|9.07.20| Елкина Галина, ИВТ 2 курс, 3 пг.

Вариант 32

Производные:

1) $y = \arccos^3\left(\sqrt{\ln\frac{\sqrt{8+x}}{x+2}}\right)$

$y' = 3\arccos^2\left(\sqrt{\ln\frac{\sqrt{8+x}}{x+2}}\right) \cdot$

$\cdot \frac{1}{2\sqrt{\ln\frac{\sqrt{8+x}}{x+2}}} \cdot \frac{\cancel{x+2}}{\sqrt{8+x}} \cdot$

$\cdot \frac{\frac{1}{2\sqrt{8+x}}\cdot(x+2) - \sqrt{8+x}}{(x+2)^{\cancel{2}}} \cdot \left(-\frac{1}{\sqrt{1-\ln\frac{\sqrt{8+x}}{x+2}}}\right) =$

$= -\frac{3\arccos^2\left(\sqrt{\frac{1}{2}\ln(8+x) - \ln(x+2)}\right)}{2\sqrt{\ln\frac{\sqrt{8+x}}{x+2}}\sqrt{1-\ln\frac{\sqrt{8+x}}{x+2}}} \cdot$

$\cdot \frac{x+2-2(8+x)}{2(8+x)(x+2)} =$

$= \frac{3(x+14)\arccos^2\left(\sqrt{\frac{1}{2}\ln(8+x) - \ln(x+2)}\right)}{4(8+x)(x+2)\sqrt{\ln\frac{\sqrt{8+x}}{x+2}}\sqrt{1-\ln\frac{\sqrt{8+x}}{x+2}}}$

2) $y = \left(\sqrt{x^4 - 10x + 6}\right)^{\arcsin(x+x^2)}$

$y' = \left(\sqrt{x^4 - 10x + 6}\right)^{\arcsin(x+x^2)} \cdot$

$\cdot \frac{2x+1}{\sqrt{1-(x+x^2)^2}} \cdot \ln\sqrt{x^4 - 10x + 6} +$

$+ \left(\sqrt{x^4 - 10x + 6}\right)^{\arcsin(x+x^2) - 1} \cdot$

$\cdot \frac{4x^3 - 10}{2\sqrt{x^4 - 10x + 6}} \cdot \arcsin(x+x^2) =$

$= \left(\sqrt{x^4 - 10x + 6}\right)^{\arcsin(x+x^2)} \cdot$

$\cdot \left(\frac{(2x+1)\ln(x^4 - 10x + 6)}{2\sqrt{1-(x+x^2)^2}} + \right.$

$\left. + \frac{(4x^3 - 10)\arcsin(x+x^2)}{2(x^4 - 10x + 6)} \right) =$

$= \frac{1}{2}(x^4 - 10x + 6)^{\frac{1}{2}\arcsin(x+x^2)} \cdot$

$\cdot \left(\frac{(2x+1)\ln(x^4 - 10x + 6)}{\sqrt{1-(x+x^2)^2}} + \frac{(4x^3 - 10)\arcsin(x+x^2)}{x^4 - 10x + 6} \right)$

3) • $\sin(14+3xy-3y)+\frac{7x^4y-xy+11}{5y^2}=y^2-15x^3$

• $\frac{1}{5y^2}\left(5y^2\sin(14+3xy-3y)+7x^4y-xy+11\right)=y^2-15x^3$

• $5y^2\sin(14+3xy-3y)+7x^4y-xy+11=5y^4-75x^3y^2 \quad |()'$

• $5\cdot 2yy'\sin(14+3xy-3y)+5y^2\cos(14+3xy-3y)\cdot$
$\cdot(3y+3xy'-3y')+7y\cdot 4x^3+7x^4y'-y-xy'=$
$=5\cdot 4y^3y'-75y^2\cdot 3x^2-75x^3\cdot 2yy'$

• $10yy'\sin(14+3xy-3y)+15y^2\cos(14+3xy-3y)\cdot$
$\cdot(y+xy'-y')+7x^4y'-xy'-20y^3y'+$
$+150x^3yy'=-28x^3y+y-225x^2y^2$

• $10yy'\sin(14+3xy-3y)+15y^3\cos(14+3xy-3y)+$
$+15xy^2y'\cos(14+3xy-3y)-15y^2y'\cos(14+3xy-3y)+$
$+7x^4y'-xy'-20y^3y'+150x^3yy'=$
$=-28x^3y+y-225x^2y^2$

