Домашняя работа

Интегрирование (часть 1)

№ 8.1.29

$$\int \frac{dx}{x^2\sqrt{x}} = \int x^{-\frac{5}{2}}\,dx = \frac{x^{-\frac{5}{2}+\frac{2}{2}}}{-\frac{5}{2}+\frac{2}{2}} + C =$$

$$= \frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + C = -\frac{2}{3x\sqrt{x}} + C$$

Проверка:

$$\left(-\frac{2}{3x\sqrt{x}} + C\right)' dx = -\frac{2}{3}\left(x^{-\frac{3}{2}}\right)' dx =$$

$$= -\frac{2}{3}\cdot\left(-\frac{3}{2}x^{-\frac{3}{2}-\frac{2}{2}}\right)dx =$$

$$= -\frac{2}{3}\cdot\left(-\frac{3}{2}\right)\cdot x^{-\frac{5}{2}}dx =$$

$$= \frac{dx}{x^2\sqrt{x}}$$

№ 8.1.30

$$\int \frac{dx}{x^2+3} = \frac{1}{2}\int\frac{d(x^2)}{x^2+3} = [d(x^2) = d(x^2+3)]$$

$$= \frac{1}{2}\int\frac{d(x^2+3)}{x^2+3} = \frac{1}{2}\ln|x^2+3| + C$$

Проверка:

$$\left(\frac{1}{2}\ln|x^2+3|\right)' dx = \frac{1}{x^2+3}\cdot\frac{1}{2}\cdot 2x\,dx =$$

$$= \frac{x\,dx}{x^2+3}$$

$$\int \frac{dx}{x^3+3} = \left[\int \frac{dx}{x^2+a^2},\ a=\sqrt{3}\right] =$$

$$= \frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{x}{\sqrt{3}} + C$$

Проверка:

$$\left(\frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{x}{\sqrt{3}}\right)' dx = \frac{1}{\sqrt{3}}\cdot$$

$$\cdot\frac{1}{1+\left(\frac{x}{\sqrt{3}}\right)^2}\cdot\frac{1}{\sqrt{3}}\,dx =$$

$$= \frac{1}{3}\cdot\frac{dx}{1+\frac{x^2}{3}} = \frac{dx}{3+x^2}$$

№8.1.31

$$\int \frac{1}{5^x}\,dx = \int 5^{-x}\,dx = \int -5^{-x}\,d(-x) =$$

$$= -\int 5^{-x}\,d(-x) = -\frac{5^{-x}}{\ln 5} + C$$

Проверка:

$$\left(-\frac{5^{-x}}{\ln 5}\right)' dx = -\frac{1}{\cancel{\ln 5}} \cdot 5^{-x} \cdot \cancel{\ln 5} \cdot$$

$$\cdot (-1)\, dx = 5^{-x}\, dx = \frac{1}{5^{x}}\, dx$$

№ 8.1.32

$$\int \frac{dx}{\sqrt{4-x^2}} = \arcsin\frac{x}{2} + C$$

Проверка:

$$\left(\arcsin\frac{x}{2}\right)' dx = \frac{dx}{\sqrt{1-\left(\frac{x}{2}\right)^2}} \cdot \frac{1}{2} =$$

$$= \frac{dx}{\sqrt{4\left(1-\frac{x^2}{4}\right)}} = \frac{dx}{\sqrt{4-x^2}}$$

№ 8.1.33

$$\int \frac{dx}{\sqrt{x^2-1}} = \cancel{\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C}$$

$$= \ln\left|x + \sqrt{x^2-1}\right| + C$$

Проверка:

$$\left(\ln\left|x+\sqrt{x^2-1}\right|\right)' dx = \frac{1}{\left|x+\sqrt{x^2-1}\right|} \cdot$$

$$\cdot \left(1 + \frac{1}{2\sqrt{x^2-1}} \cdot 2x\right) dx =$$

$$= \frac{dx}{x+\sqrt{x^2 \mp 1}} \cdot \left(1 + \frac{x}{\sqrt{x^2-1}}\right) =$$

$$= dx\left(\frac{1}{x+\sqrt{x^2-1}} + \frac{x}{(x+\sqrt{x^2-1})\sqrt{x^2-1}}\right) =$$

$$= dx \frac{\cancel{\sqrt{x^2-1} + x}}{\cancel{(x+\sqrt{x^2-1})}\sqrt{x^2-1}} = \frac{dx}{\sqrt{x^2-1}}$$

