

Дипломная работа
Есекиной Татьяной,
1 курс ИВТ, 3^{н/2}
"Ранг матриц"

W 1. 3. 17.

$$\left(\begin{array}{ccccc} 1 & 3 & 1 & -14 & 22 \\ -2 & 1 & 3 & 3 & -9 \\ -4 & -3 & 11 & -19 & 17 \end{array} \right) \xrightarrow{\text{II} + 2\text{I}} \left(\begin{array}{ccccc} 1 & 3 & 1 & -14 & 22 \\ 0 & 7 & 5 & -25 & 35 \\ -4 & -3 & 11 & -19 & 17 \end{array} \right) \xrightarrow{\text{III} + 4\text{I}} \left(\begin{array}{ccccc} 1 & 3 & 1 & -14 & 22 \\ 0 & 7 & 5 & -25 & 35 \\ 0 & 9 & 15 & -75 & 105 \end{array} \right) \xrightarrow{\text{III} - 3\text{II}}$$

$$\sim \left(\begin{array}{ccccc} 1 & 3 & 1 & -14 & 22 \\ 0 & 7 & 5 & -25 & 35 \\ 0 & -12 & 0 & 0 & 0 \end{array} \right) \sim \left(\begin{array}{ccccc} 1 & 3 & 1 & -14 & 22 \\ 0 & 7 & 5 & -25 & 35 \\ 0 & 0 & 0 & 0 & -12 \end{array} \right) \Rightarrow$$

=> ранг = 3

W 1. 3. 18

$$\left(\begin{array}{cccc} 1 & 2 & 4 & -3 \\ 3 & 5 & 6 & -4 \\ 3 & 8 & 2 & -19 \end{array} \right) \xrightarrow{\text{II} - 3\text{I}} \left(\begin{array}{cccc} 1 & 2 & 4 & -3 \\ 0 & -1 & -6 & +5 \\ 3 & 8 & 2 & -19 \end{array} \right) \sim$$

$$\xrightarrow{\text{III} - 3\text{I}} \left(\begin{array}{cccc} 1 & 2 & 4 & -3 \\ 0 & -1 & -6 & +5 \\ 0 & 2 & -10 & -10 \end{array} \right) \xrightarrow{\text{III} + 2\text{II}}$$

$$\sim \left(\begin{array}{cccc} 1 & 2 & 4 & -3 \\ 0 & -1 & -6 & +5 \\ 0 & 0 & -22 & 0 \end{array} \right) \sim \left(\begin{array}{cccc} 1 & 2 & -3 & 4 \\ 0 & -1 & 5 & -6 \\ 0 & 0 & 0 & -22 \end{array} \right) \Rightarrow$$

ранг = 3 ①

W1.3.19

$$\sim \left(\begin{array}{ccccc} 3 & -1 & 3 & 2 & 5 \\ 5 & -3 & 2 & 3 & 4 \\ 1 & -3 & -5 & 0 & -7 \\ 7 & -5 & 1 & 4 & 1 \end{array} \right) \sim \left(\begin{array}{ccccc} -1 & 3 & 3 & 2 & 5 \\ -3 & 5 & 2 & 3 & 4 \\ -3 & 1 & -5 & 0 & -7 \\ -5 & 7 & 1 & 4 & 1 \end{array} \right) \begin{matrix} \text{II} + 3\text{I} \\ \text{II} + 3\text{I} \\ \text{III} + 5\text{I} \end{matrix}$$

$$\sim \left(\begin{array}{ccccc} -1 & 3 & 3 & 2 & 5 \\ 0 & 4 & 11 & 9 & 19 \\ 0 & 10 & 4 & 6 & 8 \\ 0 & 22 & 16 & 14 & 26 \end{array} \right) \sim \left(\begin{array}{ccccc} -1 & 3 & 3 & 2 & 5 \\ 0 & 4 & 10 & 6 & 8 \\ 0 & 11 & 14 & 9 & 19 \\ 0 & 16 & 22 & 14 & 26 \end{array} \right) \begin{matrix} \\ \\ 4\text{II} - 11\text{I} \\ \text{IV} - 4\text{I} \end{matrix}$$

