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Решение СЛАУ с помощью обратной матрицы. Формулы Крамера

№2.2.16

$f(x) = ax^2 + bx + c$

$f(-2) = -8$

$f(1) = 4$

$f(2) = -4$

$$\begin{cases} 4a - 2b + c = -8 \\ a + b + c = 4 \\ 4a + 2b + c = -4 \end{cases}$$

$$A = \begin{pmatrix} 4 & -2 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{pmatrix}, \quad X = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \quad B = \begin{pmatrix} -8 \\ 4 \\ -4 \end{pmatrix}$$

$$\det A = \begin{vmatrix} 4 & -2 & 1 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{vmatrix} = \cancel{4} + 2 - 8 \cancel{-4} - 8 + 2 =$$

$$= -12 \neq 0$$

I) $A^{-1} = \frac{1}{\det A} \tilde{A}$

$$A_{11} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = 1 - 2 = -1$$

$$A_{12} = -\begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} = -1 + 4 = 3$$

$$A_{13} = \begin{vmatrix} 1 & 1 \\ 4 & 2 \end{vmatrix} = 2 - 4 = -2$$

$$A_{21} = -\begin{vmatrix} -2 & 1 \\ 2 & 1 \end{vmatrix} = 2 + 2 = 4$$

$$A_{22} = \begin{vmatrix} 4 & 1 \\ 4 & 1 \end{vmatrix} = 4 - 4 = 0$$

$$A_{23} = -\begin{vmatrix} 4 & -2 \\ 4 & 2 \end{vmatrix} = -8 - 8 = -16$$

$$A_{31} = \begin{vmatrix} -2 & 1 \\ 1 & 1 \end{vmatrix} = -2 - 1 = -3$$

$$A_{32} = -\begin{vmatrix} 4 & 1 \\ 1 & 1 \end{vmatrix} = -4 + 1 = -3$$

$$A_{33} = \begin{vmatrix} 4 & -2 \\ 1 & 1 \end{vmatrix} = 4 + 2 = 6$$

$$\tilde{A} = \begin{pmatrix} -1 & 4 & -3 \\ 3 & 0 & -3 \\ -2 & -16 & 6 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-12}\begin{pmatrix} -1 & 4 & -3 \\ 3 & 0 & -3 \\ -2 & -16 & 6 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{1}{12} & -\frac{1}{3} & \frac{1}{4} \\ -\frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{6} & \frac{4}{3} & -\frac{1}{2} \end{pmatrix}$$

$$X = A^{-1}B = \begin{pmatrix} \frac{1}{12} & -\frac{1}{3} & \frac{1}{4} \\ -\frac{1}{4} & 0 & \frac{1}{4} \\ \frac{1}{6} & \frac{4}{3} & -\frac{1}{2} \end{pmatrix} \cdot \begin{pmatrix} -8 \\ 4 \\ -4 \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{2}{3} - \frac{4}{3} - 1 \\ 2 + 0 - 1 \\ -\frac{4}{3} + \frac{16}{3} + 2 \end{pmatrix} = \begin{pmatrix} -3 \\ 1 \\ 6 \end{pmatrix}$$

II)

$$\Delta_1 = \begin{vmatrix} -8 & -2 & 1 \\ 4 & 1 & 1 \\ -4 & 2 & 1 \end{vmatrix} = \cancel{-8} \cancel{+8} + 8 +$$

$$+4 + 16 + 8 = 36$$

$$x_1 = \frac{36}{-12} = -3$$

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$$\Delta_2 = \begin{vmatrix} 4 & -8 & 1 \\ 1 & 4 & 1 \\ 4 & -4 & 1 \end{vmatrix} = \cancel{-16} - 4 - 32 \cancel{-16} +$$

$$+16 + 8 = -12$$

$$x_2 = \frac{-12}{-12} = 1$$

$$\Delta_3 = \begin{vmatrix} 4 & -2 & -8 \\ 1 & 1 & 4 \\ 4 & 2 & -4 \end{vmatrix} = \cancel{-16 - 16} + 32 +$$

$$\cancel{+32} - 32 - 8 = -72$$

$$x_3 = \frac{-72}{-12} = 6$$

<u>Ответ:</u> $(-3; 1; 6)$

<u>№2.2.17</u>

$f(x) = a \cdot 3^x + bx^2 + c$

$f(0) = 2$

$f(1) = -1$

$f(2) = 4$

$a - ?,\ b - ?,\ c - ?$

$$\begin{cases} a+c=2 \\ 3a+b+c=-1 \\ 9a+4b+c=4 \end{cases}$$

$A=\begin{pmatrix} 1 & 0 & 1 \\ 3 & 1 & 1 \\ 9 & 4 & 1 \end{pmatrix}$, $X=\begin{pmatrix} a \\ b \\ c \end{pmatrix}$, $B=\begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix}$

$\det A=\begin{vmatrix} 1 & 0 & 1 \\ 3 & 1 & 1 \\ 9 & 4 & 1 \end{vmatrix}=1+12+0-9-4-0=$

$=0 \Rightarrow$ невозможно пример решить с помощью обратной матрицы и формулы Крамера.

№2.2.18

$$\begin{cases} x_1 - x_2 = 5 \\ 2x_1 + x_2 = 1 \end{cases}$$

$$A = \begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad B = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$\det A = \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 1 + 2 = 3 \neq 0$$

I) $\Delta_1 = \begin{vmatrix} 5 & -1 \\ 1 & 1 \end{vmatrix} = 5 + 1 = 6$

$$x_1 = \frac{6}{3} = 2$$

$$\Delta_2 = \begin{vmatrix} 1 & 5 \\ 2 & 1 \end{vmatrix} = 1 - 10 = -9$$

$$x_2 = \frac{-9}{3} = -3$$

II) $A^{-1} = \frac{1}{\det A} \begin{pmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{pmatrix}$

$$A^{-1} = \frac{1}{3} \begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

$$X = A^{-1} \cdot B = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} \\ -\frac{2}{3} & \frac{1}{3} \end{pmatrix} \begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{6}{3} \\ -\frac{9}{3} \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

Ответ: $(2; -3)$

№2.2.19

$$\begin{cases} x_1 - \sqrt{5}\,x_2 = 0 \\ 2\sqrt{5}\,x_1 - 5x_2 = -10 \end{cases}$$

$$A = \begin{pmatrix} 1 & -\sqrt{5} \\ 2\sqrt{5} & -5 \end{pmatrix},\quad X = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix},\quad B = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$$

$$\det A = \begin{vmatrix} 1 & -\sqrt{5} \\ 2\sqrt{5} & -5 \end{vmatrix} = -5 + 10 = 5 \neq 0$$

I) $$\Delta_1 = \begin{vmatrix} 0 & -\sqrt{5} \\ -10 & -5 \end{vmatrix} = -10\sqrt{5}$$

$$x_1 = \frac{-10\sqrt{5}}{5} = -2\sqrt{5}$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 \\ 2\sqrt{5} & -10 \end{vmatrix} = -10$$

$$x_2 = \frac{-10}{5} = -2$$

II) $$A^{-1} = \frac{1}{5}\begin{pmatrix} -5 & \sqrt{5} \\ -2\sqrt{5} & 1 \end{pmatrix} = \begin{pmatrix} -1 & \frac{\sqrt{5}}{5} \\ -\frac{2\sqrt{5}}{5} & \frac{1}{5} \end{pmatrix}$$

$$X = A^{-1}B = \begin{pmatrix} -1 & \frac{\sqrt{5}}{5} \\ -\frac{2\sqrt{5}}{5} & \frac{1}{5} \end{pmatrix}\begin{pmatrix} 0 \\ -10 \end{pmatrix} = \begin{pmatrix} -2\sqrt{5} \\ -2 \end{pmatrix}$$

Ответ: $(-2\sqrt{5}, -2)$

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№2.2.20

$$\begin{cases} \alpha x - y = 2 \\ 2x + \alpha y = 1 \end{cases}$$

$$A = \begin{pmatrix} \alpha & -1 \\ 2 & \alpha \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \end{pmatrix}, \quad B = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

$$\det A = \begin{vmatrix} \alpha & -1 \\ 2 & \alpha \end{vmatrix} = \alpha^2 + 2 \neq 0$$

при любых значениях α

I) $$\Delta_1 = \begin{vmatrix} 2 & -1 \\ 1 & \alpha \end{vmatrix} = 2\alpha + 1$$

$$x_1 = \frac{2\alpha + 1}{\alpha^2 + 2}$$

$$\Delta_2 = \begin{vmatrix} \alpha & 2 \\ 2 & 1 \end{vmatrix} = \alpha - 4$$

$$x_2 = \frac{\alpha - 4}{\alpha^2 + 2}$$

II) $$A^{-1} = \frac{1}{\alpha^2+2} \begin{pmatrix} \alpha & 1 \\ -2 & \alpha \end{pmatrix} = \begin{pmatrix} \frac{\alpha}{\alpha^2+2} & \frac{1}{\alpha^2+2} \\ -\frac{2}{\alpha^2+2} & \frac{\alpha}{\alpha^2+2} \end{pmatrix}$$

$$X = A^{-1}B = \begin{pmatrix} \frac{\alpha}{\alpha^2+2} & \frac{1}{\alpha^2+2} \\ -\frac{2}{\alpha^2+2} & \frac{\alpha}{\alpha^2+2} \end{pmatrix} \begin{pmatrix} 2 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{2\alpha}{\alpha^2+2} + \frac{1}{\alpha^2+2} \\ -\frac{4}{\alpha^2+2} + \frac{\alpha}{\alpha^2+2} \end{pmatrix} =$$

