

Домашний задор. Елкина Татьяна  
Функции и их производные УВТ, курс, 3 курс

в 6.1.52

1)  $\operatorname{ch} dx = \operatorname{ch}^2 x + \operatorname{sh}^2 x$

$$\operatorname{ch} 2x = \left( \frac{e^x + e^{-x}}{2} \right)^2 + \left( \frac{e^x - e^{-x}}{2} \right)^2$$
$$\operatorname{ch} 2x = \frac{(e^x + e^{-x})^2}{4} + \frac{(e^x - e^{-x})^2}{4}$$
$$\operatorname{ch} 2x = \frac{e^{2x} + 2e^x e^{-x} + e^{-2x}}{4} + \frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4}$$
$$\operatorname{ch} 2x = \frac{e^{2x} + e^{-2x}}{4} + \frac{e^{2x} + e^{-2x}}{4}$$
$$\operatorname{ch} 2x = \frac{de^{2x} + de^{-2x}}{\cancel{4}}$$

$$\operatorname{ch} 2x = \operatorname{ch} dx$$

2)  $\operatorname{sh} 2x = 2 \operatorname{sh} x \operatorname{ch} x$

$$\operatorname{sh} 2x = 2 \left( \frac{e^x - e^{-x}}{2} \right) \left( \frac{e^x + e^{-x}}{2} \right) =$$

$$\operatorname{sh} 2x = \frac{e(e^x - e^{-x})(e^x + e^{-x})}{4}$$

$$\operatorname{sh} 2x = \frac{e(e^{2x} - e^{-2x})}{4}$$

$$\operatorname{sh} 2x = \frac{2e^{2x} - 2e^{-2x}}{4}$$

$$\operatorname{sh} 2x = \operatorname{sh} dx$$

$$3) \operatorname{ch}^2 x = \frac{\operatorname{ch} 2x + 1}{2}$$

$$\operatorname{ch}^2 x = \left( \frac{e^x + e^{-x}}{2} \right)^2 = \frac{\operatorname{ch} 2x + 1}{2}$$

$$\frac{e^{2x} + 2e^x e^{-x} + e^{-2x}}{4} = \frac{\operatorname{ch} 2x + 1}{2}$$

$$\frac{e^{2x} + e^{-2x} + 2e^{x-x}}{2} = \frac{\operatorname{ch} 2x + 1}{2}$$

$$\frac{e^{2x} + e^{-2x} + 2e^0}{4} = \frac{\operatorname{ch} 2x + 1}{2}$$

$$\frac{e^{2x} + e^{-2x}}{2} + 1 = \frac{\operatorname{ch} 2x + 1}{2}$$

$$\frac{\operatorname{ch} 2x + 1}{2} = \frac{\operatorname{ch} 2x + 1}{2}$$

$$9) \operatorname{sh}^2 x = \frac{\operatorname{ch} 2x - 1}{2}$$

$$\left(\frac{e^x - e^{-x}}{2}\right)^2 = \frac{\operatorname{ch} 2x - 1}{2}$$

$$\frac{e^{2x} - 2e^x e^{-x} + e^{-2x}}{4} = \frac{\operatorname{ch} 2x - 1}{2}$$

$$\frac{e^{2x} + e^{-2x} - 2e^{x-x}}{4} = \frac{\operatorname{ch} 2x - 1}{2}$$

$$\frac{\frac{e^{2x} + e^{-2x}}{2} - 1}{2} = \frac{\operatorname{ch} 2x - 1}{2}$$

$$\frac{\operatorname{ch} 2x - 1}{2} = \frac{\operatorname{ch} 2x - 1}{2}$$

$$5) \operatorname{ch}x \cdot \operatorname{chy} = \frac{1}{2} [\operatorname{ch}(x+y) + \operatorname{ch}(x-y)]$$

$$\operatorname{ch}x \cdot \operatorname{chy} = \frac{1}{2} [\operatorname{ch}x \operatorname{chy} + \operatorname{sh}x \operatorname{shy} + \operatorname{ch}x \operatorname{chy} - \operatorname{sh}x \operatorname{shy}]$$

$$\operatorname{ch}x \cdot \operatorname{chy} = \frac{1}{2} [2 \operatorname{ch}x \operatorname{chy}]$$

$$\operatorname{ch}x \cdot \operatorname{chy} = \operatorname{ch}x \cdot \operatorname{chy}$$

$$6) \operatorname{sh}x \cdot \operatorname{shy} = \frac{1}{2} [\operatorname{sh}(x+y) - \operatorname{sh}(x-y)]$$

$$\operatorname{sh}x \cdot \operatorname{shy} = \frac{1}{2} [\operatorname{ch}x \operatorname{chy} + \operatorname{sh}x \operatorname{shy} - \operatorname{ch}x \operatorname{chy} + \operatorname{sh}x \operatorname{shy}]$$

$$\operatorname{sh}x \cdot \operatorname{shy} = \frac{1}{2} [2 \operatorname{sh}x \operatorname{shy}]$$

$$\operatorname{sh}x \cdot \operatorname{shy} = \operatorname{sh}x \cdot \operatorname{shy}$$

$$7) \operatorname{sh}x \cdot \operatorname{chy} = \frac{1}{2} [\operatorname{sh}(x+y) + \operatorname{sh}(x-y)]$$

$$\operatorname{sh}x \cdot \operatorname{chy} = \frac{1}{2} [\operatorname{sh}x \operatorname{chy} + \operatorname{ch}x \operatorname{shy} + \operatorname{sh}x \operatorname{chy} - \operatorname{ch}x \operatorname{shy}]$$

$$\operatorname{sh}x \cdot \operatorname{chy} = \frac{1}{2} [2 \operatorname{sh}x \cdot \operatorname{chy}]$$

$$\operatorname{sh}x \cdot \operatorname{chy} = \operatorname{sh}x \cdot \operatorname{chy}$$

W6.1.57

$$f(x) = \operatorname{ctgx}, D(x) - ?$$

$$\operatorname{ctgx} = \frac{\cos x}{\sin x}, \sin x \neq 0$$

$$\boxed{x \neq \pi k, k \in \mathbb{Z}}$$

W6.1.58

$$f(x) = \frac{x+2}{(x+2)(x-5)}, D(x) - ?$$

$$(x+2)(x-5) = 0$$

$$x+2=0 \quad \text{and} \quad x-5=0$$

$$x=-2 \quad x=5$$

Ort6et:  $x \in (-\infty; -2) \cup (-2; 5) \cup (5; +\infty)$

W6.1.59.

