Практика
(Производные, часть 1)

№ 11.3.1

$z = xy^2 - \frac{x}{y}$, $M_0(3; -2)$, $\Delta x = 0,1$,
$\Delta y = -0,05$

Т.к. $x_0 = 3$, $y_0 = -2 \Rightarrow x_0 + \Delta x = x = 3,1$;
$y_0 + \Delta y = y = -2,05$
$M_1(3,1; -2,05)$

$z(M_0) = z(3; -2) = 3(-2)^2 + \frac{3}{2} = 13,5$

$z(x_0 + \Delta x; y_0) = z(3,1; -2) = 3,1 \cdot (-2)^2 + \frac{3,1}{2} = 13,95$

$z(x_0; y_0 + \Delta y) = z(3; -2,05) = 3 \cdot (-2,05)^2 + \frac{3}{2,05} = 14,07$

$z(M_1) = z(x_0 + \Delta x; y_0 + \Delta y) =$
$= z(3,1; -2,05) = 3,1 \cdot (2,05)^2 +$
$+ \frac{3,1}{2,05} = 14,54$

$\Delta_x z = z(x_0 + \Delta x; y_0) - z(x_0; y_0) = 0,45$
$\Delta_y z = z(x_0; y_0 + \Delta y) - z(x_0; y_0) = 0,57$

$$\Delta z = z(x_0+\Delta x; y_0+\Delta y) - z(x_0; y_0) =$$
$$= 14{,}54 - 13{,}5 = 1{,}04$$

$$[\Delta z = 1{,}04 \neq 0{,}45 + 0{,}57 = 1{,}02 = \Delta_x z + \Delta_y z]$$

<u>№ 11.3.2</u>

$z = x^2 y$; $M_0(1; 2)$; $\Delta x = 0{,}1$; $\Delta y = -0{,}2$

$$\begin{matrix} x_0 = 1 \\ y_0 = 2 \end{matrix} \Rightarrow \begin{matrix} x_0 + \Delta x = 1{,}1 \\ y_0 + \Delta y = 1{,}8 \end{matrix} \Rightarrow M_1(1{,}1; 1{,}8)$$

$$z(M_0) = 1^2 \cdot 2 = 2$$

$$z(x_0+\Delta x; y_0) = z(1{,}1; 2) = (1{,}1)^2 \cdot 2 =$$
$$= 2{,}42$$

$$z(x_0; y_0+\Delta y) = z(1; 1{,}8) = 1^2 \cdot 1{,}8 =$$
$$= 1{,}8$$

$$z(M_1) = z(1{,}1; 1{,}8) = (1{,}1)^2 \cdot 1{,}8 =$$
$$= 2{,}178$$

$$\Delta_x z = z(x_0+\Delta x; y_0) - z(x_0; y_0) =$$
$$= 2{,}42 - 2 = 0{,}42$$

$$\Delta_y z = z(x_0; y_0+\Delta y) - z(x_0; y_0) =$$
$$= 1{,}8 - 2 = -0{,}2$$

$\Delta z = z(x_0+\Delta x; y_0+\Delta y) - z(x_0; y_0) =$

$= 2{,}178 - 2 = 0{,}178$

$[\Delta z = 0{,}178 \neq 0{,}22 = \Delta_x z + \Delta_y z]$ ✓

<u>№ 11.3.3.</u>

$z = \dfrac{x^2y^2}{x^2y^2-(x-y)^2}; M_0(2;2);$

$\Delta x = -0{,}2; \Delta y = 0{,}1$

$x_0 = 2 \quad\Rightarrow\quad x_0+\Delta x = 1{,}8 \quad\Rightarrow M_1(1{,}8; 2{,}1)$

$y_0 = 2 \qquad\quad y_0+\Delta y = 2{,}1$

$z(x_0;y_0) = z(M_0) = \dfrac{2^2\cdot 2^2}{2^2\cdot 2^2 - \underbrace{(2-2)^2}_{=0}} =$

$= 1$

$z(x_0+\Delta x; y_0) = z(1{,}8; 2) = \dfrac{1{,}8^2\cdot 2^2}{1{,}8^2\cdot 2^2-(1{,}8-2)^2} =$

$= \dfrac{1{,}8^2\cdot 2^2}{1{,}8^2\cdot 2^2 - 0{,}2^2} = \dfrac{12{,}96}{12{,}92} = 1{,}003$

