$$\Rightarrow \begin{cases} A+B=2 \\ 2A-5B=-3 \end{cases} \Big| \cdot(-2) \Rightarrow + \begin{cases} -2A-2B=-4 \\ 2A-5B=-3 \end{cases} \Rightarrow$$

$$\Rightarrow -7B=-7 \Rightarrow B=1 \Rightarrow A+1=2 \Rightarrow A=1 \,]$$

$$= \int \left(\frac{1}{x-5} + \frac{1}{x+2} \right) dx =$$

$$= \int \frac{dx}{x-5} + \int \frac{dx}{x+2} = \int \frac{d(x-5)}{x-5} +$$

$$+ \int \frac{d(x+2)}{x+2} = \ln|x-5| + \ln|x-2| + C$$

<u>w8. 3.14</u>

$$\int \frac{x+2}{x^2-6x+5}\,dx = [\, x^2-6x+5=0 \Rightarrow$$

$$\Rightarrow D = 36-4\cdot5=16 \Rightarrow x_1 = \frac{6+4}{2}=5;$$

$$x_2 = \frac{6-4}{2} = 1 \Rightarrow x^2-6x+5=(x-5)(x-1)\,] =$$

$$= \int \frac{x+2}{(x-5)(x-1)}\,dx = [\, \frac{x+2}{(x-5)(x-1)} =$$

$$= \frac{A}{x-5} + \frac{B}{x-1} \Rightarrow x+2 = A(x-1)+B(x-5) =$$

$$= Ax - A + Bx - 5B = (A+B)x + (-A-5B) \Rightarrow$$

$$\Rightarrow + \begin{cases} A+B=1 \\ -A-5B=2 \end{cases} \Rightarrow -4B=3 \Rightarrow B=-\frac{3}{4} \Rightarrow$$

$$\Rightarrow A - \frac{3}{4} = 1 \Rightarrow A = \frac{7}{4} \,] = \int \left(\frac{7}{4(x-5)} - \right.$$

$$\left. - \frac{3}{4(x-1)} \right) dx = \frac{7}{4} \int \frac{dx}{x-5} - \frac{3}{4} \int \frac{dx}{x-1} =$$

$$= \frac{7}{4}\int \frac{d(x-5)}{x-5} - \frac{3}{4}\int \frac{d(x-1)}{x-1} =$$

$$= \frac{7}{4}\ln|x-5| - \frac{3}{4}\ln|x-1| + C$$

№ 8.3.15

$$\int \frac{dx}{x^4+x^2} = \Big[x^4+x^2=0 \Rightarrow x^2(x^2+1)=0 \Rightarrow$$

$\Rightarrow x^2=0$ или $x^2+1=0 \Rightarrow x=0\Big] =$

$$= \int \frac{dx}{x^2(x^2+1)} = \Big[\frac{1}{x^2(x^2+1)} = \cancel{\frac{A}{x^2} + \frac{B}{x}}$$

$$\cancel{+ \frac{C}{x^2+1}\Big|_{x^2} \Rightarrow 0x^2+0x+1 = A(x^2+1) + B(x^2+1)x + Cx^2 = Ax^2+A+(Bx^2+B)x + Cx^2}$$

$$= \frac{A}{x^2}\Big|_{x^2+1} + \frac{B}{x^2+1}\Big|_{x^2} \Rightarrow 0x^2+0x+1 =$$

$$= A(x^2+1) + Bx^2 = Ax^2 + A + Bx^2 =$$

$$= x^2(A+B) + A \Rightarrow$$

$$\Rightarrow \begin{cases} A+B=0 \\ A=1 \end{cases} \Rightarrow B=-1\Big] =$$

$$= \int \left(\frac{1}{x^2} - \frac{1}{x^2+1}\right) dx =$$

$$= \int \frac{dx}{x^2} - \int \frac{dx}{x^2+1} = -\frac{1}{x} - \mathrm{arctg}\, x + C$$

w8.3.16

$$\int \frac{x^5+x^4-8}{x^3-4x}\,dx = \int \frac{x^5+x^4-8}{x(x^2-4)}\,dx =$$

$$= \int \frac{x^5+x^4-8}{x(x-2)(x+2)}\,dx$$

$$\begin{array}{r|l}
x^5+x^4+0x^3+0x^2+0x-8 & x^3+0x^2-4x \\ \cline{2-2}
-\;x^5+0x^4-4x^3 \quad\quad\quad\quad\quad & x^2+x+4 \\ \cline{1-1}
x^4+4x^3+0x^2 \quad\quad\quad & \\
-\;x^4+0x^3-4x^2 \quad\quad\quad & \\ \cline{1-1}
4x^3+4x^2+0x \quad & \\
-\;4x^3+0x^2-16x \quad & \\ \cline{1-1}
4x^2+16x-8 &
\end{array}$$

$$\int \frac{x^5+x^4-8}{x^3-4x}\,dx = \int\left(\frac{(x^3-4x)(x^2+x+4)}{x^3-4x} + 4\,\frac{x^2+4x-2}{x^3-4x}\right)dx =$$

