Интегралы (часть 2)

Домашняя работа.

№8.2.33

$$\int \cos(6x+1)\,dx = \left[t = 6x+1 \Rightarrow \quad dt = d(6x+1) = 6dx \Rightarrow dx = \frac{1}{6}dt \right] =$$

$$= \int \cos t \cdot \frac{1}{6}dt = \frac{1}{6}\int \cos t\,dt = \frac{1}{6}\sin t + C =$$

$$= \frac{1}{6}\sin(6x+1) + C$$

№8.2.34

$$\int \frac{dx}{\sqrt[3]{(5x-2)^4}} = \left[t = 5x-2 \Rightarrow dt = 5dx \Rightarrow \Rightarrow dx = \frac{1}{5}dt \right] =$$

$$= \int \frac{\frac{1}{5}dt}{t^{\frac{4}{3}}} = \frac{1}{5}\int t^{-\frac{4}{3}}dt = \frac{1}{5}\frac{t^{-\frac{1}{3}}}{-\frac{1}{3}} + C =$$

$$= -\frac{3}{5}t^{-\frac{1}{3}} + C = -\frac{3}{5\sqrt[3]{5x-2}} + C$$

№8.2.35

$$\int \frac{\sqrt{\operatorname{tg} x}\,dx}{\cos^2 x} = \cancel{\int (\operatorname{tg}^2 x + 1)\sqrt{\operatorname{tg} x}\,dx =}$$

$$\cancel{= \int \left(\operatorname{tg}^{\frac{5}{2}} x + \sqrt{\operatorname{tg} x}\right)dx} = \left[t = \operatorname{tg} x \Rightarrow dt = (\operatorname{tg} x)'dx = \frac{dx}{\cos^2 x} \right] =$$

$$= \int \sqrt{t}\,dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C = \frac{2}{3}t^{\frac{3}{2}} + C =$$

$= \frac{2}{3} tg^{\frac{3}{2}} x + C$

№ 8.2.36

$$\int \frac{e^x dx}{e^{2x}+9} = [t = e^x \Rightarrow dt = e^x dx] =$$

$$= \int \frac{dt}{t^2+9} = \frac{1}{3} arctg \frac{t}{3} + C =$$

$$= \frac{1}{3} arctg \frac{e^x}{3} + C$$

№ 8.2.37

$$\int \frac{x^5 dx}{\sqrt{x^6+7}} = [t = x^6 + 7 \Rightarrow dt = 6x^5 dx \Rightarrow$$

$$\Rightarrow \frac{1}{6} dt = x^5 dx] = \int \frac{\frac{1}{6} dt}{\sqrt{t}} =$$

$$= \frac{1}{6} \int \frac{dt}{\sqrt{t}} = \frac{1}{6} \cdot 2\sqrt{t} + C =$$

$$= \frac{1}{3} \sqrt{x^6+7} + C$$

№ 8.2.38

$$\int \frac{dx}{\arccos x \sqrt{1-x^2}} = [t = \arccos x \Rightarrow$$

$$\Rightarrow dt = -\frac{dx}{\sqrt{1-x^2}} \Rightarrow -dt = \frac{dx}{\sqrt{1-x^2}}] =$$

$$= \int \frac{-dt}{t} = -\int \frac{dt}{t} = -\ln|t| + C =$$

$$= -\ln|\arccos x| + C$$

№ 8. 2. 39

$$\int \frac{(2x+3)dx}{(x^2+3x-1)^4} = [t = x^2+3x-1 \Rightarrow$$

$$\Rightarrow dt = (2x+3)dx] =$$

$$= \int \frac{dt}{t^4} = \int t^{-4} dt = \frac{t^{-3}}{-3} + C =$$

$$= -\frac{1}{3}(x^2+3x-1)^{-3} + C$$

№ 8. 2. 40

$$\int \cos^{11} 2x \cdot \sin 2x \, dx = [t = \cos 2x \Rightarrow$$

$$\Rightarrow dt = (\cos 2x)' dx = -2\sin 2x \, dx \Rightarrow$$

$$\Rightarrow -\frac{1}{2} dt = \sin 2x \, dx] =$$

$$= \int t^{11} dt \cdot \left(-\frac{1}{2}\right) = -\frac{1}{2} \int t^{11} dt =$$

$$= -\frac{1}{2} \cdot \frac{t^{12}}{12} + C = -\frac{1}{24} \cos^{12} 2x + C$$

№ 8. 2. 41

$$\int \frac{7^{\sqrt{x}} dx}{\sqrt{x}} = [t = \sqrt{x} \Rightarrow dt = \frac{dx}{2\sqrt{x}} \Rightarrow$$

$$\Rightarrow 2dt = \frac{dx}{\sqrt{x}} \Big] = \int 7^t \cdot 2\,dt =$$

$$= 2\int 7^t dt = \frac{2\cdot 7^t}{\ln 7} + C =$$

$$= \frac{2\cdot 7^{\sqrt{x}}}{\ln 7} + C$$

<u>№8.2.42</u>

$$\int \frac{e^{\frac{1}{x}}}{x^2}dx = \Big[t = \frac{1}{x} \Rightarrow dt = -\frac{1}{x^2}dx \Rightarrow$$

$$\Rightarrow -dt = \frac{dx}{x^2} \Big] = -\int e^t dt = -e^t + C =$$

$$= -e^{\frac{1}{x}} + C$$

<u>№8.2.43</u>

$$\int \frac{\ln 5x\,dx}{x} = \Big[t = \ln 5x \Rightarrow dt = \frac{5}{x}dx \Rightarrow$$

