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ИВТ, курс 3м

* Однородное и неоднородное
системы линейных
уравнений

№ 2.3.26

$$\begin{cases} 2x_1 - x_2 = 0 \\ -4x_1 + 2x_2 = 0 \end{cases}$$

$$\text{Решение: } \left(\begin{array}{cc|c} 2 & -1 & 0 \\ -4 & 2 & 0 \end{array} \right) \xrightarrow{I + 2II} \sim \left(\begin{array}{cc|c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array} \right) \Rightarrow$$

$$\Rightarrow r(A) = r(A|B) = 1, n=2 \Rightarrow$$

⇒ существует собств. и неспр.

$$M_1 = |2| = 2 \neq 0 \rightarrow x_1 - \text{шагн.},
x_2 = t - \text{своб.}$$

$$2x_1 - t = 0$$

~~$$2x_1 - t = 0$$~~

$$x_1 = \frac{t}{2}$$

Общ. в-е: $\left(\frac{t}{2}; t \right)$

ФСР: $\left\{ \left(\frac{1}{2}; 0 \right) \right\}$

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Wd. 3.24

$$\begin{cases} x - \sqrt{3}y = 0 \\ \sqrt{3}x - 3y = 0 \end{cases}$$

$$\begin{pmatrix} 1 & -\sqrt{3} \\ \sqrt{3} & -3 \end{pmatrix} \text{III} - \sqrt{3}\text{I} \sim \begin{pmatrix} 1 & -\sqrt{3} \\ 0 & 0 \end{pmatrix} \Rightarrow$$

$$\Rightarrow r(A) = r(A|B) = 1, n=2 \Rightarrow$$

→ curv. обр. и неодн.

$$M_1 = |1| = 1 \neq 0 \Rightarrow x - \text{мабд.}$$

$$\left\{ \begin{array}{l} y = t - \text{чод.} \\ y = t \end{array} \right.$$

$$x - \sqrt{3}t = 0$$

$$x = \sqrt{3}t$$

Одн. р-ие: $(\sqrt{3}t; t)$

ФПР: $\{(\sqrt{3}; 1)\}$

Wd. 3.28

$$\begin{cases} 3x + 4y = 0 \\ 4x - 3y = 0 \end{cases}$$

$$\begin{pmatrix} 3 & 4 \\ 4 & -3 \end{pmatrix} 3\text{II} - 4\text{I} \sim \begin{pmatrix} 3 & 4 \\ 0 & -25 \end{pmatrix} \Rightarrow$$

$$\Rightarrow r(A) = r(A|B) = 2, n=2 \Rightarrow$$

②

\Rightarrow CUCt. colm. u. auf.

$$\begin{array}{l} \text{Methode der Hauptkomponenten:} \\ \left\{ \begin{array}{l} 3x + 4y = 0 \\ -5x - 4y = 0 \end{array} \right. \\ \left\{ \begin{array}{l} x = 0 \\ y = 0 \end{array} \right. \end{array}$$

Obige p-ue: $(0; 0)$
QCP: $\{(0, 0)\}$

W2.3.29

$$\begin{array}{l} \left\{ \begin{array}{l} x_1 + 2x_2 = 0 \\ -\sqrt{3}x_1 - \sqrt{12}x_2 = 0 \\ 2x_1 + 4x_2 = 0 \end{array} \right. \end{array}$$

$$\left(\begin{array}{cc|c} 1 & 2 & I \\ -\sqrt{3} & -\sqrt{12} & II + \sqrt{3}I \\ 2 & 4 & III - 2I \end{array} \right) \sim \left(\begin{array}{cc|c} 1 & 2 & I \\ 0 & 0 & II \\ 0 & 0 & III \end{array} \right) \Rightarrow$$

$$\Rightarrow r(A) = r(A|B) = 1, n = d_B = 2 \Rightarrow$$

③ \Rightarrow CUCt. colm. u. reotp.