$$= \int (x^2+x+4)\,dx + 4\int \frac{x^2+4x-2}{x^3-4x}\,dx =$$

$$= [1)\quad \frac{x^2+4x-2}{x(x-2)(x+2)} = \frac{A}{x} + \frac{B}{x-2} + \frac{C}{x+2} \Rightarrow$$

$$\Rightarrow x^2+4x-2 = A(x^2-4) + Bx(x+2) +$$

$$+ Cx(x-2) = Ax^2 - 4A + Bx^2 + 2Bx +$$

$+Cx^2-2Cx = x^2(A+B+C)+x(2B-2C)+$

$-4A \Rightarrow$

$$\Rightarrow \begin{cases} A+B+C=1 \\ 2B-2C=4 \\ -4A=-2 \end{cases} \Leftrightarrow \begin{cases} A=\frac{1}{2} \\ \begin{cases} B+C=\frac{1}{2} \\ B-C=2 \end{cases} \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} B=C+2 \\ \cancel{B+B}\,C+2+C=\frac{1}{2} \end{cases} \Rightarrow$$

$\Rightarrow 2C=-\frac{3}{2} \Rightarrow C=-\frac{3}{4} \Rightarrow B=-\frac{3}{4}+2=$

$=\frac{5}{4} \Rightarrow A=\frac{1}{2},\ B=\frac{5}{4},\ C=-\frac{3}{4}\,]=$

$=\int(x^2+x+4)\,dx+4\int\left(\frac{1}{2x}+\frac{5}{4(x-2)}-\frac{3}{4(x+2)}\right)dx=$

$=\int x^2dx+\int x\,dx+4\int dx+\frac{4}{2}\int\frac{dx}{x}+\frac{4\cdot5}{4}\int\frac{dx}{x-2}-$

$-\frac{4\cdot3}{4}\int\frac{dx}{x+2}=$ [т.к. $dx=d(x+2)=d(x-2)$] $=$

$=\frac{x^3}{3}+\frac{x^2}{2}+4x+\frac{4}{2}\ln|x|+\frac{4\cdot5}{4}\ln|x-2|-\frac{4\cdot3}{4}\ln|x+2|+C=$

$=\frac{x^3}{3}+\frac{x^2}{2}+4x+2\ln|x|+5\ln|x-2|-3\ln|x+2|+C$

№ 8.3.17

$$\int \frac{dx}{x^3-8} = \int \frac{dx}{(x-2)(x^2+2x+4)} =$$

$$= \left[\frac{1}{(x-2)(x^2+2x+4)} = \frac{A}{x-2} + \frac{Bx+C}{x^2+2x+4} \Rightarrow \right.$$

$$\Rightarrow A(x^2+2x+4) + (Bx+C)(x-2) = 1 \Rightarrow$$

$$\Rightarrow 0x^2 + 0x + 1 = Ax^2 + 2Ax + 4A + Bx^2 + Cx - 2Bx - 2C = x^2(A+B) + x(2A+2B+C) + (4A-2C) \Rightarrow$$

$$\Rightarrow \begin{cases} A+B=0 \\ 2A+2B+C=0 \\ 4A-2C=1 \end{cases} \Rightarrow \begin{cases} A=-B \\ -2B+2B+C=0 \Rightarrow \\ C=0 \end{cases}$$

$$\Rightarrow \begin{cases} 4A=1, \ A=\frac{1}{4} \\ C=0 \\ B=-\frac{1}{4} \end{cases} \qquad \left. \right] =$$

$$= \int \left(\frac{1}{4(x-2)} + \frac{-\frac{1}{4}x+0}{x^2+2x+4} \right) dx =$$

$$= \frac{1}{4}\int \frac{d(x-2)}{x-2} - \frac{1}{4}\int \frac{x\,dx}{x^2+2x+4} =$$

$$= \frac{1}{4}\left(\int \frac{d(x-2)}{x-2} - \int \frac{\frac{1}{2}(2x+2) - \frac{1\cdot 2}{2}}{x^2+2x+4}\, dx \right) =$$

$$= \frac{1}{4}\left(\int\frac{d(x-2)}{x-2} - \frac{1}{2}\int\frac{(2x+2)dx}{x^2+2x+4} + \int\frac{dx}{x^2+2x+4}\right) =$$

$$= \left[\begin{matrix} 2) \; t = x^2+2x+4 \Rightarrow dt = (2x+2)dx \\ 3) \; y = x+1 \Rightarrow dy = dx, \quad a = \sqrt{4-\frac{4}{4}} = \sqrt{3} \end{matrix}\right] =$$

$$= \frac{1}{4}\left(\int\frac{d(x-2)}{x-2} - \frac{1}{2}\int\frac{dt}{t} + \int\frac{dy}{y^2+3}\right) =$$

$$= \frac{1}{4}\ln|x-2| - \frac{1}{8}\ln|t| + \frac{1}{4}\cdot\frac{1}{\sqrt{3}}\,\mathrm{arctg}\,\frac{y}{\sqrt{3}} + C =$$

$$= \frac{1}{4}\ln|x-2| - \frac{1}{8}\ln|x^2+2x+4| + \frac{\sqrt{3}}{12}\,\mathrm{arctg}\,\frac{x+1}{\sqrt{3}} + C.$$

