Производные

№7.1.98

$$y = 5\sqrt{x} + \frac{13}{x^4} - \frac{2}{\sqrt[3]{x}}$$

$$y' = (5\sqrt{x})' + \left(\frac{13}{x^4}\right)' - \left(\frac{2}{\sqrt[3]{x}}\right)' =$$

$$= \frac{5}{2\sqrt{x}} + (13x^{-4})' - (2x^{-\frac{1}{3}})' =$$

$$= \frac{5}{2\sqrt{x}} + 13\cdot(-4)\cdot x^{-5} - 2\cdot\left(-\frac{1}{3}\right)\cdot x^{-\frac{4}{3}} =$$

$$= \frac{5}{2\sqrt{x}} - \frac{52}{x^5} + \frac{2}{3\sqrt[3]{x^4}}$$

№7.1.99

$$y = 10x^6 - \frac{4}{x} + 3\sqrt[5]{x}$$

$$y' = (10x^6)' - \left(\frac{4}{x}\right)' + (3\sqrt[5]{x})' =$$

$$= 10\cdot 6x^5 + \frac{4}{x^2} + 3\cdot\frac{1}{5}x^{-\frac{4}{5}} =$$

$$= 60x^5 + \frac{4}{x^2} + \frac{3}{5\sqrt[5]{x^4}}$$

№7.1.100

$y = 2\,ctg x - 3\sin x$

$y' = (2\,ctg x)' - (3\sin x)' =$

$= 2\cdot\left(-\frac{1}{\sin^2 x}\right) - 3\cos x =$

$= -\frac{2}{\sin^2 x} - 3\cos x$

№7.1.101

$y = arctg x + 7e^x$

$y' = (arctg x)' + (7e^x)' =$

$= \frac{1}{1+x^2} + 7e^x$

№7.1.102

$y = 19^x - 8\arcsin x$

$y' = (19^x)' - (8\arcsin x)' =$

$= 19^x \ln 19 - 8\cdot\frac{1}{\sqrt{1-x^2}} =$

$= 19^x \ln 19 - \frac{8}{\sqrt{1-x^2}}$

№7.1.103

$y = (x^2-1)(x^3-x)$

$y' = ((x^2-1)(x^3-x))' = (x^2-1)'(x^3-x) +$
$+ (x^2-1)(x^3-x)' =$
$= 2x(x^3-x) + (x^2-1)(3x^2-1) =$
$= 2x^4 - 2x^2 + 3x^4 - 3x^2 - x^2 + 1 =$
$= 5x^4 - 6x^2 + 1$

№7.1.104

$\varphi(\alpha) = 3\arcsin\alpha - 4\arccos\alpha +$
$+ 14\sqrt[7]{\alpha}$

$\varphi'(\alpha) = (3\arcsin\alpha)' - (4\arccos\alpha)' +$
$+ (14\sqrt[7]{\alpha})' = \frac{3}{\sqrt{1-\alpha^2}} + \frac{4}{\sqrt{1-\alpha^2}} +$

$+ 14\alpha^{-\frac{8}{7}} \cdot \left(\frac{1}{7}\right) = \frac{3}{\sqrt{1-\alpha^2}} + \frac{4}{\sqrt{1-\alpha^2}} -$

$- \frac{2}{\sqrt[7]{\alpha^8}}$

№7.1.105

$f(t) = \frac{t}{1-t^2}$

$$f'(t) = \left(\frac{t}{1-t^2}\right)' = \frac{t'(1-t^2) - t(1-t^2)'}{(1-t^2)^2} =$$

$$= \frac{1-t^2+2t^{4}}{(1-t^2)^2} = \frac{-t^2+2t+1}{(1-t^2)^2}$$

W7.1.106

$$y = 3\sin^2 x - \lg x + 3\cos^2 x$$

$$y' = (3\sin^2 x)' - (\lg x)' + (3\cos^2 x)' =$$

$$= 3\cdot 2\sin x\cos x - \frac{1}{x\ln 10} + 3\cdot(-2\sin x\cos x) =$$

$$= 6\sin x\cos x - 6\sin x\cos x - \frac{1}{x\ln 10} =$$

$$= -\frac{1}{x\ln 10}$$

W7.1.107

$$y = \left(\frac{1}{2}\right)^x - \frac{1}{3^x} + 4^x$$

$$y' = \left(\left(\frac{1}{2}\right)^x\right)' - \left(\frac{1}{3^x}\right)' + (4^x)' =$$

$$= \left(\frac{1}{2}\right)^x \ln\frac{1}{2} - \frac{1}{3^x}\ln\frac{1}{3} + 4^x\ln 4 =$$

$$= -\frac{1}{2^x}\ln 2 + \frac{1}{3^x}\ln 3 + 4^x \ln 4$$

w 7.1.108

$$y = \frac{e^x + \ln x}{e^x - \ln x}.$$

$$y' = \left(\frac{e^x + \ln x}{e^x - \ln x}\right)' = \frac{(e^x + \ln x)'(e^x - \ln x)}{(e^x - \ln x)^2} -$$

$$- \frac{(e^x + \ln x)(e^x - \ln x)'}{(e^x - \ln x)^2} =$$

$$= \frac{\left(e^x + \frac{1}{x}\right)(e^x - \ln x) - (e^x + \ln x)\left(e^x - \frac{1}{x}\right)}{(e^x - \ln x)^2} =$$

$$= \frac{\cancel{e^{2x}} + \frac{e^x}{x} - e^x \ln x - \cancel{\frac{\ln x}{x}} - \cancel{e^{2x}} - e^x \ln x}{(e^x - \ln x)^2} +$$

$$+ \frac{\frac{e^x}{x} + \cancel{\frac{\ln x}{x}}}{(e^x - \ln x)^2} = \frac{\frac{2e^x}{x} - 2e^x \ln x}{(e^x - \ln x)^2} =$$

$$= \frac{2e^x - 2e^x x \ln x}{x(e^x - \ln x)^2} = \frac{2e^x(1 - x\ln x)}{x(e^x - \ln x)^2}$$