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$$= \begin{pmatrix} \frac{2\alpha+1}{\alpha^2+2} \\ \frac{\alpha-4}{\alpha^2+2} \end{pmatrix}$$

<u>Ответ:</u> $\left(\frac{2\alpha+1}{\alpha^2+2}; \frac{\alpha-4}{\alpha^2+2}\right)$

<u>№2.2.21</u>

$$\begin{cases} ax+3by=1 \\ bx+3ay=1 \end{cases}$$

$$A=\begin{pmatrix} a & 3b \\ b & 3a \end{pmatrix},\ X=\begin{pmatrix} x \\ y \end{pmatrix},\ B=\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\det A = \begin{vmatrix} a & 3b \\ b & 3a \end{vmatrix} = 3a^2 - 3b^2 = 3(a-b)(a+b)$$

При ~~$\alpha \neq k$ и $b \neq k$~~ $a=b$ $\det A = 0$, т.е. нельзя решить пример с помощью обратной матрицы и формул Крамера

$\exists$ $a \neq b$, тогда

I) $\Delta_1 = \begin{vmatrix} 1 & 3b \\ 1 & 3a \end{vmatrix} = 3a-3b$

$$x_1 = \frac{3a^2-3b^2}{3a-3b} = \frac{3(a^2-b^2)}{3(a-b)} =$$

$$= \frac{(a-b)(a+b)}{a-b} = a+b$$

$$x\,\Delta_2 = \begin{vmatrix} a & 1 \\ b & 1 \end{vmatrix} = a-b$$

$$x_2 = \frac{a-b}{a-b}$$

$$\Delta_1 = \begin{vmatrix} 1 & 3b \\ 1 & 3a \end{vmatrix} = 3a-3b = 3(a-b)$$

$$x_1 = \frac{3(a-b)}{3(a-b)(a+b)} = \frac{1}{a+b} = (a+b)^{-1}$$

$$\Delta_2 = \begin{vmatrix} a & 1 \\ b & 1 \end{vmatrix} = a-b$$

$$x_2 = \frac{a-b}{3(a-b)(a+b)} = \frac{1}{3a+3b}$$

II) $$A^{-1} = \frac{1}{3a^2-3b^2}\begin{pmatrix} 3a & -3b \\ -b & a \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{a}{a^2-b^2} & \frac{-b}{a^2-b^2} \\ \frac{-b}{3a^2-3b^2} & \frac{a}{3a^2-3b^2} \end{pmatrix}$$

$$X = A^{-1}B = \begin{pmatrix} \frac{a}{a^2-b^2} & \frac{-b}{a^2-b^2} \\ \frac{-b}{3a^2-3b^2} & \frac{a}{3a^2-3b^2} \end{pmatrix}\begin{pmatrix} 1 \\ 1 \end{pmatrix} =$$

$$=\begin{pmatrix} \dfrac{a-b}{(a-b)(a+b)} \\ \dfrac{a-b}{3(a+b)(a-b)} \end{pmatrix} = \begin{pmatrix} \dfrac{1}{a+b} \\ \dfrac{1}{3a+3b} \end{pmatrix}$$

Ответ: $\left(\dfrac{1}{a+b};\ \dfrac{1}{3a+3b}\right)$

№2.2.22

$$\begin{cases} x+2y+3z=8 \\ 4x+5y+6z=19 \\ 7x+8y=1 \end{cases}$$

$$A=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix},\ X=\begin{pmatrix} x \\ y \\ z \end{pmatrix},\ B=\begin{pmatrix} 8 \\ 19 \\ 1 \end{pmatrix}$$

$$\det A=\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix}=27\neq 0 \ (\text{см. №2.2.8})$$

I) $\Delta_1=\begin{vmatrix} 8 & 2 & 3 \\ 19 & 5 & 6 \\ 1 & 8 & 0 \end{vmatrix}=0+19\cdot 8\cdot 3+2\cdot 6\cdot 1-$

$-1\cdot 5\cdot 3-8\cdot 8\cdot 6-0=456+12-15-384=$

$=69$

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$\cancel{x} = \frac{69}{27} = \frac{23}{9} = 2\frac{5}{9}$

$\Delta_2 = \begin{vmatrix} 1 & 8 & 3 \\ 4 & 19 & 6 \\ 7 & 1 & 0 \end{vmatrix} = 0 + 4\cdot3\cdot1 + 8\cdot6\cdot7 -$

$- 7\cdot19\cdot3 - 1\cdot1\cdot6 - 0 = 12 + 336 - 399 - 6 =$

$= -57$

$\cancel{y} = \frac{-57}{27} = \frac{-19}{9} = -2\frac{1}{9}$

$\Delta_3 = \begin{vmatrix} 1 & 2 & 8 \\ 4 & 5 & 19 \\ 7 & 8 & 1 \end{vmatrix} = 1\cdot5\cdot1 + 4\cdot8\cdot8 + 2\cdot19\cdot7 -$

$- 7\cdot5\cdot8 - 8\cdot19\cdot1 - 4\cdot2\cdot1 = 5 + 256 + 266 -$

$- 280 - 152 - 8 = 87$

$\cancel{z}\, z = \frac{87}{27} = \frac{29}{9} = 3\frac{2}{9}$

$\underline{\text{II}})\ A^{-1} = \frac{1}{\det A}\tilde{A}$

$\cancel{A_{11} = \begin{vmatrix} 5 & 6 \\ 8 & 0 \end{vmatrix} = -48}$

$A^{-1} = \begin{pmatrix} -\frac{16}{9} & \frac{8}{9} & -\frac{1}{9} \\ \frac{14}{9} & -\frac{7}{9} & \frac{2}{9} \\ -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \end{pmatrix}$ (см. №2.2.8)

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$$X = A^{-1}B = \begin{pmatrix} -\frac{16}{9} & \frac{8}{9} & -\frac{1}{9} \\ \frac{14}{9} & -\frac{7}{9} & \frac{2}{9} \\ -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \end{pmatrix} \begin{pmatrix} 8 \\ 19 \\ 1 \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{16}{9}\cdot 8 + \frac{8}{9}\cdot 19 - \frac{1}{9}\cdot 1 \\ \frac{14}{9}\cdot 8 - \frac{7}{9}\cdot 19 + \frac{2}{9}\cdot 1 \\ -\frac{1}{9}\cdot 8 + \frac{2}{9}\cdot 19 - \frac{1}{9}\cdot 1 \end{pmatrix} = \begin{pmatrix} \frac{23}{9} \\ -\frac{19}{9} \\ \frac{29}{9} \end{pmatrix} = \begin{pmatrix} 2\frac{5}{9} \\ -2\frac{1}{9} \\ 3\frac{2}{9} \end{pmatrix}$$

P.S $\det A = \begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{vmatrix} = 0 + 4\cdot 8\cdot 3 + 2\cdot 6\cdot 7 -$

$- 7\cdot 5\cdot 3 - 8\cdot 6\cdot 1 - 0 = 96 + 84 - 105 - 48 =$

$= 27$

$$A^{-1} = \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 4 & 5 & 6 & 0 & 1 & 0 \\ 7 & 8 & 0 & 0 & 0 & 1 \end{array}\right) \begin{matrix} \\ II - 4I \\ III - 7I \end{matrix} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & -3 & -6 & -4 & 1 & 0 \\ 0 & -6 & -21 & -7 & 0 & 1 \end{array}\right) \begin{matrix} \\ \cdot(-1) \\ \cdot(-1) \end{matrix} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 3 & 6 & 4 & -1 & 0 \\ 0 & 6 & 21 & 7 & 0 & -1 \end{array}\right) \begin{matrix} \\ \\ III - 2II \end{matrix} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 3 & 6 & 4 & -1 & 0 \\ 0 & 0 & 9 & -1 & 2 & -1 \end{array}\right) \begin{matrix} \\ \cdot\frac{1}{3} \\ \cdot\frac{1}{9} \end{matrix} \sim$$

$$\sim\left(\begin{array}{ccc|ccc} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 1 & 2 & \frac{4}{3} & -\frac{1}{3} & 0 \\ 0 & 0 & 1 & -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \end{array}\right) \begin{array}{l} \mathrm{I} - 3\,\mathrm{III} \\ \mathrm{II} - 2\,\mathrm{III} \\ \end{array} \sim$$

$$\sim\left(\begin{array}{ccc|ccc} 1 & 2 & 0 & \frac{4}{3} & -\frac{2}{3} & \frac{1}{3} \\ 0 & 1 & 0 & \frac{14}{9} & -\frac{7}{9} & \frac{2}{9} \\ 0 & 0 & 1 & -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \end{array}\right) \begin{array}{l} \mathrm{I} - 2\,\mathrm{II} \\ \\ \end{array} \sim$$

$$\sim\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{16}{9} & \frac{8}{9} & -\frac{1}{9} \\ 0 & 1 & 0 & \frac{14}{9} & -\frac{7}{9} & \frac{2}{9} \\ 0 & 0 & 1 & -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \end{array}\right)$$

Ответ: $\left(2\frac{5}{9};\ -2\frac{1}{9};\ 3\frac{2}{9}\right)$

№2.2.23

$$\begin{cases} x_1 + 2x_2 + 3x_3 = 4 \\ 2x_1 + 6x_2 + 4x_3 = -6 \\ 3x_1 + 10x_2 + 8x_3 = -8 \end{cases}$$

