Интегралы (часть 3)

Домашняя работа

№8.3.19

$$\int \frac{5dx}{x+\sqrt{2}} = 5\ln|x+\sqrt{2}| + C$$

[I тип]

№8.3.20

$$\int \frac{4dx}{(x-\frac{1}{2})^3} = [\text{II тип}] = \frac{4}{1-3} \cdot \frac{1}{(x-\frac{1}{2})^2} + C = -\frac{2}{(x-\frac{1}{2})^2} + C$$

№8.3.21

$$\int \frac{7dx}{(x+3)^6} = [\text{II тип}] = \frac{7}{1-6} \cdot \frac{1}{(x+3)^5} + C = -\frac{7}{5(x+3)^5} + C$$

№8.3.22

$$\int \frac{dx}{(3x+2)^4} = \frac{1}{3}\int \frac{d(3x+2)}{(3x+2)^4} = \frac{1}{3} \cdot \frac{(3x+2)^{-3}}{-3} + C = -\frac{1}{9(3x+2)^3} + C$$

№8.3.23

$$\int \frac{dx}{x^2-4x+8} = [\text{III тип}; A=0, B=1, p=-4, q=8]$$

$= \left[y = x + \frac{-4}{2} = x-2 \Rightarrow dy = dx \right] =$

$= \int \frac{dy}{y^2+a^2} = \left[a = \sqrt{8 - \frac{4^2}{4}} = 2 \right] =$

$= \int \frac{dy}{y^2+2^2} = \frac{1}{2} \operatorname{arctg} \frac{y}{2} + C =$

$= \frac{1}{2} \operatorname{arctg} \frac{x-2}{2} + C$

№8.3.24

$\int \frac{dx}{x^2+x+1} = [\text{III тип}; p=1, q=1] =$

$= \left[y = x + \frac{1}{2} \Rightarrow dy = dx \right] =$

$= \int \frac{dy}{y^2+a^2} = \left[a = \sqrt{1 - \frac{1^2}{4}} = \frac{1}{2} \right] =$

$= \int \frac{dy}{y^2+\left(\frac{1}{2}\right)^2} = 2 \operatorname{arctg} 2y + C =$

$= 2 \operatorname{arctg} 2\left(x+\frac{1}{2}\right) + C =$

$= 2 \operatorname{arctg} (2x+1) + C$

№8.3.25

$\int \frac{6x+1}{x^2-8x+25} dx = [\text{III тип}, A=6,$

$B=1, p=-8, q=25] =$

$$= \left[6x+1 = \frac{6}{2}(2x-8)+1-\frac{6\cdot(-8)}{2} = \right.$$

$$\left. = 3(2x-8)+25\right] =$$

$$= 3\int\frac{(2x-8)\,dx}{x^2-8x+25} + 25\int\frac{dx}{x^2-8x+25} =$$

$$= \left[1)\ t = x^2-8x+25 \Rightarrow dt = (2x-8)dx;\right.$$

$$\left. 2)\ y = x-4,\ a = \sqrt{25-\frac{64}{4}} = 3\right] =$$

$$= 3\int\frac{dt}{t} + 25\int\frac{dy}{y^2+3^2} =$$

$$= 3\ln|t| + 25\cdot\frac{1}{3}\operatorname{arctg}\frac{y}{3} + C =$$

$$= 3\ln|x^2-8x+25| + \frac{25}{3}\operatorname{arctg}\frac{x-4}{3} + C$$

<u>W 8.3.26</u>

$$\int\frac{5x+2}{x^2+2x+10}dx = \left[\text{ш} \ \text{пп};\ A=5, B=2;\right.$$

$$p=2,\ q=10;\quad 5x+2 = \frac{5}{2}(2x+2)+$$

$$\left. +2-\frac{5\cdot 2}{2} = \frac{5}{2}(2x+2)-3\right] =$$

$$= \frac{5}{2}\int\frac{2x-2}{x^2+2x+10}dx - 3\int\frac{dx}{x^2+2x+10} =$$

$$= \left[1)\ t = x^2+2x+10 \Rightarrow dt = (2x-2)dx;\right.$$

$$\left. 2)\ y = x+1,\ a = \sqrt{10-\frac{2^2}{4}} = 3\right] =$$

$$= \frac{5}{2}\int \frac{dt}{t} - 3\int \frac{dy}{y^2+3^2} =$$

$$= \frac{5}{2}\ln|t| - 3\cdot\frac{1}{3}\,\mathrm{arctg}\,\frac{y}{3} + C =$$

$$= \frac{5}{2}\ln|x^2+2x+10| - \mathrm{arctg}\,\frac{x+1}{3} + C$$

№8.3.27

$$\int \frac{x+2}{x^2+3x+5}\,dx = [\ \text{III тип};$$

$$A=1,\ B=2,\ p=3,\ q=5;$$

$$x+2 = \frac{1}{2}(2x+3) + 2 - \frac{3}{2} = \frac{1}{2}(2x+3) + \frac{1}{2}\ ] =$$

$$= \frac{1}{2}\int \frac{(2x+3)\,dx}{x^2+3x+5} + \frac{1}{2}\int \frac{dx}{x^2+3x+5} + C =$$

$$= [\ 1)\ t = x^2+3x+5 \Rightarrow dt = (2x+3)\,dx;$$

$$2)\ y = x+\frac{3}{2};\ a = \sqrt{5-\frac{9}{4}} = \frac{\sqrt{11}}{2}\ ] =$$

$$= \frac{1}{2}\int \frac{dt}{t} + \frac{1}{2}\int \frac{dy}{y^2+\left(\frac{\sqrt{11}}{2}\right)^2} =$$

$$= \frac{1}{2}\ln|x^2+3x+5| + \frac{1}{2}\cdot\frac{2}{\sqrt{11}}\cdot \mathrm{arctg}\,\frac{2y}{\sqrt{11}} + C =$$

$$= \frac{1}{2}\ln|x^2+3x+5| + \frac{1}{\sqrt{11}}\,\mathrm{arctg}\,\frac{2x+3}{\sqrt{11}} + C$$

№8.3.28

$$\int \frac{2x-1}{5x^2+2x+1}\,dx = \cancel{\text{III тип}} \int \frac{2x-1}{5\left(x^2+\frac{2}{5}x+\frac{1}{5}\right)}\,dx$$

$$= \frac{1}{5}\int \frac{2x-1}{x^2+\frac{2}{5}x+\frac{1}{5}}\,dx = [\ \text{III тип};$$

$$A=2,\ B=-1,\ p=\frac{2}{5},\ q=\frac{1}{5};$$

$$2x-1 = \frac{2}{2}\left(2x+\frac{2}{5}\right) - 1 - \frac{2\cdot 2}{5\cdot 2} =$$

$$= \left(2x+\frac{2}{5}\right) - \frac{7}{5}\ ] = \frac{1}{5}\int \frac{2x+\frac{2}{5}}{x^2+\frac{2}{5}x+\frac{1}{5}}\,dx -$$

$$- \frac{7}{5}\cdot\frac{1}{5}\int \frac{dx}{x^2+\frac{2}{5}x+\frac{1}{5}} = [\ 1)\ t = x^2+\frac{2}{5}x+\frac{1}{5}$$

$$\Rightarrow dt = \left(2x+\frac{2}{5}\right)dx;$$

$$2)\ y = x+\frac{2}{5\cdot 2} = x+\frac{1}{5};\ a=\sqrt{\frac{1}{5}-\frac{2^2}{5^2\cdot 4}} = \frac{1}{5}\ ] =$$

$$= \frac{1}{5}\int\frac{dt}{t} - \frac{7}{25}\int\frac{dy}{y^2+\left(\frac{1}{5}\right)^2} =$$

$$= \frac{1}{5}\ln|t| - \frac{7}{25}\cdot 5\,\mathrm{arctg}\,5y + C =$$

$$= \frac{1}{5}\ln\left|x^2+\frac{2}{5}x+\frac{1}{5}\right| - \frac{7}{5}\,\mathrm{arctg}(5x+1)+C$$

