Домашняя работа
Елкиной Галины
ИВТ, 1 курс, 3 п/г
"Обратная матрица"

№1. 4. 37

$$A = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 2 \\ 0 & 0,5 & 0 \end{pmatrix}$$

1) $\det A = 1 \cdot 2 \cdot 0,5 = 1 \neq 0 \Rightarrow A^{-1}$ сущ-ет.

2) $A_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 2 \\ 0,5 & 0 \end{vmatrix} = 0 - 2 \cdot 0,5 = -1$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = 0$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 0 \\ 0 & 0,5 \end{vmatrix} = 0$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 0 \\ 0,5 & 0 \end{vmatrix} = 0$

$A_{22} = (-1)^{2+2} \begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$A_{23} = (-1)^{2+3} \begin{vmatrix} -1 & 0 \\ \cancel{0,5} & 0,5 \end{vmatrix} = -0,5$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 0 & 0 \\ 0 & 2 \end{vmatrix} = 0$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} -1 & 0 \\ 0 & 2 \end{vmatrix} = -1\cdot 2 - 0 = +\cancel{-}2$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} -1 & 0 \\ 0 & 0 \end{vmatrix} = 0$$

3) $\tilde{A} = (A_{ij})^T$

$$\tilde{A} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & +2 \\ 0 & -0{,}5 & 0 \end{pmatrix}$$

4) $A^{-1} = \frac{1}{\det A}\cdot\tilde{A}$

$$A^{-1} = \frac{1}{1}\begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & +2 \\ 0 & -0{,}5 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & +2 \\ 0 & -0{,}5 & 0 \end{pmatrix}$$

<u>№ 1.4.38</u>

$$A = \begin{pmatrix} 1 & 1 & -1 \\ 8 & 3 & -6 \\ -4 & -1 & 3 \end{pmatrix}$$

1) $\det A = -1\cdot\overset{8}{(-1)\cdot 8} + \overset{9}{1\cdot 3\cdot 3} - \overset{\cancel{24}}{6\cdot(-4)\cdot 1} + \overset{-6}{1\cdot(-6)\cdot 1} + \underset{-12}{1\cdot 3\cdot(-4)} - \overset{=\cancel{24}}{1\cdot 8\cdot 3} = -1 \neq 0 \Rightarrow$

$\Rightarrow A^{-1}$ сущ-ет.

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2) $A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & -6 \\ -1 & 3 \end{vmatrix} = 9 - 6 = 3$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 8 & -6 \\ -4 & 3 \end{vmatrix} = 24 - 24 = 0$

$A_{13} = (-1)^{1+3} \begin{vmatrix} 8 & 3 \\ -4 & -1 \end{vmatrix} = -8 + 12 = 4$

$A_{21} = (-1)^{2+1} \begin{vmatrix} 1 & -1 \\ -1 & 3 \end{vmatrix} = -(3-1) = -2$

$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & -1 \\ -4 & 3 \end{vmatrix} = 3 - 4 = -1$

$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 1 \\ -4 & -1 \end{vmatrix} = -(-1+4) = -3$

$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 1 \\ 8 & 3 \end{vmatrix} = 3 - 8 = -5$

$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & -1 \\ 8 & -6 \end{vmatrix} = -(-6+8) = -2$

$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & -1 \\ 3 & -6 \end{vmatrix} = -6 + 3 = -3$

3) $\tilde{A} = (A_{ij})^T$

$$\tilde{A} = \begin{pmatrix} 3 & -2 & -3 \\ 0 & -1 & -2 \\ 4 & -3 & -5 \end{pmatrix}$$

4) $A^{-1} = \dfrac{1}{\det A} \tilde{A} = \dfrac{1}{-1} \begin{pmatrix} 3 & -2 & -3 \\ 0 & -1 & -2 \\ 4 & -3 & -5 \end{pmatrix} =$

$$= \begin{pmatrix} -3 & 2 & 3 \\ 0 & 1 & 2 \\ -4 & 3 & 5 \end{pmatrix}$$

№1.4.39

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 2 & -1 & 2 \\ 4 & 1 & 4 \end{pmatrix}$$

