Интегралы часть 5
Домашняя работа

№8.5.19

$$\int \frac{dx}{\cos x} = [\text{табл}] = \ln\left|tg\left(\frac{x}{2}+\frac{\pi}{4}\right)\right| + C$$

№8.5.20

$$\int \frac{dx}{1-\sin x} = \left[t = tg\frac{x}{2};\ \sin x = \frac{2t}{1+t^2};\ dx = \frac{2dt}{1+t^2}\right] = \int \frac{1}{1-\frac{2t}{1+t^2}} \cdot \frac{2dt}{1+t^2} =$$

$$= \int \frac{1+t^2}{t^2-2t+1} \cdot \frac{2dt}{1+t^2} = \int \frac{2dt}{(t-1)^2} =$$

$$= 2\int \frac{d(t-1)}{(t-1)^2} = -\frac{2}{t-1} + C =$$

$$= -\frac{2}{tg\frac{x}{2}-1} + C = \frac{2}{1-tg\frac{x}{2}} + C$$

№8.5.21

$$\int \frac{dx}{5+4\sin x} = \left[t = tg\frac{x}{2};\ \sin x = \frac{2t}{1+t^2};\ dx = \frac{2dt}{1+t^2}\right] = \int \frac{1}{5+4\cdot\frac{2t}{1+t^2}} \cdot \frac{2dt}{1+t^2} =$$

$$= \int \frac{2dt}{5t^2+8t+5} = 2\int \frac{dt}{5t^2+8t+5} =$$

$= [\,5t^2+8t+5=0 \Rightarrow D = 64-4\cdot 5\cdot 5 =$

$= -36 \Rightarrow$ корней нет $\Rightarrow$ $\underline{\text{III}}$ тип $] =$

$$= \frac{2}{5}\int \frac{dt}{t^2+\frac{8}{5}t+1} = \left[y = t+\frac{4}{5};\ dy = dt\right] =$$

$$= \frac{2}{5}\int \frac{dy}{y^2+a^2} = \left[a = \sqrt{1-\frac{8^2}{5^2\cdot 4}} = \frac{3}{5}\right] =$$

$$= \frac{2}{5}\int \frac{dy}{y^2+\left(\frac{3}{5}\right)^2} = \frac{2}{5}\cdot\frac{5}{3}\,\mathrm{arctg}\,\frac{5y}{3}+C =$$

$$= \frac{2}{3}\,\mathrm{arctg}\,\frac{5t+4}{3}+C = \frac{2}{3}\,\mathrm{arctg}\,\frac{5\,\mathrm{tg}\frac{x}{2}+4}{3}+C$$

№8.5.22

$$\int \frac{2+\sin x}{2-\cos x}\,dx = \left[t = \mathrm{tg}\frac{x}{2},\right.$$

$$\sin x = \frac{2t}{1+t^2};\quad \cos x = \frac{1-t^2}{1+t^2};$$

$$\left. dx = \frac{2dt}{1+t^2}\right] = \int \frac{2+\frac{2t}{1+t^2}}{2-\frac{1-t^2}{1+t^2}}\cdot\frac{2dt}{1+t^2} =$$

$$= \int 2\cdot\frac{t^2+t+1}{1+t^2}\cdot\frac{2dt}{1+t^2} =$$

$$= 4\int \frac{t^2+t+1}{(t^2+1)^2}\,dt = 4\int\left(\frac{t}{t^2+1}+1\right)\frac{dt}{t^2+1} =$$

$$= 4\int \frac{t\,dt}{(t^2+1)^2} + 4\int\frac{dt}{t^2+1} =$$

$$= 2\int \frac{d(t^2+1)}{(t^2+1)^2} + 4\int\frac{dt}{t^2+1} =$$

$$= -\frac{2}{t^2+1} + 4\,\mathrm{arctg}\,t + C =$$

$$= -\frac{2}{tg^2\frac{x}{2}+1} + 4\,\mathrm{arctg}\,tg\frac{x}{2} + C$$

№ 8.5.23

$$\int\frac{dx}{2\sin x-\cos x+5} = \Big[\, t = tg\frac{x}{2};$$

$$\sin x = \frac{2t}{1+t^2};\quad \cos x = \frac{1-t^2}{1+t^2};$$

$$dx = \frac{2dt}{1+t^2}\Big] = \int\frac{1}{\frac{4t}{1+t^2}-\frac{1-t^2}{1+t^2}+5}\cdot\frac{2dt}{1+t^2} =$$