$$\sim \left(\begin{array}{ccccc} -1 & 3 & 3 & 2 & 5 \\ 0 & 4 & 10 & 6 & 8 \\ 0 & 0 & -54 & 30 & -12 \\ 0 & 0 & -18 & -10 & -6 \end{array} \right) \sim \left(\begin{array}{ccccc} -1 & 3 & 5 & 2 & 3 \\ 0 & 4 & 8 & 6 & 10 \\ 0 & 0 & -12 & -30 & -54 \\ 0 & 0 & -6 & -10 & -18 \end{array} \right) \begin{matrix} \\ \\ \\ \text{III} - \text{II} \end{matrix}$$

$$\sim \left(\begin{array}{ccccc} -1 & 3 & 5 & 2 & 3 \\ 0 & 4 & 8 & 6 & 10 \\ 0 & 0 & -12 & -30 & -54 \\ 0 & 0 & 0 & +10 & 18 \end{array} \right) \xrightarrow{\text{rangs} = 3}$$

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w 1. 3. 20.

$$\sim \left(\begin{array}{cccccc} 24 & 19 & 36 & 72 & -38 \\ 48 & 40 & 73 & 147 & -80 \\ 73 & 59 & 98 & 219 & -118 \\ 47 & 36 & 41 & 141 & -72 \end{array} \right) \cdot 24 \sim \left(\begin{array}{cccccc} 24 & 19 & 36 & 72 & -38 \\ 1176 & 960 & 1752 & 3518 & -1920 \\ 1752 & 1416 & 2352 & 5256 & -2832 \\ 1128 & 864 & 1704 & 3384 & -1728 \end{array} \right) \xrightarrow{\text{II}-\text{III}}$$
$$\sim \left(\begin{array}{cccccc} 24 & 19 & 36 & 72 & -38 \\ 0 & 29 & -12 & 0 & 58 \\ 0 & 29 & -276 & 0 & 58 \\ 0 & -29 & +12 & 0 & -58 \end{array} \right) \xrightarrow{\text{III}-\text{II}} \sim \left(\begin{array}{cccccc} 24 & 19 & 36 & 72 & -38 \\ 0 & 29 & -12 & 0 & 58 \\ 0 & 0 & -264 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \xrightarrow{\text{IV}+\text{II}}$$

$\Rightarrow \text{rang} = 3$.

w 1. 3. 21

$$\sim \left(\begin{array}{cccccc} 4 & 3 & -5 & 2 & 3 & \\ 8 & 6 & -7 & 4 & 2 & \\ 9 & 3 & -8 & 2 & 7 & \\ 4 & 3 & 1 & 2 & -5 & \\ 8 & 6 & -1 & 4 & -6 & \end{array} \right) \xrightarrow{\text{II}-\text{I}} \sim \left(\begin{array}{cccccc} 4 & 3 & -5 & 2 & 3 & \\ 0 & 0 & 3 & 0 & -4 & \\ 0 & 0 & -3 & 0 & 4 & \\ 0 & 0 & 6 & 0 & -8 & \\ 0 & 0 & 0 & 0 & -12 & \end{array} \right) \xrightarrow{\text{III}+\text{II}}$$
$$\sim \left(\begin{array}{cccccc} 4 & 3 & -5 & 2 & 3 & \\ 0 & 0 & 3 & 0 & -4 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \end{array} \right) \xrightarrow{\text{IV}-\text{II}}$$
$$\sim \left(\begin{array}{cccccc} 4 & 3 & -5 & 2 & 3 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \end{array} \right) \xrightarrow{\text{V}-3\text{II}}$$
$$\sim \left(\begin{array}{cccccc} 4 & 3 & -5 & 2 & 3 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \\ 0 & 0 & 0 & 0 & 0 & \end{array} \right) \Rightarrow \text{rang} = 2.$$

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w 1.3.22.

$$\left(\begin{array}{cccccc} 17 & -28 & 45 & 11 & 39 \\ 24 & -37 & 61 & 13 & 50 \\ 25 & -7 & 32 & -18 & -11 \\ 31 & 12 & 19 & -43 & -55 \\ 42 & 13 & 29 & -55 & -68 \end{array} \right) \cdot 17$$

$$\sim \left(\begin{array}{cccccc} 17 & -28 & 45 & 11 & 39 \\ 408 & 1036 & 2745 & 143 \\ 1175 & 196 & 1940 & -198 \\ 527 & -336 & 855 & -473 \\ 714 & 364 & 1305 & -605 \end{array} \right)$$
$$\sim \left(\begin{array}{cccccc} 17 & -28 & 45 & 11 & 39 \\ 408 & -629 & 1037 & 221 & 850 \\ 925 & -119 & 544 & -306 & -187 \\ 527 & 204 & 323 & -731 & -935 \\ 714 & 221 & 493 & -935 & -1156 \end{array} \right) \begin{matrix} \text{II} - 24\text{I} \\ \text{III} - 25\text{I} \\ \text{IV} - 31\text{I} \\ \text{V} - 42\text{I} \end{matrix}$$