№7.1.109

$y = (x+1)(x+2)(x+3)$

$y' = ((x+1)(x+2)(x+3))' =$

$= (\underbrace{((x+1)(x+2))}\underbrace{(x+3)})' =$

$= ((x+1)(x+2))'(x+3) +$

$+ (x+1)(x+2)(x+3)' =$

$= ((x+1)'(x+2) + (x+1)(x+2)')(x+3) +$

$+ (x+1)(\cancel{x}+2)(x+3)' =$

$= (\overbrace{x+2+x+1}^{2x+3})(x+3) + (x+1)(x+2) =$

$= 2x^2+3x+6x+9+x^2+x+2x+2 =$

$= 3x^2+12x+11$

№7.1.110

$y = (x^2-1)(x^2-3)(x^2-5)$

$y' = ((x^2-1)(x^2-3)(x^2-5))' =$

$=$ [по №7.1.109] $=$

$= ((x^2-1)'(x^2-3)+(x^2-1)(x^2-3)')\cdot$

$\cdot (x^2-5) + (x^2-1)(x^2-3)(x^2-5)' =$

$= 2x(x^2-3)(x^2-5) + 2x(x^2-1)(x^2-5) +$

$+ 2x(x^2-1)(x^2-3) = 2x(x^4-3x^2-5x^2+$

$+15 + x^4 - x^2 - 5x^2 + 5 + x^4 - x^2 - 3x^2 + 3) =$

$= 2x(3x^4 - 18x^2 + 23)$

<u>№7.1.111</u>

$$f(x) = \frac{x^2 - x + 2}{x^3 + 4}$$

$$f'(x) = \left(\frac{x^2 - x + 2}{x^3+4}\right)' = \frac{(x^2-x+2)'(x^3+4) - (x^2-x+2)(x^3+4)'}{(x^3+4)^2} =$$

$$= \frac{(2x-1)(x^3+4) - (x^2-x+2)(3x^2)}{(x^3+4)^2} =$$

$$= \frac{2x^4 - x^3 + 8x - 4 - 3x^4 + 3x^3 - 6x^2}{(x^3+4)^2} =$$

$$= \frac{-x^4 + 2x^3 - 6x^2 + 8x - 4}{(x^3+4)^2} =$$

$$= \frac{-2x^4 + x^3 - 3x^2 + 4x - 2}{2(x^3+4)^2}$$

w7.1.112

$$y = \frac{3}{x^4+2}$$

$$y' = \left(\frac{3}{x^4+2}\right)' = -\frac{3\left(x^4+2\right)'}{\left(x^4+2\right)^2} =$$

$$= -\frac{3 \cdot 4x^3}{\left(x^4+2\right)^2} = -\frac{12x^3}{\left(x^4+2\right)^2}$$

w7.1.113

$$y = \sqrt{x}\left(x^5+\sqrt{x}-2\right)$$

$$y' = \left(\sqrt{x}\left(x^5+\sqrt{x}-2\right)\right)' =$$

$$= \left(\sqrt{x}\right)'\left(x^5+\sqrt{x}-2\right) + \sqrt{x}\left(x^5+\sqrt{x}-2\right)' =$$

$$= \frac{1}{2\sqrt{x}}\left(x^5+\sqrt{x}-2\right) + \sqrt{x}\left(5x^4+\frac{1}{2\sqrt{x}}\right) =$$

$$= \frac{x^5+\sqrt{x}-2}{2\sqrt{x}} + 5x^4\sqrt{x} + \frac{1}{2} =$$

$$= \frac{x^5+\sqrt{x}-2+10x^5+\sqrt{x}}{2\sqrt{x}} =$$

$$= \frac{11x^5+2\sqrt{x}-2}{2\sqrt{x}}$$

№7.1.114

$$y = \frac{3^{2x}}{2^{2x}} - \sqrt[5]{x}\ln x^5$$

$$y' = \left(\frac{3^{2x}}{2^{2x}}\right)' - \left(\sqrt[5]{x}\ln x^5\right)' =$$

$$= \frac{9}{4}\left(\left(\frac{3}{2}\right)^x\right)' - 5\left(x^{\frac{1}{5}}\ln x\right)' =$$

$$= \cancel{\left(\frac{9}{4}\right)^x \ln\frac{9}{4}}\ \frac{9}{4}\cdot\left(\frac{3}{2}\right)^x \ln\frac{3}{2} - 5\cdot$$

$$\cdot\left(\left(x^{\frac{1}{5}}\right)'\ln x + x^{\frac{1}{5}}(\ln x)'\right) =$$

$$= \frac{9\cdot 3^x}{4\cdot 2^x}\ln\frac{3}{2} - 5\left(\frac{\ln x\sqrt[5]{x}}{5x} + \frac{\sqrt[5]{x}}{x}\right) =$$

$$= \frac{3^{x+2}}{2^{x+2}}\ln\frac{3}{2} - \frac{\ln x\sqrt[5]{x} + 5\sqrt[5]{x}}{x} =$$

$$= \left(\frac{3}{2}\right)^{x+2}\ln\frac{3}{2} - \frac{\sqrt[5]{x}(\ln x + 5)}{x}$$

№7.1.115

$$f(x) = \frac{x^2}{x^3+1},\ x_0 = 1$$

$$f'(x) = \left(\frac{x^2}{x^3+1}\right)' = \frac{2x(x^3+1) - x^2\cdot 3x^2}{(x^3+1)^2}$$

$$f'(x_0) = \frac{2\cdot 1(1^3+1) - 1^2 \cdot 3\cdot 1^2}{(1^3+1)^2} =$$

$$= \frac{2\cdot 2 - 3}{2^2} = \frac{1}{4} = 0{,}25$$

Ответ: 0,25

<u>№7.1.116</u>

$f(x) = 4x + 6\sqrt[3]{x}, \quad x_0 = 8$

$$f'(x) = 4 + \frac{6\sqrt[3]{x}}{3x} \quad \left[\sqrt[n]{x}' = \frac{\sqrt[n]{x}}{nx}\right] =$$

$$= 4 + \frac{2\sqrt[3]{x}}{x}$$

$$f'(x_0) = 4 + \frac{2\cdot\sqrt[3]{8}}{\cancel{8}_4} = 4 + \frac{2}{4} = 4{,}5$$

