Домашняя работа
Производные (часть 4)

№11.3.29

$$z = (5x^2y - y^3 + 7)^3$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

$$\frac{\partial z}{\partial x} = 3(5x^2y - y^3 + 7)^2 \cdot 10xy =$$
$$= 30xy(5x^2y - y^3 + 7)^2$$

$$\frac{\partial z}{\partial y} = 3(5x^2y - y^3 + 7)^2 \cdot (5x^2 - 3y^2) =$$
$$= 3(5x^2 - 3y^2)(5x^2y - y^3 + 7)^2$$

$$\frac{\partial z}{\partial x} = 750x^5y^3 - 300x^3y^5 + 2100x^3y^2 + 30xy^7 -$$
$$- 420xy^4 + 1470xy$$

$$\frac{\partial z}{\partial y} = 375x^6y^2 - 375x^4y^4 + 1050x^4y - 105x^2y^6 -$$
$$- 840x^2y^3 + 735x^2 - 9y^8 + 126y^5 - 441y^2$$

$$dz = 30xy(5x^2y - y^3 + 7)^2dx + 3(5x^2 - 3y^2)(5x^2y -$$
$$- y^3 + 7)^2dy$$

w 11.3.30

$$v = \operatorname{arctg} \frac{u}{t}$$

$$dv = \frac{\partial v}{\partial u} du + \frac{\partial v}{\partial t} dt$$

$$\frac{\partial v}{\partial u} = \frac{1}{1+\left(\frac{u}{t}\right)^2} \cdot \frac{1}{t} = \frac{1}{t + \frac{u^2}{t}} =$$

$$= \frac{1}{\frac{t^2+u^2}{t}} = \frac{t}{t^2+u^2}$$

$$\frac{\partial v}{\partial t} = \frac{1}{1+\frac{u^2}{t^2}} \cdot \left(-\frac{u}{t^2}\right) =$$

$$= -\frac{u}{t^2+u^2}$$

$$dv = \frac{t}{t^2+u^2} du - \frac{u}{t^2+u^2} dt =$$

$$= \frac{t\,du - u\,dt}{t^2+u^2}$$

W11.3.31

$$z = x\sqrt{y} + \frac{y}{\sqrt[3]{x}}$$

$$\frac{\partial z}{\partial x} = \sqrt{y} + y\left(x^{-\frac{1}{3}}\right)' = \sqrt{y} + y\cdot\left(-\frac{1}{3}\right)x^{-\frac{4}{3}} =$$

$$= \sqrt{y} - \frac{1}{3}yx^{-\frac{4}{3}} = \sqrt{y} - \frac{y}{3x\sqrt[3]{x}}$$

$$\frac{\partial z}{\partial y} = \frac{x}{2\sqrt{y}} + \frac{1}{\sqrt[3]{x}}$$

$$dz = \left(\sqrt{y} - \frac{y}{3x\sqrt[3]{x}}\right)dx + \left(\frac{x}{2\sqrt{y}} + \frac{1}{\sqrt[3]{x}}\right)dy$$

W11.3.32

$$z = \ln \operatorname{tg}\frac{x}{y}$$

$$\frac{\partial z}{\partial x} = \frac{1}{\operatorname{tg}\frac{x}{y}}\cdot\frac{1}{\cos^2\frac{x}{y}}\cdot\frac{1}{y} =$$

$$= \frac{1}{y\cos^2\frac{x}{y}\operatorname{tg}\frac{x}{y}}$$

$$\frac{\partial z}{\partial y} = \frac{1}{\operatorname{tg}\frac{x}{y}}\cdot\frac{1}{\cos^2\frac{x}{y}}\cdot\left(-\frac{x}{y^2}\right) =$$

$$z - \frac{x}{y^2\cos^2\frac{x}{y}\,tg\frac{x}{y}}$$

$$dz = \frac{dx - \frac{x}{y}dy}{y\cos^2\frac{x}{y}\,tg\frac{x}{y}}$$

№11.3.33

$$z = \sqrt{u+\sqrt{u^2+v^2}} = \left(u+(u^2+v^2)^{\frac{1}{2}}\right)^{\frac{1}{2}}$$

