Дифференциал

№7.2.16

$y = 2^{\cos x}$

$dy = y'dx = (2^{\cos x})'dx$ ②

$= 2^{\cos x} \cdot \ln 2 \cdot (-\sin x)\,dx$ ②

$= -2^{\cos x} \ln 2 \cdot \sin x\,dx$

№7.2.17.

$y = \ln^3 \sin x$

$dy = (\ln^3 \sin x)'dx = 3\ln^2 \sin x \cdot \frac{1}{\sin x} \cdot \cos x\,dx = 3\ln^2 \sin x \cdot \operatorname{ctg} x\,dx$

№7.2.18

$f(x) = \sqrt[3]{x^5 - 1}$

$d(f(x)) = f'(x)dx = \left(\sqrt[3]{x^5-1}\right)'dx =$

$= \frac{\sqrt[3]{x^5-1} \cdot 5x^4}{3(x^5-1)}dx = \frac{5x^4 \sqrt[3]{x^5-1}}{3x^5-3}dx$

№7.2.19

$S(t) = \frac{\sqrt{t}}{t-1}$

$$d(S(t)) = S'(t)dt = \left(\frac{\sqrt{t}}{t-1}\right)' dt =$$

$$= \frac{(\sqrt{t})'(t-1) - \sqrt{t}(t-1)'}{(t-1)^2} dt =$$

$$= \frac{\frac{1}{2\sqrt{t}}(t-1) - \sqrt{t}}{(t-1)^2} dt =$$

$$= \frac{t-1-2t}{2\sqrt{t}(t-1)^2} dt = \frac{-t-1}{2\sqrt{t}(t-1)^2} dt$$

wł. 2.20

$y = 4x^2 + 1, \quad x_0 = 1, \quad \Delta x = 0{,}02$

$\Delta y = y(x + \Delta x) - y(x)$

$\Delta y = 4(x + \Delta x)^2 + 1 - 4x^2 - 1 =$

$= 4x^2 + 8x\Delta x + 4(\Delta x)^2 - 4x^2 =$

$= 8x\Delta x + 4(\Delta x)^2$

$dy = 8x\Delta x = 8x\,dx$

$\Delta y(x_0) = 8 \cdot 1 \cdot 0{,}02 + 4 \cdot (0{,}02)^2 =$

$= 0{,}16 + 0{,}0016 = 0{,}1616$

$d(y(x_0)) = 8 \cdot 1 \cdot 0{,}02 = 0{,}16$

wł. 2.21

$y = |x|, \quad x_0 = 10, \quad \Delta x = -0{,}1$

$\Delta y = |x + \Delta x| - |x|$

$dy = (|x|)' dx = \frac{x}{|x|} dx$

$\Delta y(x_0) = |10 - 0{,}1| - |10| = 9{,}9 - 10 = -0{,}1$

$d(y(x_0)) = \frac{10}{|10|} \cdot (-0{,}1) = -0{,}1$

№7.2.21

$\sin 29^\circ = f(x_0 + \Delta x)$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x$

$x_0 = 30, \quad \Delta x = -1$

$\sin 29^\circ \approx \sin 30^\circ + (\sin 30^\circ)' \cdot (-1) =$

$= 0{,}5 + \cos 30^\circ \cdot (-1) =$

$= 0{,}5 - \frac{\sqrt{3}}{2} = \frac{1-\sqrt{3}}{2} \approx -0{,}366$

№7.2.23

$\operatorname{arctg} 1{,}05 = f(x_0 + \Delta x)$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x$

$x_0 = 1, \quad \Delta x = 0{,}05$

$\operatorname{arctg} 1{,}05 \approx \operatorname{arctg} 1 + (\operatorname{arctg} 1)' \cdot 0{,}05 =$

$= \frac{\pi}{4} + \frac{1}{1+1^2} \cdot 0{,}05 \approx 0{,}785 + 0{,}025 =$

$= 0{,}81$

№7.2.24

$(0{,}99)^4 = f(x_0 + \Delta x)$

$f(x_0 + \Delta x) \approx f(x_0) + f'(x_0)\Delta x$

$x_0 = 1, \quad \Delta x = -0{,}01$

$(0{,}99)^4 \approx 1^4 + (1^4)' \cdot (-0{,}01) =$

$= 1 + 4 \cdot 1^3 \cdot (-0{,}01) = 1 - 0{,}04 = 0{,}96$

№7.2.25

$y = \frac{x-1}{x+1}$. Найти: dy, d^2y

$y' = \left(\frac{x-1}{x+1}\right)' = \frac{(x-1)'(x+1) - (x-1)(x+1)'}{(x+1)^2} =$

$= \frac{x+1-x+1}{x^2+2x+1} = \frac{2}{x^2+2x+1}$

$dy = \frac{2}{x^2+2x+1}\,dx$

$y'' = \left(\frac{2}{(x+1)^2}\right)' = -\frac{2 \cdot ((x+1)^2)'}{(x+1)^4} =$

$= -\frac{2 \cdot 2(x+1)}{(x+1)^4} = -\frac{4}{(x+1)^3}$

$d^2y = -\frac{4}{(x+1)^3}\,dx^2$

№7.2.26

$y = x(\ln x - 1)$. Найти: dy, d^2y

$y' = (x(\ln x - 1))' = x'(\ln x - 1) + x(\ln x - 1)' =$

$= \ln x - 1 + 1 = \ln x$

$dy = \ln x\,dx$

$$y'' = (\ln x)' = \frac{1}{x}$$

$$d^2y = \frac{1}{x} dx^2 = \frac{dx^2}{x}$$

№7.2.27

$y = x^n$. Найти: dy, d^2y, d^3y

$$y' = (x^n)' = n x^{n-1}$$

$$dy = n x^{n-1} dx$$

$$d^2y = (n x^{n-1})' dx^2 = n(n-1) x^{n-2} dx^2$$

$$d^3y = (n(n-1) x^{n-2})' dx^3 =$$

$$= n(n-1)(n-2) x^{n-3} dx^3$$