$$f(x) = \arccos 3x, D(x) = ?$$

$$-1 \leq 3x \leq 1 \quad | :3$$

$$-\frac{1}{3} \leq x \leq \frac{1}{3}$$

Ort bei:  $x \in [-\frac{1}{3}; \frac{1}{3}]$

w 6.1.60  
 $f(x) = \frac{1}{\log x}$ ,  $D(x) - ?$

$\log x \neq 0$ ,  $x \neq 1$

Ort bei:  $x \in (0; 1) \cup (1; +\infty)$

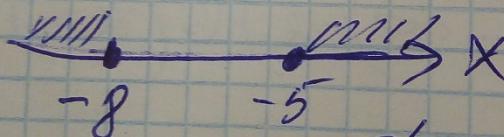
w 6.1.61

$f(x) = \sqrt{x+5} - \sqrt{-8-x}$ ,  $D(x) - ?$

~~$x+5 \geq 0$~~  u  ~~$-8-x \geq 0$~~

~~$x \geq -5$~~   $-x \geq 8$

$x \geq -5$   $x \leq -8$



Ort bei:  $x \in \emptyset$

w 6.1.62

⑥  $f(x) = \frac{\log x}{\sqrt[3]{x-3}}$ ,  $D(x) - ?$

$$x > 0, \text{ i.k.} \\ \log_3 x$$

$$x - 3 \neq 0 \\ x \neq 3$$

$$\begin{array}{c} \cancel{0} \quad \cancel{3} \\ 0 \quad 3 \end{array}$$

Ortbeit:  $x \in (3; +\infty) \cup (0; 3)$

W6.1.63  
 $f(x) = e^{\ln x}, D(x) = ?$

~~x > 0~~, i.k.  $\ln x$   
Ortbeit:  $x \in (0; +\infty)$

W6.1.64.

$$f(x) = \arcsin(\log_3 x)$$

$$0 \leq \log_3 x \leq \bar{u}$$

$$x = 3^{\log_3 x}$$

$$3^0 \leq x \leq 3^{\bar{u}}$$

$$1 \leq x \leq 3^{\bar{u}}$$

Ortbeit:  $x \in [1; 3^{\bar{u}}]$

$$\text{wG. I. 65} \\ \frac{f(x) = \sqrt{1-x^2} \cdot \arctg \frac{1}{x}}{x > 0}$$

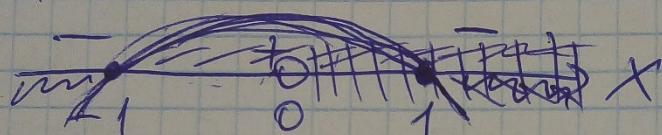
$$1-x^2 \geq 0$$

$$x^2 \leq 1$$

$$x \leq \pm 1$$

$$x > 0$$

$$x \neq 0$$



Ortbeit:  $x \in [1; +\infty)$

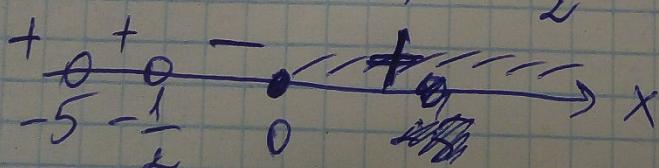
wG. I. 66

$$f(x) = \sqrt{\frac{x}{2x+1}} - \sqrt[3]{\frac{x-2}{x+5}}$$

$$\frac{x}{2x+1} \geq 0, \quad 2x+1 \neq 0, \quad x+5 \neq 0$$

$$\cancel{x \neq 0}, \quad 2x \neq -1, \quad x \neq -5$$

$$x \neq -\frac{1}{2}$$



Ortbeit:  $x \in [0; +\infty) \cup (-\infty; -5) \cup (-5; -\frac{1}{2})$

W6.1.67

$$f(x) = \cos \frac{1}{x} + \ln(x+1) + \sqrt[3]{\pi-x}$$

$$\begin{array}{lll} x \neq 0 & x+1 > 0 & \pi-x \geq 0 \\ & x > -1 & x \leq \pi \end{array}$$



Ortset:  $x \in (-1; 0) \cup (0; \pi]$

W6.1.68

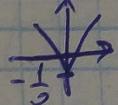
$$f(x) = 4 - x^2, \text{ вершина вниз, т.к. } a = -1$$

$x=0, y=4$  - коопр. вершина (оригинус)

Ortset:  $y \in [-\infty; 4]$

W6.1.69

$$f(x) = |x| - \frac{1}{3}$$



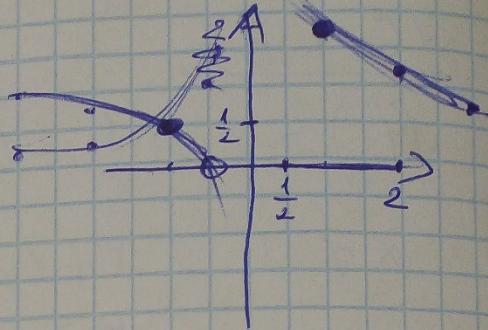
$x=0, y=-\frac{1}{3}$  - вершина функции

Ortset:  $y \in \left[-\frac{1}{3}; +\infty\right)$

$$\text{W6.1.70}$$

$$f(x) = d \frac{1}{x}$$

$x$	-3	-2	-1	1	2	3
$y$	$\frac{1}{3\sqrt{2}}$	$\frac{1}{2\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	2	$\sqrt{2}$	$\frac{3\sqrt{2}}{2}$



Oberet:  $y \in (0; 1) \cup (1; +\infty)$

W6.1.71

$$f(x) = \ln(x^2 + 1)$$

$$x^2 + 1 \geq 0; x=0, y=0$$

$$x^2 > -1 \quad \cancel{x \neq y}$$

Oberet:  $y \in [0; +\infty)$

W6.1.72

$$f(x) = e^{x^2 - 2x - 3}$$

$$x^2 - 2x - 3 \geq 0$$

$$x_1 + x_2 = 2$$

$$x_1 \cdot x_2 = -3$$

$$x_0 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{2 \pm \sqrt{4 + 12}}{2} = 1 \pm \sqrt{4} = 1 \pm 2$$

$$y = 1^2 - 2 \cdot 1 - 3 = -4$$

nur  $x = 1$

$$\mathcal{D}(f) = (-4; +\infty)$$

$$E(f) = (e^{-4}; +\infty)$$

Oberet:  $(e^{-4}; +\infty)$

$$W6.1.73 - \sqrt{1+x^2} = \sqrt{7}$$

W6.1.73

$$f(x) = \frac{x}{|x|}$$

$x \neq 0$

upu  $x < 0, y = -1$   
upu  $x > 0, y = 1$

Obereit:  $y \in [-1; -1] \cup [1; 1]$

W6.1.74

$$f(x) = \sin x \cdot \cos x$$

$$\sin x \cdot \cos x = \frac{\sin x \cos x}{2} = \frac{\sin 2x}{2}$$

$$x \in \left[-\frac{\pi}{4} + 2\pi k; \frac{\pi}{4} + 2\pi k\right], k \in \mathbb{Z}$$

