Домашняя работа
Производные, часть 5

№ 11.4.42

$z = \operatorname{arctg}\frac{y}{x}$, $x = e^{2t}+1$, $y = e^{2t}-1$

$$\frac{\partial z}{\partial x} = \frac{1}{1+\left(\frac{y}{x}\right)^2}\cdot y\cdot\left(-\frac{1}{x^2}\right) = -\frac{x^2 y}{(x^2+y^2)\cdot x^2}$$

$$= -\frac{y}{x^2+y^2}$$

$$\frac{\partial z}{\partial y} = \frac{x^2}{x^2+y^2}\cdot\frac{1}{x} = \frac{x}{x^2+y^2}$$

$$\frac{dx}{dt} = 2e^{2t}; \quad \frac{dy}{dt} = 2e^{2t}$$

$$\frac{dz}{dt} = -\frac{y}{x^2+y^2}\cdot 2e^{2t} + \frac{x}{x^2+y^2}\cdot 2e^{2t} =$$

$$= \frac{2e^{2t}(x-y)}{x^2+y^2}$$

w11.4.43

$$z = x^4 + y^4 - 4x^2y^2, \quad x = e^{2t}, \quad y = e^{2t}$$

$$z = e^{8t} + e^{8t} - 4e^{4t}e^{4t} = 2e^{8t} - 4e^{8t} =$$
$$= -2e^{8t}$$

$$\frac{dz}{dt} = -16e^{8t}$$

w11.4.44

$$z = x\cancel{y} + \frac{x}{y}, \quad x = tg\,t, \quad y = \ln t$$

$$\frac{\partial z}{\partial x} = y + \frac{1}{y}; \quad \frac{\partial z}{\partial y} = x \cancel{+} - \frac{x}{y^2}$$

$$\frac{dx}{dt} = \frac{1}{\cos^2 t}; \quad \frac{dy}{dt} = \frac{1}{t}$$

$$\frac{dz}{dt} = \frac{y + \frac{1}{y}}{\cos^2 t} + \frac{x - \frac{x}{y^2}}{t} =$$

$$= \frac{y}{\cos^2 t} + \frac{1}{y\cos^2 t} + \frac{x}{t} - \frac{x}{ty^2}$$

w 11.4.45

$$z = \frac{x}{y^2}, \quad x = \operatorname{arctg} 2t, \quad y = \arcsin t$$

$$\frac{\partial z}{\partial x} = \frac{1}{y^2}; \quad \frac{\partial z}{\partial y} = -\frac{2x}{y^3}$$

$$\frac{dx}{dt} = \frac{1}{1+4t^2} \cdot 2 = \frac{2}{1+4t^2}$$

$$\frac{dy}{dt} = \frac{1}{\sqrt{1-t^2}}$$

$$\frac{dz}{dt} = \frac{2}{y^2(1+4t^2)} - \frac{2x}{y^3\sqrt{1-t^2}}$$

w 11.4.46

$$z = \frac{x}{\sqrt{x^2+y^2}}, \quad x = 5^{t^2}, \quad y = \arccos 2t$$

$$\frac{\partial z}{\partial x} = \frac{\sqrt{x^2+y^2} - x\left(\sqrt{x^2+y^2}\right)'}{x^2+y^2} =$$

$$= \frac{\sqrt{x^2+y^2} - \frac{x}{2\sqrt{x^2+y^2}} \cdot 2x}{x^2+y^2} =$$

$$= \frac{x^2+y^2-x^2}{\sqrt{x^2+y^2}} \cdot \frac{1}{x^2+y^2} = \frac{y^2}{(x^2+y^2)^{\frac{3}{2}}}$$

$$\frac{\partial z}{\partial y} = x \cdot \left(-\frac{1}{x^2+y^2}\right) \cdot \frac{1}{2\sqrt{x^2+y^2}} \cdot 2y = -\frac{xy}{(x^2+y^2)^{\frac{3}{2}}}$$

