«Предел функции»

I.

№6.4.76

$$\lim_{x\to 2}\frac{3x-1}{2x+5}=\frac{3\cdot 2-1}{2\cdot 2+5}=\frac{5}{9}$$

№6.4.77

$$\lim_{x\to 2,5}\sqrt{4x-1}=\sqrt{4\cdot 2,5-1}=\sqrt{9}=3$$

№6.4.78

$$\lim_{x\to -1}\frac{\sqrt[3]{x+2}+1}{\sqrt{x+5}}=\frac{\sqrt[3]{-1+2}+1}{\sqrt{-1+5}}=\frac{1+1}{\sqrt{4}}=$$

$$=\frac{2}{2}=1$$

№6.4.79

$$\lim_{x\to\sqrt{2}}\left(x^2+\frac{1}{x^4}-3\right)=(\sqrt{2})^2+\frac{1}{(\sqrt{2})^4}-3=$$

$$=2+\frac{1}{4}-3=-\frac{3}{4}$$

№6.4.80

$$\lim_{x\to\frac{1}{3}}\frac{3x^2+5x-2}{3x-1}=\frac{3\cdot\left(\frac{1}{3}\right)^2+5\cdot\frac{1}{3}-2}{3\cdot\frac{1}{3}-1}=$$

$$= \frac{\frac{1}{3} + \frac{5}{3} - \frac{6}{3}}{1-1} = \left[\frac{0}{0}\right]$$

$$\lim_{x \to \frac{1}{3}} \frac{3x^2+5x-2}{3x-1} = \lim_{x \to \frac{1}{3}} (x+2) =$$

$$= \frac{1}{3} + 2 = \frac{7}{3}$$

$$\begin{array}{r|l} 3x^2+5x-2 & 3x-1 \\ \underline{3x^2-x} \quad & x+2 \\ 6x-2 & \\ \underline{6x-2} & \\ 0 & \end{array}$$

№6.4.81

$$\lim_{x \to 1} \frac{x^2-2x+1}{x^3-1} = \left[\frac{0}{0}\right]$$

$$\lim_{x \to 1} \frac{x^2-2x+1}{x^3-1} = \lim_{x \to 1} \frac{(x-1)^2}{x^3-1} =$$

$$= \lim_{x \to 1} \frac{(x-1)^{\cancel{2}}}{\cancel{(x-1)}(x^2+x+1)} = \frac{x-1}{x^2+x+1} = \frac{0}{3} =$$

$$= 0$$

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№6.4.82

$$\lim_{x\to2}\left(\frac{1}{x-2}-\frac{4}{x^2-4}\right)=\lim_{x\to2}\left(\frac{1}{x-2}-\frac{4}{(x-2)(x+2)}\right)=$$

$$=\lim_{x\to2}\frac{x+2-4}{(x-2)(x+2)}=\lim_{x\to2}\frac{\cancel{x-2}}{\cancel{(x-2)}(x+2)}=\lim_{x\to2}\frac{1}{x+2}=$$

$$=\frac{1}{2+2}=\frac{1}{4}$$

№6.4.83

$$\lim_{x\to0}\frac{x^5-3x^3+x^2}{x^4+2x^2}=\lim_{x\to0}\frac{x^2(x^3-3x+1)}{x^2(x^2+2)}=$$

$$=\lim_{x\to0}\frac{x^3-3x+1}{x^2+2}=\cancel{\lim_{x\to0}\frac{x-\frac{3}{x}+\frac{1}{x^2}}{1+\frac{2}{x^2}}}=$$

$$=\cancel{\frac{0-0+0}{1+0}}\;\frac{0-0+1}{0+2}=\frac{1}{2}$$

№6.4.84

$$\lim_{y\to1}\frac{y^3+4y-5}{y^3+2y^2-y-2}=\lim_{y\to1}\frac{\cancel{(y-1)}(y^2+y+5)}{\cancel{(y-1)}(y^2+3y+2)}=$$

$$= \lim_{y \to 1} \frac{y^2 + y + 5}{y^2 + 3y + 2} = \frac{1 + 1 + 5}{1 + 3 + 2} = \frac{7}{6}$$