Ответ: 4,5

<u>№7.1.117</u>

$f(x) = x^2 + 3\sin x - \pi x, \quad x_0 = \frac{\pi}{2}$

$f'(x) = 2x + 3\cos x - \pi$

$f'(x_0) = 2\cdot\frac{\pi}{2} + 3\cos\frac{\pi}{2} - \pi =$

$$= 3\cos\frac{\pi}{2} = 0$$

Ответ: 0

№7.1.118

$$f(x) = e^{x+1}\cdot(4x-5),\quad x_0 = \ln 2$$

$$f'(x) = \left(e\cdot e^{x}\cdot(4x-5)\right)' =$$

$$= ee^{x}(4x-5) + ee^{x}\cdot 4 =$$

$$= e^{x+1}(4x-5) + 4e^{x+1}$$

$$f'(x_0) = e^{\ln 2+1}(4\ln 2-5) +$$

$$+ 4e^{\ln 2+1} = 2e(4\ln 2-5) +$$

$$+ 8e = 2e(4\ln 2-1)$$

Ответ: $2e(4\ln 2-1)$

№7.1.119

$$y = 10^{x^2+1}$$

$$y' = 10^{x^2+1}\ln(10)\cdot(x^2+1)' =$$

$$= 10^{x^2+1}\ln 10\cdot 2x = 2\cdot 10^{x^2+1}x\ln 10$$

w7.1.120

$y = tg\,4x$

$y' = \frac{1}{\cos^2 4x} \cdot (4x)' = \frac{4}{\cos^2 4x}$

w 7.1.121

$y = ch^4 \frac{x}{2} = \left(ch \frac{x}{2}\right)^4$

$y' = \left(\left(ch \frac{x}{2}\right)^4\right)' = 4\left(ch \frac{x}{2}\right)^3 \cdot$

$\cdot \left(ch \frac{x}{2}\right)' = 4\, ch^3 \frac{x}{2} \cdot sh \frac{x}{2} \cdot \left(\frac{x}{2}\right)' =$

$= 4\, ch^3 \frac{x}{2}\, sh \frac{x}{2} \cdot \frac{1}{2} = 2\, ch^3 \frac{x}{2}\, sh \frac{x}{2}$

w7.1.122.

$y = \ln(5x^3 - x)$

$y' = (\ln(5x^3 - x))' = \frac{(5x^3 - x)'}{5x^3 - x} =$

$= \frac{15x^2 - 1}{5x^3 - x}$

w7.1.123

$$y = \cos^4 x - \sin^4 x$$

$$y' = 4\cos^3 x \cdot (\cos x)' - 4\sin^3 x \cdot (\sin x)' =$$

$$= -4(\cos^3 x \sin x + \sin^3 x \cos x)$$

w7.1.124

$$y = \sqrt{4-7x^2}$$

$$y' = \frac{1}{2\sqrt{4-7x^2}} \cdot (4-7x^2)' =$$

$$= \frac{-14x}{2\sqrt{4-7x^2}} = -\frac{7x}{\sqrt{4-7x^2}}$$

w7.1.125

$$y = \sqrt[5]{1+\operatorname{ctg} 10x}$$

$$y' = \frac{\sqrt[5]{1+\operatorname{ctg}10x}}{5(1+\operatorname{ctg}10x)} \cdot (1+\operatorname{ctg}10x)' =$$

$$= \frac{\sqrt[5]{1+\operatorname{ctg}10x}}{5(\cancel{1+}\operatorname{ctg}10x)} \cdot \left(-\frac{1}{\sin^2 10x}\right) \cdot (10x)' =$$

$$= -\frac{2\sqrt[5]{1+\operatorname{ctg}10x}}{\operatorname{ctg}10x \sin^2 10x}$$

w 7.1.126

$y = (\sin 3x - \cos 3x)^2$

$y' = 2(\sin 3x - \cos 3x) \cdot (\sin 3x - \cos 3x)' =$

$= 2(\sin 3x - \cos 3x)(3\cos 3x + 3\sin 3x) =$

$= 6(\sin 3x - \cos 3x)(\cos 3x + \sin 3x) =$

$= 6(\sin^2 3x - \cos^2 3x)$

w 7.1.127

$x = \ln^4 \sin 3t$

$x' = 4\ln^3 \sin 3t \cdot (\ln(\sin 3t))' =$

$= 4\ln^3 \sin 3t \cdot \frac{1}{\sin 3t} \cdot (\sin 3t)' =$

$= 4\ln^3 \sin 3t \cdot \frac{1}{\sin 3t} \cdot \cos 3t \cdot 3 =$

$= \frac{12\ln^3(\sin 3t)\cos 3t}{\sin 3t} = 12\ln^3(\sin 3t) \cdot$

$\cdot \operatorname{ctg} 3t$

w 7.1.128

$f(h) = \operatorname{arctg}\sqrt{h}$

$f'(h) = \frac{1}{1+h} \cdot (\sqrt{h})' = \frac{1}{2(1+h)\sqrt{h}}$

$$= \frac{1}{2}\sqrt{h}\,(1+h)$$

№ 7.1.129

$$y = \frac{1}{\arcsin x}$$

$$y' = \left(\arcsin^{-1} x\right)' = -\frac{1}{\arcsin^2 x}\cdot$$

$$\cdot\left(\arcsin x\right)' = -\frac{1}{\arcsin^2 x\,\sqrt{1-x^2}}$$

№ 7.1.130

$$y = \frac{\sin x}{1+\operatorname{tg} x}$$

$$y' = \frac{(\sin x)'(1+\operatorname{tg} x) - \sin x\,(1+\operatorname{tg} x)'}{(1+\operatorname{tg} x)^2} =$$

$$= \frac{\cos x\,(1+\operatorname{tg} x) - \frac{\sin x}{\cos^2 x}}{(1+\operatorname{tg} x)^2} =$$

$$\Rightarrow \frac{\cos x + \sin x - \frac{tg x}{\cos x}}{(1+tg x)^2} =$$

$$= \frac{\cos^2 x + \sin x \cos x - tg x}{\cos x (1+tg x)^2}$$

№7.1.131

$$y = \frac{x \ln x}{x-1}$$

$$y' = \frac{(x\ln x)'(x-1) - x\ln x (x-1)'}{(x-1)^2} =$$

$$= \frac{(\ln x + 1)(x-1) - x\ln x}{(x-1)^2} =$$

$$= \frac{\cancel{x\ln x} - \ln x + x - 1 - \cancel{x\ln x}}{(x-1)^2} =$$

$$= \frac{x - \ln x - 1}{x^2 - x + 1}$$

w7.1.132

$y = sh(\ln(tg\,2x))$

$y' = ch(\ln(tg\,2x)) \cdot (\ln(tg\,2x))' \cdot$

$\cdot (tg\,2x)' \cdot (2x)' = ch(\ln(tg\,2x)) \cdot$

$\cdot \frac{1}{tg\,2x} \cdot \frac{1}{\cos^2 2x} \cdot 2 =$

$= \frac{2ch(\ln(tg\,2x))}{tg\,2x \cos^2 2x}$

w7.1.133

$y = x \arcsin x + \sqrt{1-x^2}$

$y' = \arcsin x + x \cdot \frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{1-x^2}} \cdot$

