№ 8.2.11

$$\int \frac{4x+3}{\sqrt{x^2-5}}\,dx = 4\int \frac{x\,dx}{\sqrt{x^2-5}} + 3\int \frac{dx}{\sqrt{x^2-5}} =$$

$$= \left[\, 2)\ \int \frac{dx}{\sqrt{x^2+a}} = \ln\left|x+\sqrt{x^2+a}\right| + C,\ a=-5;\right.$$

$$1)\ t = x^2-5 \Rightarrow dt = d(x^2-5) = 2x\,dx,$$

$$\left. x\,dx = \frac{1}{2}\,dt \,\right] =$$

$$= 4 \cdot \int \frac{\frac{1}{2} dt}{\sqrt{t}} + 3 \int \frac{dx}{\sqrt{x^2-5}} =$$

$$= 2 \int \frac{dt}{\sqrt{t}} + 3 \int \frac{dx}{\sqrt{x^2-5}} =$$

$$= 2 \cdot 2\sqrt{t} + 3 \ln |x + \sqrt{x^2-5}| + C =$$

$$= 4\sqrt{x^2-5} + 3 \ln |x + \sqrt{x^2-5}| + C$$

w 8.2.12

$$\int e^{\sin^2 x} \cdot \sin 2x \, dx = [t = \sin^2 dx \Rightarrow$$

$$\Rightarrow dt = d(\sin^2 x) = 2 \sin x \cdot \cos x \, dx =$$

$$= \sin 2x \, dx] = \int e^t dt =$$

$$= e^t + C = e^{\sin^2 x} + C$$

w 8.2.13

$$\int \frac{1 - 2\sin x}{\cos^2 x} dx = \int \frac{dx}{\cos^2 x} - \int \frac{2 \sin x \, dx}{\cos^2 x} =$$

$$= [t = \cos x \Rightarrow dt = d(\cos x) =$$

$$= -\sin x \, dx \Rightarrow \sin x \, dx = -dt] =$$

$$= \int \frac{dx}{\cos^2 x} + 2 \int \frac{dt}{t^2} =$$

$$= \operatorname{tg} x + 2 \cdot \left(-\frac{1}{t}\right) + C =$$

Практика (продолжение)

Интегрирование (часть 2)

$= tg x - \frac{2}{\cos x} + C$

№8.2.14

$\int \frac{3x-4}{x^2-4} dx = 3\int \frac{x dx}{x^2-4} - 4\int \frac{dx}{x^2-4} =$

$= [1)\ t = x^2-4 \Rightarrow dt = 2x dx \Rightarrow$

$\Rightarrow x dx = \frac{1}{2} dt$; 2) табличный $] =$

$= 3\int \frac{\frac{1}{2} dt}{t} - 4\int \frac{dx}{x^2-4} =$

$= \frac{3}{2} \ln|t| - 4 \cdot \frac{1}{2 \cdot 2} \ln\left|\frac{x-2}{x+2}\right| + C =$

$= \frac{3}{2} \ln|x^2-4| - \ln\left|\frac{x-2}{x+2}\right| + C$

№8.2.16

$\int \sqrt{9-x^2} dx = [x = 3\sin t \Rightarrow$

$\Rightarrow dx = d(3\sin t) = 3\cos t\, dt] =$

$= \int \sqrt{9-9\sin^2 t} \cdot 3\cos t\, dt =$

$= \int \sqrt{9\cos^2 t} \cdot 3\cos t\, dt =$

$= \int 3\cos t \cdot 3\cos t\, dt =$

$= \cancel{9\int \cos^2 x\, dt} = 9\int \cos^2 t\, dt =$

$= 9\int \frac{1+\cos 2t}{2}\, dt =$

$= \frac{9}{2}\left(\int dt + \int \cos 2t\, dt\right) =$

$= \frac{9}{2}\left(\int dt + \frac{1}{2}\int \cos 2t\, d(2t)\right) =$

$= \frac{9}{2}\left(t + \frac{1}{2}\sin 2t\right) + C =$

$= \frac{9}{2}\left(\arcsin\frac{x}{3} + \frac{1}{2}\sin\left(2\arcsin\frac{x}{3}\right)\right) + C =$