$$\Rightarrow \frac{1}{5}dt = \frac{dx}{x} \Big] = \int \frac{1}{5} t\,dt =$$

$$= \frac{1}{5}\int t\,dt = \frac{t^2}{5\cdot 2} + C = \frac{\ln^2 5x}{10} + C$$

<u>№8.2.44</u>

$$\int ctg\,x\,dx = [\text{табличный}] =$$

$$= \ln|\sin x| + C$$

или

$= \int \frac{\cos x}{\sin x} dx = [t = \sin x \Rightarrow$

$\Rightarrow dt = \cos x \, dx] = \int \frac{dt}{t} =$

$= \ln|t| + C = \ln|\sin x| + C$

№8.2.45

$\int 4x \cdot \sqrt[3]{x^2+8} \, dx = [t = x^2+8 \Rightarrow$

$\Rightarrow dt = 2x \, dx] = \int 2 \cdot t^{\frac{1}{3}} dt =$

$= 2 \int t^{\frac{1}{3}} dt = 2 \frac{t^{\frac{4}{3}}}{\frac{4}{3}} + C =$

$= \frac{3}{2} t^{\frac{4}{3}} + C = \frac{3}{2} (x^2+8)^{\frac{4}{3}} + C$

№8.2.46

$\int \frac{\cos x \, dx}{\sin^2 x} = [t = \sin x \Rightarrow dt =$

$= \cos x \, dx] = \int \frac{dt}{t^2} = -\frac{1}{t} + C =$

$= -\frac{1}{\sin x} + C$

№8.2.47

$\int tg 2x \, dx = [t = 2x \Rightarrow dt = 2dx \Rightarrow$

$\Rightarrow \frac{1}{2} dt = dx] = \frac{1}{2} \int tg\, t \, dt =$

$= -\frac{1}{2} \ln|\cos t| + C = -\frac{1}{2} \ln|\cos 2x| + C$

w8.2.48

$$\int \frac{x\,dx}{x^4+1} = [t = x^2 \Rightarrow dt = 2x\,dx \Rightarrow$$

$$\Rightarrow \frac{1}{2}dt = x\,dx] = \frac{1}{2}\int \frac{dt}{t^2+1} =$$

$$= \frac{1}{2}\operatorname{arctg} t + C = \frac{1}{2}\operatorname{arctg} x^2 + C$$

w8.2.49

$$\int e^{-x^3} \cdot x^2\,dx = [t = -x^3 \Rightarrow$$

$$\Rightarrow dt = -3x^2\,dx \Rightarrow -\frac{1}{3}dt = x^2\,dx] =$$

$$= \int e^t \cdot \left(-\frac{1}{3}\right)dt = -\frac{1}{3}\int e^t\,dt =$$

$$= -\frac{1}{3}e^t + C = -\frac{1}{3}e^{-x^3}$$

w8.2.50

$$\int \frac{x^2\,dx}{\sqrt{x^6-4}} = [t = x^3 \Rightarrow dt = 3x^2\,dx \Rightarrow$$

$$\Rightarrow \frac{1}{3}dt = x^2\,dx] = \frac{1}{3}\int \frac{dt}{\sqrt{t^2-4}} =$$

$$= \frac{1}{3}\ln\left|t + \sqrt{t^2-4}\right| + C =$$

$$= \frac{1}{3}\ln\left|x^3 + \sqrt{x^6-4}\right| + C$$

№8.2.51

$$\int \left(8\cos\frac{x}{3}-5\right)^2 \sin\frac{x}{3}\,dx =$$

$$= \left[t = 8\cos\frac{x}{3} - 5 \Rightarrow dt = 8 \cdot \sin\frac{x}{3} \cdot \frac{1}{3}\,dx \Rightarrow \frac{3}{8}\,dt = \sin\frac{x}{3}\,dx\right] =$$

$$= \int t^2 \cdot \frac{3}{8}\,dt = \frac{3}{8}\int t^2\,dt =$$

$$= \frac{3}{8}\cdot\frac{t^3}{3} + C = \frac{t^3}{8} + C =$$

$$= \frac{1}{8}\left(8\cos\frac{x}{3}-5\right)^3 + C$$

№8.2.52

$$\int \frac{(3x^2-2x+7)\,dx}{\sqrt{x^3-x^2+7x-2}} = \left[t = x^3-x^2+7x-2 \Rightarrow\right.$$

$$\left.\Rightarrow dt = (3x^2 - \cancel{3}2x + 7)\,dx\right] =$$

$$= \int \frac{dt}{\sqrt{t}} = 2\sqrt{t} + C = 2\sqrt{x^3-x^2+7x-2} + C$$

№8.2.53

$$\int x(2x+1)^{35}\,dx = \left[t = 2x+1 \Rightarrow\right.$$

$$\Rightarrow dt = 2\,dx \Rightarrow \frac{1}{2}\,dt = dx \Rightarrow$$

$$\left.\Rightarrow x = \frac{t-1}{2}\right] = \int \frac{t-1}{2}\, t^{35} \cdot \frac{1}{2}\,dt =$$

$$= \frac{1}{4}\int (t-1)t^{35}\,dt = \frac{1}{4}\Big(\int t^{36}\,dt -$$

$$-\int t^{35}\,dt\Big) = \frac{1}{4}\left(\frac{t^{37}}{37} - \frac{t^{36}}{36}\right) + C =$$

$$= \frac{1}{148}(2x+1)^{37} - \frac{1}{144}(2x+1)^{36} + C$$

<u>W8.2.54</u>

$$\int (x-2)\sqrt{x+4}\,dx = [t = x+4 \Rightarrow$$

$$\Rightarrow dt = dx \Rightarrow x = t-4] =$$

$$= \int (t-4-2)\sqrt{t}\,dt = \int (t-6)\sqrt{t}\,dt =$$

$$= \int t^{\frac{3}{2}}\,dt - 6\int \sqrt{t}\,dt =$$

$$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 6\cdot\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C =$$

$$= \frac{2}{5}t^{\frac{5}{2}} - 4t^{\frac{3}{2}} + C = \frac{2}{5}(x+4)^{\frac{5}{2}} -$$

$$-4(x+4)^{\frac{3}{2}} + C$$

<u>W8.2.55</u>

$$\int \frac{3\sqrt{x} - 2\cos\frac{1}{x^2}}{x^3}\,dx =$$

$$= 3\int \frac{3\sqrt{x}}{x^3}\,dx - 2\int \frac{\cos\frac{1}{x^2}}{x^3}\,dx =$$

$$= 3\int x^{-\frac{5}{2}}dx - 2\int \frac{\cos\frac{1}{x^2}}{x^3}dx =$$

$$= \left[2)\ t = \frac{1}{x^2} \Rightarrow dt = -\frac{2}{x^3}dx \Rightarrow \right.$$
$$\left. \Rightarrow -\frac{1}{2}dt = \frac{dx}{x^3} \right] = 3\int x^{-\frac{5}{2}}dx -$$

$$-2\int \cos t \cdot \left(-\frac{1}{2}\right)dt =$$

$$= 3\int x^{-\frac{5}{2}}dx + \int \cos t\,dt =$$

$$= 3\frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + \sin t + C = -2x^{-\frac{3}{2}} +$$
$$+ \sin\frac{1}{x^2} + C$$