1) $\det A = \overset{4}{2 \cdot 1 \cdot 2} - \overset{-4}{1 \cdot 1 \cdot 4} + \overset{8}{1 \cdot 2 \cdot 4} - \overset{-2}{1 \cdot 2 \cdot 1} +$
$+ \overset{8}{1 \cdot 2 \cdot 4} - \overset{-8}{1 \cdot 2 \cdot 4} = 6 \neq 0 \Rightarrow A^{-1}$ сущ-ет

2) $A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 2 \\ 1 & 4 \end{vmatrix} = -4 - 2 = -6$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 2 \\ 4 & 4 \end{vmatrix} = -(8 - 8) = 0$

$A_{13} = \begin{vmatrix} 2 & -1 \\ 4 & 1 \end{vmatrix} = 2 + 4 = 6$

$A_{21} = -\begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = -(4 - 2) = -2$

$A_{22} = \begin{vmatrix} 1 & 2 \\ 4 & 4 \end{vmatrix} = 4 - 8 = -4$

$A_{23} = -\begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} = -(1 - 4) = 3$

$A_{31} = \begin{vmatrix} 1 & 2 \\ -1 & 2 \end{vmatrix} = 2 + 2 = 4$

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№1.4.39 (продолжение)

2) $A_{32} = -\begin{vmatrix} 1 & 2 \\ 2 & 2 \end{vmatrix} = -(2-4) = 2$

$A_{33} = \begin{vmatrix} 1 & 1 \\ 2 & -1 \end{vmatrix} = -1-2 = -3$

3) $\tilde{A} = \begin{pmatrix} -6 & -2 & 4 \\ 0 & -4 & 2 \\ 6 & 3 & -3 \end{pmatrix}$

4) $A^{-1} = \frac{1}{6} \begin{pmatrix} -6 & -2 & 4 \\ 0 & -4 & 2 \\ 6 & 3 & -3 \end{pmatrix} =$

$= \begin{pmatrix} -1 & -\frac{1}{3} & \frac{2}{3} \\ 0 & -\frac{2}{3} & \frac{1}{3} \\ 1 & \frac{1}{2} & -\frac{1}{2} \end{pmatrix}$

№1.4.40

$A = \begin{pmatrix} 3 & 4 & 2 \\ 2 & -4 & -3 \\ 1 & 5 & 1 \end{pmatrix}$

1) $\det A = \overset{20}{2\cdot 2\cdot 5} - \overset{-12}{4\cdot 3\cdot 1} - \overset{-12}{3\cdot 4\cdot 1} + \overset{45}{3\cdot 5\cdot 3} +$
$+ \overset{8}{4\cdot 2\cdot 1} - \overset{-8}{2\cdot 4\cdot 1} = 41 \neq 0 \Rightarrow A^{-1}$ сущ-ет.

2) $A_{11} = \begin{vmatrix} -4 & -3 \\ 5 & 1 \end{vmatrix} = -4 + 15 = 11$

$$A_{12} = -\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -(2+3) = -5$$

$$A_{13} = \begin{vmatrix} 2 & -4 \\ 1 & 5 \end{vmatrix} = 10 + 4 = 14$$

$$A_{21} = -\begin{vmatrix} 4 & 2 \\ 5 & 1 \end{vmatrix} = -(4 \cancel{+} -10) = 6$$

$$A_{22} = \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = 3 - 2 = 1$$

$$A_{23} = -\begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} = -(15 - 4) = -11$$

$$A_{33} = \begin{vmatrix} 3 & 4 \\ 2 & -4 \end{vmatrix} = -12 - 8 = -20$$

$$A_{32} = -\begin{vmatrix} 3 & 2 \\ 2 & -3 \end{vmatrix} = -(-9 - 4) = 13$$

$$A_{31} = \begin{vmatrix} 4 & 2 \\ -4 & -3 \end{vmatrix} = -12 + 8 = -4$$

3) $$\tilde{A} = \begin{pmatrix} 11 & 6 & -4 \\ -5 & 1 & 13 \\ 14 & -11 & -20 \end{pmatrix}$$

4) $$A^{-1} = \frac{1}{41}\begin{pmatrix} 11 & 6 & -4 \\ -5 & 1 & 13 \\ 14 & -11 & -20 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{11}{41} & \frac{6}{41} & -\frac{4}{41} \\ -\frac{5}{41} & \frac{1}{41} & \frac{13}{41} \\ \frac{14}{41} & -\frac{11}{41} & -\frac{20}{41} \end{pmatrix}$$