$\cdot (1-x^2)' = \arcsin x + \frac{x}{\sqrt{1-x^2}} +$

$+ \frac{1}{2\sqrt{1-x^2}} \cdot 2x =$

$= \arcsin x + \frac{2x}{\sqrt{1-x^2}}$

№7.1.134

$$y = 3^{\sin^3 2x + 4\sin 2x}$$

$$y' = 3^{\sin^3 2x + 4\sin 2x} \cdot \ln 3 \cdot$$

$$\cdot (\sin^3 2x + 4\sin 2x)'$$

$$(\sin^3 2x + 4\sin 2x)' =$$

$$= 3\sin^2 2x \cdot \cos 2x \cdot 2 + 4\cos 2x \cdot 2 =$$

$$= 6\sin^2 2x \cdot \cos 2x + 8\cos 2x$$

$$y' = 3^{\sin^3 2x + 4\sin 2x} \ln 3 \cdot \cos 2x \cdot$$

$$\cdot (6\sin^2 2x + 8)$$

№7.1.135

$$y = e^{-\ln\frac{x+2}{x-3}} - \frac{x-3}{x+2}$$

$$y' = e^{-\ln\frac{x+2}{x-3}} \cdot \left(-\ln\frac{x+2}{x-3}\right)' \cdot \left(\frac{x+2}{x-3}\right)' -$$

$$-\left(\frac{x-3}{x+2}\right)' =$$

$$= e^{-\ln\frac{x+2}{x-3}} \cdot \frac{x-3}{x+2} \cdot \frac{x-3-x-2}{(x-3)^2} -$$

$$-\frac{\cancel{x}+2-\cancel{x}+3}{(x+2)^2} =$$

$$= e^{-\ln\frac{x+2}{x-3}} \cdot \frac{1}{x+2} \cdot \frac{-5}{x-3} -$$

$$-\frac{5}{(x+2)^2} = e^{-\ln\frac{x+2}{x-3}} \cdot \frac{5}{(x+2)(x-3)} -$$

$$-\frac{5}{(x+2)^2} = \frac{5e^{-\ln\frac{x+2}{x-3}}(x+2) - 5(x-3)}{(x+2)^2(x-3)}$$

№7.1.136

$$y = \arcsin\sqrt{1-x^2}$$

$$y' = \frac{1}{\sqrt{1-\sqrt{1-x^2}}} \cdot \left(\sqrt{1-x^2}\right)' \cdot (1-x^2)' =$$

$$= \frac{\cancel{2}x}{\cancel{2}\sqrt{1-x^2}\sqrt{1-\sqrt{1-x^2}}} =$$

$$= \frac{x}{(1-x^2)^{\frac{1}{2}}\left(1-(1-x^2)^{\frac{1}{2}}\right)^{\frac{1}{2}}}$$

W7.1.137

$y = x \cdot 2^{\sqrt{x}}$

$y' = 2^{\sqrt{x}} + x \cdot 2^{\sqrt{x}} \ln 2 \cdot (\sqrt{x})' =$

$= 2^{\sqrt{x}} + x 2^{\sqrt{x}} \ln 2 \cdot \frac{1}{2\sqrt{x}} =$

$= 2^{\sqrt{x}} \left(1 + \frac{x \ln 2}{2\sqrt{x}}\right)$

W7.1.138

$y = 5^{\frac{1}{\log_5 x}}$

$y' = 5^{\frac{1}{\log_5 x}} \cdot \ln 5 \cdot \left(\frac{1}{\log_5 x}\right)' \cdot$

$\cdot (\log_5 x)' = 5^{1/\log_5 x} \cdot \cancel{\ln 5} \cdot$

$\cdot \left(-\frac{1}{\log_5^2 x}\right) \cdot \frac{1}{x \cancel{\ln 5}} =$

$= -\frac{5^{1/\log_5 x}}{x \log_5^2 x}$

№7.1.139

$$y = \frac{1}{6} \ln \frac{x-3}{x+3}$$

$$y' = \frac{1}{6} \cdot \frac{x+3}{x-3} \cdot \left(\frac{x-3}{x+3}\right)' =$$

$$= \frac{\cancel{x+3}}{\cancel{6}(x-3)} \cdot \frac{\cancel{x}+\cancel{3}-\cancel{x}+\cancel{3}}{(x+3)^{\cancel{2}}} =$$

$$= \frac{1}{(x-3)(x+3)} = \frac{1}{x^2-9}$$

№7.1.140

$$y = \ln(e^{2x}+1) - 2\,\mathrm{arctg}\, e^x$$

$$y' = \frac{1}{e^{2x}+1} \cdot (e^{2x}+1)' \cdot (2x)' -$$

$$- 2\,\frac{1}{1+e^{2x}} \cdot (e^x)' =$$

$$= \frac{2e^{2x}}{e^{2x}+1} - \frac{2e^x}{1+e^{2x}} = \frac{2e^x(e^x-1)}{e^{2x}+1}$$

№ 7.1.141

$$y = \frac{x^2}{2\sqrt{1-x^4}}$$

$$y' = \frac{2x\left(2\sqrt{1-x^4}\right) - \cancel{2}x^2 \cdot \frac{1}{\cancel{2}\sqrt{1-x^4}}}{\left(2\sqrt{1-x^4}\right)^2} =$$

$$= \frac{4x\sqrt{1-x^4} - \frac{x^2}{\sqrt{1-x^4}}}{4(1-x^4)} =$$

$$= \frac{4x(1-x^4) - x^2}{4(1-x^4)\sqrt{1-x^4}}$$

№ 7.1.142

$$y = \frac{tg\,3x + \ln\cos^2 3x}{3}$$

$$y' = \frac{3(tg\,3x + \ln\cos^2 3x)' - 0 \cdot (\dots)}{3^2} =$$

$$= \frac{1}{\cancel{3}}\left(\frac{1}{\cos^2 3x} \cdot \cancel{3} + \right.$$

$$\left. + \frac{1}{\cos^2 3x} \cdot 2\cos 3x \cdot (-\sin 3x) \cdot \cancel{3}\right) =$$