№ 8.2.56

$$\int \frac{7x+2}{\sqrt{x^2+10}}dx = 7\int \frac{x\,dx}{\sqrt{x^2+10}} + 2\int \frac{dx}{\sqrt{x^2+10}} =$$

$$= \frac{7}{2}\int \frac{d(x^2+10)}{\sqrt{x^2+10}} + 2\int \frac{dx}{\sqrt{x^2+10}} =$$

$$= \frac{7}{2}2\sqrt{x^2+10} + 2\ln\left|x + \sqrt{x^2+10}\right| + C =$$

$$= 7\sqrt{x^2+10} + 2\ln\left|x + \sqrt{x^2+10}\right| + C$$

№ 8.2.57

$$\int \frac{dx}{e^x+e^{-x}} = \int \frac{dx}{e^x+\frac{1}{e^x}} =$$

$$= \int \frac{dx}{\frac{e^{2x}+1}{e^x}} = \int \frac{e^x dx}{e^{2x}+1} =$$

$$= [t = e^{2x}+1 \Rightarrow dt = 2e^x dx \Rightarrow$$

$$\Rightarrow \frac{1}{2} dt = e^x dx] = \frac{1}{2}\int \frac{dt}{t} =$$

$$= \frac{1}{2}\ln|t| + C = \frac{1}{2}\ln|e^{2x}+1| + C$$

w8.2.58

$$\int \frac{x+8}{x^2+3} dx = \int \frac{x dx}{x^2+3} + 8\int \frac{dx}{x^2+3} =$$

$$= \frac{1}{2}\int \frac{d(x^2+3)}{x^2+3} + 8\int \frac{dx}{x^2+3} =$$

$$= \frac{1}{2}\ln|x^2+3| + 8\cdot\frac{1}{\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}} + C =$$

$$= \frac{1}{2}\ln|x^2+3| + \frac{8}{\sqrt{3}}\operatorname{arctg}\frac{x}{\sqrt{3}} + C$$

w8.2.59

$$\int \frac{x+4\sqrt{\arcsin x}}{\sqrt{1-x^2}} dx =$$

$$= \int \frac{\cancel{x}\,dx}{\sqrt{1-x^2}} + 4\int \frac{\sqrt{\arcsin x}\,dx}{\sqrt{1-x^2}} =$$

$$= \left[2)\ t = \arcsin x \Rightarrow dt = \frac{dx}{\sqrt{1-x^2}} \right] =$$

$$= -\frac{1}{2}\int \frac{d(1-x^2)}{\sqrt{1-x^2}} + 4\int \sqrt{t}\,dt =$$

$$= -\frac{1}{2}\cdot 2\sqrt{1-x^2} + 4\frac{t^{\frac{3}{2}}}{\frac{3}{2}} + C =$$

$$= -\sqrt{1-x^2} + \frac{8}{3}(\arcsin x)^{\frac{3}{2}} + C =$$

$$= \frac{8}{3}\arcsin^{\frac{3}{2}} x - \sqrt{1-x^2} + C$$

№ 8.2.60

$$\int \frac{1-6x}{(x+1)(x-1)}\,dx = \int \frac{dx}{x^2-1} - 3\int \frac{2x\,dx}{x^2-1} =$$

$$= \int \frac{dx}{x^2-1} - 3\int \frac{d(x^2-1)}{x^2-1} = \frac{1}{2}\ln\left|\frac{x-1}{x+1}\right| -$$

$$-3\ln|x^2-1| + C = \frac{1}{2}\left(\ln\left|\frac{x-1}{x+1}\right| -\right.$$

$$\left.-\ln(|x^2-1|)^6\right) + C = \frac{1}{2}\left(\ln\left|\frac{x-1}{(x+1)(x^2-1)^6}\right|\right) + C =$$

$$= \frac{1}{2}\ln\left|\frac{x-1}{(x+1)(x-1)^6(x+1)^6}\right| + C =$$

$$= \frac{1}{2}\ln\left|\frac{1}{(x+1)^7(x-1)^5}\right| + C =$$

$= \frac{1}{2}\left(-7\ln|x+1| - 5\ln|x-1|\right) + C =$

$= -\frac{7}{2}\ln|x+1| - \frac{5}{2}\ln|x-1| + C$

w 8.2.61

$\int (\cos^2 x - \sin^2 x)\sqrt[3]{1+\sin 2x}\,dx =$

$= \int \cos 2x \sqrt[3]{1+\sin 2x}\,dx =$

$= [\, t = 1+\sin 2x \Rightarrow dt = 2\cos 2x \Rightarrow$

$\Rightarrow \frac{1}{2}dt = \cos 2x\,dx \,] =$

$= \int \frac{1}{2}\sqrt[3]{t}\,dt = \frac{1}{2}\int t^{\frac{1}{3}}\,dt =$

$= \frac{1}{2}\,\frac{t^{\frac{4}{3}}}{\frac{4}{3}} + C = \frac{3}{8}(1+\sin 2x)^{\frac{4}{3}} + C$

w 8.2.62

$\int \frac{e^{tgx} - 7\sin x + 5\sin 2x}{\cos^2 x}\,dx =$

$= \int \frac{e^{tgx}}{\cos^2 x}\,dx - 7\int \frac{\sin x\,dx}{\cos^2 x} + 10\int \frac{\sin x\cos x}{\cos^2 x}\,dx$

$= \int \frac{e^{tgx}\,dx}{\cos^2 x} - 7\int \frac{\sin x\,dx}{\cos^2 x} + 10\int tgx\,dx =$

$= [\, 1)\ t = tgx \Rightarrow dt = \frac{dx}{\cos^2 x}\ ;\ 2)\ z = \cos x \Rightarrow$

$\Rightarrow dz = -\sin x\,dx \Rightarrow -dz = \sin x\,dx \,] =$

$$= \int e^t dt + 7\int \frac{dz}{z^2} + 10\int tgx\,dx =$$

$$= e^t - \frac{7}{z} - 10\ln|\cos x| + C =$$

$$= e^{tgx} - \frac{7}{\cos x} - 10\ln|\cos x| + C$$

<u>№ 8.2.63</u>

$$\int\sqrt{16 - x^2}\,dx = [x = 4\sin t] =$$

$$\cancel{= \int\sqrt{16 - 16\sin^2 t}\,4\cos t\,dt =}$$

$$\cancel{= \int\sqrt{15 + 1 - \sin^2 t}\,\cos t\,dt =}$$

$$\cancel{= \int\sqrt{15 + \cos^2 t}\,\cos t\,dt =}$$

$$\cancel{= 16\int\sqrt{16(1 - \sin^2 t)}\,\cos t\,dt}$$

$$\cancel{= 16\int 4\cos^2 t\,dt = 64\int\cos^2 t\,dt}$$

$$\cancel{= [z = \cos t \Rightarrow dz = -\sin t\,dt}$$

$$= \int\sqrt{16 - 16\sin^2 t}\cdot 4\cos t\,dt =$$

$$= 4\int\sqrt{16(1 - \sin^2 t)}\,\cos t\,dt =$$

$$= 4\int 4\cos^2 t\,dt = 16\int\cos^2 t\,dt =$$

$$= \left[t = \arcsin\frac{x}{4} \Rightarrow dt = \frac{d\left(\frac{x}{4}\right)}{\sqrt{1 - \frac{x^2}{16}}}\right] =$$