$$\frac{\partial z}{\partial u} = \frac{1}{\cancel{2}}\left(u+(u^2+v^2)^{\frac{1}{2}}\right)^{-\frac{1}{2}} \cdot$$

$$\cdot \left(1+\frac{1}{2}(u^2+v^2)^{-\frac{1}{2}}\right)\cdot \cancel{2}u =$$

$$= \left(u+(u^2+v^2)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(u+\frac{u}{2}(u^2+v^2)^{-\frac{1}{2}}\right)$$

$$\frac{\partial z}{\partial v} = \frac{1}{\cancel{2}}\left(u+(u^2+v^2)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\cdot\frac{1}{2}(u^2+v^2)^{-\frac{1}{2}}\cdot$$

$$\cdot\cancel{2}v = \frac{v}{2}\left(\left(u+(u^2+v^2)^{\frac{1}{2}}\right)(u^2+v^2)\right)^{-\frac{1}{2}}$$

$$dz = \left(u+(u^2+v^2)^{\frac{1}{2}}\right)^{-\frac{1}{2}}\left(u+\frac{u}{2}(u^2+v^2)^{-\frac{1}{2}}\right)du +$$

$$+\frac{v}{2}\left(\left(u+(u^2+v^2)^{\frac{1}{2}}\right)(u^2+v^2)\right)^{-\frac{1}{2}}dv$$

№11.3.34

$$z = \ln \frac{\sqrt{x^2+y^2}-x}{\sqrt{x^2+y^2}+x} = \ln\left(\sqrt{x^2+y^2}-x\right) -$$

$$- \ln\left(\sqrt{x^2+y^2}+x\right)$$

$$\frac{\partial z}{\partial x} = \left[\frac{1}{\sqrt{x^2+y^2}-x} \cdot \left(\frac{2x}{2\sqrt{x^2+y^2}} - 1 \right) = \right.$$

$$= \frac{x-\sqrt{x^2+y^2}}{\left(\sqrt{x^2+y^2}-x\right)\sqrt{x^2+y^2}} = \frac{x-\sqrt{x^2+y^2}}{x^2+y^2-x\sqrt{x^2+y^2}} \cdot$$

$$\cancel{\frac{\partial z}{\partial y}} \frac{1}{\sqrt{x^2+y^2}+x} \cdot \left(\frac{2x}{2\sqrt{x^2+y^2}} + 1 \right) =$$

$$\left. = \frac{x+\sqrt{x^2+y^2}}{x^2+y^2+x\sqrt{x^2+y^2}} \right] =$$

$$= \frac{x-\sqrt{x^2+y^2}}{x^2+y^2-x\sqrt{x^2+y}} - \frac{x+\sqrt{x^2+y^2}}{x^2+y^2+x\sqrt{x^2+y^2}}$$

$$\frac{\partial z}{\partial y} = \left[\frac{1}{\sqrt{x^2+y^2}-x} \cdot \frac{2y}{2\sqrt{x^2+y^2}} = \right.$$

$$= \frac{y}{x^2+y^2-x\sqrt{x^2+y^2}} \Bigg] =$$

$$= \frac{y}{x^2+y^2-x\sqrt{x^2+y^2}} - \frac{y}{x^2+y^2+x\sqrt{x^2+y^2}}$$

$$dz = \left(\frac{x-\sqrt{x^2+y^2}}{x^2+y^2-x\sqrt{x^2+y^2}} - \frac{x+\sqrt{x^2+y^2}}{x^2+y^2+x\sqrt{x^2+y^2}}\right)dx +$$

$$+\left(\frac{y}{x^2+y^2-x\sqrt{x^2+y^2}} - \frac{y}{x^2+y^2+x\sqrt{x^2+y^2}}\right)dy$$

