Интегралы, часть 6.

№ 9.1.3

$$\int_0^{\pi}(2x+\sin 2x)\,dx = 2\int_0^{\pi}x\,dx + \frac{1}{2}\int_0^{\pi}\sin 2x\,d(2x) =$$

$$= x^2\Big|_0^{\pi} - \frac{1}{2}\cos 2x\Big|_0^{\pi} = \pi^2 - \frac{1}{2}(\cos 2\pi - \cos 0) = \pi^2 - \frac{1}{2}\cdot 0 = \pi^2$$

№ 9.1.4

$$\int_0^{\lg 2} 2^x\cdot 5^x\,dx = \int_0^{\lg 2} 10^x\,dx =$$

$$= \frac{10^x}{\ln 10}\Big|_0^{\lg 2} = \frac{10^{\lg 2}}{\ln 10} - \frac{10^0}{\ln 10} =$$

$$= \frac{1}{\ln 10}$$

№ 9.1.5

$$\int_2^5 \frac{dx}{2x-3} = \frac{1}{2}\ln|2x-3|\Big|_2^5 =$$

$$= \frac{1}{2}\left(\ln|7| - \ln|1|\right) = \frac{1}{2}\ln 7$$

№ 9.1.6

$$\int_1^2 \frac{x+2}{3-x}\,dx = \int_1^2\left(-\frac{x}{x-3} - \frac{2}{x-3}\right)dx =$$

$$= \int_1^2 \left(-1 - \frac{3}{x-3} - \frac{2}{x-3}\right) dx =$$

$$= -\int_1^2 dx - 5\int_1^2 \frac{dx}{x-3} = -x\Big|_1^2 -$$

$$-5\ln|x-3|\Big|_1^2 = -2+1-5(\ln 1 - \ln 2) =$$

$$= 5\ln 2 - 1$$

<u>№ 9.1.7</u>

$$\int_1^e \frac{x+\sqrt{x}}{x\cdot\sqrt{x}} dx = \int_1^e \left(\frac{1}{\sqrt{x}} + \frac{1}{x}\right) dx =$$

$$= 2\sqrt{x}\Big|_1^e + \ln|x|\Big|_1^e = 2\sqrt{e} - 2 +$$

$$+ \ln e - \ln 1 = 2\sqrt{e} - 1$$

<u>№ 9.1.8</u>

$$\int_0^1 \frac{x-4}{\sqrt{x}-2} dx = [x = t^2,\ dx = 2t\,dt] =$$

$$= \int_0^1 \frac{t^2-4}{t-2} \cdot 2t\,dt = \int_0^1 \frac{(t-2)(t+2)}{t-2} 2t\,dt =$$

$$= 2\int_0^1 (t^2+2t)\,dt = 2\left(\frac{t^3}{3} + \frac{2t^2}{2}\right)\Big|_0^1 =$$

$$= \frac{2t^3}{3}\Big|_0^1 + 2t^2\Big|_0^1 = \frac{2}{3} + 2 = \frac{8}{3}$$

w 9.1.9

$$\int_1^5 \frac{x}{1+x^2}dx = \left[t = 1+x^2;\ dt = 2x\,dx \Rightarrow \right.$$

$$\left. \Rightarrow \frac{1}{2}dt = x\,dx \right] = \frac{1}{2}\int_1^5 \frac{dt}{t} =$$

$$= \frac{1}{2}\ln|t|\Big|_1^5 = \frac{1}{2}(\ln 5 - \ln 1) =$$

$$= \frac{1}{2}\ln 5$$

w 9.1.10

$$\int_{\frac{1}{2}}^1 \sqrt{4x-2}\,dx = \frac{1}{4}\cdot\frac{2}{3}(4x-2)^{\frac{3}{2}}\Big|_{\frac{1}{2}}^1 =$$

$$= \frac{1}{6}\left(2\sqrt{2} - 0\right) = \frac{\sqrt{2}}{3}$$

w 9.1.11

$$\int_0^2 x\sqrt{9-\frac{9}{4}x^2}\,dx = 3\int_0^2 x\sqrt{1-\frac{1}{4}x^2}\,dx =$$

$$= \left[t = 1 - \frac{x^2}{4};\ dt = -\frac{1}{2}x\,dx \Rightarrow \right.$$

$$\left. \Rightarrow -2dt = x\,dx \right] =$$

$$= \int_1^2 \left(-1 - \frac{3}{x-3} - \frac{2}{x-3}\right) dx =$$

$$= -\int_1^2 dx - 5\int_1^2 \frac{dx}{x-3} = -x\Big|_1^2 -$$

$$-5\ln|x-3|\Big|_1^2 = -2+1-5(\ln 1 - \ln 2) =$$

$$= 5\ln 2 - 1$$

<u>№9.1.7</u>

$$\int_1^e \frac{x+\sqrt{x}}{x\sqrt{x}}\, dx = \int_1^e \left(\frac{1}{\sqrt{x}} + \frac{1}{x}\right) dx =$$

$$= 2\sqrt{x}\Big|_1^e + \ln|x|\Big|_1^e = 2\sqrt{e} - 2 +$$

$$+ \ln e - \ln 1 = 2\sqrt{e} - 1$$

<u>№9.1.8</u>

$$\int_0^1 \frac{x-4}{\sqrt{x}-2}\, dx = [x = t^2,\ dx = 2t\,dt] =$$

$$= \int_0^1 \frac{t^2-4}{t-2} \cdot 2t\,dt = \int_0^1 \frac{(t-2)(t+2)}{t-2}\, 2t\,dt =$$

$$= 2\int_0^1 (t^2+2t)\, dt = 2\left(\frac{t^3}{3} + \frac{2t^2}{2}\right)\Big|_0^1 =$$

$$= \frac{2t^3}{3}\Big|_0^1 + 2t^2\Big|_0^1 = \frac{2}{3} + 2 = \frac{8}{3}$$

<u>w 9.1.9</u>

$$\int_1^5 \frac{x}{1+x^2}dx = [t = 1+x^2;\ dt = 2xdx \Rightarrow$$

$$\Rightarrow \frac{1}{2}dt = xdx] = \frac{1}{2}\int_1^5 \frac{dt}{t} =$$

$$= \frac{1}{2}\ln|t|\Big|_1^5 = \frac{1}{2}(\ln 5 - \ln 1) =$$

$$= \frac{1}{2}\ln 5$$

<u>w 9.1.10</u>

$$\int_{\frac{1}{2}}^1 \sqrt{4x-2}\,dx = \frac{1}{4}\cdot\frac{2}{3}(4x-2)^{\frac{3}{2}}\Big|_{\frac{1}{2}}^1 =$$

$$= \frac{1}{6}\left(2\sqrt{2} - 0\right) = \frac{\sqrt{2}}{3}$$

<u>w 9.1.11</u>

$$\int_0^2 x\sqrt{9 - \frac{9}{4}x^2}\,dx = 3\int_0^2 x\sqrt{1 - \frac{1}{4}x^2}\,dx =$$

$$= [t = 1 - \frac{x^2}{4};\ dt = -\frac{1}{2}xdx \Rightarrow$$

$$\Rightarrow -2dt = xdx] =$$

$$= -6\int_0^2 \sqrt{t}\,dt = -6\cdot\frac{2}{3}\, t\sqrt{t}\Big|_0^2 =$$

$$= -4\left(2\sqrt{2} - 0\right) = -8\sqrt{2}$$

W 9.1.13

$$\int_0^{\pi}\left(\cos^3 x - \frac{3}{4}\cos x\right)dx =$$

$$= \int_0^{\pi} \cos x\left(\cos^2 x - \frac{3}{4}\right)dx =$$

$$= \int_0^{\pi}\left(\frac{1}{4} - \sin^2 x\right)\cos x\,dx =$$

$$= [t = \sin x;\ dt = \cos x\,dx] =$$

$$= \int_0^{\pi}\left(\frac{1}{4} - t^2\right)dt = \frac{1}{4}t\Big|_0^{\pi} - \frac{t^3}{3}\Big|_0^{\pi} = \frac{\pi}{4} - \frac{\pi^3}{3}$$

W 9.1.14

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 x - \sin^4 x} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 x\left(1 - \sin^2 x\right)} =$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\sin^2 x \cos^2 x} = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{dx}{\frac{1}{2}\sin^2 2x} =$$

$$= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \frac{2\,dx}{\sin^2 2x} = -\operatorname{ctg} 2x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{3}} =$$

$$= -\operatorname{ctg}\frac{2\pi}{3} + \operatorname{ctg}\frac{\pi}{3} = \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} = 0$$

№ 9.1.15

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \operatorname{tg}^2 x\,dx = \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \left(\frac{1}{\cos^2 x} - 1\right) dx =$$

$$= \operatorname{tg} x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} - x \Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} = \sqrt{3} - 1 - \frac{\pi}{3} + \frac{\pi}{4} =$$

$$= \sqrt{3} - 1 - \frac{\pi}{12}$$

№ 9.1.16

$$\int_0^{\frac{\pi}{6}} \sin 2x \cos 8x\,dx = \int_0^{\frac{\pi}{6}} \frac{1}{2}\Big(\sin(2x-8x) +$$

$$+ \sin(2x+8x)\Big) dx = \frac{1}{2}\int_0^{\frac{\pi}{6}} \big(-\sin 6x +$$

$$+ \sin 10x\big) dx = -\frac{1}{2}\int_0^{\frac{\pi}{6}} \sin 6x\,dx +$$

$$+ \frac{1}{2}\int_0^{\frac{\pi}{6}} \sin 10x\,dx = \frac{1}{10}\int_0^{\frac{\pi}{6}} \sin 10x\,d(10x) -$$

$$-\frac{1}{12}\int_0^{\frac{\pi}{6}} \sin 6x \, d(6x) - \frac{1}{10}\cos 10x \Big|_0^{\frac{\pi}{6}} +$$

$$+\frac{1}{12}\cos 6x \Big|_0^{\frac{\pi}{6}} = \frac{1}{12}(\cos\pi - \cos 0) -$$

$$-\frac{1}{10}\left(\cos \frac{5\pi}{3} - \cos 0\right) =$$

$$= \frac{1}{12}(-1-1) - \frac{1}{10}\left(\frac{1}{2}-1\right) =$$

$$= -\frac{1}{6} + \frac{1}{40} = -\frac{17}{120}$$

№ 9. 1.17

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{dx}{1-\cos 6x} = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{dx}{\frac{1-\cos 6x}{2}} =$$

$$= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{dx}{\sin^2 3x} = \frac{1}{6}\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{d(3x)}{\sin^2 3x} =$$

$$= -\frac{1}{6}\operatorname{ctg} 3x \Big|_{\frac{\pi}{6}}^{\frac{\pi}{4}} = -\frac{1}{6}\left(\operatorname{ctg}\frac{3\pi}{4} -\right.$$

$$\left.-\operatorname{ctg}\frac{\pi}{2}\right) = -\frac{1}{6}(-1-0) = \frac{1}{6}$$