$= \frac{9}{2}\left(\arcsin\frac{x}{3} + \frac{1}{2}\cdot 2\sin\left(\arcsin\frac{x}{3}\right)\cdot\right.$

$\left.\cdot\cos\left(\arcsin\frac{x}{3}\right)\right) + C =$

$= \frac{9}{2}\left(\arcsin\frac{x}{3} + \frac{x}{3}\cdot\sqrt{1-\frac{x^2}{9}}\right) + C =$

$= \frac{9}{2}\left(\arcsin\frac{x}{3} + \frac{x}{3}\sqrt{\frac{9-x^2}{9}}\right) + C =$

$= \frac{9}{2}\left(\arcsin\frac{x}{3} + \frac{\sqrt{9-x^2}}{3}\cdot\frac{x}{3}\right) + C =$

$= \frac{9}{2}\left(\arcsin\frac{x}{3} + \frac{x\sqrt{9-x^2}}{9}\right) + C =$

$= \frac{9}{2}\arcsin\frac{x}{3} + \frac{x\sqrt{9-x^2}}{2} + C$

№ 8.2.17

$$\int \frac{dx}{x\sqrt{x+1}} = \Big[x = -\sin^2 t \Rightarrow dx = d(-\sin^2 t)$$

$$= (-\sin^2 t)' dt = -2\sin t \cos t \, dt =$$

$$= -\sin 2t \, dt; \; t = \arcsin\sqrt{-x} \Big] =$$

$$= \int \frac{-\sin 2t \, dt}{-\sin^2 t \sqrt{1-\sin^2 t}} =$$

$$= \int \frac{\sin 2t \, dt}{\sin^2 t \sqrt{\cos^2 t}} = \int \frac{\sin 2t \, dt}{\sin^2 t \cos t} =$$

$$= \int \frac{\cancel{\sin 2t} \, dt}{\sin t \cdot \frac{1}{2} \cdot \cancel{2\sin t \cos t}} =$$

$$= \int \frac{2dt}{\sin t} = 2\ln\left|tg \frac{t}{2}\right| + C =$$

$$= 2\ln\left|tg\left(\frac{1}{2}\arcsin\sqrt{-x}\right)\right| + C =$$

$$= 2\ln\left|\frac{\sin(\arcsin\sqrt{-x})}{1+\cos(\arcsin\sqrt{-x})}\right| + C =$$

$$= 2\ln\left|\frac{\sqrt{-x}}{1+\sqrt{1-(\sqrt{-x})^2}}\right| + C =$$

$$= 2\ln\left|\frac{\sqrt{-x}}{1+\sqrt{1+x}}\right| + C =$$

$$= \ln \frac{-x}{(1+\sqrt{1+x})^2} + C$$

№ 8.2.18

$\int x\sqrt{2-x}\,dx = [x = 2\sin^2 t \Rightarrow$

$\Rightarrow dx = (2\sin^2 t)'\,dt =$

$= 2\sin 2t\,dt = 4\sin t\cos t\,dt;$

$t = \arcsin\sqrt{\frac{x}{2}}\,] =$

$= \int 2\sin^2 t\sqrt{2(1-\sin^2 t)}\cdot 4\sin t\cos t\,dt =$

$= \int 8\sqrt{2}\sin^3 t\cos^2 t\,dt =$

$= 8\sqrt{2}\int \sin^3 t\cos^2 t\,dt =$

$= 8\sqrt{2}\int \sin^2 t\cos^2 t\sin t\,dt =$

$= [\sin t\,dt = -d(\cos t)] =$

$= -8\sqrt{2}\int \sin^2 t\cos^2 t\,d(\cos t) =$

$= -8\sqrt{2}\int (1-\cos^2 t)\cos^2 t\,d(\cos t) =$

$= -8\sqrt{2}\int (\cos^2 t - \cos^4 t)\,d(\cos t) =$

$= [\cos t = z] =$

$= -8\sqrt{2}\left(\int z^2\,dz - \int z^4\,dz\right) =$

$= -8\sqrt{2}\left(\frac{z^3}{3} - \frac{z^5}{5}\right) + C =$

$= -8\sqrt{2}\left(\frac{\cos^3 t}{3} - \frac{\cos^5 t}{5}\right) + C =$

$$= -8\sqrt{2}\left(\frac{\left(\cos\left(\arcsin\sqrt{\frac{x}{2}}\right)\right)^3}{3} - \frac{\left(\cos\left(\arcsin\sqrt{\frac{x}{2}}\right)\right)^5}{5}\right) + C =$$

