Практика

(Производные, часть 5)

№11.4.1

$\frac{dz}{dt} = ?$, $z = e^{x^2+y^2}$ $x = a\cos t$, $y = a\sin t$

$z(t) = e^{a(\cos^2 t + \sin^2 t)} = e^a$

$\frac{dz}{dt} = 0$ [т.к. e^a – число]

№11.4.2

$\frac{dz}{dt} = ?$, $z = x^5 + 2xy - y^3$, $x = \cos 2t$,

$y = \operatorname{arctg} t$

$\frac{\partial z}{\partial x} = 5x^4 + 2y$, $\frac{\partial z}{\partial y} = 2x - 3y^2$

$\frac{dx}{dt} = -2\sin 2t$, $\frac{dy}{dt} = \frac{1}{1+t^2}$

$\frac{dz}{dt} = -2(5x^4 + 2y)\sin 2t + (2x - 3y^2)\frac{1}{1+t^2}$

№11.4.3

$z = xy + xyv + yuv$, $x = \sin t$, $y = \ln t$,

$u = e^t$, $v = \operatorname{arctg} t$, $\frac{dz}{dt} = ?$

$$\frac{\partial z}{\partial x} = y + yv, \quad \frac{\partial z}{\partial y} = x + xv + uv,$$

$$\frac{\partial z}{\partial u} = yv, \quad \frac{\partial z}{\partial v} = xy + uy$$

$$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = \frac{1}{t}, \quad \frac{du}{dt} = e^t, \quad \frac{dv}{dt} = \frac{1}{1+t^2}$$

$$\frac{dz}{dt} = y(1+v)\cos t + (x + xv + uv)\frac{1}{t} +$$
$$+ yve^t + \frac{x+u}{1+t^2}y$$

№ 11.4.4

$$z = x^2 + y^2 + xy, \quad x = a\sin t, \quad y = a\cos t$$

$$z = a^2(\sin^2 t + \cos^2 t + \sin t\cos t) =$$
$$= a^2(1 + \sin t\cos t) =$$
$$= a^2\left(1 + \frac{1}{2}\sin 2t\right) = a^2 + \frac{a^2}{2}\sin 2t$$

$$\frac{dz}{dt} = a^2\cos 2t$$

№ 11.4.5

$$z = \cos(2t + 4x^2 - y), \quad x = \frac{1}{t}, \quad y = \frac{\sqrt{t}}{\ln t}$$

$$\frac{\partial z}{\partial x} = -\sin(2t+4x^2-y)\cdot 8x$$

$$\frac{\partial z}{\partial y} = -\sin(2t+4x^2-y)\cdot(-1)$$

$$\frac{dz}{dx} = -\frac{1}{t^2}, \quad \frac{dz}{dy} = \frac{(\sqrt{t})'\ln t - \sqrt{t}(\ln t)'}{\ln^2 t}$$

$$= \frac{\frac{\ln t}{2\sqrt{t}} - \frac{\sqrt{t}}{\ln t}}{\ln^2 t} = \frac{\ln^2 t - 2t}{2\sqrt{t}\ln^3 t}$$

$$\frac{dz}{dt} = -\sin\left(2t + \frac{4}{t^2} - \frac{\sqrt{t}}{\ln t}\right)\cdot$$

$$\cdot\left(2 + \frac{8}{t^3} - \frac{\ln^2 t - 2t}{2\sqrt{t}\ln^3 t}\right)$$

№ 11.4.6

$z = x^2 y^3 u, \; x = t, \; y = t^2, \; u = \sin t$

$z = t^2 \cdot t^6 \cdot \sin t = t^8 \sin t$

$$\frac{dz}{dt} = 8t^7 \sin t + t^8 \cos t = t^7(8\sin t + t\cos t)$$

№ 11.4.7

$z = e^{xy}\ln(x+y), \; x = t^3, \; y = 1-t^3$

$z = e^{t^3(1-t^3)}\ln(t^3+1-t^3) =$

$= e^{t^3(1-t^3)} \ln 1 = 0 \Rightarrow \frac{dz}{dt} = 0.$

№ 11.4.8

$z = xy\,\mathrm{arct}(xy), \; x = t^2+1, \; y = t^3$

$\frac{\partial z}{\partial x} = y\,\mathrm{arct}(xy) + xy \cdot \frac{1}{1+(xy)^2} \cdot y =$