$z(x_0; y_0+\Delta y) = z(2; 2{,}1) = \dfrac{2^2\cdot 2{,}1^2}{2^2\cdot 2{,}1^2-(2-2{,}1)^2} =$

$= \dfrac{17{,}64}{17{,}63} = 1{,}00057$

$z(x_0+\Delta x; y_0+\Delta y) = z(M_1) = z(1{,}8; 2{,}1) =$

$= \dfrac{1{,}8^2\cdot 2{,}1^2}{1{,}8^2\cdot 2{,}1^2-(1{,}8-2{,}1)^2} = \dfrac{14{,}2884}{14{,}1984} = 1{,}00634$

$\Delta_x z = 1{,}003 - 1 = 0{,}003$

$\Delta_y z = 1{,}00057 - 1 = 0{,}00057$

$\Delta z = 1{,}00634 - 1 = 0{,}00634$

$[\Delta z = 0{,}00634 \neq 0{,}00357 = \Delta_x z + \Delta_y z] \checkmark$

<u>№ 11.3.4</u>

$z = \left(\dfrac{x^2+y^2}{xy}\right)^2$; $M_0(1;1)$; $\Delta x = -0{,}1$;
$\Delta y = -0{,}1$

$\begin{matrix} x_0 = 1 \\ y_0 = 1 \end{matrix} \Rightarrow \begin{matrix} x_0 + \Delta x = 0{,}9 \\ y_0 + \Delta y = 0{,}9 \end{matrix} \Rightarrow M_1(0{,}9; 0{,}9)$

$z(M_0) = \left(\dfrac{1^2+1^2}{1\cdot 1}\right)^2 = 4$

$z(x_0+\Delta x; y_0) = \left(\dfrac{0{,}9^2+1^2}{0{,}9\cdot 1}\right)^2 = 3{,}6864$

$z(x_0; y_0+\Delta y) = \left(\dfrac{1^2+0{,}9^2}{1\cdot 0{,}9}\right)^2 = 3{,}6864$

$z(M_1) = \left(\dfrac{0{,}9^2+0{,}9^2}{0{,}9\cdot 0{,}9}\right)^2 = 4$

$\Delta_x z = 3{,}6864 - 4 = \cancel{0{,}6864}\ -0{,}3136$

$\Delta_y z = 3{,}6864 - 4 = -0{,}3136$

$\Delta z = 4 - 4 = 0$

$[\Delta z = 0 \neq -0{,}6272 = \Delta_x z + \Delta_y z] \checkmark$

№11.3.5

$z = 3x^2 + xy - y^2 + 1;\ M_0(2;1);$

$\Delta x = 0{,}1;\ \Delta y = 0{,}2$

$x_0 = 2$, $y_0 = 1$ $\Rightarrow$ $x_0 + \Delta x = 2{,}1$, $y_0 + \Delta y = 1{,}2$ $\Rightarrow M_1(2{,}1;1{,}2)$

$z(M_0) = 3 \cdot 2^2 + 2 \cdot 1 - 1^2 + 1 = 14$

$z(M_1) = 3 \cdot 2{,}1^2 + 2{,}1 \cdot 1{,}2 - 1{,}2^2 + 1 =$

$= 15{,}31$

$\Delta z = 15{,}31 - 14 = 1{,}31$

№11.3.6

$z = 3x^2 + xy - y^2 + 1;\ M_0(2;1);$

$\Delta x = 0{,}01;\ \Delta y = 0{,}02$

$x_0 = 2$, $y_0 = 1$ $\Rightarrow$ $x_0 + \Delta x = 2{,}01$, $y_0 + \Delta y = 1{,}02$ $\Rightarrow M_1(2{,}01;1{,}02)$