$$\sim \left(\begin{array}{cccccc} 17 & -28 & 45 & 11 & 39 \\ 0 & 43 & -43 & -43 & -86 \\ 0 & 581 & -581 & -581 & -1162 \\ 0 & 1072 & -1072 & -1072 & -2144 \\ 0 & 1397 & -1397 & -1397 & -2794 \end{array} \right) \begin{matrix} :43 \\ :581 \\ :1072 \\ :1397 \end{matrix}$$

(4)

№

Демонстрация работы

"Ранг матрицы"

Синхрон
Таня

№1. 3.22 (продолжение)

1 к. УВТ
5ч/2

$$\sim \left(\begin{array}{ccccc} 17 & -28 & 45 & 11 & 39 \\ 0 & 1 & -1 & -1 & -2 \\ 0 & 1 & -1 & -1 & -2 \\ 0 & 1 & -1 & -1 & -2 \\ 0 & 1 & -1 & -1 & -2 \end{array} \right) \xrightarrow{\text{III} - \text{II}} \sim$$

$$\sim \left(\begin{array}{ccccc} 17 & -28 & 45 & 11 & 39 \\ 0 & 1 & -1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array} \right) \Rightarrow \underline{\text{rang} = 2}$$

№1.3.23

$$\begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{pmatrix} = A$$

$$M_1^{(1)} = |3| = 3 \neq 0 \Rightarrow r(A) \geq 1$$

$$M_2^{(1)} = \begin{vmatrix} 3 & -1 \\ 4 & -3 \end{vmatrix} = -9 + 4 = -5 \neq 0 \Rightarrow r(A) \geq 2 \quad \textcircled{5}$$

$$M_3^{(1)} = \begin{vmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{vmatrix} = 0 + 24 - 3 - 27 + 6 + 0 = 0 \Rightarrow$$

$\Rightarrow \text{rang } A \geq 2$

$M_2^{(1)} - \delta. u.$

w 1, 3, 24

$$\begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 2 \end{pmatrix} = A$$

$$M_1^{(1)} = |3| = 3 \neq 0 \Rightarrow \text{rang } A \geq 1$$

$$M_2^{(1)} = \begin{vmatrix} 3 & -1 \\ 4 & -3 \end{vmatrix} = -5 \neq 0 \Rightarrow \text{rang } A \geq 2$$

$$M_3^{(1)} = \begin{vmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 2 \end{vmatrix} = -18 + 24 - 3 - 27 + 6 + 8 = -10 \Rightarrow$$

$\Rightarrow \text{rang } A = 3$

$M_2^{(1)} - \delta. u.$

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w 1.3.25

$$\begin{pmatrix} 2 & -1 & 5 & 6 \\ 1 & 1 & 3 & 5 \\ 1 & -5 & 1 & -3 \end{pmatrix} = A$$

$$M_1^{(1)} = |2| = 2 \neq 0 \Rightarrow \text{rang } A \geq 1$$

$$M_2^{(1)} = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2 + 1 - 3 \neq 0 \Rightarrow \text{rang } A \geq 2$$

$$M_3^{(1)} = \begin{vmatrix} 2 & -1 & 5 \\ 1 & 1 & 3 \\ 1 & -5 & 1 \end{vmatrix} = 2 \cdot 25 - 3 \cdot -5 + 30 + 1 =$$

$$= 0$$

$$M_3^{(2)} = \begin{vmatrix} 2 & -1 & 6 \\ 1 & 1 & 5 \\ 1 & -5 & -3 \end{vmatrix} = -6 - 30 - 5 + 50 - 6 - 3 =$$

$$= 0 \Rightarrow \underline{\text{rang } A = 2}.$$

$$M_2^{(1)} - \text{D.M.}$$

(7)

