Интегрирование (часть 4)

Домашняя работа

№ 8.4.12

$$\int \frac{dx}{x+\sqrt[3]{x^2}} = \left[x = t^3 \Rightarrow dx = 3t^2dt\right] =$$

$$= \int \frac{3t^2dt}{t^3+t^2} = 3\int \frac{t^2dt}{t^2(t+1)} =$$

$$= 3\int \frac{dt}{t+1} = 3\ln|t+1| + C =$$

$$= 3\ln|\sqrt[3]{x}+1| + C$$

№ 8.4.13

$$\int \frac{\sqrt{x}}{1+\sqrt[4]{x^3}}dx = \left[\begin{matrix} n=2 \\ q=4 \end{matrix}\right| \Rightarrow НОК(2,4) = 4 \Rightarrow$$

$$\Rightarrow x = t^4 \Rightarrow dx = 4t^3dt\Big] = \int \frac{\sqrt{t^4}}{1+\sqrt[4]{(t^4)^3}} \cdot$$

$$\cdot 4t^3dt = 4\int \frac{t^2 \cdot t^3dt}{1+t^3} =$$

$$= 4\int \frac{t^5dt}{t^3+1} = 4\int \frac{t^2(t^3+1)-t^2}{t^3+1}dt =$$

$$= 4\int t^2dt - \int \frac{t^2}{t^3+1}dt =$$

$$= 4\int t^2dt - \frac{1}{3}\int \frac{d(t^3+1)}{t^3+1} =$$

$$= 4\frac{t^3}{3} - \frac{1}{3}\ln|t^3+1| + C =$$

$$= \frac{4}{3}\left(x^{\frac{1}{4}}\right)^3 - \frac{1}{3}\ln\left|\left(x^{\frac{1}{4}}\right)^3+1\right| + C =$$

$$= \frac{4}{3}x^{\frac{3}{4}} - \frac{1}{3}\ln\left|x^{\frac{3}{4}}+1\right| + C$$

w8. 4.14

$$\int \frac{x+\sqrt[3]{x^2}+\sqrt[6]{x}}{x(1-\sqrt[3]{x})}dx = \left[\begin{array}{c} n=3 \\ q=6 \end{array}\right| \Rightarrow НОК(3,6)=6 \Rightarrow$$

$$\Rightarrow x = t^6 \Rightarrow dx = 6t^5dt\,\Big] =$$

$$= \int \frac{t^6+t^4+t}{t^6(1-t^2)}\cdot 6t^5dt =$$

$$= 6\int\frac{t^6(t^5+t^3+1)}{t^6(1-t^2)}dt = 6\int\frac{t^5+t^3+1}{1-t^2}dt =$$

$$= -6\int\frac{t^5+t^3+1}{t^2-1}dt = -6\int\frac{(t^2-1)(t^3+2t)}{t^2-1}dt +$$

$$-6\int\frac{2t+1}{t^2-1}dt =$$

$$= -6\left(\int\frac{t^3}{t^2-1}dt + 2\int\frac{2t}{t^2-1}dt + \int\frac{dt}{t^2-1}\right) =$$

$$= -6\left(\int\frac{t^2-1+1}{t^2-1}t\,dt + 2\int\frac{d(t^2-1)}{t^2-1} + \int\frac{dt}{t^2-1}\right) =$$

$$= -6\left(\int t\,dt + \int\frac{t\,dt}{t^2-1} + 2\int\frac{d(t^2-1)}{t^2-1} + \int\frac{dt}{t^2-1}\right) =$$

$$= -6\left(\int t\,dt + \frac{1}{2}\int \frac{d(t^2-1)}{t^2-1} + 2\int \frac{d(t^2-1)}{t^2-1} +\right.$$

$$\left. + \int \frac{dt}{t^2-1}\right) = -6\left(\int t\,dt + \frac{5}{2}\int \frac{d(t^2-1)}{t^2-1} +\right.$$

$$+ \frac{dt}{t^2-1} = -6\left(\frac{t^2}{2} + \frac{5}{2}\ln|t^2-1| +\right.$$

$$\left. + \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right|\right) + C = -3t^2 - 15\ln|t^2-1| -$$

$$- 3\ln\left|\frac{t-1}{t+1}\right| + C = -3x^{\frac{2}{6}} - 15\ln|x^{\frac{2}{6}}-1| -$$

$$- 3\ln\left|\frac{x^{\frac{1}{6}}-1}{x^{\frac{1}{6}}+1}\right| + C = -3\sqrt[3]{x} -$$

$$-15\ln|\sqrt[3]{x}-1| - 3\ln\left|\frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1}\right| + C$$

№8.4.15

$$\int \frac{\sqrt{x}\,dx}{x - \sqrt[3]{x^2}} = \left[\begin{array}{l} n=2 \\ q=3 \end{array}\right| \Rightarrow НОК(2,3)=6 \Rightarrow$$

$$\Rightarrow x = t^6 \Rightarrow dx = 6t^5\,dt \Big] =$$

$$= \int \frac{t^3 \cdot 6t^5\,dt}{t^6 - t^4} = \int \frac{6t^8\,dt}{t^4(t^2-1)} =$$

$$= 6\int \frac{t^4\,dt}{t^2-1} = 6\int \frac{t^4-1+1}{t^2-1}\,dt =$$

$$= 6\int \frac{(t^2-1)(t^2+1)}{t^2-1}\,dt + 6\int \frac{dt}{t^2-1} =$$

$$= 6\int t^2\,dt + 6\int dt + 6\int \frac{dt}{t^2-1} =$$

$$= 6\,\frac{t^3}{3} + 6t + 6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + C =$$

$$= 2t^3 + 6t + 3\ln\left|\frac{t-1}{t+1}\right| + C =$$

$$= 2\sqrt{x} + 6\sqrt[6]{x} + 3\ln\left|\frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1}\right| + C$$

№ 8.4.16

$$\int \frac{\sqrt{x}\,dx}{1+\sqrt{x}} = [x = t^2 \Rightarrow dx = 2t\,dt] =$$

$$= \int \frac{t\cdot t\,dt}{1+t} = \int \frac{t^2-1+1}{t+1}\,dt =$$

$$= \int (t-1)\,dt + \int \frac{dt}{t+1} =$$

$$= \int t\,dt - \int dt + \int \frac{dt}{t+1} =$$

$$= \frac{t^2}{2} - t + \ln|t+1| + C =$$

$$= \frac{1}{2}x - \sqrt{x} + \ln|\sqrt{x}+1| + C$$

№ 8.4.17

$$\int \frac{\sqrt{x}\,dx}{1-\sqrt[3]{x}} = [x = t^6 \Rightarrow dx = 6t^5\,dt] =$$

$$= \int \frac{t^3 \cdot 6t^5\,dt}{1-t^2} = 6\int \frac{t^8-1+1}{1-t^2}\,dt =$$

$$= 6\left(-\int \frac{(t^2-1)(t^2+1)(t^4+1)}{t^2-1}\,dt - \int \frac{dt}{t^2-1}\right) =$$