$= y\left(\mathrm{arct}(xy) + \frac{xy}{1+x^2y^2}\right)$

$\frac{\partial z}{\partial y} = x\left(\mathrm{arct}(xy) + \frac{xy}{1+x^2y^2}\right)$

$\frac{dx}{dt} = 2t \; ; \; \frac{dy}{dt} = 3t^2$

$\frac{dz}{dt} = \left(\mathrm{arctg}(xy) + \frac{xy}{1+x^2y^2}\right)\left(2yt + 3xt^2\right)$

№ 11.4.9

$z = e^{2x-3y}, \; x = \mathrm{tg}\,t, \; y = t^2 - t$

$\frac{\partial z}{\partial x} = 2e^{2x-3y} \; ; \; \frac{\partial z}{\partial y} = e^{2x-3y} \cdot (-3)$

$\frac{dx}{dt} = \frac{1}{\cos^2 t} \; ; \; \frac{dy}{dt} = 2t - 1$

$$\frac{dz}{dt} = \frac{2e^{2x-3y}}{\cos^2 t} - (3e^{2x-3y}) \cdot (2t-1)$$

w11.4.10

$z = x^y$, $x = \ln t$, $y = \sin t$

$$\frac{\partial z}{\partial x} = yx^{y-1}; \quad \frac{\partial z}{\partial y} = x^y \ln x$$

$$\frac{dx}{dt} = \frac{1}{t}; \quad \frac{dy}{dt} = \cos t$$

$$\frac{dz}{dt} = \frac{yx^{y-1}}{t} + x^y \ln x \cos t$$

w11.4.11

$\frac{\partial z}{\partial u}, \frac{\partial z}{\partial v} = ?$, $z = 3^{x^2} \operatorname{arctg} y$,

$x = \frac{u}{v}$, $y = uv$

$$\frac{\partial z}{\partial x} = 3^{x^2} \cdot 2x \ln 3 \operatorname{arctg} y$$

$$\frac{\partial z}{\partial y} = \frac{3^{x^2}}{1+y^2}$$

$$\frac{\partial x}{\partial u} = \frac{1}{v}; \quad \frac{\partial x}{\partial v} = -\frac{u}{v^2};$$

$$\frac{\partial y}{\partial u} = v \quad ; \quad \frac{\partial y}{\partial v} = u$$

$$\frac{\partial z}{\partial u} = 3^{x^2} \cdot \frac{2x \ln 3 \operatorname{arctg} y}{v} + \frac{3^{x^2}}{1+y^2} v$$

$$\frac{\partial z}{\partial v} = -3^{x^2} \cdot \frac{2xu \ln 3 \operatorname{arctg} y}{v^2} + \frac{3^{x^2}}{1+y^2} u$$

<u>№ 11.4.12</u>

$$z = \frac{x^2}{y}, \quad x = u - 2v, \quad y = 2u + v$$

$$dz = \frac{\partial z}{\partial x} dx + \frac{\partial z}{\partial y} dy$$

$$\frac{\partial z}{\partial x} = \frac{2x}{y}, \quad \frac{\partial z}{\partial y} = -\frac{x^2}{y^2}$$

$$dx = \frac{\partial x}{\partial u} du + \frac{\partial x}{\partial v} dv = du - 2dv$$

$$dy = \frac{\partial y}{\partial u} du + \frac{\partial y}{\partial v} dv = 2du + dv$$

$$dz = 2\frac{x}{y}(du - 2dv) - \frac{x^2}{y^2}(2du + dv) =$$

$$= \frac{u - 2v}{(2u+v)^2}[2(u+3v)du - (9u+2v)dv]$$

№ 11.4.13

$z = \ln(u^2+v^2)$, $u = x\cos y$, $v = y\sin x$

$$dz = \frac{\partial z}{\partial u} du + \frac{\partial z}{\partial v} dv$$

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$$

$$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy$$

$$\frac{\partial z}{\partial u} = \frac{2u}{u^2+v^2}$$

$$\frac{\partial u}{\partial x} = \cos y; \quad \frac{\partial u}{\partial y} = -x\sin y$$

$$\frac{\partial z}{\partial v} = \frac{2v}{u^2+v^2}$$

$$\frac{\partial v}{\partial x} = y\cos x; \quad \frac{\partial v}{\partial y} = \sin x$$