$M_1 = 1/1 = 1 \neq 0 \Rightarrow x_1 - \text{wahrh.}$
 $x_2 = t - \text{ebdd.}$

$$x_1 + dt = 0$$

$$x_1 = -dt$$

Dsg. p-ue: $(-dt, t)$
PCP: $\{(-1, 1)\}$

Wk. 3.80

$$\begin{cases} 2x - y - z = 0 \\ 4x - 2y - 2z = 0 \end{cases}$$

$$\left(\begin{array}{ccc} 2 & -1 & -1 \\ 4 & -2 & -2 \end{array} \right) \xrightarrow{\text{II} - 2\text{I}} \sim \left(\begin{array}{ccc} 2 & -1 & -1 \\ 0 & 0 & 0 \end{array} \right) \Rightarrow$$

$$\Rightarrow \alpha(A) = r(A/B) = 1, n = 3 \Rightarrow$$

\Rightarrow unbest. column u recomp.

$M_1 = 1/2 = 1/2 \neq 0 \Rightarrow x - \text{wahrh.}$

$$y = t_1, z = t_2 -$$

$$2x - t_1 - t_2 = 0$$

$$x = \frac{t_1 + t_2}{2}$$

$$x = \frac{1}{2}t_1 + \frac{1}{2}t_2$$

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Одн. п-ие: $\left(\frac{t_1+t_2}{2}; t_1; t_2 \right)$

QCP: $\left\{ \left(\frac{1}{2}; 1; 0 \right), \left(\frac{1}{2}; 0; 1 \right) \right\}$

№ 3.31.

$$\begin{cases} 2x - y - z = 0 \\ x + 2y + 3z = 0 \end{cases}$$

$$\begin{pmatrix} 2 & 1 & -1 \\ 1 & 2 & 3 \end{pmatrix} \text{I} - \text{II} \sim \begin{pmatrix} 0 & -5 & -7 \\ 1 & 2 & 3 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 2 & 3 \\ 0 & -5 & -7 \end{pmatrix} \Rightarrow r(A) = r(A|B) = 2$$

\Rightarrow 2, $n = 3 \Rightarrow$ существ. общ. и неоп.

$$M_2 = \begin{vmatrix} 1 & 2 \\ 0 & -5 \end{vmatrix} = -5 \neq 0 \Rightarrow$$

$\Rightarrow x, y - \text{множ.}, z = t - \text{общ.}$

$$\begin{cases} x + 2y + 3z = 0 \\ -5y - 4t = 0 \end{cases}$$

$$\begin{cases} x = -\frac{1}{5}t \\ y = -\frac{4}{5}t \end{cases} \Rightarrow$$

$$\begin{cases} x = -\frac{1}{5}t \\ y = -\frac{4}{5}t \\ z = t \end{cases} \Rightarrow \text{Одн. п-ие: } \left(-\frac{1}{5}t; -\frac{4}{5}t; t \right)$$

⑤ QCP: $\left\{ \left(-\frac{1}{5}; -\frac{4}{5}; 1 \right) \right\}$

Wd. 3. 3d

$$\begin{cases} 3x_1 + 2x_2 + x_3 = 0 \\ 2x_1 + 5x_2 + 3x_3 = 0 \\ 3x_1 + 4x_2 + 2x_3 = 0 \end{cases}$$

$$\begin{pmatrix} 3 & 2 & 1 \\ 2 & 5 & 3 \\ 3 & 4 & 2 \end{pmatrix} \xrightarrow{\text{III} - \text{I}} \begin{pmatrix} 3 & 2 & 1 \\ 0 & 11 & 7 \\ 0 & 2 & 1 \end{pmatrix} \xrightarrow{\text{II} - 2\text{I}}$$

$$\sim \begin{pmatrix} 3 & 2 & 1 \\ 0 & 11 & 7 \\ 0 & 0 & -3 \end{pmatrix} \Rightarrow r(A) = r(A|B) = 3, n=3 \Rightarrow \text{cuer. soluci. u' n'f.}$$

$$\begin{cases} 3x_1 + 2x_2 + x_3 = 0 \\ 11x_2 + 7x_3 = 0 \\ -3x_3 = 0 \end{cases}$$