<u>№ 8.1.34</u>

$$\int \frac{dx}{x^2-25} = \frac{1}{2\cdot 5}\ln\left|\frac{x-5}{x+5}\right| + C =$$

$$= \frac{1}{10}\ln\left|\frac{x-5}{x+5}\right| + C$$

<u>Проверка:</u>

$$\left(\frac{1}{10}\ln\left|\frac{x-5}{x+5}\right|\right)' dx = \frac{1}{10} \cdot \frac{1}{\frac{x-5}{x+5}} \cdot$$

$$\cdot \frac{(x+5)-(x-5)}{(x+5)^2}\, dx = \frac{1}{\cancel{10}} \cdot \frac{\cancel{x+5}}{x-5} \cdot$$

$$\cdot \frac{\cancel{10}}{(x+5)^{\cancel{2}}}\, dx = \frac{dx}{(x-5)(x+5)} = \frac{dx}{x^2-25}$$

w8.1.35

$$\int\left(x+\frac{2}{x}\right)^2 dx = \int\left(x^2+4+\frac{4}{x^2}\right)dx =$$

$$= \int x^2 dx + \int 4 dx + \int \frac{4}{x^2} dx =$$

$$= \int x^2 dx + 4\int dx + 4\int \frac{dx}{x^2} =$$

$$= \frac{x^3}{3} + 4x \cancel{4} - \frac{4}{x} + C$$

w8.1.36

$$\int \frac{dx}{4x^2+1} = \int \frac{dx}{4\left(x^2+\frac{1}{4}\right)} = \frac{1}{4}\int \frac{dx}{x^2+\left(\frac{1}{2}\right)^2} =$$

$$= \frac{1}{4}\cdot 2\,\mathrm{arctg}\,2x + C =$$

$$= \frac{1}{2}\,\mathrm{arctg}\,2x + C$$

w8.1.37

$$\int\left(7^x - \frac{8}{x} + 4\cos x\right)dx =$$

$$= \int 7^x dx - 8\int\frac{dx}{x} + 4\int \cos x\, dx =$$

$$= \frac{7^x}{\ln 7} - 8\ln|x| + 4\sin x + C$$

№8.1.38

$$\int \left(\frac{\sqrt{3}}{\cos^2 x} - \sqrt[3]{x} - \frac{2}{x^4}\right) dx =$$

$$= \sqrt{3} \int \frac{dx}{\cos^2 x} - \int x^{\frac{1}{3}} dx - 2\int x^{-4} dx =$$

$$= \sqrt{3}\, tg x - \frac{x^{\frac{1}{3}+\frac{3}{3}}}{\frac{1}{3}+\frac{3}{3}} - 2 \cdot \frac{x^{-4+1}}{-4+1} + C =$$

$$= \sqrt{3}\, tg x - \frac{3x^{4/3}}{4} + \frac{2}{3} x^{-3} + C$$

№8.1.39

$$\int \frac{\sqrt{x} - 3\sqrt[5]{x^2} + 1}{\sqrt[4]{x}} dx =$$

$$= \int \frac{dx}{\sqrt{x}} - 3\int x^{\frac{3}{20}} dx + \int x^{-\frac{1}{4}} dx =$$

$$= 2\sqrt{x} - x^{\frac{23}{20}} \cdot \frac{3 \cdot 20}{23} + \frac{4}{3} x^{\frac{3}{4}} + C =$$

$$= 2\sqrt{x} - \frac{60}{23} x^{\frac{23}{20}} + \frac{4}{3} x^{\frac{3}{4}} + C$$

№8.1.40

$$\int \left(0{,}7 x^{-0{,}1} + 0{,}2 \cdot (0{,}5)^x\right) dx =$$

$$= 0{,}7\int x^{-0{,}1}dx + 0{,}2\int (0{,}5)^x dx =$$

$$= 0{,}7 \cdot \frac{1}{0{,}9} \cdot x^{0{,}9} + 0{,}2\frac{0{,}5^x}{\ln 0{,}5} + C =$$