№ 9 1.18

$$\int_{\frac{1}{\pi}}^{\frac{2}{\pi}} \frac{\sin\frac{1}{x}}{x^2}dx = \left[t = \frac{1}{x};\ -dt = \frac{dx}{x^2}\right] =$$

$$= -\int_{\frac{1}{\pi}}^{\frac{2}{\pi}} \sin t \, dt = \cos t \Big|_{\frac{1}{\pi}}^{\frac{2}{\pi}} =$$

$$= \cos \frac{1}{x} \Big|_{\frac{1}{\pi}}^{\frac{2}{\pi}} =$$

$$= \cos \frac{\pi}{2} - \cos \pi = 0 + 1 = 1$$

№ 9.1.19

$$\int_0^{\frac{\pi}{2}} \cos^3 \varphi \, d\varphi = \int_0^{\frac{\pi}{2}} (1 - \sin^2 \varphi) \cos \varphi \, d\varphi =$$

$$= [t = \sin \varphi ; \; dt = \cos \varphi \, d\varphi] =$$

$$= \int_0^{\frac{\pi}{2}} (1 - t^2) \, dt = t \Big|_0^{\frac{\pi}{2}} - \frac{t^3}{3} \Big|_0^{\frac{\pi}{2}} =$$

$$= \sin \varphi \Big|_0^{\frac{\pi}{2}} - \frac{\sin^3 \varphi}{3} \Big|_0^{\frac{\pi}{2}} =$$

$$= \sin \frac{\pi}{2} - \sin 0 - \frac{\sin^3 \frac{\pi}{2}}{3} + \frac{\sin^3 0}{3} =$$

$$= 1 - 0 - \frac{1}{3} + 0 = \frac{2}{3}$$

№ 9.1.21

$$\int_1^3 \frac{dx}{x^2 + x} = \int_1^3 \frac{dx}{x(x+1)} = \Big[\frac{1}{x(x+1)} =$$

$$= \frac{A}{x} + \frac{B}{x+1} ; \; \Big\{ 1 = Ax + A + Bx ;$$

$$\left.\begin{cases} A+B=0 \\ A=1 \end{cases} \Rightarrow B=-1 \right] =$$

$$= \int_1^3 \left(\frac{1}{x} - \frac{1}{x+1}\right) dx = \ln|x|\Big|_1^3 -$$

$$- \ln|x+1|\Big|_1^3 = \ln 3 - \ln 1 - \ln 4 + \ln 2 =$$

$$= \ln \frac{3 \cdot 2}{4} = \ln \frac{3}{2}$$

<u>w 9.1.22</u>

$$\int_1^3 \frac{dx}{x^3+x} = \int_1^3 \frac{dx}{x(x^2+1)} = \left[\frac{1}{x(x^2+1)} = \right.$$

$$= \frac{A}{x} + \frac{Bx+C}{x^2+1} \Rightarrow 1 = Ax^2 + A$$

$$+ Bx^2 + Cx = x^2(A+B) + Cx + A \Rightarrow$$

$$\left. \Rightarrow \begin{cases} A+B=0 \\ C=0 \\ A=1 \end{cases} \Rightarrow \begin{matrix} B=-1 \\ C=0 \\ A=1 \end{matrix} \right] =$$

$$= \int_1^3 \left(\frac{1}{x} - \frac{x}{x^2+1}\right) dx =$$

$$= \int_1^3 \frac{dx}{x} - \int_1^3 \frac{x\,dx}{x^2+1} = \ln|x|\Big|_1^3 -$$

$$- \ln|x^2+1|\Big|_1^3 = \ln 3 - \ln 1 - \ln 10 +$$

$$+ \ln 2 = \ln\frac{3\cdot 2}{10} = \ln 0{,}6 = \ln\frac{3}{5}$$

№ 9.1.23

$$\int_3^5 \frac{x^2+5}{x-2}\,dx = \int_3^5 \frac{(x-2)(x+2)+9}{x-2}\,dx =$$

$$= \int_3^5 (x+2)\,dx + 9\int_3^5 \frac{dx}{x-2} =$$

$$= \int_3^5 x\,dx + 2\int_3^5 dx + 9\int_3^5 \frac{d(x-2)}{x-2} =$$

$$= \frac{x^2}{2}\Big|_3^5 + 2x\Big|_3^5 + 9\ln|x-2|\Big|_3^5 =$$

$$= \frac{25}{2} - \frac{9}{2} + 10 - 6 + 9\ln 3 - 9\ln 1 =$$

$$= 12 + 9\ln 3$$

№ 9.1.24

$$\int_1^3 \frac{dx}{x^2+6x+10} = [x^2+6x+10=0 \Rightarrow$$

$\Rightarrow D = 36 - 4 \cdot 10 = -4 \Rightarrow$ корней нет $\Rightarrow$

$\Rightarrow y = x+3;\ dy = dx;\ a = \sqrt{10 - \frac{36}{4}} =$

$= 1] = \int_1^3 \frac{dy}{y^2+1} = \operatorname{arctg} y \Big|_1^3 =$

$= \operatorname{arctg}(x+3) \Big|_1^3 = \operatorname{arctg} 6 - \operatorname{arctg} 4 =$

$= \operatorname{arctg} \frac{2}{25} = \operatorname{arctg} 0{,}08$

<u>№ 9.1.25</u>

$$\int_2^3 \frac{x^2+1}{x^3-x}\,dx = \int_2^3 \frac{x^2+1}{x(x^2-1)(x+1)}\,dx =$$

$$= \int_2^3 \frac{x^2\,dx}{x(x^2-1)} + \int_2^3 \frac{dx}{x(x-1)(x+1)} =$$

$$= \int_2^3 \frac{x\,dx}{x^2-1} + \int_2^3 \frac{dx}{x(x-1)(x+1)} =$$

$$= [1) \frac{1}{x(x-1)(x+1)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+1} \Rightarrow$$

$$\Rightarrow 1 = Ax^2 - A + Bx^2 + Bx + Cx^2 - Cx =$$

$$= x^2(A+B+C) + x(B-C) - A \Rightarrow$$

$$\Rightarrow \begin{cases} A+B+C=0 \\ B-C=0 \\ -A=1 \end{cases} \Rightarrow \left.\begin{cases} C=\frac{1}{2} \\ B=C \\ A=-1 \end{cases}\right] =$$

$$= \frac{1}{2}\int_2^3 \frac{d(x^2-1)}{x^2-1} - \int_2^3 \frac{dx}{x} + \frac{1}{2}\int_2^3 \frac{dx}{x-1} +$$

$$+ \frac{1}{2}\int_2^3 \frac{dx}{x+1} = \frac{1}{2}\ln|x^2-1|\Big|_2^3 - \ln|x|\Big|_2^3 +$$

$$+ \frac{1}{2}\ln|x-1|\Big|_2^3 + \frac{1}{2}\ln|x+1|\Big|_2^3 =$$

$$= \frac{1}{2}\ln 8 - \frac{1}{2}\ln 3 - \ln 3 + \ln 2 + \frac{1}{2}\ln 2 -$$

$$- \frac{1}{2}\ln 1 + \frac{1}{2}\ln 4 - \frac{1}{2}\ln 3 =$$

$$= \frac{1}{2}\ln 8 + \frac{1}{2}\ln 4 - 4\ln 3 + \frac{3}{2}\ln 2 =$$

$$= \ln 4 + \ln 2 - \ln 81 + \ln 2\sqrt{2} =$$

$$= \ln \frac{4\cdot 2\cdot 2\sqrt{2}}{81} = \ln \frac{16\sqrt{2}}{81}$$

№ 9.1.27

$$\int_1^{e^2} \frac{\ln^3 x}{3x}\,dx =$$

$= [t = \ln x;\ dt = \frac{dx}{x}] =$

$= \frac{1}{3}\int_1^{e^2} t^3\,dt = \frac{1}{3}\cdot\frac{t^4}{4}\Big|_1^{e^2} =$

$= \frac{1}{12}\ln^4 x\Big|_1^{e^2} = \frac{1}{12}\ln^4 e^2 - \frac{1}{12}\ln^4 1 =$

$= \frac{4}{3}$

<u>№ 9.1.28</u>

$\int_{\pi}^{2\pi} \frac{x+\cos x}{x^2+2\sin x}\,dx = [t = x^2 + 2\sin x;$

$dt = (x^2+2\sin x)'dx = 2(x+\cos x)dx \Rightarrow$

$\Rightarrow \frac{1}{2}dt = (x+\cos x)dx\ ] =$

$= \int_{\pi}^{2\pi} \frac{1}{2}\frac{dt}{t} = \frac{1}{2}\ln|t|\Big|_{\pi}^{2\pi} =$

$= \frac{1}{2}\ln|x^2+2\sin x|\Big|_{\pi}^{2\pi} =$

$= \frac{1}{2}\ln\left|\frac{4\pi^2+2\sin 2\pi}{\pi^2+2\sin\pi}\right| =$

$= \frac{1}{2}\ln\left|\frac{4\pi^2}{\pi^2}\right| = \frac{1}{2}\ln 4 = \ln 2$

№ 9.1.29

$$\int_0^1 \frac{4\operatorname{arctg}x - x}{1+x^2}\,dx =$$

$$= \int_0^1 \frac{4\operatorname{arctg}x}{1+x^2}\,dx - \int_0^1 \frac{x\,dx}{1+x^2} =$$