$$\frac{dx}{dt} = 5^{t^2}\ln 5 \cdot 2t = 2t\,5^{t^2}\ln 5$$

$$\frac{dy}{dt} = -\frac{1}{\sqrt{1-4t^2}} \cdot 2 = -\frac{2}{\sqrt{1-4t^2}}$$

$$\frac{dz}{dt} = \frac{y^2}{(x^2+y^2)^{\frac{3}{2}}} \cdot 2t\,5^{t^2}\ln 5 + \frac{2}{\sqrt{1-4t^2}} \cdot \frac{xy}{(x^2+y^2)^{\frac{3}{2}}} =$$

$$= \frac{2y}{(x^2+y^2)^{\frac{3}{2}}} \cdot \left(yt\,5^{t^2}\ln 5 + \frac{x}{\sqrt{1-4t^2}}\right)$$

№ 11.4.47

$$z = x\sin(x+y),\quad x = \frac{1}{t^3},\quad y = (t-1)^2$$

$$\frac{\partial z}{\partial x} = \cos(x+y);\quad \frac{\partial z}{\partial y} = x\cos(x+y)$$

$$\frac{dx}{dt} = -\frac{3}{t^4};\quad \frac{dy}{dt} = 2(t-1)$$

$$\frac{dz}{dt} = 2x(t-1)\cos(x+y) - \frac{3\cos(x+y)}{t^4}$$

__№ 11.4.48__

$z = \frac{\cos x^2}{y}, \quad x = \ln(t+2), \; y = tg\,t$

$\frac{\partial z}{\partial x} = -\frac{\sin x^2}{y} \cdot 2x = -\frac{2x}{y}\sin x^2$

$\frac{\partial z}{\partial y} = -\frac{\cos x^2}{y^2}$

$\frac{dx}{dt} = \frac{1}{t+2}; \quad \frac{dy}{dt} = \frac{1}{\cos^2 t}$

$\frac{dz}{dt} = -\frac{2x\sin x^2}{y(t+2)} - \frac{\cos^{\cancel{2}} x^2}{y^2\cos^2 t}$

__№ 11.4.49__

$z = tg\,\frac{x^2}{y}, \quad x = \cos^2 t, \; y = \sin 2t$

$z = tg\,\frac{\cos^4 t}{\sin 2t} = tg\,\frac{\cos^4 t}{2\cancel{\cos t}\sin t} =$

$= tg\,\frac{\cos^3 t}{2\sin t} = tg\left(\frac{1}{2}\,ctg\,t\right) \cdot \cos^2 t$

$= tg\,\frac{1}{2} \cdot tg(ctg\,t) \cdot \cos^2 t$

$\cancel{\frac{dz}{dt} = \frac{1}{\cos^2(ctg\,t)} \cdot \left(-\frac{1}{\sin^2 t}\right) \cdot tg\,\frac{1}{2}}$

