$= \sqrt{17-1} = 4] = \frac{1}{2}\ln|t| + 7\cdot\frac{1}{4}\cdot$

$\cdot \operatorname{arctg}\frac{y}{4} + C = \frac{1}{2}\ln(x^2-2x+17) +$ т.к. ветви вверх и нет корней

$+ \frac{7}{4}\operatorname{arctg}\frac{x-1}{4} + C$

№ 8.37

$\int \frac{(4x-1)dx}{x^2+x+1} = [x^2+x+1=0 \Rightarrow$

$\Rightarrow D = 1-4\cdot1 = -3 < 0 \Rightarrow$ корней нет,

$A=4,\ B=-1,\ p=1,\ q=1] =$

$= \int \frac{\frac{4}{2}(2x+1) + \left(-1-\frac{4\cdot1}{2}\right)}{x^2+x+1}dx =$

$= \int \frac{2(2x+1)-3}{x^2+x+1}dx = 2\int\frac{(2x+1)dx}{x^2+x+1} -$

$-3\int\frac{dx}{x^2+x+1} = [\ 1)\ t = x^2+x+1 \Rightarrow dt = (2x+1)dx;\ 2)\ y = x+\frac{1}{2} \Rightarrow dy = dx \Rightarrow a = \frac{\sqrt3}{2}\ ]$

$= 2\int\frac{dt}{t} - 3\int\frac{dy}{y^2+\left(\frac{\sqrt3}{2}\right)^2} =$

$= 2\ln|t| - 3\cdot\frac{2}{\sqrt3}\operatorname{arctg}\frac{2y}{\sqrt3} + C =$

$= 2\ln|x^2+x+1| - 2\sqrt3\operatorname{arctg}\frac{2x+1}{\sqrt3} + C =$

$= [$т.к. x^2+x+1 – пар, ветви вверх и корней не имеет $] =$

$= 2\ln(x^2+x+1) - 2\sqrt{3}\,\mathrm{arctg}\,\frac{2x+1}{\sqrt{3}} + C$

N 8.3.9

$\int \frac{dx}{(x^2+1)^3} = [x^2+1=0 \Rightarrow D = 0-4\cdot 1 = -4 < 0 \Rightarrow$

$\Rightarrow$ корней нет $\Rightarrow A=0, B=1, p=0, q=1 \Rightarrow$

$\Rightarrow$ IV тип, исп-ем ф. понижения степ.$] =$

$= \frac{1}{2(3-1)\cdot 1^2} \cdot \frac{x}{(x^2+1)^2} + \frac{1}{1^2} \cdot \frac{2\cdot 3-3}{2\cdot 3-2} \cdot$

$\cdot \int \frac{dx}{(x^2+1)^2} = \frac{x}{4(x^2+1)^2} + \frac{3}{4}\int \frac{dx}{(x^2+1)^2} =$

$= \frac{x}{4(x^2+1)^2} + \frac{3}{4}\left(\frac{1}{2(2-1)\cdot 1^2} \cdot \frac{x}{(x^2+1)} + \frac{2\cdot 2-3}{2\cdot 2-2} \cdot\right.$

$\left.\cdot \int \frac{dx}{x^2+1}\right) = \frac{x}{4(x^2+1)^2} + \frac{3}{4}\left(\frac{x}{2(x^2+1)} +\right.$

$\left. + \frac{1}{2}\int \frac{dx}{x^2+1}\right) = \frac{x}{4(x^2+1)^2} + \frac{3}{4}\left(\frac{x}{2(x^2+1)} +\right.$

$\left.\frac{1}{2}\mathrm{arctg}\,x\right) + C = \frac{x}{4(x^2+1)^2} + \frac{3x}{8(x^2+1)} +$