W1.3.26

$$\left(\begin{array}{ccccc} 1 & -2 & 3 & -4 & 4 \\ 0 & 1 & -1 & 1 & -3 \\ 1 & 3 & 0 & -3 & 1 \\ 0 & -7 & 3 & 1 & -3 \end{array} \right) \sim A$$

$$M_1^{(1)} = \begin{vmatrix} 1 \end{vmatrix} = 1 \neq 0 \Rightarrow \text{rang } A \geq 1$$

$$M_2^{(1)} = \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1 + 0 = 1 \neq 0 \Rightarrow \text{rang } A \geq 2$$

$$M_3^{(1)} = \begin{vmatrix} 1 & -2 & 3 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{vmatrix} = 0 + 0 + 2 + 3 - 3 + 0 = 2 \neq 0$$

$$\Rightarrow \text{rang } A \geq 3$$

$$M_4^{(1)} = \begin{vmatrix} 1 & -2 & 3 & -4 \\ 0 & 1 & -1 & 1 \\ 1 & 3 & 0 & -3 \\ 0 & -7 & 3 & 1 \end{vmatrix} \stackrel{R1 \downarrow}{=} \begin{vmatrix} 1 & -1 & 1 \\ 3 & 0 & -3 \\ -7 & 3 & 1 \end{vmatrix} + \begin{vmatrix} -2 & 3 & -4 \\ 1 & -1 & 1 \\ -7 & 3 & 1 \end{vmatrix}$$

$$= 9 + 0 - 21 + 9 + 0 + 25 - 12 + 2 - 21 + 6 + 28 - 3 =$$

$$= 0$$

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W1.3.26 "Ранг матрицы"
(упрощение)

Секция Тадж
ИВТ, 1к., 3ч/2

$$M_4^{(2)} = \begin{vmatrix} 1 & -2 & 3 & 4 \\ 0 & 1 & -1 & -3 \\ 1 & 3 & 0 & 1 \\ 0 & -7 & 3 & -3 \end{vmatrix} = \begin{vmatrix} 1 & -1 & -3 \\ 3 & 0 & 1 \\ -7 & 3 & -3 \end{vmatrix} + \begin{vmatrix} -2 & 3 & 4 \\ 1 & -1 & -3 \\ -7 & 3 & -3 \end{vmatrix} =$$

$$= -27 + 0 + 7 - 3 + 0 - 9 + 12 - 6 + 63 - 18 - 28 + 92$$

$$= 0 \Rightarrow \text{rang } A \geq 3$$

$$M_3^{(1)} = 5. \text{млн.}$$

W1.3.27

$$\begin{pmatrix} 1 & -2 & 1 & -1 & 1 \\ 2 & 1 & -1 & 2 & -3 \\ 3 & -2 & -1 & 1 & -2 \\ 2 & -5 & 1 & -2 & 2 \end{pmatrix} = A$$

$$M_1^{(1)} = |1| = 1 \neq 0 \Rightarrow \text{rang } A \geq 1$$

$$M_2^{(1)} = \begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} = 1 + 4 - 5 \neq 0 \Rightarrow \text{rang } A \geq 2$$

$$M_3^{(1)} = \begin{vmatrix} 1 & -2 & 1 \\ 2 & 1 & -1 \\ 3 & -2 & -1 \end{vmatrix} = -4 - 1 + 6 - 2 - 3 - 4 = -8 \neq 0 \Rightarrow$$

⑨

$$\Rightarrow \text{rang } A \geq 3$$

$$\begin{aligned}
 M_4^{(1)} &= \begin{vmatrix} 1 & -2 & 1 & -1 \\ 2 & 1 & -1 & 2 \\ 3 & -2 & -1 & 1 \\ 2 & -5 & 1 & -2 \end{vmatrix} = \begin{vmatrix} 1 & 2 & \\ 3 & -2 & 1 \\ 2 & -5 & -2 \end{vmatrix} + \\
 &+ \begin{vmatrix} 1 & -2 & -1 \\ 3 & -2 & 1 \\ 2 & -5 & -2 \end{vmatrix} - \begin{vmatrix} 1 & -2 & -1 \\ 2 & 1 & 2 \\ 2 & -5 & -2 \end{vmatrix} - \begin{vmatrix} 1 & -2 & -1 \\ 2 & 1 & 2 \\ 3 & -2 & 1 \end{vmatrix} = \\
 &= -30 + 8 + 2 + 10 + 8 + 6 - 30 + 4 - 4 + 5 - 4 - 12 - \\
 &- (10 + 2 - 8 + 10 - 2 - 8) - (4 + 1 - 12 + 4 + 3 + 4) = \\
 &= -45 \neq 0 \Rightarrow \underline{\text{rang } A = 4}
 \end{aligned}$$

$M_4^{(1)} = 8.$ M.