Obereit:  $y \in \left[-\frac{1}{2}; \frac{1}{2}\right]$

W6.1.75

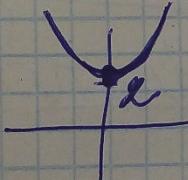
$$f(x) = \sqrt{x^2 + 4}$$

$$x^2 + 4 \geq 0$$

$$x^2 \geq -4 \Rightarrow x \in \mathbb{R}$$

upu  $x = 0, y = \sqrt{4} = 2$  беришика

Obereit:  $y \in [2; +\infty)$



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W6. 1.76

$$f(x) = x^2 - 4x + 3, x \in [0, 5]$$

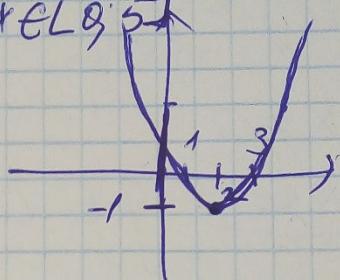
$$x^2 - 4x + 3 = 0$$

$$\Delta = 4^2 - 4 \cdot 3 = 4$$

$$x_1 = \frac{4+2}{2} = 3$$

$$x_2 = \frac{4-2}{2} = 1$$

$$K_0 = \frac{4}{2} = 2, y = 2^2 - 4 \cdot 2 + 3 = -1 \text{ - Minimum}$$



~~Obere Grenze~~

$$x=0, y=0^2 - 4 \cdot 0 + 3 = 3$$

$$x=5, y=5^2 - 4 \cdot 5 + 3 = 8$$

Obere Grenze  $y \in [-1, 8]$

Hauptsätze:  $y(0), y(\frac{1}{2}), y(\frac{x}{2}), y(t^2)$   
 $y(5x), y(5^x)$  genauso  $y(x)$ :

W6. 1.77

$$y(t^2) = \sqrt{2t+7}$$

$$y(0) = \sqrt{d \cdot 0 + f} = \sqrt{f}$$

$$y(2) = \sqrt{d \cdot 2 + f} = \sqrt{11}$$

$$y\left(\frac{x}{2}\right) = \sqrt{d \cdot \frac{x}{2} + f} = \sqrt{\frac{dx}{2} + f}$$

$$y(t^2) = \sqrt{d t^2 + f}$$

$$y(5x) = \sqrt{d \cdot 5x + f} = \sqrt{10x + f}$$

w6.1.78

$$y(x) = \begin{cases} -1 & \text{wenn } x < 2 \\ 0 & \text{wenn } x = 2 \\ 1 & \text{wenn } x > 2 \end{cases}$$

$$y(0) = -1, \quad y(2) = 0$$

$$y\left(\frac{x}{2}\right) = \begin{cases} -1 & \text{wenn } x < 4 \\ 0 & \text{wenn } x = 4 \\ 1 & \text{wenn } x > 4 \end{cases}$$

$$y(t^2) = \begin{cases} -1 & \text{wenn } |t| < \sqrt{2} \\ 0 & \text{wenn } |t| = \sqrt{2} \\ 1 & \text{wenn } |t| > \sqrt{2} \end{cases}$$

$$3y(5x) = \begin{cases} -3 & \text{wenn } x < \frac{2}{5} \\ 0 & \text{wenn } x = \frac{2}{5} \\ 3 & \text{wenn } x > \frac{2}{5} \end{cases}$$

W 6. 1.79

$$f(x) = 4x^3 - 4x + 1$$

$$f(1) = f(1) - p = \text{pfpue}$$

$$4x^3 - 4x + 1 = 4 \cdot 1^3 - 4 \cdot 1 + 1$$

$$4x^3 - 4x + 1 = 1$$

$$4x^3 - 4x = 0$$

$$4x(x^2 - 1) = 0$$

$$4x = 0 \quad \text{and} \quad x^2 - 1 = 0$$

$$x = 0$$

$$(x-1)(x+1) = 0$$

$$x-1 = 0 \quad \text{and} \quad x+1 = 0$$

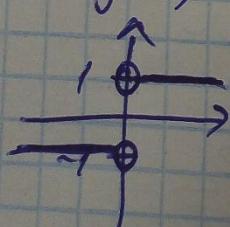
$$x = 1$$

$$x = -1$$

ObereT!  $x = 0, x = -1, x = 1$

W 6. 1.80

$$1) y(x) = \frac{|x|}{x}$$



wpy  $x > 0, y = 1$

wpy  $x < 0, y = -1$

$x \neq 0, y \neq 0$

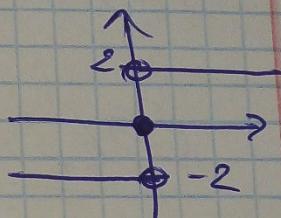
$$f(-x) = \frac{-x}{|x|} = -\frac{|x|}{x} = -f(x)$$

если отн.  $(0, 0)$

Ответ: ф-ия нечетная

2)  $y(x) = |x+1| - |x-1|$

$x$	-10	-5	-2	0	2	5	10
$y$	-2	-2	-2	0	2	2	2



$$y(x) = \begin{cases} -2 & \text{при } x < 0 \\ 0 & \text{при } x = 0 \\ 2 & \text{при } x > 0 \end{cases}$$

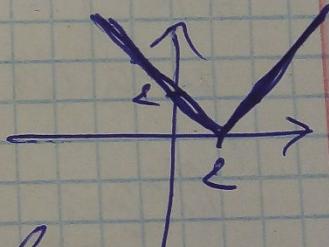
если отн.  $(0, 0)$

Ответ: ф-ия четная

3)  $g(t) = |t-2|$

$t$	-4	-2	0	2	4	6	8
$g$	6	4	2	0	2	4	6

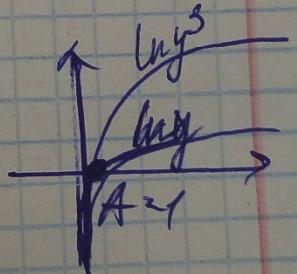
не симм.



Ответ: ф-ия общего вида

4)  $z(y) = \ln y^3$

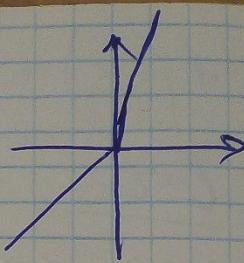
~~$\neq z(-y)$~~  не симм-ст.  
не член.