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№1.4.41

$$A = \begin{pmatrix} 3 & -1 & 2 \\ 4 & -3 & 3 \\ 1 & 3 & 0 \end{pmatrix}$$

1) $\det A = \overset{24}{4 \cdot 3 \cdot 2} + 0 \overset{-3}{- 1 \cdot 3 \cdot 1} - \overset{-27}{3 \cdot 3 \cdot 3} + \overset{6}{1 \cdot 3 \cdot 2} - 0 =$
$= 0 \Rightarrow A^{-1}$ не сущ-ет.

№1.4.42.

$$A = \begin{pmatrix} 5 & 8 & -1 \\ 2 & -3 & 2 \\ 1 & 2 & 3 \end{pmatrix}$$

1) $\det A = \overset{-4}{2 \cdot 2 \cdot (-1)} - \overset{-45}{3 \cdot 3 \cdot 5} + \overset{16}{8 \cdot 2 \cdot 1} - \overset{-20}{2 \cdot 2 \cdot 5} -$
$- \overset{-3}{1 \cdot (-3) \cdot (-1)} - \overset{-48}{2 \cdot 8 \cdot 3} = -104 \Rightarrow A^{-1}$ сущ-ет.

2) $A_{11} = \begin{vmatrix} -3 & 2 \\ 2 & 3 \end{vmatrix} = -9 - 4 = -13$

$A_{12} = -\begin{vmatrix} 2 & 2 \\ 1 & 3 \end{vmatrix} = -(6-2) = -4$

$A_{13} = \begin{vmatrix} 2 & -3 \\ 1 & 2 \end{vmatrix} = 4 + 3 = 7$

$A_{21} = -\begin{vmatrix} 8 & -1 \\ 2 & 3 \end{vmatrix} = -(24+2) = -26$

$A_{22} = \begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix} = 15 + 1 = 16$

$$A_{23} = -\begin{vmatrix} 5 & 8 \\ 1 & 2 \end{vmatrix} = -(10-8) = -2$$

$$A_{31} = \begin{vmatrix} 8 & -1 \\ -3 & 2 \end{vmatrix} = 16 \cancel{4} - 3 = 13$$

$$A_{32} = -\begin{vmatrix} 5 & -1 \\ 2 & 2 \end{vmatrix} = -(10+2) = -12$$

$$A_{33} = \begin{vmatrix} 5 & 8 \\ 2 & -3 \end{vmatrix} = -15-16 = -31$$

3) $$\tilde{A} = \begin{pmatrix} -13 & -26 & 13 \\ -4 & 16 & -12 \\ 7 & -2 & -31 \end{pmatrix}$$

4) $$A^{-1} = \frac{1}{-104}\begin{pmatrix} -13 & -26 & 13 \\ -4 & 16 & -12 \\ 7 & -2 & -31 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{13}{104} & +\frac{26}{104} & -\frac{13}{104} \\ +\frac{4}{104} & -\frac{16}{104} & \frac{12}{104} \\ -\frac{7}{104} & \frac{2}{104} & \frac{31}{104} \end{pmatrix} = \begin{pmatrix} \frac{1}{8} & \frac{1}{4} & -\frac{1}{8} \\ \frac{1}{26} & -\frac{2}{13} & \frac{3}{26} \\ -\frac{7}{104} & \frac{1}{52} & \frac{31}{104} \end{pmatrix}$$

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№1.4.43 $\det A = 1 \Rightarrow A^{-1}$ сущ-ет.

$$\left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ -1 & 2 & 1 & 0 & 1 & 0 \\ -1 & 1 & 2 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \text{II}+\text{I} \\ \text{III}+\text{I} \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right) \begin{array}{l} \text{I}+\text{II}+\text{III} \\ \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} 3 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}$$