$$\cancel{z = \frac{tg\,0{,}5}{\cos^2(ctg\,t)\sin^2 t}}$$

$$\frac{dz}{dt} = -\frac{tg\,0{,}5\cdot\cos^2 t}{\cos^2(ctg\,t)\sin^2 t} -$$

$$- 2\cos t\cdot\sin t\cdot tg\,0{,}5\cdot$$

$$\cdot\, tg(ctg\,t) =$$

$$= -\frac{tg\,0{,}5\; ctg^2 t}{\cos^2(ctg\,t)} - 2\sin 2t\,\cdot$$

$$\cdot\, tg\,0{,}5\cdot tg(ctg\,t)$$

№ 11.4.62

$$y^4 - 6x^2y^2 + arctg^2 x = 0$$

$$y^4 - 6x^2y^2 = -arctg^2 x$$

$$y^2(y^2 - 6x^2) = -arctg^2 x$$

$$\begin{cases} y^2 = -arctg^2 x \\ y^2 - 6x^2 = -arctg^2 x \end{cases}$$

$$\begin{cases} y^2 = -arctg^2 x \\ y^2 = 6x^2 - arctg^2 x \end{cases}$$

№11.4.62

$$y^4 - 6x^2y^2 + \operatorname{arctg}2x = 0$$

$$(y^2)^2 - 6x^2 \cdot y^2 + \operatorname{arctg}2x = 0$$

$$D = (-6x^2)^2 - 4 \cdot 1 \cdot \operatorname{arctg}2x =$$

$$= 36x^4 - 4\operatorname{arctg}2x$$

$$y^2 = \frac{6x^2 \pm \sqrt{36x^4 - 4\operatorname{arctg}2x}}{2}$$

$$y = \pm\sqrt{\frac{6x^2 \pm \sqrt{36x^4 - 4\operatorname{arctg}2x}}{2}}$$

№11.4.63

$$e^{-x+y^3} - 20x - 18x^3 - 1 = 0$$

$$e^{-x+y^3} = 20x + 18x^3 + 1 \quad | \ln(\ )$$

$$-x + y^3 = \ln(20x + 18x^3 + 1)$$

$$y^3 = \ln(20x + 18x^3 + 1) + x$$

$$y = \sqrt[3]{\ln(20x + 18x^3 + 1) + x}$$

w11.4.64

$tg(x^2+y^4)-3x^2-17=0$

$tg(x^2+y^4)=3x^2+17$

$x^2+y^4=arctg(3x^2+17)$

$y^4=arctg(3x^2+17)-x^2$

$y=\pm\sqrt[4]{arctg(3x^2+17)-x^2}$

w11.4.65

$x^2y^4-3y^3-6y^2+3y+x^2=0$

$\exists\ x^2=C=const$

$x^2y^4-3y^3-6y^2+3y+x^2=0 \mid :y^2$

$x^2y^2-3y-6+\frac{3}{y}+\frac{x^2}{y^2}=0$

$x^2y^2+\frac{x^2}{y^2}-3y+\frac{3}{y}-6=0$

$x^2\left(y^2+\frac{1}{y^2}\right)-3\left(y-\frac{1}{y}\right)-6=0$

$x^2\left(y^2+\frac{1}{y^2}-2\right)+2x^2-3\left(y-\frac{1}{y}\right)-6=0$

$\exists\ y - \frac{1}{y} = z \Rightarrow y^2 + \frac{1}{y^2} - 2 = z^2$

$x^2 z^2 - 3z + 2x^2 - 6 = 0$

$D = 9 - 4\cdot(2x^2 - 6) = 9 - 8x^2 + 24 =$
$= 33 - 8x^2$

$z = \frac{3 \pm \sqrt{33 - 8x^2}}{2}$

$y - \frac{1}{y} = \frac{3 \pm \sqrt{33 - 8x^2}}{2}$

$\frac{y^2 - 1}{y} = \frac{3 \pm \sqrt{33 - 8x^2}}{2}$

$y^2 - 1 = \frac{1}{2} y \left(3 \pm \sqrt{33 - 8x^2}\right)$

$y^2 - \frac{1}{2} y \left(3 \pm \sqrt{33 - 8x^2}\right) - 1 = 0$

$D = \frac{1}{4}\left(3 \pm \sqrt{33 - 8x^2}\right)^2 - 4\cdot(-1) =$
$= \frac{1}{4}\left(\left(3 \pm \sqrt{33 - 8x^2}\right)^2 + 16\right)$

$y = \frac{1}{4}\left(3 \pm \sqrt{33 - 8x^2} \pm \sqrt{\left(3 \pm \sqrt{33 - 8x^2}\right)^2 + 16}\right)$

№ 11.4.67

$z = u^2 \ln v, \quad u = \frac{y}{x}, \quad v = x^2 + y^2$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}$$

$$\frac{\partial z}{\partial u} = 2u \ln v; \quad \frac{\partial z}{\partial v} = \frac{u^2}{v}$$

$$\frac{\partial u}{\partial x} = -\frac{y}{x^2}; \quad \frac{\partial u}{\partial y} = \frac{1}{x}$$

$$\frac{\partial v}{\partial x} = 2x; \quad \frac{\partial v}{\partial y} = 2y$$

$$\frac{\partial z}{\partial x} = 2u \ln v \cdot \left(-\frac{y}{x^2}\right) + \frac{u^2}{v} \cdot 2x = 2u\left(\frac{xu}{v} - \frac{y \ln v}{x^2}\right)$$

$$\frac{\partial z}{\partial y} = 2u \ln v \cdot \frac{1}{x} + \frac{u^2}{v} \cdot 2y = 2u\left(\frac{\ln v}{x} + \frac{yu}{v}\right)$$

w11.4.67

$z = u^2 \ln v, \quad u = \frac{y}{x}, \quad v = x^2 + y^2$

$\frac{\partial z}{\partial u} = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$