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 6 & 4 \\ 3 & 10 & 8 \end{pmatrix},\quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix},\quad B = \begin{pmatrix} 4 \\ -6 \\ -8 \end{pmatrix}$$

$$\det A = \begin{vmatrix} 1 & 2 & 3 \\ 2 & 6 & 4 \\ 3 & 10 & 8 \end{vmatrix} = 6\cdot8\cdot1 + 2\cdot10\cdot3 +$$

$$+2\cdot4\cdot3 - 3\cdot3\cdot6 - 10\cdot4\cdot1 - 2\cdot2\cdot8 =$$

$$= 48 + 60 + 24 - 54 - 40 - 32 = 6 \neq 0$$

I) $$\Delta_1 = \begin{vmatrix} 4 & 2 & 3 \\ -6 & 6 & 4 \\ -8 & 10 & 8 \end{vmatrix} = 4\cdot6\cdot8 - 6\cdot10\cdot3 -$$

$$-8\cdot2\cdot4 + 8\cdot6\cdot3 - 10\cdot4\cdot4 + 6\cdot8\cdot2 =$$

$$= 192 - 180 - 64 + 144 - 160 + 96 = 28$$

$$x_1 = \frac{28}{6} = \frac{14}{3} = 4\frac{2}{3}$$

$$\Delta_2 = \begin{vmatrix} 1 & 4 & 3 \\ 2 & -6 & 4 \\ 3 & -8 & 8 \end{vmatrix} = -6\cdot1\cdot8 - 8\cdot2\cdot3 +$$

$$+4\cdot4\cdot3 + 6\cdot3\cdot3 + 8\cdot4\cdot1 - 2\cdot4\cdot8 =$$

$$= -48 - \cancel{48} + \cancel{48} + 54 + 32 - 64 = -26$$

$$x_2 = \frac{-26}{6} = -\frac{13}{3} = -4\frac{1}{3}$$

$$\Delta_3 = \begin{vmatrix} 1 & 2 & 4 \\ 2 & 6 & -6 \\ 3 & 10 & -8 \end{vmatrix} = -8\cdot6\cdot1 - 6\cdot2\cdot3 +$$

$$+2\cdot10\cdot4 - 3\cdot6\cdot4 + 6\cdot10\cdot1 + 8\cdot2\cdot2 =$$

$$= -48 - 36 + 80 - 72 + 60 + 32 = 16$$

$$X_3 = \frac{16}{6} = \frac{8}{3} = 2\frac{2}{3}$$

II) $A^{-1} = \frac{1}{\det A}\tilde{A}$

$$A_{11} = \begin{vmatrix} 6 & 4 \\ 10 & 8 \end{vmatrix} = 48 - 40 = 8$$

$$A_{12} = -\begin{vmatrix} 2 & 4 \\ 3 & 8 \end{vmatrix} = -16 + 12 = -4$$

$$A_{13} = \begin{vmatrix} 2 & 6 \\ 3 & 10 \end{vmatrix} = 20 - 18 = 2$$

$$A_{21} = -\begin{vmatrix} 2 & 3 \\ 10 & 8 \end{vmatrix} = -16 + 30 = 14$$

$$A_{22} = \begin{vmatrix} 1 & 3 \\ 3 & 8 \end{vmatrix} = 8 - 9 = -1$$

$$A_{23} = -\begin{vmatrix} 1 & 2 \\ 3 & 10 \end{vmatrix} = -10 + 6 = -4$$

$$A_{31} = \begin{vmatrix} 2 & 3 \\ 6 & 4 \end{vmatrix} = 8 - 18 = -10$$

$$A_{32} = -\begin{vmatrix} 1 & 3 \\ 2 & 4 \end{vmatrix} = -4 + 6 = 2$$

$$A_{33} = \begin{vmatrix} 1 & 2 \\ 2 & 6 \end{vmatrix} = 6 - 4 = 2$$

$$\tilde{A} = \begin{pmatrix} 8 & 14 & -10 \\ -4 & -1 & 2 \\ 2 & -4 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{6}\begin{pmatrix} 8 & 14 & -10 \\ -4 & -1 & 2 \\ 2 & -4 & 2 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{4}{3} & \frac{7}{3} & -\frac{5}{3} \\ -\frac{2}{3} & -\frac{1}{6} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

$$X = A^{-1}B = \begin{pmatrix} \frac{4}{3} & \frac{7}{3} & -\frac{5}{3} \\ -\frac{2}{3} & -\frac{1}{6} & \frac{1}{3} \\ \frac{1}{3} & -\frac{2}{3} & \frac{1}{3} \end{pmatrix}\begin{pmatrix} 4 \\ -6 \\ -8 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{4}{3}\cdot 4 + \frac{7}{3}\cdot(-6) - \frac{5}{3}\cdot(-8) \\ -\frac{2}{3}\cdot 4 - \frac{1}{6}\cdot(-6) + \frac{1}{3}\cdot(-8) \\ \frac{1}{3}\cdot 4 - \frac{2}{3}\cdot(-6) + \frac{1}{3}\cdot(-8) \end{pmatrix} = \begin{pmatrix} \frac{14}{3} \\ -\frac{13}{3} \\ \frac{8}{3} \end{pmatrix} = \begin{pmatrix} 4\frac{2}{3} \\ -4\frac{1}{3} \\ 2\frac{2}{3} \end{pmatrix}$$

Ответ: $\left(4\frac{2}{3}; -4\frac{1}{3}; 2\frac{2}{3}\right)$

№ 2.2.24

$$\begin{cases} 3x + 2y + z = 1 \\ 6x + 5y + 4z = -2 \\ 9x + 8y + 7z = 3 \end{cases}$$

$$A=\begin{pmatrix}3&2&1\\6&5&4\\9&8&7\end{pmatrix},\ X=\begin{pmatrix}x\\y\\z\end{pmatrix},\ B=\begin{pmatrix}1\\-2\\3\end{pmatrix}$$

$$\det A=\begin{vmatrix}3&2&1\\6&5&4\\9&8&7\end{vmatrix}=3\cdot5\cdot7+6\cdot8\cdot1+2\cdot4\cdot9-$$

$$-9\cdot5\cdot1-6\cdot2\cdot7-8\cdot4\cdot3=105+48+$$

$$+72-45-84-96=0\ =>$$

=> пример нельзя решить с помощью обратной матрицы и формулы Крамера

№ 2.2.25

$$\begin{cases}3x+2y+z=-8\\2x+3y+z=-3\\2x+y+3z=-1\end{cases}$$

$$A=\begin{pmatrix}3&2&1\\2&3&1\\2&1&3\end{pmatrix},\ X=\begin{pmatrix}x\\y\\z\end{pmatrix},\ B=\begin{pmatrix}-8\\-3\\-1\end{pmatrix}$$

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$$\det A = \begin{vmatrix} 3 & 2 & 1 \\ 2 & 3 & 1 \\ 2 & 1 & 3 \end{vmatrix} = 3\cdot3\cdot3 + 2\cdot1\cdot1 +$$

$$+2\cdot2\cdot1 - 2\cdot3\cdot1 - 2\cdot2\cdot3 - 1\cdot1\cdot3 =$$

$$= 27 + 2 + 4 - 6 - 12 - 3 = 12 \neq 0$$

I) $$\Delta_1 = \begin{vmatrix} -8 & 2 & 1 \\ -3 & 3 & 1 \\ -1 & 1 & 3 \end{vmatrix} = -8\cdot3\cdot3 - 3\cdot1\cdot1 -$$

$$-1\cdot1\cdot2 + 1\cdot3\cdot1 + 8\cdot1\cdot1 + 3\cdot2\cdot3 =$$

$$= -72 - 3 - 2 + 3 + 8 + 18 = -48$$

$$x = \frac{-48}{12} = -4$$

$$\Delta_2 = \begin{vmatrix} 3 & -8 & 1 \\ 2 & -3 & 1 \\ 2 & -1 & 3 \end{vmatrix} = -3\cdot3\cdot3 - 1\cdot2\cdot1 -$$

$$-8\cdot1\cdot2 + 3\cdot2\cdot1 + 8\cdot2\cdot3 + 1\cdot1\cdot3 =$$

$$= -27 - 2 - 16 + 6 + 48 + 3 = 12$$

$$y = \frac{12}{12} = 1$$

$$\Delta_3 = \begin{vmatrix} 3 & 2 & -8 \\ 2 & 3 & -3 \\ 2 & 1 & -1 \end{vmatrix} = -1\cdot3\cdot3 - 8\cdot2\cdot1 -$$

$$-3\cdot2\cdot2 + 8\cdot3\cdot2 + 3\cdot1\cdot3 + 1\cdot2\cdot2 =$$

$$= \cancel{-9} - 16 - 12 + 48 \cancel{+9} + 4 = 24$$

$z = \frac{24}{12} = 2$

II) $A^{-1} = \frac{1}{\det A} \tilde{A}$

$A_{11} = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = 9 - 1 = 8$

$A_{12} = -\begin{vmatrix} 2 & 1 \\ 2 & 3 \end{vmatrix} = -6 + 2 = -4$

$A_{13} = \begin{vmatrix} 2 & 3 \\ 2 & 1 \end{vmatrix} = 2 - 6 = -4$

$A_{21} = -\begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = -6 + 1 = -5$