W 1.3.28

$$\begin{pmatrix} 2 & 1 & -1 & -1 & 1 \\ 1 & -1 & 1 & 1 & -2 \\ 3 & 3 & -3 & -3 & 4 \\ 4 & 5 & -5 & -5 & 7 \end{pmatrix} = A$$

$$M_1^{(1)} = |2| = 2 \neq 0 \Rightarrow \text{rang } A \geq 1$$

$$\textcircled{10} \quad M_2^{(1)} = \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -2 - 1 = -3 \neq 0 \Rightarrow \\
 \Rightarrow \text{rang } A \geq 2$$

$$M_3^{(1)} = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 3 & 3 & -3 \end{vmatrix} = -3 + 6 + 3 - 6 - 3 + 3 = 0$$

$$M_3^{(2)} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -2 \\ 3 & 3 & 4 \end{vmatrix} = 3 - 8 - 6 + 12 + 3 - 4 = 0$$

$$M_3^{(3)} = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -1 & 1 \\ 4 & 5 & -5 \end{vmatrix} = -5 + 10 + 4 - 10 - 4 + 5 = 0$$

$$M_3^{(4)} = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -2 \\ 4 & 5 & 7 \end{vmatrix} = 5 - 14 - 8 + 20 + 4 - 7 = 0$$

$\Rightarrow \underline{\text{rang } A = 2}$

$M_2^{(1)} = \text{D.M.}$

~~w1. 3. 29~~

$$\begin{pmatrix} 1 & -3 & 2 & 0 \\ 2 & -3 & -1 & 3 \\ 3 & -6 & -1 & \lambda \\ 1 & -2 & 0 & 1 \end{pmatrix} = A$$

(11)

$$M_1^{(1)} = \begin{vmatrix} 1 & 1 & 1 \end{vmatrix} = 1 \neq 0 \Rightarrow \text{rang } A \geq 1$$

$$M_2^{(1)} = \begin{vmatrix} 1 & -3 \\ 2 & -3 \end{vmatrix} = -3 + 6 = 3 \neq 0 \Rightarrow \text{rang } A \geq 2$$

$$M_3^{(1)} = \begin{vmatrix} 1 & -3 & 2 \\ 2 & -3 & -1 \\ 3 & -6 & -1 \end{vmatrix} = -24 + 5 + 9 - 6 + 18 - 6 = -6 \Rightarrow \text{rang } A \geq 3$$

$$M_4^{(1)} = \begin{vmatrix} 1 & -3 & 2 & 0 \\ 2 & -3 & -1 & 3 \\ 3 & -6 & -1 & \lambda \\ 1 & -2 & 0 & 1 \end{vmatrix} = 3 \begin{vmatrix} 1 & -3 & 2 \\ 3 & -6 & -1 \\ 1 & -2 & 0 \end{vmatrix} - \lambda \cdot$$

$$\begin{vmatrix} 1 & -3 & 2 \\ 2 & -3 & -1 \\ 1 & -2 & 0 \end{vmatrix} + \begin{vmatrix} 1 & -3 & 2 \\ 2 & -3 & -1 \\ 3 & -6 & -1 \end{vmatrix} =$$

$$= 3(0 - 12 + 3 - 2 + 12 + 0) - \lambda(-8 + 0 + 3 - 2 + 6 + 0) = -6 = 3 - 6 + \lambda =$$

$$= \lambda - 3$$

$$\lambda - 3 = 0 \Rightarrow \text{if } \lambda = 3 \text{ rang } A = 3$$

(12)

$$\lambda \neq 3 \Rightarrow \text{if } \lambda \neq 3 \text{ rang } A = 4$$

Ранг матрицы⁴

wt. 3. 30

Евгения Тарх
УВТ, 1к., 3н/2

$$\begin{pmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{pmatrix} = A$$

$$M_1^{(1)} = |1| = 1 \neq 0 \Rightarrow \text{rang } A \geq 1$$

$$M_2^{(1)} = \begin{vmatrix} 1 & 1 \\ 4 & 10 \end{vmatrix} = 10 - 4 = 6 \neq 0 \Rightarrow \text{rang } A \geq 2$$