$$= -6\int (t^6+t^4+t^2+1)\,dt - 6\int \frac{dt}{t^2-1} =$$

$$= -6\cdot\frac{t^7}{7} - 6\cdot\frac{t^5}{5} - 6\cdot\frac{t^3}{3} - 6t -$$

$$-6\cdot\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + C = -\frac{6}{7}t^7 - \frac{6}{5}t^5 -$$

$$-2t^3 - 6t - 3\ln\left|\frac{t-1}{t+1}\right| + C =$$

$$= -\frac{6}{7}x\sqrt[6]{x} - \frac{6}{5}\sqrt[6]{x^5} - 2\sqrt{x} - 6\sqrt[6]{x} -$$

$$-3\ln\left|\frac{\sqrt[6]{x}-1}{\sqrt[6]{x}+1}\right| + C$$

<u>№8.4.18</u>

$$\int \frac{\sqrt{x+2}}{x}\,dx = \int \sqrt{\frac{x+2}{x^2}}\,dx =$$

$$= \left[\ \frac{x+2}{x^2} = t^2;\quad x+2 = t^2x^2;\right.$$

$$\cancel{x = x^2t^2 - 2;\quad x(1 - xt^2) = -2;}$$

$$x = t^2x^2 - 2;\quad t^2x^2 - x - 2 = 0;$$

$$D = 1 - 4\cdot(-2)t^2 = 1 + 8t^2;$$

$$\cancel{x_1 = \frac{+1+\sqrt{1+8t^2}}{2t^2} = \frac{\sqrt{1+8t^2}+1}{2t^2}}$$

$$\cancel{x_2 = \frac{-\sqrt{1+8t^2}+1}{2t^2} = -\frac{\sqrt{1+8t^2}-1}{2t^2}}$$

$$\cancel{x = \frac{1 \pm \sqrt{1+8t^2}}{2t^2};}$$

$$\cancel{dx = \left(\frac{1-\sqrt{1+8t^2}}{2t^2}\right)' dt}$$

(2) $\left[\, x+2 = t^2;\ x = t^2-2;\ dx = 2t\,dt \,\right] =$

$$= \int \frac{t}{t^2-2}\, t\,dt = \int \frac{t^2-2+2}{t^2-2}\, dt =$$

$$= \int dt + 2\int \frac{dt}{t^2-2} = t + 2\cdot \frac{1}{2\sqrt{2}} \cdot$$

$$\cdot \ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + C = t + \frac{1}{\sqrt{2}} \ln\left|\frac{t-\sqrt{2}}{t+\sqrt{2}}\right| + C =$$

$$= \sqrt{x+2} + \frac{1}{\sqrt{2}} \ln\left|\frac{\sqrt{x+2}-\sqrt{2}}{\sqrt{x+2}+\sqrt{2}}\right| + C$$

№8.4.19

$$\int \frac{x\,dx}{\sqrt{x+1}+\sqrt[3]{x+1}} = \left[\, x+1 = t^6 \Rightarrow x = t^6-1;\right.$$

$$\left. dx = 6t^5\,dt \,\right] = \int \frac{t^6-1}{t^3+t^2} \cdot 6t^5\,dt =$$

$$= 6\int \frac{t^5(t^6-1)}{t^2(t+1)}\,dt = 6\int \frac{t^3(t^6-1)}{t+1}\,dt =$$

$$= 6\int \frac{t^9 - t^3}{t+1}\,dt = 6\int \frac{t^9dt}{t+1} - 6\int \frac{t^3}{t+1}\,dt =$$

$$= 6\int\left(t^8 - t^7 + t^6 - t^5 + t^4 - t^3 + t^2 - t + 1 - \right.$$

$$\left. - \frac{1}{t+1}\right)dt - 6\int\left(t^2 - t + 1 - \frac{1}{t+1}\right)dt =$$

$$= 6\left(\int t^8dt - \int t^7dt + \int t^6dt - \int t^5dt + \right.$$

$$\left. + \int t^4dt - \int t^3dt\right) = 6\left(\frac{t^9}{9} - \frac{t^8}{8} + \right.$$

$$\left. + \frac{t^7}{7} - \frac{t^6}{6} + \frac{t^5}{5} - \frac{t^4}{4}\right) + C =$$

$$= \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{3}{4}(x+1)^{\frac{4}{3}} + \frac{6}{7}(x+1)^{\frac{7}{6}} -$$

$$-(x+1) + \frac{6}{5}(x+1)^{\frac{5}{6}} - \frac{3}{2}(x+1)^{\frac{2}{3}} + C =$$

$$= \frac{2}{3}(x+1)^{\frac{3}{2}} - \frac{3}{4}(x+1)^{\frac{4}{3}} + \frac{6}{7}(x+1)^{\frac{7}{6}} + \frac{6}{5}(x+1)^{\frac{5}{6}} -$$

$$- \frac{3}{2}(x+1)^{\frac{2}{3}} - x - 1 + C$$

№ 8.4.20

$$\int \frac{dx}{(x+1)^{3/2} + (x+1)^{1/2}} = \left[x+1 = t^2;\ dx = 2t\,dt\right] =$$

$$= 2\int \frac{t\,dt}{t^3+t} = 2\int \frac{t\,dt}{t(t^2+1)} = 2\int \frac{dt}{t^2+1} =$$

$= 2\operatorname{arctg} t + C = 2\operatorname{arctg}\sqrt{x+1} + C$

№8.4.21

$$\int \frac{\sqrt{1+x}+1}{\sqrt{1+x}-1}\,dx = \int \frac{\sqrt{1+x}-1+2}{\sqrt{1+x}-1}\,dx =$$

$$= \int dx + \cancel{2}\int \frac{2dx}{\sqrt{1+x}-1} = \left[2)\ 1+x = t^2 \Rightarrow \Rightarrow dx = 2t\,dt\right] = \int dx + 4\int \frac{t\,dt}{t-1} =$$

$$= \int dx + 4\int \frac{t-1+1}{t-1}\,dt = \int dx + 4\int dt + 4\int \frac{dt}{t-1} = x + 4t + 4\ln|t-1| + C = x + 4\sqrt{1+x} + 4\ln|\sqrt{1+x}-1| + C$$