№ 11.3.35

$$z = \arccos\frac{\sqrt{x^2-y^2}}{\sqrt{x^2+y^2}} = \arccos\sqrt{\frac{x^2-y^2}{x^2+y^2}}$$

$$\boxed{\frac{\partial z}{\partial x}} = \cancel{-\frac{1}{\left(1-\left(\frac{x^2-y^2}{x^2+y^2}\right)\right)^{\frac{1}{2}}}} - \left(1-\frac{x^2-y^2}{x^2+y^2}\right)^{-\frac{1}{2}} \cdot$$

$$\cdot\left(\frac{2x(x^2+y^2)-2x(x^2-y^2)}{(x^2+y^2)^2}\right)\cdot\frac{1}{2}\left(\frac{x^2-y^2}{x^2+y^2}\right)^{-\frac{1}{2}} =$$

$$= -\frac{1}{\cancel{2}}\left(1-\frac{x^2-y^2}{x^2+y^2}\right)^{\frac{1}{2}}\left(\frac{x^2-y^2}{x^2+y^2}\right)^{-\frac{1}{2}}\cdot\cancel{2}x\,\cdot$$

$$\cdot \frac{x^2+y^2-x^2+y^2}{(x^2+y^2)^2} =$$

$$= -2\frac{xy^2}{(x^2+y^2)^2}\left(\left(1-\frac{x^2-y^2}{x^2+y^2}\right)\left(\frac{x^2-y^2}{x^2+y^2}\right)\right)^{-\frac{1}{2}}$$

$$\boxed{\frac{\partial z}{\partial y}} = -\left(1-\frac{x^2-y^2}{x^2+y^2}\right)^{-\frac{1}{2}} \cdot \frac{1}{2}\left(\frac{x^2-y^2}{x^2+y^2}\right)^{-\frac{1}{2}} \cdot$$

$$\cdot \frac{2y(x^2+y^2)-2y(x^2-y^2)}{(x^2+y^2)^2} =$$

$$= -2\frac{y^3}{(x^2+y^2)^2}\left(\left(1-\frac{x^2-y^2}{x^2+y^2}\right)\left(\frac{x^2-y^2}{x^2+y^2}\right)\right)^{-\frac{1}{2}}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

№ 11.336

$$z = \sin\frac{x}{y} \cdot \cos\frac{y}{x}$$

$$\boxed{\frac{\partial z}{\partial y}} = \cos\frac{x}{y} \cdot \left(-\frac{x}{y^2}\right) \cdot \cos\frac{y}{x} -$$

$$- \sin\frac{x}{y}\sin\frac{y}{x} \cdot \frac{1}{x} =$$

$$= -\frac{x}{y^2}\cos\frac{x}{y}\cos\frac{y}{x} - \frac{1}{x}\sin\frac{x}{y}\sin\frac{y}{x}$$

$$\boxed{\frac{\partial z}{\partial x}} = \cos\frac{x}{y}\cdot\frac{1}{y}\cos\frac{y}{x} - \sin\frac{x}{y}\cdot\sin\frac{y}{x}\cdot$$

$$\cdot\left(-\frac{y}{x^2}\right) = \frac{1}{y}\cos\frac{x}{y}\cos\frac{y}{x} +$$

$$+\frac{y}{x^2}\sin\frac{x}{y}\sin\frac{y}{x}$$

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

N11.3.37

$$z = (x^2+y^2)\frac{1-\sqrt{x^2+y^2}}{1+\sqrt{x^2+y^2}}$$

$$\exists\ x^2+y^2 = t \Rightarrow$$

$$z = t\cdot\frac{1-\sqrt{t}}{1+\sqrt{t}}\cdot\frac{(1-\sqrt{t})}{(1-\sqrt{t})} =$$

$$= \frac{t(1-\sqrt{t})^2}{1-t} = \frac{t(1-2\sqrt{t}+t)}{1-t} =$$

$$= \frac{t-2t\sqrt{t}+t^2}{1-t} = \frac{t-2t\sqrt{t}+t^2+t+\sqrt{t}-t-\sqrt{t}}{1-t} =$$

$$= -\frac{t^2 - 2t\sqrt{t} + t - t - \sqrt{t}}{t-1} - \frac{t+\sqrt{t}}{t-1} =$$