$$= \frac{1}{\cos^2 3x} - \frac{2\cos 3x \sin 3x}{\cos^2 3x} =$$

$$= \frac{1 - 2\cos 3x \sin 3x}{\cos^2 3x} =$$

$$= \frac{1 - \sin 6x}{\cos^2 3x}$$

№ 7.1.143

$$y = \ln\sqrt{\frac{1-\cos x}{1+\cos x}}$$

$$y' = \left(\frac{1-\cos x}{1+\cos x}\right)^{-\frac{1}{2}} \cdot \frac{1}{2} \cdot \left(\frac{1-\cos x}{1+\cos x}\right)^{-\frac{1}{2}} \cdot$$

$$\cdot \frac{(1-\cos x)'(1+\cos x) - (1-\cos x)(1+\cos x)'}{(1+\cos x)^2} =$$

$$= \frac{1+\cos x}{2(1-\cos x)} \cdot \frac{\sin x(1+\cos x) + \sin x(1-\cos x)}{(1+\cos x)^2} =$$

$$= \frac{1+\cos x}{2(1-\cos x)} \cdot \frac{\sin x(1+\cos x + 1 - \cos x)}{(1+\cos x)^2} =$$

$$= \frac{1+\cos x}{2(1-\cos x)} \cdot \frac{2\sin x}{(1+\cos x)^2} =$$

$$= \frac{\sin x}{1-\cos^2 x} = \frac{\sin x}{\sin^2 x} = \frac{1}{\sin x}$$

<u>№7.1.144</u>

$$f(x) = \frac{\operatorname{arctg} x}{2} - \frac{x}{2(1+x^2)}$$

$$f'(x) = \frac{1}{2}\left(\operatorname{arctg} x - \frac{x}{1+x^2}\right)' =$$

$$= \frac{1}{2}\left(\frac{1}{1+x^2} - \frac{1+x^2-2x^2}{(1+x^2)^2}\right) =$$

$$= \frac{1}{2}\left(\frac{\cancel{1+x^2} - \cancel{1-x^2} + 2x^2}{(1+x^2)^2}\right) =$$

$$= \frac{2x^2}{2(1+x^2)^2} = \frac{x^2}{(1+x^2)^2} = \left(\frac{x}{1+x^2}\right)^2$$

№7.1.145

$$f(x) = \frac{\sqrt{x^2-1}}{x} + \operatorname{arctg}\frac{1}{x}$$

$$f'(x) = \frac{\sqrt{x^2-1} - x\left(\sqrt{x^2-1}\right)'\cdot\left(x^2-1\right)'}{x^2} +$$

$$+ \frac{1}{1+\frac{1}{x^2}}\cdot\left(\frac{1}{x}\right)' =$$

$$= \frac{\sqrt{x^2-1} - x\cdot\frac{1}{\cancel{2}\sqrt{x^2-1}}\cdot\cancel{2}x}{x^2} + \frac{1}{\left(1+\frac{1}{x^2}\right)\cdot\left(-x^2\right)} =$$

$$= \frac{\sqrt{x^2-1}}{x^2} - \frac{1}{\sqrt{x^2-1}} \cancel{+} - \frac{1}{x^2+1}$$

№7.1.146

$$y = 14\arcsin\frac{x+1}{2} - \frac{(3x-19)\sqrt{3-2x-x^2}}{2}$$

$$y' = \frac{14}{\sqrt{1-\left(\frac{x+1}{2}\right)^2}}\cdot\frac{1}{2} - \frac{1}{2}\cdot\left(3\sqrt{3-2x-x^2} +\right.$$

$$\left.+ (3x-19)\frac{1}{2\sqrt{3-2x-x^2}}\cdot(-2x-2)\right) =$$

$$= \frac{1}{2}\left(\frac{14}{2\sqrt{3-2x-x^2}} - 3\sqrt{3-2x-x^2} - \frac{-6x^2+32x+38}{2\sqrt{3-2x-x^2}}\right. =$$

$$= \frac{1}{2}\left(\frac{14-6(3-2x-x^2)+6x^2-32x-38}{2\sqrt{3-2x-x^2}}\right) =$$

$$= \frac{12x^2-20x-42}{4\sqrt{3-2x-x^2}}$$

<u>№7.1.147</u>

$$y = \ln\left(\frac{x^2+2}{2}\right) + \frac{2-x}{4(x^2+2)} - \frac{1}{4\sqrt{2}} \cdot$$

$$\cdot \operatorname{arctg}\frac{x}{\sqrt{2}}$$

$$y' = \frac{1}{\cancel{2}(x^2+2)} \cdot \cancel{2}x + \frac{1}{4}\left(\frac{x^2+2-2x(2-x)}{(x^2+2)^2}\right) -$$

$$- \frac{1}{4\sqrt{\cancel{2}}} \cdot \frac{1}{1+\left(\frac{x}{\sqrt{2}}\right)^2} \cdot \left(\frac{\sqrt{\cancel{2}}}{2}\right) =$$

$$= \frac{x}{x^2+2} + \frac{1}{4}\left(\frac{3x^2-4x+2}{(x^2+2)^2}\right) - \frac{2}{x^2+2} =$$

$$= \frac{4x(x^2+2)+3x^2-4x+2-8(x^2+2)}{4(x^2+2)^2} =$$

$$= \frac{4x^3-5x^2-4x-14}{4x^4+16x^2+16}$$

№7.1.148

$y = x^{\operatorname{arctg}x}$

$y' = x^{\operatorname{arctg}x} \ln x \cdot \frac{1}{1+x^2} +$

$+ x^{\operatorname{arctg}x - 1} \cdot \operatorname{arctg}x =$

$= \frac{x^{\operatorname{arctg}x} \ln x}{1+x^2} + x^{\operatorname{arctg}x-1} \operatorname{arctg}x.$

№7.1.149

$y = (x^2+1)^{\sqrt{x}}$

$y' = (x^2+1)^{\sqrt{x}} \ln(x^2+1) \cdot \frac{1}{2\sqrt{x}} +$

$+ (x^2+1)^{\sqrt{x}-1} \cdot 2x \cdot \sqrt{x} =$

$= \frac{(x^2+1)^{\sqrt{x}} \ln(x^2+1)}{2\sqrt{x}} +$

$+ 2x\sqrt{x}(x^2+1)^{\sqrt{x}-1}$

№7.1.150

$y = \frac{e^x \cdot (x+4)^4}{\sqrt{5x-1}} = e^x \cdot (x+4)^4 \cdot (5x-1)^{-\frac{1}{2}}$