$= [1) t_1 = \operatorname{arctg}x;\ dt_1 = \frac{dx}{1+x^2};$

$2) t_2 = x^2+1;\ \frac{1}{2}dt_2 = x\,dx] =$

$$= \int_0^1 t_1\,dt_1 - \frac{1}{2}\int_0^1 \frac{dt_2}{t_2} = \frac{t_1^2}{2}\Big|_0^1 - \frac{1}{2}\ln|t_2|\Big|_0^1 =$$

$$= \frac{\operatorname{arctg}^2 x}{2}\Big|_0^1 - \frac{1}{2}\ln|x^2+1|\Big|_0^1 =$$

$$= \frac{\pi^2}{32} - 0 - \frac{1}{2}\ln 2 - \frac{1}{2}\ln 1 =$$

$$= \frac{\pi^2}{32} - \ln\sqrt{2}$$

№ 9.1.30

$$\int_1^e \frac{\sin\ln x}{x}\,dx = [t = \ln x;\ dt = \frac{dx}{x}] =$$

$$= \int_1^e \sin t\,dt = -\cos t\Big|_1^e = \cos\ln 1 -$$

$$-\cos \ln e = \cos 0 - \cos 1 = 1 - \cos 1$$

№ 9.1.31

$$\int_{-1}^{0} \frac{3^x - 2^x}{6^x}\,dx = \int_{-1}^{0} \frac{1}{2^x}\,dx - \int_{-1}^{0} \frac{1}{3^x}\,dx =$$

$$= -\int_{-1}^{0} 2^{-x}\,d(-x) + \int_{-1}^{0} 3^{-x}\,d(-x) =$$

$$= -\frac{2^{-x}}{\ln 2}\Big|_{-1}^{0} + \frac{3^{-x}}{\ln 3}\Big|_{-1}^{0} =$$

$$= \frac{1}{3^x \ln 3}\Big|_{-1}^{0} - \frac{1}{2^x \ln 2}\Big|_{-1}^{0} =$$

$$= \frac{1}{\ln 3} - \frac{3}{\ln 3} - \frac{1}{\ln 2} + \frac{2}{\ln 2} =$$

$$= -\frac{2}{\ln 3} + \frac{1}{\ln 2} = \frac{1}{\ln 2} - \frac{2}{\ln 3}$$

№ 9.1.32

$$\int_{0}^{1} \frac{x\,dx}{\sqrt{x^4 + x^2 + 1}} = \left[t = x^2;\ \tfrac{1}{2}dt = x\,dx \right] =$$

$$= \frac{1}{2}\int_{0}^{1} \frac{dt}{\sqrt{t^2 + t + 1}} = \left[y = t + \frac{1}{2};\ dt = dy; \right.$$

$$a = \sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2}\Big] = \frac{1}{2}\int_0^1 \frac{dy}{\sqrt{y^2+\left(\frac{\sqrt{3}}{2}\right)^2}} =$$

$$= \frac{1}{2}\int_0^1 \frac{dy}{\sqrt{y^2+\frac{3}{4}}} = \frac{1}{2}\ln\left|y+\sqrt{y^2+\frac{3}{4}}\right|\Bigg|_0^1 =$$

$$= \frac{1}{2}\ln\left|x^2+\frac{1}{2}+\sqrt{\left(x^2+\frac{1}{2}\right)^2+\frac{3}{4}}\right|\Bigg|_0^1 =$$

$$= \frac{1}{2}\ln\left|x^2+\frac{1}{2}+\sqrt{x^4+x^2+\frac{1}{4}+\frac{3}{4}}\right|\Bigg|_0^1 =$$

$$= \frac{1}{2}\ln\left|x^2+\frac{1}{2}+\sqrt{x^4+x^2+1}\right|\Bigg|_0^1 =$$

$$= \frac{1}{2}\ln\left|1+\frac{1}{2}+\sqrt{1+1+1}\right| - \frac{1}{2}\ln\left|0+\frac{1}{2}+\right.$$

$$\left.+\sqrt{0+0+1}\right| = \frac{1}{2}\ln\left(\frac{3}{2}+\sqrt{3}\right) - \frac{1}{2}\ln\frac{3}{2} =$$

$$= \frac{1}{2}\ln\left(\frac{3+2\sqrt{3}}{2}\right) - \frac{1}{2}\ln\frac{3}{2} =$$

$$= \frac{1}{2}\ln\left(\frac{3+2\sqrt{3}}{2}\cdot\frac{2}{3}\right) = \ln\sqrt{1+\frac{2}{\sqrt{3}}}$$

№ 9.1.33

$$\int_0^{\frac{\pi}{4}} tg^3 x\,dx = \int_0^{\frac{\pi}{4}}\left(\frac{1}{\cos^2 x}-1\right) tg x\,dx =$$

$$= \int_0^{\frac{\pi}{4}}\left(\frac{tg x}{\cos^2 x} - tg x\right) dx =$$

$$= \int_0^{\frac{\pi}{4}} \frac{\sin x}{\cos^3 x}\,dx - \int_0^{\frac{\pi}{4}} \operatorname{tg} x\,dx =$$

$$= [\cos x = t, -dt = \sin x\,dx] =$$

$$= -\int_0^{\frac{\pi}{4}} \frac{dt}{t^3} - \int_0^{\frac{\pi}{4}} \operatorname{tg} x\,dx =$$

$$= \cancel{\ast}\frac{1}{2t^{\cancel{2}}}\Big|_0^{\frac{\pi}{4}} + \ln|\cos x|\Big|_0^{\frac{\pi}{4}} =$$

$$= \frac{1}{2\cos^2 x}\Big|_0^{\frac{\pi}{4}} + \ln|\cos x|\Big|_0^{\frac{\pi}{4}} =$$

$$= \frac{1}{2\cos^2\frac{\pi}{4}} - \frac{1}{2\cos^2 0} + \ln|\cos\frac{\pi}{4}| -$$

$$- \ln|\cos 0| = \frac{1}{2\cdot\frac{1}{2}} - \frac{1}{2\cdot 1} +$$

$$+ \ln\frac{\sqrt{2}}{2} - \ln 1 = \frac{1}{2} + \ln\frac{\sqrt{2}}{2} = \frac{1}{2} - \frac{1}{2}\ln 2 =$$

$$= \frac{1}{2}(1 - \ln 2)$$

№ 9.1.34

$$\int_0^{\frac{\pi}{3}} \operatorname{tg} x \cdot \ln\cos x\,dx = [\cos x = t;$$

$$-dt = \sin x\,dx] = \int_0^{\frac{\pi}{3}} \frac{\ln t}{t}\,dt =$$

$$= -\int_0^{\frac{\pi}{3}} \ln t \, d(\ln t) = -\frac{1}{2}\ln^2 t \Big|_0^{\frac{\pi}{3}} =$$

$$= -\frac{1}{2}\ln^2 \cos x \Big|_0^{\frac{\pi}{3}} = -\frac{1}{2}\ln^2 \cos\frac{\pi}{3} +$$

$$+ \frac{1}{2}\ln^2 \cos 0 = -\frac{1}{2}\ln^2\frac{1}{2} + \frac{1}{2}\ln^2 1 =$$

$$= -\frac{1}{2}\ln^2 2$$

№ 91.35

$$\cancel{\int_0^2 \frac{x\,dx}{\sqrt{x+2}+\sqrt{3x+2}}} = \cancel{\int_0^2 \frac{x+2-2}{\sqrt{x+2}+\sqrt{3x+2}}}$$

$$\cancel{\int_0^2 \sqrt{x+2}\,dx + 2\int \frac{dx}{\sqrt{x+2}+\sqrt{3x+2}}}$$

$$= \left[\, 3x+2 = t^2;\ \frac{2}{3}t\,dt = dx;\ x = \frac{t^2-2}{3} \right] =$$

$$= \int_0^2 \frac{\frac{1}{3}(t^2-2)\cdot\frac{2}{3}t\,dt}{\sqrt{\frac{t^2-2}{3}+2}+\sqrt{t^2}} = \frac{2}{9}\int_0^2 \frac{t^3-2t}{\frac{\sqrt{t^2+4}}{\sqrt{3}}+t}\,dt =$$

$$= \frac{2}{9}\int_0^2 \frac{\sqrt{3}\,(t^3-2t)}{\sqrt{t^2+4}+\sqrt{3}\,t}\,dt = \frac{2\sqrt{3}}{9}\int_0^2 \frac{t^3-2t}{\sqrt{t^2+4}+t\sqrt{3}}$$

$$= \frac{2\sqrt{3}}{9}\int_0^2 \frac{\sqrt{y^2-4}\,(y^2-4-2)}{y+\sqrt{y^2-4}\,\sqrt{3}} \cdot \frac{2y\,dy}{\sqrt{y^2-4}} =$$

$$= \frac{4\sqrt{3}}{9}\int_0^2 \frac{y(y^2-6)}{3y+\sqrt{3y^2-12}}\,dy$$

w 9.1.35

$$\int_0^2 \frac{x\,dx}{\sqrt{x+2}+\sqrt{3x+2}} = \int_0^2 \frac{x(\sqrt{x+2}+\sqrt{3x+2})}{4x+4+2\sqrt{3x^2+8x+4}}dx =$$

$$=$$

w 9.1.47

$$\int_0^{\ln 2} \frac{dz}{e^z+1} = [e^z = t;\ z = \ln(t);$$

$$dz = \frac{dt}{t}] = \int_0^{\ln 2} \frac{dt}{t(t+1)} =$$

$$= [\frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1} \Rightarrow$$

$$1 = At + A + Bt \Rightarrow A = 1,\ B = -1] =$$

$$= \int_0^{\ln 2} \frac{dt}{t} - \int_0^{\ln 2} \frac{dt}{t+1} = \ln t\Big|_0^{\ln 2} -$$