$$= 16\int\cos^2\arcsin\frac{x}{4}\cdot\frac{1}{\sqrt{1 - \frac{x^2}{16}}}\,d\left(\frac{x}{4}\right) =$$

$$= 16\int \left(1-\frac{x^2}{16}\right)\frac{d\left(\frac{x}{4}\right)}{\left(1-\frac{x^2}{16}\right)^{\frac{1}{2}}} =$$

$$= 16\int \left(1-\left(\frac{x}{4}\right)^2\right)^{\frac{1}{2}} d\left(\frac{x}{4}\right) =$$

$$= -8\int \left(1-\left(\frac{x}{4}\right)^2\right)^{\frac{1}{2}} d\left(1-\left(\frac{x}{4}\right)^2\right) =$$

$$= -\frac{16}{3}\left(1-\frac{x^2}{16}\right)^{\frac{3}{2}} + C$$

w8.2.64

$$\int \frac{dx}{1+\sqrt{x}} = [x = t^2 \Rightarrow dx = 2t\,dt] =$$

$$= 2\int \frac{t\,dt}{1+t} = 2\int \frac{t+1-1}{t+1}\,dt =$$

$$= 2\left(\int dt - \int \frac{dt}{t+1}\right) =$$

$$= 2\left(\int dt - \int \frac{d(t+1)}{t+1}\right) =$$

$$= 2t - 2\ln|t+1| + C = [t = \sqrt{x}] =$$

$$= 2\sqrt{x} - 2\ln|\sqrt{x}+1| + C$$

w8.2.65

$$\int x\sqrt{x+3}\,dx = [x = t-3 \Rightarrow$$

$$\Rightarrow dx = dt;\ t = x+3] =$$

$$= \int (t-3)\sqrt{t}\,dt = \int \left(t^{\frac{3}{2}} - 3\sqrt{t}\right)dt =$$

$$= \int t^{\frac{3}{2}}dt - 3\int t^{\frac{1}{2}}dt = \frac{2t^{\frac{5}{2}}}{5} - \frac{6\cdot t^{\frac{3}{2}}}{3} + C =$$

$$= \frac{2t^{\frac{5}{2}}}{5} - 2t^{\frac{3}{2}} + C = \frac{2(x+3)^{\frac{5}{2}}}{5} -$$

$$- 2(x+3)^{\frac{3}{2}} + C$$

<u>№8.2.66</u>

$$\int \frac{dx}{(x+1)\sqrt{x}} = \left[x = t^2 \Rightarrow dx = 2t\,dt \Rightarrow\right.$$

$$\left.\Rightarrow t = \sqrt{x}\right] = \int \frac{2t\,dt}{(t^2+1)t} = 2\int \frac{dt}{t^2+1} =$$

$$= 2\operatorname{arctg} t + C = 2\operatorname{arctg}\sqrt{x} + C$$

<u>№8.2.67</u>

$$\int \frac{x\,dx}{\sqrt{1-x}} = \left[x = 1-t \Rightarrow dx = -dt\right] =$$

$$= \int \frac{(1-t)\cdot(-dt)}{\sqrt{1-1+t}} = -\int \frac{1-t}{\sqrt{t}}\,dt =$$

$$= -\left(\int \frac{dt}{\sqrt{t}} - \int \sqrt{t}\,dt\right) = \int t^{\frac{1}{2}}dt - \int \frac{dt}{\sqrt{t}} =$$

$$= \frac{2}{3}t^{\frac{3}{2}} - 2\sqrt{t} + C = \frac{2}{3}(1-x)^{\frac{3}{2}} - 2\sqrt{1-x} + C$$

№8.2.68

$$\int \frac{x^2 dx}{\sqrt{1-x^2}} = \left[x = \sin t \Rightarrow dx = \cos t\, dt ;\ t = \arcsin x \right] =$$

$$= \int \frac{\sin^2 t \cos t\, dt}{\sqrt{1-\sin^2 t}} = \int \frac{\sin^2 t \cos t\, dt}{\cos t} =$$

$$= \int \sin^2 t\, dt = \cancel{\int \sin^2 \arcsin x\, d(\arcsin x) =}$$

$$\cancel{= \int \frac{x^2 dx}{\sqrt{1-x^2}}} = \int \frac{1}{2}(1-\cos 2t)\, dt =$$

$$= \frac{1}{2}\left(\int dt - \int \cos 2t\, dt\right) =$$

$$= \frac{1}{2}\left(\int dt - \frac{1}{2}\int \cos 2t\, d(2t)\right) =$$

$$= \frac{1}{2} t - \frac{1}{4} \sin 2t + C =$$

$$= \frac{1}{2} \arcsin x - \frac{1}{4} \cdot 2 \sin \arcsin x \cdot \cos \arcsin x + C =$$

$$= \frac{1}{2} \arcsin x - \frac{1}{2} x \sqrt{1-x^2} + C$$

№8.2.69

$$\int x \ln x\, dx = \left[\begin{matrix} u = \ln x \\ v' = x \end{matrix} \Rightarrow \begin{matrix} u' = \frac{1}{x} \\ v = \frac{x^2}{2} \end{matrix} \right] =$$

$$= \frac{x^2}{2} \ln x - \int \frac{x^2}{2x}\, dx =$$

$$= \frac{1}{2} x^2 \ln x - \frac{1}{2} \int x\, dx = \frac{1}{2} x^2 \ln x - \frac{1}{4} x^2 + C$$

№8.2.70

$$\int (2x+3)\cdot\cos x\,dx = \left[\begin{matrix} u = 2x+3 \\ v' = \cos x \end{matrix} \Rightarrow \begin{matrix} u' = 2 \\ v = \sin x \end{matrix}\right] = (2x+3)\sin x - \int 2\sin x\,dx = (2x+3)\sin x + 2\cos x + C = 2x\sin x + 3\sin x + 2\cos x + C$$

№8.2.71

$$\int x\cdot \mathrm{sh}\,5x\,dx = \left[\begin{matrix} u = x \\ v' = \mathrm{sh}\,5x \end{matrix} \Rightarrow \begin{matrix} u' = 1 \\ v = \frac{1}{5}\mathrm{ch}\,5x \end{matrix}\right] = \frac{1}{5}x\,\mathrm{ch}\,5x - \int \frac{1}{5}\mathrm{ch}\,5x\,dx = \frac{1}{5}x\,\mathrm{ch}\,5x - \frac{1}{25}\int \mathrm{ch}\,5x\,d(5x) = \frac{1}{5}x\,\mathrm{ch}\,5x - \frac{1}{25}\mathrm{sh}\,5x + C$$