$$= \left[\cos\arcsin\sqrt{\frac{x}{2}} = \sqrt{1-\left(\sqrt{\frac{x}{2}}\right)^2} = \right.$$

$$\left. = \sqrt{1-\frac{x}{2}} \right] =$$

$$= -8\sqrt{2}\left(\frac{1}{3}\left(\sqrt{1-\frac{x}{2}}\right)^3 - \frac{1}{5}\left(\sqrt{1-\frac{x}{2}}\right)^5\right) + C =$$

$$= -8\sqrt{2}\left(\frac{1}{3}\frac{\sqrt{(2-x)^3}}{2\sqrt{2}} - \frac{1}{5}\frac{\sqrt{(2-x)^5}}{4\sqrt{2}}\right) + C =$$

$$= -\frac{4\sqrt{(2-x)^3}}{3} + \frac{2\sqrt{(2-x)^5}}{5} + C =$$

$$= \frac{2}{5}\sqrt{(2-x)^5} - \frac{4}{3}\sqrt{(2-x)^3} + C$$

<u>w8. 2.19</u>

$$\int \frac{\sqrt{x}\,dx}{x+16} = \left[\begin{matrix} x = t^2 \Rightarrow dx = 2t\,dt; \\ x+16 = t^2+16 \end{matrix}\right] =$$

$$= \int \frac{2t^2\,dt}{t^2+16} = 2\int \frac{t^2\,dt}{t^2+16} = 2\int \frac{t^2+16-16}{t^2+16}dt$$

$$= 2\int\left(\frac{t^2+16}{t^2+16} - \frac{16}{t^2+16}dt\right) =$$

$$= 2\left(\int dt - 16\int\frac{dt}{t^2+16}\right) =$$

$= 2\left(t - 16 \cdot \frac{1}{4} \operatorname{arctg} \frac{t}{4}\right) + C =$

$= 2\sqrt{x} - 8 \operatorname{arctg} \frac{\sqrt{x}}{4} + C$

2сп.

$\int \frac{\sqrt{x}}{x+16} dx = [x+16 = t \Rightarrow x = t-16] =$

$= \int \frac{\sqrt{t-16}}{t} dt = \ldots$

W8.2.2~~1~~

$\int x \sin x \, dx = \left[\begin{matrix} u = x \\ v' = \sin x \end{matrix} \Rightarrow \begin{matrix} u' = 1 \\ v = \int \sin x \, dx = -\cos x \end{matrix}\right] =$

$= -x\cos x - \int 1 \cdot (-\cos x) \, dx =$

$= -x\cos x + \int \cos x \, dx =$

$= -x\cos x + \sin x + C = \sin x - x\cos x + C$

W8.2.22

$\int (2x-1) e^{3x} dx = \left[\begin{matrix} u = 2x-1 \\ v' = e^{3x} \end{matrix} \Rightarrow\right.$

$\left.\Rightarrow \begin{matrix} u' = 2 \\ v = \int e^{3x} dx = \frac{1}{3}\int e^{3x} d(3x) = \frac{1}{3} e^{3x} \end{matrix}\right] =$

$= (2x-1) \cdot \frac{1}{3} e^{3x} - \int \frac{1}{3} e^{3x} \cdot 2 \, dx =$

$$= \frac{1}{3}e^{3x}(2x-1) - \frac{2}{3}\int e^{3x}dx =$$

$$= \frac{1}{3}e^{3x}(2x-1) - \frac{2}{3}\cdot\frac{1}{3}\int e^{3x}d(3x) =$$

$$= \frac{1}{3}e^{3x}(2x-1) - \frac{2}{9}\cdot e^{3x} + C =$$

$$= \frac{1}{3}e^{3x}\left(2x-1-\frac{2}{3}\right) + C =$$

$$= \frac{1}{3}e^{3x}\left(2x-\frac{5}{3}\right) + C$$

<u>w8.2.23</u>

$$\int \frac{\ln x\,dx}{x^2} = \left[\begin{matrix} u = \ln x \\ v' = \frac{1}{x^2}\end{matrix} \Rightarrow \begin{matrix} u' = \frac{1}{x} \\ v = \cancel{\frac{x^3}{3}} -\frac{1}{x}\end{matrix}\right] =$$

