Интеграли, част 2

Практика

№ 8.2.2

$$\int \sqrt{4x-5}\,dx = \left[\begin{matrix} t = 4x-5 \Rightarrow dt = d(4x-5) \\ = 4dx \Rightarrow dx = \frac{1}{4}dt \end{matrix}\right]$$

$$= \int \sqrt{t} \cdot \frac{1}{4}\,dt = \frac{1}{4}\int \sqrt{t}\,dt = \frac{1}{4} \cdot \frac{2t^{3/2}}{3} + C =$$

$$= \frac{t^{3/2}}{6} + C = \frac{(4x-5)^{3/2}}{6} + C$$

№ 8.2.3

$$\int \frac{dx}{(3x+2)^4} = \left[\begin{matrix} t = 3x+2 \\ dx = \frac{1}{3}dt \end{matrix}\right] = \int \frac{\frac{1}{3}dt}{t^4} =$$

$$= \frac{1}{3}\int t^{-4}\,dt = \frac{1}{3}\,\frac{t^{-3}}{-3} + C =$$

$$= -\frac{1}{9t^3} + C = -\frac{1}{9(3x+2)^3} + C$$

№ 8.2.4

$$\int \sin^3 x \cdot \cos x\,dx = \left[\begin{matrix} t = \sin x \Rightarrow dt = \\ = d(\sin x) = \cos x\,dx \Rightarrow \end{matrix}\right.$$

$\Rightarrow dx = \frac{dt}{\cos x}] =$

$= \int t^3 \cos x \frac{dt}{\cos x} = \int t^3 dt =$

$= \frac{t^4}{4} + C = \frac{\sin^4 x}{4} + C$

№ 8.2.5

$\int e^{x^3} \cdot x^2 dx = [t = x^3 \Rightarrow dt = d(x^3) = 3x^2 dx \Rightarrow$

$\Rightarrow x^2 dx = \frac{dt}{3} = \frac{1}{3} dt] = \int e^t \cdot \frac{1}{3} dt =$

$= \frac{1}{3} \int e^t dt = \frac{1}{3} e^t + C = \frac{e^{x^3}}{3} + C$

№ 8.2.6

$\int \frac{\ln^5 x \, dx}{x} = [t = \ln x \Rightarrow dt = \frac{1}{x} dx] =$

$= \int t^5 dt = \frac{t^6}{5} + C = \frac{1}{5} \ln^6 x + C$

№ 8.2.7

$\int \frac{\sin x \, dx}{\cos x + 1} = [t = \cos x + 1 \Rightarrow$

$\Rightarrow dt = -\sin x dx \Rightarrow$

$\Rightarrow \sin x dx = -dt] =$

$= \int \frac{-dt}{t} = -\int \frac{dt}{t} = -\ln|t| + C =$

$= -\ln|\cos x + 1| + C$

№ 8.2.8

$$\int \frac{x^2 dx}{x^3+1} = \left[t = x^3+1 \Rightarrow dt = 3x^2 dx \Rightarrow x^2 dx = \frac{1}{3} dt \right] = \frac{1}{3} \int \frac{dt}{t} =$$

$$= \frac{1}{3} \ln|t| + C = \frac{1}{3} \ln|x^3+1| + C$$

№ 8.2.9

$$\int \frac{arctg\, x\, dx}{x^2+1} = \left[t = arctg\, x \Rightarrow dt = \frac{dx}{x^2+1} \right] = \int t\, dt =$$

$$= \frac{t^2}{2} + C = \frac{1}{2} arctg^2 x + C$$