$$= \int\frac{2\,dt}{6t^2+4t+4} = \int\frac{dt}{3t^2+2t+2} =$$

$= [\, 3t^2+2t+2=0 \Rightarrow D=-20 \Leftrightarrow$ корней $] =$

$$= 3\int\frac{dt}{t^2+\frac{2}{3}t+\frac{2}{3}} = \Big[\, y = t+\frac{1}{3};\ a = \sqrt{\frac{2}{3}-\frac{2^2}{3^2\cdot 4}} = \frac{\sqrt{5}}{3}\Big] =$$

$$= 3\int \frac{dy}{y^2+\left(\frac{\sqrt{5}}{3}\right)^2} = \frac{9}{\sqrt{5}} \operatorname{arctg} \frac{3y}{\sqrt{5}} + C =$$

$$= \frac{9}{\sqrt{5}} \operatorname{arctg} \frac{3t+1}{\sqrt{5}} + C =$$

$$= \frac{9}{\sqrt{5}} \operatorname{arctg} \frac{3\operatorname{tg}\frac{x}{2}+1}{\sqrt{5}} + C$$

<u>№8.5.24</u>

$$\int \frac{1+\sin x}{(1+\cos x)\sin x}\,dx \; \textcircled{=} \; \Big[\cancel{t = \operatorname{tg} x;}$$

$$\cancel{dx = \frac{dt}{1+t^2}; \quad \sin x = \frac{t}{\sqrt{t^2+1}};}$$

$$\cancel{\cos x = \frac{1}{\sqrt{t^2+1}} \Big] = \int \frac{1+\frac{t}{\sqrt{t^2+1}}}{\left(1+\frac{1}{\sqrt{t^2+1}}\right)\frac{t}{\sqrt{t^2+1}}} \cdot}$$

$$\cancel{\cdot \frac{dt}{1+t^2} = \int \frac{(t+\sqrt{t^2+1})(1+\sqrt{t^2+1})\,dt}{t(t^2+1)\sqrt{t^2+1}}}$$

$$\cancel{= \int \frac{t+\sqrt{t^2+1}}{t\sqrt{t^2+1}\,(1+\sqrt{t^2+1})}\,dt =}$$

$$\textcircled{=} \int \frac{1+\sin x}{\sin x + \sin x \cos x}\,dx = \Big[t = \sin x;$$

$$dx = \frac{dt}{\sqrt{1-t^2}}; \; \cos x = \sqrt{1-t^2} \Big] =$$

$$= \int \frac{1+t}{t+t\sqrt{1-t^2}} \cdot \frac{dt}{\sqrt{1-t^2}} = \int \frac{(1+t)\,dt}{t(1+\sqrt{1-t^2})\sqrt{1-t}\sqrt{1+t}} =$$

$$= \int \frac{\sqrt{1+t}\,dt}{t\sqrt{1-t}\,(1+\sqrt{1-t^2})}$$

№ 8.5.25

$$\int \frac{dx}{5\sin^2 x - 3\cos^2 x + 4} =$$

$$= \int \frac{dx}{5\sin^2 x - 3 + 3\sin^2 x + 4} =$$

$$= \int \frac{dx}{8\sin^2 x + 1} = \Big[\, tg x = t;$$

$$dx = \frac{dt}{t^2+1}; \quad \sin^2 x = \frac{t^2}{t^2+1} \Big] =$$

$$= \int \frac{1}{8\frac{t^2}{t^2+1}+1} \cdot \frac{dt}{t^2+1} =$$

$$= \int \frac{dt}{9t^2+1} = \frac{1}{3}\int \frac{d(3t)}{(3t)^2+1} = \frac{1}{3}\,arctg\, 3t + C =$$

$$= \frac{1}{3}\,arctg(3 tg x) + C$$

№ 8.5.26

$$\int \frac{dx}{4\sin^2 x + 9\cos^2 x} = \int \frac{dx}{4\sin^2 x + 9 - 9\sin^2 x} =$$

$$= \int \frac{dx}{9-5\sin^2 x} = \Big[\operatorname{tg} x = t;$$

$$dx = \frac{dt}{t^2+1}; \quad \sin^2 x = \frac{t^2}{t^2+1} \Big] =$$

$$= \int \frac{1}{9 - \frac{5t^2}{t^2+1}} \cdot \frac{dt}{t^2+1} =$$

$$= \int \frac{dt}{4t^2+9} = \frac{1}{2}\int \frac{2dt}{(2t)^2+3^2} =$$

$$= \frac{1}{2} \cdot \frac{1}{3} \operatorname{arctg} \frac{2t}{3} + C =$$

$$= \frac{1}{6} \operatorname{arctg}\left(\frac{2}{3} \operatorname{tg} x\right) + C$$