№ 8.3.29

$$\int\frac{x-1}{(x^2+2x+3)^2}\,dx = [\ \text{IV тип};\ A=1,\ B=-1;$$

$$p=2,\ q=3,\ x-1 = \frac{1}{2}(2x+2) - 1 - \frac{2}{2} =$$

$$= \frac{1}{2}(2x+2) - 2\ ] = \frac{1}{2}\int\frac{2x+2}{(x^2+2x+3)^2}\,dx -$$

$$-2\int\frac{dx}{(x^2+2x+3)^2} = [\ 1)\ t = x^2+2x+3,\ dt=(2x+2)\,dx;$$

$2)\ y = x+1;\ a = \sqrt{3-\frac{2^2}{4}} = \sqrt{2}\ ] =$

$= \frac{1}{2}\int\frac{dt}{t^2} - 2\int\frac{dy}{(y^2+2)^2} =$

$= [\int\frac{dy}{(y^2+2)^2} = \frac{1}{2(2-1)\cdot 2}\cdot\frac{y}{(y^2+2)} +$

$+ \frac{1}{2}\cdot\frac{2\cdot 2-3}{2\cdot 2-2}\cdot\int\frac{dy}{y^2+2} =$

$= \frac{y}{4y^2+8} + \frac{1}{4}\cdot\frac{1}{2}\operatorname{arctg}\frac{x+1}{2} + C =$

$= \frac{1}{4}\left(\frac{y}{y^2+2} + \frac{1}{2}\operatorname{arctg}\frac{x+1}{2}\right) + C\ ] =$

$= -\frac{1}{2}\cdot\frac{1}{x^2+2x+3} - 2\cdot\frac{1}{4}\left(\frac{y}{y^2+2} + \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x+1}{\sqrt{2}}\right) +$

$+ C = -\frac{1}{2}\left(\frac{1}{x^2+2x+3} + \frac{y}{y^2+2} + \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x+1}{\sqrt{2}}\right) + C =$

$= -\frac{1}{2}\left(\frac{1}{x^2+2x+3} + \frac{x+1}{x^2+2x+3} + \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x+1}{\sqrt{2}}\right) + C =$

$= -\frac{1}{2}\left(\frac{x+2}{x^2+2x+3} + \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x+1}{\sqrt{2}}\right) + C$

<u>№ 8.3.30</u>

$\int\frac{2x+1}{(x^2+2x+5)^2}dx = [$ IV тип; $A=2,\ B=1,$

$p=2,\ q=5;\ 2x+1 = \frac{2}{2}(2x+2)+1-$

$$- \frac{2 \cdot 2}{2} = (2x+2) - 1] =$$

$$= \int \frac{2x+2}{(x^2+2x+5)^2} dx - \int \frac{dx}{(x^2+2x+5)^2} =$$

$$= [1) \ t = x^2+2x+5 \Rightarrow dt = (2x+2)\,dx;$$

$$2) \ y = x+1, \ a = \sqrt{5 - \frac{2^2}{4}} = 2] =$$

$$= \int \frac{dt}{t^2} - \int \frac{dy}{(y^2+2^2)^2} =$$

$$= \cancel{\int \frac{dt}{t^2}} \left[\int \frac{dy}{(y^2+2^2)^2} = \frac{1}{2(2-1)\cdot 4} \cdot \frac{y}{y^2+4} + \right.$$

$$+ \frac{1}{4} \cdot \frac{2\cdot 2 - 3}{2 \cdot 2 - 2} \cdot \int \frac{dy}{y^2+2^2} = \frac{1}{8} \cdot \frac{y}{y^2+4} +$$

$$\left. + \frac{1}{8} \int \frac{dy}{y^2+2^2} = \frac{1}{8} \cdot \frac{y}{y^2+4} + \frac{1}{8} \cdot \frac{1}{2} \operatorname{arctg} \frac{y}{2} \right] =$$

$$= \cancel{\int \frac{dt}{t^2}} - \frac{1}{t} - \frac{1}{8} \left(\frac{y}{y^2+4} + \frac{1}{2} \operatorname{arctg} \frac{y}{2} \right) + C =$$

$$= - \frac{1}{x^2+2x+5} - \frac{1}{8} \left(\frac{x+1}{x^2+2x+5} + \frac{1}{2} \operatorname{arctg} \frac{x+1}{2} \right) + C =$$

$$= - \frac{8+x+1}{8(x^2+2x+5)} - \frac{1}{16} \operatorname{arctg} \frac{x+1}{2} + C =$$

$$= - \frac{x+9}{8(x^2+2x+5)} - \frac{1}{16} \operatorname{arctg} \frac{x+1}{2} + C$$

№8.3.31.

$$\int \frac{dx}{(x^2+1)^4} = \Big[x^2+1=0 \text{ — корней нет} \Rightarrow \text{IV тип; } p=0,\ q=1 \Big] =$$

$$= \frac{1}{2(4-1)\cdot 1}\cdot\frac{x}{(x^2+1)^3}+\frac{2\cdot 4-3}{2\cdot 4-2}\cdot$$

$$\cdot\int\frac{dx}{(x^2+1)^3} = \frac{1}{6}\cdot\frac{x}{(x^2+1)^3}+\frac{5}{6}\int\frac{dx}{(x^2+1)^3} =$$

$$= \frac{x}{6(x^2+1)^3}+\frac{5}{6}\left(\frac{1}{2\cdot(3-1)}\cdot\frac{x}{(x^2+1)^2}+\frac{2\cdot 3-3}{2\cdot 3-2}\cdot\right.$$

$$\left.\cdot\int\frac{dx}{(x^2+1)^2}\right) = \frac{x}{6(x^2+1)^3}+\frac{5}{6}\left(\frac{1}{4}\cdot\frac{x}{(x^2+1)^2}+\right.$$

$$\left.+\frac{3}{4}\int\frac{dx}{(x^2+1)^2}\right) = \frac{x}{6(x^2+1)^3}+\frac{5x}{24(x^2+1)^2}+$$

$$+\frac{15}{24}\int\frac{dx}{(x^2+1)^2} = \Big[\int\frac{dx}{(x^2+1)^2} =$$

$$= \frac{1}{2(2-1)}\cdot\frac{x}{x^2+1}+\frac{2\cdot 2-3}{2\cdot 2-2}\cdot\int\frac{dx}{x^2+1} =$$

$$= \frac{x}{2x^2+2}+\frac{1}{2}\int\frac{dx}{x^2+1} = \frac{x}{2x^2+2}+\frac{1}{2}\operatorname{arctg}x\Big] =$$

$$= \frac{x}{6(x^2+1)^3}+\frac{5x}{24(x^2+1)^2}+\frac{15}{24}\cdot\frac{x}{2x^2+2}+$$

$$+\frac{1}{2}\operatorname{arctg}x + C$$

W8.3.32

$$\int \frac{3x+2}{(x^2-3x+3)^2}\,dx = \Big[\ \text{IV тип};$$

$$A=3,\ B=2,\ p=-3,\ q=3;$$

$$3x+2 = \frac{3}{2}(2x-3)+3-\frac{3\cdot(-3)}{2} =$$

$$= \frac{3}{2}(2x-3)+\frac{15}{2}\ \Big] =$$

$$= \frac{3}{2}\int\frac{(2x-3)\,dx}{(x^2-3x+3)^2} + \frac{15}{2}\int\frac{dx}{(x^2-3x+3)^2} =$$

$$= \Big[\ 1)\ t = x^2-3x+3 \Rightarrow dt = (2x-3)\,dx;$$

$$2)\ y = x-\frac{3}{2};\ a=\sqrt{3-\frac{9}{4}} = \frac{\sqrt{3}}{2}\ \Big] =$$

$$= \frac{3}{2}\int\frac{dt}{t^2} + \frac{15}{2}\int\frac{dy}{\left(y^2+\left(\frac{\sqrt{3}}{2}\right)^2\right)^2} =$$

$$= \Big[\int\frac{dy}{\left(y^2+\left(\frac{\sqrt{3}}{2}\right)^2\right)^2} = \frac{1}{2\cdot(2-1)\cdot\frac{3}{4}}\cdot\frac{y}{y^2+\frac{3}{4}} +$$

$$+ \frac{4}{3}\cdot\frac{2\cdot 2-3}{2\cdot 2-2}\int\frac{dy}{y^2+\frac{3}{4}} =$$