$$\begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{cases} \Rightarrow \begin{array}{l} \text{Obig. p-ue: } (0; 0; 0) \\ \text{PCP: } \{(0; 0; 0)\} \end{array}$$

wd. 3.33

$$\begin{cases} x_1 - 2x_2 - 3x_3 = 0 \\ 2x_1 + 3x_2 + x_3 = 0 \\ 5x_1 - 3x_2 - 8x_3 = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 2 & 3 & 1 & 0 \\ 5 & -3 & -8 & 0 \end{array} \right) \xrightarrow{\text{I} - 2\text{I}} \sim \left(\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 7 & 7 & 0 \\ 0 & 7 & 7 & 0 \end{array} \right) \xrightarrow{\cdot \frac{1}{7}}$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array} \right) \xrightarrow{\text{III} - \text{II}} \left(\begin{array}{ccc|c} 1 & -2 & -3 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \Rightarrow$$

$$\Rightarrow r(A) = r(A|B) = 2, n = 3 \Rightarrow$$

\Rightarrow существуют x_1, x_2 и x_3 такие что

$$\begin{cases} x_1 - 2x_2 - 3x_3 = 0 \\ x_2 + x_3 = 0 \end{cases}$$

$$\begin{cases} x_1 = x_3 \\ x_2 = -x_3 \end{cases}$$

$\exists x_3 = t$, тогда $\text{решение: } (t; -t; t)$

$$\textcircled{7} \quad \underline{\text{QCP: }} \{(1; -1; 1)\}$$

wd. 3.34

$$\begin{cases} x_1 - 2x_2 + 3x_3 = 0 \\ -x_1 + dx_2 - 3x_3 = 0 \\ dx_1 - 4x_2 + 6x_3 = 0 \end{cases}$$

$$\left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ -1 & d & -3 & 0 \\ 2 & -4 & 6 & 0 \end{array} \right) \xrightarrow{\text{I} + \text{II}} \sim \left(\begin{array}{ccc|c} 1 & -2 & 3 & 0 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\Rightarrow r(A) = r(A|B) \Rightarrow \begin{matrix} 1 \\ 0 \\ 1 \end{matrix}, \begin{matrix} 0 \\ 1 \\ 0 \end{matrix}, \begin{matrix} 0 \\ 0 \\ 1 \end{matrix} \Rightarrow$$

→ cver. sestav. u neop.

$$M_A = |1| = 1 \neq 0 \Rightarrow x_1 \text{ f. neabn.}$$

$$x_2 = t_1, x_3 = t_2 - \text{obos}$$

$$x_1 - 2t_1 + 3t_2 = 0$$

$$x_1 = 2t_1 - 3t_2$$

$$\text{Osys. p-ve: } (2t_1 - 3t_2; t_1; t_2)$$

$$\text{QCP: } \{(2; 1; 0), (-3; 0; 1)\}$$

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wh. 3.35

$$\begin{cases} 2x_1 + x_2 - x_3 = 0 \\ x_1 - 2x_2 + x_3 = 0 \\ x_1 + 3x_2 - 2x_3 = 0 \\ x_1 + 8x_2 - 5x_3 = 0 \end{cases}$$

⋮

$$\left(\begin{array}{ccc|c} 2 & 1 & -1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 3 & -2 & 0 \\ 1 & 8 & -5 & 0 \end{array} \right) \xrightarrow{I \leftrightarrow II} \text{...}$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 2 & 1 & -1 & 0 \\ 1 & 3 & -2 & 0 \\ 1 & 8 & -5 & 0 \end{array} \right) \xrightarrow{II - 2I} \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 5 & -3 & 0 \\ 1 & 3 & -2 & 0 \\ 1 & 8 & -5 & 0 \end{array} \right) \xrightarrow{III - I} \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 5 & -3 & 0 \\ 0 & 5 & -3 & 0 \\ 1 & 8 & -5 & 0 \end{array} \right) \xrightarrow{IV - I} \sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 5 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 5 & -3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right) \Rightarrow r(A) = r(A|B) = 2, n = 3 \Rightarrow$$