• $y'\left(10y\sin(14+3xy-3y)+15xy^2\cos(14+3xy-3y)-\right.$
$\left.-15y^2\cos(14+3xy-3y)+7x^4-x-20y^3+150x^3y\right)$
$=y\left(1-15y^2\cos(14+3xy-3y)-28x^3-225x^2y\right)$

- $$y' = \frac{y\left(1-15y^2\cos(14+3xy-3y)-28x^3-225x^2y\right)}{5\left(2y\sin(14+3xy-3y)+3xy^2\cos(14+3xy-3y)-3y^2\cos(14+3xy-3y)-4y^3+30x^3y\right)+7x^4-x}$$

*простите за перенос знаменателя

[Для сокращения записи:

- $\sin(14+3xy-3y) = \sin(\ldots)$
- $\cos(14+3xy-3y) = \cos(\ldots)$]

- $$y' = \frac{y\left(1-15y^2\cos(\ldots)-28x^3-225x^2y\right)}{5\left(2y\sin(\ldots)+3xy^2\cos(\ldots)-3y^2\cos(\ldots)-4y^3+30x^3y\right)+x(7x^3-1)}$$

- $$y' = \frac{y\left(1-15y^2\cos(\ldots)-28x^3-225x^2y\right)}{5y\left(2\sin(\ldots)+3xy\cos(\ldots)-3y\cos(\ldots)-4y^2+30x^3\right)+x(7x^3-1)}$$

Интегралы:

1) $$\int \frac{x\,dx}{(11-7x^2)^9} =$$

$$= \left[t = 11-7x^2;\ dt = -14x\,dx \Rightarrow x\,dx = -\frac{dt}{14}\right] =$$

$$= -\frac{1}{14}\int \frac{dt}{t^9} = -\frac{1}{14}\cdot\left(-\frac{1}{8t^8}\right)+C =$$

$$= \frac{1}{112\,t^8}+C = \frac{1}{112(11-7x^2)^8}+C$$

2) $\int (x^5+9x)\ln x\,dx = \left[\begin{matrix} u = \ln x \\ v' = x^5+9x \end{matrix}\right. \Rightarrow$

$\Rightarrow \begin{matrix} u' = \frac{1}{x} \\ v = \int (x^5+9x)\,dx = \int x^5 dx + 9\int x\,dx = \frac{x^6}{6} + \frac{9x^2}{2} \end{matrix} \Bigg] =$

$= \frac{x^2}{2}\ln x\left(\frac{x^4}{3}+9\right) - \int \frac{x}{2}\left(\frac{x^4}{3}+9\right)dx =$

$= \frac{x^2}{2}\ln x\left(\frac{x^4}{3}+9\right) - \frac{1}{2}\int\left(\frac{x^5}{3}+9x\right)dx =$

$= \frac{x^2}{2}\ln x\left(\frac{x^4}{3}+9\right) - \frac{1}{6}\int x^5 dx - \frac{9}{2}\int x\,dx =$

$= \frac{x^2}{2}\ln x\left(\frac{x^4}{3}+9\right) - \frac{x^6}{36} - \frac{9x^2}{4} + C =$

$= \frac{x^2}{2}\left(\ln x\left(\frac{x^4}{3}+9\right) - \frac{x^4}{18} - \frac{9}{2}\right) + C$

3) $\int \frac{x - \frac{17}{5}}{x^3-2x^2+3x-6}\,dx = \Big[$

$$\begin{array}{r|l} x^3-2x^2+3x-6 & x-2 \\ \cline{2-2} -\ x^3-2x^2 \qquad\quad & x^2+3 \\ \hline 3x-6 & \\ -\ 3x-6 & \\ \hline 0 & \end{array} \Rightarrow$$