№1.4.44

$$A = \begin{pmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{pmatrix}$$

$$\det A = \overset{45}{3\cdot 3\cdot 5} + \overset{54}{2\cdot 9\cdot 3} + \overset{28}{7\cdot 4\cdot 1} - \overset{-40}{2\cdot 5\cdot 4} - \overset{-27}{1\cdot 9\cdot 3} - \overset{-63}{3\cdot 7\cdot 3} = -3 \neq 0 \Rightarrow A^{-1} \text{ сущ-ет.}$$

$$A^{-1} = \left(\begin{array}{ccc|ccc} 2 & 7 & 3 & 1 & 0 & 0 \\ 3 & 9 & 4 & 0 & 1 & 0 \\ 1 & 5 & 3 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \mathrm{I} \leftrightarrow \mathrm{III} \end{array} \sim$$

$$\left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 3 & 9 & 4 & 0 & 1 & 0 \\ 2 & 7 & 3 & 1 & 0 & 0 \end{array}\right) \begin{array}{l} \\ \mathrm{II} - 3\mathrm{I} \sim \\ \mathrm{III} - 2\mathrm{I} \end{array}$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & -6 & -5 & 0 & 1 & -3 \\ 0 & -3 & -3 & 1 & 0 & -2 \end{array}\right) \begin{array}{l} \\ \\ \mathrm{III} - \frac{1}{2}\mathrm{II} \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & -6 & -5 & 0 & 1 & -3 \\ 0 & 0 & -\frac{1}{2} & 1 & -\frac{1}{2} & -\frac{1}{2} \end{array}\right) \begin{array}{l} \\ \cdot\left(-\frac{1}{6}\right) \\ \cdot(-2) \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & 1 & \frac{5}{6} & 0 & -\frac{1}{6} & \frac{1}{2} \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) \begin{array}{l} \\ \mathrm{II} - \frac{5}{6}\mathrm{III} \\ \\ \end{array} \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 5 & 3 & 0 & 0 & 1 \\ 0 & 1 & 0 & \frac{5}{3} & -1 & -\frac{1}{3} \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right) \begin{array}{l} \mathrm{I} - 5\mathrm{II} - 3\mathrm{III} \\ \sim \\ \\ \end{array}$$

$$\sim\left(\begin{array}{ccc|ccc} 1 & 0 & 0 & -\frac{7}{3} & 2 & -\frac{1}{3} \\ 0 & 1 & 0 & \frac{5}{3} & -1 & -\frac{1}{3} \\ 0 & 0 & 1 & -2 & 1 & 1 \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} -\frac{7}{3} & 2 & -\frac{1}{3} \\ \frac{5}{3} & -1 & -\frac{1}{3} \\ -2 & 1 & 1 \end{pmatrix}$$

<u>№1.4.45</u>

$$A = \begin{pmatrix} 1 & 2 & -2 & 4 \\ 2 & 6 & 1 & 0 \\ 3 & 0 & 1 & 2 \\ -1 & 4 & 5 & -4 \end{pmatrix} \leftarrow$$

$$\det A = (-1)^{2+1}\, 2 \begin{vmatrix} 2 & -2 & 4 \\ 0 & 1 & 2 \\ 4 & 5 & -4 \end{vmatrix} + (-1)^{2+2}\, 6 \cdot$$

$$\cdot \begin{vmatrix} 1 & -2 & 4 \\ 3 & 1 & 2 \\ -1 & 5 & -4 \end{vmatrix} + (-1)^{2+3}\, 1 \begin{vmatrix} 1 & 2 & 4 \\ 3 & 0 & 2 \\ -1 & 4 & -4 \end{vmatrix} =$$

$$= -2(0 - 8 - 16 \overset{120}{-20} - 16 - 0) +$$

$$+6(60 - 4 \overset{180}{+4} - 10 + 4 - 24) -$$

$$-(48 + 0 - 4 \overset{-60}{-8} - 0 + 24) = 240 \neq 0 \Rightarrow$$

$\Rightarrow A^{-1}$ сущ-ет.