$$dz = \frac{2u}{u^2+v^2}(\cos y\,dx - x\sin y\,dy) +$$

$$+ \frac{2v}{u^2+v^2}(y\cos x\,dx + \sin x\,dy) =$$

$$= \left\{ \frac{1}{u^2+v^2}(\cos y \cdot 2u + 2vy\cos x) \right\} dx +$$

$+\dfrac{1}{u^2+v^2}(-2ux\sin y+2v\sin x)dy$

<u>№11.4.14</u>

$z=x^3+y^3,\ x=uv,\ y=\dfrac{u}{v}$

$$\boxed{\begin{aligned} dz&=\frac{\partial z}{\partial x}dx+\frac{\partial z}{\partial y}dy\\ dx&=\frac{\partial x}{\partial u}du+\frac{\partial x}{\partial v}dv\\ dy&=\frac{\partial y}{\partial u}du+\frac{\partial y}{\partial v}dv\end{aligned}}$$

$\dfrac{\partial z}{\partial x}=3x^2;\quad \dfrac{\partial z}{\partial y}=3y^2$

$\dfrac{\partial x}{\partial u}=v;\quad \dfrac{\partial y}{\partial u}=\dfrac{1}{v}$

$\dfrac{\partial x}{\partial v}=u;\quad \dfrac{\partial y}{\partial v}=-\dfrac{u}{v^2}$

$dx=v\,du+u\,dv$

$dy=\dfrac{du}{v}-\dfrac{u\,dv}{v^2}$

$dz=3x^2(v\,du+u\,dv)+3y^2\left(\dfrac{du}{v}-\dfrac{u\,dv}{v^2}\right)$

$= 3x^2 v\,du + 3x^2 u\,dv + \frac{3y^2}{v}du - \frac{3y^2u}{v^2}dv =$

$= 3\left(x^2 v + \frac{y^2}{v}\right)du + 3\left(x^2 u - \frac{y^2 u}{v^2}\right)dv =$

$= 3\left(u^2 v^3 + \frac{u^2}{v^3}\right)du + 3\left(u^3 v^2 - \frac{u^3}{v^4}\right)dv =$

$= 3u^2\left(v^3 + \frac{1}{v^3}\right)du + 3u^3\left(v^2 - \frac{1}{v^4}\right)dv$

№ 11.4.15

$z = \sqrt{x^2 - y^2}, \quad x = u^v, \quad y = u \ln v$

$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{x^2 - y^2}} \cdot 2x = \frac{x}{\sqrt{x^2 - y^2}}$

$\frac{\partial z}{\partial y} = \frac{1}{2\sqrt{x^2 - y^2}} \cdot -2y = -\frac{y}{\sqrt{x^2 - y^2}}$

$\frac{\partial x}{\partial u} = v u^{v-1}; \quad \frac{\partial x}{\partial v} = u^v \ln u$

$\frac{\partial y}{\partial u} = \ln v; \quad \frac{\partial y}{\partial v} = \frac{u}{v}$

$dx = v u^{v-1} du + u^v \ln u\, dv$

$dy = \ln v\, du + \frac{u}{v} dv$

$dz = \frac{x}{\sqrt{x^2 - y^2}}\left(v u^{v-1} du + u^v \ln u\, dv\right)$

$+ \cancel{\frac{y}{\sqrt{x^2-y^2}}}$

$$= \frac{y}{\sqrt{x^2-y^2}}\left(\ln v\,du + \frac{u}{v}\,dv\right) =$$

$$= \left(\frac{x}{\sqrt{x^2-y^2}}\, v u^{v-1} - \frac{y}{\sqrt{x^2-y^2}} \ln v\right) du +$$

$$+ \left(\frac{x}{\sqrt{x^2-y^2}}\, u^{v} \ln u - \frac{y}{\sqrt{x^2-y^2}} \cdot \frac{u}{v}\right) dv =$$

$$= \frac{1}{\sqrt{x^2-y^2}}\left(x u^{u-1} v - y \ln v\right) du +$$

$$+ \frac{1}{\sqrt{x^2-y^2}}\left(x u^{v} \ln u - y\frac{u}{v}\right) dv$$

№ 11.4.16

$z = \cos xy$, $x = ue^{v}$, $y = v \ln u$

$$\frac{\partial z}{\partial x} = \cancel{\cos} -\sin xy \cdot y$$

$$\frac{\partial z}{\partial y} = -\sin xy \cdot x$$

$$\frac{\partial x}{\partial u} = e^{v}; \quad \frac{\partial x}{\partial v} = ue^{v}$$

$$\frac{\partial y}{\partial u} = \frac{v}{u}; \quad \frac{\partial y}{\partial v} = \ln u$$

