$$= \frac{x^2}{2}\operatorname{arctg}x + \frac{1}{2}\int\frac{x^2dx}{1+x^2} =$$

$$= \left[\int\frac{x^2dx}{1+x^2} = \int\frac{x^2+1-1}{1+x^2}dx =\right.$$

$$= \int\frac{x^2+1}{x^2+1}dx - \int\frac{dx}{1+x^2} =$$

$$\left.= \int dx - \int\frac{dx}{x^2+1} = x - \operatorname{arctg}x\right] =$$

$$= \frac{x^2}{2}\operatorname{arctg}x + \frac{1}{2}(x - \operatorname{arctg}x) + C =$$

$$= \frac{1}{2}\left(x^2\operatorname{arctg}x + x - \operatorname{arctg}x\right) + C$$

№8.2.28

$$\int e^x\sin x\,dx = \left[\begin{matrix} u = e^x \\ v' = \sin x\end{matrix} \Rightarrow \begin{matrix} u' = e^x \\ v = -\cos x\end{matrix}\right] =$$

$$= -e^x\cos x - \int e^x(-\cos x)dx =$$

$$= -e^x\cos x + \int e^x\cos x\,dx =$$

$$= \left[\begin{matrix} u = e^x \\ v' = \cos x\end{matrix} \Rightarrow \begin{matrix} u' = e^x \\ v = \sin x\end{matrix}\right] =$$

$$= -e^x\cos x + e^x\sin x - \int e^x\sin x\,dx$$

$$\int e^x\sin x\,dx = -e^x\cos x + e^x\sin x - \int e^x\sin x + C$$

$$\int e^x\sin x\,dx + \int e^x\sin x\,dx = e^x\sin x - e^x\cos x + C$$

$$2\int e^x\sin x\,dx = e^x\sin x - e^x\cos x + C$$

$$\int e^x \sin x\, dx = \frac{1}{2} e^x (\sin x - \cos x) + C$$

№ 8.2.29

$$\int \sin(\ln x)\, dx = \left[t = \ln x \Rightarrow dt = d(\ln x) = \frac{dx}{x} \Rightarrow x = e^t \right]$$

$$= \int \frac{\sin(\ln x)\, dx}{x} = \int e^t \sin t\, dt =$$

$= [$см. решение №8.2.28$] =$

$$= \frac{1}{2} e^t (\sin t - \cos t) + C =$$

$$= \frac{1}{2} e^{\ln x} (\sin \ln x - \cos \ln x) + C =$$

$$= \frac{1}{2} x (\sin \ln x - \cos \ln x) + C$$

Дом. $\int 1 \cdot \sin(\ln x)\, dx =$ [см. справ.]

№ 8.2.31

$$\int \arcsin x\, dx = \left[\begin{matrix} u = \arcsin x \\ v' = 1 \end{matrix} \Rightarrow \begin{matrix} u' = \frac{1}{\sqrt{1-x^2}} \\ v = x \end{matrix} \right] = x \arcsin x -$$

$$- \int \frac{x\, dx}{\sqrt{1-x^2}} = x \arcsin x + \frac{1}{2} \int \frac{-2x\, dx}{\sqrt{1-x^2}} =$$

$$= x\arcsin x + \frac{1}{2}\int \frac{d(1-x^2)}{\sqrt{1-x^2}} =$$

$$= x\arcsin x + \frac{1}{2}\int (1-x^2)^{-\frac{1}{2}} d(1-x^2) =$$

$$= x\arcsin x + \frac{1}{2}\cdot \frac{(1-x^2)^{\frac{1}{2}}}{\frac{1}{2}} + C =$$

$$= x\arcsin x + \sqrt{1-x^2} + C$$

w8.2.32

$$\int \frac{\ln(\ln x)}{x} dx = \left[\frac{dx}{x} = d(\ln x)\right] =$$

$$= \int \ln(\ln x)\, d(\ln x) = \left[\begin{matrix} u = \ln(\ln x) \\ v' = 1 \end{matrix} \Rightarrow \begin{matrix} u' = \frac{1}{\ln x} \\ v = \ln x \end{matrix}\right] = \ln(\ln x)\ln x -$$