$A_{22} = \begin{vmatrix} 3 & 1 \\ 2 & 3 \end{vmatrix} = 9 - 2 = 7$

$A_{23} = -\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} = -3 + 4 = 1$

$A_{31} = \begin{vmatrix} 2 & 1 \\ 3 & 1 \end{vmatrix} = 2 - 3 = -1$

$A_{32} = -\begin{vmatrix} 3 & 1 \\ 2 & 1 \end{vmatrix} = -3 + 2 = -1$

$A_{33} = \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = 9 - 4 = 5$

$$\tilde{A} = \begin{pmatrix} 8 & -5 & -1 \\ -4 & 7 & -1 \\ -4 & 1 & 5 \end{pmatrix}$$

$$A^{-1} = \frac{1}{12}\begin{pmatrix} 8 & -5 & -1 \\ -4 & 7 & -1 \\ -4 & 1 & 5 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{2}{3} & -\frac{5}{12} & -\frac{1}{12} \\ -\frac{1}{3} & \frac{7}{12} & -\frac{1}{12} \\ -\frac{1}{3} & \frac{1}{12} & \frac{5}{12} \end{pmatrix}$$

$$X = A^{-1}B$$

$$X = \begin{pmatrix} \frac{2}{3} & -\frac{5}{12} & -\frac{1}{12} \\ -\frac{1}{3} & \frac{7}{12} & -\frac{1}{12} \\ -\frac{1}{3} & \frac{1}{12} & \frac{5}{12} \end{pmatrix} \cdot \begin{pmatrix} -8 \\ -3 \\ -1 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{2}{3}\cdot(-8) - \frac{5}{12}\cdot(-3) - \frac{1}{12}\cdot(-1) \\ -\frac{1}{3}\cdot(-8) + \frac{7}{12}\cdot(-3) - \frac{1}{12}\cdot(-1) \\ -\frac{1}{3}\cdot(-8) + \frac{1}{12}\cdot(-3) + \frac{5}{12}\cdot(-1) \end{pmatrix} = \begin{pmatrix} -\frac{48}{12} \\ \frac{12}{12} \\ \frac{24}{12} \end{pmatrix}$$

Ответ: $(-4; 1; 2)$

№2.2.26

$$\begin{cases} ax + by + z = 1 \\ x + aby + z = b \\ x + by + az = 1 \end{cases}$$

$$A = \begin{pmatrix} a & b & 1 \\ 1 & ab & 1 \\ 1 & b & a \end{pmatrix}, \quad X = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \quad B = \begin{pmatrix} 1 \\ b \\ 1 \end{pmatrix}$$

$$\det A = \begin{vmatrix} a & b & 1 \\ 1 & ab & 1 \\ 1 & b & a \end{vmatrix} = a^3 b + b + b - ab -$$

$$- ab - ab = a^3 b + 2b - 3ab = b(a^3 + 2 - 3a)$$

При $b = 0$ $\det A = 0$ => пример нельзя решить с помощью формул Крамера и обратной матрицы.

] $b \neq 0$, тогда

I) $$\Delta_1 = \begin{vmatrix} 1 & b & 1 \\ b & ab & 1 \\ 1 & b & a \end{vmatrix} = a^2 b + b^2 \cancel{+ b} -$$

$$- ab \cancel{- b} - ab^2 = a^2 b - ab^2 + b^2 -$$

$$- ab = b(a^2 - ab + b - a)$$

$$x = \frac{b(a^2-ab+b-a)}{b(a^3-3a+2)} = \frac{a^2-ab+b-a}{a^3-3a+2}$$

$$\Delta_2 = \begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & a \end{vmatrix} = a^2b+1+1-b-a-a =$$

$$= a^2b-2a-b+2$$

$$y = \frac{a^2b-2a-b+2}{\cancel{b}a^3b-3ab+2b}$$

$$\Delta_3 = \begin{vmatrix} a & b & 1 \\ 1 & ab & b \\ 1 & b & 1 \end{vmatrix} = a^2b+\cancel{b}+b^2-$$

$$-ab-ab^2-\cancel{b} = a^2b-ab^2+b^2-ab =$$

$$\cancel{z} = \underbrace{a^2b-ab^2}+\underbrace{b^2-ab}$$

$$= b(a^2-ab+b-a)$$

$$z = \frac{b(a^2-ab+b-a)}{b(a^3-3a+2)} =$$

$$= \frac{a^2-ab+b-a}{a^3-3a+2}$$

II) $A^{-1} = \frac{1}{\det A}\tilde{A}$

$$A_{11} = \begin{vmatrix} ab & 1 \\ b & a \end{vmatrix} = a^2b - b = b(a^2-1)$$

$$A_{12} = -\begin{vmatrix} 1 & 1 \\ 1 & a \end{vmatrix} = -a+1 = 1-a$$

$$A_{13} = \begin{vmatrix} 1 & ab \\ 1 & b \end{vmatrix} = b - ab = b(1-a)$$

$$A_{21} = -\begin{vmatrix} b & 1 \\ b & a \end{vmatrix} = -ab+b = b(1-a)$$

$$A_{22} = \begin{vmatrix} a & 1 \\ 1 & a \end{vmatrix} = a^2-1$$

$$A_{23} = -\begin{vmatrix} a & b \\ 1 & b \end{vmatrix} = -ab+b = b(1-a)$$

$$A_{31} = \begin{vmatrix} b & 1 \\ ab & 1 \end{vmatrix} = b - ab = b(1-a)$$

$$A_{32} = -\begin{vmatrix} a & 1 \\ 1 & 1 \end{vmatrix} = -a+1 = 1-a$$

$$A_{33} = \begin{vmatrix} a & b \\ 1 & ab \end{vmatrix} = a^2b - b = b(a^2-1)$$

$$\tilde{A} = \begin{pmatrix} b(a^2-1) & b(1-a) & b(1-a) \\ 1-a & a^2-1 & 1-a \\ b(1-a) & b(1-a) & b(a^2-1) \end{pmatrix}$$

$$A^{-1}=\frac{1}{b(a^3-3a+2)}\begin{pmatrix} b(a^2-1) & b(1-a) & b(1-a) \\ 1-a & a^2-1 & 1-a \\ b(1-a) & b(1-a) & b(a^2-1) \end{pmatrix}=$$

$$=\begin{pmatrix} \frac{a^2-1}{a^3-3a+2} & \frac{1-a}{a^3-3a+2} & \frac{1-a}{a^3-3a+2} \\ \frac{1-a}{a^3b-3ab+2b} & \frac{a^2-1}{a^3b-3ab+2b} & \frac{1-a}{a^3b-3ab+2b} \\ \frac{1-a}{a^3-3a+2} & \frac{1-a}{a^2-3a+2} & \frac{a^2-1}{a^2-3a+2} \end{pmatrix}$$

$$X=A^{-1}B$$

$$X=\begin{pmatrix} \frac{a^2-1}{a^3-3a+2} & \frac{1-a}{a^3-3a+2} & \frac{1-a}{a^3-3a+2} \\ \frac{1-a}{a^3b-3ab+2b} & \frac{a^2-1}{a^3b-3ab+2b} & \frac{1-a}{a^3b-3ab+2b} \\ \frac{1-a}{a^3-3a+2} & \frac{1-a}{a^3-3a+2} & \frac{a^2-1}{a^2-3a+2} \end{pmatrix}\cdot\begin{pmatrix} 1 \\ b \\ 1 \end{pmatrix}=$$

$$=\begin{pmatrix} \frac{a^2-ab+b-a}{a^3-3a+2} \\ \frac{a^2b-2a-b+2}{a^3b-3ab+2b} \\ \frac{a^2-ab+b-a}{a^3-3a+2} \end{pmatrix}$$

Ответ: $\left(\frac{a^2-ab+b-a}{a^3-3a+2};\ \frac{a^2b-2a-b+2}{a^3b-3ab+2b};\ \frac{a^2-ab+b-a}{a^3-3a+2}\right)$

W2.2.27

$$\begin{cases} 2x_1 + x_2 + 4x_3 + 8x_4 = 0 \\ x_1 + 3x_2 - 6x_3 + 2x_4 = 0 \\ 3x_1 - 2x_2 + 2x_3 - 2x_4 = 0 \\ 2x_1 - x_2 + 2x_3 = 0 \end{cases}$$

$$A = \begin{pmatrix} 2 & 1 & 4 & 8 \\ 1 & 3 & -6 & 2 \\ 3 & -2 & 2 & -2 \\ 2 & -1 & 2 & 0 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \quad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\det A = \begin{vmatrix} 2 & 1 & 4 & 8 \\ 1 & 3 & -6 & 2 \\ 3 & -2 & 2 & -2 \\ 2 & -1 & 2 & 0 \end{vmatrix} \leftarrow = 2\cdot(-1)^{4+1} \begin{vmatrix} 1 & 4 & 8 \\ 3 & -6 & 2 \\ -2 & 2 & -2 \end{vmatrix} -$$

$$-1\cdot(-1)^{4+2} \begin{vmatrix} 2 & 4 & 8 \\ 1 & -6 & 2 \\ 3 & 2 & -2 \end{vmatrix} + 2\cdot(-1)^{4+3} \begin{vmatrix} 2 & 1 & 8 \\ 1 & 3 & 2 \\ 3 & -2 & -2 \end{vmatrix} + 0\cdot(-1)^{4+4} \begin{vmatrix} \dots \end{vmatrix} =$$

$$= -2(12 + 48 - 16 - 96 - 4 + 24) - (24 + 16 + 24 +$$

$$+ 144 \cancel{-8} \cancel{+8}) - 2(-12 \cancel{-16} + 6 - 72 + \cancel{2} \cancel{+8}) =$$

$$= 64 - 208 + 168 = 24 \neq 0$$

I) $\Delta_1 = \begin{vmatrix} 0 & 1 & 4 & 8 \\ 0 & 3 & -6 & 2 \\ 0 & -2 & 2 & -2 \\ 0 & -1 & 2 & 0 \end{vmatrix} = 0$