№8.5.~~21~~

$$\int \frac{dx}{1+3\cos^2 x} = \Big[t = \operatorname{tg} x; \; dx = \frac{dt}{t^2+1};$$

$$\cos^2 x = \frac{1}{t^2+1} \Big] = \int \frac{1}{1+\frac{3}{t^2+1}} \cdot \frac{dt}{t^2+1} =$$

$$= \int \frac{dt}{t^2+4} = \frac{1}{2} \operatorname{arctg} \frac{t}{2} + C =$$

$$= \frac{1}{2} \operatorname{arctg}\left(\frac{1}{2} \operatorname{tg} x\right) + C$$

№8.5.28

$$\int \frac{dx}{\sin^4 x} = \left[\sin^2 x = \frac{1-\cos 2x}{2}\right] =$$

$$= \int \frac{dx}{\left(\frac{1-\cos 2x}{2}\right)^2} = 4\int \frac{dx}{(1-\cos 2x)^2} =$$

$$= 4\int \frac{dx}{1-2\cos 2x+\cos^2 2x} =$$

$$= \Big[t = \operatorname{tg} 2x;\ dx = \frac{dt}{2(1+t^2)};$$

$$\cos 2x = \frac{1}{\sqrt{t^2+1}}\Big] = 2\int \frac{1}{1-\frac{2}{\sqrt{t^2+1}}+\frac{1}{t^2+1}} \cdot$$

$$\cdot \frac{dt}{1+t^2} = \int \frac{dt}{t^2-2\sqrt{t^2+1}+2} = \Big[y = \sqrt{t^2-1};$$

$$t = \sqrt{y^2-1};\ dt = \frac{y\,dy}{\sqrt{y^2-1}}\Big] =$$

$$= \int \frac{1}{y^2-1-2y+2} \cdot \frac{y\,dy}{\sqrt{y^2+1}} = \int \frac{y\,dy}{(y^2-1)^{\frac{3}{2}}} =$$

$$= \int \frac{d(y^2-1)}{(y^2-1)^{\frac{3}{2}}} = \int (y^2-1)^{-\frac{3}{2}} d(y^2-1) =$$

$$= \frac{(y^2-1)^{-\frac{1}{2}}}{-\frac{1}{2}} + C = -\frac{2}{\sqrt{y^2-1}} + C =$$

$$= -\frac{2}{t} + C = -\frac{2}{tg^2 x} + C$$

<u>№8.5.29</u>

$$\int \sin^5 x\,dx = \left[\sin^2 x = \frac{1-\cos 2x}{2}\right] =$$

$$= \int \left(\frac{1-\cos 2x}{2}\right)^2 \sin x\,dx =$$

$$= \int \left(\frac{1-2\cos^2 x+1}{2}\right)^2 \sin x\,dx =$$

$$= \int (1-\cos^2 x)^2 \sin x\,dx =$$

$$= [t = \cos x;\ -dt = \sin x\,dx] =$$

$$= -\int (1-t^2)^2\,dt =$$

$$= -\int (1-2t^2+t^4)\,dt =$$

$$= -\int dt + 2\int t^2\,dt - \int t^4\,dt =$$

$$= 2\frac{t^3}{3} - t - \frac{t^5}{5} + C =$$

$$= \frac{2}{3}\cos^3 x - \cos x - \frac{1}{5}\cos^5 x + C$$

№8.5.30

$$\int \sin^4 x \cdot \cos^5 x \, dx =$$

$$= \int \sin^4 x \left(1 - \sin^2 x\right)^2 \cos x \, dx =$$

$$= \left[t = \sin x \; ; \; dt = \cos x \, dx \right] =$$

$$= \int t^4 \left(1 - t^2\right)^2 dt = \int t^4 \left(1 - 2t^2 + t^4\right) dt =$$