$$= \frac{2}{3}\cdot\frac{4y}{4y^2+3} + \frac{2}{3}\int\frac{dy}{y^2+\frac{3}{4}} =$$

$$= \frac{2}{3}\left(\frac{4y}{4y^2+3} + \frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2y}{\sqrt{3}}\right)\Big] =$$

$$= \frac{3}{2} \cdot \left(-\frac{1}{t}\right) + \frac{15}{2} \cdot \frac{2}{3} \cdot$$

$$\cdot \left(\frac{4y}{4y^2+3} + \frac{2}{\sqrt{3}} \operatorname{arctg} \frac{2y}{\sqrt{3}}\right) + C =$$

$$= -\frac{3}{2(x^2-3x+3)} + 5\left(\frac{4\left(x-\frac{3}{2}\right)}{4\left(x-\frac{3}{2}\right)^2+3} + \frac{2}{\sqrt{3}} \cancel{\operatorname{arctg}} \cdot\right.$$

$$\left.\cdot \operatorname{arctg} \frac{2x-3}{\sqrt{3}}\right) + C =$$

$$= -\frac{3}{2(x^2-3x+3)} + 5\left(\frac{6}{4x^2-12x+12} + \frac{2}{\sqrt{3}} \cdot\right.$$

$$\left.\cdot \operatorname{arctg} \frac{2x-3}{\sqrt{3}}\right) + C =$$

$$= \frac{-4+30}{4(x^2-3x+3)} + \frac{10}{\sqrt{3}} \operatorname{arctg} \frac{2x-3}{\sqrt{3}} + C =$$

$$= \frac{26}{4(x^2-3x+3)} + \frac{10}{\sqrt{3}} \operatorname{arctg} \frac{2x-3}{\sqrt{3}} + C$$

<u>W8.3.33</u>

$$\int \frac{2x-3}{(x-1)(x+2)} dx = \left[\frac{2x-3}{(x-1)(x+2)} = \right.$$

$$= \frac{A}{x-1} + \frac{B}{x+2} \Rightarrow 2x-3 = A(x+2) + B(x-1) =$$

$$= Ax + 2A + Bx - B = x(A+B) + (2A-B) \Rightarrow$$

$$\Rightarrow \begin{cases} A+B=2 \\ 2A-B=-3 \end{cases} \quad 3A = -1 \Rightarrow$$

$\Rightarrow A = -\frac{1}{3} \Rightarrow -\frac{1}{3} + B = 2;$

$B = 2 + \frac{1}{3} = \frac{7}{3} \Big] =$

$= \int \left(-\frac{1}{3(x-1)} + \frac{7}{3(x+2)} \right) dx =$

$= -\frac{1}{3} \int \frac{dx}{x-1} + \frac{7}{3} \int \frac{dx}{x+2} =$

$= \frac{7}{3} \ln|x+2| - \frac{1}{3} \ln|x-1| + C$

№ 8.3.34

$\int \frac{x-4}{(x-2)(x-3)} dx = \Big[\frac{x-4}{(x-2)(x-3)} = \frac{A}{x-2} +$

$+ \frac{B}{x-3} \Rightarrow x-4 = A(x-3) + B(x-2) =$

$= Ax - 3A + Bx - 2B = x(A+B) - 3A - 2B \Rightarrow$

$\Rightarrow \begin{cases} A + B = 1 \quad |\cdot 2 \\ -3A - 2B = -4 \end{cases} \Rightarrow + \begin{cases} 2A + 2B = 2 \\ -3A - 2B = -4 \end{cases} \Rightarrow$

$\Rightarrow A = -2 \Rightarrow -2 + B = 1;\ B = 3 \Big] =$

$= \int \left(-\frac{2}{x-2} + \frac{3}{x-3} \right) dx =$

$= -2 \int \frac{dx}{x-2} + 3 \int \frac{dx}{x-3} =$

$= 3 \ln|x-3| - 2 \ln|x-2| + C$

w8.3.35

$$\int \frac{x\,dx}{x^2-4x-5} = [x^2-4x-5=0 \Rightarrow$$

$$\Rightarrow D = 16-4\cdot(-5) = 36 \Rightarrow$$

$$\Rightarrow x_1 = \frac{4+6}{2} = 5,\ x_2 = \frac{4-6}{2} = -1] =$$

$$= \int \frac{x\,dx}{(x-5)(x+1)} = \left[\frac{x}{(x-5)(x+1)} = \frac{A}{x-5} + \frac{B}{x+1} \Rightarrow\right.$$

$$\Rightarrow x = A(x+1) + B(x-5) =$$

$$= Ax + A + Bx - 5B = x(A+B) + A - 5B \Rightarrow$$

$$\Rightarrow \begin{cases} A+B=1 \\ A-5B=0 \end{cases} \Rightarrow 6B = 1 \Rightarrow B = \frac{1}{6} \Rightarrow$$

$$\Rightarrow A = 1-\frac{1}{6} = \frac{5}{6}] =$$

$$= \int \left(\frac{5}{6(x-5)} + \frac{1}{6(x+1)}\right) dx =$$

$$= \frac{5}{6}\int \frac{dx}{x-5} + \frac{1}{6}\int \frac{dx}{x+1} =$$

$$= \frac{5}{6}\ln|x-5| + \frac{1}{6}\ln|x+1| + C$$

w8.3.36

$$\int \frac{2x^2-11}{x^2+x-6}dx = \int \frac{2(x^2+x-6)-2x+1}{x^2+x-6}dx =$$

$$= 2\int dx - \int \frac{2x-1}{x^2+x-6}dx =$$

$$= \left[x^2 + x - 6 = 0 \Rightarrow D = 1 - 4 \cdot (-6) = 25 \Rightarrow \right.$$

$$\left. \Rightarrow x_1 = \frac{-1+5}{2} = 2,\ x_2 = \frac{-1-5}{2} = -3 \right] =$$

$$= 2\int dx - \int \frac{2x-1}{(x-2)(x+3)}\,dx =$$

$$= \left[\frac{2x-1}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3} \Rightarrow \right.$$

$$\Rightarrow 2x - 1 = A(x+3) + B(x-2) =$$

$$= Ax + 3A + Bx - 2B =$$

$$= x(A+B) + 3A - 2B \Rightarrow$$

$$\Rightarrow \begin{cases} A + B = 2 & \big| \cdot 2 \\ 3A - 2B = -1 \end{cases} \Rightarrow + \begin{cases} 2A + 2B = 4 \\ 3A - 2B = -1 \end{cases} \Rightarrow$$

$$\left. \Rightarrow 5A = 3 \Rightarrow A = \frac{3}{5} \Rightarrow B = \frac{7}{5} \right] =$$

$$= 2\int dx - \int \left(\frac{3}{5(x-2)} + \frac{7}{5(x+3)} \right) dx =$$

$$= 2\int dx - \frac{3}{5}\int \frac{dx}{x-2} + \frac{7}{5}\int \frac{dx}{x+3} =$$

$$= 2x - \frac{3}{5}\ln|x-2| + \frac{7}{5}\ln|x+3| + C$$

<u>W8.3.37</u>

$$\int \frac{-3x^2 + x + 19}{(x-4)(x-2)(x+1)}\,dx =$$

$= \Big[\frac{-3x^2+x+19}{(x-4)(x-2)(x+1)} = \frac{A}{x-4} + \frac{B}{x-2} + \frac{C}{x+1} \Rightarrow$

$\Rightarrow -3x^2+x+19 = A(x-2)(x+1) +$

$+ B(x-4)(x+1) + C(x-4)(x-2) =$

$= Ax^2 - 2Ax + Ax - 2A + Bx^2 - 4Bx + Bx -$

$-4B + Cx^2 - 4Cx - 2Cx + 8C =$

$= x^2(A+B+C) + x(-A-3B-6C) +$

$+ (-2A-4B+8C) \Rightarrow$

Простите, посчитала A, B, C в Maxima

$\Rightarrow \begin{cases} A+B+C = -3 \\ -A-3B-6C = 1 \\ -2A-4B+8C = 19 \end{cases} \Rightarrow \begin{matrix} A = -\frac{5}{2} \\ B = -\frac{3}{2} \\ C = 1 \end{matrix} \Big] =$