⋮

$$\Rightarrow \text{сист. симм. и неопр.}$$

⑨ $M_2 = \begin{vmatrix} 1 & -2 \\ 0 & 5 \end{vmatrix} = 5 \neq 0 \Rightarrow x_1, x_2 - \text{независимы.}$
 $x_3 = t - \text{своб.}$

$$\begin{cases} x_1 - 2x_2 + t \geq 0 \\ 5x_2 - 3t \geq 0 \end{cases}$$

$$\begin{cases} x_1 = \frac{1}{5}t \\ x_2 = \frac{3}{5}t \end{cases}$$

Optim. p-werte: $\left(\frac{1}{5}t; \frac{3}{5}t; t\right)$

QCP: $\left\{\left(\frac{1}{5}; \frac{3}{5}; 1\right)\right\}$

Wd. 3.86

$$\left\{ \begin{array}{l} x_1 - x_3 + x_5 \geq 0 \\ x_2 - x_4 + x_6 \geq 0 \\ x_1 - x_2 + x_5 - x_6 \geq 0 \\ x_2 - x_3 + x_6 \geq 0 \\ x_1 - x_4 + x_5 \geq 0 \end{array} \right.$$

$$\left(\begin{array}{cccccc|c} 1 & 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 & 1 \\ 1 & -1 & 0 & 0 & 1 & -1 & 0 \\ 0 & 1 & -1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & -1 & 1 & 0 & 0 \end{array} \right) \xrightarrow{\text{III} - \text{I}} \sim \xrightarrow{\text{IV} - \text{II}}$$

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$$\sim \left(\begin{array}{cccccc} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & -1 & 1 & 0 & 0 & -1 \\ 0 & 1 & -1 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{array} \right) \left. \begin{array}{l} \text{III} + \text{II} \\ \text{IV} - \text{II} \end{array} \right\} \sim$$

$$\sim \left(\begin{array}{cccccc} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \end{array} \right) \left. \begin{array}{l} \text{IV} + \text{III} \\ \text{V} - \text{III} \end{array} \right\} \sim$$

$$\sim \left(\begin{array}{cccccc} 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right) \Rightarrow r(A) = r(A|B) = 3, \\ n = 6 \Rightarrow$$

\Rightarrow существует 3 решения

~~т.к.~~ $n - r = 3$, QCP будет иметь 3 решения

(11)

$$M_3 = \begin{vmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} = 1 \neq 0 \Rightarrow$$

x_1, x_2, x_3 - мажюсе

$x_4 = t_1, x_5 = t_2, x_6 = t_3$ - чод.

$$\begin{cases} x_1 - x_3 + t_2 = 0 \\ x_2 - t_1 + t_3 = 0 \\ x_3 - t_1 = 0 \end{cases}$$

$$\begin{cases} x_1 = t_1 - t_2 \\ x_2 = t_1 - t_3 \\ x_3 = t_1 \end{cases}$$

Одн. п-ве: $(t_1 - t_2; t_1 - t_3; t_1; t_1; t_2; t_3)$

$$\underline{\text{ФСР:}} \quad \left\{ \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \right\}$$

№ 2.3.37

$$\begin{cases} x_1 + x_2 - x_3 + 2x_4 = 0 \\ x_1 + 3x_2 - 3x_3 + 4x_4 = 0 \\ 3x_1 + 2x_2 + x_3 = 0 \\ x_1 + 3x_2 - 5x_3 = 0 \end{cases}$$

$$\left(\begin{array}{cccc} 1 & 1 & -1 & 2 \\ 1 & 3 & -3 & 4 \\ 3 & 2 & 1 & 0 \\ 1 & 3 & 0 & -5 \end{array} \right) \xrightarrow{\text{I}-\text{I}} \sim \left(\begin{array}{cccc} 1 & 1 & -1 & 2 \\ 0 & 2 & -2 & 2 \\ 0 & -1 & 4 & -6 \\ 0 & 2 & 1 & -7 \end{array} \right) \cdot \frac{1}{2} \sim$$

$$\sim \left(\begin{array}{cccc} 1 & 1 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & -1 & 4 & -6 \\ 0 & 2 & 1 & -7 \end{array} \right) \xrightarrow{\text{III}+\text{II}} \sim \left(\begin{array}{cccc} 1 & 1 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 3 & -5 \\ 0 & 0 & 3 & -9 \end{array} \right) \sim \xrightarrow{\text{IV}-2\text{II}}$$

$$\sim \left(\begin{array}{cccc} 1 & 1 & -1 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 3 & -5 \\ 0 & 0 & 0 & -4 \end{array} \right) \Rightarrow M(A) = M(A|B) = 4, n=4 \Rightarrow$$

\Rightarrow существует собств. числа и eig.