$$= \int t^4 dt - 2 \int t^6 dt + \int t^8 dt =$$

$$= \frac{t^5}{5} - \frac{2t^7}{7} + \frac{t^9}{9} + C =$$

$$= \frac{1}{5} \sin^5 x - \frac{2}{7} \sin^7 x + \frac{1}{9} \sin^9 x + C$$

№8.5.31

$$\int \frac{\sin 2x \, dx}{\cos^7 x} = \int \frac{2 \cos x \sin x \, dx}{\cos^7 x} =$$

$$= 2 \int \frac{\sin x \, dx}{\cos^6 x} = \left[t = \cos x \; ; \right.$$

$$\left. -dt = \sin x \, dx \right] = -2 \int \frac{dt}{t^6} =$$

$$= \frac{2}{7t^7} + C = \frac{2}{7 \cos^7 x} + C$$

№8.5.32

$$\int \frac{\sin^4 x\,dx}{\cos x} = \Big[t = tg x;$$

$$dx = \frac{dt}{1+t^2}; \cos x = \frac{1}{\sqrt{1+t^2}};$$

$$\sin = \frac{t}{\sqrt{1+t^2}} \Big] =$$

$$= \int \frac{t^4}{(1+t^2)^2} \cdot \sqrt{1+t^2} \cdot \frac{dt}{1+t^2} =$$

$$= \int \frac{t^4\,dt}{(1+t^2)^{\frac{5}{2}}} = \Big[1+t^2 = y;$$

$$t = \sqrt{y-1}; \quad dt = \frac{dy}{2\sqrt{y-1}} \Big] =$$

$$= \int \frac{(y-1)^2\,dy}{2y\sqrt{y-1}} =$$

$$= \frac{1}{2} \int (y-1)^{\frac{3}{2}} \frac{dy}{y}$$

w8.5.33

$$\int \sin^6 x\,dx = \int \left(\frac{1-\cos 2x}{2}\right)^3 dx =$$

$$= \frac{1}{8}\int (1-\cos 2x)^3 dx =$$

$$= \frac{1}{8}\int (1-\cos^3 2x + 3\cos^2 2x - 3\cos 2x)\,dx =$$

$$= \frac{1}{8}\int dx - \frac{1}{8}\int \cos^3 2x\,dx + \frac{3}{8}\int \cos^2 2x\,dx -$$

$$- \frac{3}{8}\int \cos 2x\,dx = \frac{1}{8}\int dx - \frac{3}{16}\int \cos 2x \cdot 2dx -$$

$$- \frac{1}{8}\int \cos^3 2x\,dx + \frac{3}{8}\int \cos^2 2x\,dx =$$

$$= [\ 3)\ y = \sin \cancel{2x};\ dx = \frac{1}{2}\cdot\frac{dy}{\sqrt{1-y^2}};$$

$$\cos 2x = \sqrt{1-y^2};\ 4)\ \cos^2 2x =$$

$$= \frac{1+\cos 4x}{2}\ ] = \frac{1}{8}\int dx - \frac{3}{16}\int \cos 2x\,d(2x) -$$

$$- \frac{1}{8}\int (1-y^2)\sqrt{1-y^2}\cdot\frac{1}{2}\cdot\frac{dy}{\sqrt{1-y^2}} +$$

$$+ \frac{3}{8}\int \frac{1+\cos 4x}{2}\,dx =$$

$$= \frac{1}{8}\int dx - \frac{3}{16}\int \cos 2x\, d(2x) - \frac{1}{16}\int dy +$$

$$+ \frac{1}{16}\int y^2 dy + \frac{3}{16}\int dx + \frac{3}{16}\int \cos 4x\, dx =$$

$$= \frac{5}{16}\int dx - \frac{1}{16}\int dy - \frac{3}{16}\int \cos 2x\, d(2x) +$$

$$+ \frac{1}{16}\int y^2 dy + \frac{3}{64}\int \cos 4x\, d(4x) =$$

$$= \frac{5}{16}x - \frac{1}{16}y - \frac{3}{16}\sin 2x + \frac{1}{48}y^3 +$$

$$+ \frac{3}{64}\sin 4x + C = \frac{5}{16}x - \frac{1}{16}\sin 2x -$$

$$- \frac{3}{16}\sin 2x + \frac{1}{48}\sin^3 2x + \frac{3}{64}\sin 4x + C =$$

$$= \frac{5}{16}x - \frac{1}{4}\sin 2x + \frac{1}{48}\sin^3 2x + \frac{3}{64}\sin 4x + C$$