№8.2.72

$$\int \frac{x\cdot\cos x\,dx}{\sin^3 x} = \cancel{\left[\begin{matrix} u = \frac{1}{\sin^3 x} \\ v' = x\cos x \end{matrix} \Rightarrow \begin{matrix} u' = \frac{3}{\sin^4 x} \\ v = \int x\cos x\,dx \end{matrix}\right]} = \int \frac{x\,\mathrm{ctg}\,x\,dx}{\sin^2 x} =$$

$$= \left[\begin{matrix} u = x \\ v' = \frac{\mathrm{ctg}\,x}{\sin^2 x} \end{matrix} \Rightarrow \begin{matrix} u' = 1 \\ v = \int \frac{\mathrm{ctg}\,x\,dx}{\sin^2 x} \end{matrix}\right. =$$

$$= \left[t = \mathrm{ctg}\,x \Rightarrow dt = -\frac{dx}{\sin^2 x} \Rightarrow -dt = \frac{dx}{\sin^2 x}\right] =$$

$$= -\int t\,dt = -\frac{1}{2}t^2 + C = -\frac{1}{2}\operatorname{ctg}^2 x + C \Big] =$$

$$= -\frac{1}{2}x\operatorname{ctg}^2 x - \int -\frac{1}{2}\operatorname{ctg}^2 x\,dx =$$

$$= -\frac{1}{2}x\operatorname{ctg}^2 x + \frac{1}{2}\int \operatorname{ctg}^2 x\,dx =$$

$$= \Big[\begin{matrix} x = \operatorname{arcctg} t \\ dx = -\frac{dt}{1+t^2} \end{matrix} \Rightarrow \int \operatorname{ctg}^2 x\,dx =$$

$$= -\int \frac{t^2}{1+t^2}\,dt = -\int \frac{1-1+t^2}{1+t^2}\,dt =$$

$$= -\int dt + \int \frac{dt}{t^2+1} = -t + \operatorname{arctg} t =$$

$$= \operatorname{arctg}(\operatorname{ctg} x) - \operatorname{ctg} x \Big] =$$

$$= -\frac{1}{2}x\operatorname{ctg}^2 x + \frac{1}{2}\operatorname{arctg}(\operatorname{ctg} x) - \frac{1}{2}\operatorname{ctg} x + C =$$

$$= -\frac{1}{2}\operatorname{ctg} x\,(x\operatorname{ctg} x + 1) + \frac{1}{2}\operatorname{arctg}(\operatorname{ctg} x) + C$$

w8.2.73

$$\int x^2 \ln x\,dx = \Big[\begin{matrix} u = \ln x \\ v' = x^2 \end{matrix} \Rightarrow \begin{matrix} u' = \frac{1}{x} \\ v = \frac{x^3}{3} \end{matrix} \Big] =$$

$$= \frac{1}{3}x^3 \ln x - \int \frac{1}{3}x^2\,dx = \frac{1}{3}x^3 \ln x -$$

$$- \frac{1}{3}\cdot\frac{x^3}{3} + C = \frac{1}{3}x^3\left(\ln x - \frac{1}{3}\right) + C$$

w8.2.74

$$\int (x^2-4x+1)e^{-x}dx = \left[\begin{matrix} u = x^2-4x+1 \\ v' = e^{-x} \end{matrix} \Rightarrow \begin{matrix} u' = 2x-4 \\ v = -e^{-x} \end{matrix} \right] = -(x^2-4x+1)e^{-x} +$$

$$+\int (2x-4)e^{-x}dx = \left[\begin{matrix} u = 2x-4 \\ v' = e^{-x} \end{matrix} \Rightarrow \begin{matrix} u' = 2 \\ v = -e^{-x} \end{matrix} \right] = -(x^2-4x+1)e^{-x} -$$

$$- (2x-4)e^{-x} + \int 2e^{-x}dx =$$

$$= (-x^2+4x-1)e^{-x} + (4-2x)e^{-x} -$$

$$-2e^{-x} + C = -e^{-x}(x^2-4x+1+2x-4+2) +$$

$$+ C = -e^{-x}(x^2-2x-1) + C$$

№8.2.75

$$\int x^3e^x dx = \left[\begin{matrix} u = x^3 \\ v' = e^x \end{matrix} \Rightarrow \begin{matrix} u' = 3x^2 \\ v = e^x \end{matrix} \right] =$$

$$= x^3e^x - \int 3x^2e^x dx = \left[\begin{matrix} u = x^2 \\ v' = e^x \end{matrix} \Rightarrow \begin{matrix} u' = 2x \\ v = e^x \end{matrix} \right] = x^3e^x - 3\left(x^2e^x - \right.$$

$$\left. - \int 2xe^x dx\right) = \left[\begin{matrix} u = x \\ v' = e^x \end{matrix} \Rightarrow \begin{matrix} u' = 1 \\ v = e^x \end{matrix} \right] =$$

$$= x^3e^x - 3x^2e^x + 6xe^x - 6\int e^x dx =$$

$$= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$$

№8.2.76

$$\int \frac{\arccos x\, dx}{\sqrt{1+x}} = \left[\begin{matrix} u = \arccos x \\ v' = \frac{1}{\sqrt{1+x}} \end{matrix} \Rightarrow \right.$$

$$u' = -\frac{1}{\sqrt{1-x^2}}$$
$$v = \int \frac{dx}{\sqrt{1+x}} = 2\sqrt{1+x} \Bigg] =$$

$$= 2\arccos x \sqrt{1+x} + \int \frac{2\sqrt{1+x}}{\sqrt{1-x^2}} dx =$$

$$= 2\arccos x \sqrt{1+x} + 2\int \frac{\sqrt{1+x}}{\sqrt{1-x}\sqrt{1+x}} dx =$$

$$= 2\arccos x \sqrt{1+x} + 2\int \frac{dx}{\sqrt{1-x}} =$$

$$= 2\arccos x \sqrt{1+x} - 2 \cdot 2\sqrt{1-x} + C =$$

$$= 2\arccos x \sqrt{1+x} - 4\sqrt{1-x} + C$$

№8.2.77

$$\int \frac{\arcsin\sqrt{x}}{\sqrt{1-x}} dx = \left[\begin{aligned} u &= \arcsin\sqrt{x} \\ v' &= \frac{1}{\sqrt{1-x}} \end{aligned} \right. \Rightarrow$$

$$\rightarrow \left. \begin{aligned} u' &= \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}\sqrt{1-x}} = \frac{1}{2\sqrt{x-x^2}} \\ v &= \int \frac{dx}{\sqrt{1-x}} = -2\sqrt{1-x} \end{aligned} \right] =$$