Ответ: ф-ия общего вида

$$5) A(x) = \begin{cases} x^3 & \text{при } x \geq 0 \\ x & \text{при } x < 0 \end{cases}$$

$f(x) = x \neq \begin{cases} x^3 \\ x \end{cases}$

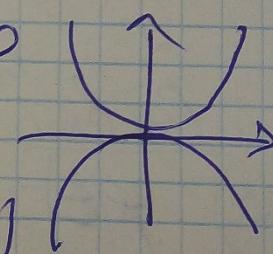


Обрати ф-ия общей буга

$$6) f(t) = \begin{cases} t^2 & \text{при } t \geq 0 \\ -t^2 & \text{при } t < 0 \end{cases}$$

$$f(-t) = -t^2 = -f(t)$$

ф-ия симм. очн.  $(0;0)$



Обрати кривизна ф-ии

$$7) h(x) = \frac{\arctg^2 x}{x-1}$$

$$f(-x) = \frac{\arctg^2(-x)}{-x-1} =$$

$$= \frac{-\arctg^2 x}{-(x+1)} = \frac{\arctg^2 x}{x+1} \neq h(x)$$

Обрати: ф-ия общей буга  $\neq -h(x)$

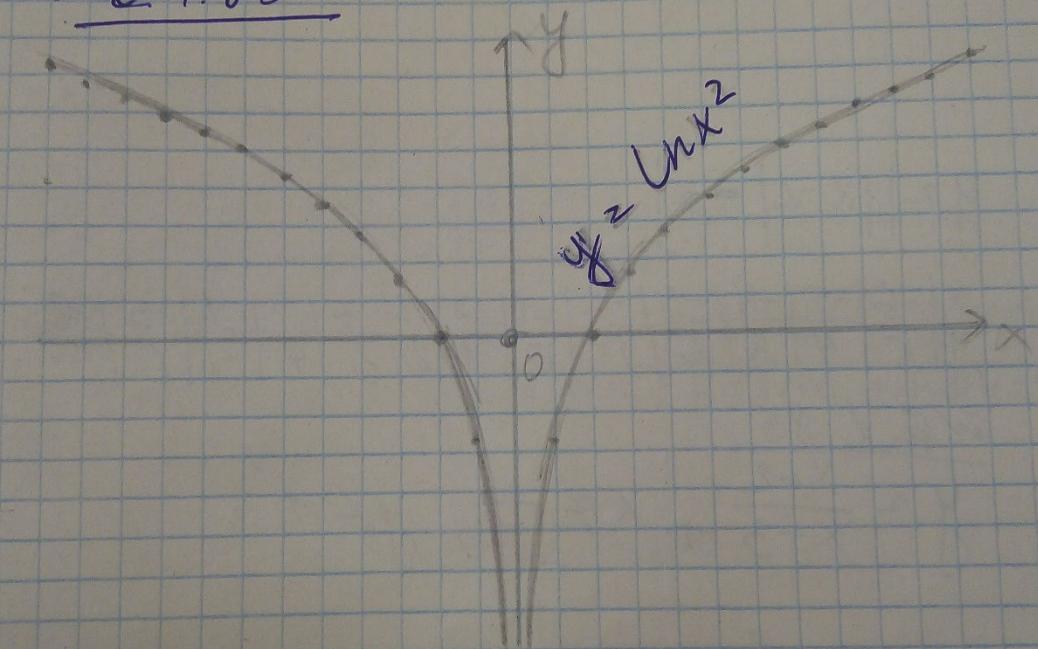
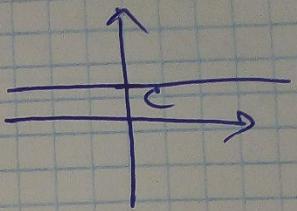
$$8) f(x) = c$$

qd-urz cenn. oth. Gy

$$f(-x) = c = f(x)$$

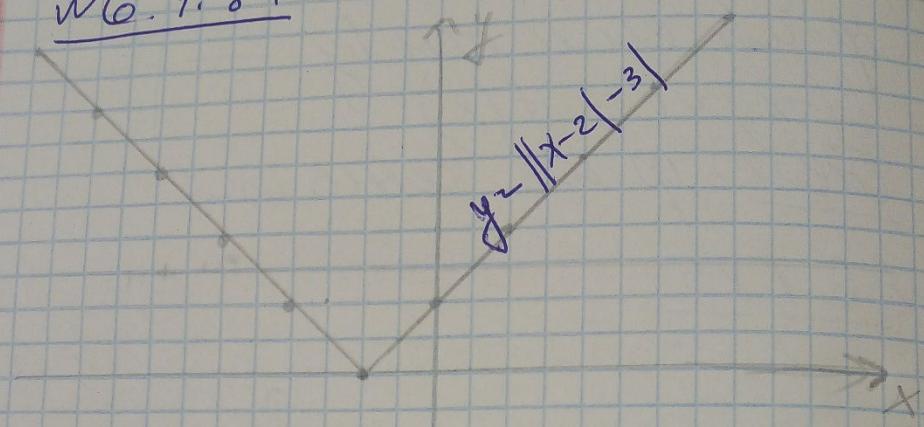
Ober: qd-urz cethar

W 6.189



X	$\pm 6$	$\pm 5.5$	$\pm 5$	$\pm 4.5$	$\pm 4$	$\pm 3.5$	$\pm 3$	$\pm 2.5$	$\pm 2$	$\pm 1.5$	$\pm 1$	$\pm 0.5$	$\pm 0.25$
y	36	34	3	24	5	3.75	2.5	3.19	1.83	1.4	0.81	0	-0.25

W6. 1. 84

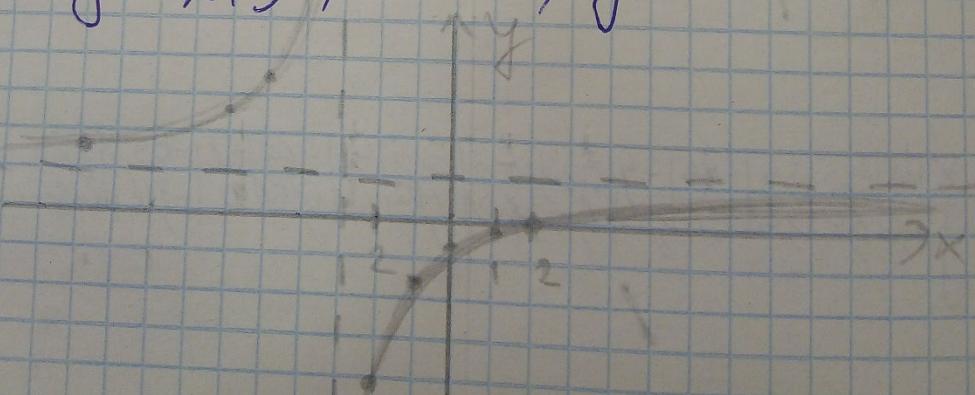


$$y = ||x - 2| - 3|$$

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
y	5	4	3	2	1	0	1	2	3	4	5	6	<del>7</del>