<u>W 8.5.34</u>

$$\int \sin^2 x \cos^4 x\, dx = \int \frac{1-\cos 2x}{2} \cdot$$

$$\cdot \left(\frac{1+\cos 2x}{2}\right)^2 dx = \frac{1}{8}\int (1-\cos 2x) \cdot$$

$$\cdot (1+\cos 2x)^2 dx = \frac{1}{8}\int (1-\cos 2x) \cdot$$

$$\cdot (1+2\cos 2x + \cos^2 2x)\, dx =$$

$$= \frac{1}{8}\int\left(1+2\cos 2x+\cos^2 2x-\cos 2x-\right.$$
$$\left.-2\cos^2 2x-\cos^3 2x\right)dx =$$
$$= \frac{1}{8}\int\left(1+\cos 2x-\cos^2 2x-\cos^3 2x\right)dx =$$
$$= \frac{1}{8}\left(\int dx+\int\cos 2x\,dx-\int\cos^2 2x\,dx-\right.$$
$$\left.-\int\cos^3 2x\,dx\right) = \left[4)\ y=\sin 2x \Rightarrow\right.$$
$$\left.\Rightarrow dx=\frac{dy}{2\sqrt{1-y^2}};\ \cos 2x=\sqrt{1-y^2}\right] =$$
$$= \frac{1}{8}\left(\int dx+\frac{1}{2}\int\cos 2x\,d(2x)-\int\frac{1+\cos 4x}{2}dx-\right.$$
$$\left.-\int(1-y^2)\sqrt{1-y^2}\cdot\frac{dy}{2\sqrt{1-y^2}}\right) =$$
$$= \frac{1}{8}\left(\int dx+\frac{1}{2}\int 2\cos 2x\,dx-\frac{1}{2}\int dx-\frac{1}{2}\int\cos 4x\,dx-\right.$$
$$\left.-\frac{1}{2}\int dy+\frac{1}{2}\int y^2dy\right) = \frac{1}{8}\left(\frac{1}{2}\int dx+\right.$$
$$+\frac{1}{2}\int 2\cos 2x\,dx-\frac{1}{8}\int 4\cos 4x\,dx-\frac{1}{2}\int dy+$$
$$\left.+\frac{1}{2}\int y^2dy\right) = \frac{1}{8}\left(\frac{1}{2}x+\frac{1}{2}\sin 2x-\frac{1}{8}\sin 4x-\right.$$
$$\left.-\frac{1}{2}y+\frac{1}{2}\cdot\frac{y^3}{3}\right)+C = \frac{1}{16}x+\frac{1}{16}\sin 2x-$$
$$-\frac{1}{64}\sin 4x-\frac{1}{16}\sin 2x+\frac{1}{48}\sin^3 2x+C =$$

$$= \frac{x}{16} - \frac{1}{64}\sin 4x - \frac{1}{48}\sin^3 2x + C$$

№ 8.5.35

$$\int \sin^4 x \cos^4 x \, dx = \int \left(\frac{1-\cos 2x}{2}\right)^2 \cdot$$

$$\cdot \left(\frac{1+\cos 2x}{2}\right)^2 dx = \frac{1}{16}\int (1-\cos 2x)^2 \cdot$$

$$\cdot (1+\cos 2x)^2 dx = \frac{1}{16}\int (1-\cos^2 2x)^2 dx =$$

$$= \frac{1}{16}\int (\cos^4 2x - 2\cos^2 2x + 1) dx =$$

$$= \frac{1}{16}\int \left(\left(\frac{1+\cos 4x}{2}\right)^2 - 2\,\frac{1+\cos 4x}{2} + 1\right) dx =$$

$$= \frac{1}{16}\int \frac{1}{4}(1 + 2\cos 4x + \cos^2 4x) - 1 -$$

$$- \cos 4x + 1) dx = \frac{1}{16}\int \frac{1}{4} + \frac{1}{2}\cos 4x +$$

$$+ \frac{1+\cos 8x}{4\cdot 2} - \cos 4x) dx =$$

$$= \frac{1}{16}\int \left(\frac{1}{8}\cos 8x - \frac{1}{2}\cos 4x + \frac{3}{8}\right) dx =$$