$$= -\arcsin\sqrt{x} \cdot 2\sqrt{1-x} - \int \left(-\frac{2\sqrt{1-x}}{2\sqrt{x}\sqrt{1-x}} \right) dx =$$

$$= -2\arcsin\sqrt{x}\sqrt{1-x} + \int \frac{dx}{\sqrt{x}} =$$

$$= -2\sqrt{1-x}\arcsin\sqrt{x} + 2\sqrt{x} + C$$

№ 8.2.78

$$\int \frac{x^2\,dx}{(x^2-1)^2} = \left[\begin{matrix} u = x \\ v' = \frac{x}{(x^2-1)^2} \end{matrix} \Rightarrow \begin{matrix} u' = 1 \\ v = \int \frac{x\,dx}{(x^2-1)^2} = \end{matrix} \right.$$

$$= \frac{1}{2}\int \frac{d(x^2-1)}{(x^2-1)^2} = -\frac{1}{2(x^2-1)} \Big] =$$

$$= -\frac{x}{2(x^2-1)} + \int \frac{dx}{2(x^2-1)} = \frac{1}{2}\cdot\frac{1}{2}\ln\left|\frac{x-1}{x+2}\right| -$$

$$- \frac{x}{2(x^2-1)} + C = \frac{1}{4}\ln\left|\frac{x-1}{x+1}\right| -$$

$$- \frac{x}{2(x^2-1)} + C$$

№8.2.79

$$\int \cos \ln x \, dx = \left[\begin{array}{l} u = \cos \ln x \\ v' = 1 \end{array} \Rightarrow \begin{array}{l} u' = -\sin \ln x \cdot \frac{1}{x} \\ v = x \end{array}\right] =$$

$$= x \cos \ln x + \int \sin \ln x \, dx =$$

$$= x \cos \ln x + x \sin \ln x - \int \cos \ln x \, dx$$

$$\int \cos \ln x \, dx = x \cos \ln x + x \sin \ln x - \\ - \int \cos \ln x \, dx$$

$$2\int \cos \ln x \, dx = x \cos \ln x + x \sin \ln x + C$$

$$\int \cos \ln x \, dx = \frac{x}{2}(\cos \ln x + \sin \ln x) + C$$

№8.2.80

$$\int e^{3x} \cdot \cos^2 x \, dx = \left[\begin{array}{l} u = \cos^2 x \\ v' = e^{3x} \end{array} \Rightarrow \begin{array}{l} u' = 2\cos x \sin x \\ v = \frac{1}{3} e^{3x} \end{array}\right] = \cancel{\frac{1}{3} e^{3x} \cos^2 x}$$

$$\cancel{\frac{2}{3} \int e^{3x} \sin x \cos x \, dx = \left[u = \cos x \right.}$$

② $\frac{1}{3}e^{3x}\cos^2 x - \int \frac{1}{3}e^{3x} \cdot 2\cos x \sin x\, dx =$

$= \frac{1}{3}e^{3x}\cos^2 x - \frac{1}{3}\int e^{3x} \cdot \sin 2x\, dx =$

$= \left[\begin{matrix} u = \sin 2x \\ v' = e^{3x} \end{matrix} \Rightarrow \begin{matrix} u' = \cos 2x \cdot 2 = 2\cos 2x \\ v = \frac{1}{3}e^{3x} \end{matrix}\right] =$

$= \frac{1}{3}e^{3x}\cos^2 x - \frac{1}{3}\left(\frac{1}{3}e^{3x}\sin 2x - \int \frac{2}{3}e^{3x}\cos 2x\, dx\right) =$

$= \left[\begin{matrix} u = \cos 2x \\ v' = e^{3x} \end{matrix} \Rightarrow \begin{matrix} u' = -2\sin 2x \\ v = \frac{1}{3}e^{3x} \end{matrix}\right] =$

$= \frac{1}{3}e^{3x}\cos^2 x - \frac{1}{3}\left(\frac{1}{3}e^{3x}\sin 2x - \right.$

$\left. - \frac{2}{3} \cdot \frac{1}{3}e^{3x}\cos 2x - \frac{2}{3}\int -\frac{2}{3}\sin 2x \cdot e^{3x}\, dx\right);$

$\int e^{3x}\sin 2x\, dx = \frac{1}{3}e^{3x}\sin 2x - \frac{2}{9}e^{3x} \cdot$

$\cdot \cos 2x + \frac{4}{9}\int e^{3x}\sin 2x\, dx$

$-\frac{5}{9}\int e^{3x}\sin 2x\, dx = \frac{1}{3}e^{3x}\left(\sin 2x - \frac{2}{3}\cos 2x\right) + C$

$\int e^{3x}\sin 2x\, dx = -\frac{3}{5}e^{3x}\left(\sin 2x - \frac{2}{3}\cos 2x\right) + C$

$\int e^{3x}\cos^2 x\, dx = \frac{1}{3}e^{3x}\cos^2 x + \frac{1}{5}e^{3x}\left(\frac{2}{2}\sin 2x - \right.$

$\left. - \frac{2}{3}\cos 2x\right) + C$

№8.2.81

$$\int e^{\sqrt{x}}\,dx = \left[\cancel{t=\sqrt{x} \Rightarrow dt=\frac{dx}{2\sqrt{x}}}\right.$$

$$\left. x=t^2 \Rightarrow dx = 2t\,dt\right] =$$

$$= \int e^t\, 2t\,dt = \left[\begin{matrix} u=t \\ v'=e^t\end{matrix} \Rightarrow \begin{matrix} u'=1 \\ v=e^t\end{matrix}\right] =$$

$$= 2te^t - 2\int e^t dt =$$

$$= 2te^t - 2e^t + C =$$

$$= 2e^t(t-1) + C = [t=\sqrt{x}] =$$

$$= 2e^{\sqrt{x}}(\sqrt{x}-1) + C$$

№8.2.82

$$\int \frac{x\,dx}{\cos^2 x} = \left[\begin{matrix} u=x \\ v'=\frac{1}{\cos^2 x}\end{matrix} \Rightarrow\right.$$

$$\left.\Rightarrow \begin{matrix} u'=1 \\ v=\operatorname{tg}x\end{matrix}\right] = x\operatorname{tg}x - \int \operatorname{tg}x\,dx =$$

$$= x\operatorname{tg}x + \ln|\cos x| + C$$

№8.2.83

$$\int x^3 e^{x^2} dx = \left[t=x^2 \Rightarrow \cancel{dt=2xdx}\right.$$

$$\left.\Rightarrow x=\sqrt{t} \Rightarrow dx = \frac{dt}{2\sqrt{t}}\right] =$$

$$= \int t^{\frac{3}{2}} e^t \frac{dt}{2\sqrt{t}} = 2\int t^2 e^t dt =$$

$$= \left[\begin{matrix} u = t^2 \\ v' = e^t \end{matrix} \Rightarrow \begin{matrix} u' = 2t \\ v = e^t \end{matrix}\right] =$$