$$-\ln(t+1)\Big|_0^{\ln 2} = \ln e^z\Big|_0^{\ln 2} - \ln(e^z+1)\Big|_0^{\ln 2} =$$

$$= \ln 2 - 0 - \ln 3 - \ln 2 = -\ln 3$$

№ 9.1.48

$$\int_{-1}^{1} \frac{x\,dx}{\sqrt{5-4x}} = \Big[\, 5-4x = t^2;\ x = \frac{-t^2+5}{4} =$$

$$= \frac{5-t^2}{4};\quad dx = \frac{1}{4}(5-t^2)'dt = -\frac{t}{2}dt \Big] =$$

$$= \int_{-1}^{1} \frac{5-t^2}{4}\cdot\left(-\frac{t\,dt}{2t}\right) = -\frac{1}{8}\int_{-1}^{1}(5-t^2)dt =$$

$$= -\frac{5}{8}\int_{-1}^{1} dt + \frac{1}{8}\int_{-1}^{1} t^2\,dt = \frac{1}{8}\cdot\frac{t^3}{3}\Big|_{-1}^{1} - \frac{5}{8}t\Big|_{-1}^{1} =$$

$$= \frac{1}{24}(5-4x)^3\Big|_{-1}^{1} - \frac{5}{8}(5-4x)\Big|_{-1}^{1} =$$

$$= \frac{1}{24}(5-4\cdot 1)^3 - \frac{1}{24}(5-4\cdot(-1))^3 -$$

$$- \frac{5}{8}(5-4) + \frac{5}{8}(5-4\cdot(-1)) =$$

$$= \frac{1}{24} - \frac{729}{24} - \frac{5}{8} + \frac{45}{8} = -\frac{728}{24} + \frac{40}{8} =$$

$$= -\frac{91}{3} + 5 = -\frac{76}{3}$$

№ 9.1.49

$$\int_1^{16}\frac{dx}{x+\sqrt[4]{x}} = [t^4 = x;\ dx = 4t^3dt] =$$

$$= \int_1^{16}\frac{4t^3\,dt}{t^4+t} = 4\int_1^{16}\frac{t^2dt}{t^3+1} =$$

$$= 4\int_1^{16}\frac{d(t^3+1)}{t^3+1} = 4\ln|t^3+1|\Big|_1^{16} =$$

$$= 4\ln|\sqrt[4]{x^3}+1|\Big|_1^{16} = 4\ln|\sqrt[4]{16^3}+1| -$$

$$-4\ln|\sqrt[4]{1^3}+1| = 8\ln 3 - 4\ln 2 =$$

$$= 4\ln\frac{9}{2}$$

№ 9.1.50

$$\int_{-1}^{7}\frac{dx}{1+\sqrt[3]{x+1}} = [t^3 = x+1;\ x = t^3-1;$$

$$dx = 3t^2dt] = \int_{-1}^{7}\frac{3t^2dt}{1+t} =$$

$$= 3\int_{-1}^{7}\frac{t^2-1+1}{t+1}dt = 3\int_{-1}^{7}\frac{(t-1)(t+1)}{t+1}dt +$$

$$+3\int_{-1}^{7}\frac{dt}{t+1} = 3\int_{-1}^{7}t\,dt - 3\int_{-1}^{7}dt + 3\int\frac{dt}{t+1} =$$

$= 3t^2\Big|_{-1}^{7} - 3t\Big|_{-1}^{7} + 3\ln|t+1|\Big|_{-1}^{7} \circeq$

$\cancel{= 3\cdot 7^2 - 3\cdot(-1)^2 - 3\cdot 7 + 3\cdot(-1) +}$

$\cancel{+ 3\ln 8 - 3\ln 0}$

$\circeq 3\sqrt[3]{(x+1)^2}\Big|_{-1}^{7} - 3\sqrt[3]{(x+1)}\Big|_{-1}^{7} + 3\ln|\sqrt[3]{x+1}+1|\Big|_{-1}^{7} =$

$= 3\cdot\sqrt[3]{864} - 3\sqrt[3]{0} - 3\sqrt[3]{8} + 3\sqrt[3]{0} +$

$+ 3\ln|\sqrt[3]{8}+1| - 3\ln|\sqrt[3]{0}+1| =$

$= 12 - 6 + 3\ln 3 = 6 + 3\ln 3$

<u>№9.1.53</u>

$\int_{\frac{1}{2}}^{1} x\sqrt[4]{2x-1}\,dx = [\,2x-1 = t^4;$

$x = \frac{1}{2}(t^4+1);\ dx = \frac{1}{2}\cdot 4t^3 dt = 2t^3 dt\,] =$

$= \int_{\frac{1}{2}}^{1} \frac{1}{2}(t^4+1)\cdot 2t^3 dt = \int_{\frac{1}{2}}^{1} (t^7+t^3)dt =$

$= \int_{\frac{1}{2}}^{1} t^7 dt + \int_{\frac{1}{2}}^{1} t^3 dt = \frac{1}{8}t^8\Big|_{\frac{1}{2}}^{1} +$

$+ \frac{1}{4}t^4\Big|_{\frac{1}{2}}^{1} = \frac{1}{8}(2x-1)^2\Big|_{\frac{1}{2}}^{1} + \frac{1}{4}(2x-1)\Big|_{\frac{1}{2}}^{1} =$

$= \frac{1}{8} + \frac{1}{4} = \frac{3}{8}$

w 9.1.54

$$\int_1^9 \frac{x\,dx}{\sqrt{2x+7}} = [\,2x+7=t^2;\ x=\frac{1}{2}(t^2-7)$$

$$dx=\frac{1}{2}\cdot 2t\,dt = t\,dt\ ] =$$

$$=\int_1^9 \frac{1}{2}\,\frac{(t^2-7)\cancel{t}\,dt}{\cancel{t}} = \frac{1}{2}\int_1^9 t^2dt -$$

$$-\frac{7}{2}\int_1^9 dt = \frac{1}{2}\cdot\frac{t^3}{3}\Big|_1^9 - \frac{7}{2}t\Big|_1^9 =$$

$$=\frac{1}{6}\left(\sqrt{2x+7}\right)^3\Big|_1^9 - \frac{7}{2}\left(\sqrt{2x+7}\right)\Big|_1^9 =$$

$$\cancel{= \frac{15625}{6} - \frac{729}{6} - \frac{175}{2} + \frac{63}{2} =}$$

$$=\frac{125}{6}-\frac{27}{6}-\frac{35}{2}+\frac{21}{2}=$$

$$=\frac{98}{6}-\frac{14}{2}=\frac{56}{6}=\frac{28}{3}$$

w 9.1.55

$$\int_{-0,4}^{0} (2+5x)^4dx = [\,t=2+5x;\ dx=\frac{1}{5}dt\,] =$$

$$=\frac{1}{5}\int_{-0,4}^{0} t^4\,dt = \frac{1}{5}\cdot\frac{t^5}{5}\Big|_{-0,4}^{0} =$$

$$=\frac{(2+5x)^5}{25}\Big|_{-0,4}^{0} = \frac{32}{25} - 0 = \frac{32}{25} = 1,28$$

№ 9.1.56

$$\int_0^{\pi} \sin\left(\frac{5}{4}x - \frac{\pi}{4}\right)dx = \left[t = \frac{5}{4}x - \frac{\pi}{4};\ \frac{4}{5}dt = dx\right] = \frac{4}{5}\int_0^{\pi} \sin t\,dt =$$

$$= -\frac{4}{5}\cos t\Big|_0^{\pi} = -\frac{4}{5}\cos\left(\frac{5}{4}x - \frac{\pi}{4}\right)\Big|_0^{\frac{\pi}{4}} =$$

$$= -\frac{4}{5}\cdot(-1) + \frac{4}{5}\cdot\frac{\sqrt{2}}{2} = \frac{4+2\sqrt{2}}{5}$$

№ 9.1.57

$$\int_0^1 \frac{x^2dx}{(x+1)^3} = [t = x+1;\ dx = dt] =$$

$$= \int_0^1 \frac{(t-1)^2dt}{t^3} = \int_0^1 \frac{t^2-2t+1}{t^3}dt =$$

$$= \int_0^1 \frac{dt}{t} - 2\int_0^1 \frac{dt}{t^2} + \int_0^1 \frac{dt}{t^3} =$$

$$= \ln|t|\Big|_0^1 - 2\cdot\left(-\frac{1}{t}\right)\Big|_0^1 \cancel{+} - \frac{1}{2t^2}\Big|_0^1 =$$

$$= \ln|x+1|\Big|_0^1 + \frac{2}{x+1}\Big|_0^1 - \frac{1}{2x+2}\Big|_0^1 =$$

$$= \ln 2 - \ln 1 + 1 - 2 - \frac{1}{4} + \frac{1}{2} = \ln 2 - \frac{3}{4}$$

№9.1.58

$$\int_0^3 x^2\sqrt{9-x^2}\,dx = \left[x=3\sin t;\ dx=3\cos t\,dt;\ t=\arcsin\frac{x}{3}\right]=$$

$$=\int_0^3 9\sin^2 t\sqrt{9-9\sin^2 t}\cdot 3\cos t\,dt=$$

$$=\int_0^3 27\sin^2 t\sqrt{\cos^2 t}\cdot 3\cos t\,dt=$$

$$=81\int_0^3 \sin^2 t\cos^2 t\,dt=$$

$$=81\int_0^3 \frac{1-\cos 2t}{2}\cdot\frac{1+\cos 2t}{2}\,dt=$$

$$=\frac{81}{4}\int_0^3 (1-\cos^2 2t)\,dt=$$

$$=\frac{81}{4}\int_0^3 \sin^2 2t\,dt=\frac{81}{4}\int_0^3 \frac{1-\cos 4t}{2}\,dt=$$