№6. 4.85

$$\lim_{a \to 0} \frac{(x+a)^3 - x^3}{a} = \lim_{a \to 0} \frac{x^3 - x^3 + 3x^2a + 3xa^2 + a^3}{a} =$$

$$= \lim_{a \to 0} \frac{3x^2a + 3a^2x + a^3}{a} = \lim_{a \to 0} (3x^2 + 3ax + a^2) =$$

$$= 3x^2$$

№6 4.86

$$\lim_{x \to 8} \frac{x^2 - 8x}{\sqrt{x+1} - 3} = \lim_{x \to 8} \frac{x(x-8)(\sqrt{x+1} + 3)}{(\sqrt{x+1} - 3)(\sqrt{x+1} + 3)} =$$

$$= \lim_{x \to 8} \frac{x\cancel{(x-8)}(\sqrt{x+1} + 3)}{\cancel{x + 1 - 9}} =$$

$$= \lim_{x \to 8} x(\sqrt{x+1} + 3) = 8 \cdot (\sqrt{8+1} + 3) =$$

$$= 8 \cdot 6 = 48$$

4

№6.4.87

$$\lim_{x\to 5}\frac{\sqrt{9-x}-2}{3-\sqrt{x+4}}=\lim_{x\to 5}\frac{(\sqrt{9-x}-2)(3+\sqrt{x+4})}{(3-\sqrt{x+4})(3+\sqrt{x+4})}=$$

$$=\lim_{x\to 5}\frac{(\sqrt{9-x}-2)(3+\sqrt{x+4})}{9-x-4}=$$

$$=\lim_{x\to 5}\frac{(\sqrt{9-x}-2)(\sqrt{9-x}+2)(3+\sqrt{x+4})}{(9-x-4)(\sqrt{9-x}+2)}=$$

$$=\lim_{x\to 5}\frac{\cancel{(9-x-4)}(3+\sqrt{x+4})}{\cancel{(9-x-4)}(\sqrt{9-x}+2)}=$$

$$=\lim_{x\to 5}\frac{3+\sqrt{x+4}}{\sqrt{9-x}+2}=\frac{3+\sqrt{5+4}}{\sqrt{9-5}+2}=$$

$$=\frac{3+3}{2+2}=\frac{6}{4}=\frac{3}{2}$$

№6.4.88

$$\lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}=\lim_{h\to 0}\frac{(\sqrt{x+h}-\sqrt{x})(\sqrt{x+h}+\sqrt{x})}{h(\sqrt{x+h}+\sqrt{x})}=$$

$$=\lim_{h\to 0}\frac{x+h-x}{h(\sqrt{x+h}+\sqrt{x})}=\lim_{h\to 0}\frac{\cancel{h}}{\cancel{h}(\sqrt{x+h}+\sqrt{x})}=$$

$$=\lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}}=\frac{1}{\sqrt{x}+\sqrt{x}}=\frac{1}{2\sqrt{x}}$$

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№6.4.89

$$\lim_{x\to\sqrt{3}} \frac{\sqrt{x^2+1}-2}{\sqrt{x^2+6}-3} =$$

$$= \lim_{x\to\sqrt{3}} \frac{\left(\sqrt{x^2+1}-2\right)\left(\sqrt{x^2+1}+2\right)\left(\sqrt{x^2+6}+3\right)}{\left(\sqrt{x^2+6}-3\right)\left(\sqrt{x^2+6}+3\right)\left(\sqrt{x^2+1}+2\right)} =$$

$$= \lim_{x\to\sqrt{3}} \frac{\left(\underbrace{\cancel{x^2+1-4}}_{=-3}\right)\left(\sqrt{x^2+6}+3\right)}{\left(\underbrace{\cancel{x^2+6-9}}_{=-3}\right)\left(\sqrt{x^2+1}+2\right)} =$$

$$= \lim_{x\to\sqrt{3}} \frac{\sqrt{x^2+6}+3}{\sqrt{x^2+1}+2} = \frac{\sqrt{(\sqrt{3})^2+6}+3}{\sqrt{(\sqrt{3})^2+1}+2} =$$