№8.4.22

$$\int \frac{x-1}{\sqrt{2x-1}}\,dx = \left[2x-1 = t^2;\ x = \frac{t^2+1}{2};\ dx = \frac{1}{2}\cdot 2t\,dt = t\,dt\right] =$$

$$= \int \frac{\frac{t^2+1}{2}-1}{t}\cdot t\,dt = \int \frac{1}{2}(t^2-1)\,dt =$$

$$= \frac{1}{2}\int t^2\,dt - \frac{1}{2}\int dt = \frac{1}{2}\cdot\frac{t^3}{3} - \frac{1}{2}t + C =$$

$$= \frac{1}{6}t^3 - \frac{1}{2}t + C = \frac{1}{6}\sqrt{(2x-1)^3} - \frac{1}{2}\sqrt{2x-1} + C =$$

$$= \frac{1}{2}\sqrt{2x-1}\left(\frac{1}{3}(2x-1) - 1\right) + C =$$

$$= \frac{1}{2}\sqrt{2x-1}\left(\frac{2}{3}x - \frac{4}{3}\right) + C =$$

$$= \sqrt{2x-1}\left(\frac{1}{3}x - \frac{2}{3}\right) + C =$$

$$= \frac{1}{3}\sqrt{2x-1}\,(x-2) + C$$

№8.4.23

$$\int \frac{dx}{\sqrt{1-2x} - \sqrt[4]{1-2x}} = \Big[\, 1-2x = t^4;$$

$$x = \frac{1-t^4}{2} \Rightarrow dx = \frac{1}{2}\cdot 4t^3\,dt = 2t^3\,dt \Big] =$$

$$= \int \frac{2t^3\,dt}{t^2 - t} = \int \frac{2t^2\,dt}{t-1} =$$

$$= 2\int \frac{t^2-1+1}{t-1}\,dt = 2\int\left(t+1 + \frac{1}{t-1}\right)dt =$$

$$= 2\cdot\frac{t^2}{2} + 2t + 2\ln|t-1| + C =$$

$$= t^2 + 2t + 2\ln|t-1| + C =$$

$$= \sqrt{1-2x} + 2\sqrt[4]{1-2x} + 2\ln\left|\sqrt[4]{1-2x} - 1\right| + C$$

№8.4.24

$$\int \frac{1}{(2-x)^2}\cdot\sqrt{\frac{2-x}{2+x}}\,dx =$$

$$= \left[\frac{2-x}{2+x} = t^2 \Rightarrow x = 2 \cdot \frac{1-t^2}{1+t^2} \Rightarrow \right.$$

$$= dx = 2 \cdot \frac{-2t(1+t^2) - 2t(1-t^2)}{(1+t^2)^2} dt =$$

$$= 2 \cdot (-2t) \cdot \frac{1+t^2+1-t^2}{(1+t^2)^2} dt =$$

$$= -4t \cdot \frac{2}{1+2t^2+t^4} dt =$$

$$\left. = -\frac{8t\,dt}{(t^2+1)^2} \right] =$$

$$= \int \frac{1}{\left(2 - 2 \cdot \frac{1-t^2}{1+t^2}\right)^2} \cdot t \cdot \left(-\frac{8t\,dt}{(t^2+1)^2}\right) =$$

$$= -\int \frac{2t^2\,dt}{\left(1 - \frac{1-t^2}{1+t^2}\right)^2 (t^2+1)^2} =$$

$$= -2 \int \frac{t^2\,dt}{\left(\frac{2}{1+t^2}\right)^2 (t^2+1)^2} =$$

$$= -2 \int \frac{1}{4} t^2\,dt = -\frac{1}{2} \int t^2\,dt =$$

$$= -\frac{1}{2} \cdot \frac{t^3}{3} + C = -\frac{1}{6} t^3 + C =$$

$$= -\frac{1}{6} \sqrt{\left(\frac{2-x}{2+x}\right)^3} + C$$

w8.4.25

$$\int \frac{dx}{\sqrt{(x-1)^3(x-2)}} = (x-1)^{-\frac{3}{2}}(x-2)^{-\frac{1}{2}}dx =$$

$$= [t^2 = x-1;\ x = t^2+1;\ dx = 2t\,dt] =$$

$$= \int t^{-3}(t^2-1)^{-\frac{1}{2}}\cancel{d}\cdot 2t\,dt \ ②$$

$$= \int (t^6(t^2-1))^{-\frac{1}{2}}\cdot 2t\,dt =$$

$$= \int (t^8-t^6)^{-\frac{1}{2}}\cdot 2t\,dt =$$

$$= \int (t^4-t^3)(t^4+t^3)$$

$$② \ 2\int t^{-2}(t^2-1)^{-\frac{1}{2}}dt =$$

$$= [\ m=-2,\ n=2,\ p=-\frac{1}{2} \Rightarrow$$

$$\Rightarrow 1)\ p\notin\mathbb{Z};\quad 2)\ \frac{m+1}{n} = \frac{-2+1}{2} = -\frac{1}{2}\notin\mathbb{Z};$$

$$3)\ \frac{m+1}{n}+p = -\frac{1}{2}-\frac{1}{2} = -1\in\mathbb{Z} \Rightarrow$$

$$\Rightarrow -1t^{-2}+1 = y^2 \Rightarrow$$

$$-\frac{1}{t^2}+1 = y^2;\quad \frac{1}{t^2} = 1-y^2;$$

$$t^2 = \frac{1}{1-y^2};\quad t = \sqrt{\frac{1}{1-y^2}} = \frac{1}{\sqrt{1-y^2}}$$

$$dt = -\frac{y}{(1-y^2)\sqrt{1-y^2}}dy\ ] =$$

$$= 2\int (1-y^2)\left(\frac{1}{1-y^2}-1\right)^{-\frac{1}{2}} \cdot$$

$$\cdot \left(-\frac{y}{(1-y^2)\sqrt{1-y^2}}\right)dy =$$

$$= -2\int \cancel{(1-y^2)} \cdot \frac{\cancel{\sqrt{1-y^2}}}{\cancel{y}} \cdot \frac{\cancel{y}}{\cancel{(1-y^2)}\cancel{\sqrt{1-y^2}}}dy =$$

$$= -2\int dy = -2y + C =$$

$$= -2\sqrt{1-\frac{1}{t^2}} + C = -2\frac{\sqrt{t^2-1}}{t} + C =$$

$$= -2\frac{\sqrt{x-2}}{\sqrt{x-1}} + C$$

<u>№ 8.4.27</u>

$$\int \frac{dx}{(1-x)\sqrt{1-x^2}} = \int \frac{dx}{(1-x)\sqrt{(1-x)(1+x)}} =$$

$$= \cancel{[\, t^2 = 1-x;\ x = 1-t^2} = \int (1-x)^{-1}(1-x^2)^{-\frac{1}{2}}dx =$$

$$= \int (1-x)^{-1}(1-x)^{-\frac{1}{2}}(1+x)^{-\frac{1}{2}}dx =$$

$$= \int (1-x)^{-\frac{3}{2}}(1+x)^{-\frac{1}{2}}dx = [\, t^2 = 1+x,$$

$$x = t^2-1;\ dx = 2t\,dt\,] =$$

$$= \int t^{-1}(1-t^2+1)^{-\frac{3}{2}} \cdot 2t\,dt = 2\int (2-t^2)^{-\frac{3}{2}}dt =$$

$= \left[m = 0,\ n = 2,\ p = -\frac{3}{2} \Rightarrow \right.$

$\Rightarrow \nexists$ 1) $p \notin \mathbb{Z}$; 2) $\frac{m+1}{n} = \frac{0+1}{2} \notin \mathbb{Z}$;

3) $\frac{m+1}{n} + p = \frac{1}{2} - \frac{3}{2} = -1 \in \mathbb{Z} \Rightarrow$