$$A^{-1} \sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 2 & 6 & 1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 0 & 1 & 2 & 0 & 0 & 1 & 0 \\ -1 & 4 & 5 & -4 & 0 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \underline{II} - 2\underline{I} \\ \underline{III} - 3\underline{I} \\ \underline{IV} + \underline{I} \end{array} \sim$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 5 & -8 & -2 & 1 & 0 & 0 \\ 0 & -6 & 7 & -10 & -3 & 0 & 1 & 0 \\ 0 & 6 & 3 & 0 & 1 & 0 & 0 & 1 \end{array}\right) \begin{array}{l} \sim \\ \underline{III} + 3\underline{II} \\ \underline{IV} - 3\underline{II} \end{array}$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 2 & 5 & -8 & -2 & 1 & 0 & 0 \\ 0 & 0 & 22 & -34 & -9 & 3 & 1 & 0 \\ 0 & 0 & -12 & 24 & 7 & -3 & 0 & 1 \end{array}\right) \begin{array}{l} \\ \cdot\frac{1}{2} \\ \cdot\frac{1}{22} \\ \end{array} \sim$$

$$\sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{5}{2} & -4 & -1 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & -\frac{17}{11} & -\frac{9}{22} & \frac{3}{22} & \frac{1}{22} & 0 \\ 0 & 0 & -12 & 24 & 7 & -3 & 0 & 1 \end{array}\right) \begin{array}{l} \sim \\ \\ \underline{IV} + 12\underline{III} \end{array}$$

(12) $$\sim \left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{5}{2} & -4 & -1 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & -\frac{17}{11} & -\frac{9}{22} & \frac{3}{22} & \frac{1}{22} & 0 \\ 0 & 0 & 0 & \frac{60}{11} & \frac{23}{11} & -\frac{15}{11} & \frac{6}{11} & 1 \end{array}\right) \cdot\frac{11}{60} \sim$$

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№1.4.45 (продолжение)

$$\sim\left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{5}{2} & -4 & -1 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 1 & -\frac{17}{11} & -\frac{9}{22} & \frac{3}{22} & \frac{1}{22} & 0 \\ 0 & 0 & 0 & 1 & \frac{23}{60} & -\frac{1}{4} & \frac{1}{10} & \frac{11}{60} \end{array}\right) \begin{array}{l} I - 4\,IV \\ II + 4\,IV \\ III + \frac{17}{11}\,IV \end{array} \sim$$

$$\sim\left(\begin{array}{cccc|cccc} 1 & 2 & -2 & 0 & -\frac{8}{15} & 1 & -\frac{2}{5} & -\frac{11}{15} \\ 0 & 1 & \frac{5}{2} & 0 & \frac{8}{15} & -\frac{1}{2} & \frac{2}{5} & \frac{11}{15} \\ 0 & 0 & 1 & 0 & \frac{11}{60} & -\frac{1}{4} & \frac{1}{5} & \frac{17}{60} \\ 0 & 0 & 0 & 1 & \frac{23}{60} & -\frac{1}{4} & \frac{1}{10} & \frac{11}{60} \end{array}\right) \begin{array}{l} I + 2\,III \\ II - \frac{5}{2}\,III \end{array} \sim$$

$$\sim\left(\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & -\frac{1}{6} & \frac{1}{2} & 0 & -\frac{1}{6} \\ 0 & 1 & 0 & 0 & \frac{3}{40} & \frac{1}{8} & -\frac{1}{10} & \frac{1}{40} \\ 0 & 0 & 1 & 0 & \frac{11}{60} & -\frac{1}{4} & \frac{1}{5} & \frac{17}{60} \\ 0 & 0 & 0 & 1 & \frac{23}{60} & -\frac{1}{4} & \frac{1}{10} & \frac{11}{60} \end{array}\right) \begin{array}{l} I - 2\,II \end{array} \sim$$

$$\sim\left(\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & -\frac{19}{60} & \frac{1}{4} & \frac{1}{5} & -\frac{13}{60} \\ 0 & 1 & 0 & 0 & \frac{3}{40} & \frac{1}{8} & -\frac{1}{10} & \frac{1}{40} \\ 0 & 0 & 1 & 0 & \frac{11}{60} & -\frac{1}{4} & \frac{1}{5} & \frac{17}{60} \\ 0 & 0 & 0 & 1 & \frac{23}{60} & -\frac{1}{4} & \frac{1}{10} & \frac{11}{60} \end{array}\right)$$

$$A^{-1} = \begin{pmatrix} -\frac{19}{60} & \frac{1}{4} & \frac{1}{5} & -\frac{13}{60} \\ \frac{3}{40} & \frac{1}{8} & -\frac{1}{10} & \frac{1}{40} \\ \frac{11}{60} & -\frac{1}{4} & \frac{1}{5} & \frac{17}{60} \\ \frac{23}{60} & -\frac{1}{4} & \frac{1}{10} & \frac{11}{60} \end{pmatrix}$$

"Матричные уравнения"

№1.4.50

$$X \cdot \underbrace{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}}_{B}$$

1) $\det A = \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2 \Rightarrow A^{-1}$ сущ-ет.