$$= \frac{3+3}{2+2} = \frac{3}{2}$$

№6.4.90

$$\lim_{t\to0} \frac{\sqrt[3]{1+t}-1}{t} =$$

$$= \lim_{t\to0} \frac{\left(\sqrt[3]{1+t}-1\right)\left((1+t)^{\frac{2}{3}}+\sqrt[3]{1+t}+1\right)}{t\left((1+t)^{\frac{2}{3}}+\sqrt[3]{1+t}+1\right)} =$$

$$\lim_{t\to0} \frac{\cancel{1+t-1}}{t\left((1+t)^{\frac{2}{3}}+\sqrt[3]{1+t}+1\right)} =$$

6

$$= \lim_{t\to 0} \frac{1}{(1+t)^{\frac{2}{3}} + \sqrt[3]{1+t} + 1} = \frac{1}{1+1+1} = \frac{1}{3}$$

№ 6.4.91

$$\lim_{y\to 1} \frac{y-1}{\sqrt[4]{y}-1} = \lim_{y\to 1} \frac{(y-1)(\sqrt[4]{y}+1)(\sqrt{y}+1)}{(\sqrt[4]{y}-1)(\sqrt[4]{y}+1)(\sqrt{y}+1)} =$$

$$= \lim_{y\to 1} \frac{\cancel{(y-1)}(\sqrt[4]{y}+1)(\sqrt{y}+1)}{\cancel{(y-1)}} =$$

$$= \lim_{y\to 1} (\sqrt[4]{y}+1)(\sqrt{y}+1) = (1+1)(1+1) =$$

$$= 4$$

№ 6.4.94

$$\lim_{x\to\infty} \frac{(x^2-3)(2x+9)}{(x^2+x+1)(3x^2-4)} = \lim_{x\to\infty} \frac{2x^3+9x^2-6x-27}{3x^4+3x^3-x^2-4x-4} =$$

$$= \lim_{x\to\infty} \frac{\frac{2}{x}+\frac{9}{x^2}-\frac{6}{x^3}-\frac{27}{x^4}}{3+\frac{3}{x}-\frac{1}{x^2}-\frac{4}{x^3}-\frac{4}{x^4}} = \frac{0+0-0-0}{3+0-0-0-0} =$$

$$= \frac{0}{3} = 0$$

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W6.4.92

$$\lim_{x\to\infty} \frac{5x^4-2x+3}{x^2-3x^4} = \lim_{x\to\infty} \frac{5-\frac{2}{x^3}+\frac{3}{x^4}}{\frac{1}{x^2}-3} =$$

$$= \frac{5-0+0}{0-3} = \frac{5}{-3} = -\frac{5}{3}$$

W6.4.93

$$\lim_{x\to\infty} \frac{7x^3-x^2+3x-1}{10x^2+x} =$$

$$\lim_{x\to\infty} \frac{7-\frac{1}{x}+\frac{3}{x^2}-\frac{1}{x^3}}{\frac{10}{x}+\frac{1}{x^2}} = \frac{7-0+0-0}{0+0} =$$

$$= \frac{7}{0} = +\infty$$

W6.4.95

$$\lim_{x\to-\infty} \left(\sqrt{x^2+4}-10x\right) =$$

$$= \lim_{x\to-\infty} \frac{\left(\sqrt{x^2+4}-10x\right)\left(\sqrt{x^2+4}+10x\right)}{\sqrt{x^2+4}+10x} =$$

$$= \lim_{x\to\infty} = \frac{x^2+4-100x^2}{\sqrt{x^2+4}+10x} =$$

$$= \lim_{x\to-\infty} \frac{4-99x^2}{\sqrt{x^2+4}+10x} = \lim_{x\to-\infty} \frac{\frac{4}{x^2}-99}{\sqrt{1+\frac{4}{x^2}}+\frac{10}{x}} =$$

$$= \frac{0-99}{\sqrt{1+0}+0} = -99$$

№6.4.96

$$\lim_{x\to\infty}\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right) =$$