$\frac{\partial u}{\partial x} = -\frac{y}{x^2}; \quad \frac{\partial u}{\partial y} = \frac{1}{x}$

$\frac{\partial v}{\partial x} = 2x, \quad \frac{\partial v}{\partial y} = 2y$

$\frac{\partial z}{\partial x} =$

$z = \left(\frac{y}{x}\right)^2 \ln(x^2+y^2) = \frac{y^2}{x^2} \ln(x^2+y^2)$

$\frac{\partial z}{\partial x} = -\frac{2y^2}{x^3} \ln(x^2+y^2) + \frac{y^2}{x^2} \cdot \frac{1}{x^2+y^2} \cdot$

$\cdot 2x = -\frac{2y^2}{x^3} \ln(x^2+y^2) + \frac{2y^2}{x^3+y^2 x}$

w11.4.68

$z = f(u; v), \quad u = \frac{2y}{x+y}, \quad v = x^2 - 3y$

$$dz = \frac{\partial z}{\partial u} du + \frac{\partial z}{\partial v} dv$$

$$du = \frac{\partial u}{\partial x} dx + \frac{\partial u}{\partial y} dy$$

$$dv = \frac{\partial v}{\partial x} dx + \frac{\partial v}{\partial y} dy$$

$$\frac{\partial u}{\partial x} = -\frac{2y}{(x+y)^2}; \quad \cancel{\frac{\partial u}{\partial y}} =$$

$$\frac{\partial u}{\partial y} = \frac{2(x+y) - 2y}{(x+y)^2} = \frac{2(x+y-y)}{(x+y)^2} = \frac{2x}{(x+y)^2}$$

$$\frac{\partial v}{\partial x} = 2x; \quad \frac{\partial v}{\partial y} = -3 \}$$

$$\{ dz = \frac{\partial z}{\partial u}\left(-\frac{2y}{(x+y)^2} dx + \frac{2(x+y) - 2y}{(x+y)^2} \cdot dy\right) + \frac{\partial z}{\partial v}(2x dx - 3 dy) =$$

$$= -\frac{\partial z}{\partial u}\frac{2y\,dx}{(x+y)^2} + \frac{\partial z}{\partial v} 2x dx + \frac{\partial z}{\partial u}\frac{(2(x+y) - 2y) dy}{(x+y)^2} - 3\frac{\partial z}{\partial v} dy \} 2$$

$= 2\partial z\left(-\dfrac{y}{\partial u(x+y)^2} + \dfrac{x}{\partial v}\right)dx +$

$+ \partial z\left(\dfrac{2x}{(x+y)^2\partial u} - \dfrac{3}{\partial v}\right)dy$

№11.4.69

$z = f(u; v),\ u = \ln(x^2 - y^2),\ v = xy^2$

$\dfrac{\partial z}{\partial x} = ?,\ \dfrac{\partial z}{\partial y} = ?$

$dz = \dfrac{\partial z}{\partial u}\left(\dfrac{\partial u}{\partial x}dx + \dfrac{\partial u}{\partial y}dy\right) + \dfrac{\partial z}{\partial v}\left(\dfrac{\partial v}{\partial x}dx +\right.$

$\left. + \dfrac{\partial v}{\partial y}dy\right) = \dfrac{\partial z}{\partial x}dx + \dfrac{\partial z}{\partial y}dy +$