(13)

$$\begin{cases} x_1 + x_2 - x_3 + 2x_4 \geq 0 \\ x_2 - x_3 + x_4 \geq 0 \\ 3x_3 - 5x_4 \geq 0 \\ -4x_4 \geq 0 \end{cases}$$

$$\begin{cases} x_1 \geq 0 \\ x_2 \geq 0 \\ x_3 \geq 0 \\ x_4 \geq 0 \end{cases} \Rightarrow$$

Darstellung: $(0; 0; 0; 0)$

QCP: $\{ (0; 0; 0; 0) \}$

Wk. 3.38

$$\begin{cases} 2x_1 - 4x_2 + 5x_3 + 3x_4 \geq 0 \\ 3x_1 - 6x_2 + 4x_3 + 2x_4 \geq 0 \\ 4x_1 - 8x_2 + 17x_3 + 11x_4 \geq 0 \end{cases}$$

$$\left(\begin{array}{cccc} 2 & -4 & 5 & 3 \\ 3 & -6 & 4 & 2 \\ 4 & -8 & 17 & 11 \end{array} \right) \xrightarrow{\text{II} - 3\text{I}} \sim \quad \left(\begin{array}{cccc} 2 & -4 & 5 & 3 \\ 0 & 0 & -7 & -5 \\ 4 & -8 & 17 & 11 \end{array} \right) \xrightarrow{\text{III} - 2\text{I}} \sim$$

$$\left(\begin{array}{cccc} 2 & -4 & 5 & 3 \\ 0 & 0 & -7 & -5 \\ 0 & 0 & 7 & 5 \end{array} \right) \xrightarrow{\cdot(-1)} \sim \quad \left(\begin{array}{cccc} 2 & -4 & 5 & 3 \\ 0 & 0 & 7 & 5 \\ 0 & 0 & -7 & -5 \end{array} \right) \xrightarrow{\text{III} + \text{II}}$$

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$\sim \begin{pmatrix} 2 & -4 & 5 & 3 \\ 0 & 0 & 17 & 15 \\ 0 & 0 & 0 & 0 \end{pmatrix} \Rightarrow r(A) = r(A|B) = 2, n = 3 \Rightarrow$
 \Rightarrow exist. solu. u. neouf.

$$M_2 = \begin{vmatrix} 5 & 3 \\ 7 & 5 \end{vmatrix} = 25 - 21 = 4 \neq 0 \Rightarrow$$
 $\Rightarrow \cancel{x_1 = t_1}, x_2 = t_2 - cb_1,$
 $x_3, x_4 = \text{arbitrary.}$

$$\begin{cases} 2t_1 - 4t_2 + 5x_3 + 3x_4 = 0 \\ 7x_3 + 5x_4 = 0 \end{cases}$$

$$\begin{cases} x_3 = 4t_2 - 2t_1 - 3x_4 \\ x_4 = -\frac{7}{5}x_3 \end{cases}$$

$$x_3 = 4t_2 - 2t_1 - 3(-\frac{7}{5}x_3)$$

$$x_3 = 4t_2 - 2t_1 + \frac{21}{5}x_3$$

$$-\frac{16}{5}x_3 = 4t_2 - 2t_1$$

$$x_3 = -\frac{5}{4}t_2 + \frac{5}{8}t_1$$

$$x_4 = -\frac{7}{5}x_3$$

$$x_4 = -\frac{7}{5} \left(-\frac{5}{4}t_2 + \frac{5}{8}t_1 \right)$$

$$x_4 = \frac{7}{4}t_2 - \frac{7}{8}t_1$$

$$\begin{cases} x_3 = \frac{5}{8}t_1 - \frac{5}{4}t_2 \\ x_4 = \frac{7}{4}t_2 - \frac{7}{8}t_1 \end{cases}$$