$z(M_0) = 3 \cdot 2^2 + 2 \cdot 1 - 1^2 + 1 = 14$

$z(M_1) = 3 \cdot 2{,}01^2 + 2{,}01 \cdot 1{,}02 - 1{,}02^2 + 1 =$

$= 14{,}1301$

$\Delta z = 14{,}1301 - 14 = 0{,}1301$

№11.3.7

$z = x^2 - xy + y^2$; $M_0(2;1)$; $M_1(2,1;1,2)$

$z(M_0) = 2^2 - 2\cdot 1 + 1^2 = 3$

$z(M_1) = 2,1^2 - 2,1\cdot 1,2 + 1,2^2 = 2,55$

$\Delta z = 2,55 - 3 = -0,45$

№11.3.8

$z = \lg(x^2+y^2)$; $M_0(2;1)$; $M_1(2,1;0,9)$

$z(M_0) = \lg(2^2+1^2) = \lg(5)$

$z(M_1) = \lg(2,1^2 + 0,9^2) = \lg(5,22)$

$\Delta z = \cancel{\lg(5)}\ \lg(5,22) - \lg(5) =$

$= \lg\frac{5,22}{5} = \lg 1,044$

№11.3.9

$$z = \frac{x}{y^3} + \frac{y}{x^3} - \frac{1}{6x^2y}$$

$$z'_x = \frac{1}{y^3}(x)' + y\left(\frac{1}{x^3}\right)' - \frac{1}{6y}\left(\frac{1}{x^2}\right)' =$$

$$= \frac{1}{y^3} - \frac{3y}{x^4} + \frac{1}{3x^3y}$$

$$z'_y = x\left(\frac{1}{y^3}\right)' + \frac{1}{x^3}(y)' - \frac{1}{6x^2}\left(\frac{1}{y}\right)' =$$

$$z = -\frac{3x}{y^4} + \frac{1}{x^3} + \frac{1}{6x^2y^2}$$

№ 11.3.10

$$z = \frac{x^2 - 2xy}{y^2 + 2xy + 1}$$

$$z'_x = \frac{(2x-2y)(y^2+2x+1) - (x^2-2xy)2y}{(y^2+2x+1)^2}$$

$$z'_y = \frac{-2x(y^2+2xy+1) - (2y-2x)(x^2-2xy)}{(y^2+2x+1)^2}$$

№ 11.3.11

$$z = e^{x^2+y^2}$$

$$z'_x = e^{x^2+y^2} \cdot (x^2+y^2)' = e^{x^2+y^2} \cdot 2x$$

$$z'_y = e^{x^2+y^2} \cdot (x^2+y^2)' = 2y\, e^{x^2+y^2}$$

№ 11.3.12

$$u = t^5 \sin^3 z$$

$$u'_z = t^5 \cdot 3\sin^2 z \cdot (\sin z)' = 3t^5 \sin^2 z \cdot$$
$$\cdot \cos z$$

$$u'_t = 5t^4 \sin^3 z$$

№ 11.3.13

$v = x^4 \cos^2 y - y^4 \sin^3 x^5$

$v'_x = 4x^3 \cos^2 y - y^4 \cdot 3\sin^2 x^5 \cdot$
$\cdot (\sin x^5)' \cdot (\cancel{1}x^5)' = 4x^3\cos^2 y -$
$- 15x^4 y^4 \sin^2 x^5 \cos x^5$

$v'_y = -2x^4 \cos y \sin y - 4y^3 \sin^3 x^5$

№ 11.3.14

$z = x^2 \cos 2xy - y^2 \sin(x+y)$

$z'_x = -2x \sin 2xy \cdot 2y - y^2 \cos(x+y) \cdot 1 =$
$= -4xy \sin 2xy - y^2 \cos(x+y)$

$z'_y = -2x^3 \sin 2xy - 2y \cos(x+y)$

№ 11.3.15

$u = x^y + (xy)^z + z^{xy}$

$u'_x = yx^{y-1} + z(xy)^{z-1} \cdot y + z^{xy} \ln z \cdot y =$
$= yx^{y-1} + yz(xy)^{z-1} + z^{xy} y \ln z$

$u'_y = x^y \ln x + xz(xy)^{z-1} + z^{xy} x \ln z$

$u'_z = (xy)^z \ln xy + xy \cdot z^{xy-1}$

№ 11.3.16

$z = \cos\frac{x^2+y^2}{x^3+y^3}$

Найти: z'_x, z'_y, $d_x z$, $d_y z$, dz

$z'_x = -\sin\frac{x^2+y^2}{x^3+y^3}\cdot\left(\frac{x^2+y^2}{x^3+y^3}\right)' =$

$= -\sin\frac{x^2+y^2}{x^3+y^3}\cdot\frac{2x(x^3+y^3)-3x^2(x^2+y^2)}{(x^3+y^3)^2}$

$z'_y = -\sin\frac{x^2+y^2}{x^3+y^3}\cdot\frac{2y(x^3+y^3)-3y^2(x^2+y^2)}{(x^3+y^3)^2}$

