right) \cdot 2x = \frac{2xy}{(x^2+y^2)^2}$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial}{\partial y}\left(-\frac{y}{x^2+y^2}\right) =$$

$$\frac{\partial^2 z}{\partial y \partial x} = \frac{\partial}{\partial x}\left(\frac{x}{x^2+y^2}\right) = \frac{x^2+y^2 - x\cdot 2x}{(x^2+y^2)^2} =$$

$$= \frac{y^2-x^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 z}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{x}{x^2+y^2}\right) = x\cdot\left(-\frac{1}{(x^2+y^2)^2}\right)\cdot 2y$$

$$= -\frac{2xy}{(x^2+y^2)^2}$$

3) $$d^2z = \frac{\partial^2 z}{\partial x^2}dx^2 + 2\frac{\partial^2 z}{\partial x \partial y}dx dy +$$

$$+ \frac{\partial^2 z}{\partial y^2}dy^2$$

$$d^2z = \frac{2xy}{(x^2+y^2)^2}dx^2 + 2\frac{y^2-x^2}{(x^2+y^2)^2}\cdot$$

$$\cdot dx\,dy - \frac{2xy}{(x^2+y^2)^2}dy^2$$