Ortsvektoren: $(t_1; t_2; \frac{5}{8}t_1 - \frac{5}{4}t_2; \frac{7}{4}t_2 - \frac{7}{8}t_1)$

OPCP: $\{(1; 0; \frac{5}{8}; -\frac{7}{8}); (0; 1; -\frac{5}{4}; \frac{7}{4})\}$

W.L. 3. 39.

$$\begin{cases} 5x_1 + 6x_2 - 2x_3 + 7x_4 + 4x_5 = 0 \\ 2x_1 + 3x_2 - x_3 + 4x_4 + 2x_5 = 0 \\ 5x_1 + 9x_2 - 3x_3 + 8x_4 + 6x_5 = 0 \\ 7x_1 + 9x_2 - 3x_3 + 5x_4 + 6x_5 = 0 \end{cases}$$

$$\left(\begin{array}{ccccc|c} 5 & 6 & -2 & 7 & 4 \\ 2 & 3 & -1 & 4 & 2 \\ 5 & 9 & -3 & 1 & 6 \\ 7 & 9 & -3 & 5 & 6 \end{array} \right) \xrightarrow{\begin{matrix} 5I - 2I \\ I - I \\ 5I - 7I \end{matrix}}$$

(10)

$$\sim \left(\begin{array}{ccccc} 5 & 6 & -2 & 7 & 4 \\ 0 & 3 & -1 & -6 & 2 \\ 0 & 3 & -1 & -6 & -2 \\ 0 & 3 & -1 & -24 & 2 \end{array} \right) \xrightarrow{\text{III} - \text{II}} \sim$$

$$\sim \left(\begin{array}{ccccc} 5 & 6 & -2 & 7 & 4 \\ 0 & 3 & -1 & 6 & 2 \\ 0 & 0 & 0 & -12 & -4 \\ 0 & 0 & 0 & -30 & 0 \end{array} \right) \xrightarrow{2\text{IV} - 5\text{III}} \sim$$

$$\sim \left(\begin{array}{ccccc} 5 & 6 & -2 & 7 & 4 \\ 0 & 3 & -1 & 6 & 2 \\ 0 & 0 & 0 & -12 & -4 \\ 0 & 0 & 0 & 0 & 20 \end{array} \right) \xrightarrow{\begin{matrix} r(A) | (A|B) = 4 \\ : -4 \\ n=5 \rightarrow \end{matrix}}$$

\Rightarrow сист. обр. и реш.

$$M_4 = \begin{vmatrix} 5 & 6 & 7 & 4 \\ 0 & 3 & 6 & 2 \\ 0 & 0 & +3 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 45 \neq 0 \Rightarrow$$

$\Rightarrow x_1, x_2, x_4, x_5$ - свободн.

(*) $x_3 = t$ - свобд.

$$\left\{ \begin{array}{l} 5x_1 + 6x_2 - 2t + 7x_4 + 4x_5 = 0 \\ 3x_2 - t + 6x_4 + 2x_5 = 0 \\ 3x_4 + x_5 = 0 \\ x_5 = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} 5x_1 + 6x_2 - dt + 7x_4 + 4x_5 = 0 \\ 3x_2 - t + 6x_4 + 2x_5 = 0 \\ x_4 = 0 \\ x_5 = 0 \end{array} \right.$$

$$\left\{ \begin{array}{l} x_1 = 0 \\ x_2 = \frac{1}{3}t \\ x_4 = 0 \\ x_5 = 0 \end{array} \right.$$