$+ \frac{3}{8}\mathrm{arctg}\,x + C = \frac{2x + 3x(x^2+1)}{8(x^2+1)^2} +$

$+ \frac{3}{8}\mathrm{arctg}\,x + C$

w8.3.10

$$\int \frac{dx}{(x^2-4x+29)^2} = [\ x^2-4x+29=0 \Rightarrow$$

$\Rightarrow D = 16-4\cdot 29 < 0 \Rightarrow$ корней нет $\Rightarrow$

$\Rightarrow$ IV тип пр.д. $\Rightarrow A=0,\ B=1,\ p=-4,\ q=29] =$

$$= [\ y = x-2 \Rightarrow dy = dx,\ a = \sqrt{29-\frac{(-4)^2}{4}} = 5] =$$

$$= \int \frac{dy}{(y^2+5^2)^2} = \frac{1}{2(2-1)\cdot 5^2} \cdot \frac{y}{(y^2+5^2)} + \frac{1}{5^2} \cdot$$

$$\cdot \frac{2\cdot 2-3}{2\cdot 2-2} \cdot \int \frac{dy}{(y^2+5^2)} = \frac{y}{50(y^2+25)} +$$

$$+ \frac{1}{50}\int \frac{dy}{y^2+5^2} = \frac{1}{50}\left(\frac{y}{y^2+25} +\right.$$

$$\left. + \frac{1}{5}\operatorname{arctg}\frac{y}{5}\right) + C = \frac{1}{50}\left(\frac{x-2}{(x-2)^2+25} +\right.$$

$$\left. + \frac{1}{5}\operatorname{arctg}\frac{x-2}{5}\right) + C =$$

$$= \frac{1}{250}\left(\frac{5(x-2)}{x^2-4x+29} + \operatorname{arctg}\frac{x-2}{5}\right) + C$$

w8.3.11

$$\int \frac{3x-2}{(x^2+6x+10)^2}\,dx = [\ x^2+6x+10=0 \Rightarrow$$

$\Rightarrow D = 36-4\cdot 10 = -4 < 0 \Rightarrow$ корней нет $\Rightarrow$

$$\Rightarrow A=3,\ B=-2,\ p=6,\ q=10,\ a=\sqrt{10-\frac{36}{4}}=1] =$$

$$= \int \frac{\frac{3}{2}(2x+6) + \left(-2 - \frac{3\cdot 6}{2}\right)}{(x^2+6x+10)^2}\,dx =$$

$$= \frac{3}{2}\int \frac{(2x+6)\,dx}{(x^2+6x+10)^2} - 11\int \frac{dx}{(x^2+6x+10)^2} =$$

$$= \left[\begin{array}{l} 1)\ t = x^2+6x+10 \Rightarrow dt = (2x+6)\,dx \\ 2)\ y = x+3 \Rightarrow dy = dx \end{array}\right] =$$

$$= \frac{3}{2}\int \frac{dt}{t^2} - 11\int \frac{dy}{(y^2+1)^2} =$$

$$= \frac{3}{2}\int \frac{dt}{t^2} - 11\left(\frac{1}{2(2-1)}\cdot\frac{y}{y^2+1} + \right.$$

$$\left. + \frac{2\cdot 2-3}{2\cdot 2-2}\int \frac{dy}{y^2+1}\right) = -\frac{3}{2t} -$$

$$- 11\left(\frac{y}{2(y^2+1)} + \frac{1}{2}\,\mathrm{arctg}\,y\right) + C =$$

$$= -\frac{3}{2(x^2+6x+10)} - 11\left(\frac{x+3}{2(x^2+6x+10)} + \frac{1}{2}\,\mathrm{arctg}(x+3)\right) +$$

$$+ C = \frac{-3-11(x+3)}{2(x^2+6x+10)} - \frac{11}{2}\,\mathrm{arctg}(x+3) + C$$

