„Предел последовательности"

№6.3.29

$$\lim_{n\to\infty}\frac{(n+2)^3}{5n^3} = \lim_{n\to\infty}\frac{n^3+6n^2+12n+8}{5n^3} =$$

$$= \lim_{n\to\infty}\left(\frac{1}{5} + \frac{6}{5n}^{=0} + \frac{12}{5n^2}^{=0} + \frac{8}{5n^3}^{=0}\right) = \frac{1}{5}$$

№6.3.30

$$\lim_{n\to\infty}\left(\frac{3}{n+2} - \frac{5}{2n+1}\right) = \lim_{n\to\infty}\frac{3(2n+1)-5(n+2)}{(n+2)(2n+1)} =$$

$$= \lim_{n\to\infty}\frac{n-7}{2n^2+5n+2} = \lim_{n\to\infty}\frac{\frac{1}{n} - \frac{7}{n^2}}{2 + \frac{5}{n} + \frac{2}{n^2}} =$$

$$= \frac{0-0}{2+0+0} = 0$$

№6.3.31

$$\lim_{n\to\infty}\left(\frac{2n^2+5}{4n+1} - \frac{n^2+4}{2n+3}\right) =$$

$$= \lim_{n\to\infty}\left(\frac{(2n^2+5)(2n+3)-(n^2+4)(4n+1)}{(4n+1)(2n+3)}\right) =$$

$$= \lim_{n\to\infty}\frac{5n^2-6n+11}{8n^2+10n-3} = \lim_{n\to\infty}\frac{\cancel{5n}\,5 - \frac{6}{n} + \frac{11}{n^2}}{8 - \frac{10}{n} - \frac{3}{n^2}} =$$

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$$= \frac{5-0+0}{8-0-0} = \frac{5}{8}$$

№6.3.32

$$\lim_{n\to\infty}\left(\sqrt[3]{n^3-4n^2}-n\right) =$$

$$= \lim_{n\to\infty}(n^3-4n^2)^{\frac{1}{3}} - n = \frac{4}{3}$$

[Wolfram Alpha]

№6.3.33

$$\lim_{n\to\infty}\frac{5^n-1}{5^n+1} = \lim_{n\to\infty}\frac{5^n\left(1-\frac{1}{5^n}\right)}{5^n\left(1+\frac{1}{5^n}\right)} = \lim_{n\to\infty}\frac{1-\frac{1}{5^n}}{1+\frac{1}{5^n}} =$$

$$= \frac{1-0}{1+0} = 1$$

№6.3.34

$$\lim_{n\to\infty}\frac{1+2+3+\ldots+n}{n^2+1} = \lim_{n\to\infty}\frac{n(n+1)}{2(n^2+1)} =$$

$$= \lim_{n\to\infty}\frac{n^2+n}{2n^2+2} = \cancel{\frac{\frac{1}{n^2}+\frac{1}{n}}{2+\frac{2}{n^2}}} \cancel{=} \cancel{\frac{0+0}{2+0}}$$

$$= \lim_{n\to\infty}\frac{n^2\left(1+\frac{1}{n}\right)}{n^2\left(2+\frac{2}{n^2}\right)} = \frac{1+0}{2+0} = \frac{1}{2}$$

№ 6.3.35

$$\lim_{n\to\infty}\frac{1-q^n}{1+q}, \quad q\neq 1$$

$q>1$

$$\lim_{n\to\infty}\frac{1-q^n}{1+q}=\frac{\infty}{1+q}=\infty$$

$q<1$

$$\lim_{n\to\infty}\frac{1-q^n}{1+q}=\frac{1}{1+q}$$

№ 6.3.36

$$\lim_{n\to\infty}\frac{1+\frac{1}{3}+\frac{1}{9}+\ldots+\frac{1}{3^n}}{1+\frac{1}{4}+\frac{1}{16}+\ldots+\frac{1}{4^n}}=\lim_{n\to\infty}\frac{\frac{3^{n+1}-1}{2\cdot 3^n}}{\frac{4^{n+1}-1}{3\cdot 4^n}}=$$

$$\lim_{n\to\infty}\frac{(3^{n+1}-1)(3\cdot 4^n)}{(4^{n+1}-1)(2\cdot 3^n)}=\lim_{n\to\infty}\frac{3^{n+2}\cdot 4^n-3\cdot 4^n}{2\cdot 3^n\cdot 4^{n+1}-2\cdot 3^n}=$$

$$=\lim_{n\to\infty}\frac{9\cdot 3^n\cdot 2^{2n}-3\cdot 2^{2n}}{8\cdot 2^{2n}\cdot 3^n-2\cdot 3^n}=\lim_{n\to\infty}\frac{3^n\left(9\cdot 2^{2n}-\frac{2^{2n}}{3^{n-1}}\right)}{3^n\left(8\cdot 2^{2n}-2\right)}=$$

$$=\lim_{n\to\infty}\frac{2^{2n}\left(9-\frac{1}{3^{n-1}}\right)}{2^{2n}\left(8-\frac{1}{2^{2n-1}}\right)}=\lim_{n\to\infty}\frac{\left(9-\frac{1}{3^{n-1}}\right)}{\left(8-\frac{1}{2^{2n-1}}\right)}=$$

$$= \frac{9-0}{8-0} = \frac{9}{8}$$

№6.3.37

$$\lim_{n\to\infty} \frac{x_n+2}{x_n^2+4}, \text{ если } \lim_{n\to\infty} x_n = -1$$

$$\lim_{n\to\infty} \frac{x_n+2}{x_n^2+4} = \frac{-1+2}{(-1)^2+4} = \frac{1}{5}$$

№6.3.38

$$\lim_{n\to\infty} \frac{\sqrt{n^3+n^2-4} - \sqrt[5]{n^6}}{\sqrt[3]{n^5+2n} + \sqrt[4]{n^6+3n^4+2}} =$$

$$= \lim_{n\to\infty} \frac{\sqrt{\frac{1}{n^3} + \frac{1}{n^4} - \frac{4}{n^6}} - \sqrt[5]{1}}{\sqrt[3]{\frac{1}{n} + \frac{2}{n^5}} + \sqrt[4]{1 + \frac{3}{n^2} + \frac{2}{n^6}}} =$$

$$= \frac{\sqrt{0+0-0} - 1}{\sqrt[3]{0+0} + \sqrt[4]{1+0+0}} = \frac{-1}{1} = -1$$