<u>№ 8.3.18</u>

$$\int\frac{7x^3-10x^2+50x-77}{(x^2+9)(x^2+x-2)}dx =$$

$= [\; x^2+9=0 \Rightarrow$ корней нет; $x^2+x-2=0 \Rightarrow$

$\Rightarrow D = 1-4\cdot(-2) = 9 \Rightarrow x_1 = \frac{-1+3}{2} = 1;$

$x_2 = \frac{-1-3}{2} = -2 \Rightarrow x^2+x-2 = (x-1)(x+2)\;] =$

$$= \int\frac{7x^3-10x^2+50x-77}{(x^2+9)(x-1)(x+2)}dx =$$

$$= \left[\frac{7x^3-10x^2+50x-77}{(x^2+9)(x-1)(x+2)} = \frac{Ax+B}{x^2+9} + \frac{C}{x-1} + \frac{D}{x+2} \Rightarrow\right.$$

$\Rightarrow 7x^3-10x^2+50x-77 = (Ax+B)(x^2+x-2) +$

$+ C(x^2+9)(x+2) + D(x^2+9)(x-1) =$

$= Ax^3 + Bx^2 + Ax^2 + Bx - 2Ax - 2B +$

$+ C(x^3 + 9x + 2x^2 + 18) + D(x^3 + 9x - x^2 - 9) =$

$= Ax^3 + Ax^2 + Bx^2 - 2Ax + Bx - 2B +$

$+ Cx^3 + 2Cx^2 + 9Cx + 18C + Dx^3 - Dx^2 + 9Dx - 9D =$

$= x^3(A + C + D) + x^2(A + B + 2C - D) +$

$+ x(-2A + B + 9C + 9D) + (-2B + 18C - 9D) \Rightarrow$

$$\Rightarrow \begin{cases} A + C + D = 7 \\ A + B + 2C - D = -10 \\ -2A + B + 9C + 9D = 50 \\ -2B + 18C - 9D = -77 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} A = 7 - C - D \\ B = -10 - 2C + D - (7 - C - D) = -17 - C + 2D \\ -2(7 - C - D) + 2D - C - 17 + 9C + 9D = 50 \\ -2(2D - C - 17) + 18C - 9D = -77 \end{cases} \Rightarrow$$

$$\Rightarrow + \begin{cases} 10C + 13D = 81 \\ 20C - 13D = -111 \end{cases} \Rightarrow 30C = -30 \Rightarrow C = -1 \Rightarrow$$

$\cancel{\Rightarrow 10 \cdot 1 + 13D = 81}$ $\cancel{A = 7 - 1 - \frac{71}{13}}$

$\cancel{13D = 71} \cancel{\Rightarrow}$ $\cancel{A = \frac{7}{13}}$

$\cancel{D = \frac{71}{13}}$ $\cancel{B = -17 - 1 + \ldots}$

$$= \int \frac{\frac{7}{13}x - \frac{92}{13}}{x^2+9}\,dx +$$

$$+ \int \frac{dx}{x-1} + \int \frac{\frac{71}{13}}{x+2}\,dx =$$

$$= \frac{7}{13}\int \frac{x\,dx}{x^2+9} - \frac{92}{13}\int \frac{dx}{x^2+9} +$$

$$+ \int \frac{dx}{x-1} + \frac{71}{13}\int \frac{dx}{x+2} =$$

$$= \frac{7}{26}\int \frac{d(x^2+9)}{x^2+9} - \frac{92}{13}\int \frac{dx}{x^2+9} +$$

$$+ \int \frac{d(x-1)}{x-1} + \frac{71}{13}\int \frac{d(x+2)}{x+2} =$$

$$=$$

$$\Rightarrow -10 + 13D = 81$$

$$13D = 91 \Rightarrow$$

$$D = 7$$

$$A = 7+1-7 = 1$$

$$B = -17+1+2\cdot 7 = -2$$

$$= \int \frac{x-2}{x^2+9}\,dx + \int \frac{-dx}{x-1} + \int \frac{7\,dx}{x+2} =$$

$$= \int \frac{x\,dx}{x^2+9} - 2\int \frac{dx}{x^2+9} - \int \frac{d(x-1)}{x-1} +$$

$$+7\int\frac{d(x+2)}{x+2} = \frac{1}{2}\int\frac{2x\,dx}{x^2+9} - 2\int\frac{dx}{x^2+9} -$$

$$-\int\frac{d(x-1)}{x-1} + 7\int\frac{d(x+2)}{x+2} = \left[\text{т.к. } 2x\,dx = d(x^2+9)\right] =$$

$$= \frac{1}{2}\ln|x^2+9| - \frac{2}{3}\operatorname{arctg}\frac{x}{3} - \ln|x-1| +$$

$$+7\ln|x+2| + C$$