$$X_1 = \frac{0}{24} = 0$$

$$\Delta_2 = \begin{vmatrix} 2 & 0 & 4 & 8 \\ 1 & 0 & -6 & 2 \\ 3 & 0 & 2 & -2 \\ 2 & 0 & 2 & 0 \end{vmatrix} = 0$$

$$X_2 = \frac{0}{24} = 0$$

$$\Delta_3 = \begin{vmatrix} 2 & 1 & 0 & 8 \\ 1 & 3 & 0 & 2 \\ 3 & -2 & 0 & -2 \\ 2 & -1 & 0 & 0 \end{vmatrix} = 0$$

$$X_3 = \frac{0}{24} = 0$$

$$\Delta_4 = \begin{vmatrix} 2 & 1 & 4 & 0 \\ 1 & 3 & -6 & 0 \\ 3 & -2 & 2 & 0 \\ 2 & -1 & 2 & 0 \end{vmatrix} = 0$$

$$X_4 = \frac{0}{24} = 0$$

II) $$A^{-1} = \frac{1}{\det A} \tilde{A}$$

$$A_{11} = \begin{vmatrix} 3 & -6 & 2 \\ -2 & 2 & -2 \\ -1 & 2 & 0 \end{vmatrix} = 0 - 8 - \cancel{12} + 4 + \cancel{12} - 4 \cdot 0 =$$

$$= -4$$

$$A_{12} = -\begin{vmatrix} 1 & -6 & 2 \\ 3 & 2 & -2 \\ 2 & 2 & 0 \end{vmatrix} = 0 - 12 - 24 + $$

$$+ 8 - 4 - 0 = -32$$

$$A_{13} = \begin{vmatrix} 1 & 3 & 2 \\ 3 & -2 & -2 \\ 2 & -1 & 0 \end{vmatrix} = 0 - 6 - 12 + 8 - 2 - 0 =$$

$$= -12$$

$$A_{14} = -\begin{vmatrix} 1 & 3 & -6 \\ 3 & -2 & 2 \\ 2 & -1 & 2 \end{vmatrix} = -(-4 + 18 + 12 - 24 + 2 - 18) =$$

$$= 14$$

$$A_{21} = -\begin{vmatrix} 1 & 4 & 8 \\ -2 & 2 & -2 \\ -1 & 2 & 0 \end{vmatrix} = -(0 - 32 + 8 + 16 + 4 - 0) =$$

$$= 4$$

$$A_{22} = \begin{vmatrix} 2 & 4 & 8 \\ 3 & 2 & -2 \\ 2 & 2 & 0 \end{vmatrix} = 0 + 48 - 16 - 32 + 8 - 0 =$$

$$= 8$$

$$A_{23} = -\begin{vmatrix} 2 & 1 & 8 \\ 3 & -2 & -2 \\ 2 & -1 & 0 \end{vmatrix} = -(0 - 24 - 4 + 32 - 4 - 0) =$$

$$= 0$$

$$A_{24} = \begin{vmatrix} 2 & 1 & 4 \\ 3 & -2 & 2 \\ 2 & -1 & 2 \end{vmatrix} = \cancel{-8} \cancel{-12} \cancel{+4} \cancel{+16} + 4 - 6 =$$

$$= -2$$

$$A_{31} = \begin{vmatrix} 1 & 4 & 8 \\ 3 & -6 & 2 \\ -1 & 2 & 0 \end{vmatrix} = 0 + \cancel{48} - 8 - \cancel{48} - 4 - 0 =$$
$$= -12$$

$$A_{32} = -\begin{vmatrix} 2 & 4 & 8 \\ 1 & -6 & 2 \\ 2 & 2 & 0 \end{vmatrix} = -(0 + 16 + 16 + 96 - 8 - 0) =$$
$$= -120$$

$$A_{33} = \begin{vmatrix} 2 & 1 & 8 \\ 1 & 3 & 2 \\ 2 & -1 & 0 \end{vmatrix} = 0 + 8 + 4 - 48 + 4 - 0 =$$
$$= -32$$

$$A_{34} = -\begin{vmatrix} 2 & 1 & 4 \\ 1 & 3 & -6 \\ 2 & -1 & 2 \end{vmatrix} = -(\cancel{12} - 4 - \cancel{12} - 24 -$$
$$-12 - 2 = +42$$

$$A_{41} = -\begin{vmatrix} 1 & 4 & 8 \\ 3 & -6 & 2 \\ -2 & 2 & -2 \end{vmatrix} = -(12 + 48 - 16 - 96 -$$
$$-4 + 24) = -42$$

$$A_{42} = \begin{vmatrix} 2 & 4 & 8 \\ 1 & -6 & 2 \\ 3 & 2 & -2 \end{vmatrix} = 24 + 16 + 24 + 144 -$$
$$-\cancel{8} + \cancel{8} = 208$$

$$A_{43} = -\begin{vmatrix} 2 & 1 & 8 \\ 1 & 3 & 2 \\ 3 & -2 & -2 \end{vmatrix} = -(-12 - \cancel{16} + \cancel{16} - 72 +$$
$$+\cancel{2} + 8) = 84$$

$$A_{44} = \begin{vmatrix} 2 & 1 & 4 \\ 1 & 3 & -6 \\ 3 & -2 & 2 \end{vmatrix} = 12 - 8 - 18 - 36 - 24 - 2 =$$
$$= -76$$

$$\tilde{A} = \begin{pmatrix} -4 & 4 & -12 & -42 \\ -32 & 8 & -120 & 208 \\ -12 & 0 & -32 & 84 \\ 14 & -2 & 42 & -76 \end{pmatrix}$$

$$A^{-1} = \frac{1}{24}\begin{pmatrix} -4 & 4 & -12 & -42 \\ -32 & 8 & -120 & 208 \\ -12 & 0 & -32 & 84 \\ 14 & -2 & 42 & -76 \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{1}{6} & \frac{1}{6} & -\frac{1}{2} & -\frac{7}{4} \\ -\frac{4}{3} & \frac{1}{3} & -5 & \frac{26}{3} \\ -\frac{1}{2} & 0 & -\frac{4}{3} & \frac{7}{2} \\ \frac{7}{12} & -\frac{1}{12} & \frac{7}{4} & -\frac{19}{6} \end{pmatrix}$$

$$X = A^{-1}B$$

$$X = \begin{pmatrix} -\frac{1}{6} & \frac{1}{6} & -\frac{1}{2} & -\frac{7}{4} \\ -\frac{4}{3} & \frac{1}{3} & -5 & \frac{26}{3} \\ -\frac{1}{2} & 0 & -\frac{4}{3} & \frac{7}{2} \\ \frac{7}{12} & -\frac{1}{12} & \frac{7}{4} & -\frac{19}{6} \end{pmatrix}\begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

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Ответ: (0;0;0;0)

№2.2.28

$$\begin{cases} x_1 + 2x_2 - 3x_3 + 4x_4 = -13 \\ -x_1 + x_3 + 2x_4 = -1 \\ 3x_1 + 4x_2 + 5x_3 = 11 \\ 5x_1 + 6x_2 + 7x_3 - 2x_4 = 19 \end{cases}$$

$$A = \begin{pmatrix} 1 & 2 & -3 & 4 \\ -1 & 0 & 1 & 2 \\ 3 & 4 & 5 & 0 \\ 5 & 6 & 7 & -2 \end{pmatrix}, \quad X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}, \quad B = \begin{pmatrix} -13 \\ -1 \\ 11 \\ 19 \end{pmatrix}$$

$$\det A = \begin{vmatrix} 1 & 2 & -3 & 4 \\ -1 & 0 & 1 & 2 \\ 3 & 4 & 5 & 0 \\ 5 & 6 & 7 & -2 \end{vmatrix} \leftarrow = 3 \cdot \begin{vmatrix} 2 & -3 & 4 \\ 0 & 1 & 2 \\ 6 & 7 & -2 \end{vmatrix} -$$

$$-4 \cdot \begin{vmatrix} 1 & -3 & 4 \\ -1 & 1 & 2 \\ 5 & 7 & -2 \end{vmatrix} + 5 \cdot \begin{vmatrix} 1 & 2 & 4 \\ -1 & 0 & 2 \\ 5 & 6 & -2 \end{vmatrix} =$$

$$= 3 \cdot (-4 + 0 - 36 - 24 - 0 - 28) -$$

$$-4 \cdot (-2 - 28 - 30 - 20 - 14 + 6) +$$

$$+5 \cdot (0 - 24 + 20 - 0 - 12 - 4) =$$

1

$= 3 \cdot \overset{-276}{(-92)} - 4 \cdot \overset{352}{(-88)} + 5 \cdot \overset{-100}{(-20)} =$

$= -24 \neq 0$

II) $A^{-1} = \frac{1}{\det A} \tilde{A}$

$A_{11} = \begin{vmatrix} 0 & 1 & 2 \\ 4 & 5 & 0 \\ 6 & 7 & -2 \end{vmatrix} = 0 + 56 + 0 - 60 - 0 + 8 =$

$= 4$

$A_{12} = -\begin{vmatrix} -1 & 1 & 2 \\ 3 & 5 & 0 \\ 5 & 7 & -2 \end{vmatrix} = -(10 + 42 + 0 - 50 -$

$- 0 + 6) = -8$

$A_{13} = \begin{vmatrix} -1 & 0 & 2 \\ 3 & 4 & 0 \\ 5 & 6 & -2 \end{vmatrix} = 8 + 36 + 0 - 40 - 0 -$

$- 0 = 4$

$A_{14} = -\begin{vmatrix} -1 & 0 & 1 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{vmatrix} = -(-28 + 18 + 0 -$