$= \int \left(-\frac{5}{2(x-4)} - \frac{3}{2(x-2)} + \frac{1}{x+1} \right) dx =$

$= -\frac{5}{2} \int \frac{dx}{x-4} - \frac{3}{2} \int \frac{dx}{x-2} + \int \frac{dx}{x+1} =$

$= -\frac{5}{2} \ln|x-4| - \frac{3}{2} \ln|x-2| + \ln|x+1| + C$

№ 8.3.38

$\int \frac{x-1}{(x+1)(x^2-4)} dx = \int \frac{x-1}{(x+1)(x-2)(x+2)} dx =$

$= \Big[\frac{x-1}{(x+1)(x-2)(x+2)} = \frac{A}{x+1} + \frac{B}{x-2} + \frac{C}{x+2} \Rightarrow$

$$\Rightarrow x-1 = A(x^2-4) + B(x+1)(x+2) +$$
$$+ C(x+1)(x-2) = Ax^2 - 4A +$$
$$+ Bx^2 + Bx + 2Bx + 2B + Cx^2 + Cx - 2Cx - 2C =$$
$$= x^2(A+B+C) + x(3B-C) - 4A + 2B - 2C \Rightarrow$$

$$\Rightarrow \begin{cases} A+B+C=0 \\ 3B-C=1 \\ -4A+2B-2C=-1 \end{cases} \qquad \Rightarrow C = 3B-1 \rightarrow$$

$$\Rightarrow \begin{cases} A+B+3B=1 \\ -4A+2B-2(3B-1)=-1 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} A = 1-4B \\ -4(1-4B)+2B-2(3B-1)=-1 \end{cases} \Rightarrow$$

$$-4+16B+2B-6B+2=-1$$
$$12B=1 \Rightarrow B=\frac{1}{12};$$
$$A = 1-4\cdot\frac{1}{12}=\frac{2}{3}; \quad C = 3\cdot\frac{1}{12}-1=-\frac{3}{4} \Big] =$$

$$= \int\left(\frac{2}{3(x+1)} + \frac{1}{12(x-2)} - \frac{3}{4(x+2)}\right)dx =$$

$$= \frac{2}{3}\ln|x+1| + \frac{1}{12}\ln|x-2| - \frac{3}{4}\ln|x+2| + C$$

w8.3.39

$$\int \frac{x^2+2}{(x^2-1)(x+1)^2}dx = \left[\frac{x^2+2}{(x^2-1)(x+1)^2} =\right.$$

$$= \frac{x^2+2}{(x-1)(x+1)^3} = \frac{A}{x-1} + \frac{B}{(x+1)^3} +$$

$$+ \frac{C}{(x+1)^2} + \frac{D}{x+1} \Rightarrow$$

$$\Rightarrow x^2+2 = A(x+1)^3 + B(x-1) +$$

$$+ C(x^2-1) + D(x-1)(x+1)^2 =$$

$$= Ax^3 + 3Ax^2 + 3Ax + A + Bx - B +$$

$$+ Cx^2 - C + Dx^3 + Dx^2 - Dx - D =$$

$$= x^3(A+D) + x^2(3A+C+D) +$$

$$+ x(3A+B-D) + A - B - C - D \Rightarrow$$

$$\Rightarrow \begin{cases} A+D=0 \\ 3A+C+D=1 \\ 3A+B-D=0 \\ A-B-C-D=2 \end{cases} \Rightarrow \begin{cases} \cancel{A=} D=-A \\ 3A+C-A=1 \\ 3A+B+A=0 \\ A-B-C+A=2 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} C=1-2A \\ 4A+B=0 \\ A-B-1+2A+A=2 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} B = -4A \\ A + 4A - 1 + 2A + A = 2 \end{cases} \Rightarrow$$

$$\Rightarrow 8A = 3 \Rightarrow A = \frac{3}{8};$$

$$B = -4 \cdot \frac{3}{8} = -\frac{3}{2};$$

$$C = 1 - 2 \cdot \frac{3}{8} = \frac{1}{4};$$

$$D = -\frac{3}{8} \Big] =$$

$$= \int \left(\frac{3}{8(x-1)} - \frac{3}{2(x+1)^3} + \frac{1}{4(x+1)^2} - \frac{3}{8(x+1)} \right) dx =$$

$$= \frac{3}{8}\int \frac{dx}{x-1} - \frac{3}{2}\int \frac{dx}{(x+1)^3} + \frac{1}{4}\int \frac{dx}{(x+1)^2} - \frac{3}{8}\int \frac{dx}{x+1} =$$

$$= \frac{3}{8}\ln|x-1| + \frac{3}{4} \cdot \frac{1}{(x+1)^2} - \frac{1}{4} \cdot \frac{1}{x+1} -$$

$$- \frac{3}{8}\ln|x+1| + C$$

№ 8.3.40

$$\int \frac{2x+3}{(x-2)^3} dx = \Big[\frac{2x+3}{(x-2)^3} =$$

$$= \frac{A}{(x-2)^3} + \frac{B}{(x-2)^2} + \frac{C}{(x-2)} = ?$$

$$\Rightarrow 2x + 3 = A + B(x-2) + C(x-2)^2 =$$

$$= A + Bx - 2B + Cx^2 - 4Cx + 4C =$$

$$= x^2(C) + x(B - 4C) + A - 2B + 4C \Rightarrow$$

$$\Rightarrow \begin{cases} C = 0 \\ B - 4C = B - 4 \cdot 0 = 2 \\ A - 2 \cdot 2 + 4 \cdot 0 = 3 \end{cases} \Rightarrow \begin{cases} C = 0 \\ B = 2 \\ A = 7 \end{cases}$$

$$= \int \left(\frac{7}{(x-2)^3} + \frac{2}{(x-2)^2} + \frac{0}{(x-2)} \right) dx =$$

$$= 7 \int \frac{dx}{(x-2)^3} + 2 \int \frac{dx}{(x-2)^2} =$$

$$= 7 \cdot \left(-\frac{1}{2(x-2)^2} \right) + 2 \cdot \left(-\frac{1}{x-2} \right) + C =$$

$$= -\frac{7}{2} \cdot \frac{1}{(x-2)^2} - 2 \cdot \frac{1}{x-2} + C =$$

$$= \frac{-7 - 4(x-2)}{2(x-2)^2} \cancel{+} C = -\frac{7 + 4x - 8}{2(x \cancel{4} - 2)^2} + C =$$

$$= -\frac{4x - 1}{2(x-2)^2} + C$$

№ 8.3.41

$$\int \frac{x^4\,dx}{(x^2-1)(x+2)} = \int \frac{(x-2)(x^2-1)(x+2) + 5x^2 + 4}{(x^2-1)(x+2)}\,dx =$$

$$= \int \frac{x-2}{(x^2-1)(x+2)}\,dx + \int \frac{5x^2+4}{(x^2-1)(x+2)}\,dx =$$

$= [$ 1) $\frac{x-2}{(x^2-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2} \Rightarrow$

$\Rightarrow x-2 = A(x+1)(x+2) + B(x-1)(x+2) +$
$+ C(x^2-1) = Ax^2 + 3Ax + 2A +$
$+ Bx^2 + Bx - 2B + Cx^2 - C =$
$= x^2(A+B+C) + x(3A+B) + 2A - 2B - C \Rightarrow$

$\Rightarrow \begin{cases} A+B+C=0 \\ 3A+B=1 \\ 2A-2B-C=-2 \end{cases} \Rightarrow \begin{cases} B=1-3A \\ C=-A-1+3A \Rightarrow \\ 2A-2(1-3A)-2A+1=-2 \end{cases}$

$\Rightarrow 2A - 2 + 6A - 2A + 1 = -2$;
$6A = -1 \Rightarrow A = -\frac{1}{6} \Rightarrow$
$\Rightarrow B = 1 + 3 \cdot \frac{1}{6} = \frac{3}{2}$; $C = 2 \cdot \left(-\frac{1}{6}\right) - 1 = -\frac{4}{3}$

2) $\frac{5x^2+4}{(x^2-1)(x+2)} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{x+2} \Rightarrow$