$$= 2t^2 e^t - 4\int t e^t dt = \left[\begin{matrix} u = t \\ v' = e^t \end{matrix} \Rightarrow \begin{matrix} u' = 1 \\ v = e^t \end{matrix}\right] = 2t^2 e^t - 4te^t +$$

$$+ 4\int e^t dt = 2t^2 e^t - 4te^t + 4e^t + C =$$

$$= 2e^t(t^2 - 2t + 2) + C = 2e^{x^2}(x^4 - 2x^2 + 2) + C$$

N8.2.84

$$\int \ln(x + \sqrt{x^2+1})\,dx = \left[\begin{matrix} u = \ln(x+\sqrt{x^2+1}) \\ v' = 1 \end{matrix} \Rightarrow \begin{matrix} u' = \frac{1}{\sqrt{x^2+1}} \\ v = x \end{matrix}\right] = x\ln(x+\sqrt{x^2+1}) -$$

$$- \int \frac{x\,dx}{\sqrt{x^2+1}} = x\ln(x+\sqrt{x^2+1}) - \frac{1}{2}\cdot$$

$$\cdot \int \frac{d(x^2+1)}{\sqrt{x^2+1}} = x\ln(x+\sqrt{x^2+1}) - \frac{1}{2}\cdot$$

$$\cdot 2\sqrt{x^2+1} + C = x\ln(x+\sqrt{x^2+1}) -$$

$$- \sqrt{x^2+1} + C$$

W8.2.85

$$\int \sin 2x \cdot \ln \sin x \, dx = ②$$

$$\cancel{= \left[\begin{matrix} u = \ln \sin x \\ v' = \sin 2x \end{matrix} \Rightarrow \begin{matrix} u' = \frac{1}{\sin x} \cdot \cos x = ctg x \\ v = -\frac{1}{2}\cos 2x \end{matrix}\right] =}$$

$$\cancel{= -\frac{1}{2}\cos 2x \ln \sin x + \frac{1}{2}\int \cos 2x \, ctg x \, dx =}$$

$$② \left[t = \sin x \Rightarrow dt = \cos x \, dx\right] =$$

$$= 2\int t \ln t \, dt = \left[\begin{matrix} u = \ln t \\ v' = t \end{matrix} \Rightarrow \begin{matrix} u' = \frac{1}{t} \\ v = \frac{t^2}{2} \end{matrix}\right] = 2 \ln t \cdot \frac{t^2}{2} -$$

$$- 2\int \frac{1}{2} t \, dt = t^2 \ln t - \int t \, dt =$$

$$= t^2 \ln t - \frac{1}{2} t^2 + C = t^2\left(\ln t - \frac{1}{2}\right) + C =$$

$$= \sin^2 x \left(\ln \sin x - \frac{1}{2}\right) + C$$

W8.2.86

$$\int x^2 \arccos 3x \, dx = \left[\begin{matrix} u = \cancel{\cos}\arccos 3x \\ v' = x^2 \end{matrix} \Rightarrow \begin{matrix} u' = -\frac{3}{\sqrt{1-9x^2}} \\ v = \frac{1}{3}x^3 \end{matrix}\right] = \frac{1}{3}x^3 \arccos 3x +$$

$$+ \int \frac{x^3 \, dx}{\sqrt{1-9x^2}} = \cancel{\left[\begin{matrix} u = x^3 \\ v' = \frac{1}{\sqrt{1-9x^2}} \end{matrix} \Rightarrow\right]}$$

$$\int \frac{x^3dx}{\sqrt{1-9x^2}} = \frac{1}{2}\int \frac{x^2dx^2}{\sqrt{1-9x^2}} = -\frac{1}{162}\cdot$$

$$\cdot\int \frac{-9x^2 d(-9x^2)}{\sqrt{1-9x^2}} = -\frac{1}{162}\cdot\int \frac{1-9x^2-1}{\sqrt{1-9x^2}}\cdot$$

$$\cdot d(1-9x^2) = -\frac{1}{162}\Big(\int \sqrt{1-9x^2}\, d(1-9x^2) -$$

$$-\int \frac{d(1-9x^2)}{\sqrt{1-9x^2}}\Big) = -\frac{2}{3\cdot 162}(1-9x^2)^{\frac{3}{2}} +$$

$$+\frac{2}{162}\sqrt{1-9x^2} + C = \frac{1}{243}\Big(3\sqrt{1-9x^2} -$$

$$-(1-9x^2)^{\frac{3}{2}}\Big) + C = \frac{\sqrt{1-9x^2}}{243}(3-1-9x^2) =$$

$$= \frac{1}{243}\sqrt{1-9x^2}(2-9x^2) + C \Rightarrow$$

$$\int x^2 \arccos 3x\, dx = \frac{1}{3}x^3 \arccos 3x +$$

$$+\frac{1}{243}\sqrt{1-9x^2}(2-9x^2) + C$$

№ 8.2.87

$$\int x \sin\sqrt{x}\, dx = [t=\sqrt{x},\ x=t^2;$$
$$dx = 2tdt] = \int t^2 \sin t \cdot 2tdt =$$

$$= 2\int t^3 \sin t\,dt = \left[\begin{matrix} u = t^3 \\ v' = \sin t \end{matrix} \Rightarrow \begin{matrix} u' = 3t^2 \\ v = -\cos t \end{matrix}\right] = -2t^3\cos t +$$

$$+ 2\int 3t^2 \cos t\,dt = \left[\begin{matrix} u = t^2 \\ v' = \cos t \end{matrix} \Rightarrow \begin{matrix} u' = 2t \\ v = \sin t \end{matrix}\right] = -2t^3\cos t + 6t^2\sin t -$$

$$- 6\int 2t \sin t\,dt = \left[\begin{matrix} u = t \\ v' = \sin t \end{matrix} \Rightarrow \begin{matrix} u' = 1 \\ v = -\cos t \end{matrix}\right] = -2t^3\cos t + 6t^2\sin t +$$

$$+ 12\,t\cos t - 12\int \cos t\,dt =$$

$$= -2t^3\cos t + 6t^2\sin t + 12t\cos t - 12\sin t + C =$$

$$= -2(t^3 - 6t)\cos t + 6(t^2 - 2)\sin t + C =$$

$$= 6(t^2-2)\sin t - 2(t^3 - 6t)\cos t + C =$$

$$= 6(t^2-2)\sin t - 2t(t^2-6)\cos t + C$$

<u>№ 8.2.88</u>

$$\int \arcsin^2 x\,dx = \left[\begin{matrix} u = \arcsin^2 x \\ v' = 1 \end{matrix} \Rightarrow \begin{matrix} u' = 2\arcsin x \cdot \frac{1}{\sqrt{1-x^2}} \\ v = x \end{matrix}\right] =$$