2) $A^{-1} = \frac{1}{-2} \cdot \begin{pmatrix} 4 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}$

3) $X = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$

№1.4.51

$$X \cdot \underbrace{\begin{pmatrix} 4 & 3 \\ -5 & -4 \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{B}$$

1) $\det A = -16 + 15 = -1 \neq 0 \Rightarrow A^{-1}$ сущ.

2) $A^{-1} = \frac{1}{-1} \begin{pmatrix} -4 & -3 \\ 5 & 4 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ -5 & -4 \end{pmatrix}$

3) $X = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 4 & 3 \\ -5 & -4 \end{pmatrix} = \begin{pmatrix} 1 \cdot 4 & 1 \cdot 3 \\ -5 \cdot 1 & 1 \cdot (-4) \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ -5 & -4 \end{pmatrix}$

№1.4.52

$$\overset{A=}{\begin{pmatrix}1&1\\1&1\end{pmatrix}}\cdot X = \overset{B=}{\begin{pmatrix}2\\3\end{pmatrix}}$$

1) $\det A = 1-1=0 \Rightarrow A^{-1}$ не сущ-ет. $\Rightarrow$

$\Rightarrow$ решений нет

№1.4.53

$$\overset{A=}{\begin{pmatrix}1&1\\1&1\end{pmatrix}}\cdot X = \overset{B=}{\begin{pmatrix}2\\2\end{pmatrix}}$$

1) $\det A = 1-1=0 \Rightarrow A^{-1}$ не сущ-ет $\Rightarrow$

$\Rightarrow$ решений нет.

№1.4.54.

$$\overset{A=}{\begin{pmatrix}1&-1\\2&3\end{pmatrix}}\cdot X\cdot \overset{C=}{\begin{pmatrix}-5&6\\-4&5\end{pmatrix}} = \overset{B=}{\begin{pmatrix}1&-1\\2&3\end{pmatrix}}$$

$$X = A^{-1}\cdot B\cdot C^{-1}$$

1) $\det A = 3+2=5\neq 0 \Rightarrow A^{-1}$ сущ-ет.

$\det C = -25+24=-1\neq 0 \Rightarrow C^{-1}$ сущ-ет.

2) $A^{-1} = \frac{1}{5}\begin{pmatrix}3&1\\-2&1\end{pmatrix} = \begin{pmatrix}\frac{3}{5}&\frac{1}{5}\\-\frac{2}{5}&\frac{1}{5}\end{pmatrix}$

$C^{-1} = \frac{1}{-1}\begin{pmatrix}5&-6\\4&-5\end{pmatrix} = \begin{pmatrix}-5&6\\-4&5\end{pmatrix}$

3) $A^{-1}\cdot B = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{pmatrix} \begin{pmatrix} \cancel{3}1 & \cancel{6}-1 \\ 2 & 3 \end{pmatrix} =$

$= \begin{pmatrix} \frac{3}{5}\cdot 1+\frac{1}{5}\cdot 2 & \frac{3}{5}\cdot(-1)+\frac{1}{5}\cdot 3 \\ -\frac{2}{5}\cdot 1+\frac{1}{5}\cdot 2 & -\frac{2}{5}\cdot(-1)+\frac{1}{5}\cdot 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$A^{-1}\cdot B\cdot C^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix} = \begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix}$

$X = \begin{pmatrix} -5 & 6 \\ -4 & 5 \end{pmatrix}$

w1.4.55

$\overset{A=}{\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}} \cdot X \cdot \overset{C=}{\begin{pmatrix} 2 & -2 \\ -4 & 5 \end{pmatrix}} = \overset{B=}{\begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix}}$

1) $\det A = 3+2 = 5 \neq 0 \Rightarrow A^{-1}$ сущ-ет.

$\det C = 10-8 = 2 \neq 0 \Rightarrow C^{-1}$ сущ-ет.