$z'_x = -\sin\frac{x^2+y^2}{x^3+y^3}\cdot\frac{-x^4-3x^2y^2+2xy^3}{(x^3+y^3)^2}$

$z'_y = -\sin\frac{x^2+y^2}{x^3+y^3}\cdot\frac{-y^4-3x^2y^2+2x^3y}{(x^3+y^3)^2}$

$d_x z = \frac{x^4+3x^2y^2-2xy^3}{(x^3+y^3)^2}\cdot\sin\frac{x^2+y^2}{y^3+x^3}dx$

$d_y z = \frac{y^4+3x^2y^2-2x^3y}{(x^3+y^3)^2}\cdot\sin\frac{x^2+y^2}{x^3+y^3}dy$

$dz = (x^3+y^3)^{-2}\sin\frac{x^2+y^2}{x^3+y^3}\cdot\big(x(x^3+3xy^2-$
$-2y^3)dx + y(y^3+3x^2y-2x^3)dy\big)$

№11.3.17

$$u = \frac{x}{\sqrt{y^2+z^2}}, \quad du = ?$$

$$u'_x = \frac{1}{\sqrt{y^2+z^2}}$$

$$u'_y = \frac{-xy}{\sqrt{(y^2+z^2)^3}}$$

$$u'_z = \frac{-xz}{\sqrt{(y^2+z^2)^3}}$$

$$du = \frac{dx}{\sqrt{y^2+z^2}} - \frac{xy\,dy + xz\,dz}{\sqrt{(y^2+z^2)^3}}$$

№11.3.18

$$1{,}07^{3{,}97} \approx \; ?$$

$$f(x;y) = x^y \Rightarrow x = 1{,}07, \; y = 3{,}97$$

$$f(1;4) = 1 \Rightarrow x_0 = 1, \; y_0 = 4$$

$$\Delta x = x - x_0 = 0{,}07$$

$$\Delta y = y - y_0 = -0{,}03$$

$$f(x+\Delta x; y+\Delta y) \approx f(x_0;y_0) +$$

$$+ df(x_0;y_0)$$

$$df(x_0;y_0) = f'_x(x_0;y_0)\Delta x + f'_y(x_0;y_0)\Delta y$$

$f'_x = yx^{y-1}$, $f'_y = x^y \ln x$,

$f'_x(1,4) = 4$, $f'_y(1;4) = 0$

$df(1;4) = 4 \cdot 0{,}07 + 0 \cdot (-0{,}03) = 0{,}28$

$\Rightarrow 1{,}07^{3,97} \approx 1 + 0{,}28 = 1{,}28$

№ 11.3.19

$1{,}04^{2,03} \approx ?$

$f(x;y) = x^y \Rightarrow x = 1{,}04;\ y = 2{,}03$

$f(1;2) = 1 \Rightarrow x_0 = 1,\ y_0 = 2$

$\Delta x = x - x_0 = 1{,}04 - 1 = 0{,}04$

$\Delta y = y - y_0 = 2{,}03 - 2 = 0{,}03$

$f(x+\Delta x;\ y+\Delta y) \approx f(x_0;y_0) +$

$+ f'_x(x_0;y_0)\Delta x + f'_y(x_0;y_0)\Delta y$

$f'_x = yx^{y-1}$, $f'_y = x^y \ln x$

$f'_x(1;2) = 2$, $f'_y(1;2) = 0$

$1{,}04^{2,03} \approx 1 + 2 \cdot 0{,}04 + 0 \cdot 0{,}03 = 1{,}08$

№ 11.3.20

$\sqrt{(1{,}04)^2 + (3{,}01)^2} \approx ?$

$f(x;y)=\sqrt{x^2+y^2} \Rightarrow x=1{,}04;\ y=3{,}01$

$\exists\ x_0=1;\ y_0=3 \Rightarrow$

$\Rightarrow f(x_0;y_0)=\sqrt{1^2+3^2}=\sqrt{10}$

$\Delta x=x-x_0=1{,}04-1=0{,}04$

$\Delta y=y-y_0=3{,}01-3=0{,}01$

$f'_x=\dfrac{1}{2\sqrt{x^2+y^2}}\cdot(x^2+y^2)'=$

$=\dfrac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2}}=\dfrac{x}{\sqrt{x^2+y^2}}$