$$= \lim_{x\to\infty} \frac{\left(\sqrt{x^2+1}-\sqrt{x^2-1}\right)\left(\sqrt{x^2+1}+\sqrt{x^2-1}\right)}{\sqrt{x^2+1}+\sqrt{x^2-1}} =$$

$$= \lim_{x\to\infty} \frac{x^2+1-x^2+1}{\sqrt{x^2+1}+\sqrt{x^2-1}} =$$

$$= \lim_{x\to\infty} \frac{2}{\sqrt{x^2+1}+\sqrt{x^2-1}} = \lim_{x\to\infty} \frac{\frac{2}{x}}{\sqrt{1+\frac{1}{x^2}}+\sqrt{1-\frac{1}{x^2}}} =$$

$$= \frac{0}{\sqrt{1+0}+\sqrt{1-0}} = \frac{0}{2} = 0$$

~~№6.4.97~~

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№6.4.97

$$\lim_{x\to\infty}\left(\frac{x^3}{5x^2+1}-\frac{x^2}{5x-3}\right)=$$

$$=\lim_{x\to\infty}\frac{x^3(5x-3)-x^2(5x^2+1)}{(5x^2+1)(5x-3)}=$$

$$=\lim_{x\to\infty}\frac{5x^4-3x^3-5x^4+x^2}{25x^3+5x-15x^2-3}=$$

$$=\lim_{x\to\infty}\frac{x^2-3x^3}{25x^3-15x^2+5x-3}=$$

$$=\lim_{x\to\infty}\frac{\frac{1}{x}-3}{25-\frac{15}{x}+\frac{5}{x^2}-\frac{3}{x^3}}=$$

$$=\frac{0-3}{25-0+0-0}=-\frac{3}{25}=-0{,}12$$

~~№6.4.98~~

№6.4.98

$$\lim_{x\to 0}\frac{tg\,3x}{x} = 3$$ (по замеч. пр.)

№6.4.99

$$\lim_{x\to \pi}\frac{\sin 3x}{\sin 2x} = \lim_{x\to \pi}\frac{\sin 3x\cdot x}{\sin 2x\cdot x} =$$

$$= -\frac{3}{2}$$

№6.4.100

$$\lim_{x\to 0}\frac{tg x-\sin x}{x^3} = \frac{\sin x\left(\frac{1-\cos x}{\cos x}\right)}{x^3} =$$

$$= \lim_{x\to 0}\frac{\sin x\cdot 2\sin^2\frac{x}{2}}{x^3\cos x} =$$

$$= \lim_{x\to 0}\frac{\sin x\cdot 2\cdot \sin\frac{x}{2}\cdot \sin\frac{x}{2}}{x\cdot x\cdot x\cdot \cos x} =$$

$$= \frac{1\cdot 2\cdot\frac{1}{2}\cdot\frac{1}{2}}{1} = \frac{1}{2}$$ (по зам. пр.)

W6.4.101

$$\lim_{h\to 0}\frac{h-\sin h}{h+\sin h}=\frac{0-\sin 0}{0+\sin 0}=\left[\frac{0}{0}\right]$$

$$\lim_{h\to 0}\frac{h-\sin h}{h+\sin h}=\lim_{h\to 0}\frac{\frac{h-\sin h}{h}}{\frac{h+\sin h}{h}}=$$

$$=\lim_{h\to 0}\frac{1-\frac{\sin h}{h}}{1+\frac{\sin h}{h}}=\frac{1-1}{1+1}=\frac{0}{2}=0$$

W6.4.102

$$\lim_{x\to 2}(x-2)\cancel{ctg}\,\pi x=\lim_{x\to 2}(x-2)\cdot$$

$$\cdot\frac{\cos\pi x}{\sin\pi x}=\lim_{x\to 2}\frac{x\cos\pi x-2\cos\pi x}{\sin\pi x}=$$

$$=\lim_{x\to 2}\left(\frac{x\cos\pi x}{\sin\pi x}-\frac{2\cos\pi x}{\sin\pi x}\right)=$$

$$=\lim_{x\to 2}\left(\frac{x\underbrace{\cos\pi x}_{=1}}{\underbrace{\sin\pi x}_{=\frac{1}{\pi}}}-\frac{2x\underbrace{\cos\pi x}_{=1}}{x\underbrace{\sin\pi x}_{=\frac{1}{\pi}}}\right)=$$

$$\cancel{=\frac{2\cos 2\pi}{\sin 2\pi}-\frac{2\cdot 2\cos 2\pi}{2\cdot\sin 2\pi}}$$