W6. 1. 85

$$y = \frac{x-2}{x+3}, x \neq -3, y \neq 1$$

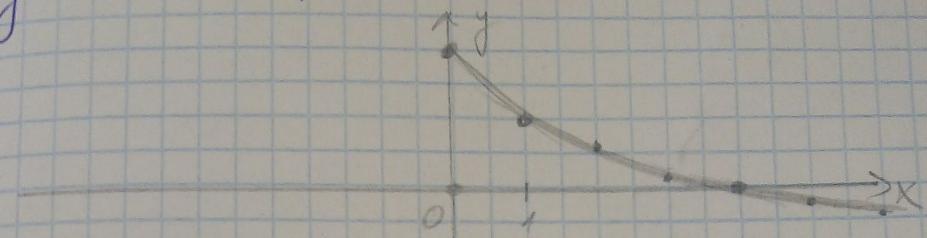


(8)

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	12	-10
y	<del>108</del>	<del>55</del>	6	-4	-1,5	-0,6666666666666667	0	1	2	3	4	5	<del>108</del>	<del>55</del>

W6. 1.86.

$$y = -\sqrt{x} + 2, \quad x \geq 0$$



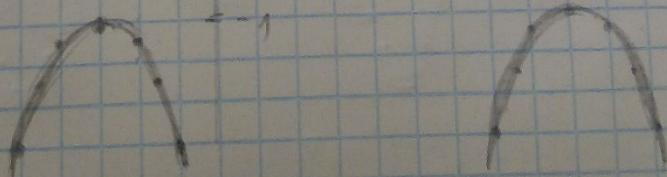
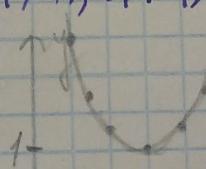
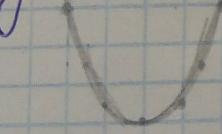
X	+6	5	4	3	2	1	0	-2	-1	Y
	-9.45	-0.24	0	1.92	2.95	3.81	4.2	-9.65	-9.83	

27  
1/2

W6. 1.87

$$y = \operatorname{cosec} x$$

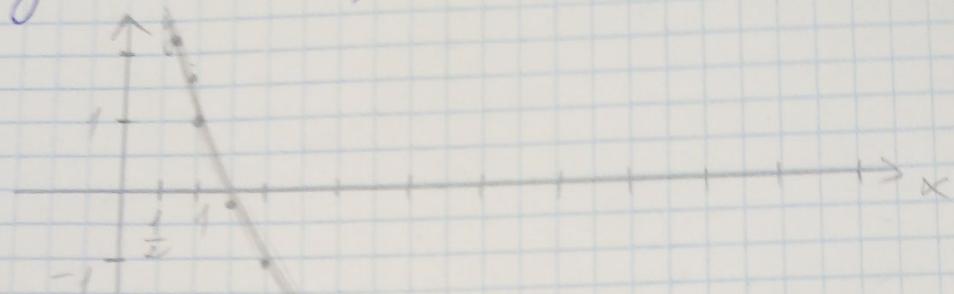
X	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{4\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$	$\frac{7\pi}{4}$	$\frac{4\pi}{3}$	$\frac{3\pi}{2}$	$\frac{5\pi}{3}$	$\frac{7\pi}{6}$	$\frac{\pi}{2}$	-1	Y
	2.0	1.41	1.15	1	0.15	0.41	2	-2.41	-1.15	-1	-0.41	-0.15	0		



X	$\pm \frac{\pi}{6}$	$\pm 1$	$\pm 1.5$	$\pm 2$	$\pm 2.5$	$\pm 3$	$\pm 3.5$	$\pm 4$	$\pm 4.5$	$\pm 5$	$\pm 5.5$	$\pm 6$	Y
	-1.08	-1.18	-1	-1.09	-1.67	-2.08	-2.85	-3.2	-3.92	-4.42	-4.93	-5.56	

$$\text{W6.1.88}$$

$$y = \frac{1}{1-x} \ln x, \quad x > 0$$



x	1	2	3	4	5	$\frac{1}{6}$	$\frac{2}{7}$	$\frac{6}{9}$	$\frac{5}{10}$	$\frac{5}{15}$	$\frac{35}{35}$	$\frac{35}{45}$	$\frac{5}{55}$
y	-1	-0.5	-0.23	-0.15	-0.10	-0.086	-0.075	-0.065	-0.058	-0.051	-0.044	-0.039	-0.035