$+ \dfrac{\partial z}{\partial x}dx + \dfrac{\partial z}{\partial y}dy$

$dz = 2\dfrac{\partial z}{\partial x}dx + 2\dfrac{\partial z}{\partial y}dy$

$2dz = \dfrac{\partial z}{\partial x}dx + \dfrac{\partial z}{\partial y}dy$

$$\cancel{\frac{\partial u}{\partial x} = \frac{1}{x-y}}$$

$$u = \ln(x^2 - y^2) = \ln((x-y)(x+y)) =$$

$$= \ln(x-y) + \ln(x+y)$$

$$\frac{\partial u}{\partial x} = \frac{1}{x-y} + \frac{1}{x+y}$$

$$\frac{\partial v}{\partial x} = y^2$$

$$\frac{\partial u}{\partial y} = -\frac{1}{x-y} + \frac{1}{x+y}$$

$$\frac{\partial v}{\partial y} = 2xy$$

$$\frac{\partial z}{\partial u}\left(\left(\frac{1}{x+y} + \frac{1}{x-y}\right)dx + \left(\frac{1}{x+y} - \frac{1}{x-y}\right)dy\right) +$$

$$+ \frac{\partial z}{\partial v}\left(y^2 dx + 2xy\,dy\right) = dz$$

$$dz = \frac{\partial z}{\partial u}\left(\frac{1}{x+y} + \frac{1}{x-y}\right)dx + \frac{\partial z}{\partial v} y^2 dx +$$

$$+ \frac{\partial z}{\partial u}\left(\frac{1}{x+y} - \frac{1}{x-y}\right)dy + \frac{\partial z}{\partial v} 2xy\,dy$$

$$\frac{\partial z}{\partial x} = \left(\frac{\partial z}{\partial u}\left(\frac{1}{x+y} + \frac{1}{x-y}\right) + \frac{\partial z}{\partial v} y^2\right)$$

$$\frac{\partial z}{\partial y} = \left(\frac{\partial z}{\partial u}\left(\frac{1}{x+y} - \frac{1}{x-y}\right) + \frac{\partial z}{\partial v} 2xy\right)$$

№ 11.4.70

$$z = u^2 v - u v^2, \quad u = x\sin y, \quad v = y\cos x$$

$$dz = \frac{\partial z}{\partial u}\left(\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy\right) + \frac{\partial z}{\partial v} \cdot$$

$$\cdot \left(\frac{\partial v}{\partial x}dx + \frac{\partial v}{\partial y}dy\right)$$

$$\frac{\partial u}{\partial x} = \sin y; \quad \frac{\partial u}{\partial y} = x\cos y$$

$$\frac{\partial v}{\partial x} = -y\sin x; \quad \frac{\partial v}{\partial y} = \cos x$$

$$\frac{\partial z}{\partial u} = 2uv - v^2; \quad \frac{\partial z}{\partial v} = u^2 - 2uv$$

$$dz = (2uv - v^2)(\sin y\,dx + x\cos y\,dy) +$$
$$+ (u^2 - 2uv)(-y\sin x\,dx + \cos x\,dy) =$$
$$= 2uv\sin y\,dx - v^2\sin y\,dx + 2uvx\cos y\,dy -$$
$$- v^2 x\cos y\,dy - u^2 y\sin x\,dx + 2uvy\sin x\,dx +$$
$$+ u^2\cos x\,dy - 2uv\cos x\,dy =$$
$$= (2uv\sin y - v^2\sin y - u^2 y\sin x +$$
$$+ 2uvy\sin x)\,dx + (2uvx\cos y -$$
$$- v^2 x\cos y + u^2\cos x - 2uv\cos x)\,dy =$$
$$= (v\sin y + uy\sin x)(2u - v - u + 2v)\,dx +$$
$$+ (vx\cos y + u\cos x)(2u - v + u - 2v)\,dy =$$
$$= (v\sin y + uy\sin x)(u + v)\,dx +$$
$$+ (vx\cos y + u\cos x)(3u - 3v)\,dy$$

№11.4.71

$z = f(u; v)$, $u = \cos(xy)$, $v = x^5 - 7y$

$\frac{\partial u}{\partial x} = -\sin(xy) \cdot y = -y\sin(xy)$

$\frac{\partial u}{\partial y} = -\sin(xy) \cdot x = -x\sin(xy)$

$\frac{\partial v}{\partial x} = 5x^4$; $\frac{\partial v}{\partial y} = -7$

$dz = \frac{\partial z}{\partial u}(-y\sin(xy)dx - x\sin(xy)dy) +$

$+ \frac{\partial z}{\partial v}(5x^4 dx - 7dy) =$

$= -\frac{\partial z}{\partial u} y\sin(xy)dx + \frac{\partial z}{\partial v} 5x^4 dx -$