№8.3.13

$$\int \frac{2x-3}{(x-5)(x+2)}\,dx = \left[\frac{2x-3}{(x-5)(x+2)} = \right.$$

$$= \frac{A}{x-5} + \frac{B}{x+2} \Rightarrow 2x-3 = A(x+2) +$$

$$+ B(x-5) = Ax + 2A + Bx - 5B =$$

$$= (A+B)x + (2A-5B) \Rightarrow$$

$$\Rightarrow \begin{cases} A+B=2 \\ 2A-5B=-3 \end{cases} \Big| \cdot (-2) \Rightarrow + \begin{cases} -2A-2B=-4 \\ 2A-5B=-3 \end{cases} \Rightarrow$$

$$\Rightarrow -7B=-7 \Rightarrow B=1 \Rightarrow A+1=2 \Rightarrow A=1] =$$

$$= \int \left(\frac{1}{x-5} + \frac{1}{x+2} \right) dx =$$

$$= \int \frac{dx}{x-5} + \int \frac{dx}{x+2} = \int \frac{d(x-5)}{x-5} +$$

$$+ \int \frac{d(x+2)}{x+2} = \ln|x-5| + \ln|x-2| + C$$

<u>w8. 3.14</u>

$$\int \frac{x+2}{x^2-6x+5} dx = [x^2-6x+5=0 \Rightarrow$$

$$\Rightarrow D = 36-4\cdot 5 = 16 \Rightarrow x_1 = \frac{6+4}{2} = 5;$$

$$x_2 = \frac{6-4}{2} = 1 \Rightarrow x^2-6x+5 = (x-5)(x-1)] =$$

$$= \int \frac{x+2}{(x-5)(x-1)} dx = [\frac{x+2}{(x-5)(x-1)} =$$

$$= \frac{A}{x-5} + \frac{B}{x-1} \Rightarrow x+2 = A(x-1) + B(x-5) =$$

$$= Ax - A + Bx - 5B = (A+B)x + (-A-5B) \Rightarrow$$

$$\Rightarrow + \begin{cases} A+B=1 \\ -A-5B=2 \end{cases} \Rightarrow -4B=3 \Rightarrow B=-\frac{3}{4} \Rightarrow$$

$$\Rightarrow A - \frac{3}{4} = 1 \Rightarrow A = \frac{7}{4}] = \int \left(\frac{7}{4(x-5)} - \right.$$

$$\left. - \frac{3}{4(x-1)} \right) dx = \frac{7}{4} \int \frac{dx}{x-5} - \frac{3}{4} \int \frac{dx}{x-1} =$$

$$= \frac{7}{4}\int \frac{d(x-5)}{x-5} - \frac{3}{4}\int \frac{d(x-1)}{x-1} =$$

$$= \frac{7}{4}\ln|x-5| - \frac{3}{4}\ln|x-1| + C$$

№ 8.3.15

$$\int \frac{dx}{x^4+x^2} = \Big[x^4+x^2=0 \Rightarrow x^2(x^2+1)=0 \Rightarrow$$

$\Rightarrow x^2=0$ или $x^2+1=0 \Rightarrow x=0\Big] =$

$$= \int \frac{dx}{x^2(x^2+1)} = \Big[\frac{1}{x^2(x^2+1)} = \cancel{\frac{A}{x^2} + \frac{B}{x}}$$

$$\cancel{+ \frac{C}{x^2+1} \Rightarrow 0x^2+0x+1 = A(x^2+1) +}$$

$$\cancel{+ B(x^2+1)x + Cx^2 = Ax^2+A+(Bx^2+B)x+}$$

$$\cancel{+ Cx^2}$$

$$= \frac{A}{x^2}^{\,|x^2+1} + \frac{B}{x^2+1}^{\,|x^2} \Rightarrow 0x^2+0x+1 =$$

$$= A(x^2+1)+Bx^2 = Ax^2+A+Bx^2 =$$

$$= x^2(A+B)+A \Rightarrow$$

$$\Rightarrow \begin{cases} A+B=0 \\ A=1 \end{cases} \Rightarrow B=-1\Big] =$$

$$= \int\left(\frac{1}{x^2} - \frac{1}{x^2+1}\right)dx =$$

$$= \int\frac{dx}{x^2} - \int\frac{dx}{x^2+1} = -\frac{1}{x} - \operatorname{arctg}x + C$$