$\Rightarrow x^3-2x^2+3x-6 = (x-2)(x^2+3) \Big] =$

$= \int \frac{x-\frac{17}{5}}{(x-2)(x^2+3)}\,dx = \Big[\frac{x-\frac{17}{5}}{(x-2)(x^2+3)} =$

$$= \frac{A}{x-2} + \frac{Bx+C}{x^2+3} \Rightarrow$$

$$\Rightarrow x - \frac{17}{5} = A(x^2+3) + (Bx+C)(x-2) =$$

$$= Ax^2 + 3A + Bx^2 - 2Bx + Cx - 2C =$$

$$= x^2(A+B) + x(C-2B) + 3A - 2C \Rightarrow$$

$$\Rightarrow \begin{cases} A+B=0 \\ C-2B=1 \\ 3A-2C=-\frac{17}{5} \end{cases} \Rightarrow \begin{cases} A=-B \\ C=2B+1 \\ 3\cdot(-B)-2(2B+1)=-\frac{17}{5} \end{cases} \Rightarrow$$

$$\Rightarrow -3B-4B-2=-\frac{17}{5}$$

$$-7B=-\frac{7}{5} \Rightarrow B=\frac{1}{5}$$

$$A=-\frac{1}{5};\ C=\frac{7}{5} \ ] =$$

$$= \int\left(-\frac{1}{5(x-2)} + \frac{\frac{1}{5}x+\frac{7}{5}}{x^2+3}\right)dx =$$

$$= \frac{1}{5}\int\frac{x+7}{x^2+3}dx - \frac{1}{5}\int\frac{dx}{x-2} =$$

$$= \frac{1}{5}\int\frac{x\,dx}{x^2+3} + \frac{7}{5}\int\frac{dx}{x^2+3} - \frac{1}{5}\int\frac{d(x-2)}{x-2} =$$

$$= \frac{1}{10}\int\frac{d(x^2+3)}{x^2+3} + \frac{7}{5}\int\frac{dx}{x^2+3} - \frac{1}{5}\int\frac{d(x-2)}{x-2} =$$

$$= \frac{1}{10}\ln|x^2+3| + \frac{7}{5}\cdot\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}} - \frac{1}{5}\ln|x-2| + C=$$

$= [$ т.к. x^2+3 — параб., ветви вверх $] =$

$= \frac{1}{10}\ln(x^2+3) + \frac{7}{5\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}} - \frac{1}{5}\ln|x-2| + C$

4) $\int \frac{dx}{1+\sqrt{2x+1}} = [\, t^2 = 2x+1;\ x = \frac{t^2-1}{2};$

$dx = \frac{1}{2}\cdot 2t\,dt = t\,dt \,] =$

$= \int \frac{t\,dt}{t+1} = \int \frac{t+1-1}{t+1}\,dt =$

$= \int dt - \int \frac{d(t+1)}{t+1} = t - \ln|t+1| + C =$

$= \sqrt{2x+1} - \ln|\sqrt{2x+1}+1| + C =$

$= [$ т.к. $\sqrt{2x+1}+1 > 0 \,] =$

$= \sqrt{2x+1} - \ln(\sqrt{2x+1}+1) + C$

5) $\int \frac{dx}{3\cos^2 x + 5\sin^2 x} = \int \frac{dx}{3\cos^2 x + 5(1-\cos^2 x)} =$

$= \int \frac{dx}{5 - 2\cos^2 x} = [\, t = \operatorname{tg} x \Rightarrow$

$\Rightarrow \cos^2 x = \frac{1}{t^2+1};\ x = \operatorname{arctg} t \Rightarrow$

$\Rightarrow dx = \frac{dt}{t^2+1} \,] = \int \frac{dt}{\left(5 - \frac{2}{t^2+1}\right)(t^2+1)} =$

$$= \int \frac{dt}{\frac{5t^2+3}{t^2+1}\cdot(t^2+1)} = \int \frac{dt}{5t^2+3} =$$

$$= \int \frac{dt}{(\sqrt{5}t)^2+(\sqrt{3})^2} = \frac{1}{\sqrt{5}} \int \frac{d(\sqrt{5}t)}{(\sqrt{5}t)^2+(\sqrt{3})^2} =$$

$$= \frac{1}{\sqrt{5}} \cdot \frac{1}{\sqrt{3}} \operatorname{arctg} \frac{t\sqrt{5}}{\sqrt{3}} + C =$$

$$= \frac{1}{\sqrt{15}} \operatorname{arctg}\left(\operatorname{tg}x \cdot \sqrt{\frac{5}{3}}\right) + C$$