$$= \frac{7}{9} x^{\frac{9}{10}} + \frac{1}{-5\cdot 2^x \ln 2} + C =$$

$$= \frac{7}{9} x^{\frac{9}{10}} - \frac{1}{5\cdot 2^x \ln 2} + C$$

№ 8.1.41

$$\int (5\,\mathrm{sh}\,x - 7\,\mathrm{ch}\,x + 1)\,dx =$$

$$= 5\int \mathrm{sh}\,x\,dx - 7\int \mathrm{ch}\,x\,dx + \int dx =$$

$$= 5\,\mathrm{ch}\,x - 7\,\mathrm{sh}\,x + x + C$$

№ 8.1.42

$$\int (x^2-1)(\sqrt{x}+4)\,dx =$$

$$= \int \left(x^2\sqrt{x} - \sqrt{x} + 4x^2 - 4\right) dx =$$

$$= \int x^{\frac{5}{2}}dx - \int \sqrt{x}\,dx + 4\int x^2 dx - 4\int dx$$

$$= \frac{2}{7} x^{\frac{7}{2}} - \frac{2}{3} x^{\frac{3}{2}} + \frac{4}{3} x^3 - 4x + C$$

w8.1.43

$$\int \frac{7-\sqrt{x^2+\pi}}{\sqrt{x^2+\pi}}\,dx =$$

$$= 7\int \frac{dx}{\sqrt{x^2+\pi}} - \int dx =$$

$$= 7\ln\left|x+\sqrt{x^2+\pi}\right| - x + C$$

w8.1.44

$$\int \left(\frac{\sqrt{x}-5}{x}\right)^3 dx = \int \frac{(\sqrt{x}-5)^3}{x^3}\,dx =$$

$$= \int \frac{x^{3/2}-15x+75\sqrt{x}-125}{x^3}\,dx =$$

$$= \int \left(x^{-\frac{3}{2}} - 15x^{-2} + 75x^{-\frac{5}{2}} - 125x^{-3}\right)dx =$$

$$= \int x^{-\frac{3}{2}}dx - 15\int x^{-2}dx + 75\int x^{-\frac{5}{2}}dx -$$

$$-125\int x^{-3}dx = -2\frac{1}{\sqrt{x}} + 15\frac{1}{x} + 75\cdot\left(-\frac{2}{3}\right)x^{-\frac{3}{2}}$$

$$- 125\cdot\left(-\frac{1}{2}\right)x^{-2} + C =$$

$$= -\frac{2}{\sqrt{x}} + \frac{15}{x} - 50x^{-\frac{3}{2}} + \frac{125}{2}x^{-2} + C$$

w 8.1.45

$$\int \sin 7x\,dx = \frac{1}{7}\int \sin 7x\,d(7x) =$$

$$= \frac{1}{7}(-\cos 7x) + C = -\frac{1}{7}\cos 7x + C$$

w 8.1.46

$$\int \sqrt[5]{2x-8}\,dx = \frac{1}{2}\int \sqrt[5]{2x-8}\,d(2x-8) =$$

$$= \frac{1}{2}\cdot\frac{5}{6}\cdot(2x-8)^{\frac{6}{5}} + C = \frac{5}{12}(2x-8)^{\frac{6}{5}} + C$$

w 8.1.47

$$\int (1-4x)^{2001}dx = -\frac{1}{4}\int (1-4x)^{2001}d(1-4x) =$$

$$= -\frac{1}{4}\cdot\frac{1}{2002}\cdot(1-4x)^{2002} + C =$$

$$= -\frac{1}{8008}(1-4x)^{2002} + C$$

w 8.1.48

$$\int \frac{dx}{9x+7} = \frac{1}{9}\int \frac{d(9x+7)}{9x+7} = \frac{1}{9}\ln|9x+7| + C$$

w 8.1.49

$$\int \frac{dx}{(6x+11)^4} = \frac{1}{6}\int (6x+11)^{-4}d(6x+11) =$$

$$= \frac{1}{6} \cdot \left(-\frac{1}{3}\right)(6x+11)^{-3} + C =$$

$$= -\frac{1}{18}(6x+11)^{-3} + C$$

w8.1.50

$$\int \frac{dx}{25x^2+1} = \frac{1}{5}\int \frac{d(5x)}{(5x)^2+1} =$$

$$= \frac{1}{5}\,\mathrm{arctg}(5x) + C$$

w8.1.51

$$\int 3^{2-11x}\,dx = -\frac{1}{11}\int 3^{2-11x}\,d(2-11x) =$$

$$= -\frac{3^{2-11x}}{11\ln 3} + C$$

w8.1.52

$$\int \frac{dx}{(4x^2-1)^{\frac{1}{2}}} = \frac{1}{2}\int \frac{d(2x)}{\sqrt{(2x)^2-1}} =$$