$$= -\cancel{t} - \cancel{\sqrt{t}} - \frac{t+\sqrt{t}}{t-1}$$

$$t'_x = 2x \quad ; \quad t'_y = 2y$$

$$z'_x = -1 \cdot t'_x - \frac{1}{2\sqrt{t}} \cdot t'_x - \frac{(t+\sqrt{t})'_x (t-1)}{(t-1)^2} -$$

$$- \frac{(t+\sqrt{t})(t-1)'_x}{(t-1)^2} = \left[(t+\sqrt{t})'_x = 1 \cdot t'_x + \frac{1}{2\sqrt{t}} \cdot t'_x \; ; \; (t-1)'_x = 1 \cdot t'_x \right] =$$

$$= -t'_x - \frac{t'_x}{2\sqrt{t}} - \frac{\left(t'_x + \frac{t'_x}{2\sqrt{t}}\right)(t-1)}{(t-1)^2} -$$

$$- \frac{t'_x (t+\sqrt{t})}{(t-1)^2} = -2x - \frac{2x}{2\sqrt{x^2+y^2}} -$$

$$- \frac{\left(2x + \frac{2x}{2\sqrt{x^2+y^2}}\right)\cancel{(x^2+y^2-1)}}{\cancel{(x^2+y^2-1)^2}} -$$

$$- \frac{2x\left(x^2+y^2+\sqrt{x^2+y^2}\right)}{(x^2+y^2-1)^2} =$$

$$= -2x - \frac{x}{\sqrt{x^2+y^2}} - \frac{2x + \frac{x}{\sqrt{x^2+y^2}}}{x^2+y^2-1} -$$

$$- \frac{2x\left(x^2+y^2+\sqrt{x^2+y^2}\right)}{\left(x^2+y^2-1\right)^2} =$$

$$= \left(2x + \frac{x}{\sqrt{x^2+y^2}}\right)\left(-1 - \frac{1}{x^2+y^2-1}\right) -$$

$$- \frac{2x\left(x^2+y^2+\sqrt{x^2+y^2}\right)}{\left(x^2+y^2-1\right)^2}$$

Учитывая симметричность пер-ых x и y:

$$z'_y = \left(2y + \frac{y}{\sqrt{x^2+y^2}}\right)\left(-1 - \frac{1}{x^2+y^2-1}\right) -$$

$$- \frac{2y\left(x^2+y^2+\sqrt{x^2+y^2}\right)}{\left(x^2+y^2-1\right)^2}$$

$$dz = z'_x\,dx + z'_y\,dy$$

№11.3.38

$$u = x^3 + yz^2 + 3yx - x + z$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$\frac{\partial u}{\partial x} = 3x^2 + 3y - 1$$

$$\frac{\partial u}{\partial y} = z^2 + 3x$$

$$\frac{\partial u}{\partial z} = 2yz + 1$$

$$du = (3x^2 + 3y - 1)dx + (z^2 + 3x)dy + $$
$$+ (2yz + 1)dz$$

№ 11.3.39

$$u = x^{\frac{y}{z}}$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$\frac{\partial u}{\partial x} = \frac{y}{z} x^{\frac{y}{z} - 1}$$

$$\frac{\partial u}{\partial y} = x^{\frac{y}{z}} \ln x \cdot \frac{1}{z} = \frac{1}{z} x^{\frac{y}{z}} \ln x$$

$$\frac{\partial u}{\partial z} = x^{\frac{y}{z}} \ln x \cdot \left(-\frac{y}{z^2}\right) = -\frac{y}{z^2} \ln x \, x^{\frac{y}{z}}$$

$$du = \frac{y}{z} x^{\frac{y}{z} - 1} dx + \frac{1}{z} x^{\frac{y}{z}} \ln x \, dy - \frac{y}{z^2} x^{\frac{y}{z}} \ln x \, dz$$

№11.3.40

$u = x^{y^z}$

$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$

$\frac{\partial u}{\partial x} = y^z x^{(y^z-1)}$

$\frac{\partial u}{\partial y} = x^{y^z}\ln x \cdot z \cdot y^{z-1} = x^{y^z} y^{z-1} z \ln x$

$\frac{\partial u}{\partial z} = x^{y^z}\ln x \cdot y^z \ln y = \cancel{y}\, x^{y^z} y^z \ln x \ln y$