$- \frac{\partial z}{\partial u} x\sin(xy)dy - \frac{\partial z}{\partial v} \cdot 7dy =$

$= \left(\frac{\partial z}{\partial v} 5x^4 - \frac{\partial z}{\partial u} y\sin(xy)\right)dx -$

$- \left(\frac{\partial z}{\partial u} x\sin(xy) + \frac{\partial z}{\partial v} \cdot 7\right)dy$

№11.4.72

$z = f(u; v)$, $u = \sin\frac{x}{y}$, $v = \sqrt{\frac{x}{y}}$

$$\frac{\partial u}{\partial x} = \cos\frac{x}{y}\cdot\frac{1}{y} = \frac{1}{y}\cos\frac{x}{y}$$

$$\frac{\partial u}{\partial y} = \cos\frac{x}{y}\cdot\left(-\frac{x}{y^2}\right) = -\frac{x}{y^2}\cos\frac{x}{y}$$

$$\frac{\partial v}{\partial x} = \frac{1}{2\sqrt{\frac{x}{y}}}\cdot\frac{1}{y} = \frac{1}{2y\sqrt{\frac{x}{y}}}$$

$$\frac{\partial v}{\partial y} = \frac{1}{2\sqrt{\frac{x}{y}}}\cdot\left(-\frac{x}{y^2}\right) = -\frac{x}{2y^2\sqrt{\frac{x}{y}}}$$

$$dz = \left(\frac{\partial z}{\partial u}\cdot\frac{1}{y}\cos\frac{x}{y} + \frac{\partial z}{\partial v}\cdot\frac{1}{2y\sqrt{\frac{x}{y}}}\right)dx +$$

$$+\left(\frac{\partial z}{\partial u}\cdot\left(-\frac{x}{y^2}\cos\frac{x}{y}\right) - \frac{\partial z}{\partial v}\cdot\frac{x}{2y\sqrt{\frac{x}{y}}}\right)dy =$$

$$= \left(\frac{\partial z}{y\,\partial u}\cos\frac{x}{y} + \frac{\partial z}{\partial v\,2y\sqrt{\frac{x}{y}}}\right)dx -$$

$$-\left(\frac{x\,\partial z}{y^2\,\partial u}\cos\frac{x}{y} + \frac{x\,\partial z}{2y\,\partial v\sqrt{\frac{x}{y}}}\right)dy$$

№ 11.4.~~7~~3

$x = \frac{u^2+v^2}{2}, \; y = \frac{u^2-v^2}{2}, \; z = uv$

$x+y = \frac{1}{2}(u^2+v^2+u^2-v^2)$

$x+y = \frac{1}{2}\cdot 2u^2 \Rightarrow u = \sqrt{x+y}$

$x = \frac{x+y+v^2}{2}$

$v^2 = 2x-x-y \Rightarrow v = \sqrt{x-y}$

$z = \sqrt{x+y}\sqrt{x-y} = \sqrt{x^2 \cancel{+} - y^2}$

$\frac{\partial z}{\partial x} = \frac{1}{2\sqrt{x^2-y^2}} \cdot 2x = \frac{x}{\sqrt{x^2-y^2}}$

$\frac{\partial z}{\partial y} = \frac{1}{2\sqrt{x^2-y^2}} \cdot 2y = \frac{y}{\sqrt{x^2-y^2}}$

$dz = \frac{x\,dx}{\sqrt{x^2-y^2}} + \frac{y\,dy}{\sqrt{x^2-y^2}}$

№ 11.4.74

$x = \sqrt{a}(\sin u + \cos v)$

$y = \sqrt{a}(\cos u - \sin v)$

$z = 1 + \sin(u - v)$

$$\begin{cases} x = \sqrt{a}(\sin u + \cos v) \\ y = \sqrt{a}(\cos u - \sin v) \end{cases}$$

$$+\begin{cases} x^2 = a(1 + 2\sin u \cos v) \\ y^2 = a(1 - 2\sin v \cos u) \end{cases}$$