$$= \frac{1}{2}\ln\left|2x + \sqrt{4x^2-1}\right| + C$$

w8.1.53

$$\int \sin^2 3x\,dx = \int \frac{1-\cos 6x}{2}\,dx =$$

$= \frac{1}{2}\int(1-\cos 6x)\,dx =$

$= \frac{1}{2}\left(\int dx - \frac{1}{6}\int \cos 6x\, d(6x)\right) =$

$= \frac{1}{2}\left(x - \frac{1}{6}\sin 6x\right) + C =$

$= \frac{x}{2} - \frac{1}{12}\sin 6x + C$

№ 8.1.54

$\int \cos^2 8x\, dx = \int \frac{1+\cos 16x}{2}\, dx =$

$= \frac{1}{2}\int(1+\cos 16x)\,dx =$

$= \frac{1}{2}\left(\int dx + \frac{1}{16}\int \cos 16x\, d(16x)\right) =$

$= \frac{1}{2}\left(x + \frac{1}{16}\sin 16x\right) + C = \frac{x}{2} + \frac{1}{32}\sin 16x + C$

№ 8.1.55

$\int tg^2 x\, dx = \int((tg^2 x + 1) - 1)\, dx =$

$= \int(tg^2 x + 1)\, dx - \int dx =$

$= \int \frac{dx}{\cos^2 x} - \int dx = tg x - x$

№ 8.1.56

$$\int \frac{4x+1}{x-5}dx = \int \frac{4x}{x-5}dx +$$

$$+ \int \frac{dx}{x-5} = 4\int \frac{x\,dx}{x-5} + \int \frac{d(x-5)}{x-5} =$$

$$= 4\int \frac{x+5-5}{x-5}dx + \int \frac{dx}{x-5} =$$

$$= 4\int dx + 4\cdot 5\int \frac{dx}{x-5} +$$

$$+ \int \frac{dx}{x-5} = 4\int dx + 21\int \frac{dx}{x-5} =$$

$$= 4\int dx + 21\int \frac{d(x-5)}{x-5} = 4x + 21\ln|x-5| + C$$

№ 8.1.57

$$\int (3\,tg\,x - 2\,ctg\,x)^2 dx =$$

$$= \int (9\,tg^2 x - 12\,tg\,x\,ctg\,x + 4\,ctg^2 x)\,dx =$$

$$= 9\int tg^2 x\,dx - 12\int dx + 4\int ctg^2 x\,dx =$$

$$= 9\int ((tg^2 x + 1) - 1)\,dx - 12\int dx +$$

$$+ 4\int ((ctg^2 x + 1) - 1)\,dx =$$

$$= 9\int\frac{dx}{\cos^2 x} - 9\int dx - 12\int dx +$$

$$+ 4\int\frac{dx}{\sin^2 x} - 4\int dx =$$

$$= 9\int\frac{dx}{\cos^2 x} + 4\int\frac{dx}{\sin^2 x} - 25\int dx =$$

$$= 9\,tg x - 4\,ctg x - 25x + C$$

№ 8.1.58

$$\int\frac{4\sqrt{1-x^2}+3x^2}{x^2-1}dx =$$

$$= -4\int\frac{\sqrt{1-x^2}\,dx}{1-x^2} + 3\int\frac{x^2dx}{x^2-1} =$$

$$= -4\int\frac{dx}{\sqrt{1-x^2}} + 3\int\frac{x^2-1+1}{x^2-1}dx =$$

$$= -4\int\frac{dx}{\sqrt{1-x^2}} + 3\int dx + 3\int\frac{dx}{x^2-1} =$$

$$= -4\arcsin x + 3x + 3\cdot\frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| + C =$$

$$= -4\arcsin x + 3x + \frac{3}{2}\ln\left|\frac{x-1}{x+1}\right| + C$$

№ 8.1.59

$$\int\frac{\cos 2x\,dx}{\sin^2 x\cos^2 x} = \int\frac{\cos^2 x - \sin^2 x}{\cos^2 x\sin^2 x}dx =$$

$$= \int \frac{dx}{\sin^2 x} - \int \frac{dx}{\cos^2 x} =$$

$$= -\mathrm{ctg}\, x - \mathrm{tg}\, x + C$$

<u>№ 8.1.60</u>

$$\int \frac{\sin 2x}{\cos x} dx = 2\int \frac{\sin x \cos x}{\cos x} dx =$$

$$= 2\int \sin x \, dx = -2\cos x + C$$