$\Rightarrow 2t^{-2} - 1 = y^2$; $\cancel{\frac{2-t^2}{t^2} = y^2}$

$\frac{2}{t^2} = y^2 + 1$; $t^2 = \frac{2}{y^2+1}$;

$t = \frac{\sqrt{2}}{\sqrt{y^2+1}} \Rightarrow dt = \sqrt{2} \cdot -\frac{1}{2(y^2+1)\sqrt{y^2+1}} \cdot$

$\left. \cdot\, 2y\,dy = -\frac{\sqrt{2}\,y\,dy}{(y^2+1)\sqrt{y^2+1}} \right] =$

$= 2\int \left(2 - \frac{2}{y^2+1}\right)^{-\frac{3}{2}} \cdot \left(-\frac{\sqrt{2}\,y\,dy}{(y^2+1)\sqrt{y^2+1}}\right) =$

$= -\cancel{2}\int \frac{\cancel{(y^2+1)\sqrt{y^2+1}}}{\cancel{2\sqrt{2}}\, y^{\cancel{3}\,2}} \cdot \frac{\cancel{\sqrt{2}}\, \cancel{y}\, dy}{\cancel{(y^2+1)\sqrt{y^2+1}}} =$

$= -\cancel{2}\int \frac{dy}{y^2} = \frac{1}{y} + C =$

$= \frac{t}{\sqrt{2-t^2}} + C = \frac{\sqrt{1+x}}{\sqrt{2-1-x}} + C =$

$= \frac{\sqrt{1+x}}{\sqrt{1-x}} + C = \sqrt{\frac{1+x}{1-x}} + C$

W8. 4.26

$$\int \frac{dx}{\sqrt[3]{(x-1)^2(x+1)}} = \int (x-1)^{-\frac{2}{3}} (x+1)^{-\frac{1}{3}} dx =$$

$$= \left[\ t^3 = x+1,\ x = t^3-1,\ dx = 3t^2 dt \right] =$$

$$= \int (t^3-2)^{-\frac{2}{3}}\, t^{-1} \cdot 3t^2 dt =$$

$$= 3\int t(t^3-2)^{-\frac{2}{3}} dt =$$

$$= \left[\ m=1,\ n=3,\ p=-\frac{2}{3} \Rightarrow\right.$$

$$\Rightarrow 1)\ p \notin \mathbb{Z};\quad 2)\ \frac{m+1}{n} = \frac{1+1}{3} = \frac{2}{3} \notin \mathbb{Z};$$

$$3)\ \frac{m+1}{n} + p = \frac{2}{3} - \frac{2}{3} = 0 \in \mathbb{Z} \Rightarrow$$

$$\Rightarrow -2t^{-3} + 1 = y^3$$

$$-\frac{2}{t^3} + 1 = y^3;\quad -\frac{2}{t^3} = y^3 - 1;$$

$$\frac{2}{t^3} = 1 - y^3;\quad t^3 = \frac{2}{1-y^3}$$

$$dt = \left(\left(\frac{2}{1-y^3}\right)^{\frac{1}{3}}\right)' dy = \frac{1}{\cancel{3}}\left(\frac{2}{1-y^3}\right)^{-\frac{2}{3}} \cdot$$

$$\cdot\left(-\frac{2}{(1-y^3)^2} \cdot \cancel{3}y^2\right) dy =$$

$$= -\frac{(1-y^3)^{\frac{2}{3}}}{\sqrt[3]{4}} \cdot \frac{2y^2}{(1-y^3)^2}\, dy =$$

$$= -\sqrt[3]{2}\, y^2 (1-y^3)^{\frac{1}{3}}\, dy \;] =$$

$$= 3\int \frac{\sqrt[3]{2}}{\cancel{(1-y^3)^{\frac{1}{3}}}} \left(\frac{2}{1-y^3}-2\right)^{-\frac{2}{3}} \cdot$$

$$\cdot \left(-\sqrt[3]{2}\, y^2\right) \cancel{(1-y^3)^{\frac{1}{3}}}\, dy =$$

$$= -3\int \frac{\sqrt[3]{4}\, y^2}{\sqrt[3]{4}\, y^2} \cdot (1-y^3)^{\frac{2}{3}}\, dy =$$

$$= -3\int (1-y^3)^{\frac{2}{3}}\, dy = [\, z^3 = 1-y^3;$$

$$dy = \left((-z^3+1)^{\frac{1}{3}}\right)' dz = \frac{1}{3}(1-z^3)^{-\frac{2}{3}}$$

$$dt = \left(\left(\frac{2}{1-y^3}\right)^{\frac{1}{3}}\right)' dy =$$

$$= \frac{\sqrt[3]{2}\, y^2}{(1-y^3)^{4/3}}\, dy \;] =$$

$$= 3\int \left(\frac{2}{1-y^3}\right)^{\frac{1}{3}} \left(\frac{2}{1-y^3}-2\right)^{-\frac{2}{3}} \cdot$$

$$\cdot \frac{\sqrt[3]{2}\, y^2}{(1-y^3)^{4/3}}\, dy =$$

$$= 3\int \frac{\cancel{\sqrt[3]{2}}}{\cancel{(1-y^3)^{\frac{1}{3}}}} \cdot \frac{\cancel{(1-y^3)^{\frac{2}{3}}}}{\cancel{\sqrt[3]{4}}\;\cancel{y^2}} \cdot$$

$$\cdot \frac{\cancel{\sqrt[3]{2}}\;\cancel{y^2}}{(1-y^3)\cancel{\sqrt[3]{1-y^3}}}\,dy = 3\int \frac{dy}{1-y^3} =$$

$$= 3\int \frac{dy}{(1-y)(1+y+y^2)} = \Bigg[\ \frac{1}{(1-y)(y^2+y+1)} =$$

$$= \frac{A}{1-y} + \frac{By+C}{y^2+y+1} \Rightarrow 1 = Ay^2+Ay+A -$$

$$- By^2+By-Cy+C = y^2(A-B) + y(A+B-C) + A+C \Rightarrow$$

$$\begin{cases} A-B=0 \\ A+B-C=0 \\ A+C=1 \end{cases} \Rightarrow \begin{cases} A=B \\ B+B-1+B=0 \\ C=1-A=1-B \end{cases}$$

$$3B=1 \Rightarrow B=\frac{1}{3},\ A=\frac{1}{3},\ C=\frac{2}{3}\ \Bigg] =$$

$$= 3\int \left(\frac{1}{3(1-y)} + \frac{\frac{1}{3}y+\frac{2}{3}}{y^2+y+1}\right) dy =$$

$$= \int \left(\frac{1}{1-y} + \frac{y+2}{y^2+y+1}\right) dy =$$

$$= \int \frac{dy}{1-y} + \int \frac{y+2}{y^2+y+1}\,dy =$$

$= \left[2) \ \overline{\text{III}} \text{ тип}; \ A=1, B=2, p=1, q=1; \right.$

$\left. y+2 = \frac{1}{2}(2y+1) + 2 - \frac{1}{2} = \frac{1}{2}(2y+1) + \frac{3}{2} \right] =$