№ 7.1.150

$$y = \frac{e^x \cdot (x+4)^4}{\sqrt{5x-1}} \quad \Big| \ln$$

$$\ln y = \ln\left(\frac{e^x \cdot (x+4)^4}{\sqrt{5x-1}}\right)$$

$$\ln y = \ln e^x + \ln(x+4)^4 - \ln\sqrt{5x-1}$$

$$\ln y = x + 4\ln(x+4) - \frac{1}{2}\ln(5x-1) \Big| ()'$$

$$(\ln y)' = x' + 4(\ln(x+4))' - \frac{1}{2}(\ln(5x-1))'$$

$$\frac{y'}{y} = 1 + 4 \cdot \frac{1}{x+4} - \frac{1}{2} \cdot \frac{1}{5x-1}$$

$$\frac{y'}{y} = 1 + \frac{4}{x+4} - \frac{1}{10x-2}$$

$$y' = \frac{e^x \cdot (x+4)^4}{\sqrt{5x-1}}\left(1 + \frac{4}{x+4} - \frac{1}{10x-2}\right)$$

№ 7.1.151

$$y = \frac{x^3\sqrt{x-10}}{(x^2+4)^3\sqrt[7]{x-6}} \quad \Big| \ln(\ )$$

$$\ln y = \ln\frac{x^3\sqrt{x-10}}{(x^2+4)^3\sqrt[7]{x-6}}$$

$\ln y = 3\ln x^{3} + \ln\sqrt{x-10} - 3\ln(x^2+4) -$
$- \frac{1}{7}\ln(x-6)$

$(\ln y)' = 3(\ln x)' + \frac{1}{2}(\ln (x-10))' -$
$- 3(\ln(x^2+4))' - \frac{1}{7}(\ln(x-6))'$

$\cancel{(\ln y)'}\ \frac{y'}{y} = \frac{3}{x} + \frac{1}{2x-20} - \frac{3}{x^2+4} -$
$- \frac{1}{7x-42}$

$y' = \frac{x^3\sqrt{x-10}}{(x^2+4)^3\sqrt[7]{x-6}}\left(\frac{3}{x} + \frac{1}{2x-20} - \frac{3}{x^2+4} - \frac{1}{7x-42}\right)$

№7.1-152

$y = 3^x \cdot x^5 \cdot \sqrt{x^4+x} \quad | \ln()$

$\ln y = \ln(3^x \cdot x^5 \cdot \sqrt{x^4+x})$

$\ln y = \ln 3^x + \ln x^5 + \ln(\sqrt{x^4+x})$

$\ln y = x\ln 3 + 5\ln x + \frac{1}{2}\ln(x^4+x)$

$\ln y = x\ln 3 + 5\ln x + \frac{1}{2}\ln x + \frac{1}{2}\ln(x^3+1) \quad |()'$

$(\ln y)' = x'\ln 3 + 5(\ln x)' +$
$+ \frac{1}{2}(\ln x)' + \frac{1}{2}(\ln(x^3+1))'$

$$\frac{y'}{y} = \ln 3 + \frac{5}{x} + \frac{1}{2x} + \frac{1}{2x^3+2}$$

$$y' = 3^x \cdot x^5 \cdot \sqrt{x^4+x}\left(\ln 3 + \frac{5}{x} + \frac{1}{2x} + \frac{1}{2x^3+2}\right)$$

№ 7.1.153

$$f(t) = t^{\frac{1}{\ln t}} \quad \Big| \ln()$$

$$\ln(f(t)) = \frac{1}{\ln t} \cdot \ln t$$

$$\ln(f(t)) = 1 \quad \Big| ()'$$

$$\frac{f'(t)}{f(t)} = 0$$

$$f'(t) = 0$$

$$\cancel{f'(t) = \left(t^{\frac{1}{\ln t}}\right)' = \frac{1}{\ln t}\, t^{\frac{1}{\ln t}-1}}$$

№ 7.1.154

$$\sqrt{x} + \sqrt{y} = \sqrt{5} \quad \Big| ()^2$$

$$\cancel{x + y = 5}$$

$$\cancel{y = 5 - x}$$

$$\cancel{y' = -1}$$

$$\sqrt{y} = \sqrt{5} - \sqrt{x}$$

$$y = (\sqrt{5} - \sqrt{x})^2 \quad \Big| ()'$$

$$y' = -2(\sqrt{5} - \sqrt{x}) \cdot \frac{1}{2\sqrt{x}}$$

$$y' = \frac{-2\sqrt{5} + 2\sqrt{x}}{2\sqrt{x}}$$

№7.1.155

$x^2+3y^2-4xy+10=0 \quad |()'$

$2x+6yy'-4(x'y+xy')=0$

$2x+6yy'-4y+4xy'=0$

$6yy'+4xy'=4y-2x$

$y'(6y+4x)=4y-2x$

$y'=\frac{2(2y-x)}{2(3y+2x)}$

$y'=\frac{2y-x}{3y+2x}$

№7.1.156

$\arcsin\frac{x}{y}=y\ln x \quad |()'$

$\frac{1}{\sqrt{1-\frac{x^2}{y^2}}}\cdot\left(\frac{x}{y}\right)'=y'\ln x+y\cdot\frac{1}{x}$

$\left(y'\ln x+\frac{y}{x}\right)\sqrt{1-\frac{x^2}{y^2}}=\frac{y-xy'}{y^2}$

$y'\ln x\sqrt{1-\frac{x^2}{y^2}}+\frac{y\sqrt{1-\frac{x^2}{y^2}}}{x}=\frac{1}{y}-\frac{xy'}{y^2}$

$$y' \ln x \sqrt{1-\frac{x^2}{y^2}} + \frac{xy'}{y^2} = \frac{1}{y} - \frac{y}{x}\sqrt{1-\frac{x^2}{y^2}}$$

$$y' \left(\ln x \sqrt{1-\frac{x^2}{y^2}} + \frac{x}{y^2}\right) = \frac{1}{y} - \frac{y}{x}\sqrt{1-\frac{x^2}{y^2}}$$

$$y' = \frac{\frac{1}{y} - \frac{y}{x}\sqrt{1-\frac{x^2}{y^2}}}{\ln x \cdot \sqrt{1-\frac{x^2}{y^2}} + \frac{x}{y^2}}$$

$$y' = \frac{x - y^2\sqrt{1-\frac{x^2}{y^2}}}{xy \cdot \ln x \cdot \sqrt{1-\frac{x^2}{y^2}} + \frac{x^2}{y}}$$