$$=\frac{81}{8}\int_0^3 (1-\cos 4t)\,dt=\frac{81}{8}\int_0^3 dt-$$

$$-\frac{81}{32}\int_0^3 4\cos 4t\,dt=\frac{81}{8}t\Big|_0^3-\frac{81}{32}\sin 4t\Big|_0^3=$$

$$=\frac{81}{8}\arcsin\frac{x}{3}\Big|_0^3-\frac{81}{32}\sin 4\left(\arcsin\frac{x}{3}\right)\Big|_0^3=$$

$$= \frac{81}{8}\cdot\frac{\pi}{2} - \frac{81}{8}\cdot 0 - \frac{81}{32}\cdot 0 + \frac{81}{32}\cdot 0 =$$

$$= \frac{81\pi}{16}$$

<u>№9.1.60</u>

$$\int_{\frac{3}{4}}^{1} \frac{2dx}{x\sqrt{x^2+1}} = \Big[t^2 = x^2+1;\ x = \sqrt{t^2-1},$$

$$dx = \frac{1}{2\sqrt{t^2-1}}\cdot 2t\,dt = \frac{t\,dt}{\sqrt{t^2-1}} \Big] =$$

$$= \int_{\frac{3}{4}}^{1} \frac{2t\,dt}{\sqrt{t^2-1}\,t\,\sqrt{t^2-1}} = 2\int_{\frac{3}{4}}^{1} \frac{dt}{t^2-1} =$$

$$= 2\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\Bigg|_{\frac{3}{4}}^{1} = \ln\left|\frac{\sqrt{x^2+1}-1}{\sqrt{x^2+1}+1}\right|\Bigg|_{\frac{3}{4}}^{1} =$$

$$= \ln\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right| - \ln\left|\frac{\frac{5}{4}-1}{\frac{5}{4}+1}\right| =$$

$$= \ln\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right| - \ln\left|\frac{\frac{1}{4}}{\frac{9}{4}}\right| =$$

$$= \ln\left|\frac{\sqrt{2}-1}{\sqrt{2}+1}\right| - \ln\left|\frac{1}{9}\right| = \ln\left|9\cdot\frac{\sqrt{2}-1}{\sqrt{2}+1}\right| =$$

$$= \ln\left(27-18\sqrt{2}\right)$$

w9.1.62

$$\int_0^{\sqrt{3}} \frac{dx}{(1+x^2)^3} = [\cancel{t = 1+x^3;\ x = \sqrt{t-1}}$$

$$\cancel{dx = \frac{dt}{2\sqrt{t-1}}} \quad [\text{понижение степ.}] =$$

$$= \frac{1}{4} \cdot \frac{x}{(x^2+1)^2}\Big|_0^{\sqrt{3}} + \frac{3}{4}\int_0^{\sqrt{3}} \frac{dx}{(x^2+1)^2} =$$

$$= \frac{x}{4(x^2+1)^2}\Big|_0^{\sqrt{3}} + \frac{3}{4} \cdot \frac{1}{2} \cdot \frac{x}{x^2+1}\Big|_0^{\sqrt{3}} +$$

$$+ \frac{3}{4} \cdot \frac{1}{2} \int_0^{\sqrt{3}} \frac{dx}{x^2+1} = \frac{x}{4(x^2+1)^2}\Big|_0^{\sqrt{3}} + \frac{3x}{8(x^2+1)}\Big|_0^{\sqrt{3}} +$$

$$+ \frac{3}{8} \operatorname{arctg} x \Big|_0^{\sqrt{3}} =$$

$$= \frac{\sqrt{3}}{4 \cdot 16} + \frac{3\sqrt{3}}{8 \cdot 4} + \frac{3}{8} \cdot \frac{\pi}{3} =$$

$$= \frac{7\sqrt{3}}{64} + \frac{\pi}{8}$$

w9.1.63

$$\int_0^1 \sqrt{\frac{1-x}{1+x}}\, dx = [\cancel{x = \cos t;\ t = \arccos x;}$$

$$\cancel{dx = -\sin t\, dt\,] = -\int_0^1 \sqrt{\frac{1-\cos t}{1+\cos t}}}$$

$\cdot \cancel{\sin t\,dt} = \left[\frac{1-x}{1+x} = t^2;\ x = \frac{1-t^2}{1+t^2};\right.$

$\left. dx = \frac{-4t}{(1+t^2)^2}\,dt;\ t = \sqrt{\frac{1-x}{1+x}} \right] =$

$= \int_0^1 t^2 \cdot \frac{-4t}{(1+t^2)^2}\,dt = -4\int_0^1 \frac{t^2\,dt}{(1+t^2)^2} =$

$= -4\int_0^1 \frac{t^2+1-1}{(t^2+1)^2}\,dt = -4\int_0^1 \frac{dt}{t^2+1} +$

$+ 4\int_0^1 \frac{dt}{(t^2+1)^2} = \left[2)\ \frac{1}{2}\cdot\frac{t}{t^2+1} + \frac{1}{2}\,\mathrm{arctg}\,t \right] =$

$= -4\,\mathrm{arctg}\,t\Big|_0^1 + \frac{2t}{t^2+1}\Big|_0^1 + 2\,\mathrm{arctg}\,t\Big|_0^1 =$

$= \frac{2t}{t^2+1}\Big|_0^1 - 2\,\mathrm{arctg}\,t\Big|_0^1 = 1 - \frac{\pi}{2}$

№ 9.1.64

$\int_1^{\sqrt{2}} \sqrt{2-x^2}\,dx = \left[x = \sqrt{2}\cos t;\ t = \arccos\frac{x}{\sqrt{2}};\right.$

$\left. dx = -\sqrt{2}\sin t\,dt \right] = \int^{\sqrt{2}} -\sqrt{2}\sin t\,\sqrt{2-2\cos^2 t}\,dt =$

$= -\int_1^{\sqrt{2}} 2\sin t\,\sqrt{\sin^2 t}\,dt = -2\int_1^{\sqrt{2}} \sin^2 t\,dt =$

$$= -2\int_1^{\sqrt{2}} \frac{1-\cos 2t}{2}\,dt = -\int_1^{\sqrt{2}} (1-\cos 2t)\,dt =$$

$$= -\int_1^{\sqrt{2}} dt + \frac{1}{2}\int_1^{\sqrt{2}} 2\cos 2t\,dt =$$

$$= \frac{1}{2}\sin 2t\Big|_1^{\sqrt{2}} - t\Big|_1^{\sqrt{2}} =$$

$$= \frac{1}{2}\sin 2\left(\arccos\frac{x}{\sqrt{2}}\right)\Big|_1^{\sqrt{2}} -$$

$$- \arccos\frac{x}{\sqrt{2}}\Big|_1^{\sqrt{2}} = \frac{1}{2}\cdot 0 - \frac{1}{2}\cdot 1 -$$

$$- 0 + \frac{\pi}{4} = \frac{\pi}{4} - \frac{1}{2}$$

<u>№ 9.1.65</u>

$$\int_0^{\frac{\pi}{4}} tg^3 x\,dx = \int_0^{\frac{\pi}{4}} \left(\frac{1}{\cos^2 x} - 1\right)\frac{\sin x}{\cos x}\,dx =$$

$$= \int_0^{\frac{\pi}{4}} \left(\frac{\sin x}{\cos^3 x} - \frac{\sin x}{\cos x}\right)dx =$$

$$= [t = \cos x;\ x = \arccos t;$$

$$-dt = \sin x\,dx\,] = \int_0^{\frac{\pi}{4}} \frac{dt}{t} - \int_0^{\frac{\pi}{4}} \frac{dt}{t^3} =$$

$$= \ln|t|\Big|_0^{\frac{\pi}{4}} + \frac{1}{2t^2}\Big|_0^{\frac{\pi}{4}} =$$

$$= \ln\cos x\Big|_0^{\frac{\pi}{4}} + \frac{1}{2\cos^2 x}\Big|_0^{\frac{\pi}{4}} =$$

$$= -\ln\sqrt{2} - \ln 1 + 1 - \frac{1}{2} =$$

$$= \frac{1}{2} - \frac{1}{2}\ln 2$$

<u>№ 9.1.6~~6~~</u>

$$\int_0^{\frac{\pi}{2}} \frac{5dx}{1+\cos x} = [t = \cos x + 1;\ x = \arccos(t-1)$$

$$dx = -\frac{dt}{\sqrt{1-(t-1)^2}} = -\frac{dt}{\sqrt{1-t^2+2t-1}} =$$

$$= -\frac{dt}{\sqrt{2t-t^2}}] = -5\int_0^{\frac{\pi}{2}} \frac{dt}{t\sqrt{t}\sqrt{2-t}} =$$

$$= -5\int_0^{\frac{\pi}{2}} t^{-\frac{3}{2}}(2-t)^{-\frac{1}{2}}\,dt = [m = -\frac{3}{2},$$

$$n = 1,\ p = -\frac{1}{2} \notin \mathbb{Z} \Rightarrow \nexists\ 2)\ \frac{m+1}{n} =$$

$$= \frac{-\frac{3}{2}+1}{1} = -\frac{1}{2} \notin \mathbb{Z} \Rightarrow \nexists\ 3)\ \frac{m+1}{n} + p =$$

$$= -\frac{1}{2} - \frac{1}{2} = -1 \in \mathbb{Z} \Rightarrow 2\cdot t^{-1} - 1 = y^2;$$

$$y^2 = \frac{2}{t} - 1;\ t = \frac{2}{y^2+1};\ dt = -\frac{4y\,dy}{(y^2+1)^2}] =$$

$$= -5\int_0^{\frac{\pi}{4}} \frac{(y^2+1)\sqrt{y^2+1}}{2\sqrt{2}} \cdot \frac{1}{\sqrt{2-\frac{2}{y^2+1}}} \cdot \left(-\frac{4y\,dy}{(y^2+1)^2}\right)$$

$$= -5\int_0^{\frac{\pi}{2}} \frac{\cancel{(y^2+1)\sqrt{y^2+1}}}{\cancel{2\sqrt{2}}} \cdot \frac{\cancel{\sqrt{2}\sqrt{y^2+1}}}{\cancel{y}} \cdot$$

