№ 11.5.4

$z = \frac{xy}{x-y}$

$d^2 z = ?$

Док-ть: $z''_{x^2} + 2z''_{xy} + z''_{y^2} = \frac{2}{x-y}$

$z'_x = \frac{\partial z}{\partial x}$, $z'_y = \frac{\partial z}{\partial y}$

$z''_{xx} = \frac{\partial^2 z}{\partial x^2}$, $z''_{yy} = \frac{\partial^2 z}{\partial y^2}$

$z''_{xy} = \frac{\partial^2 z}{\partial x \partial y}$, $z''_{yx} = \frac{\partial^2 z}{\partial y \partial x}$

$d^2 z = \frac{\partial^2 z}{\partial x^2} dx^2 + \frac{2\partial^2 z}{\partial x \partial y} dx dy + \frac{\partial^2 z}{\partial y^2} dy^2$

$z = \frac{xy}{x-y}$

1) $z'_x = \frac{\partial z}{\partial x} = \left(\frac{xy}{x-y}\right)'_x = \frac{(xy)'_x(x-y)}{(x-y)^2} -$

$- \frac{xy(x-y)'_x}{(x-y)^2} = \frac{y(x-y) - xy}{(x-y)^2} =$

$= \frac{yx - y^2 - xy}{(x-y)^2} = \frac{-y^2}{(x-y)^2}$

$$z'_y = \frac{\partial z}{\partial y} = \left(\frac{xy}{x-y}\right)'_y = \frac{(xy)'_y(x-y) - xy(x-y)'_y}{(x-y)^2}$$

$$= \frac{x(x-y)+xy}{(x-y)^2} = \frac{x^2-xy+xy}{(x-y)^2} = \frac{x^2}{(x-y)^2}$$

$$z''_{x^2} = \left(\frac{\partial z}{\partial x}\right)'_x = \left(-\frac{y^2}{(x-y)^2}\right)'_x = -y^2 \cdot \frac{-2}{(x-y)^3} \cdot$$

$$\cdot (1-0) = \frac{2y^2}{(x-y)^3}$$

$$z''_{xy} = \left(\frac{\partial z}{\partial x}\right)'_y = \left(-\frac{y^2}{(x-y)^2}\right)'_y = -\frac{(y^2)'_y(x-y)^2 - y^2((x-y)^2)'_y}{(x-y)^4}$$

$$= -\frac{2y(x-y)^2 + 2y^2(x-y)}{(x-y)^4} =$$

$$= -\frac{(x-y)(2y(x-y)+2y^2)}{(x-y)^4} =$$

$$= -\frac{\cancel{(x-y)}(2xy-2y^2+2y^2)}{(x-y)^{\cancel{4}\,3}} =$$

$$= -\frac{2xy}{(x-y)^3}$$

$$z''_{y^2} = \left(\frac{\partial z}{\partial y}\right)'_y = \left(\frac{x^2}{(x-y)^2}\right)'_y = x^2 \frac{-2}{(x-y)^3} \cdot$$

$$\cdot (-1) = \frac{2x^2}{(x-y)^3}$$

$$d^2z = z''_{x^2}dx^2 + 2z''_{xy}dxdy + z''_{y^2}dy^2$$

$$d^2z = \frac{2y^2dx^2}{(x-y)^3} + 2\left(-\frac{2xy}{(x-y)^3}\right)dxdy +$$

$$+ \frac{2x^2}{(x-y)^3}dy^2 = \frac{2y^2dx^2 - 4xy\,dxdy + 2x^2dy^2}{(x-y)^3}$$

2) $z''_{x^2} + 2z''_{xy} + z''_{y^2} = \frac{2}{x-y}$

$$\frac{2y^2 - 4xy + 2x^2}{(x-y)^3} = \left[z''_{x^2} + 2z''_{xy} + z''_{y^2}\right] =$$

$$= \frac{(x\sqrt{2} - y\sqrt{2})^2}{(x-y)^3} = \frac{2\cancel{(x-y)^2}}{(-y+x)^{\cancel{3}}} =$$

$$= \frac{2}{x-y}, \text{ ч.т.д.}$$

w11.5.5

$z = \frac{xy}{x+y}$, $d^3z = ?$

$$d^3z = \frac{\partial^3 z}{\partial x^3}dx^3 + 3\frac{\partial^3 z}{\partial x^2 \partial y}dx^2 dy +$$

$$+3\frac{\partial^3 z}{\partial x \partial y^2}dx dy^2 + \frac{\partial^3 z}{\partial y^3}dy^3$$

$$\frac{\partial z}{\partial x} = \left(\frac{xy}{x+y}\right)'_x = \frac{y(x+y)-xy}{(x+y)^2} =$$

$$= \frac{y^2}{(x+y)^2}$$

$$\frac{\partial z}{\partial y} = \left(\frac{xy}{x+y}\right)'_y = \frac{x^2}{(x+y)^2}$$