6) $\int x \arcsin(5x)\,dx =$

$$= \left[\begin{matrix} u = \arcsin(5x) \\ v' = x \end{matrix} \Rightarrow \begin{matrix} u' = \frac{5}{\sqrt{1-25x^2}} \\ v = \frac{x^2}{2} \end{matrix} \right] =$$

$$= \frac{x^2}{2} \arcsin(5x) - \frac{5}{2} \int \frac{x^2\,dx}{\sqrt{1-25x^2}} =$$

$$= \Big[t^2 = 1-25x^2;\ x^2 = \frac{1-t^2}{25};$$

$$dx = \frac{1}{2\sqrt{\frac{1-t^2}{25}}} \cdot \left(-\frac{1}{25} \cdot dt^2\right) =$$

$$= -\frac{t\,dt}{5\sqrt{1-t^2}} \Big] = \frac{x^2}{2} \arcsin(5x) +$$

$$+ \frac{5}{2} \int \frac{(1-t^2)^{1/2}\,\cancel{t}\,dt}{\cancel{t} \cdot 25 \cdot \cancel{5}\,\cancel{\sqrt{1-t^2}}} = \frac{x^2}{2} \arcsin(5x) +$$

$$+\frac{1}{50}\int\sqrt{1-t^2}\,dt = \Big[\ \sqrt{1-t^2} =$$

$$= t^0(1-1t^2)^{\frac{1}{2}} \Rightarrow m=0,\ n=2,\ p=\frac{1}{2}\notin\mathbb{Z};$$

$$\nexists\ \frac{m+1}{n}=\frac{1}{2}\notin\mathbb{Z} \Rightarrow \nexists\ \frac{m+1}{n}+p=1\in\mathbb{Z}\Rightarrow$$

$$\Rightarrow t^{-2}-1=y^2;\ y^2=\frac{1-t^2}{t^2};\ y=\frac{\sqrt{1-t^2}}{t};$$

$$t=\frac{1}{\sqrt{y^2+1}};\ dt=-\frac{2y\,dy}{2(y^2+1)\sqrt{y^2+1}}=-\frac{y\,dy}{(y^2+1)\sqrt{y^2+1}}\Big] =$$

$$=\frac{x^2}{2}\arcsin(5x)+\frac{1}{50}\int\sqrt{1-\frac{1}{y^2+1}}\cdot\left(-\frac{y\,dy}{(y^2+1)\sqrt{y^2+1}}\right)=$$

$$=\frac{x^2}{2}\arcsin(5x)-\frac{1}{50}\int\frac{y}{\sqrt{y^2+1}}\cdot\frac{y\,dy}{(y^2+1)\sqrt{y^2+1}}=$$

$$=\frac{x^2}{2}\arcsin(5x)-\frac{1}{50}\int\frac{y^2\,dy}{(y^2+1)^2}=$$

$$=\frac{x^2}{2}\arcsin(5x)-\frac{1}{50}\int\frac{y^2+1-1}{(y^2+1)^2}\,dy=$$

$$=\frac{x^2}{2}\arcsin(5x)-\frac{1}{50}\int\frac{dy}{y^2+1}+\frac{1}{50}\int\frac{dy}{(y^2+1)^2}=$$

= [ф-ла понижения степени:

$$\int\frac{dy}{(y^2+1)^2}=\frac{1}{2}\cdot\frac{y}{y^2+1}+\frac{1}{2}\int\frac{dy}{y^2+1}=$$

$$= \frac{1}{2}\left(\frac{y}{y^2+1} + \operatorname{arctg} y\right)\Big] =$$

$$= \frac{x^2}{2}\arcsin(5x) - \frac{1}{50}\operatorname{arctg} y +$$

$$+ \frac{1}{100}\left(\frac{y}{y^2+1} + \operatorname{arctg} y\right) \cancel{+} C =$$

$$= \frac{x^2}{2}\arcsin(5x) - \frac{1}{100}\operatorname{arctg} y + \frac{y}{100(y^2+1)} + C =$$

$$= \cancel{\frac{x^2}{2}\arcsin(5x)}\Big[y = \frac{\sqrt{1-t^2}}{t};$$

$$t = \sqrt{1-25x^2} \Rightarrow y = \frac{\sqrt{1-1+25x^2}}{\sqrt{1-25x^2}} =$$

$$= \frac{5x}{\sqrt{1-25x^2}}\Big] = \frac{x^2}{2}\arcsin(5x) -$$

$$- \frac{1}{100}\operatorname{arctg}\frac{5x}{\sqrt{1-25x^2}} + \frac{5x}{100\sqrt{1-25x^2}\cdot\left(\frac{25x^2}{1-25x^2}+1\right)} + C =$$

$$= \frac{x^2}{2}\arcsin(5x) - \frac{1}{100}\operatorname{arctg}\frac{5x}{\sqrt{1-25x^2}} +$$