$$\cdot\left(-\frac{\cancel{4}\cancel{y}\,dy}{\cancel{(y^2+1)^2}}\right) = 5\int_0^{\frac{\pi}{2}} dy =$$

$$= 5y\Big|_0^{\frac{\pi}{2}} = 5\sqrt{\frac{2}{t}-1}\,\Big|_0^{\frac{\pi}{2}} =$$

$$= 5\sqrt{\frac{2}{\cos x+1}-1}\,\Big|_0^{\frac{\pi}{2}} =$$

$$= 5 - 0 = 5$$

<u>№ 91.67</u>

$$\int_0^{\ln 4} \sqrt{e^x-1}\,dx = \Big[\, t^2 = e^x - 1\,;$$

$$x = \ln(t^2+1)\,;\ dx = \frac{dt}{t^2+1}\Big] =$$

$$= \int_0^{\ln 4} \frac{t\,dt}{t^2+1} = \frac{1}{2}\int_0^{\ln 4} \frac{d(t^2+1)}{t^2+1} =$$

$$= \frac{1}{2}\ln|t^2+1|\,\Big|_0^{\ln 4} = \frac{1}{2}x\Big|_0^{\ln 4} =$$

$$= \frac{1}{2}\ln 4 = \ln 2$$

№ 9.1.68

$$\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos x+\sin x} = \left[\cancel{\sin x}\ t = tg\frac{x}{2};\right.$$

$$\sin x = \frac{2t}{1+t^2}; \quad \cos x = \frac{1-t^2}{1+t^2};$$

$$\left. dx = \frac{2dt}{1+t^2} \right] = \int_0^{\frac{\pi}{2}} \frac{1}{1+\frac{1-t^2}{1+t^2}+\frac{2t}{1+t^2}} \cdot \frac{2dt}{1+t^2} =$$

$$= \int_0^{\frac{\pi}{2}} \frac{1+t^2}{2(t+1)} \cdot \frac{2dt}{1+t^2} = \int_0^{\frac{\pi}{2}} \frac{dt}{t+1} =$$

$$= \ln|t+1| \Big|_0^{\frac{\pi}{2}} = \ln\left|tg\frac{x}{2}+1\right| \Big|_0^{\frac{\pi}{2}} =$$

$$= \ln 2 - \ln 1 = \ln 2$$

№ 9.1.69

$$\int_0^{\frac{\pi}{2}} \frac{\cos x\,dx}{6-5\sin x+\sin^2 x} = [t = \sin x;\ dt = \cos x\,dx]$$

$$= \int_0^{\frac{\pi}{2}} \frac{dt}{6-5t+t^2} = [t^2+5t-6=0;$$

$$D = 25-4\cdot 6 = 1 \Rightarrow t_1 = \frac{-5+1}{2} = -2;$$

$$t_2 = \frac{-5-1}{2} = -3] =$$

$$= \int_0^{\frac{\pi}{2}} \frac{dt}{(t+2)(t+3)} = \left[\frac{1}{(t+2)(t+3)} = \frac{A}{t+2} + \frac{B}{t+3} \Rightarrow \right.$$

$$\Rightarrow 1 = At + 3A + Bt + 2B;$$

$$\begin{cases} A + B = 0 \\ 3A + 2B = 1 \end{cases} \Rightarrow \begin{cases} A + \frac{1}{2}(1-3A) = 0 \\ B = \frac{1-3A}{2} \end{cases};$$

$$A + \frac{1}{2}(1-3A) = 0; \quad -\frac{1}{2}A + \frac{1}{2} = 0;$$

$$A = 1; \quad B = \frac{1-3}{2} = -1 \,] =$$

$$= \int_0^{\frac{\pi}{2}} \left(\frac{1}{t+2} - \frac{1}{t+3} \right) dt =$$

$$= \int_0^{\frac{\pi}{2}} \frac{dt}{t+2} - \int_0^{\frac{\pi}{2}} \frac{dt}{t+3} = \ln|t+2| \Big|_0^{\frac{\pi}{2}} -$$

$$- \ln|t+3| \Big|_0^{\frac{\pi}{2}} = \ln\left|\frac{t+2}{t+3}\right| \Big|_0^{\frac{\pi}{2}} =$$

$$= \ln\left|\frac{\sin x + 2}{\sin x + 3}\right| \Big|_0^{\frac{\pi}{2}} = \ln\left|\frac{3}{4}\right| - \ln\left|\frac{2}{3}\right| =$$

$$= \ln\left|\frac{3}{4} \cdot \frac{3}{2}\right| = \ln\left|\frac{9}{8}\right| =$$

$$= 2\ln\frac{3}{2\sqrt{2}}$$

№ 9.1.70

$$\int_1^2 3x(1-x)^{17} dx = [t = 1-x; \; dx = -dt] =$$

$$= \int_1^2 -3(1-t)t^{17} dt = -3\int_1^2 (t^{17} - t^{18}) dt =$$

$$= -3\int_1^c t^{17}dt + 3\int_1^2 t^{18}dt =$$

$$= -3\cdot\frac{t^{18}}{18}\Big|_1^2 + 3\cdot\frac{t^{19}}{19}\Big|_1^2 =$$

$$= -\frac{1}{6}(1-x)^{18}\Big|_1^2 + \frac{3}{19}(1-x)^{19}\Big|_1^2 =$$

$$= -\frac{1}{6} - 0 - \frac{3}{19} + 0 = -\frac{37}{114}$$

№ 9.1.80

$$\int_{\sqrt{2}}^{x}\frac{dt}{t\sqrt{t^2-1}} = \frac{\pi}{12}$$

$$\int_{\sqrt{2}}^{x}\frac{dt}{t\sqrt{t^2-1}} = \left[y^2 = t^2-1;\ t = \sqrt{y^2+1};\ dt = \frac{\cancel{2}y\,dy}{\cancel{2}\sqrt{y^2+1}} = \frac{y\,dy}{\sqrt{y^2+1}}\right] =$$

$$= \int_{\sqrt{2}}^{x}\frac{1}{\sqrt{y^2+1}\,\cancel{y}}\cdot\frac{\cancel{y}\,dy}{\sqrt{y^2+1}} = \int_{\sqrt{2}}^{x}\frac{dy}{y^2+1} =$$

$$= \operatorname{arctg} y\Big|_{\sqrt{2}}^{x} = \operatorname{arctg}\sqrt{t^2-1}\Big|_{\sqrt{2}}^{x} =$$

$$= \operatorname{arctg}\sqrt{x^2-1} - \operatorname{arctg}\sqrt{(\sqrt{2})^2-1} =$$

$$= \operatorname{arctg}\sqrt{x^2-1} - \frac{\pi}{4}$$

$\operatorname{arctg}\sqrt{x^2-1} - \frac{\pi}{4} = \frac{\pi}{12}$

$\operatorname{arctg}\sqrt{x^2-1} = \frac{\pi}{3}$

$\sqrt{x^2-1} = \sqrt{3}$

$x^2-1 = 3$

$x^2 = 4 \Rightarrow x = \pm 2$

<u>№ 9.1.87</u>

$$\int_{-1}^{0} xe^{-x}dx = \left[\begin{matrix} u = x \\ v' = e^{-x} \end{matrix} \Rightarrow \begin{matrix} u' = 1 \\ v = -e^{-x} \end{matrix}\right] =$$

$$= -xe^{-x}\Big|_{-1}^{0} - \int_{-1}^{0} -e^{-x}dx =$$

$$= -xe^{-x}\Big|_{-1}^{0} - e^{-x}\Big|_{-1}^{0} = -e^{-x}(x+1)\Big|_{-1}^{0} =$$

$$= -1 + 0 = -1$$

<u>№ 9.1.88</u>

$$\int_{0}^{2} \ln(x^2+4)dx = \left[\begin{matrix} u = \ln(x^2+4) \\ v' = 1 \end{matrix} \Rightarrow \right.$$

$$\left.\Rightarrow \begin{matrix} u' = \frac{2x}{x^2+4} \\ v = x \end{matrix}\right] = x\ln(x^2+4)\Big|_{0}^{2} -$$

$$- \int_{0}^{2} \frac{2x^2dx}{x^2+4} = \left[\int_{0}^{2} \frac{x^2dx}{x^2+4} = \right.$$

$$= \int_0^2 \frac{x^2+4-4}{x^2+4}\,dx = \int_0^2 dx - 4\int_0^2 \frac{dx}{x^2+4} =$$

$$= x\Big|_0^2 - 4\cdot\frac{1}{2}\,\mathrm{arctg}\,\frac{x}{2}\Big|_0^2 =$$

$$= x\Big|_0^2 - 2\,\mathrm{arctg}\,\frac{x}{2}\Big|_0^2] =$$

$$= \ln(x^2+4)\Big|_0^2 + x\Big|_0^2 - 2\,\mathrm{arctg}\,\frac{x}{2}\Big|_0^2 =$$

$$= \ln 8 - \ln 4 + 2 - 0 - \frac{\pi}{2} + 0 =$$

$$= \ln 2 + 2 - \frac{\pi}{2}$$

<u>№ 9.1.89</u>

$$\int_1^e \frac{\ln^3 x}{x^2}\,dx = \left[\begin{matrix} u = \ln^3 x & \Rightarrow & u' = 3\ln^2 x\cdot\frac{1}{x} \\ v' = \frac{1}{x^2} & & v = -\frac{1}{x}\end{matrix}\right] =$$

$$= -\frac{\ln^3 x}{x}\Big|_1^e + 3\int_1^e \frac{\ln^2 x}{x^2}\,dx =$$

$$= \left[\begin{matrix} u = \ln^2 x & \Rightarrow & u' = 2\ln x\cdot\frac{1}{x} \\ v' = \frac{1}{x^2} & & v = -\frac{1}{x}\end{matrix}\right] =$$

$$= -\frac{\ln^3 x}{x}\Big|_1^e - 3\,\frac{\ln^2 x}{x}\Big|_1^e + 6\int_1^e \frac{\ln x}{x^2}\,dx =$$