$f'_y=\dfrac{y}{\sqrt{x^2+y^2}}$ [учитывая симметричность двух переменных]

$f'_x(1;3)=\dfrac{1}{\sqrt{1^2+3^2}}=\dfrac{1}{\sqrt{10}}=\dfrac{\sqrt{10}}{10}$

$f'_y(1;3)=\dfrac{3}{\sqrt{1^2+3^2}}=\dfrac{3}{\sqrt{10}}=\dfrac{3\sqrt{10}}{10}$

$\sqrt{(1{,}04)^2+(3{,}01)^2}\approx\sqrt{10}+\dfrac{\sqrt{10}}{10}\cdot 0{,}04+$

$+\dfrac{3\sqrt{10}}{10}\cdot 0{,}01=\sqrt{10}+\dfrac{4\sqrt{10}}{1000}+\dfrac{3\sqrt{10}}{1000}=$

$=\sqrt{10}+0{,}007\sqrt{10}=$

$=1{,}007\sqrt{10}\approx 3{,}184$

№ 11.3.21

$\sin 28^\circ \cdot \cos 61^\circ \approx ?$

$f(x;y) = \sin x \cos y \Rightarrow x = 28,\ y = 61$

$\exists\, x_0 = 30,\ y_0 = 60 \Rightarrow f(30;60) =$

$= \sin 30 \cos 60 = \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4} = 0{,}25$

$\Delta x = -2^\circ;\ \Delta y = 1^\circ \Rightarrow \Delta x = -0{,}035;\ \Delta y = 0{,}017$

$f'_x = \cos x \cos y;\quad f'_y = -\sin x \sin y$

$f'_x(30;60) = \cos 30 \cos 60 = \frac{\sqrt{3}}{2} \cdot \frac{1}{2} =$

$= \frac{\sqrt{3}}{4} \approx 0{,}433$

$f'_y(30;60) = -\sin 30 \sin 60 = -\frac{1}{2} \cdot \frac{\sqrt{3}}{2} =$

$= -\frac{\sqrt{3}}{4} \approx -0{,}433$

$\sin 28^\circ \cos 61^\circ \approx \cancel{\frac{1}{4} + \frac{\sqrt{3}}{4} \cdot (-2) +}$

$\cancel{+ \left(-\frac{\sqrt{3}}{4}\right) \cdot 1 = \frac{1}{4} - \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4} =}$

$\cancel{= \frac{1 - 3\sqrt{3}}{4} \approx -1{,}049}$

$\cancel{\approx 0{,}25 + 0{,}433 \cdot (-2) + (-0{,}433) \cdot 1 =}$

$\cancel{= -1{,}049}$?

$\approx 0{,}25 + 0{,}433 \cdot (-0{,}035) + (-0{,}433) \cdot 0{,}017 = 0{,}227484$ ✓

№ 11.3.22

$\sqrt{(\sin^2 1{,}55 + 8e^{0{,}015})^5} \approx ?$

1) $f(x;y) = (\sin^2 x + 8e^y)^{\frac{5}{2}}$

$x_0 = 1{,}571 = \frac{\pi}{2}$

$y_0 = 0$

$x = 1{,}55 \quad ; \quad \Delta x = 1{,}55 - 1{,}571 = -0{,}021$

$y = 0{,}015 \quad ; \quad \Delta y = 0{,}015$

2) $f(x_0;y_0) = (\sin^2\frac{\pi}{2} + 8e^0)^{\frac{5}{2}} =$ [redacted] 243

3) $f'_x = \frac{5}{2}(\sin^2 x + 8e^y)^{\frac{3}{2}} \cdot \sin 2x$

$f'_y = \frac{5}{2}(\sin^2 x + 8e^y)^{\frac{3}{2}} \cdot 8e^y$

$f'_x(x_0;y_0) = 0$ [т.к. $\sin 2x_0 = \sin\pi = 0$]