$\Rightarrow 5x^2+4 = A(x-1)(x+2) + B(x+1)(x+2) +$
$+ C(x^2-1) = Ax^2 + Ax - 2A + Bx^2 + 3Bx + 2B +$
$+ Cx^2 - C = x^2(A+B+C) + x(A+3B) +$
$+ (-2A+2B-C) \Rightarrow$

$$\Rightarrow \begin{cases} A+B+C=5 \\ A+3B=0 \\ -2A+2B-C=4 \end{cases} \Rightarrow \begin{aligned} &A=-3B; \\ &C=5+3B-B= \\ &=5+2B; \end{aligned}$$

$$-2(-3B)+2B-5-2B=4;$$

$$6B+2B-5-2B=4$$

$$6B=9 \Rightarrow B=\frac{3}{2} \Rightarrow$$

$$A=-3\cdot\frac{3}{2}=-\frac{9}{2};$$

$$C=5+2\cdot\frac{3}{2}=8 \ \Big] =$$

$$=\int\left(-\frac{1}{6}\cdot\frac{1}{x-1}+\frac{3}{2}\cdot\frac{1}{x+1}-\frac{4}{3}\cdot\frac{1}{x+2}\right)dx+$$

$$+\int\left(-\frac{9}{2}\cdot\frac{1}{x+1}+\frac{3}{2}\cdot\frac{1}{x-1}+8\cdot\frac{1}{x+2}\right)dx=$$

$$=\frac{4}{3}\int\frac{dx}{x-1}-3\int\frac{dx}{x+1}+\frac{20}{3}\int\frac{dx}{x+2}=$$

$$=\frac{4}{3}\ln|x-1|-3\ln|x+1|+\frac{20}{3}\ln|x+2|+C$$

№ 8.3.42

$$\int\frac{dx}{x^3-1}=\int\frac{dx}{(x-1)(x^2+x+1)}=\left[\frac{1}{(x-1)(x^2+x+1)}=\right.$$

$$=\frac{A}{x-1}+\frac{Bx+C}{x^2+x+1}\Rightarrow$$

$$\Rightarrow 0x^2+0x+1 = A(x^2+x+1)+(Bx+C)(x-1) =$$

$$= Ax^2+Ax+A+Bx^2+Cx-Bx-C =$$

$$= x^2(A+B)+x(A+C-B)+A-C \Rightarrow$$

$$\Rightarrow \begin{cases} A+B=0 \\ A-B+C=0 \\ A-C=1 \end{cases} \rightarrow \begin{cases} A=C+1 \\ B=-C-1 \\ C+1+C+1+C=0 \end{cases} \rightarrow$$

$$\Rightarrow C=-\frac{2}{3},\ A=\frac{1}{3},\ B=-\frac{1}{3} \Big] =$$

$$= \int\left(\frac{1}{3(x-1)}+\frac{-\frac{1}{3}x\ \cancel{4}-\frac{2}{3}}{x^2+x+1}\right)dx =$$

$$= \frac{1}{3}\int\frac{dx}{x-1}-\frac{1}{3}\int\frac{x+2}{x^2+x+1}dx =$$

$= \Big[$ 2) $\overline{\text{III}}$ тип, $A=1,\ B=\cancel{\bullet}2,\ p=1,\ q=1;$

$$x+2 = \frac{1}{2}(2x+1)\cancel{4}+2-\frac{1}{2} = \frac{1}{2}(2x+1)+\frac{3}{2};$$

$$\int\frac{x+2}{x^2+x+1}dx = \frac{1}{2}\int\frac{2x+1}{x^2+x+1}dx\ \cancel{4}+\frac{3}{2}\int\frac{dx}{x^2+x+1} =$$

$$= \Big[1)\ t=x^2+x+1 \Rightarrow dt=(2x+1)dx;$$

$$2)\ y=x+\frac{1}{2},\ a=\sqrt{1-\frac{1}{4}}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}\Big] =$$

$$= \frac{1}{2}\int\frac{dt}{t}+\frac{3}{2}\int\frac{dy}{y^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{2}\ln|x^2+x+1|+$$

$$+\frac{3}{2}\cdot\frac{2}{\sqrt{3}}\operatorname{arctg}\frac{2x+1}{\sqrt{3}}+C\Big] =$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|x^2+x+1| +$$

$$+ \frac{3}{3\sqrt{3}}\,\mathrm{arctg}\,\frac{2x+1}{\sqrt{3}} + C =$$

$$= \frac{1}{3}\ln|x-1| - \frac{1}{6}\ln|x^2+x+1| +$$

$$+ \frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{2x+1}{\sqrt{3}} + C$$

№8.3.43

$$\int \frac{x\,dx}{(x^2-1)(x^2+1)} = \left[\frac{x}{(x^2-1)(x^2+1)} =\right.$$

$$= \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+1} \Rightarrow$$

$$\Rightarrow 0x^3 + 0x^2 + 1x + 0 =$$

$$= A(x+1)(x^2+1) + B(x-1)(x^2+1) +$$

$$+ (Cx+D)(x^2-1) = Ax^3 + Ax^2 +$$

$$+ Ax + A + Bx^3 - Bx^2 + Bx - B +$$

$$+ Cx^3 + Dx^2 - Cx - D =$$

$$= x^3(A+B+C) + x^2(A-B+D) +$$

$$+ x(A+B-C) + A - B - D \Rightarrow$$

$$\Rightarrow \begin{cases} A+B+C=0 \\ A-B+D=0 \end{cases} +$$

$$\begin{cases} A+B-C=1 \\ A-B-D=0 \end{cases} + \longrightarrow$$

$$\Rightarrow - \begin{cases} 2A+C+D=0 \\ 2A-C-D=1 \end{cases} \Rightarrow$$

$$\Rightarrow 2C+2D=-1 \Rightarrow C=-\frac{1}{2}-D;$$

$$D = [\text{из } 2\text{ур.}] = B-A \Rightarrow$$

$$A+B \cancel{=} \frac{1}{2} - B + A = 0$$

$$\cancel{A}\quad 2A = +\frac{1}{2}$$

$$A = +\frac{1}{4};$$

$$A-B-D=0$$

$$+\frac{1}{4} - B - B + \left(+\frac{1}{4}\right) = 0$$

$$-2B = \cancel{2}-\frac{1}{2}$$

$$B = +\frac{1}{4};$$

$$D = +\frac{1}{4} - \left(+\frac{1}{4}\right) = 0;$$

$$C = -\frac{1}{2} \Big] =$$

$$= \int \left(\frac{1}{4(x-1)} + \frac{1}{4(x+1)} - \frac{x}{2(x^2+1)} \right) dx =$$

$$= \frac{1}{4}\int \frac{dx}{x-1} + \frac{1}{4}\int \frac{dx}{x+1} - \frac{1}{2}\int \frac{x\,dx}{x^2+1} =$$

$$= \frac{1}{4}\int \frac{d(x-1)}{x-1} + \frac{1}{4}\int \frac{d(x+1)}{x+1} - \frac{1}{4}\int \frac{d(x^2+1)}{x^2+1} =$$

$$= \frac{1}{4}\left(\ln|x-1| + \ln|x+1| - \ln|x^2+1|\right) + C =$$

$$= \frac{1}{4}\ln\left|\frac{x^2-1}{x^2+1}\right| + C$$

<u>№ 8.3.44</u>

$$\int \frac{dx}{(x^2+1)(x^2+4)} = ;$$

$$\frac{1}{(x^2+1)(x^2+4)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+4};$$

$$(Ax+B)(x^2+4) + (Cx+D)(x^2+1) = 1$$

$$Ax^3 + Bx^2 + 4Ax + 4B + Cx^3 + Dx^2 + Cx + D = 1$$

$$x^3(A+C) + x^2(B+D) + x(4A+C) + 4B+D = 1.$$

$$\begin{cases} A+C=0 \\ B+D=0 \\ 4A+C=0 \\ 4B+D=1 \end{cases} \Rightarrow \begin{array}{l} -\begin{cases} A+C=0 \\ 4A+C=0 \end{cases} \\ -\begin{cases} B+D=0 \\ 4B+D=1 \end{cases} \end{array} \Rightarrow$$

$$\Rightarrow -3A=0 \Rightarrow A=0 \Rightarrow C=0$$

$$-3B=-1 \Rightarrow B=\frac{1}{3} \Rightarrow D=-\frac{1}{3}$$

$$\int \frac{dx}{(x^2+1)(x^2+4)} = \int \left(\frac{1}{3(x^2+1)} - \frac{1}{3(x^2+4)} \right) dx =$$

$$= \frac{1}{3} \int \frac{dx}{x^2+1} - \frac{1}{3} \int \frac{dx}{x^2+4} =$$

$$= \frac{1}{3} \operatorname{arctg} x - \frac{1}{3} \cdot \frac{1}{2} \operatorname{arctg} \frac{x}{2} + C =$$

$$= \frac{1}{3} \operatorname{arctg} x - \frac{1}{6} \operatorname{arctg} \frac{x}{2} + C$$