$$= \left[\begin{matrix} u = \ln x & \Rightarrow & u' = \frac{1}{x} \\ v' = \frac{1}{x^2} & & v = -\frac{1}{x}\end{matrix}\right] =$$

$$= -\frac{\ln^3 x}{x}\Big|_1^e - 3\frac{\ln^2 x}{x}\Big|_1^e -$$

$$-6\frac{\ln x}{x}\Big|_1^e + 6\int_1^e \frac{dx}{x^2} =$$

$$= -\frac{\ln^3 x}{x}\Big|_1^e - 3\frac{\ln^2 x}{x}\Big|_1^e - 6\frac{\ln x}{x}\Big|_1^e - \frac{6}{x}\Big|_1^e =$$

$$= -\frac{\ln^3 x + 3\ln^2 x + 6\ln x + 6}{x}\Big|_1^e =$$

$$= -\frac{16}{e} + 6 = 6 - \frac{16}{e}$$

w9.1.90

$$\int_{-1}^{0} 9x^2\ln(x+2)\,dx = \left[\begin{matrix} u = \ln(x+2) \\ v' = x^2 \end{matrix} \Rightarrow \begin{matrix} u' = \frac{1}{x+2} \\ v = \frac{x^3}{3} \end{matrix}\right] = 3x^3\ln(x+2)\Big|_{-1}^{0} -$$

$$-3\int_{-1}^{0}\frac{x^3\,dx}{x+2} = \left[\int_{-1}^{0}\frac{x^3+8-8}{x+2}\,dx =\right.$$

$$= \int_{-1}^{0}(x^2-2x+4)\,dx - 8\int_{-1}^{0}\frac{dx}{x+2} =$$

$$= \left(\frac{x^3}{3} - x^2 + 4x\right)\Big|_{-1}^{0} - 8\ln|x+2|\Big|_{-1}^{0}\Big] =$$

$= 3x^3 \ln(x+2)\Big|_{-1}^{0} - \left(\frac{x^3}{3} - x^2 + 4x\right)\Big|_{-1}^{0} +$

$+ 8\ln|x+2|\Big|_{-1}^{0} = 0 + 3\ln 1 - 0 + \frac{1}{3} + 1 +$

$+ 4 + 8\ln 2 - 8\ln 1 = \frac{16}{3} + 8\ln 2$

№ 9.1.92

$\int_0^1 4x \arcsin x\, dx = \left[\begin{array}{l} u = \arcsin x \\ v' = 4x \end{array}\right. \Rightarrow$

$\Rightarrow \left.\begin{array}{l} u' = \frac{1}{\sqrt{1-x^2}} \\ v = 2x^2 \end{array}\right] = 2x^2 \arcsin x\Big|_0^1 -$

$- \int_0^1 \frac{2x^2 dx}{\sqrt{1-x^2}} = \left[t = \sqrt{1-x^2};\ x = \sqrt{1-t^2};\right.$

$\left. dx = -\frac{2t dt}{2\sqrt{1-t^2}} = -\frac{t dt}{\sqrt{1-t^2}} \right] =$

$= 2x^2 \arcsin x\Big|_0^1 + 2\int_0^1 \frac{1-t^2}{t} \cdot \frac{t dt}{\sqrt{1-t^2}} =$

$= \pi + 2\int_0^1 \sqrt{1-t^2}\, dt = [t = \sin y;\ dt = \cos y\, dy] =$

$= \pi + 2\int_0^1 \sqrt{1-\sin^2 y}\, \cos y\, dy = \pi +$

$+ 2\int_0^1 \cos^2 y\, dy = \pi + \int_0^1 (1 + \cos 2y)\, dy =$

$$= \pi + \int_0^1 dy + \frac{1}{2}\int_0^1 2\cos 2y\,dy = \pi + y\Big|_0^1 +$$

$$+ \frac{1}{2}\sin 2y\Big|_0^1 = \pi + \arcsin t\Big|_0^1 +$$

$$+ \frac{1}{2}\sin 2(\arcsin t)\Big|_0^1 = \pi + \arcsin\sqrt{1-x^2}\Big|_0^1 +$$

$$+ \frac{1}{2}\sin 2(\arcsin\sqrt{1-x^2})\Big|_0^1 =$$

$$= \pi + 0 - \frac{\pi}{2} + 0 - 0 = \frac{\pi}{2}$$

№ 9.1.93

$$\int_0^1 (\arcsin x)^2 dx = \Big[\cancel{t = \arcsin x,}$$

$$\cancel{x = \sin t,\ dx = \cos t\,dt,}$$

$$u = (\arcsin x)^2 \qquad u' = 2\arcsin x \cdot \frac{1}{\sqrt{1-x^2}}$$

$$v' = 1 \quad \Rightarrow \quad v = x \Big] =$$

$$= x\arcsin^2 x\Big|_0^1 - \int_0^1 \frac{2\arcsin x \cdot x}{\sqrt{1-x^2}}\,dx =$$

$$= \Big[u = 2\arcsin x \qquad u' = \frac{2}{\sqrt{1-x^2}}$$

$$v' = \frac{x}{\sqrt{1-x^2}} \Rightarrow v = \int \frac{x\,dx}{\sqrt{1-x^2}} = -2\sqrt{1-x^2} \Big] =$$

$$= x\arcsin^2 x\Big|_0^1 + 4\arcsin x\sqrt{1-x^2}\Big|_0^1 -$$

$$\cancel{4} - \int_0^1 \frac{4\sqrt{1-x^2}}{\sqrt{1-x^2}}dx = x\arcsin^2 x\Big|_0^1 +$$

$$+4\arcsin x\sqrt{1-x^2}\Big|_0^1 - 4x\Big|_0^1 =$$

$$= \frac{\pi^2}{4} \cancel{*} - 0 + 0 - 0 - 4 + 0 = \frac{\pi^2}{4} - 4$$

<u>№ 9.1.94</u>

$$\int_0^1 \frac{x\,\mathrm{arctg}x}{\sqrt{1+x^2}}dx = \left[\begin{array}{l} u = \mathrm{arctg}x \\ v' = \frac{x}{\sqrt{1+x^2}} \end{array} \Rightarrow\right.$$

$$\Rightarrow \left.\begin{array}{l} u' = \frac{1}{1+x^2} \\ v = \int \frac{x\,dx}{\sqrt{1+x^2}} = +2\sqrt{1+x^2} \end{array}\right] =$$

$$= 2\sqrt{1+x^2}\,\mathrm{arctg}x\Big|_0^1 - \int_0^1 \frac{2\sqrt{1+x^2}}{1+x^2}dx =$$

$$= 2\sqrt{2}\cdot\frac{\pi}{4} - 2\cdot 0 - 2\int_0^1 \frac{dx}{\sqrt{1+x^2}} =$$

$$= \frac{\pi\sqrt{2}}{2} - 2\ln\left|x+\sqrt{x^2+1}\right|\Big|_0^1 =$$

$$= \frac{\pi\sqrt{2}}{2} - 2\ln\left|1+\sqrt{2}\right| + 2\ln 1 = \frac{\pi\sqrt{2}}{2} - 2\ln(1+\sqrt{2})$$

<u>№ 9.1.96</u>

$$\int_0^{\frac{\pi}{4}} x\cos^2 x\,dx = \left[\begin{array}{l} u = \cos^2 x \\ v' = x \end{array} \Rightarrow \begin{array}{l} u' = 2\cos x\sin x \\ v = \frac{1}{2}x^2 \end{array}\right] =$$

$$= \frac{1}{2}x^2\cos^2 x\Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}} x^2\cos x\sin x\,dx$$

$$= \left[\begin{matrix} u = x \\ v' = \cos^2 x \end{matrix} \Rightarrow \begin{matrix} u' = 1 \\ v = \int\cos^2 x\,dx = \end{matrix}\right.$$

$$= \frac{1}{2}\int(1+\cos 2x)\,dx = \frac{1}{2}\int dx + \frac{1}{4}\int 2\cos 2x\,dx =$$

$$= \frac{1}{2}x + \frac{1}{4}\sin 2x \ \Big] =$$

$$= x\left(\frac{1}{2}x + \frac{1}{4}\sin 2x\right)\Big|_0^{\frac{\pi}{4}} - \int_0^{\frac{\pi}{4}}\left(\frac{x}{2} +\right.$$

$$\left. + \frac{1}{4}\sin 2x\right)dx = \frac{\pi}{32}(2+\pi) - \int_0^{\frac{\pi}{4}}\frac{x}{2}\,dx -$$

$$- \frac{1}{4}\int_0^{\frac{\pi}{4}}\sin 2x\,dx = \frac{\pi}{16} + \frac{\pi^2}{32} - \frac{x^2}{4}\Big|_0^{\frac{\pi}{4}} +$$

$$+ \frac{1}{8}\cos 2x\Big|_0^{\frac{\pi}{4}} = \frac{\pi}{16} + \frac{\pi^2}{32} - \frac{\pi^2}{64} - \frac{1}{8} =$$

$$= \frac{\pi^2}{32} + \frac{\pi}{16} - \frac{1}{8} = \frac{1}{8}\left(\frac{\pi^2}{4} + \frac{\pi}{2} - 1\right)$$

<u>w9.1.97</u>

$$\int_0^{\frac{\pi}{2}} x^3\sin x\,dx = \left[\begin{matrix} u = x^3 \\ v' = \sin x \end{matrix} \Rightarrow \begin{matrix} u' = 3x^2 \\ v = -\cos x \end{matrix}\right] =$$

$$= -x^3\cos x\Big|_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} 3x^2\cos x\,dx =$$

$$= 0 + 3x^2\sin x\Big|_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} 6x\sin x\,dx =$$

$$= \frac{3\pi^2}{4} + 6x\cos x\Big|_0^{\frac{\pi}{2}} - 6\int_0^{\frac{\pi}{2}} \cos x\,dx =$$

$$= \frac{3\pi^2}{4} + 0 - 6\sin x\Big|_0^{\frac{\pi}{2}} = \frac{3\pi^2}{4} - 6$$