№ 7.1.157

$$\operatorname{arctg} y = x^2 y \quad \Big| (\;)'$$

$$\frac{1}{1+y^2} \cdot y' = (x^2)' y + x^2 y'$$

$$\frac{y'}{1+y^2} = 2xy + x^2 y'$$

$$y' = 2xy + 2xy^3 + x^2 y' + x^2 y^2 y'$$

$$y' - x^2 y' - x^2 y^2 y' = 2xy + 2xy^3$$

$$y' (1 - x^2 - x^2 y^2) = 2xy(1+y^2)$$

$$y'\ln x\sqrt{1-\frac{x^2}{y^2}}+\frac{xy'}{y^2}=\frac{1}{y}-\frac{y}{x}\sqrt{1-\frac{x^2}{y^2}}$$

$$y'\left(\ln x\sqrt{1-\frac{x^2}{y^2}}+\frac{x}{y^2}\right)=\frac{1}{y}-\frac{y}{x}\sqrt{1-\frac{x^2}{y^2}}$$

$$y'=\frac{\frac{1}{y}-\frac{y}{x}\sqrt{1-\frac{x^2}{y^2}}}{\ln x\cdot\sqrt{1-\frac{x^2}{y^2}}+\frac{x}{y^2}}$$

$$y'=\frac{x-y^2\sqrt{1-\frac{x^2}{y^2}}}{xy\cdot\ln x\cdot\sqrt{1-\frac{x^2}{y^2}}+\frac{x^2}{y}}$$

№7.1.157

$$\operatorname{arctg} y=x^2y \quad \Big|\,(\;)'$$

$$\frac{1}{1+y^2}\cdot y'=(x^2)'y+x^2y'$$

$$\frac{y'}{1+y^2}=2xy+x^2y'$$

$$y'=2xy+2xy^3+x^2y'+x^2y^2y'$$

$$y'-x^2y'-x^2y^2y'=2xy+2xy^3$$

$$y'(1-x^2-x^2y^2)=2xy(1+y^2)$$

$$y' \ln x \sqrt{1-\frac{x^2}{y^2}} + \frac{xy'}{y^2} = \frac{1}{y} - \frac{y}{x}\sqrt{1-\frac{x^2}{y^2}}$$

$$y'\left(\ln x \sqrt{1-\frac{x^2}{y^2}} + \frac{x}{y^2}\right) = \frac{1}{y} - \frac{y}{x}\sqrt{1-\frac{x^2}{y^2}}$$

$$y' = \frac{\frac{1}{y} - \frac{y}{x}\sqrt{1-\frac{x^2}{y^2}}}{\ln x \cdot \sqrt{1-\frac{x^2}{y^2}} + \frac{x}{y^2}}$$

$$y' = \frac{x - y^2\sqrt{1-\frac{x^2}{y^2}}}{xy \cdot \ln x \cdot \sqrt{1-\frac{x^2}{y^2}} + \frac{x^2}{y}}$$

№7.1.157

$$\operatorname{arctg} y = x^2 y \quad \Big| (\ )'$$

$$\frac{1}{1+y^2} \cdot y' = (x^2)' y + x^2 y'$$

$$\frac{y'}{1+y^2} = 2xy + x^2 y'$$

$$y' = 2xy + 2xy^3 + x^2 y' + x^2 y^2 y'$$

$$y' - x^2 y' - x^2 y^2 y' = 2xy + 2xy^3$$

$$y'(1 - x^2 - x^2 y^2) = 2xy(1+y^2)$$

$$y' = \frac{2xy(1+y^2)}{1-x^2(1-y^2)}$$

$$y' = \frac{2xy(1+y^2)}{1-x^2(1-y^2)}$$

<u>w7.1.158</u>

$x^y \cdot y^x = 1 \quad |()'$

$(x^y)' y^x + x^y (y^x)' = 0$

$y x^{y-1} \cdot y^x + x^y x y^{x-1} \cdot y' = 0$

$x^{y-1} y^{x+1} + x^{y+1} y^{x-1} \cdot y' = 0$

$x^{y+1} y^{x-1} y' = -x^{y-1} y^{x+1}$

$$y' = -\frac{x^{y-1} y^{x+1}}{x^{y+1} y^{x-1}}$$

$$y' = -\frac{y^2}{x^2}$$

<u>w7.1.159</u>

$x^2 + y^2 = 4$

$y^2 = 4 - x^2$

$y = \sqrt{4-x^2} \quad |()'$

$$y' = \frac{1}{2\sqrt{4-x^2}} \cdot (-2x)$$

$$y' = -\frac{x}{\sqrt{4-x^2}}$$

$$y' = -\frac{-\sqrt{2}}{\sqrt{4-(-\sqrt{2})^2}} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

$$y' = 1$$

№7.1.160

$x = t^3 \quad y = 3t$

$$y'(x) = \frac{y'(t)}{x'(t)}$$

$$y'(x) = \frac{3}{3t^2} = \frac{1}{t^2}$$

№7.1.161

$x = \cos^3 t, \quad y = \sin^3 t$

$$y'(x) = \frac{y'(t)}{x'(t)}$$

$y'(t) = 3\sin^2 t \cos t$

$x'(t) = -3\cos^2 t \sin t$

$$y'(x) = \frac{3\sin^2 t\cos t}{-3\cos^2 t\sin t} = -\frac{\sin t}{\cos t} =$$

$$= -\mathrm{tg}\, t$$

w 7.1.162

$$x = \frac{t+1}{t}, \quad y = \frac{t-1}{t}$$

$$x'(t) = \frac{(t+1)'t - (t+1)t'}{t^2} = \frac{t - t - 1}{t^2} = -\frac{1}{t^2}$$

$$y'(t) = \frac{(t-1)'t - (t-1)t'}{t^2} = \frac{t - t + 1}{t^2} = \frac{1}{t^2}$$