$= \cancel{\lim} \frac{1}{4} - \frac{1}{4} = 0$

№ 6.4.103.

$$\lim_{x\to 0} \frac{1-\cos 4x}{2x \cdot tg\, 2x} = \lim_{x\to 0} \frac{\cancel{1}\sin^2 2x}{2x \cdot tg 2x} =$$

$$= \lim_{x\to 0} \frac{\sin 2x \cdot \sin 2x \cdot \cos 2x}{x \cdot \cancel{tg 2x}\, \sin 2x} =$$

$= \cancel{\lim}\ 2 \cdot 1 = 2$

№ 6.4.104.

$$\lim_{x\to 0} \frac{\arcsin 7x}{\sin 4x} = \lim_{x\to 0} \frac{\frac{\arcsin 7x}{x}}{\frac{\sin 4x}{x}} =$$

$= \cancel{\lim_{x\to 0}} \frac{7}{4}$ (по экв. пр.)

№ 6.4.105

$$\lim_{x\to\infty} x \sin\left(\frac{2}{x}\right) = \lim_{x\to\infty} \frac{\frac{x \sin\left(\frac{2}{x}\right)}{x}}{\frac{1}{x}} =$$

$$= \lim_{x\to\infty} \frac{\sin\left(\frac{2}{x}\right)}{\frac{1}{x}} = 2$$

№6.4.106

$$\lim_{x\to 0} \frac{7^x-1}{4^x-1} \qquad \left[\lim_{x\to 0} \frac{a^x-1}{x} = \ln a\right]$$

$$\lim_{x\to 0} \frac{7^x-1}{4^x-1} = \lim_{x\to 0} \frac{\frac{7^x-1}{x}}{\frac{4^x-1}{x}} =$$

$$= \frac{\ln 7}{\ln 4} = \frac{1}{2}\log_2 7$$

№6.4.107.

$$\lim_{x\to 0} (1+\operatorname{tg}x)^{\operatorname{ctg}x} = \lim_{x\to 0} (1+\operatorname{tg}x)^{\frac{1}{\operatorname{tg}x}} =$$

$$= e$$

№6.4.108

$$\lim_{x\to\infty} \left(\frac{x^2+2}{x^2-2}\right)^{x^2} = \lim_{x\to\infty} \left(\frac{x^2\left(1+\frac{2}{x^2}\right)}{x^2\left(1-\frac{2}{x^2}\right)}\right)^{x^2} =$$

$$= \lim_{x\to\infty} \frac{\left(1+\frac{2}{x^2}\right)^{x^2}}{\left(1+\frac{(-2)}{x^2}\right)^{x^2}} = \frac{e^2}{e^{-2}} = e^4$$

№6.4.109

$$\lim_{x\to 0} \frac{e^{3x}-1}{\sin x} = \lim_{x\to 0} \frac{x\,(e^{3x}-1)}{\sin x \cdot x} = 3$$

№6.4.110

$$\lim_{x\to\infty} \left(\frac{8+x}{10+x}\right)^{2x+3} = \lim_{x\to\infty} \left(\left(\frac{8+x}{10+x}\right)^{2x} \cdot \left(\frac{8+x}{10+x}\right)^{3}\right) = \lim_{x\to\infty} \left(\frac{1+\frac{8}{x}}{1+\frac{10}{x}}\right)^{2x} \cdot \left(\frac{1+\frac{8}{x}}{1+\frac{10}{x}}\right)^{3} =$$

$$= \frac{e^{16}}{e^{20}} \cdot \left(\frac{1+0}{1+0}\right)^3 = e^{-4} \cdot 1 = e^{-4}$$