№ 8.3.45

$$\int \frac{2x^2-3x-3}{(x^2-2x+5)(x-1)} dx;$$

$$x^2-2x+5=0$$

$$D=(-2)^2-4\cdot 5 = 4-20 = -16 < 0 -$$

корней нет

$$\frac{2x^2-3x-3}{(x^2-2x+5)(x-1)} = \frac{Ax+B}{x^2-2x+5} + \frac{C}{x-1}$$

$$2x^2-3x-3 = (Ax+B)(x-1) + C(x^2-2x+5) =$$

$$\cancel{2x^2} = Ax^2 + Bx - Ax - B + Cx^2 - 2Cx + 5C =$$

$$= x^2(A+C) + x(B-A-2C) - B + 5C;$$

$$\begin{cases} A+C=2 \\ -A+B-2C=-3 \\ -B+5C=-3 \end{cases}$$

$[\text{из } 3\text{ур}]: B = 3+5C$

$1\text{ур} + 2\text{ур}:$

$C+3+5C-2C = -1$

$4C = -4 \Rightarrow C = -1$

$B = 3+5\cdot(-1) = -2$

$A - 1 = 2 \Rightarrow A = 3$

$$\int \frac{2x^2-3x-3}{(x^2-2x+5)(x-1)}\,dx = \int\left(\frac{3x-2}{x^2-2x+5} - \frac{1}{x-1}\right)dx = \int \frac{3x-2}{x^2-2x+5}\,dx - \int\frac{dx}{x-1};$$

$$\int \frac{3x-2}{x^2-2x+5}\,dx = \Big[\text{III тип}; A = 3, B = -2; p = -2,\ q = 5,\ a = \sqrt{5-\tfrac{4}{4}} = 2;$$

$$3x-2 = \frac{3}{2}(2x-2) - 2 - \frac{3\cdot(-2)}{2} = \frac{3}{2}(2x-2)+1\Big] = \frac{3}{2}\int\frac{2x-2}{x^2-2x+5}\,dx + \int\frac{dx}{x^2-2x+5};$$

1) $$\int\frac{2x-2}{x^2-2x+5}\,dx = \Big[t = x^2-2x+5 \Rightarrow dt = (2x-2)\,dx\Big] =$$

$$= \int \frac{dt}{t} = \ln|t| + C_1 = \ln|x^2-2x+5| + C_1;$$

2) $$\int \frac{dx}{x^2-2x+5} = [y = x - 1] =$$

$$= \int \frac{dy}{y^2+2^2} = \frac{1}{2}\operatorname{arctg}\frac{x-1}{2} + C_2$$

$$\int \frac{3x-2}{x^2-2x+5}\,dx - \int \frac{dx}{x-1} =$$

$$= \frac{3}{2}\ln|x^2-2x+5| + \frac{1}{2}\operatorname{arctg}\frac{x-1}{2} -$$

$$- \ln|x-1| + C$$

<u>№8.3.4б</u>

$$\int \frac{x^4+x^3+x^2+x+1}{(x^2+1)^2\cdot x}\,dx$$

$$\frac{x^4+x^3+x^2+x+1}{(x^2+1)^2\cdot x} = \frac{Ax+B}{(x^2+1)^2} + \frac{Cx+D}{x^2+1} + \frac{E}{x}$$

$$x^4+x^3+x^2+x+1 = (Ax+B)\cdot x + (Cx+D)(x^2+1)x +$$

$$+ E(x^2+1)^2 = Ax^2+Bx + Cx^4 + Dx^3 + Cx^2 + Dx +$$

$$+ Ex^4 + 2Ex^2 + E = x^4(C+E) +$$

$$+ x^3(D) + x^2(A+C+2E) + x(B+D) + E;$$

$$\begin{cases} C+E=1 \\ D=1 \\ A+C+2E=1 \\ B+D=1 \\ E=1 \end{cases} \Rightarrow \begin{cases} C=0 \\ D=1 \\ A=-1 \\ B=0 \\ E=1 \end{cases}$$

$$\int \frac{x^4+x^3+x^2+x+1}{(x^2+1)^2\cdot x}\,dx = \int \left(-\frac{1x+0}{(x^2+1)^2} + \right.$$

$$\left. + \frac{0x+1}{x^2+1} + \frac{1}{x}\right)dx = -\int \frac{x\,dx}{(x^2+1)^2} +$$

$$+ \int \frac{dx}{x^2+1} + \int \frac{dx}{x} = \frac{1}{2(x^2+1)} + \operatorname{arctg} x +$$

$$+ \ln|x| + C$$

W8.3.47

$$\int \frac{x^4-2x^3+3x+4}{x^3+1}\,dx = \int \frac{x^4-2x^3+5x+4}{(x+1)(x^2-x+1)}\,dx =$$

$$= \int \frac{(x-2)(x^3+1)+2x+6}{(x+1)(x^2-x+1)}\,dx =$$

$$= \int \frac{x-2}{(x+1)(x^2-x+1)}\,dx + \int \frac{2x+6}{(x+1)(x^2-x+1)}\,dx =$$

$$= 3$$

$$\frac{f(x)}{(x+1)(x^2-x+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}$$

$$f(x) = A(x^2-x+1) + (Bx+C)(x+1) =$$
$$= Ax^2 - Ax + A + Bx^2 + Cx + Bx + C =$$
$$= x^2(A+B) + x(-A+B+C) + A+C$$

1) $\begin{cases} A+B=0 \\ -A+B+C=1 \\ A+C=-2 \end{cases} \Rightarrow \begin{cases} A=-B \\ B+B-2+B=1 \\ C=-2-A \end{cases}$

$3B = 3 \Rightarrow B=1,\ A=-1,\ C=-1$

$$\int \frac{x-2}{(x+1)(x^2-x+1)}dx = \int\left(-\frac{1}{x+1} + \frac{x-1}{x^2-x+1}\right)dx =$$
$$= -\int\frac{dx}{x+1} + \int\frac{x-1}{x^2-x+1}dx = (1)$$

$$\int\frac{x-1}{x^2-x+1}dx = [\text{III тип};\ A=1,\ B=-1,$$
$$p=-1,\ q=1,\ a=\sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{2};$$
$$x-1 = \frac{1}{2}(2x-1)-1+\frac{1}{2} = \frac{1}{2}(2x-1)-\frac{1}{2}] =$$
$$= \frac{1}{2}\int\frac{2x-1}{x^2-x+1}dx - \frac{1}{2}\int\frac{dx}{x^2-x+1} = [y = x-\tfrac{1}{2}] =$$
$$= \frac{1}{2}\int\frac{2x-1}{x^2-x+1}dx - \frac{1}{2}\int\frac{dy}{y^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{1}{2}\ln|x^2-x+1| -$$

$$-\frac{1}{2}\cdot\frac{2}{\sqrt{3}}\,\mathrm{arctg}\,\frac{2x-1}{\sqrt{3}}+C=$$

$$=\frac{1}{2}\ln|x^2-x+1|-\frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{2x-1}{\sqrt{3}}+C$$

$$1 = -\ln|x+1|+\frac{1}{2}\ln|x^2-x+1|-$$

$$-\frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{2x-1}{\sqrt{3}}+C$$

2) $$\begin{cases} A+B=0 \\ -A+B+C=2 \\ A+C=6 \end{cases} \Rightarrow \begin{cases} A=-B \\ B+B+6+B=2 \\ C=6-A=6+B \end{cases}$$