$$= \frac{1}{128}\int \cos 8x \, dx - \frac{1}{32}\int \cos 4x \, dx +$$

$$+ \frac{3}{128}\int dx = \frac{1}{1024}\int \cos 8x \, d(8x) -$$

$$-\frac{1}{128}\int \cos 4x\, d(4x) + \frac{3}{128}\int dx =$$

$$= \frac{1}{1024}\sin 8x - \frac{1}{128}\sin 4x + \frac{3}{128}x + C$$

№ 8.5.36

$$\int \sin x \cdot \sin 3x\, dx = \int \frac{1}{2}(\cos(1-3)x -$$

$$- \cos(1+3)x)\, dx = \int \frac{1}{2}(\cos 2x - \cos 4x)\, dx =$$

$$= \frac{1}{2}\int \cos 2x\, dx - \frac{1}{2}\int \cos 4x\, dx =$$

$$= \frac{1}{4}\int \cos 2x\, d(2x) - \frac{1}{8}\int \cos 4x\, d(4x) =$$

$$= \frac{1}{4}\sin 2x - \frac{1}{8}\sin 4x + C$$

№ 8.5.37

$$\int \sin\frac{x}{12} \cdot \cos\frac{x}{3}\, dx = \int \frac{1}{2}\left(\sin\left(\frac{x}{12} - \frac{x}{3}\right) +\right.$$

$$\left. + \sin\left(\frac{x}{12} + \frac{x}{3}\right)\right) dx = \frac{1}{2}\int\left(-\sin\frac{x}{4} + \sin\frac{5x}{12}\right) dx$$

$$= \frac{1}{2}\int \sin\frac{5x}{12}\, dx - \frac{1}{2}\int \sin\frac{x}{4}\, dx =$$

$$= -\frac{6}{5}\cos\frac{5x}{12} + 2\cos\frac{x}{4} + C =$$

$$= 2\cos\frac{x}{4} - \frac{6}{5}\cos\frac{5x}{12} + C$$

№8.5.38

$\int \cos x \cdot \cos 3x\,dx = \int \frac{1}{2}\left(\cos(1-3)x +\right.$
$\left.+ \cos(1+3)x\right)dx = \frac{1}{2}\int\left(\cos 2x +\right.$
$\left.+ \cos 4x\right)dx = \frac{1}{2}\int \cos 2x dx + \frac{1}{2}\int \cos 4x dx =$
$= \frac{1}{4}\sin 2x + \frac{1}{8}\sin 4x + C$

№8.5.39

$\int \cos x \cdot \cos 3x \cdot \cos 5x\,dx =$
$= [$по №8.5.38 $\cos x \cdot \cos 3x =$
$= \frac{1}{2}(\cos 2x + \cos 4x)\,] =$
$= \frac{1}{2}\int(\cos 2x \cos 5x + \cos 4x \cos 5x)dx =$
$= \frac{1}{2}\int\left(\frac{1}{2}(\cos 3x + \cos 7x) + \frac{1}{2}(\cos x + \cos 9x)\right)dx =$
$= \frac{1}{4}\int(\cos 3x + \cos 7x + \cos x + \cos 9x)dx =$
$= \frac{1}{4}\int \cos 3x dx + \frac{1}{4}\int \cos 7x\,dx + \frac{1}{4}\int \cos x dx +$
$+ \frac{1}{4}\int \cos 9x\,dx = \frac{1}{12}\sin 3x + \frac{1}{28}\sin 7x +$
$+ \frac{1}{4}\sin x + \frac{1}{36}\sin 9x + C$

№8.5.40

$$\int ctg^6 x\,dx = \int\left(\frac{1}{\sin^2 x}-1\right)^3 dx =$$

$$= \int\left(\frac{1}{\sin^6 x} - 3\cdot\frac{1}{\sin^4 x} + 3\cdot\frac{1}{\sin^2 x} - 1\right)dx =$$

$$= \int\frac{dx}{\sin^6 x} - 3\int\frac{dx}{\sin^4 x} + 3\int\frac{dx}{\sin^2 x} - \int dx =$$