$du = y^z x^{(y^z-1)}dx + x^{y^z} y^{z-1} z \ln x\, dy + x^{y^z} y^z \ln x \ln y\, dz$

№11.3.41

$u = \ln(1+x+y^2+z^3)$

$u'_x = \frac{1}{1+x+y^2+z^3}$

$u'_y = \frac{1}{1+x+y^2+z^3} \cdot 2y$

$u'_z = \frac{1}{1+x+y^2+z^3} \cdot 3z^2$

$\cancel{u'_x}$

$x_0 = y_0 = z_0 = 1$

$u'_x(x_0, y_0, z_0) = \frac{1}{1+1+1+1} = \frac{1}{4} = 0,25$

$u'_y(x_0, y_0, z_0) = \frac{1}{1+1+1+1} \cdot 2 \cdot 1 = \frac{2}{4} = 0,5$

$u'_z(x_0, y_0, z_0) = \frac{1}{1+1+1+1} \cdot 3 \cdot 1^2 = \frac{3}{4} = 0,75$

$u'_x + u'_y + u'_z = 0,25 + 0,5 + 0,75 = 1,5$

№11.3.42

$x_0 = 1, \quad y_0 = 2$

$z = x^3 y - x y^3$

$z'_x = 3x^2 y - y^3; \quad z'_x(x_0, y_0) = 6 - 8 = -2$

$z'_y = x^3 - 3y^2 x; \quad z'_y(x_0, y_0) = 1 - 12 = -11$

$$\frac{z'_x + z'_y}{z'_x z'_y} = \frac{-2 - 11}{-2 \cdot (-11)} = -\frac{13}{22}$$

№11.3.43

$x_0 = 0, \quad y_0 = 0, \quad z_0 = \frac{\pi}{4}$

$u = \sqrt{\sin^2 x + \sin^2 y + \sin^2 z}$

$$\frac{\partial u}{\partial z} = \frac{2 \sin z \cos z}{2\sqrt{\sin^2 x + \sin^2 y + \sin^2 z}} = \frac{\sin 2z}{2\sqrt{\sin^2 x + \sin^2 y + \sin^2 z}}$$

$$\frac{\partial u}{\partial z}(x_0, y_0, z_0) = \frac{\sin 2 \cdot \frac{\pi}{4}}{2\sqrt{\sin^2 0 + \sin^2 0 + \sin^2 \frac{\pi}{4}}} =$$

$$= \frac{1}{2\cdot\sqrt{1+1+\frac{1}{4}}} = \frac{1}{2\cdot 1,5} = \frac{1}{3}$$