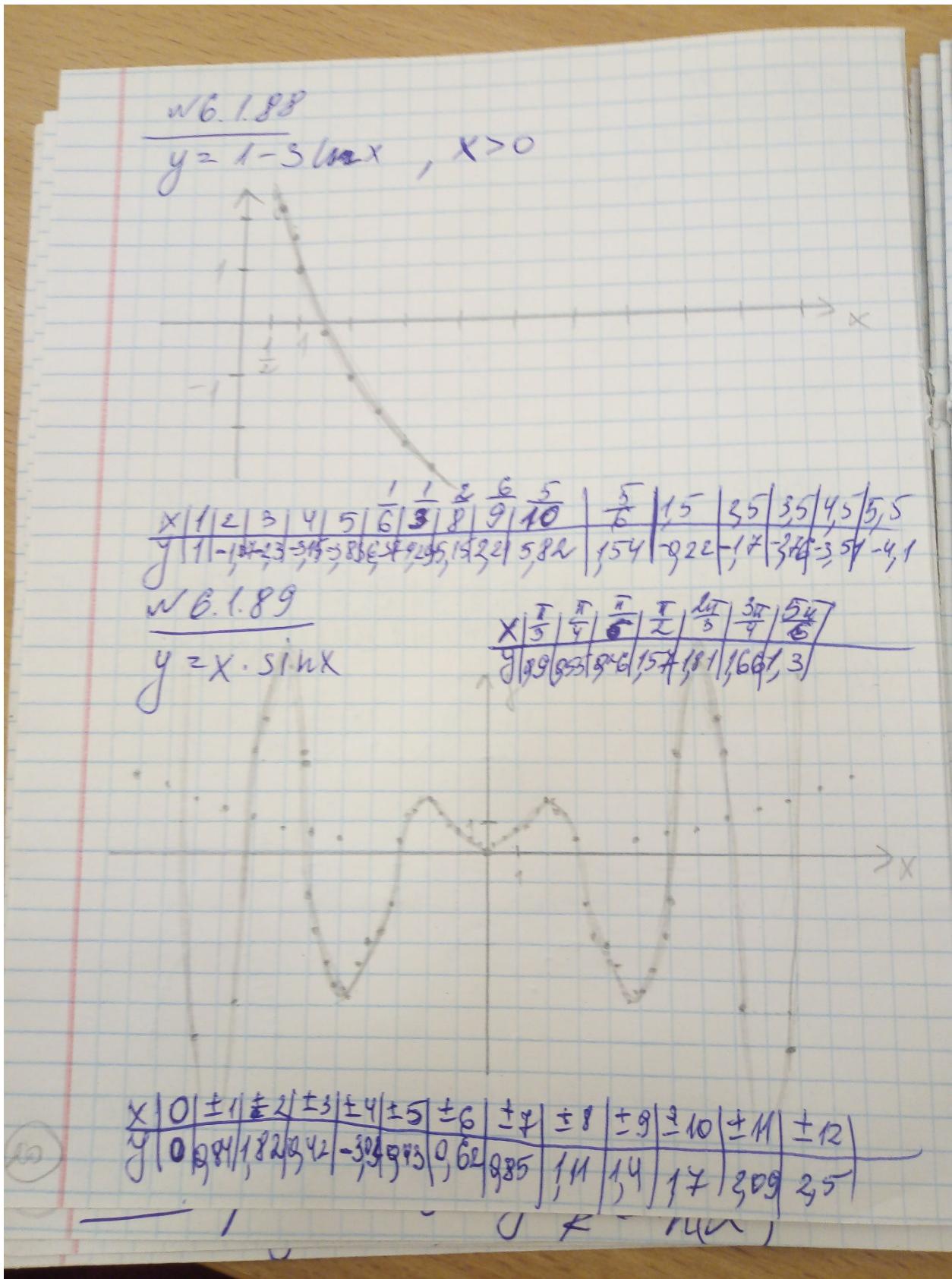
$$\text{W6.1.89}$$

$$y = x \cdot \sin x$$

x	$\frac{\pi}{3}$	$\frac{\pi}{4}$	$\frac{\pi}{6}$	$\frac{\pi}{2}$	$\frac{4\pi}{3}$	$\frac{3\pi}{4}$	$\frac{5\pi}{6}$
y	0.89	0.707	0.433	1.57	-1.57	-0.707	0.433

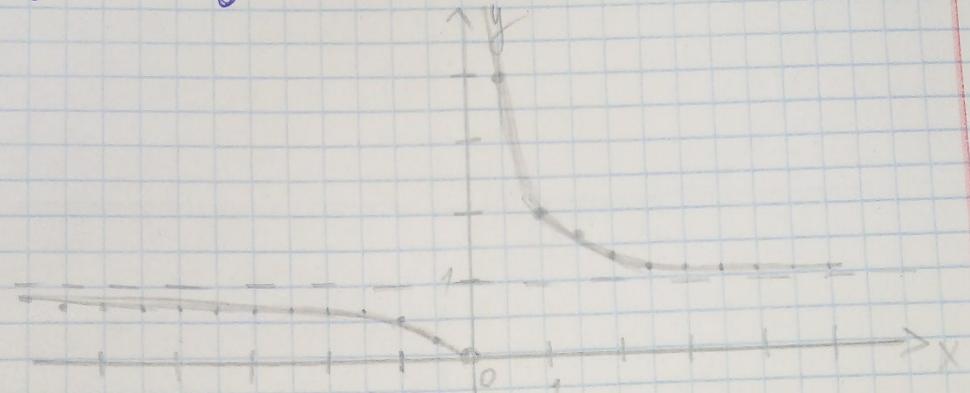
(2)

x	0	$\pm 1$	$\pm 2$	$\pm 3$	$\pm 4$	$\pm 5$	$\pm 6$	$\pm 7$	$\pm 8$	$\pm 9$	$\pm 10$	$\pm 11$	$\pm 12$
y	0	0.84	1.82	3.42	-3.05	0.93	0.62	0.85	1.11	1.4	1.7	2.09	2.5



$$\text{W 6.1.90}$$

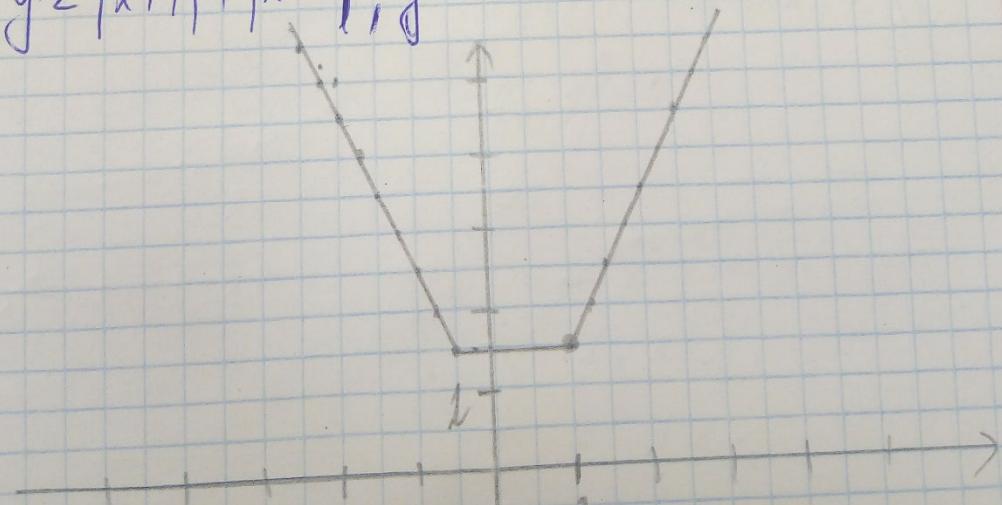
$$y = 2^{\frac{1}{x}}, y > 0$$



x	-6	-5	-5	-4	-4	-3	-3	-2	-2	-1	-1	-0.5	0	0.5	1	1.5	2	2.5	3	3.5	4	5	5.5
y	9.9	9.88	9.87	9.85	9.83	9.81	9.82	9.79	9.78	9.75	9.72	9.64	9.59	9.51	9.45	9.34	9.21	9.08	8.94	8.81	8.67	8.53	8.39

W 6.1.91

$$y = |x+1| + |x-2|, y > 0$$



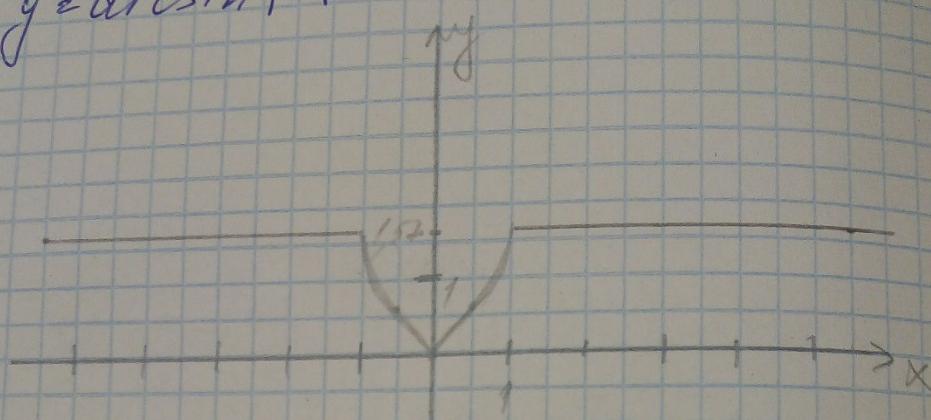
x	-5	-4	-4	-3	-3	-2	-2	-1	-1	-0.5	0	0.5	1	1.5	2	2.5	3	3.5	4
y	11	10	9	8	7	6	5	4	3	3	3	3	3	3	3	3	3	3	3