$$y'(x) = \frac{\frac{1}{t^2}}{-\frac{1}{t^2}} = -1$$

w7.1.163

$$x = t - \mathrm{arctg}\, t, \quad y = \frac{t^3}{3} + 1$$

$$x'(t) = 1 - \frac{1}{1+t^2} = \frac{1+t^2-1}{1+t^2} = \frac{t^2}{1+t^2}$$

$$y'(t) = \frac{1}{3}\cdot 3t^2 = t^2$$

$$y'(x) = \frac{t^2}{\frac{t^2}{1+t^2}} = 1+t^2$$

w 7.1.171

$y = \ln\cos x$, $y'' = ?$

$y' = \frac{1}{\cos x} \cdot (-\sin x) = -tg x$

$y'' = (-tg x)' = -\frac{1}{\cos^2 x}$

w 7.1.172

$y = \sin^2 x$, $y'' = ?$

$y' = 2\sin x \cos x = \sin 2x$

$y'' = (\sin 2x)' = 2\cos 2x$

w 7.1.173

$y = 5^x$, $y'' = ?$

$y' = 5^x \ln 5$

$y'' = 5^x \ln^2 5$

w 7.1.174

$y = \frac{1}{4x-1}$, $y'' = ?$

$y' = ((4x-1)^{-1})' = -\frac{1}{(4x-1)^2} \cdot 4 =$

$= -\frac{4}{(4x-1)^2}$

$$y'' = \left(-\frac{4}{(4x-1)^2}\right)' = \frac{4}{(4x-1)^4} \cdot 2(4x-1) \cdot$$

$$\cdot 4 = \frac{32\cancel{(4x-1)}}{(4x-1)^{\cancel{4}3}} = \frac{32}{(4x-1)^3}$$

$$y'' = \frac{32}{(4x-1)^3}$$

<u>№7.1.175</u>

$f(x) = xe^x$, $f'''(x) = ?$

$$f'(x) = x'e^x + x(e^x)' = e^x + xe^x$$

$$f''(x) = (e^x)' + (xe^x)' = e^x + e^x + xe^x =$$

$$= 2e^x + xe^x$$

$$f'''(x) = (2e^x)' + (xe^x)' =$$

$$= 3e^x + xe^x$$

<u>№7.1.176</u>

$r(\varphi) = \cos\varphi$, $r^{(IV)}(\varphi) = ?$

$$r'(\varphi) = -\sin\varphi$$

$$r''(\varphi) = -\cos\varphi$$

$$r'''(\varphi) = \sin\varphi$$

$$r^{(IV)}(\varphi) = \cos\varphi$$

№7.1.177

$y = \ln x, \quad y^{(n)} = ?$

$y' = \frac{1}{x} = x^{-1}$

$y'' = -\frac{1}{x^2} = -x^{-2}$

$y''' = -x^{-2} = 2x^{-3}$

$y'''' = -6x^{-4}$

$y^{(n)} = (-1)^{n-1}(n-1)!\,x^{-n}$

№7.1.178

$x = \cos^3 t, \quad y = \sin^3 t, \quad y''_{xx} = ?$

$$y''_{xx} = \frac{y''_t \cdot x'_t - x''_t \cdot y'_t}{(x'_t)^3}$$

$x'_t = 3\cos^2 t \cdot (-\sin t) = -3\cos^2 t \sin t$

$x''_t = -3\left((\cos^2 t)'\sin t + \cos^2 t(\sin t)'\right) =$

$= -3(-2\sin t \cdot \sin t + \cos^2 t \cdot \cos t) =$

$= 6\sin^2 t - 3\cos^3 t$

$y'_t = 3\sin^2 t \cos t$

$y''_t = 3\left((\sin^2 t)'\cos t + \sin^2 t(\cos t)'\right) =$

$$= 6\cos^2 t - 3\sin^3 t$$

$$y''_{xx} = \frac{(6\cos^2 t - 3\sin^3 t)\cdot(-3\cos^2 t\sin t)}{(-3\cos^2 t\sin t)^3} -$$

$$- \frac{(6\sin^2 t - 3\cos^3 t)(3\sin^2 t\cos t)}{(-3\cos^2 t\sin t)^3} =$$

$$= \frac{-18\cos^4 t\sin t + 9\sin^4 t\cos^2 t}{(-3\cos^2 t\sin t)^3} -$$

$$- \frac{18\sin^4 t\cos t - 9\cos^4 t\sin^2 t}{(-3\cos^2 t\sin t)^3} =$$

$$= \frac{-9(2\cos^4 t\sin t - \sin^4 t\cos^2 t + 2\sin^4 t\cos t - \cos^4 t\sin^2 t)}{-27(\cos^2 t\sin t)^3} =$$

$$= \frac{2\cos^4 t\sin t - \sin^4 t\cos^2 t + 2\sin^4 t\cos t - \cos^4 t\sin^2 t}{-3\{\cos^6 t\sin^3 t\}} =$$

$$= \frac{\cos t\sin t(2\cos^3 t - \sin^3 t\cos t + 2\sin^3 t - \cos^3 t\sin t)}{-3\cos^6 t\sin^3 t} =$$

$$= \frac{2\cos^3 t - \sin^3 t\cos t + 2\sin^3 t - \cos^3 t\sin^2 t}{-3\cos^5 t\sin^3 t} =$$

$$= \frac{\cos^3 t(2 - \sin t) - \sin^3 t(\cos t - 2)}{-3\cos^5 t\sin^3 t}$$

№ 7.1.179

$x = e^{3t} \quad y = e^{5t} \quad y''_{xx} = ?$

$$y''_{xx} = \frac{y''_t \cdot x'_t - x''_t \cdot y'_t}{(x'_t)^3}$$

$$x'_t = (e^{3t})' = 3e^{3t}$$

$$x''_t = (3e^{3t})' = 9e^{3t}$$

$$y'_t = (e^{5t})' = 5e^{5t}$$

$$y''_t = (5e^{5t})' = 25e^{5t}$$

$$y''_{xx} = \frac{25e^{5t} \cdot 3e^{3t} - 9e^{3t} \cdot 5e^{5t}}{(3e^{3t})^3} =$$

$$= \frac{75e^{8t} - 45e^{8t}}{27e^{9t}} = \frac{30e^{8t}}{27e^{9t}} =$$

$$= \frac{10}{9e^t} = \frac{10}{9}e^{-t}$$