$= \int \frac{dy}{1-y} + \frac{1}{2} \int \frac{2y+1}{y^2+y+1} dy + \frac{3}{2} \int \frac{dy}{y^2+y+1} =$

$= \left[3) \ z = y + \frac{1}{2}; \ a = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2} \right] =$

$= \int \frac{dy}{1-y} + \frac{1}{2} \int \frac{(2y+1)dy}{y^2+y+1} + \frac{3}{2} \int \frac{dz}{z^2 + \frac{3}{4}} =$

$= -\ln|y-1| + \frac{1}{2} \ln|y^2+y+1| +$

$+ \frac{3}{2} \cdot \frac{2}{\sqrt{3}} \operatorname{arctg} \frac{2y+1}{\sqrt{3}} + C =$

$= -\ln|y-1| + \frac{1}{2} \ln|y^2+y+1| +$

$+ \sqrt{3} \operatorname{arctg} \frac{2y+1}{\sqrt{3}} + C =$

$= \left[y = \frac{\sqrt[3]{t^3-2}}{t}, \ t = \sqrt[3]{x+1} \right] =$

$= -\ln\left| \sqrt[3]{\frac{x-1}{x+1}} - 1 \right| + \frac{1}{2} \ln \left| \sqrt[3]{\left(\frac{x-1}{x+1}\right)^2} + \right.$

$\left. + \sqrt[3]{\frac{x-1}{x+1}} + 1 \right| + \sqrt{3} \operatorname{arctg} \left(\frac{2\sqrt[3]{x-1}}{\sqrt{3}\sqrt[3]{x+1}} + \frac{1}{\sqrt{3}} \right) +$

$+ C$

№ 8.4.28

$$\int \frac{dx}{x(1+\sqrt[3]{x})^3} = \int x^{-1}(1+x^{\frac{1}{3}})^{-3}dx =$$

$$= \Big[m=-1,\ n=\frac{1}{3},\ p=-3 \Rightarrow p\in\mathbb{Z} \Rightarrow$$

$$\Rightarrow x=t^3;\ dx=3t^2dt =$$

$$= \int t^{-3}(1+t)^{-3}\cdot 3t^2 dt =$$

$$= 3\int\frac{dt}{t(1+t)^3} = \Big[\frac{1}{t(1+t)^3} =$$

$$= \frac{A}{t} + \frac{B}{(1+t)^3} + \frac{C}{(1+t)^2} + \frac{D}{1+t} \Rightarrow$$

$$\Rightarrow 1 = A(1+t)^3 + Bt + C(1+t)t +$$

$$+ D(1+t)^2 t = At^3 + 3At^2 + 3At + A +$$

$$+ Bt + Ct^2 + Ct + Dt^3 + 2Dt^2 + Dt =$$

$$= t^3(A+D) + t^2(3A+C+2D) + t(3A+B+C+D) +$$

$$+ A \rightarrow \begin{cases} A+D=0 \\ 3A+C+2D=0 \\ 3A+B+C+D=0 \\ A=1 \end{cases} \Rightarrow \begin{cases} D=-1 \\ C=-1 \\ B=-1 \\ A=1 \end{cases}\Big] =$$

$$= 3\int\left(\frac{1}{t} - \frac{1}{(1+t)^3} - \frac{1}{(1+t)^2} - \frac{1}{1+t}\right)dt =$$

$$= 3\left(\ln|t| + \frac{1}{3}\cdot\frac{1}{(t+1)^2} + \frac{1}{2}\cdot\frac{1}{t+1} - \ln|t+1|\right) + C =$$

$$= 3\ln|t| + \frac{1}{(t+1)^2} + \frac{3}{2}\cdot\frac{1}{t+1} - 3\ln|t+1| + C$$

<u>w8.4.29</u>

$$\int x^3\cdot\sqrt{1+x^2}\,dx = \Big[\ m=3,\ n=2,\ p=\frac{1}{2} \Rightarrow$$

$$\Rightarrow 1)\ p\notin\mathbb{Z};\quad 2)\ \frac{m+1}{n} = \frac{4}{2} =$$

$$= 2 \in \mathbb{Z} \Rightarrow 1+x^2 = t^2 \Rightarrow$$

$$\Rightarrow x = \sqrt{t^2-1};\quad dx = \frac{2t}{2\sqrt{t^2-1}}\,dt =$$

$$= \frac{t\,dt}{\sqrt{t^2-1}}\ \Big] = \int\sqrt{(t^2-1)^3}\cdot t\cdot\frac{t\,dt}{\sqrt{t^2-1}} =$$

$$= \int (t^2-1)\cancel{\sqrt{t^2-1}}\,\frac{t^2\,dt}{\cancel{\sqrt{t^2-1}}} =$$

$$= \int (t^2-1)\,t^2\,dt = \int (t^4-t^2)\,dt =$$

$$= \int t^4\,dt - \int t^2\,dt = \frac{t^5}{5} - \frac{t^3}{3} + C =$$

$$= \frac{(1+x^2)\sqrt{1+x^2}}{5} - \frac{(1+x^2)\sqrt{1+x^2}}{3} + C =$$

$$= (1+x^2)\sqrt{1+x^2}\left(\frac{1}{5}(1+x^2) - \frac{1}{3}\right) + C =$$

$$= (1+x^2)\sqrt{1+x^2}\left(\frac{1}{5}x^2 - \frac{2}{15}\right) + C$$

№8.4.30

$$\int \frac{dx}{x^{11}\cdot\sqrt{x^4+1}} = \int x^{-11}\cdot(x^4+1)^{-\frac{1}{2}}\,dx =$$

$$= \Big[\; m=-11,\; n=4,\; p=-\frac{1}{2} \notin \mathbb{Z} \Rightarrow$$

$$\Rightarrow 2)\; \frac{m+1}{n} = \frac{-11+1}{4} = \frac{5}{2} \notin \mathbb{Z};$$

$$3)\; \frac{m+1}{n} + p = \frac{5}{2} - \frac{1}{2} = 2 \in \mathbb{Z} \Rightarrow$$

$$\Rightarrow x^{-4}+1 = t^2;\quad x = \frac{1}{\sqrt[4]{t^2-1}};$$

$$dx = -\frac{2t\,dt}{4(t^2-1)\sqrt[4]{t^2-1}} = -\frac{t\,dt}{(t^2-1)\sqrt[4]{t^2-1}}\Big] =$$

$$= \int (t^2-1)^2\sqrt[4]{(t^2-1)^3}\cdot\left(\frac{1}{t^2-1}+1\right)^{-\frac{1}{2}}\cdot$$

$$\cdot\left(-\frac{t\,dt}{(t^2-1)\sqrt[4]{t^2-1}}\right) = -\int (t^2-1)^{\cancel{2}}\sqrt[4]{(t^2-1)^{\cancel{3}2}}\cdot$$