$$+ \frac{x}{20}\sqrt{1-25x^2} + C$$

7) $$\int_{-5}^{-2} \frac{dx}{\sqrt{5-4x-x^2}} = \Big[5-4x-x^2 = 0 \Rightarrow$$

$$\Rightarrow D = 16 - 4\cdot(-1)\cdot 5 = 36 \Rightarrow$$

$$x_1 = \frac{4+6}{-2} = -5;\; x_2 = \frac{4-6}{-2} = 1\Big] =$$

$$② \;[\; 5-4x-x^2 = -(x^2+4x-5) = -(x^2+4x+4)+5+4 =$$

$$= 9-(x+2)^2 = 3^2-(x+2)^2 \;] =$$

$$= \int_{-5}^{-2} \frac{d(x+2)}{\sqrt{9-(x+2)^2}} = \arcsin \frac{x+2}{3} \Big|_{-5}^{-2} =$$

$$= \arcsin \frac{-2+2}{3} - \arcsin \frac{-5+2}{3} =$$

$$= \arcsin 0 - \arcsin(-1) =$$

$$= 0 + \arcsin 1 = \frac{\pi}{2}$$

Дифференциальное уравнения:

1) $y' = -2y \;;\; y(0) = 33$

$\cancel{y = \int(-2y)} \quad \cancel{y' + 2y = 0}$

$$\frac{dy}{dx} = -2y \Rightarrow \frac{dy}{y} = -2dx \quad \Big|\int()$$

$$\ln y = -2x + C$$

$y = e^{-2x+C}$ — общее р-ие

$e^{-2\cdot 0+C} = 33$

$e^{C} = 33$

$C = \ln 33$

$y = e^{-2x+\ln 33} = e^{-2x}\cdot e^{\ln 33} =$

$= 33e^{-2x}$

$y = 33e^{-2x}$ — частное р-ие

2) $xy' = 3\sqrt{3x^2+y^2} + y \quad \Big| \cdot \frac{1}{x}$

$y' = \frac{3}{x}\sqrt{3x^2+y^2} + \frac{y}{x}$

$\exists\, y = uv \Rightarrow y' = u'v + uv'$, тогда:

$u'v + uv' = \frac{3}{x}\sqrt{3x^2+u^2v^2} + \frac{uv}{x}$

$u'v + uv' - \frac{uv}{x} = \frac{3}{x}\sqrt{3x^2+u^2v^2}$

$u'v + u\left(v' - \frac{v}{x}\right) = \frac{3}{x}\sqrt{3x^2+u^2v^2}$

$v' - \frac{v}{x} = 0$

$v' = \frac{v}{x}$

Зачёркнутое считать верным

$$\frac{dv}{dx} = \frac{v}{x}; \quad \frac{dv}{v} = \frac{dx}{x};$$

$$\ln|v| = \ln|x| + \cancel{\ln|C|} + C$$

$$v = x + C$$

J $C = 0$, тогда $v = x$

$$u'x = \frac{3}{x}\sqrt{3x^2 + u^2x^2}$$

$$u'x = \frac{3}{x} \cdot \sqrt{x^2(3+u^2)}$$

$$u'x = 3\sqrt{3+u^2}$$

$$\frac{du}{dx}x = 3\sqrt{3+u^2}$$

$$\frac{du}{\sqrt{u^2+3}} = \frac{3dx}{x} \quad \Big| \int()$$

$$\ln\left|u + \sqrt{u^2+3}\right| = 3\ln|x| + \ln|C|$$

$$u + \sqrt{u^2+3} = x^3 + C$$

$$\sqrt{u^2+3} = x^3 + C - u$$

$$u^2 + 3 = (x^3 + C - u)^2$$

$$u^2 + 3 = x^6 + 2Cx^3 - 2ux^3 - 2Cu + C^2 + u^2$$

$u^2 + 2ux^3 + 2Cu - u^2 = x^6 + 2Cx^3 + C^2 - 3$

$2u(x^3 + C) = (x^3 + C)^2 - 3$

$u = \frac{(x^3+C)^2 - 3}{2(x^3+C)} \quad [x^3 + C \neq 0]$

$y = uv = \frac{x(x^3+C)^2 - 3x}{2(x^3+C)}$

$2y(x^3+C) = x(x^3+C)^2 - 3x$ –

– общий интеграл.