$f'_y(x_0;y_0) = 20(1+8)^{\frac{3}{2}} = 540$

$df(x_0;y_0) = 540 \cdot 0{,}015 = 8{,}1$

$\sqrt{(\sin^2 1{,}55 + 8e^{0{,}0015})^5} \approx 243 + 8{,}1 = 251{,}1$

№ 11.3.23

$\operatorname{arctg}\frac{1{,}02}{0{,}95} \approx ?$

$$f(x;y) = \operatorname{arctg}\frac{x}{y}$$

$x = 1{,}02$; $y = 0{,}95$

$\exists\ x_0 = 1,\ y_0 = 1 \Rightarrow \Delta x = 0{,}02$; $\Delta y = -0{,}05$

$$f(x_0;y_0) = \operatorname{arctg}\frac{1}{1} = \operatorname{arctg} 1 = 0{,}7854$$

$$f'_x = \frac{1}{y(1+x^2)} ;$$

$$f'_y = \frac{x}{1+\left(\frac{1}{y}\right)^2} \cdot \left(\frac{1}{y}\right)' = -\frac{x}{1+\frac{1}{y^2}} \cdot$$

$$\cdot \frac{1}{y^2} = -\frac{x \cdot y^2}{y^2+1} \cdot \frac{1}{y^2} = -\frac{x}{y^2+1}$$

$$f'_x(x_0;y_0) = \frac{1}{1(1+1^2)} = \frac{1}{2} = 0{,}5$$

$$f'_y(x_0;y_0) = -\frac{1}{1^2+1} = -\frac{1}{2} = -0{,}5$$

$$\operatorname{arctg}\frac{1{,}02}{0{,}95} \approx 0{,}7854 + 0{,}5 \cdot 0{,}02 - 0{,}5 \cdot (-0{,}05) =$$

$$= 0{,}8204$$

№ 11.3.24

$\sqrt{5e^{0,02}+2,03^2} \approx ?$

$f(x;y)=\sqrt{5e^x+y^2}=(5e^x+y^2)^{\frac{1}{2}}$

$\begin{cases} x=0,02 ;\ y=2,03 \\ x_0=0 ;\ y_0=2 \end{cases} \Rightarrow$

$\Rightarrow f(x_0;y_0)=\sqrt{5e^0+2^2}=\sqrt{9}=3$

$\Delta x=0,02$

$\Delta y=0,03$

$f'_x=\frac{1}{2}(5e^x+y^2)^{-\frac{1}{2}}\cdot 5e^x$

$f'_y=\frac{1}{2}(5e^x+y^2)^{-\frac{1}{2}}\cdot 2y$

$f'_x(x_0;y_0)=\frac{1}{2}(5e^0+2^2)^{-\frac{1}{2}}\cdot 5\cdot e^0=$

$=\frac{5}{2\cdot 3}=\frac{5}{6}\approx 0,83$

$f'_y(x_0;y_0)=\frac{1}{2}(5e^0+2^2)^{-\frac{1}{2}}\cdot 2\cdot 2=$

$=\frac{4}{6}\approx 0,67$

$\sqrt{5e^{0,02}+2,03^2}\approx 3+0,83\cdot 0,02+0,67\cdot$

$\cdot 0,03=3,0367$

№11.3.25

$\ln(0{,}09^3+0{,}99^3)\approx ?$

$f(x;y)=\ln(x^3+y^3)$

$x=0{,}09\ ;\ y=0{,}99$

$\exists\ x_0=0,\ y_0=1 \Rightarrow \Delta x=0{,}09;\ \Delta y=-0{,}01;$

$f(x_0;y_0)=\ln(0^3+1^3)=0$

$f'_x=\frac{1}{x^3+y^3}\cdot 3x^2=\frac{3x^2}{x^3+y^3}$

$f'_y=\frac{3y^2}{x^3+y^3}$

$f'_x(x_0;y_0)=\frac{3\cdot 0^2}{0^3+1^3}=0$

$f'_y(x_0;y_0)=\frac{3\cdot 1^2}{0^3+1^3}=3$

$\ln(0{,}09^3+0{,}99^3)\approx 0+0\cdot 0{,}09+3\cdot(-0{,}01)=$

$=-0{,}03$

№11.3.26

$\cos 2{,}36\cdot \operatorname{arctg} 0{,}97\cdot 3^{2{,}05}$

$f(x;y;z)=\cos x\ \operatorname{arctg} y\cdot 3^z$

$x_0=\frac{3\pi}{4}=2{,}356\ ;\ x=2{,}36;\ \Delta x=0{,}004$

$y_0=1;\ y=0{,}97;\ \Delta y=-0{,}03$

$z_0 = 2$; $z = 2,05$; $\Delta z = 0,05$

$f(x_0; y_0; z_0) = \cos\frac{3\pi}{4} \cdot \operatorname{arctg} 1 \cdot 3^2 =$

$= -\frac{9\sqrt{2}}{2} \cdot \frac{\pi}{4} \approx -4,9957$

$df = -\sin x \cdot \operatorname{arctg} y \cdot 3^z \Delta x +$

$+ \frac{\cos x \cdot 3^z}{1+y^2} \Delta y + \cos x \operatorname{arctg} y \cdot 3^z \cdot$