$$3B=-4 \Rightarrow B=-\frac{4}{3},\ A=\frac{4}{3},\ C=\frac{14}{3}$$

$$\int\frac{2x+6}{(x+1)(x^2-x+1)}=\int\left(\frac{4}{3(x+1)}-\frac{4x-14}{3(x^2-x+1)}\right)dx=$$

$$=\frac{4}{3}\int\frac{dx}{x+1}-\frac{2}{3}\int\frac{2x-7}{x^2-x+1}\,dx = 2$$

$$\int\frac{2x-7}{x^2-x+1}\,dx = [\ \text{III тип};\ A=2,\ B=-7,$$

$$p=-1,\ q=1;\ a=\sqrt{1-\frac{1}{4}}=\frac{\sqrt{3}}{2}$$

$$2x-7=\frac{2}{2}(2x-1)-7+\frac{2}{2}=(2x-1)-6\ ]=$$

$$=\int\frac{2x-6}{x^2-x+1}\,dx-6\int\frac{dx}{x^2-x+1}=\left[\begin{matrix}\text{по преди.}\\ y=x-\frac{1}{2}\end{matrix}\right]=$$

$$= \ln|x^2-x+1| - 6\cdot\frac{2}{\sqrt{3}}\,\mathrm{arctg}\,\frac{2x-1}{\sqrt{3}} + C$$

(2) $$= \frac{4}{3}\ln|x+1| - \frac{2}{3}\ln|x^2-x+1| +$$

$$+ \frac{24}{3\sqrt{3}}\,\mathrm{arctg}\,\frac{2x-1}{\sqrt{3}} + C =$$

$$= \frac{4}{3}\ln|x+1| - \frac{2}{3}\ln|x^2-x+1| +$$

$$+ \frac{8}{\sqrt{3}}\,\mathrm{arctg}\,\frac{2x-1}{\sqrt{3}} + C$$

(3) $$= -\ln|x+1| + \frac{1}{2}\ln|x^2-x+1| -$$

$$- \frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{2x-1}{\sqrt{3}} + \frac{4}{3}\ln|x+1| -$$

$$- \frac{2}{3}\ln|x^2-x+1| + \frac{8}{\sqrt{3}}\,\mathrm{arctg}\,\frac{2x-1}{\sqrt{3}} + C =$$

$$= \frac{1}{3}\ln|x+1| - \frac{1}{6}\ln|x^2-x+1| +$$

$$+ \frac{7}{\sqrt{3}}\,\mathrm{arctg}\,\frac{2x-1}{\sqrt{3}} + C$$

№ 8.3.48

$$\int \frac{3x+5}{x(x^2+1)^2}\,dx = \left[\frac{3x+5}{x(x^2+1)^2} = \right.$$

$$= \frac{A}{x} + \frac{Bx+C}{(x^2+1)^2} + \frac{Dx+E}{x^2+1} \Rightarrow$$

по №8.3.46:

$$\begin{cases} D+A=0 \\ E=0 \\ B+D+2A=0 \\ C+E=3 \\ A=5 \end{cases} \Rightarrow \begin{cases} D=-5 \\ E=0 \\ B=-5 \\ C=-3 \\ A=5 \end{cases} =$$

$$= \int \left(\frac{5}{x} + \frac{-5x-3}{(x^2+1)^2} - \frac{5x}{x^2+1} \right) dx =$$

$$= 5\int \frac{dx}{x} - \int \frac{5x+3}{(x^2+1)^2}\, dx - 5\int \frac{x\,dx}{x^2+1} =$$

$$= \Big[2) \int \frac{5x+3}{(x^2+1)^2}\, dx = [\text{IV тип}; A=5,$$

$$B=3, p=0, q=1, a=\sqrt{q-\tfrac{p^2}{4}}=1] =$$

$$= \frac{5}{2}\int \frac{2x\,dx}{(x^2+1)^2} + 3\int \frac{dx}{(x^2+1)^2} =$$

$$= [\, 2) \int \frac{dx}{(x^2+1)^2} = \frac{1}{2(2-1)} \cdot \frac{x}{x^2+1} + \frac{2\cdot 2-3}{2\cdot 2-2} \cdot$$

$$\cdot \int \frac{dx}{x^2+1} = \frac{x}{2(x^2+1)} + \frac{1}{2}\operatorname{arctg} x \,] =$$

$$= \frac{5}{2} \cdot \left(-\frac{1}{x^2+1} \right) + \frac{3x}{2(x^2+1)} + \frac{3}{2}\operatorname{arctg} x + C_1 =$$

$$= \frac{3x-5}{2(x^2+1)} + \frac{3}{2}\operatorname{arctg} x + C_1 \Big] =$$

$$= 5\ln|x| - \frac{3x-5}{2(x^2+1)} - \frac{3}{2}\operatorname{arctg} x -$$

$$- \frac{5}{2}\ln|x^2+1| + C$$

<u>№ 8.3.49</u>

$$\int \frac{x^2\,dx}{x^3+5x^2+8x+4} = \Big[x^3+5x^2+8x+4=0$$

$$(x+2)(x^2+3x+2)=0$$

$$x^2+3x+2=0$$

$$D = 3^2 - 4\cdot 2 = 1$$

$$x_1 = \frac{-3+1}{2} = -1, \quad x_2 = \frac{-3-1}{2} = -2 \Big] =$$

$$= \int \frac{x^2\,dx}{(x+2)(x+1)(x+2)} = \int \frac{x^2\,dx}{(x+1)(x+2)^2} =$$

$$= \Big[\frac{x^2}{(x+2)^2(x+1)} = \frac{A}{x+1} + \frac{B}{(x+2)^2} + \frac{C}{x+2} \Rightarrow$$

$$\Rightarrow x^2 = A(x+2)^2 + B(x+1) + C(x+2)(x+1) =$$

$$= Ax^2 + 4Ax + 4A + Bx + B + Cx^2 + 3Cx + 2C =$$

$$= x^2(A+C) + x(4A+B+3C) + 4A+B+2C;$$

$$\begin{cases} A+C=1 \\ 4A+B+3C=0 \\ 4A+B+2C=0 \end{cases} \Rightarrow \begin{cases} A=1-C \\ C=0 \\ 4(1-C)+B+2C=0 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} A=1 \\ C=0 \\ B=-4 \end{cases} = \int\left(\frac{1}{x+1}-\frac{4}{(x+2)^2}\right)dx =$$

$$= \int\frac{dx}{x-1} - 4\int\frac{dx}{(x+2)^2} = \ln|x-1| -$$

$$-4\cdot\left(-\frac{1}{x+2}\right)+C = \ln|x-1| +$$

$$+\frac{4}{x+2}+C$$

<u>w8.3.50</u>

$$\int\frac{dt}{t^4-1} = \int\frac{dt}{(t^2-1)(t^2+1)} = \int\frac{dt}{(t-1)(t+1)(t^2+1)} =$$

$$= \left[\frac{1}{(t-1)(t+1)(t^2+1)} = \frac{A}{t-1}+\frac{B}{t+1}+\right.$$

$$+\frac{Ct+D}{t^2+1} \Rightarrow$$

$$1 = A(t+1)(t^2+1)+B(t-1)(t^2+1)+$$

$$+(Ct+D)(t^2-1) = At^3+At^2+At+A+$$

$$+Bt^3-Bt^2+Bt-B+Ct^3+Dt^2-$$

$$-Ct-D = t^3(A+B+C)+t^2(A-B+D)+$$

$$+t(A+B-C)+A-B-D \Rightarrow$$

$$+\begin{cases} A+B+C=0 \\ A-B+D=0 \\ A+B-C=0 \\ A-B-D=1 \end{cases}$$

$4A=1 \Rightarrow A=\frac{1}{4}$

[из 4 ур.] : $D=\frac{1}{4}-1-B=-\frac{3}{4}-B$

$\frac{1}{4}-B+\left(-\frac{3}{4}-B\right)=0$

$-2B=\frac{1}{2} \Rightarrow B=-\frac{1}{4}$

$D=-\frac{3}{4}+\frac{1}{4}=-\frac{1}{2}$

$\frac{1}{4}-\frac{1}{4}+C=0 \Rightarrow C=0$] $=$

$$=\int\left(\frac{1}{4(t-1)}-\frac{1}{4(t+1)}-\frac{1}{2(t^2+1)}\right)dt=$$

$$=\frac{1}{4}\int\frac{dt}{t-1}-\frac{1}{4}\int\frac{dt}{t+1}-\frac{1}{2}\int\frac{dt}{t^2+1}=$$

$$=\frac{1}{4}\ln|t-1|-\frac{1}{4}\ln|t+1|-\frac{1}{2}\operatorname{arctg} t+C=$$

$$=\frac{1}{4}\ln\left|\frac{t-1}{t+1}\right|-\frac{1}{2}\operatorname{arctg} t+C$$