$$- \int \frac{\ln x}{\ln x} d(\ln x) = \ln(\ln x)\ln x -$$

$$- \ln x + C = \ln x\,(\ln(\ln x) - 1) + C$$

Практика

Интегралы, часть 3

w8.3.2

$$\int \frac{4\,dx}{x+3} = 4\int \frac{d(x+3)}{x+3} = 4\ln|x+3| + C$$

w8.3.3

$$\int \frac{dx}{(x-1)^5} = \int \frac{d(x-1)}{(x-1)^5} = \int (x-1)^{-5}\,d(x-1) =$$

$$= \frac{(x-1)^{-4}}{-4} + C = -\frac{1}{4(x-1)^4} + C$$

w8.3.4

$$\int \frac{11\,dx}{(x+2)^3} = 11\int \frac{d(x+2)}{(x+2)^3} =$$

$$= 11\,\frac{(x+2)^{-2}}{-2} + C = -\frac{11}{2(x+2)^2} + C$$

w8.3.5

$$\int \frac{dx}{x^2+10x+29} = [\,x^2+10x+29=0 \Rightarrow$$

$\Rightarrow D = 100 - 4\cdot 29 < 0 \Rightarrow$ нет корней $] =$

$$= [\,A=0,\ B=1,\ p=10,\ q=29 \Rightarrow$$

$$\Rightarrow y = x + \frac{10}{2} = x+5 \Rightarrow dy = dx\,] =$$

$$= \int \frac{dy}{y^2+a^2} = [\,a = \sqrt{29 - \frac{10^2}{4}} =$$

$= \sqrt{29-25} = 2 \Big] = \int \frac{dy}{y^2+2^2} =$

$= \frac{2}{\sqrt{4\cdot 29-100}} \cdot \operatorname{arctg} \frac{2x+10}{\sqrt{4\cdot 29-100}} + C =$

$= \frac{1}{2} \operatorname{arctg}\left(\frac{x+5}{2}\right) + C$

w8.3.6

$\int \frac{(x+6)\,dx}{x^2-2x+17} = \Big[x^2-2x+17=0 \Rightarrow$

$\Rightarrow D = (-2)^2 - 4\cdot 17 < 0 \Rightarrow$ корней нет $\Rightarrow$

$\Rightarrow A=1,\ B=6,\ p=-2,\ q=17 \Big] =$

$= \int \frac{\frac{1}{2}(2x-2)+\left(6+\frac{2}{2}\right)}{x^2-2x+17}dx =$

$= \int \frac{\frac{1}{2}(2x-2)+7}{x^2-2x+17}dx =$

$= \frac{1}{2}\int \frac{2x-2}{x^2-2x+17}dx + 7\int \frac{dx}{x^2-2x+17} =$

$= \Big[1)\ t = x^2-2x+17 \Rightarrow dt = (2x-2)dx;$

$2)\ y = x-1 \Rightarrow dy = dx \Big] =$

$= \frac{1}{2}\int \frac{dt}{t} + 7\int \frac{dy}{y^2+a^2} = \Big[a = \sqrt{17-\frac{4}{4}} =$

$= \sqrt{17-1} = 4 \Big] = \frac{1}{2}\ln|t| + 7 \cdot \frac{1}{4} \cdot$

$\cdot \operatorname{arctg} \frac{y}{4} + C = \frac{1}{2}\ln(x^2 - 2x + 17) +$ т.к. ветви верх и нет корней

$+ \frac{7}{4} \operatorname{arctg} \frac{x-1}{4} + C$

№ 8.3.7

$\int \left(\frac{(4x-1)\,dx}{x^2+x+1}\right) = \Big[\, x^2 + x + 1 = 0 \Rightarrow$

$\Rightarrow D = 1 - 4 \cdot 1 = -3 < 0 \Rightarrow$ корней нет;

$A = 4,\ B = -1,\ p = 1,\ q = 1 \Big] =$

$= \int \frac{\frac{4}{2}(2x+1) + \left(-1 - \frac{4 \cdot 1}{2}\right)}{x^2+x+1}$