$x^2 + y^2 = a(1 + 2\sin u \cos v + 1 - 2\sin v \cos u)$

$x^2 + y^2 = 2a(1 + \sin(u - v))$

$1 + \sin(u - v) = \dfrac{x^2 + y^2}{2a} = z$

$z = \dfrac{x^2 + y^2}{2a}$

$dz = \dfrac{\partial z}{\partial x} dx + \dfrac{\partial z}{\partial y} dy$

$\dfrac{\partial z}{\partial x} = \dfrac{1}{2a} \cdot 2x = \dfrac{x}{a}$

$$\frac{\partial z}{\partial y} = \frac{1}{2a} \cdot 2y = \frac{y}{a}$$

$$dz = \frac{1}{a} x dx + \frac{1}{a} y dy = \frac{1}{a}(x dx + y dy)$$

<u>№ 11.4.75</u>

$x = u + v, \; y = u - v, \; z = u^2 v^2$

$$+\begin{cases} x = u + v \\ y = u - v \end{cases} \Rightarrow x + y = 2u \Rightarrow u = \frac{x+y}{2}$$

$$x = \frac{x+y}{2} + v \Rightarrow v = \frac{x-y}{2}$$

$$z = \left(\frac{x+y}{2}\right)^2 \left(\frac{x-y}{2}\right)^2 = \frac{(x+y)^2 (x-y)^2}{4} =$$

$$= \frac{(x^2 + 2xy + y^2)(x^2 - 2xy + y^2)}{4} =$$

$$= \frac{x^4 - 2x^2 y^2 + y^4}{4} = \frac{(x^2 - y^2)^2}{4}$$

$$\frac{\partial z}{\partial x} = \frac{1}{4}(4x^3 - 4y^2 x) = x^3 - xy^2$$

$$\frac{\partial z}{\partial y} = \frac{1}{4}(-4x^2 y + 4y^3) = y^3 - x^2 y$$

$dz = (x^3 - xy^2)dx + (y^3 - x^2y)dy$

№11.4.76

$x = u\cos v$, $y = u\sin v$, $z = u^2$

$+\begin{cases} x = u\cos v \\ y = u\sin v \end{cases} \Rightarrow x+y = u(\cos v + \sin v)$

$(x+y)^2 = u^2(1+\sin 2v)$

$u^2 = \dfrac{(x+y)^2}{1+\sin 2v} = z$

$z = \dfrac{(x+y)^2}{1+\sin 2v}$

$\dfrac{\partial z}{\partial x} = \dfrac{2(x+y)}{1+\sin 2v}$; $\dfrac{\partial z}{\partial y} = \dfrac{2(x+y)}{1+\sin 2v}$

$dz = \dfrac{2(x+y)}{1+\sin 2v} \cdot (dx + dy)$

№11.4.77

$x = v\cos u - u\cos u + \sin u$

$y = v\sin u - u\sin u - \cos u$

$z = (u-v)^2$

$$\begin{cases} x = \cos u\,(v-u) + \sin u \\ y = \sin(v-u) - \cos u \end{cases}$$

$$+\begin{cases} x = (\cos u + \sin u)(v-u+1) \\ y = (\sin u - \cos u)(v-u+1) \end{cases}$$

$$x+y = (v-u+1)(\cos u + \sin u + \sin u - \cos u)$$

$$x+y = 2\sin u\,(v-u+1)$$

$$x+y = -2\sin u\,(u-v-1)$$

$$u-v-1 = \frac{x+y}{-2\sin u}$$

$$u-v = 1 - \frac{x+y}{2\sin u}$$

$$(u-v)^2 = \left(1 - \frac{x+y}{2\sin u}\right)^2 = z$$

$$z = \left(1 - \frac{x+y}{2\sin u}\right)^2$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y} = \frac{1}{\sin u}\left(\frac{x+y}{2\sin u} - 1\right)$$

$$dz = \frac{1}{\sin u}\left(\frac{x+y}{2\sin u} - 1\right)(dx+dy)$$