№11.3.47

$z = x + y - \sqrt{x^2+y^2}$

$x_0 = 3,\ y_0 = 4,\ \Delta x = 0,1,\ \Delta y = 0,2$

$z'_x = f(x+\Delta x; y) - f(x;y)$

$z'_x = f(3+0,1; 4) - f(3,4) =$

$= 3,1+4-\sqrt{3,1^2+4^2} - f(3;4) =$

$= 7,1 - \sqrt{25,61} - f(3;4) = 7,1 - 5,06 - f(3;4) =$

$= 2,04 - 3+4-\sqrt{3^2+4^2} = 2,04 - 2 = 0,04$

$z'_y = f(x; y+\Delta y) - f(x;y)$

$z'_y = f(3; 4+0,2) - f(3;4) =$

$= 3+4,2 - \sqrt{3^2+4,2^2} - 3 - 4 +$

$+ \sqrt{3^2+4^2} = 0,2 - 5,16 + 5 =$

$= 0,04$

$\Delta z = z'_x \cdot \Delta x + z'_y \cdot \Delta y$

$\Delta z = 0,04 \cdot 0,1 + 0,04 \cdot 0,2 =$

$= 0,012$

$dz = f(x_0; y_0) + \Delta z$

$dz = 2 + 0,012 = 2,012$

w11.3.45

$z = e^{xy}$

$x_0 = 1,\ y_0 = 1,\ \Delta x = 0{,}15,\ \Delta y = 0{,}1$

$z'_x = f(x_0 + \Delta x;\ y) - f(x;\ y)$

$z'_x = f(1 + 0{,}15;\ 1) - f(1;1) =$

$= e^{1{,}15 \cdot 1} - e^{1 \cdot 1} = e^{1{,}15} - e = e(e^{0{,}15} - 1)$

$z'_y = f(x;\ y + \Delta y) - f(x;\ y)$

$z'_y = f(1;\ 1 + 0{,}1) - f(1;1) =$

$= e^{1 \cdot 1{,}1} - e^{1 \cdot 1} = e^{1{,}1} - e = e(e^{0{,}1} - 1)$

$\Delta z = z'_x \Delta x + z'_y \Delta y$

$\Delta z = e(e^{0{,}15} - 1) \cdot 0{,}15 + e(e^{0{,}1} - 1) \cdot 0{,}1 =$

~~$= e(0{,}15(e^{0{,}15} -$~~

$= e(0{,}15e^{0{,}15} - 0{,}15 + 0{,}1e^{0{,}1} - 0{,}1) =$

$= e(0{,}15e^{0{,}15} + 0{,}1e^{0{,}1} - 0{,}25)$

~~w11.3.46~~ $dz = f(x_0;\ y_0) + \Delta z$

$dz = e + e(0{,}15e^{0{,}15} + 0{,}1e^{0{,}1} - 0{,}25) =$

$= e(0{,}15e^{0{,}15} + 0{,}1e^{0{,}1} + 0{,}75)$

w11.3.46

$z = \dfrac{x + 3y}{y - 3x}$; $x_1 = 2,\ x_2 = 2{,}5,\ y_1 = 4,\ y_2 = 3{,}5$

Изменение функции — это:

$\Delta z = f(x+\Delta x; y+\Delta y) - f(x;y)$

$\Delta x = x_2 - x_1 = 0{,}5$

$\Delta y = y_2 - y_1 = -0{,}5$

$f(x+\Delta x; y+\Delta y) = \frac{2{,}5+3\cdot 3{,}5}{3{,}5 - 3\cdot 2{,}5} =$

$= \frac{13}{-4} = -3{,}25$

$f(x;y) = \frac{2+3\cdot 4}{3-3\cdot 2} = \frac{14}{-3} \approx -4{,}67$

$\Delta z = -3{,}25 + 4{,}67 = 1{,}42$

№11.3.47

$\sqrt{1{,}02^3 + 1{,}97^3} = f(x,y) = f(x_0+\Delta x; y_0+\Delta y)$

$f(x,y) = \sqrt{x^3+y^3}$

$\exists\ x_0 = 1,\ y_0 = 2 \Rightarrow$

$\Rightarrow \Delta x = 0{,}02,\ \Delta y = -0{,}03$

$f(x_0+\Delta x; y_0+\Delta y) \approx f(x_0;y_0) + f'_x(x_0;y_0)\cdot$

$\cdot \Delta x + f'_y(x_0;y_0)\cdot \Delta y$

$$f'_x = \frac{1}{2\sqrt{x^3+y^3}} \cdot 3x^2 = \frac{3x^2}{2\sqrt{x^3+y^3}}$$

$$f'_y = \frac{1}{2\sqrt{x^3+y^3}} \cdot 3y^2 = \frac{3y^2}{2\sqrt{x^3+y^3}}$$

$$f'_x(x_0;y_0) = \frac{3\cdot 1^2}{2\sqrt{1^3+2^3}} = \frac{3}{2\cdot 3} = \frac{1}{2}$$

$$f'_y(x_0;y_0) = \frac{3\cdot 2^2}{2\sqrt{1^3+2^3}} = \frac{3\cdot 2}{3} = 2$$

$$f(x_0;y_0) = \sqrt{1^3+2^3} = 3$$

$$f(x,y) \approx 3 + 0{,}5\cdot 0{,}02 + 2\cdot(-0{,}03) =$$
$$= 3 + 0{,}01 - 0{,}06 = \underline{2{,}95}$$