$= [1)\ y = tg\frac{x}{2};\ x = 2arctg\,y;\ dx = \frac{2dy}{1+y^2};$

$\sin^2 x = \frac{y^2}{y^2+1};$ 2) тоже, что и п. 1$] =$

$$= \int\frac{(y^2+1)^3}{y^6}\cdot\frac{2dy}{1+y^2} - 3\int\frac{(y^2+1)^2}{y^4}\cdot\frac{2dy}{1+y^2} +$$

$$+ 3\int\frac{dx}{\sin^2 x} - \int dx = 2\int\frac{(y^2+1)^2 dy}{y^6} -$$

$$- 6\int\frac{(y^2+1)dy}{y^4} + 3\int\frac{dx}{\sin^2 x} - \int dx =$$

$$= 2\int\frac{y^4+2y^2+1}{y^6}dy - 6\int\frac{y^2+1}{y^4}dy +$$

$$+ 3\int\frac{dx}{\sin^2 x} - \int dx = 2\int\frac{dy}{y^2} + 4\int\frac{dy}{y^4} +$$

$$+2\int\frac{dy}{y^6}-6\int\frac{dy}{y^2}-6\int\frac{dy}{y^4}+$$

$$+3\int\frac{dx}{\sin^2 x}-\int dx=2\int\frac{dy}{y^8}-2\int\frac{dy}{y^6}-$$

$$-4\int\frac{dy}{y^4}+3\int\frac{dx}{\sin^2 x}-\int dx=$$

$$=-\frac{2}{7y^7}+\frac{2}{5y^5}+\frac{4}{3y^3}-$$

$$-3\,ctg\,x-x+C=-\frac{2}{7}tg^{-7}\frac{x}{2}+$$

$$+\frac{2}{5}tg^{-5}\frac{x}{2}+\frac{4}{3}tg^{-3}\frac{x}{2}-3ctg\,x-x+C$$

№ 8.5.41

$$\int tg^4\frac{x}{2}\,dx=\int\left(\frac{1}{\cos^2\frac{x}{2}}-1\right)^2dx=$$

$$=\int\left(\frac{1}{\cos^4\frac{x}{2}}-\frac{2}{\cos^2\frac{x}{2}}+1\right)dx=$$

$$=\int\frac{dx}{\cos^4\frac{x}{2}}-2\int\frac{dx}{\cos^2\frac{x}{2}}+\int dx=$$

$$=\left[\ 1)\ t=tg\frac{x}{2};\ dx=\frac{2dt}{1+t^2};\right.$$

$$\left.\cos^4\frac{x}{2}=\frac{1}{(t^2+1)^2}\ \right]=$$

$$= \int (t^2+1)^2 \cdot \frac{2dt}{t^2+1} - 2\int \frac{dx}{\cos^2\frac{x}{2}} + \int dx =$$

$$= 2\int t^2 dt + 2\int dt - 4\int \frac{d\left(\frac{x}{2}\right)}{\cos^2\frac{x}{2}} + \int dx =$$

$$= \frac{2t^3}{3} + 2t - 4\,tg\frac{x}{2} + x + C =$$

$$= \frac{2}{3}\,tg^3\frac{x}{2} + 2\,tg\frac{x}{2} - 4\,tg\frac{x}{2} + x + C =$$

$$= \frac{2}{3}\,tg^3\frac{x}{2} - 2\,tg\frac{x}{2} + x + C$$

<u>№8.5.42</u>

$$\int tg^7 x\,dx = \int \left(\frac{1}{\cos^2 x} - 1\right)^3 \frac{\sin x}{\cos x}\,dx =$$

$$= \int \left(\frac{1}{\cos^6 x} - \frac{3}{\cos^4 x} + \frac{3}{\cos^2 x} - 1\right) tg x\,dx =$$

$$= \int \left(\frac{\sin x}{\cos^7 x} - \frac{3\sin x}{\cos^5 x} + \frac{3\sin x}{\cos^3 x} - \frac{\sin x}{\cos x}\right) dx =$$

$$= \left[t = tg\frac{x}{2};\ \sin x = \frac{2t}{1+t^2};\ \cos x = \frac{1-t^2}{1+t^2};\ dx = \frac{2dt}{1+t^2} \right] = \int \left(\frac{2t(1+t^2)^7}{(1+t^2)(1-t^2)^7} -\right.$$

$$\left. - \frac{3\cdot 2t(1+t^2)^5}{(1+t^2)(1-t^2)^5} + \frac{3\cdot 2t(1+t^2)^3}{(1+t^2)(1-t^2)^3} - \frac{2t(1+t^2)}{(1+t^2)(1-t^2)}\right) \cdot$$