№ 9.1.98

$$\int_0^1 x^2 3^x\,dx = \left[\begin{matrix} u = x^2 \\ v' = 3^x \end{matrix} \Rightarrow \begin{matrix} u' = 2x \\ v = \int 3^x dx = \frac{3^x}{\ln 3} \end{matrix}\right] =$$

$$= \frac{3^x x^2}{\ln 3}\Big|_0^1 - \int_0^1 \frac{2x\,3^x}{\ln 3}\,dx = \left[\begin{matrix} u = x \\ v' = 3^x \end{matrix} \Rightarrow \begin{matrix} u' = 1 \\ v = \frac{3^x}{\ln 3} \end{matrix}\right] =$$

$$= \frac{3}{\ln 3} - \frac{2 \cdot 3^x x}{\ln^2 3}\Big|_0^1 + \frac{2}{\ln 3}\int_0^1 \frac{3^x}{\ln 3}\,dx =$$

$$= \frac{3}{\ln 3} - \frac{6}{\ln^2 3} + \frac{2 \cdot 3^x}{\ln^3 3}\Big|_0^1 =$$

$$= \frac{3}{\ln 3} - \frac{6}{\ln^2 3} + \frac{6}{\ln^3 3} - \frac{2}{\ln^3 3} =$$

$$= \frac{1}{\ln 3}\left(3 - \frac{6}{\ln 3} + \frac{4}{\ln^2 3}\right)$$

№ 9.1.99

$$\int_0^{\pi} x\cos\frac{x}{2}\,dx = \left[\begin{matrix} u = x \\ v' = \cos\frac{x}{2} \end{matrix} \Rightarrow \begin{matrix} u' = 1 \\ v = \int \cos\frac{x}{2}dx = \end{matrix}\right.$$

$$\left.= 2\sin\frac{x}{2}\right] = 2x\sin\frac{x}{2}\Big|_0^{\pi} - \int_0^{\pi} 2\sin\frac{x}{2}\,dx =$$

$= 2\pi + 4\cos\frac{x}{2}\Big|_0^{\pi} = 2\pi + 4$

№ 9.1.100

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \frac{x\,dx}{\sin^2 x} = \left[\begin{array}{l} u = x \\ v' = \frac{1}{\sin^2 x} \end{array} \Rightarrow \begin{array}{l} u' = 1 \\ v = \int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x \end{array} \right] =$$

$$= -x\operatorname{ctg} x\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \operatorname{ctg} x\,dx =$$

$$= -x\operatorname{ctg} x\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} + \ln|\sin x|\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}} =$$

$$= \frac{\pi}{2\sqrt{3}} + \ln 2$$

№ 9.1.101

$$\int_0^{0{,}2} x e^{5x}\,dx = \left[\begin{array}{l} u = x \\ v' = e^{5x} \end{array} \Rightarrow \begin{array}{l} u' = 1 \\ v = \frac{1}{5}e^{5x} \end{array} \right] =$$

$$= \frac{1}{5}xe^{5x}\Big|_0^{0{,}2} - \frac{1}{5}\int_0^{0{,}2} e^{5x}\,dx =$$

$$= \frac{1}{5}xe^{5x}\Big|_0^{0{,}2} - \frac{1}{25}e^{5x}\Big|_0^{0{,}2} = \frac{1}{25}$$

~~№~~ №9.1.102

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} 4x\, tg^2x\, dx = \left[\begin{matrix} u = 4x \\ v' = tg^2x \end{matrix} \Rightarrow \begin{matrix} u' = 4 \\ v = \int tg^2x\,dx = \end{matrix} \right.$$

$$= \int \frac{dx}{\cos^2 x} - \int dx = tgx - x \Big] =$$

$$= 4x(tgx - x)\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} - 4\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} (tgx - x)\,dx =$$

$$= 4x\,tgx\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} - 4x^2\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} + 4\ln|\cos x|\,\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} + 2x^2\Big|_{\frac{\pi}{4}}^{\frac{\pi}{3}} =$$

$$= \frac{4\pi}{\sqrt{3}} - \pi - \frac{4\pi^2}{9} + \frac{\pi^2}{4} - 4\ln 2 + \ln 4 + \frac{2\pi^2}{9} -$$

$$- \frac{\pi^2}{8} = \frac{\pi}{\sqrt{3}} - \frac{7\pi^2}{72} - 2\ln 2$$

№ 9.1.103

$$\int_1^{e^2} \ln^2 x\, dx = \left[\begin{matrix} u = \ln^2 x \\ v' = 1 \end{matrix} \Rightarrow \begin{matrix} u' = 2\ln x \cdot \frac{1}{x} \\ v = x \end{matrix} \right] =$$

$$= x\ln^2 x\Big|_1^{e^2} - 2\int_1^{e^2} \ln x\, dx = \left[\begin{matrix} u = \ln x \\ v' = 1 \end{matrix} \Rightarrow \right.$$

$$\Rightarrow \left. \begin{matrix} u' = \frac{1}{x} \\ v = x \end{matrix} \right] = 4e^2 - 2x\ln x\Big|_1^{e^2} + 2\int_1^{e^2} dx =$$

$$= 4e^2 - 4e^2 + 2x\Big|_1^{e^2} = 2e^2 - 2$$

№9.1.104

$$\int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{x\cos x}{\sin^2 x} = \left[\begin{array}{l} u = x \\ v' = \dfrac{\cos x}{\sin^2 x} \end{array}\right. \Rightarrow \begin{array}{l} u' = 1 \\ v = \displaystyle\int \dfrac{\cos x\,dx}{\sin^2 x} = \end{array}$$

$$= [t = \sin x;\ dt = \cos dx] = \int \frac{dt}{t^2} = -\frac{1}{t} =$$

$$= -\frac{1}{\sin x} \Big] = -\frac{x}{\sin x}\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} +$$

$$+ \int_{\frac{\pi}{3}}^{\frac{\pi}{2}} \frac{dx}{\sin x} = -\frac{x}{\sin x}\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} + \ln\left|tg\frac{x}{2}\right|\Big|_{\frac{\pi}{3}}^{\frac{\pi}{2}} =$$

$$= -\frac{\pi}{2} + \frac{\pi}{2\sqrt{3}} + 0 + \frac{1}{2}\ln 3 = \frac{\pi - \sqrt{3}\pi}{2\sqrt{3}} + \frac{1}{2}\ln 3$$

№9.1.105

$$\int_0^2 \frac{x^3}{\sqrt{1+x^2}}\,dx = \left[\begin{array}{l} u = x^2 \\ v' = \dfrac{x}{\sqrt{1+x^2}} \end{array}\right. \Rightarrow$$

$$\Rightarrow \begin{array}{l} u' = 2x \\ v = \displaystyle\int \dfrac{x\,dx}{\sqrt{1+x^2}} = \dfrac{1}{2}\int \dfrac{d(x^2+1)}{\sqrt{x^2+1}} = \sqrt{x^2+1} \end{array} \Big] =$$

$$= x^2\sqrt{x^2+1}\Big|_0^2 - \int_0^2 2x\sqrt{x^2+1}\,dx =$$

$$= 4\sqrt{5} - \int_0^2 \sqrt{x^2+1}\,d(x^2+1) =$$

$$= 4\sqrt{5} - (x^2+1)^{3/2} \cdot \frac{2}{3}\Big|_0^2 =$$

$$= 4\sqrt{5} - \frac{10}{3}\sqrt{5} = \frac{2}{3}\sqrt{5}$$

№ 9.1.106

$$\int_0^{\sqrt{3}} \frac{x^2}{(1+x^2)^2}dx = \left[\begin{matrix} u = x \\ v' = \frac{x}{(1+x^2)^2} \end{matrix} \Rightarrow\right.$$

$$\left.\Rightarrow \begin{matrix} u' = 1 \\ v = \int \frac{x\,dx}{(1+x^2)^2} = \frac{1}{2}\int \frac{d(x^2+1)}{(x^2+1)^2} = -\frac{1}{2(x^2+1)} \end{matrix}\right] =$$

$$= -\frac{x}{2(x^2+1)}\Big|_0^{\sqrt{3}} + \int_0^{\sqrt{3}} \frac{dx}{2(x^2+1)} =$$

$$= -\frac{\sqrt{3}}{8} + \frac{1}{2}\operatorname{arctg} x\Big|_0^{\sqrt{3}} =$$

$$= -\frac{\sqrt{3}}{8} + \frac{\pi}{6} = \frac{\pi}{6} - \frac{\sqrt{3}}{8}$$

№ 9.1.107

$$\int_0^{\frac{\pi^2}{4}} \sin\sqrt{x}\,dx = [t = \sqrt{x},\ x = t^2,\ dx = 2t\,dt] =$$

$$= \int_0^{\frac{\pi^2}{4}} 2\sin t\; t\,dt = \left[\begin{matrix} u = 2t \\ v' = \sin t \end{matrix} \Rightarrow \begin{matrix} u' = 2 \\ v = -\cos t \end{matrix}\right] =$$

$$= -2\sqrt{x}\cos\sqrt{x}\Big|_0^{\frac{\pi^2}{4}} + 2\int_0^{\frac{\pi^2}{4}} \cos t\,dt =$$

$$= 0 + 2\sin\sqrt{x}\Big|_0^{\frac{\pi^2}{4}} = 2$$

<u>№ 9.1.108</u>

$$\int_0^9 e^{\sqrt{x}}dx = [t = \sqrt{x},\ dx = 2tdt] =$$

$$= \int_0^9 2te^t dt = \begin{bmatrix} u = 2t & \Rightarrow & u' = 2 \\ v' = e^t & & v = e^t \end{bmatrix} =$$

$$= 2\sqrt{x}e^{\sqrt{x}}\Big|_0^9 - 2\int_0^9 e^t dt = 6e^3 - 2e^{\sqrt{x}}\Big|_0^9 =$$

$$= 6e^3 - 2e^3 + 2 = 4e^3 + 2$$