$$\cancel{= \ln x\,\frac{x^3}{3} - \int \frac{1}{x}\cdot\frac{1}{x}dx =}$$

$$\cancel{= \frac{x^3}{3}\ln x + \int\frac{dx}{x^2} = \frac{x^3}{3}\ln x - \frac{1}{x} + C}$$

$$= -\frac{1}{x}\ln x - \int -\frac{1}{x}\cdot\frac{1}{x}\,dx =$$

$$= -\frac{\ln x}{x} + \int\frac{dx}{x^2} = -\frac{\ln x}{x} - \frac{1}{x} + C$$

<u>w8.2.24</u>

$$\int x\cdot 2^x dx = \left[\begin{matrix} u = x \\ v' = 2^x\end{matrix} \Rightarrow \begin{matrix} u' = 1 \\ v = \int 2^x dx = \frac{2^x}{\ln 2}\end{matrix}\right] =$$

$$= \frac{x2^x}{\ln 2} - \int\frac{2^x}{\ln 2}dx = \frac{x2^x}{\ln 2} - \frac{1}{\ln 2}\int 2^x dx =$$

$$= \frac{x 2^x}{\ln 2} - \frac{1}{\ln 2} \cdot \frac{2^x}{\ln 2} + C =$$

$$= \frac{2^x}{\ln 2}\left(x - \frac{1}{\ln 2}\right) + C$$

w 8.2.25

$$\int \ln^2 x\,dx = \left[\begin{matrix} u = \ln^2 x \\ v' = 1 \end{matrix} \Rightarrow \begin{matrix} u' = \frac{2\ln x}{x} \\ v = x \end{matrix}\right] =$$

$$= x\ln x - \int x \cdot \frac{2\ln x}{x}\,dx =$$

$$= x\ln^2 x - 2\int \ln x\,dx = \left[\begin{matrix} u = \ln x \\ v' = 1 \end{matrix} \Rightarrow \begin{matrix} u' = \frac{1}{x} \\ v = x \end{matrix}\right] =$$

$$= x\ln^2 x - 2\left(x\ln x - \int x \cdot \frac{1}{x}\,dx\right) =$$

$$= x\ln^2 x - 2(x\ln x - x) + C =$$

$$= x\ln^2 x - 2x\ln x - 2x + C$$

w 8.2.26

$$\int x \operatorname{arctg} x\,dx = \left[\begin{matrix} u = \operatorname{arctg} x \\ v' = x \end{matrix} \Rightarrow\right.$$

$$\left.\Rightarrow \begin{matrix} u' = -\frac{1}{1+x^2} \\ v = \int x\,dx = \frac{x^2}{2} \end{matrix}\right] =$$

$$= \frac{x^2}{2}\operatorname{arctg} x - \int -\frac{1}{1+x^2} \cdot \frac{x^2}{2}\,dx =$$

$$= \frac{x^2}{2}\operatorname{arctg}x + \frac{1}{2}\int\frac{x^2\,dx}{1+x^2} =$$

$$= \left[\int\frac{x^2\,dx}{1+x^2} = \int\frac{x^2+1-1}{1+x^2}\,dx =\right.$$

$$= \int\frac{x^2+1}{x^2+1}\,dx - \int\frac{dx}{1+x^2} =$$

$$= \left.\int dx - \int\frac{dx}{x^2+1} = x - \operatorname{arctg}x\right] =$$

$$= \frac{x^2}{2}\operatorname{arctg}x + \frac{1}{2}(x - \operatorname{arctg}x) + C =$$

$$= \frac{1}{2}\left(x^2\operatorname{arctg}x + x - \operatorname{arctg}x\right) + C$$