$$\frac{\partial^2 z}{\partial x^2} = \left(\frac{y^2}{(x+y)^2}\right)'_x = y^2 \cdot -\frac{2}{(x+y)^3} =$$

$$= -\frac{2y^2}{(x+y)^3}$$

$$\frac{\partial^2 z}{\partial y^2} = \left(\frac{x^2}{(x+y)^2}\right)'_y = x^2 \cdot -\frac{2}{(x+y)^3} =$$

$$= -\frac{2x^2}{(x+y)^3}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \left(\frac{y^2}{(x+y)^2}\right)'_y = \frac{2y(x+y)^2 - y^2 2(x+y)}{(x+y)^4} =$$

$$= \frac{(x+y)(2yx + 2y^2 - 2y^2)}{(x+y)^4} = \frac{2yx}{(x+y)^3}$$

$$\frac{\partial^3 z}{\partial x^3} = \left(-\frac{2y^2}{(x+y)^3}\right)'_x = -2y^2 \, \frac{-3}{(x+y)^4} =$$

$$= \frac{6y^2}{(x+y)^4}$$

$$\frac{\partial^3 z}{\partial y^3} = \left(-\frac{2x^2}{(x+y)^3}\right)'_y = \frac{6x^2}{(x+y)^4}$$

$$\frac{\partial^3 z}{\partial x^2 \partial y} = \left(\frac{2xy}{(x+y)^3}\right)'_x = \frac{2y(x+y)^3 - 2xy \cdot 3(x+y}{(x+y)^6} =$$

$$= \frac{\cancel{(x+y)^2}(2xy + 2y^2 - 6xy)}{(x+y)^{\cancel{6}\,4}} =$$

$$= \frac{2y^2 - 4xy}{(x+y)^4} = \frac{2y(y-2x)}{(x+y)^4}$$

$$\frac{\partial^3 z}{\partial x \partial y^2} = \left(\frac{2xy}{(x+y)^3}\right)'_y = \frac{2x^2 - 4xy}{(x+y)^4} =$$

$$= \frac{2x(x-2y)}{(x+y)^4}$$

$$d^3 z = \frac{6y^2 dx^3}{(x+y)^4} + 3\,\frac{(2y^2 - 4xy)dx^2 dy}{(x+y)^4} +$$

$$+ 3\,\frac{(2x^2 - 4xy)dx dy^2}{(x+y)^4} + \frac{6x^2 dy^3}{(x+y)^4} =$$

$$= \frac{6y^2 dx^3 + 3(2y^2 - 4xy)dx^2 dy + 3(2x^2 - 4xy)dx dy^2}{(x+y)^4}$$

$$+ \frac{6x^2 dy^3}{(x+y)^4}$$

<u>№ 11.5.6</u>

$$z = \ln(x^2 + y^2)$$

$$d^2 z = \frac{\partial^2 z}{\partial x^2} dx^2 + \frac{2\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2}$$

$$\frac{\partial z}{\partial x} = (\ln(x^2+y))'_x = \frac{1}{x^2+y^2} \cdot 2x =$$

$$= \frac{2x}{x^2+y^2}$$

$$\frac{\partial z}{\partial y} = \frac{2y}{x^2+y^2}$$

$$\frac{\partial^2 z}{\partial x^2 \cancel{\partial y}} = \left(\frac{2x}{x^2+y^2}\right)'_x = \frac{2(x^2+y^2)-2x\cdot 2x}{(x^2+y^2)^2} =$$

$$= \frac{2x^2+2y^2-4x^2}{(x^2+y^2)^2} = \frac{2y^2-2x^2}{(x^2+y^2)^2} =$$

$$\frac{\partial^2 z}{\partial y^2} = \left(\frac{2y}{x^2+y^2}\right)'_y = \frac{2x^2-2y^2}{(x^2+y^2)^2}$$

$$\frac{\partial^2 z}{\partial x \partial y} = \left(\frac{2x}{x^2+y^2}\right)'_y = 2x \cdot \frac{-1}{(x^2+y^2)^2} \cdot 2y$$

$$= -\frac{4xy}{(x^2+y^2)^2}$$

$$d^2 z = \frac{2y^2-2x^2}{(x^2+y^2)^2}dx^2 - \frac{8xy}{(x^2+y^2)^2}dxdy +$$

$$+ \frac{2x^2-2y^2}{(x^2+y^2)^2}dy^2 =$$

$$= 2\,\frac{(y^2 - x^2)\,dx^2 - 4xy\,dx\,dy + (x^2 - y^2)\,dy^2}{(x^2+y^2)^2}$$

W 11.5.23

$[y(x) =]\; x^3y^2 - xy^5 + 5x - y = 0$

$y'''(0) = ?$

$y'_x \Rightarrow$

$(x^3y^2)'_x - (xy^5)'_x + (5x)'_x - (y)'_x = 0$

$3x^2y^2 + 2x^3yy' - y^5 + 4xy^4y' + 5 - y' =$