Dsgy. p-ue: $(0; \frac{1}{3}t; t; 0; 0)$

QCP: $\{(0; \frac{1}{3}; 1; 0; 0)\}$

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Wh. 3.40

$$\begin{cases} 2x_1 + x_2 + 3x_3 = 0 \\ 4x_1 - x_2 + 7x_3 = 0 \\ x_1 + \alpha x_2 + 2x_3 = 0 \end{cases}$$

$$\left(\begin{array}{ccc} 2 & 1 & 3 \\ 4 & -1 & 7 \\ 1 & \alpha & 2 \end{array} \right) \xrightarrow{\text{III} \leftrightarrow \text{I}} \sim \left(\begin{array}{ccc} 1 & \alpha & 2 \\ 4 & -1 & 7 \\ 2 & 1 & 3 \end{array} \right) \xrightarrow{\text{II} - 4\text{I}} \sim \left(\begin{array}{ccc} 1 & \alpha & 2 \\ 0 & -1-4\alpha & -1 \\ 0 & 1+2\alpha & -1 \end{array} \right) \xrightarrow{\text{III} - \text{II}}$$

$$\sim \left(\begin{array}{ccc} 1 & \alpha & 2 \\ 0 & -1-4\alpha & -1 \\ 0 & 1+2\alpha & -1 \end{array} \right) \xrightarrow{\text{III} - \text{II}} \sim \left(\begin{array}{ccc} 1 & \alpha & 2 \\ 0 & -1-4\alpha & -1 \\ 0 & 2+2\alpha & 0 \end{array} \right) \xrightarrow{\text{II} \leftrightarrow \text{III}}$$

$$\sim \left(\begin{array}{ccc} 1 & \alpha & 2 \\ 0 & 2(1+\alpha) & 0 \\ 0 & -1-4\alpha & -1 \end{array} \right) \xrightarrow{2(1+\alpha)\text{III} - (-1-4\alpha)\text{II}}$$

$$\sim \left(\begin{array}{ccc} 1 & \alpha & 2 \\ 0 & 2(1+\alpha) & 0 \\ 0 & 0 & -2(1+\alpha) \end{array} \right) \xrightarrow[2]{-2} \sim \left(\begin{array}{ccc} 1 & \alpha & 2 \\ 0 & 4\alpha & 0 \\ 0 & 0 & 1+\alpha \end{array} \right)$$

~~$\Rightarrow r(A) = r(A|B) = 3, n=3$~~
 ~~$\Rightarrow \text{unst. LGS mit } \alpha \neq 0$~~

1) при $\alpha \neq -1$, $r(A) = r(A|B) = 3, n=3$
 \Rightarrow существ. собств. и ненул.

$$\begin{cases} x_1 + \alpha x_2 + 2x_3 = 0 \\ (1+\alpha)x_2 = 0 \\ (1+\alpha)x_3 = 0 \end{cases}$$

$$\begin{cases} x_1 = 0 \\ x_2 = 0 \\ x_3 = 0 \end{cases} \Rightarrow \begin{array}{l} \text{Основ. p-ue: } (0; 0; 0) \\ \text{ФСР: } \{(0; 0; 0)\} \end{array}$$

2) при $\alpha = -1$, $r(A) = r(A|B) = 1, n=3$
 \Rightarrow существ. собств. и ненул.

$$M_1 = |1| = 1 \neq 0 \Rightarrow x_1 - \text{запись}, \\ x_2 = t_1, x_3 = t_2 - \text{чтобы}$$

$$x_1 + \alpha t_1 + dt_2 = 0$$

$$x_1 = -\alpha t_1 - dt_2$$

$$\text{Основ. p-ue: } (-\alpha t_1, -dt_2; t_1, t_2)$$

$$\text{ФСР: } \{(-\alpha; 1; 0), (-2; 0; 1)\}$$

VL 3.41

$$\left\{ \begin{array}{l} x_1 - 3x_2 + x_3 - dx_4 = 0 \\ 3x_1 + dx_2 + 3x_4 = 0 \\ 5x_1 + 6x_2 - 4x_3 - x_4 = 0 \\ 5x_1 + 5x_2 - \lambda x_3 = 0 \end{array} \right.$$