$$= x\arcsin^2 x - 2\int \arcsin x \frac{x}{\sqrt{1-x^2}}\,dx =$$

$$= \left[\begin{matrix} u = \arcsin x \\ v' = \frac{x}{\sqrt{1-x^2}} \end{matrix} \Rightarrow \begin{matrix} u' = \frac{1}{\sqrt{1-x^2}} \\ v = -\sqrt{1-x^2} \end{matrix} \right] =$$

$$= x\arcsin^2 x + 2\arcsin x\sqrt{1-x^2} +$$

$$+ 2\int \frac{\sqrt{1-x^2}}{\sqrt{1-x^2}}dx = x\arcsin^2 x + 2 \cdot$$

$$\cdot \arcsin x\sqrt{1-x^2} + 2\int dx =$$

$$= x\arcsin^2 x + 2\arcsin x\sqrt{1-x^2} + 2x + C =$$

$$= \arcsin x\left(x\arcsin x + 2\sqrt{1-x^2}\right) + 2x + C$$

<u>№ 8.2.89</u>

$$\int \frac{\cos\sqrt{x}}{\sqrt{x}}dx = \left[x = t^2 \Rightarrow dx = 2tdt\right] =$$

$$= \int \frac{\cos t}{t} \cdot 2tdt = 2\int \cos t dt =$$

$$= 2\sin t + C = \left[t = \sqrt{x}\right] = 2\sin\sqrt{x} + C$$

<u>№ 8.2.890</u>

$$\int \operatorname{arctg}\sqrt{x}\,dx = \left[\begin{matrix} u = \operatorname{arctg}\sqrt{x} \\ v' = 1 \end{matrix} \Rightarrow \right.$$

$$\left. \Rightarrow \begin{matrix} u' = \frac{1}{1+x} \cdot \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{x}(1+x)} \\ v = x \end{matrix} \right] =$$

$$= x\operatorname{arctg}\sqrt{x} - \int \frac{xdx}{2\sqrt{x}(1+x)} =$$

$$= \left[\int \frac{x\,dx}{2\sqrt{x}(1+x)} = \int \frac{x\,d(\sqrt{x})}{1+x} = \int \frac{x+1-1}{x+1}\,d(\sqrt{x}) = \right.$$

$$= \int d\sqrt{x} - \int \frac{d(\sqrt{x})}{1+x} = \sqrt{x} -$$

$$\left. - \operatorname{arctg}\sqrt{x}\right] = x \operatorname{arctg}\sqrt{x} - \sqrt{x} +$$

$$+ \operatorname{arctg}\sqrt{x} + C = \operatorname{arctg}\sqrt{x}\,(x+1) - \sqrt{x} + C$$

№8.2.91

$$\int \frac{dx}{\sin x} = [\text{табличный}] =$$

$$= \ln\left|\operatorname{tg}\frac{x}{2}\right| + C$$

№8.2.92

$$\int \frac{\ln^2 x\,dx}{x\cdot\sqrt{3-\ln x}} = \left[t = \ln x \Rightarrow dt = \frac{dx}{x}\right] =$$

$$= \int \frac{t^2\,dt}{\sqrt{3-t}} = \left[\begin{array}{l} u = t^2 \\ v' = \frac{1}{\sqrt{3-t}} \end{array} \Rightarrow \right.$$

$$\left. \Rightarrow \begin{array}{l} u' = 2t \\ v = \int \frac{dt}{\sqrt{3-t}} = -2\sqrt{3-t} \end{array}\right] =$$

$$= -2t^2\sqrt{3-t} + \cancel{4}\int \frac{2t}{2\sqrt{3-t}}\,dt =$$

$$= -2t^2\sqrt{3-t} + \int \frac{t\,dt}{\sqrt{3-t}} =$$

$= -2t^2\sqrt{3-t} \cancel{+} \int\sqrt{3-t}\,dt + 3\int\frac{dt}{\sqrt{3-t}} =$

$= -2t^2\sqrt{3-t} + \frac{2}{3}(3-t)^{\frac{3}{2}} - 3\cdot 2\sqrt{3-t} + C =$

$= -2t^2\sqrt{3-t} + \frac{2}{3}(3-t)^{\frac{3}{2}} - 6\sqrt{3-t} + C =$

$= -2\sqrt{3-t}\,(t^2 - \frac{1}{3}(3-t) + 3) + C =$

$= -2\sqrt{3-t}\,(t^2 - 1 + \frac{1}{3}t + 3) + C =$

$= -2\sqrt{3-t}\,(t^2 + \frac{1}{3}t + 2) + C$

№8.2.93

$\int \frac{e^{arctg x} + 8x}{1+x^2}dx =$

$= \int\frac{e^{arctg x}\,dx}{1+x^2} + \int\frac{8x\,dx}{1+x^2} =$

$= [1)\ t = arctg x \Rightarrow dt = \frac{dx}{1+x^2}] =$

$= \int e^t dt + 4\int\frac{d(1+x^2)}{1+x^2} =$

$= e^t + 4\ln|1+x^2| + C = [$т.к. $1+x^2$ – параб. ветви вверх, то: $] =$

$= e^{arctg x} + 4\ln(1+x^2) + C$

№8.2.94

$$\int \frac{3x + 5\sin\left(\frac{1}{e^x}\right)}{e^x}\,dx =$$

$$= \int \frac{3x\,dx}{e^x} + \int \frac{5\sin\left(\frac{1}{e^x}\right)dx}{e^x} =$$

$$= \left[t = \frac{1}{e^x} \Rightarrow x = -\ln t ;\ dt = -\frac{dx}{e^x} \Rightarrow \right.$$

$$\left. \Rightarrow -dt = \frac{dx}{e^x} \right] = -3\int \ln t\,dt +$$

$$+ 5\int \sin t\,dt = \left[1) \begin{matrix} u = \ln t \\ v' = 1 \end{matrix} \Rightarrow \right.$$

$$\Rightarrow \begin{matrix} u' = \frac{1}{t} \\ v = t \end{matrix} \Rightarrow \int \ln t\,dt = t\ln t -$$

$$\left. - \int dt = t\ln t - t \right] =$$

$$= -3t(\ln t - 1) - 5\cos t + C =$$

$$= -\frac{3}{e^x}(-\ln e^x - 1) - 5 \cdot$$

$$\cdot \cos\frac{1}{e^x} + C = \frac{3}{e^x}(x + 1) - 5\cos\frac{1}{e^x} + C$$