$\cdot \ln 3 \cdot \Delta z$

$df(x_0; y_0; z_0) = -9\frac{\sqrt{2}\pi}{8} \cdot 0,004 -$

$- \frac{9\sqrt{2}}{4} \cdot 0,03 - 9\ln 3 \frac{\sqrt{2}}{2}\frac{\pi}{4} \cdot 0,05 \approx$

$\approx -0,0199 - 0,0954 - 0,2744 = -0,3718$

$\cos 2,36 \cdot \operatorname{arctg} 0,97 \cdot 3^{2,06} \approx -4,9957 -$

$- 0,3718 = -5,3676$

№ 11.3.27

$1,002 \cdot 2,003^2 \cdot 3,004^3 \approx ?$

$f(x;y) = xy^2z^3$

$x = 1,002$; $y = 2,003$; $z = 3,004$

$\exists\ x_0 = 1,\ y_0 = 2,\ z_0 = 3 \Rightarrow$

$\Delta x = 0,002$; $\Delta y = 0,003$; $\Delta z = 0,004$;

$f(x_0; y_0; z_0) = 1 \cdot 2^2 \cdot 3^3 = 108$

$f'_x = y^2 z^3$

$f'_y = 2yxz^3$

$f'_z = 3z^2 x y^2$

$df = y^2 z^3 \Delta x + 2xyz^3 \Delta y + 3xy^2 z^2 \Delta z$

$df(x_0; y_0; z_0) = 2^2 \cdot 3^3 \cdot 0{,}002 + 2 \cdot 1 \cdot 2 \cdot 3^3 \cdot 0{,}003 + 3 \cdot 1 \cdot 2^2 \cdot 3^2 \cdot 0{,}004 = 0{,}972$

$1{,}002 \cdot 2{,}003^2 \cdot 3{,}004^3 \approx 108 + 0{,}972 = 108{,}972$

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$$\frac{1{,}03^2}{\sqrt[3]{0{,}98 \cdot \sqrt[4]{1{,}05^3}}} \approx ?$$

$$f(x; y; z) = \frac{x^2}{\sqrt[3]{y \cdot \sqrt[4]{z^3}}} = x^2 \left(y z^{\frac{3}{4}}\right)^{-\frac{1}{3}} = x^2 y^{-\frac{1}{3}} z^{-\frac{1}{4}}$$

$x = 1{,}03;\ y = 0{,}98,\ z = 1{,}05$

$\}\ x_0 = 1;\ y_0 = 1;\ z_0 = 1 \Rightarrow$

$\Rightarrow \Delta x = 0{,}03;\ \Delta y = -0{,}02;\ \Delta z = 0{,}05;$

$f(x_0; y_0; z_0) = 1$

$f'_x = 2x\,y^{-\frac{1}{3}} z^{-\frac{1}{4}}$

$f'_y = -\frac{1}{3} x^2 y^{-\frac{4}{3}} z^{-\frac{1}{4}}$

$f'_z = -\frac{1}{4} x^2 y^{-\frac{1}{3}} z^{-\frac{5}{4}}$

$$df = 2x y^{-\frac{1}{3}} z^{-\frac{1}{4}} \Delta x - \frac{1}{3} x^2 y^{-\frac{4}{3}} z^{-\frac{1}{4}} \Delta y - \frac{1}{4} x^2 y^{-\frac{1}{3}} z^{-\frac{5}{4}} \Delta z$$

$$df(x_0; y_0; z_0) = 2\cdot 1\cdot 1\cdot 1\cdot 0{,}03 - \frac{1}{3}\cdot 1\cdot 1\cdot 1\cdot (-0{,}02) - \frac{1}{4}\cdot 1\cdot 1\cdot 1\cdot 0{,}05 \approx 0{,}0542$$

$$\frac{1{,}03^2}{\sqrt[3]{0{,}98\cdot\sqrt[4]{1{,}05^3}}} \approx 1 + 0{,}0542 = 1{,}0542$$