$$\cdot\frac{\sqrt{t^2-1}}{\cancel{t}}\cdot\frac{\cancel{t}\,dt}{\cancel{(t^2-1)\sqrt[4]{t^2-1}}} =$$

$$= -\int (t^2-1)\,dt = -\int t^2\,dt + \int dt =$$

$$= t - \frac{t^3}{3} + C = \frac{\sqrt{x^4+1}}{x^2} - \frac{(x^4+1)\sqrt{x^4+1}}{3x^6} + C =$$

$$= \frac{\sqrt{x^4+1}}{x^2}\left(1 - \frac{x^4+1}{3x^4}\right) + C = \frac{\sqrt{x^4+1}}{x^2}\left(\frac{2x^4+1}{3x^4}\right) + C =$$

$$= \frac{(2x^4+1)\sqrt{x^4+1}}{3x^6} + C$$

№ 8.4.31

$$\int \frac{dx}{\sqrt{x}(1-\sqrt{x})^2} = \int x^{-\frac{1}{2}}(1-x^{\frac{1}{2}})^{-2}dx =$$

$$= [m = -\frac{1}{2},\ n = \frac{1}{2},\ p = -2 \in \mathbb{Z} \Rightarrow$$

$$\Rightarrow 1 - x^{\frac{1}{2}} = t^2;\ x = (1-t^2)^2;$$

$$dx = 2(1-t^2)\cdot 2t\,dt = 4(1-t^2)t\,dt] =$$

$$= \int (1-t^2)^{-1}(1-1+t^2)^{-2}\cdot 4(1-t^2)t\,dt =$$

$$= 4\int \frac{dt}{t^3} = -4\cdot\frac{1}{2t^2} + C =$$

$$= -\frac{2}{t^2} + C = -\frac{2}{1-\sqrt{x}} + C$$

№ 8.4.32

$$\int x^5 \cdot \sqrt[3]{(1+x^3)^2}\,dx = x^5\cdot(1+x^3)^{\frac{2}{3}}dx =$$

$$= [m = 5,\ n = 3,\ p = \frac{2}{3} \notin \mathbb{Z} \Rightarrow$$

$$\Rightarrow 2)\ \frac{m+1}{n} = \frac{5+1}{3} = 2 \in \mathbb{Z} \Rightarrow$$

$$\Rightarrow 1+x^3 = t^3;\ x = \sqrt[3]{t^3-1};$$

$$dx = \frac{\sqrt[3]{t^3-1}}{3(t^3-1)}\cdot 3t^2dt =$$

$$= \frac{t^2 \sqrt[3]{t^3-1}}{t^3-1} dt \Big] =$$

$$= \int \cancel{(t^3-1)} \sqrt[3]{(t^3-1)^2} \left(1+t^3-1\right)^{\frac{2}{3}} \cdot$$

$$\cdot \frac{t^2 \sqrt[3]{t^3-1}}{\cancel{t^3-1}} dt = \int t^4 (t^3-1) dt =$$

$$= \int (t^7 - t^4) dt = \int t^7 dt - \int t^4 dt =$$

$$= \frac{t^8}{8} - \frac{t^5}{5} + C = t^5 \left(\frac{t^3}{8} - \frac{1}{5}\right) + C =$$

$$= (1+x^3) \sqrt[3]{(1+x^3)^2} \left(\frac{1+x^3}{8} - \frac{1}{5}\right) + C =$$

$$= (1+x^3) \sqrt[3]{(1+x^3)^2} \left(\frac{5+5x^3-8}{40}\right) + C =$$

$$= (1+x^3) \sqrt[3]{(1+x^3)^2} \left(\frac{5x^3-3}{40}\right) + C$$

№ 8.4.33

$$\int \frac{dx}{x^3 \cdot \sqrt[3]{2-x^3}} = \int x^{-3} (2-x^3)^{-\frac{1}{3}} dx =$$

$$= \Big[m = -3,\ n = 3,\ p = -\frac{1}{3} \notin \mathbb{Z} \Rightarrow$$

$$\Rightarrow \not 2)\ \frac{m+1}{n} = \frac{-3+1}{3} = -\frac{2}{3} \notin \mathbb{Z};$$

$$3)\ \frac{m+1}{n} + p = -\frac{2}{3} - \frac{1}{3} = -1 \in \mathbb{Z} \Rightarrow$$

$$\Rightarrow 2x^{-3} - 1 = t^3;\ x = \sqrt[3]{\frac{2}{t^3+1}};$$

$$dx = \frac{t^2 (t^3+1) dt}{\sqrt[3]{4} \sqrt[3]{t^3+1}} \Big] =$$

$$= \int \frac{t^3+1}{2}\left(2-\frac{2}{t^3+1}\right)^{-\frac{1}{3}}\cdot$$

$$\cdot\frac{t^2\,(t^3+1)\,dt}{\sqrt[3]{4}\,\sqrt[3]{t^3+1}} =$$

$$= \int \frac{t^3+1}{2}\cdot\frac{\cancel{\sqrt[3]{t^3+1}}}{\sqrt[3]{2}\,\cancel{t}}\cdot\frac{t^{\cancel{2}}(t^3+1)\,dt}{\sqrt[3]{4}\,\cancel{\sqrt[3]{t^3+1}}} =$$

$$= \frac{1}{4}\int (t^3+1)^2\,dt = \frac{1}{4}\int (t^6+2t^3+1)\,dt =$$

$$= \frac{1}{4}\left(\frac{t^7}{7}+\frac{2t^4}{4}+t\right)+C =$$

$$= \frac{1}{28}t^7+\frac{1}{8}t^4+t+C =$$

$$= \frac{1}{28}\cdot\frac{\sqrt[3]{(2-x^3)^7}}{x^7}+\frac{1}{8}\cdot\frac{\sqrt[3]{(2-x^3)^4}}{x^4}+$$

$$+\frac{\sqrt[3]{2-x^3}}{x}+C = \frac{(2-x^3)^{\frac{7}{3}}}{28x^7}+$$

$$+\frac{(2-x^3)^{\frac{4}{3}}}{8x^4}+\frac{(2-x^3)^{\frac{1}{3}}}{x}+C$$

№8.4.34

$$\int\sqrt{x}\,(1+\sqrt{x})^3\,dx = \left[m=\tfrac{1}{2},\ n=\tfrac{1}{2},\ p=3\in\mathbb{Z} \Rightarrow\right.$$

$$\left.\Rightarrow x=t^2;\ dx=2t\,dt\right] =$$

$$= \int t(1+t)^3 \cdot 2t\,dt = 2\int t^2(1+t)^3\,dt =$$

$$= 2\int t^2(t^3+3t^2+3t+1)\,dt =$$

$$= 2\int (t^5+3t^4+3t^3+t^2)\,dt =$$

$$= 2\left(\frac{t^6}{6}+3\cdot\frac{t^5}{5}+3\cdot\frac{t^4}{4}+\frac{t^3}{3}\right)+C =$$

$$= \frac{1}{3}t^6+\frac{6}{5}t^5+\frac{3}{2}t^4+\frac{2}{3}t^3+C =$$

$$= \frac{1}{3}x^3+\frac{6}{5}x^2\sqrt{x}+\frac{3}{2}x^2+\frac{2}{3}x\sqrt{x}+C$$