$dx = e^{v}du + ue^{v}dv$

$dy = \frac{v}{u}du + \ln u\, dv$

$dz = -y\sin xy\,(e^{v}du + ue^{v}dv) -$
$- x\sin xy\,(\frac{v}{u}du + \ln u\, dv) =$

$= -\sin xy\,(ye^{v} + \frac{xv}{u})du -$
$-\sin xy\,(yue^{v} + x\ln u)dv$

<u>№ 11.4.17</u>

$z = \operatorname{arctg} xy, \quad x = \sqrt{u^2+v^2}, \quad y = u - v$

$\frac{\partial z}{\partial x} = \frac{1}{1+x^2y^2}\cdot y = \frac{y}{1+x^2y^2} = \frac{y}{(1+xy)^2}$

$\frac{\partial z}{\partial y} = \frac{1}{1+x^2y^2}\cdot x = \frac{x}{1+x^2y^2}$

$\frac{\partial x}{\partial u} = \frac{1}{2\sqrt{u^2+v^2}}\cdot 2u = \frac{u}{\sqrt{u^2+v^2}}$

$\frac{\partial x}{\partial v} = \frac{1}{2\sqrt{u^2+v^2}}\cdot 2v = \frac{v}{\sqrt{u^2+v^2}}$

$\frac{\partial y}{\partial u} = 1, \quad \frac{\partial y}{\partial v} = -1$

$$dx = \frac{u\,du}{\sqrt{u^2+v^2}} + \frac{v\,dv}{\sqrt{u^2+v^2}}$$

$$dy = du - dv$$

$$dz = \frac{y}{1+x^2y^2}\left(\frac{u\,du}{\sqrt{u^2+v^2}} + \frac{v\,dv}{\sqrt{u^2+v^2}}\right) +$$

$$+ \frac{x}{1+x^2y^2}(du - dv) =$$

$$= \frac{1}{1+x^2y^2}\left(\frac{yu}{\sqrt{u^2+v^2}} + x\right)du +$$

$$+ \frac{1}{1+x^2y^2}\left(\frac{yv}{\sqrt{u^2+v^2}} - x\right)dv$$

№11.4.18

$$z = \sqrt{x+y},\quad x = u\,tg\,v,\quad y = u\,ctg\,v$$

$$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{x+y}};\quad \frac{\partial z}{\partial y} = \frac{1}{2\sqrt{x+y}}$$

$$\frac{\partial x}{\partial u} = tg\,v;\quad \frac{\partial x}{\partial v} = +\frac{u}{\cos^2 v}$$

$$\frac{\partial y}{\partial u} = ctg\,v;\quad \frac{\partial y}{\partial v} = -\frac{u}{\sin^2 v}$$

$$dx = \operatorname{tg} v\, du + \frac{u}{\cos^2 v}\, dv$$

$$dy = \operatorname{ctg} v\, du - \frac{u}{\sin^2 v}\, dv$$

$$dz = \frac{1}{2\sqrt{x+y}}\left(\operatorname{tg} v\, du + \frac{u}{\cos^2 v}\, dv\right) +$$

$$+ \frac{1}{2\sqrt{x+y}}\left(\operatorname{ctg} v\, du - \frac{u}{\sin^2 v}\, dv\right) =$$

$$= \frac{1}{2\sqrt{x+y}}\left(\operatorname{tg} v + \operatorname{ctg} v\right) du +$$

$$+ \frac{1}{2\sqrt{x+y}}\left(\frac{u}{\cos^2 v} - \frac{u}{\sin^2 v}\right) dv$$