$- 20 + 30 - 0) = 0$

$A_{21} = -\begin{vmatrix} 2 & -3 & 4 \\ 4 & 5 & 0 \\ 6 & 7 & -2 \end{vmatrix} = -(112 - 20 + 0 -$

$- 0 - 120 - 24) = 52$

$$A_{22} = \begin{vmatrix} 1 & -3 & 4 \\ 3 & 5 & 0 \\ 5 & 7 & -2 \end{vmatrix} = -10 + 84 + 0 -$$

$$-100 - 0 - 18 = -44$$

$$A_{23} = -\begin{vmatrix} 1 & 2 & 4 \\ 3 & 4 & 0 \\ 5 & 6 & -2 \end{vmatrix} = -(-8 + 72 + 0 -$$

$$-80 - 0 + 12) = 4$$

$$A_{24} = \begin{vmatrix} 1 & 2 & -3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{vmatrix} = 28 - 54 + 50 +$$

$$+60 - 30 - 42 = 12$$

$$A_{31} = \begin{vmatrix} 2 & -3 & 4 \\ 0 & 1 & 2 \\ 6 & 7 & -2 \end{vmatrix} = -4 + 0 - 36 - 24 -$$

$$-28 - 0 = -92$$

$$A_{32} = -\begin{vmatrix} 1 & -3 & 4 \\ -1 & 1 & 2 \\ 5 & 7 & -2 \end{vmatrix} = -(-2 - 28 - 30 -$$

$$-20 - 14 + 6) = 88$$

$$A_{33} = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 0 & 2 \\ 5 & 6 & -2 \end{vmatrix} = 0 - 24 + 20 - 0 - 12 -$$

$$-4 = -20$$

$$A_{34} = -\begin{vmatrix} 1 & 2 & -3 \\ -1 & 0 & 1 \\ 5 & 6 & 7 \end{vmatrix} = -(0+18+10-0-$$

$$-6+14) = -36$$

$$A_{41} = -\begin{vmatrix} 2 & -3 & 4 \\ 0 & 1 & 2 \\ 4 & 5 & 0 \end{vmatrix} = -(0+0-24-$$

$$-16-20-0) = 60$$

$$A_{42} = \begin{vmatrix} 1 & -3 & 4 \\ -1 & 1 & 2 \\ 3 & 5 & 0 \end{vmatrix} = 0-20-18-12-10-0 =$$

$$= -60$$

$$A_{43} = -\begin{vmatrix} 1 & 2 & 4 \\ -1 & 0 & 2 \\ 3 & 4 & 0 \end{vmatrix} = -(0-16+12-0-$$

$$-8-0) = 12$$

$$A_{44} = \begin{vmatrix} 1 & 2 & -3 \\ -1 & 0 & 1 \\ 3 & 4 & 5 \end{vmatrix} = 0+12+6-0-$$

$$-4+10 = 24$$

$$\tilde{A} = \begin{pmatrix} 4 & 52 & -92 & 60 \\ -8 & -44 & 88 & -60 \\ 4 & 4 & -20 & 12 \\ 0 & 12 & -36 & 24 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-24}\begin{pmatrix} 4 & 52 & -92 & 60 \\ -8 & -44 & 88 & -60 \\ 4 & 4 & -20 & 12 \\ 0 & 12 & -36 & 24 \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{1}{6} & -\frac{13}{6} & \frac{23}{6} & -\frac{5}{2} \\ \frac{1}{3} & \frac{11}{6} & -\frac{11}{3} & \frac{5}{2} \\ -\frac{1}{6} & -\frac{1}{6} & \frac{5}{6} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{3}{2} & -1 \end{pmatrix}$$

$$X = A^{-1}B$$

$$X = \begin{pmatrix} -\frac{1}{6} & -\frac{13}{6} & \frac{23}{6} & -\frac{5}{2} \\ \frac{1}{3} & \frac{11}{6} & -\frac{11}{3} & \frac{5}{2} \\ -\frac{1}{6} & -\frac{1}{6} & \frac{5}{6} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{3}{2} & -1 \end{pmatrix} \cdot \begin{pmatrix} -13 \\ -1 \\ 11 \\ 19 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{13}{6} + \frac{13}{6} + \frac{253}{6} - \frac{285}{6} \\ -\frac{26}{6} - \frac{11}{6} - \frac{242}{6} + \frac{285}{6} \\ \frac{13}{6} + \frac{1}{6} + \frac{55}{6} - \frac{57}{6} \\ 0 \cancel{+ \frac{6}{12} + \frac{143}{12} - \frac{228}{12}} \; \frac{1}{2} + \frac{33}{2} - \frac{38}{2} \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 2 \\ -2 \end{pmatrix}$$

II) $$\Delta_1 = \begin{vmatrix} -13 & 2 & -3 & 4 \\ -1 & 0 & 1 & 2 \\ 11 & 4 & 5 & 0 \\ 19 & 6 & 7 & -2 \end{vmatrix} =$$

$$= -1\cdot(-1)^{2+1}\cdot\begin{vmatrix} 2 & -3 & 4 \\ 4 & 5 & 0 \\ 6 & 7 & -2 \end{vmatrix} + 1\cdot(-1)^{2+3}\cdot$$

$$\cdot\begin{vmatrix} -13 & 2 & 4 \\ 11 & 4 & 0 \\ 19 & 6 & -2 \end{vmatrix} + 2\cdot(-1)^{2+4}\cdot\begin{vmatrix} -13 & 2 & -3 \\ 11 & 4 & 5 \\ 19 & 6 & 7 \end{vmatrix} =$$

$$= -20+0+112-120-0-24-(104+264+$$
$$+0-304-0+44)+2(-364-198+190+$$
$$+228+390-154) = 24$$

$$x_1 = \frac{24}{-24} = -1$$

$$\Delta_2 = \begin{vmatrix} 1 & -13 & -3 & 4 \\ -1 & -1 & 1 & 2 \\ 3 & 11 & 5 & 0 \\ 5 & 19 & 7 & -2 \end{vmatrix} = 4\cdot(-1)^{4+1}\cdot$$

$$\cdot\begin{vmatrix} -1 & -1 & 1 \\ 3 & 11 & 5 \\ 5 & 19 & 7 \end{vmatrix} + 2\cdot(-1)^{2+4}\cdot\begin{vmatrix} 1 & -13 & -3 \\ 3 & 11 & 5 \\ 5 & 19 & 7 \end{vmatrix} -$$

$$-2\cdot(-1)^{4+4}\cdot\begin{vmatrix} 1 & -13 & -3 \\ -1 & -1 & 1 \\ 3 & 11 & 5 \end{vmatrix} = -4(-77+57-25-$$

$$-55+95+21)+2(77-325-171+165-$$

$-95+273)-2(-5+33-39-9-$

$-11-65)=-4\cdot16+2\cdot(-76)-2\cdot$

$\cdot(-96)=-64-152+192=-24$

$x_2=\frac{-24}{-24}=1$

$$\Delta_3=\begin{vmatrix} 1 & 2 & -13 & 4 \\ -1 & 0 & -1 & 2 \\ 3 & 4 & 11 & 0 \\ 5 & 6 & 19 & -2 \end{vmatrix} \leftarrow =$$

$$=3\cdot(-1)^{3+1}\cdot\begin{vmatrix} 2 & -13 & 4 \\ 0 & -1 & 2 \\ 6 & 19 & -2 \end{vmatrix}+4\cdot(-1)^{3+2}\cdot$$

$$\cdot\begin{vmatrix} 1 & -13 & 4 \\ -1 & -1 & 2 \\ 5 & 19 & -2 \end{vmatrix}+11\cdot(-1)^{3+3}\cdot\begin{vmatrix} 1 & 2 & 4 \\ -1 & 0 & 2 \\ 5 & 6 & -2 \end{vmatrix}=$$

$=3(4+0-156+24-76-0)-$

$-4(2-76-130+20-38+26)+$

$+11(0-24+20-0-12-4)=$

$=3\cdot(-204)-4\cdot(-196)+11\cdot(-20)=$

$=-612+784-220=-48$

$x_3=\frac{-48}{-24}=2$

(7)

$$\Delta_4 = \begin{vmatrix} 1 & 2 & -3 & -13 \\ -1 & 0 & 1 & -1 \\ 3 & 4 & 5 & 11 \\ 5 & 6 & 7 & 19 \end{vmatrix} = -1\cdot(-1)^{2+1}\cdot \begin{vmatrix} 2 & -3 & -13 \\ 4 & 5 & 11 \\ 6 & 7 & 19 \end{vmatrix} + 1\cdot(-1)^{2+3}\cdot \begin{vmatrix} 1 & 2 & -13 \\ 3 & 4 & 11 \\ 5 & 6 & 19 \end{vmatrix} -$$

$$- 1\cdot(-1)^{2+4}\cdot \begin{vmatrix} 1 & 2 & -3 \\ 3 & 4 & 5 \\ 5 & 6 & 7 \end{vmatrix} = 190 - 364 - 198 +$$

$$+ 390 - 154 + 228 - (76 - 234 + 110 +$$

$$+ 260 - 66 - 114) - (28 - 54 + 50 + 60 -$$

$$- 42 - 30) = 48$$

$$x_4 = \frac{48}{-24} = -2$$

<u>Ответ:</u> $(-1;\ 1;\ 2;\ -2)$

№2.2.29

$$\begin{cases} -x_1+4x_2+5x_3-4x_4=-15 \\ x_1+2x_2-2x_3+4x_4=3 \\ 2x_1+6x_2+x_3=-6 \\ 3x_1+x_3+2x_4=11 \end{cases}$$