<u>w8.4.35</u>

$$\int \sqrt[3]{x^3-4}\cdot x^2\,dx = \int x^2(x^3-4)^{\frac{1}{3}}\,dx =$$

$$= \Big[\, m=2,\ n=3,\ p=\frac{1}{3}\notin\mathbb{Z} \Rightarrow$$

$$\Rightarrow \Big\{\ 1)\ \frac{m+1}{n} = \frac{2+1}{3} = 1 \in \mathbb{Z} \Rightarrow$$

$$\Rightarrow x^3-4=t^3;\ x^3=t^3+4;\ x=\sqrt[3]{t^3+4};$$

$$dx = \frac{t^2\sqrt[3]{t^3+4}}{t^3+4}\,dt\ \Big] =$$

$$= \int \sqrt[3]{(t^3+4)^2}\,(t^3+4-4)^{\frac{1}{3}}\,\frac{t^2\sqrt[3]{t^3+4}}{t^3+4}\,dt =$$

$$= \int t^3\,dt = \frac{t^4}{4}+C = \frac{1}{4}(x^3-4)\sqrt[3]{x^3-4}+C$$

<u>w8.4.36</u>

$$\int\frac{dx}{\sqrt{1-2x-x^2}} = \int\frac{dx}{\sqrt{(1-x)^2}} = \int\frac{dx}{1-x} =$$

$= -\ln|1-x| + C$

w 8.4.37

$$\int \frac{(x-2)\,dx}{\sqrt{x^2-10x+29}} = \int (x-2)\left((x-5)^2+4\right)^{-\frac{1}{2}}dx =$$

$$= \left[y = x-2,\ dy = dx\right] = \int y\left((y-3)^2+4\right)^{-\frac{1}{2}}dy =$$

$= [\, m=1,\ n=2,\ p=-\frac{1}{2} \notin \mathbb{Z} \Rightarrow$

$\Rightarrow$ ﾂｧ 2) $\frac{m+1}{n} = \frac{1+1}{2} = 1 \in \mathbb{Z} \Rightarrow$

$\Rightarrow (y-3)^2+4 = t^2;\ y = \sqrt{t^2-4}+3;$

$$dy = \frac{t\,dt}{\sqrt{t^2-4}}\Big] = \int \left(\sqrt{t^2-4}+3\right) t^{-1} \cdot$$

$$\cdot \frac{t\,dt}{\sqrt{t^2-4}} = \int \frac{\sqrt{t^2-4}+3}{\sqrt{t^2-4}}\,dt =$$

$$= \int dt + 3\int \frac{dt}{\sqrt{t^2-4}} =$$

$$= t + 3\ln\left|t+\sqrt{t^2-4}\right| + C =$$

$$= \sqrt{x^2-10x+29} + 3\ln\left|\sqrt{x^2-10x+29} + x - 5\right| + C$$

w 8.4.38

$$\int \frac{3x-5}{\sqrt{x^2-4x+5}}\,dx = \int \frac{\frac{3}{2}(2x-4)-5-\frac{3(-4)}{4}}{\sqrt{x^2-4x+5}}\,dx$$

$$= \int \frac{\frac{3}{2}(2x-4)-2}{\sqrt{x^2-4x+5}}\,dx = \frac{3}{2}\int\frac{(2x-4)\,dx}{\sqrt{x^2-4x+5}} -$$

$$-2\int\frac{dx}{\sqrt{x^2-4x+5}} = [\,2)\ y = x-2;\ dx = dy\,] =$$

$$a = \sqrt{5-\frac{16}{4}} = 1$$

$$= \frac{3}{2}\int\frac{(2x-4)\,dx}{\sqrt{x^2-4x+5}} - 2\int\frac{dx}{\sqrt{y^2+1}} =$$

$$= 3\sqrt{x^2-4x+5} - 2\ln\left|y+\sqrt{y^2+1}\right| + C =$$

$$= 3\sqrt{x^2-4x+5} - 2\ln\left|x-2+\sqrt{(x-2)^2+1}\right| + C =$$

$$= 3\sqrt{x^2-4x+5} - 2\ln\left|x-2+\sqrt{x^2-4x+5}\right| + C$$

№ 8.4.39

$$\int\frac{x+1}{\sqrt{2x-x^2}}\,dx = \int\frac{x+1}{\sqrt{x(2-x)}}\,dx =$$

$$= \int\frac{x\,dx}{\sqrt{x(2-x)}} + \int\frac{dx}{\sqrt{2-x}} =$$

$$= \int\sqrt{\frac{x}{2-x}}\,dx + \int\frac{dx}{\sqrt{2-x}} =$$

$$= [\,1)\ \frac{x}{2-x} = y^2 \Rightarrow x = \frac{2y^2}{y^2+1};$$

$$dx = \frac{4y}{(y^2+1)^2}\,dy;\quad 2)\ 2-x = z^2;$$

$$x = 2-z^2;\quad dx = 2z\,dz\,] =$$

$$= \int y \cdot \frac{4y}{(y^2+1)^2}\, dy + \int \frac{2z\, dz}{z} =$$

$$= \int \frac{4y^2\, dy}{(y^2+1)^2} + 2\int dz =$$

$$= \left[\frac{4y^2}{(y^2+1)^2} = \frac{Ay+B}{(y^2+1)^2} + \frac{Cy+D}{y^2+1} \Rightarrow \right.$$

$$\Rightarrow 4y^2 = (Ay+B)\cancel{(y^2+1)} + (Cy+D)(y^2+1) =$$

$$= Ay + B + Cy^3 + Dy^2 + Cy + D \Rightarrow$$

$$\begin{cases} C = 0 \\ D = 4 \\ A + C = 0 \\ B + D = 0 \end{cases} \Rightarrow \left. \begin{cases} C = 0 \\ D = 4 \\ A = 0 \\ B = -4 \end{cases} \right] =$$

$$= -\int \frac{4}{(y^2+1)^2}\, dy + \int \frac{4}{y^2+1}\, dy + 2\int dz =$$

$$= \left[(1) \int \frac{dy}{(y^2+1)^2} = \frac{1}{2(2-1)} \cdot \frac{y}{y^2+1} + \right.$$

$$\left. + \frac{2\cdot 2-3}{2\cdot 2-2} \int \frac{dy}{y^2+1} = \frac{y}{2(y^2+1)} + \frac{1}{2}\int \frac{dy}{y^2+1} \right] =$$

$$= -4 \cdot \frac{y}{2(y^2+1)} - 4 \cdot \frac{1}{2} \int \frac{dy}{y^2+1} +$$

$$+ 4 \int \frac{dy}{y^2+1} + 2\int dz =$$

$$= -\frac{2y}{y^2+1} + 2\int \frac{dy}{y^2+1} + 2\int dz =$$

$$= 2z + 2\,\mathrm{arctg}\, y - \frac{2y}{y^2+1} + C =$$

$$= 2\sqrt{2-x} + 2\,\mathrm{arctg}\sqrt{\frac{x}{2-x}} - \frac{2\sqrt{\frac{x}{2-x}}}{\frac{x}{2-x}+1} + C$$