$$M_3^{(1)} = \begin{vmatrix} 1 & 1 & 4 \\ 4 & 10 & 1 \\ 2 & 4 & 3 \end{vmatrix} = 64 + 30 + 2 - 4 - 80 - 12 = 0$$

$$M_3^{(2)} = \begin{vmatrix} 1 & 1 & 4 \\ 4 & 10 & 1 \\ 7 & 17 & 3 \end{vmatrix} = 2 \cdot 2 + 30 + 7 - 17 - 280 - 12 = 0$$

$$M_3^{(3)} = \begin{vmatrix} 3 & 1 & 1 \\ \lambda & 4 & 10 \\ 2 & 2 & 4 \end{vmatrix} = 2\lambda + 48 + 20 - 60 - 8 - 41 = -2\lambda \Rightarrow$$

\Rightarrow при $\lambda = 0$ $\text{rang } A = 2$, при $\lambda \neq 0$ ③

$\text{rang } A \geq 3$

tyczka $\lambda \neq 0$, moga

$$M_4 = \begin{vmatrix} 3 & 1 & 1 & 4 \\ \lambda & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \\ 2 & 2 & 4 & 3 \end{vmatrix} - \lambda \cdot$$

$$\begin{vmatrix} 1 & 1 & 4 \\ 7 & 17 & 3 \\ 2 & 4 & 3 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 4 \\ 4 & 10 & 1 \\ 2 & 4 & 3 \end{vmatrix} - 2 \begin{vmatrix} 1 & 1 & 4 \\ 4 & 10 & 1 \\ 7 & 17 & 3 \end{vmatrix} =$$

$$= 3(28+204+60-48-34-210) - \lambda(112+51+6-12-136-21) + 64+30+2-4-80-120+12 =$$

$$= 0 \rightarrow \text{rang } A < 4$$

Odbierz: npu $\lambda = 0$ $\text{rang } A = 2$
 npu $\lambda \neq 0$ $\text{rang } A = 3$.

w 1.3.31

$$\begin{pmatrix} \lambda & 1 & 1 & 1 \\ 1 & \lambda & 1 & \lambda \\ 1 & 1 & \lambda & \lambda^2 \end{pmatrix} = A$$

(14)

$$M_1 = \begin{vmatrix} 1 \end{vmatrix} = 1 \neq 0 \Rightarrow \text{rang } A \geq 1$$

$$M_2^{(1)} = \begin{vmatrix} 1 & 1 \\ \lambda & 1 \end{vmatrix} = 1 - \lambda$$

$$\text{npu } \lambda = 1 \text{ rang } A = 1$$

npu $\lambda \neq 1$ $\text{rang } A \geq 2$. т.к. $\lambda \neq 1$, т.к.

$$M_3^{(1)} = \begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 1 \\ 1 & 1 & \lambda \end{vmatrix} = 1 + \lambda^3 + 1 - \lambda - \lambda - \lambda =$$

$$= \lambda^3 - 3\lambda + 2$$

$$\lambda^3 - 3\lambda + 2 = 0$$

$$\begin{array}{r} \lambda^3 - 3\lambda + 2 \\ \lambda^3 - \lambda^2 \\ \hline -\lambda^2 - 3\lambda + 2 \end{array} \left| \begin{array}{c} \lambda - 1 \\ \hline \lambda^2 - \lambda - 2 \end{array} \right.$$

$$\begin{array}{r} -\lambda^2 + \lambda \\ \hline -2\lambda + 2 \end{array}$$

$$\begin{array}{r} -2\lambda + 2 \\ \hline 0 \end{array}$$

$$(\lambda^2 - \lambda - 2)(\lambda - 1) = 0$$

$$\lambda^2 - \lambda - 2 = 0 \quad \text{или} \quad \lambda = 1$$

$$\Delta = (-1)^2 - 4 \cdot (-2) = 9$$

(15)

$$\lambda_1 = \frac{-1+3}{2} = 1$$

$$\lambda_2 = \frac{-1-3}{2} = -2$$

\Rightarrow npu $\lambda = 1$ u $\lambda = -2$ $\text{rang } A \leq 3$,
npu $\lambda \neq 1$ u $\lambda \neq -2$ $\text{rang } A \geq 3$

Ombeen: npu $\lambda = 1$ $\text{rang } A = 1$;
npu $\lambda = -2$ $\text{rang } A = 2$;
npu $\lambda \neq 1$ u $\lambda \neq -2$
 $\text{rang } A = 3$.