$$A=\begin{pmatrix} -1 & 4 & 5 & -4 \\ 1 & 2 & -2 & 4 \\ 2 & 6 & 1 & 0 \\ 3 & 0 & 1 & 2 \end{pmatrix},\ X=\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix},\ B=\begin{pmatrix} -15 \\ 3 \\ -6 \\ 11 \end{pmatrix}$$

$$\det A=\begin{vmatrix} -1 & 4 & 5 & -4 \\ 1 & 2 & -2 & 4 \\ 2 & 6 & 1 & 0 \\ 3 & 0 & 1 & 2 \end{vmatrix} \leftarrow = 2\cdot\begin{vmatrix} 4 & 5 & -4 \\ 2 & -2 & 4 \\ 0 & 1 & 2 \end{vmatrix} -$$

$$-6\cdot\begin{vmatrix} -1 & 5 & -4 \\ 1 & -2 & 4 \\ 3 & 1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & 4 & -4 \\ 1 & 2 & 4 \\ 3 & 0 & 2 \end{vmatrix} =$$

$$=2(-16-8+0-0-16-20)-$$

$$-6(\cancel{4}-\cancel{4}+60-24+4-10)+$$

$$+(-4+0+48+24-0-8)=$$

$$=\overset{-120}{2\cdot(-60)}-\overset{-180}{6\cdot 30}+60=-240$$

1

I) $A^{-1} = \frac{1}{\det A} \tilde{A}$

$$A_{11} = \begin{vmatrix} 2 & -2 & 4 \\ 6 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix} = 4 + 24 + 0 - 0 - 0 + 24 = 52$$

$$A_{12} = -\begin{vmatrix} 1 & -2 & 4 \\ 2 & 1 & 0 \\ 3 & 1 & 2 \end{vmatrix} = -(2 + 8 + 0 - 12 - 0 + 8) = -6$$

$$A_{13} = \begin{vmatrix} 1 & 2 & 4 \\ 2 & 6 & 0 \\ 3 & 0 & 2 \end{vmatrix} = 12 + 0 + 0 - 72 - 0 - 8 = -68$$

$$A_{14} = -\begin{vmatrix} 1 & 2 & -2 \\ 2 & 6 & 1 \\ 3 & 0 & 1 \end{vmatrix} = -(6 + 0 + 6 + 36 - 0 - 4) = -44$$

$$A_{21} = -\begin{vmatrix} 4 & 5 & -4 \\ 6 & 1 & 0 \\ 0 & 1 & 2 \end{vmatrix} = -(8 - 24 + 0 - 0 - 0 - 60) = 76$$

$$A_{22} = \begin{vmatrix} -1 & 5 & -4 \\ 2 & 1 & 0 \\ 3 & 1 & 2 \end{vmatrix} = -2 - 8 + 0 + 12 - 0 - 20 = -18$$

$$A_{23} = -\begin{vmatrix} -1 & 4 & -4 \\ 2 & 6 & 0 \\ 3 & 0 & 2 \end{vmatrix} = -(-12 + 0 + 0 + 72 - 0 - 16) = -44$$

$$A_{24} = \begin{vmatrix} -1 & 4 & 5 \\ 2 & 6 & 1 \\ 3 & 0 & 1 \end{vmatrix} = -6 + 0 + 12 - 90 - 0 - 8 = -92$$

$$A_{31} = \begin{vmatrix} 4 & 5 & -4 \\ 2 & -2 & 4 \\ 0 & 1 & 2 \end{vmatrix} = -16-8+0-0-16-20 = -60$$

$$A_{32} = -\begin{vmatrix} -1 & 5 & -4 \\ 1 & -2 & 4 \\ 3 & 1 & 2 \end{vmatrix} = -(\cancel{4}-\cancel{4}+60-24+4-10) = -30$$

$$A_{33} = \begin{vmatrix} -1 & 4 & -4 \\ 1 & 2 & 4 \\ 3 & 0 & 2 \end{vmatrix} = -4+0+48+24-0-8 = 60$$

$$A_{34} = -\begin{vmatrix} -1 & 4 & 5 \\ 1 & 2 & -2 \\ 3 & 0 & 1 \end{vmatrix} = -(-2+0-24-30-0-4) = 60$$

$$A_{41} = -\begin{vmatrix} 4 & 5 & -4 \\ 2 & -2 & 4 \\ 6 & 1 & 0 \end{vmatrix} = -(0-8+120-48-16-0) = -48$$

$$A_{42} = \begin{vmatrix} -1 & 5 & -4 \\ 1 & -2 & 4 \\ 2 & 1 & 0 \end{vmatrix} = 0-4+40-16+4-0 = 24$$

$$A_{43} = -\begin{vmatrix} -1 & 4 & -4 \\ 1 & 2 & 4 \\ 2 & 6 & 0 \end{vmatrix} = -(0-\cancel{24}+32+16+\cancel{24}-0) = -48$$

$$A_{44} = \begin{vmatrix} -1 & 4 & 5 \\ 1 & 2 & -2 \\ 2 & 6 & 1 \end{vmatrix} = -2+30-16-20-12-4 = -24$$

$$\tilde{A} = \begin{pmatrix} 52 & 76 & -60 & -48 \\ -6 & -18 & -30 & 24 \\ -68 & -44 & 60 & -48 \\ -44 & -92 & 60 & -24 \end{pmatrix}$$

$$A^{-1}=\frac{1}{-240}\begin{pmatrix} 52 & 76 & -60 & -48 \\ -6 & -18 & -30 & 24 \\ -68 & -44 & 60 & -48 \\ -44 & -92 & 60 & -24 \end{pmatrix}=$$

$$=\begin{pmatrix} -\frac{13}{60} & -\frac{19}{60} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{40} & \frac{3}{40} & \frac{1}{8} & -\frac{1}{10} \\ \frac{17}{60} & \frac{11}{60} & -\frac{1}{4} & \frac{1}{5} \\ \frac{11}{60} & \frac{23}{60} & -\frac{1}{4} & \frac{1}{10} \end{pmatrix}$$

$$X=A^{-1}B=\begin{pmatrix} -\frac{13}{60} & -\frac{19}{60} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{40} & \frac{3}{40} & \frac{1}{8} & -\frac{1}{10} \\ \frac{17}{60} & \frac{11}{60} & -\frac{1}{4} & \frac{1}{5} \\ \frac{11}{60} & \frac{23}{60} & -\frac{1}{4} & \frac{1}{10} \end{pmatrix}\cdot\begin{pmatrix} -15 \\ 3 \\ -6 \\ 11 \end{pmatrix}=$$

$$=\begin{pmatrix} \frac{195}{60}-\frac{57}{60}-\frac{90}{60}+\frac{132}{60} \\ -\frac{15}{40}+\frac{9}{40}-\frac{30}{40}-\frac{44}{40} \\ -\frac{255}{60}+\frac{33}{60}+\frac{90}{60}+\frac{132}{60} \\ -\frac{165}{60}+\frac{69}{60}+\frac{90}{60}+\frac{66}{60} \end{pmatrix}=\begin{pmatrix} 3 \\ -2 \\ 0 \\ 1 \end{pmatrix}$$

VI) $\Delta_1 = \begin{vmatrix} -15 & 4 & 5 & -4 \\ 3 & 2 & -2 & 4 \\ -6 & 6 & 1 & 0 \\ 11 & 0 & 1 & 2 \end{vmatrix} \leftarrow =$

$= -6 \cdot (-1)^{3+1} \cdot \begin{vmatrix} 4 & 5 & -4 \\ 2 & -2 & 4 \\ 0 & 1 & 2 \end{vmatrix} + 6 \cdot (-1)^{3+2} \cdot$

$\cdot \begin{vmatrix} -15 & 5 & -4 \\ 3 & -2 & 4 \\ 11 & 1 & 2 \end{vmatrix} + 1 \cdot (-1)^{3+3} \cdot \begin{vmatrix} -15 & 4 & -4 \\ 3 & 2 & 4 \\ 11 & 0 & 2 \end{vmatrix} =$

$= -6(-16-8+0-0-16-20) - 6(60-12+$

$+220-88+60-30) + (-60+0+176+$

$+88-0-24) = -6 \cdot (-60) - 6 \cdot 210 + 180 =$

$= 360 - 1260 + 180 = -720$

$x_1 = \frac{-720}{-240} = 3$

$\Delta_2 = \begin{vmatrix} -1 & -15 & 5 & -4 \\ 1 & 3 & -2 & 4 \\ 2 & -6 & 1 & 0 \\ 3 & 11 & 1 & 2 \end{vmatrix} \leftarrow =$

$= 2 \cdot \begin{vmatrix} -15 & 5 & -4 \\ 3 & -2 & 4 \\ 11 & 1 & 2 \end{vmatrix} + 6 \cdot \begin{vmatrix} -1 & 5 & -4 \\ 1 & -2 & 4 \\ 3 & 1 & 2 \end{vmatrix} + \begin{vmatrix} -1 & -15 & -4 \\ 1 & 3 & 4 \\ 3 & 1 & 2 \end{vmatrix} =$