№ 11.4.19

$$z = \ln \sqrt[7]{x^2 + 3y^5}, \quad x = u\cos v, \; y = u\sin v$$

$$\frac{\partial z}{\partial x} = \frac{1}{(x^2+3y^5)^{\frac{1}{7}}} \cdot \left(-\frac{1}{7}\right)(x^2+3y^5)^{-\frac{8}{7}} \cdot 2x =$$

$$= -\frac{2}{7}\,\frac{x}{(x^2+3y^5)^{\frac{9}{7}}}$$

$$\frac{\partial z}{\partial y} = \frac{1}{(x^2+3y^5)^{\frac{1}{7}}} \cdot \left(-\frac{1}{7}\right)(x^2+3y^5)^{-\frac{8}{7}} \cdot$$

$$\cdot 15y^4 = -\frac{15}{7}\,\frac{y^4}{(x^2+3y^5)^{\frac{9}{7}}}$$

$$\frac{\partial x}{\partial u} = \cos v; \quad \frac{\partial x}{\partial v} = -u\sin v$$

$$\frac{\partial y}{\partial u} = \sin v; \quad \frac{\partial y}{\partial v} = +u\cos v$$

$$dx = \cos v\,du - u\sin v\,dv$$

$$dy = \sin v\,du + u\cos v\,dv$$

$$dz = -\frac{2}{7}\frac{x}{(x^2+3y^5)^{\frac{9}{7}}}(\cos v\,du - u\sin v\,dv) - \frac{15}{7}\frac{y^4}{(x^2+3y^5)^{\frac{9}{7}}}\cdot(\sin v\,du + u\cos v\,dv) =$$

$$= -\frac{1}{7(x^2+3y^5)^{\frac{9}{7}}}(2x\cos v + 15y^4\sin v)du - \frac{1}{7(x^2+3y^5)^{\frac{9}{7}}}(2xu\sin v + 15y^4u\cos v)dv$$

w11.4.20

$$2x^2 - 3y^2 + 5xy - y^3x + x^5 = 7$$

$$(x_0, y_0) = (2; -3)$$

$\exists$ $y = y(x)$ при $x = 2$?

$y'(x)$, $y'(2)$ - ?

$F(x;y) = 2x^2 - 3y^2 + 5xy - y^3x + x^5 - 37$

$F(2;-3) = 0$

$F'_y = -6y + 5x - 3y^2x$

$F'_x = 4x + 5y - y^3 + 5x^4$

$F'_x(2;-3) = 100$

$F'_y(2;-3) = -26$

$F'_y(x_0;y_0) \neq 0 \Rightarrow \exists\, y = y(x)$
в $U(x_0 = 2)$

$$y'(x) = -\frac{F'_x(x,y)}{F'_y(x,y)} = \frac{4x + 5y - y^3 + 5x^4}{6y - 5x + 3y^2x}$$

$$y'(2) = \frac{100}{26} = \frac{50}{13}$$

№ 11.4.21

$-8x^2 + xy^2 + 2x^3y - 7 = 0$

Сущ. линия U $x = 1$ в неск. ~~местах~~ точках

Сколько $\exists\, y = y(x)$ в $U(x=1)$?

Сост. ур. кас.

Решение:

1) $\exists$ $F(x;y) = -8x^2 + xy^2 + 2x^3y - 7$

$\Rightarrow F(1;y) = y^2 + 2y - 15$;

$y^2 + 2y - 15 = 0$ при $y = 3$, $y = -5$ $\Rightarrow$

$\Rightarrow F(x;y) = 0$ в $U(x=1)$ опр-ет 2 функции.

Проверка:

$$\frac{\partial F}{\partial y} = 2xy + 2x^3$$

$$\frac{\partial F}{\partial y}(1;3) = 8; \quad \frac{\partial F}{\partial y}(1;-5) = -8 \Rightarrow$$

$\Rightarrow F(x;y)$ опр-ет $y = y_1(x)$, $y = y_2(x)$; $y_1(1) = 3$, $y_2(1) = -5$

2) $$\frac{\partial F}{\partial x} = -16x + y^2 + 6x^2y$$

$$\frac{\partial F}{\partial x}(1;3) = 11; \quad \frac{\partial F}{\partial x}(1;-5) = -21$$

$$\Rightarrow y_1'(1) = -\frac{11}{8}, \quad y_2'(1) = +\frac{21}{8}$$

3) Касат t_1 к $y_1(x)$ в $(1;3)$ и
t_2 к $y_2(x)$ в $(1;-5)$:

$y-y_0=k(x-x_0)$, где $k=y_1'(1)$
или $y_2'(1)$ =>

=> (t_1): $y-3=-\frac{11}{8}(x-1)$

$11x+8y-35=0$

(t_2): $y+5=\frac{21}{8}(x-1)$

$21x-8y-61=0$

<u>№ 11.4.22</u>

$xe^{2y}-y\ln x=8$, $y'(x)=?$

$xe^{2y}-y\ln x-8=F(x;y)$

$F'_x=e^{2y}-\frac{y}{x}$

$F'_y=2xe^{2y}-\ln x$

$$y'(x)=-\frac{F'_x}{F'_y}=-\frac{e^{2y}-\frac{y}{x}}{2xe^{2y}-\ln x}=$$

$$=-\frac{xe^{2y}-y}{2x^2e^{2y}-x\ln x}=-\frac{xe^{2y}-y}{2x^2e^{2y}-x\ln x}$$

№11.4.23

$e^y + 9x^2 e^{-y} - 26x = 0$, $y'(x) = ?$

$F'_x = 18xe^{-y} - 26$

$F'_y = e^y - 9x^2 e^{-y}$

$$y'(x) = -\frac{F'_x}{F'_y} = -\frac{18xe^{-y} - 26}{e^y - 9x^2 e^{-y}}$$

$$= \frac{26e^y - 18x}{e^y} \cdot \frac{e^y}{e^{2y} - 9x^2} =$$

$$= \frac{26e^y - 18x}{e^{2y} - 9x^2}$$

№11.4.24

$$\ln\frac{\sqrt{x^2+y^2}}{2} = \operatorname{arctg}\frac{y}{x}$$

$$\frac{1}{2}\ln(x^2+y^2) - \ln 2 - \operatorname{arctg}\frac{y}{x} = 0$$

$$F(x;y) = \frac{1}{2}\ln(x^2+y^2) - \operatorname{arctg}\frac{y}{x} - \ln 2$$

$$F'_x = \frac{1}{2(x^2+y^2)} \cdot 2x - \frac{1}{1+\left(\frac{y}{x}\right)^2} \cdot \cancel{y}$$

$$\cancel{= \frac{x}{x^2+y^2} - \frac{x^2 y}{x^2+y^2} = \frac{x - x^2 y}{x^2+y^2}}$$

$$\cancel{F'_y = \frac{1}{2(x^2+y^2)}\cdot 2y}$$

$$\cdot\left(-\frac{y}{x^2}\right) = \frac{x}{x^2+y^2} + \frac{\cancel{x^2}\cdot y}{(x^2+y^2)\cdot\cancel{x^2}} =$$

$$= \frac{x+y}{x^2+y^2} \cancel{= \frac{x+y}{(x+y)^2-2xy} = \frac{1}{x+y}}$$

$$F'_y = \frac{1}{2(x^2+y^2)}\cdot 2y - \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot\frac{1}{x} =$$

$$= \frac{y}{x^2+y^2} - \frac{x^2}{x(x^2+y^2)} = \frac{y-x}{x^2+y^2}$$

$$y'(x) = -\frac{F'_x}{F'_y} = -\frac{x+y}{x^2+y^2}\cdot\frac{x^2+y^2}{y-x} =$$

$$= \frac{x+y}{x-y}$$

№ 11.4.25

$$x^2\ln y - y^2\ln x = 0, \quad y'(x) = ?$$

$$F'_x = 2x\ln y - \frac{y^2}{x} = \frac{2x^2\ln y - y^2}{x}$$

$$F'_y = \frac{x^2}{y} - 2y\ln x = \frac{x^2-2y^2\ln x}{y}$$

$$y'(x) = -\frac{2x^2\ln y - y^2}{\cancel{y}x}\cdot\frac{y}{x^2-2y^2\ln x} =$$

$$= \frac{y^3-2x^3y\ln y}{x^3-2y^3x\ln x}$$

№ 11.4.26

$1 + xy - \ln(e^{xy}+e^{-xy}) = 0$, $y'(x) = ?$

$$F'_x = y - \frac{1}{e^{xy}+e^{-xy}}\cdot(ye^{xy}-ye^{-xy}) =$$

$$= y\left(1-\frac{e^{xy}-e^{-xy}}{e^{xy}+e^{-xy}}\right)$$

$$F'_y = x\left(1-\frac{e^{xy}-e^{-xy}}{e^{xy}+e^{-xy}}\right)$$

$$y'(x) = -\frac{y\left(1-\frac{e^{xy}-e^{-xy}}{e^{xy}+e^{-xy}}\right)}{x\left(1-\frac{e^{xy}-e^{-xy}}{e^{xy}+e^{-xy}}\right)} = -\frac{y}{x}$$

№ 11.4.33

Найти $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$, dz для

$z = z(x,y)$: $z^3 + 3x^2y + xz + y^2z^2 + y - 2x = 0$

Решение:

] $F(x;y;z) = z^3 + 3x^2y + xz + y^2z^2 + y - 2x$

Способ 1:

$F'_x = 6yx + z - 2$

$F'_y = 3x^2 + 2yz^2 + 1$

$F'_z = 3z^2 + x + 2y^2z$

$$\frac{\partial z}{\partial x} = -\frac{F'_x}{F'_z} = -\frac{6xy + z - 2}{3z^2 + x + 2y^2z}$$

$$\frac{\partial z}{\partial y} = -\frac{F'_y}{F'_z} = -\frac{3x^2 + 2yz^2 + 1}{3z^2 + x + 2y^2z}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy =$$

$$= \frac{2 - z - 6xy}{3z^2 + x + 2y^2z}dx - \frac{3x^2 + 2yz^2 + 1}{3z^2 + x + 2y^2z}dy$$

Способ 2:

$z^3(x;y) + 3x^2y + xz(x;y) + y^2z^2(x;y) + y - 2x \equiv 0$

$3z^2z'_x + 6xy + z + xz'_x + 2zy^2z'_x - 2 \equiv 0$

$3z^2z'_y + 3x^2 + xz'_y + 2yz^2 + 2y^2zz'_y + 1 \equiv 0$

$$z'_x = -\frac{6xy + z - 2}{3z^2 + x + 2y^2 z}$$

$$z'_y = -\frac{3x^2 + 2yz^2 + 1}{3z^2 + x + 2y^2 z}$$

№11.4.34

z'_x, z'_y = ?, если $x+y+z = e^{-(x+y+z)}$

$$1 + z'_x = e^{-(x+y+z)}(-1 - z'_x)$$

$$1 + z'_y = e^{-(x+y+z)}(-1 - z'_y)$$

$$\exists\ e^{-(x+y+z)} = x+y+z$$

$$1 + z'_x = (x+y+z)(-1 - z'_x)$$

$$1 + z'_y = (x+y+z)(-1 - z'_y)$$

$$z'_x = \frac{-(x+y+z) - 1}{x+y+z+1} = -1$$

$$z'_y = -1$$

$$dz = -dx - dy$$

W11.4.35

$z^3 - 3xyz = R^2$

Найти $\frac{\partial z}{\partial x}$; $\frac{\partial z}{\partial y}$; dz

$F(x;y;z) = z^3 - 3xyz - R^2$

$F'_x = -3yz$

$F'_y = -3xz$

$F'_z = 3z^2 - 3xy$

$$\frac{\partial z}{\partial x} = -\frac{-3yz}{3(z^2-xy)} = \frac{yz}{z^2-xy}$$

$$\frac{\partial z}{\partial y} = -\frac{-3xz}{3(z^2-xy)} = \frac{xz}{z^2-xy}$$

$$dz = \cancel{\frac{y}{z^2}dx + \frac{x}{z^2}dy = \frac{y\,dx + x\,dy}{z^2}}$$

$$= \frac{yz\,dx + xz\,dy}{z^2-xy}$$

W11.4.36

$x + y + z = e^z$

$F(x;y;z) = x + y + z - e^z$

$F'_x = 1$, $F'_y = 1$

$$F'_z = 1 - e^z$$

$$\frac{\partial z}{\partial x} = -\frac{1}{1-e^z}; \quad \frac{\partial z}{\partial y} = -\frac{1}{1-e^z}$$

$$dz = -\frac{1}{1-e^z}dx + \frac{1}{e^z-1}dy =$$

$$= \frac{dx+dy}{e^z-1}$$

<u>w11.4.37</u>

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

$$F(x; y; z) = \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} - 1$$

$$F'_x = \frac{2x}{a^2}; \quad F'_y = \frac{2y}{b^2}; \quad F'_z = \frac{2z}{c^2}$$

$$\frac{\partial z}{\partial x} = -\frac{2x}{a^2} \cdot \frac{c^2}{2z} = -\frac{xc^2}{za^2}$$

$$\frac{\partial z}{\partial y} = -\frac{2y}{b^2} \cdot \frac{c^2}{2z} = -\frac{yc^2}{zb^2}$$

$$dz = -\frac{xc^2}{za^2}dx - \frac{yc^2}{zb^2}dy =$$

$$= -\frac{c^2}{z}\left(\frac{xdx}{a^2} + \frac{ydy}{b^2}\right)$$