№6.4.111

$$\lim_{x\to 0} \frac{\ln(1+x)}{\sin 2x} = \lim_{x\to 0} \frac{x \cdot \ln(1+x)}{\sin 2x \cdot x} = \frac{1}{2}$$

№6.4.112

$$\lim_{t\to+\infty}\left(1+\frac{1}{t^2}\right)^t = 1+0=1$$

№6.4.113

$$\lim_{x\to 0}(\cos 2x)^{\frac{1}{\sin^2 x}} =$$

$$= \lim_{x\to 0}(\cos 2x)^{\frac{1}{1-\cos^2 x}} =$$

$$= \lim_{x\to 0}\frac{\cos 2x}{(\cos 2x)^{\frac{1}{\cos^2 x}}} = 1$$

№6.4.114

$$\lim_{x\to+\infty}\left(\sqrt{x^2+2x+2}-\sqrt{x^2-2x-3}\right) =$$

$$= \lim_{x\to+\infty}\frac{\cancel{x^2}+2x+2-\cancel{x^2}+2x+3}{\sqrt{x^2+2x+2}+\sqrt{x^2-2x-3}} =$$

$$= \lim_{x\to+\infty}\frac{4x+5}{\sqrt{x^2+2x+2}+\sqrt{x^2-2x-3}} =$$

$$= \lim_{x\to+\infty}\frac{4+\frac{5}{x}}{\sqrt{1+\frac{2}{x}+\frac{2}{x^2}}+\sqrt{1-\frac{2}{x}-\frac{5}{x^2}}} =$$

$$= \frac{4+0}{1+1} = 2$$

II.

№6. 4.118

$$\lim_{x\to 0} \frac{\ln(1+x^2)}{tg^2 8x} \sim \lim_{x\to 0} \frac{x^2}{8x\cdot 8x} = \frac{1}{64}$$

№6.4.119

$$\lim_{x\to 0} \frac{2^{\sin x}-1}{x} \sim \lim_{x\to 0} \frac{2^x-1}{x} = \ln 2$$

№6.4.120

$$\lim_{x\to 0} \frac{x\cdot \arcsin\sqrt{x}}{arctg^{3/2} 2x} \sim \lim_{x\to 0} \frac{x\cdot\sqrt{x}}{(2x)^{3/2}} =$$

$$= \lim_{x\to 0} \frac{\sqrt{x^3}}{\sqrt{(2x)^3}} = \lim_{x\to 0} \sqrt{\left(\frac{x}{2x}\right)^3} =$$

$$= \lim_{x\to 0} \frac{1}{2\sqrt{2}} = \frac{1}{2\sqrt{2}}$$

№6.4.121

$$\lim_{x\to 1}\frac{e^{x-1}-1}{\sqrt{x}-1} \sim \lim_{x\to 1}\frac{x-1}{\sqrt{x}-1} = \lim_{x\to 1}\frac{(x-1)(\sqrt{x}+1)}{x-1}$$

$$= \lim_{x\to 1}\sqrt{x}+1 = 2$$

№6.4.122

$$\lim_{x\to 0}\frac{\sqrt[3]{1+x^2}-1}{1-\cos x} \sim \lim_{x\to 0}\frac{1+x^2-1}{\frac{x^2}{2}\left((1+x^2)^{2/3}+(1+x^2)^{1/3}+1\right)} =$$

$$= \lim_{x\to 0}\frac{2}{(1+x^2)^{2/3}+(1+x^2)^{\frac{1}{3}}+1} = \frac{2}{3}$$

№6.4.123

$$\lim_{x\to 0}\frac{\sqrt{1+x\sin x}-1}{x^2} \sim \lim_{x\to 0}\frac{\sqrt{1+x^2}-1}{x^2} =$$

$$= \lim_{x\to 0}\frac{1+x^2-1}{x^2(\sqrt{1+x^2}+1)} = \lim_{x\to 0}\frac{1}{\sqrt{1+x^2}+1} =$$

$$= \frac{1}{2}$$