$= 2 \cdot (60-12+220-88+60-30) +$

$$+6\left(\cancel{4}-\cancel{4}+60-24+4-10\right)+$$

$$+\left(\underset{420}{-6}-\cancel{4}-\overset{180}{180}+36+\cancel{4}+30\right)=$$

$$=2\cdot 210+6\cdot 30-120=480$$

$$X_2=\frac{480}{-240}=-2$$

$$\Delta_3=\begin{vmatrix} -1 & 4 & -15 & -4 \\ 1 & 2 & 3 & 4 \\ 2 & 6 & -6 & 0 \\ 3 & 0 & 11 & 2 \end{vmatrix}\leftarrow =$$

$$=2\cdot\begin{vmatrix} 4 & -15 & -4 \\ 2 & 3 & 4 \\ 0 & 11 & 2 \end{vmatrix}-6\cdot\begin{vmatrix} -1 & -15 & -4 \\ 1 & 3 & 4 \\ 3 & 11 & 2 \end{vmatrix}-$$

$$-6\cdot\begin{vmatrix} -1 & 4 & -4 \\ 1 & 2 & 4 \\ 3 & 0 & 2 \end{vmatrix}=2\cdot(24-88+0-0-176+$$

$$+60)-6\cdot\left(-6-\cancel{44}-180+36+\cancel{44}+30\right)-$$

$$-6\cdot(-4+0+48+24-0-8)=$$

$$=2\cdot(-180)-6\cdot(-120)-6\cdot 60=$$

$$=-360+720-360=0$$

$$X_3=\frac{0}{-240}=0$$

$$\Delta_4 = \begin{vmatrix} -1 & 4 & 5 & -15 \\ 1 & 2 & -2 & 3 \\ 2 & 6 & 1 & -6 \\ 3 & 0 & 1 & 11 \end{vmatrix} =$$

$$= 4\cdot(-1)^{1+2}\begin{vmatrix} 1 & -2 & 3 \\ 2 & 1 & -6 \\ 3 & 1 & 11 \end{vmatrix} + 2\cdot(-1)^{2+2}\cdot\begin{vmatrix} -1 & 5 & -15 \\ 2 & 1 & -6 \\ 3 & 1 & 11 \end{vmatrix} + 6\cdot(-1)^{3+2}\begin{vmatrix} -1 & 5 & -15 \\ 1 & -2 & 3 \\ 3 & 1 & 11 \end{vmatrix} =$$

$$= -4\cdot(11+6+36-9+6+44) +$$

$$+2\cdot(-11-30-90+45-6-110) -$$

$$-6\cdot(22-15+45-90+3-55) =$$

$$= -4\cdot 94 + 2\cdot(-202) - 6\cdot(-90) =$$

$$= -376 - 404 + 540 = -240$$

$$x_4 = \frac{-240}{-240} = 1$$

Ответ: $(3; -2; 0; 1)$

№2.2.30

$f(x) = ax^3 + bx^2 + c$

$f(-1) = 3$

$f(1) = 1$

$f(2) = -15$

$$\begin{cases} -a + b + c = 3 \\ a + b + c = 1 \\ 8a + 4b + c = -15 \end{cases}$$

$$A = \begin{pmatrix} -1 & 1 & 1 \\ 1 & 1 & 1 \\ 8 & 4 & 1 \end{pmatrix}, \quad X = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \quad B = \begin{pmatrix} 3 \\ 1 \\ -15 \end{pmatrix}$$

$$\det A = \begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & 1 \\ 8 & 4 & 1 \end{vmatrix} = -1 + 4 \cancel{+8} \cancel{-8} + 4 - 12$$

$= 6 \neq 0$

I) $A^{-1} = \dfrac{1}{\det A} \tilde{A}$

$A_{11} = \begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} = 1 - 4 = -3$

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$$A_{12} = -\begin{vmatrix} 1 & 1 \\ 8 & 1 \end{vmatrix} = -(1-8) = 7$$

$$A_{13} = \begin{vmatrix} 1 & 1 \\ 8 & 4 \end{vmatrix} = 4-8 = -4$$

$$A_{21} = -\begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} = -(1-4) = 3$$

$$A_{22} = \begin{vmatrix} -1 & 1 \\ 8 & 1 \end{vmatrix} = -1-8 = -9$$

$$A_{23} = -\begin{vmatrix} -1 & 1 \\ 8 & 4 \end{vmatrix} = -(-4-8) = 12$$

$$A_{31} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1-1 = 0$$

$$A_{32} = -\begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -(-1-1) = 2$$

$$A_{33} = \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = -1-1 = -2$$

$$\tilde{A} = \begin{pmatrix} -3 & 3 & 0 \\ 7 & -9 & 2 \\ -4 & 12 & -2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{6}\begin{pmatrix} -3 & 3 & 0 \\ 7 & -9 & 2 \\ -4 & 12 & -2 \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{7}{6} & -\frac{3}{2} & \frac{1}{3} \\ -\frac{2}{3} & 2 & -\frac{1}{3} \end{pmatrix}$$

$$X = A^{-1}B = \begin{pmatrix} -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{7}{6} & -\frac{3}{2} & \frac{1}{3} \\ -\frac{2}{3} & 2 & -\frac{1}{3} \end{pmatrix} \cdot \begin{pmatrix} 3 \\ 1 \\ -15 \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{3}{2} + \frac{1}{2} + 0 \\ \frac{7}{2} - \frac{3}{2} - 5 \\ -2 + 2 + 5 \end{pmatrix} = \begin{pmatrix} -1 \\ -3 \\ 5 \end{pmatrix}$$

4)

$$\Delta_1 = \begin{vmatrix} 3 & 1 & 1 \\ 1 & 1 & 1 \\ -15 & 4 & 1 \end{vmatrix} = 3 + 4 \cancel{-15} +$$

$$\cancel{+15} - 12 - 1 = -6$$

$$x_1 = \frac{-6}{6} = -1$$

4/4

$$\Delta_2 = \begin{vmatrix} -1 & 3 & 1 \\ 1 & 1 & 1 \\ 8 & -15 & 1 \end{vmatrix} = -1 - 15 + 24 - 8 - 15 - 3 =$$
$$= -18$$

$$x_2 = \frac{-18}{6} = -3$$

$$\Delta_3 = \begin{vmatrix} -1 & 1 & 3 \\ 1 & 1 & 1 \\ 8 & 4 & -15 \end{vmatrix} = 15 + 12 + 8 - 24 + 4 +$$
$$+ 15 = 30$$

$$x_3 = \frac{30}{6} = 5$$

Ответ: $(-1; -3; 5)$

№2.2.31

$$f(x) = a\log_3 x + bx + c$$
$$f(1) = 5$$
$$f(3) = 8$$
$$f(9) = 19$$

$$\begin{cases} b + c = 5 \\ a + 3b + c = 8 \\ 2a + 9b + c = 19 \end{cases}$$

$$A=\begin{pmatrix}0&1&1\\1&3&1\\2&9&1\end{pmatrix},\quad X=\begin{pmatrix}a\\b\\c\end{pmatrix},\quad B=\begin{pmatrix}5\\8\\19\end{pmatrix}$$

$$\det A=\begin{vmatrix}0&1&1\\1&3&1\\2&9&1\end{vmatrix}=9+2-6-1=4\neq 0$$

I) $A^{-1}=\dfrac{1}{\det A}\tilde{A}$

$$A_{11}=\begin{vmatrix}3&1\\9&1\end{vmatrix}=3-9=-6$$

$$A_{12}=-\begin{vmatrix}1&1\\2&1\end{vmatrix}=-1+2=1$$

$$A_{13}=\begin{vmatrix}1&3\\2&9\end{vmatrix}=9-6=3$$

$$A_{21}=-\begin{vmatrix}1&1\\9&1\end{vmatrix}=-1+9=8$$

$$A_{22}=\begin{vmatrix}0&1\\2&1\end{vmatrix}=-2$$

$$A_{23}=-\begin{vmatrix}0&1\\2&9\end{vmatrix}=+2$$

$$A_{31} = \begin{vmatrix} 1 & 1 \\ 3 & 1 \end{vmatrix} = 1 - 3 = -2$$

$$A_{32} = -\begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} = 1$$

$$A_{33} = \begin{vmatrix} 0 & 1 \\ 1 & 3 \end{vmatrix} = -1$$

$$\tilde{A} = \begin{pmatrix} -6 & 8 & -2 \\ 1 & -2 & 1 \\ 3 & 2 & -1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{4} \begin{pmatrix} -6 & 8 & -2 \\ 1 & -2 & 1 \\ 3 & 2 & -1 \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{3}{2} & 2 & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \\ \frac{3}{4} & \frac{1}{2} & -\frac{1}{4} \end{pmatrix}$$

$$X = A^{-1}B = \begin{pmatrix} -\frac{3}{2} & 2 & -\frac{1}{2} \\ \frac{1}{4} & -\frac{1}{2} & \frac{1}{4} \\ \frac{3}{4} & \frac{1}{2} & -\frac{1}{4} \end{pmatrix} \cdot \begin{pmatrix} 5 \\ 8 \\ 19 \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{15}{2} + 16 - \frac{19}{2} \\ \frac{5}{4} - 4 + \frac{19}{4} \\ \frac{15}{4} + 4 - \frac{19}{4} \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}$$

IV)
$$\Delta_1 = \begin{vmatrix} 5 & 1 & 1 \\ 8 & 3 & 1 \\ 19 & 9 & 1 \end{vmatrix} = 15+72+19-$$
$$-57-45-8=-4$$
$$x_1 = \frac{-4}{4} = -1$$
$$\Delta_2 = \begin{vmatrix} 0 & 5 & 1 \\ 1 & 8 & 1 \\ 2 & 19 & 1 \end{vmatrix} = 19+10-16-5=$$
$$=8$$
$$x_2 = \frac{8}{4} = 2$$
$$\Delta_3 = \begin{vmatrix} 0 & 1 & 5 \\ 1 & 3 & 8 \\ 2 & 9 & 19 \end{vmatrix} = 45+16-30-$$
$$-19=12$$
$$x_3 = \frac{12}{4} = 3$$
Ответ: $(-1;\ 2;\ 3)$

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