v-

v-

$$\text{w6.1.92}$$

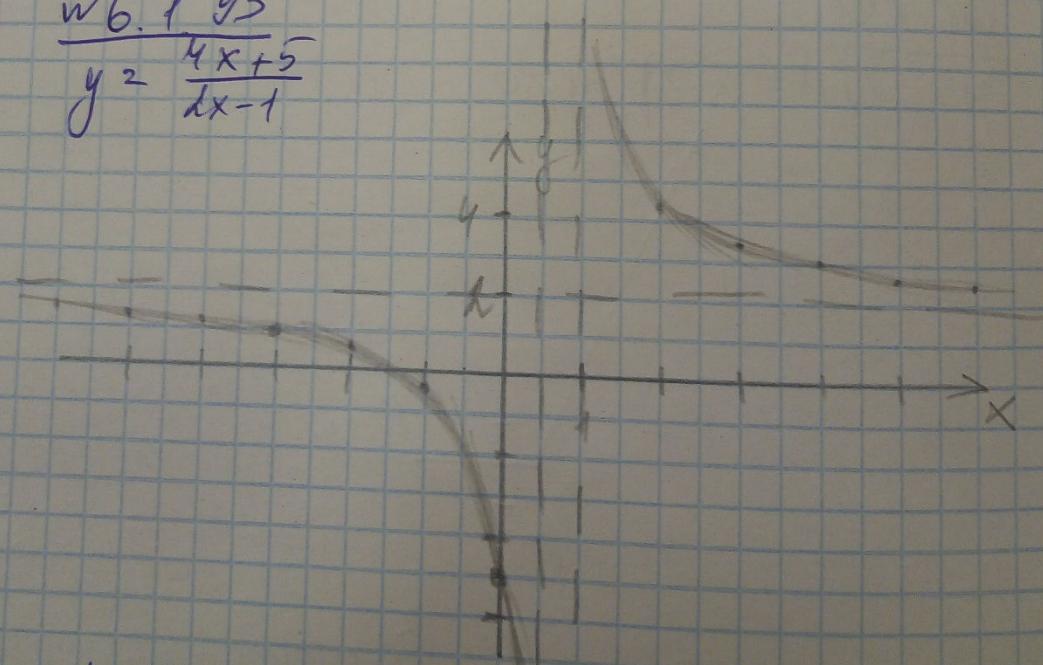
$$y = \arcsin |x|$$



$x$	-6	-5	-4	-3	-2	-1	-0.5	0	0.5	1	2	3	4	5	6
$y$	1.57	1.57	1.57	1.57	1.57	1.57	0.95	0	0.95	1.57	1.57	1.57	1.57	1.57	1.57

$$\text{w6.1.93}$$

$$y^2 = \frac{4x+5}{dx-1}$$

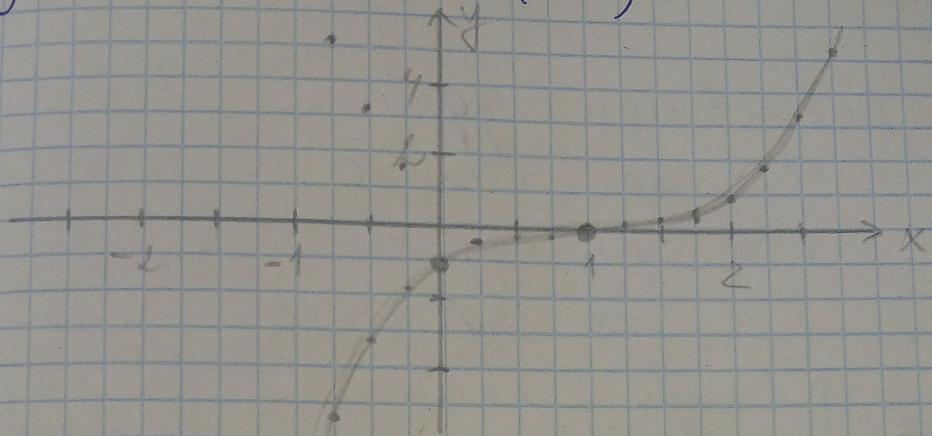


(1)

$x$	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
$y$	1.46	1.29	1.24	1	0.96	-0.55	-5	0.94	3.33	3.4	3	3.04	3.63

W6.1.94

$$y = x^3 - 3x^2 + 3x - 1 = (x-1)^3$$



6  
57

X	-3	-2,5	-2	-1,5	-1	-0,5	0,5	1,5	2	2,5	3
y	-64	-12,875	-27	-15,625	-8	-5,55	-3,375	-1,95	-1	-0,625	-0,125
X	1	1,25	1,5	1,75	2	2,25	2,5	2,75	3		
y	0	0,015	0,125	0,42	1	1,95	3,375	5,35	8		

W6.1.96  
HauTu:  $f \circ f$ ,  $f \circ g$ ,  $g \circ f$

$$1) f(x) = x^2, g(x) = x+2$$

$$f \circ f = f(f(x)) = (x^2)^2 = x^4$$

$$f \circ g = f(g(x)) = (x+2)^2$$

$$g \circ f = g(f(x)) = x^2 + 2$$

$$2) f(x) = \text{sign } x, g(x) = -2$$

$$\text{sign } x = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

$$f \circ f^{-1} \circ \text{sign}(\text{sign}(x)) =$$

$$= \begin{cases} 1 & \text{if } \text{sign}(x) > 0 \\ 0 & \text{if } \text{sign}(x) = 0 \\ -1 & \text{if } \text{sign}(x) < 0 \end{cases}$$

$$= \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

$$f \circ g = \text{sign}(-x) = -1$$

$$g \circ f = -x$$

$$(3) f(x) = \frac{1}{x-3}, g(x) = \frac{x-1}{x}$$

$$f \circ f^{-1} = \frac{1}{\frac{1}{x-3} - 3} = \frac{1}{\frac{1-3(x-3)}{x-3}} =$$

$$= \frac{x-3}{1-3x+9} = \frac{x-3}{-3x+10} \quad \cancel{\text{as } 3x+8}$$

$$f \circ g = \frac{1}{\frac{x-1}{x} - 3} = \frac{1}{\frac{x-1-3x}{x}} = \frac{x}{-1-2x}$$

$$g \circ f = \frac{\frac{1}{x-3} - 1}{\frac{1}{x-3}} = \frac{\frac{1-x+3}{x-3}}{\frac{1}{x-3}} =$$