№ 11.3.48

$$\sin 29^\circ \sin 46^\circ = f(x;y) \approx f(x_0+\Delta x; y_0+\Delta y)$$

$$f(x,y) = \sin x \sin y$$

$$\exists\ x_0 = 30^\circ = \frac{\pi}{6},\quad y_0 = 45^\circ = \frac{\pi}{4} \Rightarrow$$

$$\Rightarrow \Delta x = -1^\circ = -\frac{\pi}{180},\quad \Delta y = 1^\circ = \frac{\pi}{180}$$

$$f(x_0;y_0) = \sin\frac{\pi}{6}\sin\frac{\pi}{4} = \frac{1}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{4}$$

$$f'_x = \cos x \sin y;\quad f'_y = \cos y \sin x$$

$f'_x(x_0;y_0) = \cos\frac{\pi}{6}\sin\frac{\pi}{4} = \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2} = \frac{\sqrt{6}}{4}$

$f'_y(x_0;y_0) = \cos\frac{\pi}{4}\sin\frac{\pi}{6} = \frac{\sqrt{2}}{2}\cdot\frac{1}{2} = \frac{\sqrt{2}}{4}$

$f(x,y) \approx \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}\cdot\frac{\pi}{180} + \frac{\sqrt{2}}{4}\cdot\frac{\pi}{180} =$

$= 0{,}35 + 0{,}61\cdot 0{,}017 + 0{,}35\cdot 0{,}017 =$

$= \underline{0{,}36632}$

<u>№11.3.49</u>

$\mathrm{arctg}\left(\frac{1{,}97}{1{,}02} - 1\right) = f(x,y) = \mathrm{arctg}\left(\frac{x}{y} - 1\right)$

$] \; x_0 = 2, \; y_0 = 1 \Rightarrow$

$\Rightarrow \Delta x = -0{,}03, \; \Delta y = 0{,}02$

$f(x_0,y_0) = \mathrm{arctg}\left(\frac{2}{1} - 1\right) = \mathrm{arctg}\, 1 =$

$= \frac{\pi}{4} = 0{,}785$

$f'_x = \frac{1}{1+\left(\frac{x}{y}-1\right)^2}\cdot\frac{1}{y} = \frac{y^2}{y^2+(x-y)^2}\cdot\frac{1}{y} =$

$= \frac{y}{y^2+(x-y)^2}$

$f'_y = \frac{1}{1+\left(\frac{x}{y}-1\right)^2}\cdot\left(-\frac{x}{y^2}\right) =$

$$= \frac{y^2}{y^2+(x-y)^2} \cdot \left(-\frac{x}{y^2}\right) = -\frac{x}{y^2+(x-y)^2}$$

$$f'_x(x_0;y_0) = \frac{1}{1^2+(2-1)^2} = \frac{1}{2} = 0{,}5$$

$$f'_y(x_0;y_0) = -\frac{2}{1^2+(2-1)^2} = -\frac{2}{2} = -1$$

$$f(x,y) = 0{,}785 + 0{,}5\cdot(-0{,}03) - 1\cdot 0{,}02 =$$

$$= 0{,}75$$

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$$2{,}003^2 \cdot 3{,}998^3 \cdot 1{,}002^2 = f(x,y,z)$$

$$f(x,y,z) = x^2y^3z^2$$

$$\exists\ x_0 = 2,\ y_0 = 4,\ z_0 = 1 \Rightarrow$$

$$\Rightarrow \Delta x = 0{,}003,\ \Delta y = -0{,}002,\ \Delta z = 0{,}002$$

$$f(x_0,y_0,z_0) = 2^2 \cdot 4^3 \cdot 1^2 = 256$$

$$f'_x = 2xy^3z^2;\quad f'_x(x_0,y_0,z_0) = 256$$

$$f'_y = 3x^2y^2z^2;\quad f'_y(x_0,y_0,z_0) = 192$$

$$f'_z = 2x^2y^3z;\quad f'_z(x_0,y_0,z_0) = 512$$

$$f(x,y,z) = 256 + \overset{0{,}768}{256\cdot 0{,}003} + \overset{-0{,}384}{192\cdot(-0{,}002)} +$$

$$+ \overset{1{,}024}{512\cdot 0{,}002} = 257{,}408$$