

$$\approx \arcsin 1 = \frac{\pi}{2}$$

№ 9.2.47

$$\int_0^1 \ln x \, dx = \left[ \lim_{x \rightarrow 0} \ln x = -\infty \Rightarrow \right.$$

$\Rightarrow$  разрыв в точке при  $x=0$  ] =

$$= \lim_{\varepsilon \rightarrow 0} \int_{\varepsilon}^1 \ln x \, dx \quad \text{②}$$

$$\lim_{\varepsilon \rightarrow 0} (-\ln \varepsilon) \quad \text{②} \quad \left[ \int \ln x \, dx = \left[ \begin{array}{l} u = \ln x \\ u' = 1 \end{array} \Rightarrow \right. \right.$$

$$\Rightarrow \left. \begin{array}{l} u' = \frac{1}{x} \\ u = x \end{array} \right\} = x \ln x - \int \frac{x}{x} \, dx = x \ln x -$$

$$- \int dx = x \ln x - x \quad \left. \right] = \lim_{\varepsilon \rightarrow 0} (x \ln x - x) \Big|_{\varepsilon}^1 =$$

$$= \lim_{\varepsilon \rightarrow 0} (1 \cdot \ln 1 - 1 - \varepsilon \cdot \ln \varepsilon + \varepsilon) =$$

$$= \lim_{\varepsilon \rightarrow 0} (-1 - \varepsilon \ln \varepsilon + \varepsilon) = -1 - 0 + 0 =$$

$$= -1 \Rightarrow \text{УИТ. СХ.}$$

№ 9.2.51

$$\int_{-1}^1 \frac{dx}{x^2} = \left[ \lim_{x \rightarrow 0} \frac{1}{x^2} = \infty \Rightarrow \right.$$

$\Rightarrow$  внутренний разрыв в  $x=0$  ] =

$$\begin{aligned}
 &= \int_{-1}^0 \frac{dx}{x^2} + \int_0^1 \frac{dx}{x^2} = \lim_{\varepsilon \rightarrow 0} \left( \int_{-1}^{-\varepsilon} \frac{dx}{x^2} + \int_{\varepsilon}^1 \frac{dx}{x^2} \right) = \left[ \int \frac{dx}{x^2} = -\frac{1}{x} \right] \ominus \\
 &= \lim_{\varepsilon \rightarrow 0} \left( \left( -\frac{1}{x} \right) \Big|_{-1}^{-\varepsilon} + \left( -\frac{1}{x} \right) \Big|_{\varepsilon}^1 \right) = \\
 &= \lim_{\varepsilon \rightarrow 0} \left( -\frac{1}{\varepsilon} + \frac{1}{-1} - \frac{1}{\varepsilon} + \frac{1}{1} \right) = \\
 &= \lim_{\varepsilon \rightarrow 0} (-2) = -2
 \end{aligned}$$

$$\begin{aligned}
 \ominus \quad &\lim_{\varepsilon \rightarrow 0} \left( -\frac{1}{x} \right) \Big|_{-1}^{\varepsilon} + \lim_{\delta \rightarrow 0} \left( -\frac{1}{x} \right) \Big|_{\delta}^1 = \\
 &= \lim_{\varepsilon \rightarrow 0} \left( -\frac{1}{\varepsilon} - 1 \right) + \lim_{\delta \rightarrow 0} \left( -1 + \frac{1}{\delta} \right) = \\
 &= \lim_{\delta \rightarrow 0} \frac{1}{\delta} - 1 - \lim_{\varepsilon \rightarrow 0} \frac{1}{\varepsilon} - 1 = \infty \Rightarrow \\
 &\Rightarrow \text{интеграл расх.}
 \end{aligned}$$

№ 2.55.

$$\int_0^1 \frac{\cos^2 x}{\sqrt[3]{(1-x^2)^2}} dx = \left[ \text{разреш в точке } x=1 \right] = \int_0^1 \frac{\cos^2 x}{(1+x)^{\frac{2}{3}}} \cdot \frac{1}{(1-x)^{\frac{2}{3}}} =$$

$$\approx \left[ \varphi(x) = \frac{1}{(1-x)^{\frac{2}{3}}}, f(x) - \text{норма } p\text{-уиз}; \right. \\ \left. \int_0^1 \frac{1}{(1-x)^{\frac{2}{3}}} dx - \text{сх.}, \text{ т.к. } \alpha = \frac{2}{3} < 1 \right] =$$

$$\approx \lim_{x \rightarrow 1} \frac{\cos^2 x}{(1+x)^{\frac{2}{3}}} \cdot \frac{1}{(1-x)^{\frac{2}{3}}} \cdot \frac{(1-x)^{\frac{2}{3}}}{1} =$$

$$= \lim_{x \rightarrow 1} \frac{\cos^2 x}{(1+x)^{\frac{2}{3}}} = \frac{\cos^2 1}{2^{\frac{2}{3}}} =$$

$$= \frac{\cos^2 1}{\sqrt[3]{4}} (\neq 0, \neq \infty) \Rightarrow$$

$$\Rightarrow \text{по предельному признаку сходимости} \\ \int_0^1 f(x) dx - \text{сх.}, \text{ т.к. } \int_0^1 \varphi(x) dx - \text{сх.}$$

нр 2.58

$$\int_0^1 \frac{dx}{3x^2 + \sqrt[3]{x}}; \text{ т.к. } f(x) \text{ имеет разрыв} \\ \text{в т. } x=0. \text{ т.к. } \varphi(x) = \frac{1}{\sqrt[3]{x}}.$$

$$f(x) < \varphi(x), \int_0^1 \varphi(x) dx - \text{сх.}, \text{ т.к.}$$

$$\alpha = \frac{1}{3} < 1 \Rightarrow \int_0^1 f(x) dx = \int_0^1 \frac{dx}{3x^2 + \sqrt[3]{x}} -$$

- сх. по предельному сравнению.