$$= 2\sqrt{2-x} + 2\,\mathrm{arctg}\sqrt{\frac{x}{2-x}} - \sqrt{2x+x^2} + C$$

№ 8.4.40

$$\int \frac{\sqrt{1-x^2}}{x}\,dx = \int x^{-1}(1-x^2)^{\frac{1}{2}}\,dx =$$

$$= \Big[\, m=-1,\ n=2,\ p=\frac{1}{2} \notin \mathbb{Z} \Rightarrow$$

$$\Rightarrow \cancel{1},\ 2)\ \frac{m+1}{n} = \frac{-1+1}{2} = 0 \in \mathbb{Z} \Rightarrow$$

$$\Rightarrow 1-x^2 = t^2;\quad x = \sqrt{1-t^2};$$

$$dx = \frac{2t\,dt}{2\sqrt{1-t^2}} = \frac{t\,dt}{\sqrt{1-t^2}}\Big] =$$

$$= \int \frac{t}{\sqrt{1-t^2}} \cdot \frac{t\,dt}{\sqrt{1-t^2}} = \int \frac{t^2\,dt}{1-t^2} =$$

$$= -\int \frac{t^2-1+1}{t^2-1}\,dt = -\int dt - \int \frac{dt}{t^2-1} =$$

$$= -t - \frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| + C =$$

$$= -\sqrt{1-x^2} - \frac{1}{2}\ln\left|\frac{\sqrt{1-x^2}-1}{\sqrt{1-x^2}+1}\right| + C$$

№8.4.41

$$\int \sqrt{4-x^2}\,dx = \left[\, 4-x^2 = t^2 \Rightarrow \right.$$
$$\left. \Rightarrow x = \sqrt{4-t^2};\ dx = -\frac{t\,dt}{\sqrt{4-t^2}} \,\right] =$$
$$= -\int \frac{t^2\,dt}{\sqrt{4-t^2}}$$

$$\int \sqrt{4-x^2}\,dx = \left[\, m=0,\ n=2,\ p=\frac{1}{2} \right.$$
$$\Rightarrow \left\{ \frac{m+1}{n} = \frac{0+1}{2} = \frac{1}{2} \notin \mathbb{Z} \right\} \Rightarrow$$
$$\Rightarrow \frac{m+1}{n} + p = \frac{1}{2} + \frac{1}{2} = 1 \in \mathbb{Z} \Rightarrow$$
$$\Rightarrow 4x^{-2} - 1 = t^2;\ x = \sqrt{\frac{4}{t^2+1}} = \frac{2}{\sqrt{t^2+1}}$$
$$\left. dx = -\frac{2t\,dt}{(t^2+1)\sqrt{t^2+1}} \right] =$$
$$= -\int \sqrt{4 - \frac{4}{t^2+1}} \cdot \frac{2t\,dt}{(t^2+1)\sqrt{t^2+1}} =$$
$$= -\int \frac{2t}{\sqrt{t^2+1}} \cdot \frac{2t\,dt}{(t^2+1)\sqrt{t^2+1}} =$$
$$= -\int \frac{4t^2\,dt}{(t^2+1)^2} = [\text{по №8.4.39}] =$$
$$= \frac{2t}{t^2+1} - 2\,\mathrm{arctg}\, t + C =$$

$$= \frac{\frac{2\sqrt{4-x^2}}{x}}{\frac{4-x^2}{x^2}+1} - 2\operatorname{arctg}\frac{\sqrt{4-x^2}}{x} + C =$$

$$= \frac{x}{2}\sqrt{4-x^2} - 2\operatorname{arctg}\frac{\sqrt{4-x^2}}{x} + C$$

<u>№ 8.4.42</u>

$$\int x\cdot\sqrt[5]{x-2}\,dx = \Big[m=1,\ n=1,\ p=\frac{1}{5}\notin \mathbb{Z} \Rightarrow$$

$$\Rightarrow \frac{m+1}{n} = \frac{1+1}{1} = 2 \in \mathbb{Z} \Rightarrow x-2=t^5;$$

$$x = t^5-2;\ dx = 5t^4\,dt \Big] =$$

$$= \int (t^5-2)\,t\cdot 5t^4\,dt =$$

$$= 5\int (t^5-2)\,t^5\,dt = 5\int (t^{10}-2t^5)\,dt =$$

$$= 5\int t^{10}\,dt - 10\int t^5\,dt =$$

$$= 5\cdot\frac{t^{11}}{11} - 10\cdot\frac{t^6}{6} + C =$$

$$= \frac{5}{11}t^{11} - \frac{5}{3}t^6 + C = 5t^6\left(\frac{t^5}{11}-\frac{1}{3}\right)+C =$$

$$= 5(x-2)\sqrt{x-2}\left(\frac{x-2}{11}+\frac{1}{3}\right)+C =$$

$$= 5(x-2)\sqrt{x-2}\left(\frac{3x-6+11}{33}\right)+C =$$

$$= (5x-10)\sqrt{x-2}\left(\frac{3x+5}{33}\right)+C$$

$$= \frac{1}{33}\sqrt{x-2}\,(5x-10)(3x+5)+C$$

№ 8.4.43

$$\int \frac{dx}{x^2\sqrt{x^2+1}} = \int x^{-2}(x^2+1)^{-\frac{1}{2}}\,dx =$$

$$= \Big[\, m=-2,\ n=2,\ p=-\frac{1}{2} \notin \mathbb{Z} \Rightarrow$$

$$\Rightarrow \nexists\ \frac{m+1}{n} = \frac{-2+1}{2} = -\frac{1}{2} \notin \mathbb{Z} \Rightarrow$$

$$\Rightarrow \exists\ \frac{m+1}{n}+p = \frac{-2+1}{2} - \frac{1}{2} = -1 \in \mathbb{Z} \Rightarrow$$

$$\Rightarrow x^{2}+1 = t^2;\ x = \frac{1}{\sqrt{t^2-1}};$$

$$dx = -\frac{t\,dt}{(t^2-1)\sqrt{t^2-1}};\ t = \frac{\sqrt{x^2+1}}{x}\,\Big] =$$

$$= \int (t^2-1)\left(\frac{1}{t^2-1}+1\right)^{-\frac{1}{2}} \cdot \left(-\frac{t\,dt}{(t^2-1)\sqrt{t^2-1}}\right) =$$

$$= -\int \cancel{(t^2-1)}\,\frac{\cancel{\sqrt{t^2-1}}}{\cancel{t}} \cdot \frac{\cancel{t}\,dt}{\cancel{(t^2-1)\sqrt{t^2-1}}} =$$

$$= -\int dt = -t + C = -\frac{\sqrt{x^2+1}}{x} + C$$