$$\cdot \frac{2dt}{(1+t^2)} = \int\left(\frac{4t(1+t^2)^5}{(1-t^2)^7} - \frac{12t(1+t^2)^3}{(1-t^2)^5} + \right.$$

$$\left. + \frac{6t(1+t^2)}{(1-t^2)^3} - \frac{4t}{(1+t^2)(1-t^2)}\right) dt =$$

$$= 4\int \frac{t(1+t^2)^5}{(1-t^2)^7} dt - 12\int \frac{t(1+t^2)^3}{(1-t^2)^5} dt +$$

$$+ 6\int \frac{t(1+t^2)}{(1-t^2)^3} dt - 4\int \frac{t\,dt}{(1+t^2)(1-t^2)} =$$

$$= \left[1-t^2 = y \Rightarrow -\frac{1}{2}dy = t\,dt\right] =$$

$$= -2\int \frac{(2-y)^5}{y^7} dy + 6\int \frac{(2-y)^3}{y^5} dy -$$

$$- 3\int \frac{(2-y)}{y^3} dy \cancel{+ 2\int \frac{dy}{(2-y)y}} \cancel{+}$$

$$- 4\int \frac{t\,dt}{(1-t^4)} = \left[t^2 = 1-y\right] =$$

$$= -2\int \frac{-y^5 + 10y^4 - 40y^3 + 80y^2 - 80y + 32}{y^7} dy +$$

$$+ 6\int \frac{-y^3 + 6y^2 - 12y + 8}{y^5} dy - 3\int \frac{2-y}{y^3} dy +$$

$$+2\int\frac{dy}{-y^2+2y}=2\int\frac{dy}{y^2}-20\int\frac{dy}{y^3}+80\int\frac{dy}{y^4}-$$

$$-160\int\frac{dy}{y^5}+160\int\frac{dy}{y^6}-64\int\frac{dy}{y^7}-$$

$$-6\int\frac{dy}{y^2}+36\int\frac{dy}{y^3}-72\int\frac{dy}{y^4}+48\int\frac{dy}{y^5}-$$

$$-6\int\frac{dy}{y^3}+3\int\frac{dy}{y^2}+2\int\frac{dy}{2y-y^2}=$$

$$=\cancel{\;}2\int\frac{dy}{2y-y^2}-64\int\frac{dy}{y^7}+160\int\frac{dy}{y^6}-$$

$$-112\int\frac{dy}{y^5}+8\int\frac{dy}{y^4}+10\int\frac{dy}{y^3}-\int\frac{dy}{y^2}=$$

$$=\Big[\;1)\ \frac{1}{(2-y)y}=\frac{A}{2-y}+\frac{B}{y}\Rightarrow$$

$$\Rightarrow 1=Ay+2B-By;$$

$$\begin{cases}A-B=0\\2B=1\end{cases}\Rightarrow\begin{matrix}A=\frac{1}{2}\\B=\frac{1}{2}\end{matrix}\Big]=$$

$$=\int\frac{1}{2-y}dy+\int\frac{dy}{y}-64\int\frac{dy}{y^7}+160\int\frac{dy}{y^6}-$$

$$-112\int\frac{dy}{y^5}+8\int\frac{dy}{y^4}+10\int\frac{dy}{y^3}-\int\frac{dy}{y^2}=$$

$$=+\frac{64}{8y^8}\mp\frac{160}{7y^7}+\frac{112}{6y^6}\mp\frac{8}{5y^5}\mp$$

$$\mp\frac{10}{4y^4}+\frac{1}{3y^3}-$$

$$=\frac{64}{6y^6}-\frac{160}{5y^5}+\frac{112}{4y^4}-\frac{8}{3y^3}-$$

$$-\frac{10}{2y^2}+\frac{1}{y}+\ln|y|-\ln|y-2|+C=$$

$$=\left[y=1-tg^2\frac{x}{2}\right]=\frac{32}{3(1-tg^2\frac{x}{2})^6}-$$

$$-\frac{32}{(1-tg^2\frac{x}{2})^5}+\frac{28}{(1-tg^2\frac{x}{2})^4}-\frac{8}{3(1-tg^2\frac{x}{2})^3}-$$

$$-\frac{5}{(1-tg^2\frac{x}{2})^2}+\frac{1}{1-tg^2\frac{x}{2}}+\ln\left|1-tg^2\frac{x}{2}\right|-$$

$$-\ln\left|-1-tg^2\frac{x}{2}\right|+C$$