$$\left(\begin{array}{cccc} 1 & -3 & 1 & -d \\ 3 & 2 & 0 & 3 \\ 5 & 6 & -4 & -1 \\ 3 & 5 & 1 & 0 \end{array} \right) \xrightarrow{\text{II} - 3\text{I}} \sim \left(\begin{array}{cccc} 1 & -3 & 1 & -d \\ 0 & 11 & -3 & 9 \\ 5 & 6 & -4 & -1 \\ 3 & 5 & 1 & 0 \end{array} \right) \xrightarrow{\text{III} - 5\text{I}} \sim \left(\begin{array}{cccc} 1 & -3 & 1 & -d \\ 0 & 11 & -3 & 9 \\ 0 & 21 & -9 & 9 \\ 3 & 5 & 1 & 0 \end{array} \right) \xrightarrow{\text{IV} - 3\text{I}}$$

$$\sim \left(\begin{array}{cccc} 1 & -3 & 1 & -d \\ 0 & 11 & -3 & 9 \\ 0 & 21 & -9 & 9 \\ 0 & 16 & 1-3 & 6 \end{array} \right) \xrightarrow{11\text{II} - 21\text{I}} \sim \left(\begin{array}{cccc} 1 & -3 & 1 & -d \\ 0 & 11 & -3 & 9 \\ 0 & 0 & -36 & -90 \\ 0 & 0 & 11\lambda+12 & -78 \end{array} \right) \xrightarrow{16\text{II} - 16\text{I}}$$

$$\sim \left(\begin{array}{cccc} 1 & -3 & 1 & -d \\ 0 & 11 & -3 & 9 \\ 0 & 0 & -36 & -90 \\ 0 & 0 & 11\lambda+12 & -78 \end{array} \right) \xrightarrow{-18} \sim \left(\begin{array}{cccc} 1 & -3 & 1 & -d \\ 0 & 11 & -3 & 9 \\ 0 & 0 & +2 & 5 \\ 0 & 0 & 11\lambda+12 & -78 \end{array} \right) \xrightarrow{-\cancel{(11\lambda+12)\text{II}}}$$

(21)

$$\sim \left(\begin{array}{cccc} 1 & -3 & 1 & -2 \\ 0 & 11 & -3 & 9 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & 0 & -55 \end{array} \right)$$

1) Трн $\lambda \neq -3 \frac{51}{55}$, $r(A) = r(A|B) = 4$,
 $n=4 \Rightarrow$ сист. с обл. и опт.

Т.к. сист. опт и однород., то

$$\begin{cases} x_1=0 \\ x_2=0 \\ x_3=0 \\ x_4=0 \end{cases} \Rightarrow \begin{array}{l} \text{Общ. ф-ие: } (0; 0; 0; 0) \\ \text{ФСР: } \{(0; 0; 0; 0)\} \end{array}$$

2) Трн $\lambda = -3 \frac{51}{55}$, $r(A) = r(A|B) = 3$, $n=4 \Rightarrow$
 \Rightarrow сист. с обл. и неопт.

]) x_1, x_2, x_3 — мабд., $x_4 = t$ — свобод.

$$\begin{cases} x_1 - 3x_2 + x_3 - 2t = 0 \\ 11x_2 - 3x_3 + 9t = 0 \\ 2x_3 + 5t = 0 \end{cases}$$

$$\begin{cases} x_1 - 3x_2 + x_3 - 2t = 0 \\ 11x_2 - 3x_3 + 9t = 0 \\ x_3 = -\frac{5}{2}t \end{cases}$$

$$\begin{cases} x_1 - 3x_2 + x_3 = 0 \\ \vdots \end{cases}$$

$$x_2 = \frac{3}{22}t$$

$$x_3 = -\frac{5}{2}t$$

$$\begin{cases} x_1 = \frac{105}{22}t \\ \vdots \end{cases}$$

$$x_2 = \frac{3}{22}t$$

$$x_3 = -\frac{5}{2}t$$

$$\text{Dsg. p-ue: } \left(\frac{105}{22}t; \frac{3}{22}t; -\frac{5}{2}t; t \right)$$

$$\Phi CP: \left\{ \left(\frac{105}{22}; \frac{3}{22}; -\frac{5}{2}; 1 \right) \right\}$$