№8.3.51

$$\int\frac{e^{2x}dx}{e^{2x}+3e^x+2}=\left[t=e^x \Rightarrow dt=e^x dx\right]=$$

$$= \int \frac{t\,dt}{t^2+3t+2} = \Big[\, t^2+3t+2=0 \rightarrow$$

$$\Rightarrow D = 9-4\cdot 2 = 1 \Rightarrow t_1 = \frac{-3+1}{2} = -1,$$

$$t_2 = \frac{-3-1}{2} = -2 \,\Big] = \int \frac{t\,dt}{(t+1)(t+2)} =$$

$$= \Big[\, \frac{t}{(t+1)(t+2)} = \frac{A}{t+1} + \frac{B}{t+2} \Rightarrow$$

$$\Rightarrow t = A(t+2) + B(t+1) =$$

$$= \cancel{4}At + 2A + Bt + B = t(A+B) + 2A + B;$$

$$- \begin{cases} A+B=1 \\ 2A+B=0 \end{cases} \Rightarrow -A = 1 \Rightarrow A = -1$$

$$-1+B = 1 \Rightarrow B = 2 \,\Big] =$$

$$= \int \left(-\frac{1}{t+1} + \frac{2}{t+2}\right) dt =$$

$$= -\int \frac{dt}{t+1} + 2\int \frac{dt}{t+2} =$$

$$= 2\ln|t+2| - \ln|t+1| + C =$$

$$= 2\ln(e^x+2) - \ln|e^x+1| + C$$

W8.3.51

$$\int \frac{1+e^x}{(1-e^{2x})e^x}\,dx = \int \frac{1+e^x}{(1+e^x)(1-e^x)e^x}\,dx =$$

$$= \int \frac{dx}{(1-e^x)e^x} = [t = e^x;\ dt = e^x dx] =$$

$$= \int \frac{e^x dx}{(1-e^x)e^{2x}} = \int \frac{dt}{t^2(1-t)} = \left[\frac{1}{t^2(1-t)} = \right.$$

$$= \frac{At+B}{t^2} + \frac{C}{1-t} \Rightarrow$$

$$1 = (At+B)(1-t) + Ct^2 =$$

$$= At + B - At^2 - Bt + Ct^2 =$$

$$= t^2(C-A) + t(A-B) + B \Rightarrow$$

$$\Rightarrow \begin{cases} C - A = 0 \\ A - B = 0 \\ B = 1 \end{cases} \Rightarrow \left. \begin{cases} C = 1 \\ A = 1 \\ B = 1 \end{cases} \right] =$$

$$= \int \left(\frac{t+1}{t^2} + \frac{1}{1-t} \right) dx =$$

$$= \int \left(\frac{1}{t} + \frac{1}{t^2} + \frac{1}{1-t} \right) dt =$$

$$= \int \frac{dt}{t} + \int \frac{dt}{t^2} - \int \frac{dt}{t-1} =$$

$$= \ln|t| - \frac{1}{t} - \ln|t-1| + C =$$

$$= \ln|e^x| - \frac{1}{e^x} - \ln|e^x - 1| + C =$$

$$= x - \frac{1}{e^x} - \ln|e^x - 1| + C$$

W8.3.53

$$\int \frac{\cos x\,dx}{(\sin x-1)(\sin x+2)} = \Big[t = \sin x \Rightarrow$$

$$\Rightarrow dt = \cos x\,dx\Big] = \int \frac{dt}{(t-1)(t+2)} =$$

$$= \Big[\frac{1}{(t-1)(t+2)} = \frac{A}{t-1} + \frac{B}{t+2} \Rightarrow$$

$$\Rightarrow 1 = A(t+2) + B(t-1) = At + 2A +$$

$$+ Bt - B = t(A+B) + 2A - B \Rightarrow$$

$$\Rightarrow + \begin{cases} A+B=0 \\ 2A-B=1 \end{cases} \Rightarrow 3A = 1 \Rightarrow A = \frac{1}{3};$$

$$\frac{1}{3} + B = 0 \Rightarrow B = -\frac{1}{3}\Big] =$$

$$= \int \left(\frac{1}{3}\cdot\frac{1}{t-1} - \frac{1}{3}\cdot\frac{1}{t+2}\right)dt =$$

$$= \frac{1}{3}\int\frac{dt}{t-1} - \frac{1}{3}\int\frac{dt}{t+2} =$$

$$= \frac{1}{3}\ln|t-1| - \frac{1}{3}\ln|t+2| + C =$$

$$= \frac{1}{3}\ln|\sin x-1| - \frac{1}{3}\ln|\sin x+2| + C$$

W8.3.54

$$\int \frac{\sin^4 x\,dx}{\cos x} = \cancel{\int \frac{\sin^4 x \cos x\,dx}{\cos^2 x}}$$

$$= \int \frac{3-4\cos 2x+\cos 4x}{8\cos x}\,dx =$$

$$= \int \frac{3-4(2\cos^2 x-1)+2\cos^2 2x-1}{8\cos x}\,dx =$$

$$= \int \frac{2-8\cos^2 x+4+2(2\cos^2 x-1)^2}{8\cos x}\,dx$$

$$= \int \frac{6-8\cos^2 x+8\cos^4 x-8\cos^2 x+2}{8\cos x}\,dx =$$

$$= \int \frac{1-2\cos^2 x+\cos^4 x}{\cos x}\,dx =$$

$$= \int \frac{(1-\cos^2 x)^2}{\cos x}\,dx =$$

$$= [\, t=\cos^2 x,\ dt = 2\cos x \sin x\, dx$$

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(2) $\int \sin^3 x \,\operatorname{tg} x\, dx = \left[\begin{array}{l} u = \cancel{\sin^3 x}\ \operatorname{tg} x \\ v' = \cancel{\operatorname{tg} x}\ \sin^3 x \end{array} \right. \Rightarrow$

$\Rightarrow \begin{array}{l} u' = \frac{1}{\cos^2 x} \\ v = \cancel{\frac{\cos^3 x}{3}}\ \cancel{-\cos x}\ \int \sin^3 x\, dx = \end{array}$

$= \int (1-\cos^2 x)\sin x\, dx = [\, t = \cos x \Rightarrow$

$\Rightarrow dt = -\sin x\, dx\,] =$

$= \int \cancel{(\cos^2 x - 1)\,dt}\ (t^2-1)\, dt =$

$= \int t^2 dt - \int dt = \frac{t^3}{3} - t =$

$= \frac{\cos^3 x}{3} - \cos x\ ] =$

$$= tg x \left(\frac{\cos^3 x}{3} - \cos x\right) -$$

$$- \int \frac{1}{\cos^2 x} \cdot \left(\frac{\cos^3 x}{3} - \cos x\right) dx =$$

$$= \frac{\sin x \cos^4 x}{3} - \sin x \cos^2 x -$$

$$- \int \left(\frac{1}{3} \cos x - \frac{1}{\cos x}\right) dx =$$

$$= \frac{1}{3} \sin x \cos^4 x - \sin x \cos^2 x -$$

$$- \frac{1}{3} \int \cos x \, dx + \int \frac{dx}{\cos x} =$$

$$= \frac{1}{3} \sin x \cos^2 x \left(\cos^2 x - 1\right) -$$

$$- \frac{1}{3} \sin x + \ln \left| tg \left(\frac{x}{2} + \frac{\pi}{4}\right) \right| + C =$$

$$= - \frac{1}{3} \sin^3 x \cos^2 x - \frac{1}{3} \sin x +$$

$$+ \ln \left| tg \left(\frac{x}{2} + \frac{\pi}{4}\right) \right| + C$$