$$= \frac{4-x}{x-3} \cdot (x-3) = 4-x$$

$$4) f(x) = [x], g(x) = \frac{1}{x^2+1}$$

$$f \circ f = [[x]] = [x]$$

$$f \circ g = \left[ \frac{1}{x^2+1} \right]$$

$$g \circ f = \frac{1}{[x]^2+1}$$

w 6.1.9f

$$y = \frac{x}{1-x}$$

$$\alpha = b \times \beta$$

$$\frac{\beta}{1-\beta} = \frac{\beta}{1-\beta}$$

$$x = \frac{y}{1-y}$$

$$y = x(1-x)$$

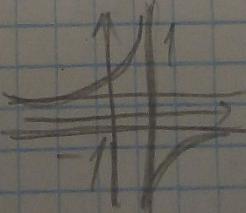
$$\frac{a}{1-a} = \frac{b}{1-b}$$

$$a/(1-b) = b/(1-a)$$

$$a - ab = b - ab$$

$$a = b - ab - y \text{ aus}$$

unterteilt so dass



(15)

(18)

$$y = x - xy$$

$$y + xy = x$$

$$y(1+x) = x$$

$y = \frac{x}{1+x}$  - обратная ф-ция

№ 6.1.98

$$y = 2^{x-3}$$

$$2^{a-3} = 2^{b-3}$$

$$\frac{2^a}{2^3} = \frac{2^b}{2^3}$$

$$2^a = 2^b$$

$$a = b$$

$$x = 2^{y-3}$$

$$x = \frac{2^y}{2^3}$$

$$2^y = 8x$$

$$\log_2 2^y = \log_2 8x$$

$$y = \log_2 8 + \log_2 x$$

$$y = 3 + \log_2 x$$

$$y = \log_2 x + 3$$

Обратная ф-ция

№ 6.1.99

$$y = \begin{cases} 2x & \text{при } x \geq 0 \\ -x^2 & \text{при } x < 0 \end{cases}$$

имеет обратную  
ф-цию

(6)

$$x^2 - dy^2 \Rightarrow y^2 = \frac{x}{2}$$

$x^2 - y^2 \Rightarrow$  не имеет реальной части

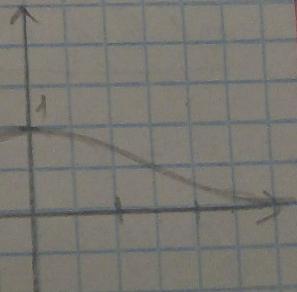
$$y^2 = \begin{cases} \frac{x}{2} & \text{при } x \geq 0 \\ -\sqrt{\xi x} & \text{при } x < 0 \end{cases}$$

н. 6.1.100

$$y = \operatorname{sign} x = \begin{cases} -1 & \text{при } x < 0 \\ 0 & \text{при } x = 0 \\ 1 & \text{при } x > 0 \end{cases}$$

нет обратной, т.к. оп-ция  
постоянна на  
разных отрезках

$$\frac{w 6.1.101}{y = 2^{-x^2}} = \frac{1}{2x^2}$$



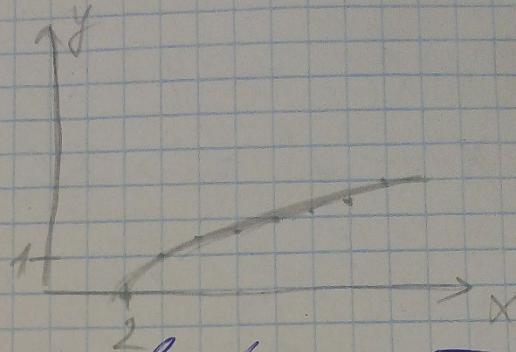
оп-ция ограничена  
сверху и снизу, т.к.  
 $y \in (0; 1]$

№ 6.1.102

$$y = \sqrt{x-2}$$

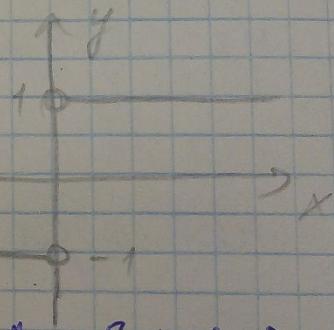
Ф-ция  
стремо

многоточна, тк возрастает  
и ограничена сверху, т. к.  $y \in [0; +\infty)$



№ 6.1.103

$$y = \frac{|x|}{x} = \begin{cases} 1 & \text{при } x > 0 \\ x \neq 0, y \neq 0 \\ -1 & \text{при } x < 0 \end{cases}$$



Ф-ция обратима

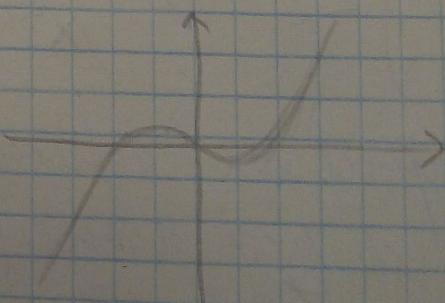
т. к.  $y \in \mathbb{R} \setminus \{0\} \cup \{-1\} \cup \{1\}$

№ 6.1.104

$$y = x^3 - x$$

Ф-ция не  
многоточна

и не ограничена



№6. 1.105

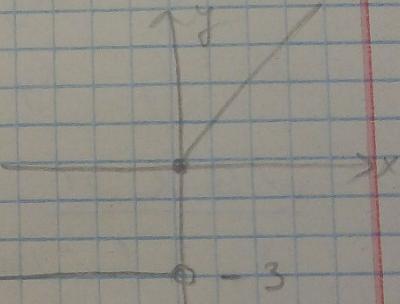
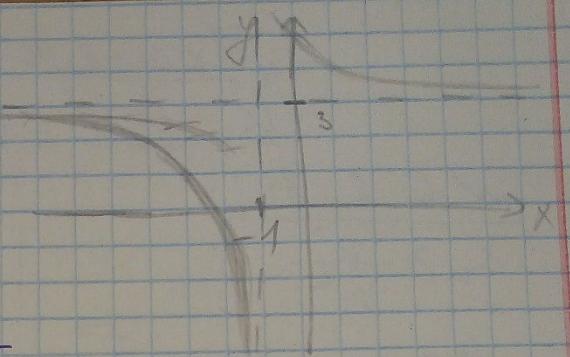
$$y = \frac{3x+5}{x+1}$$

функция не  
многоточечна  
и не ограничена

№6. 1.106

$$y = \begin{cases} -3 & \text{при } x < 0 \\ x & \text{при } x \geq 0 \end{cases}$$

функция многоточечна,  
т.к. неубывающая,  
и ограничена снизу, т.к.  $y \in \{-3\} \cup$   
 $\cup [0; +\infty)$



(2)