2) $A^{-1} = \frac{1}{5}\begin{pmatrix} 3 & +1 \\ -2 & 1 \end{pmatrix} = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{pmatrix}$

$C^{-1} = \frac{1}{2}\begin{pmatrix} 2 & -2 \\ -4 & 5 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -2 & \frac{5}{2} \end{pmatrix}$

3) $A^{-1}B = \begin{pmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$A^{-1}BC^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -2 & \frac{5}{2} \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -2 & \frac{5}{2} \end{pmatrix}$

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$$X \cdot \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}}_{A} = \underbrace{\begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{pmatrix}}_{B}$$

$X = BA^{-1}$

1) $\det A = 1 \cdot 2 \cdot 3 = 6 \neq 0 \Rightarrow A^{-1}$ сущ-ет.

2) $A^{-1} = \frac{1}{\det A} \tilde{A}$

1. $A_{11} = \begin{vmatrix} 2 & 0 \\ 0 & 3 \end{vmatrix} = 2 \cdot 3 = 6$

$A_{12} = -\begin{vmatrix} 0 & 0 \\ 0 & 3 \end{vmatrix} = 0$

$A_{13} = \begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = 0$

$A_{21} = -\begin{vmatrix} 0 & 0 \\ 0 & 3 \end{vmatrix} = 0$

$A_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 3 \end{vmatrix} = 1 \cdot 3 = 3$

$A_{23} = -\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$A_{31} = \begin{vmatrix} 0 & 0 \\ 2 & 0 \end{vmatrix} = 0$

$A_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$A_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 2 \end{vmatrix} = 1 \cdot 2 = 2$

2. $\tilde{A} = \begin{pmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix}$

3. $A^{-1} = \dfrac{1}{6} \begin{pmatrix} 6 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 2 \end{pmatrix} =$

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$

3) $X = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{1}{3} \end{pmatrix}$

$X_{11} = 0 \cdot 1 + 0 \cdot 0 + 1 \cdot 0 = 0$

$X_{12} = 0 \cdot 0 + 0 \cdot \frac{1}{2} + 1 \cdot 0 = 0$

$X_{13} = 0 \cdot 0 + 0 \cdot 0 + 1 \cdot \frac{1}{3} = \frac{1}{3}$

$X_{21} = 0 \cdot 1 + 2 \cdot 0 + 0 \cdot 0 = 0$

$X_{22} = 0 \cdot 0 + 2 \cdot \frac{1}{2} + 0 \cdot 0 = 1$

$X_{23} = 0 \cdot 0 + 2 \cdot 0 + 0 \cdot \frac{1}{3} = 0$

$X_{31} = 3 \cdot 1 + 0 \cdot 0 + 0 \cdot 0 = 3$

$X_{32} = 3 \cdot 0 + \frac{1}{2} \cdot 0 + 0 \cdot 0 = 0$

$X_{33} = 3 \cdot 0 + 0 \cdot 0 + 0 \cdot \frac{1}{3} = 0$

$X = \begin{pmatrix} 0 & 0 & \frac{1}{3} \\ 0 & 1 & 0 \\ 3 & 0 & 0 \end{pmatrix}$

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$$\overset{A=}{\begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 0 & -2 & 1 \end{pmatrix}} \cdot X = \overset{B}{\begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}}$$

$$X = A^{-1} B$$

1) $\det A = -\overset{-12}{2 \cdot 2 \cdot 3} + \overset{3}{1 \cdot 3 \cdot 1} - \overset{0}{2 \cdot (-1) \cdot 0} - \overset{-2}{1 \cdot (-2) \cdot}$
$\cdot (-1) - \overset{0}{0 \cdot 3 \cdot 3} - \overset{4}{2 \cdot (-2) \cdot 1} = -7 \neq 0 \Rightarrow$
$\Rightarrow A^{-1}$ сущ-ет.

2) $A^{-1} = \frac{1}{\det A} \tilde{A}$

1. $A_{11} = \begin{vmatrix} 3 & -1 \\ -2 & 1 \end{vmatrix} = 3 - 2 = 1$

$A_{12} = -\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} = -(2 \cdot 1 - 0 \cdot (-1)) = -2$

$A_{13} = \begin{vmatrix} 2 & 3 \\ 0 & -2 \end{vmatrix} = 2 \cdot (-2) = -4$

$A_{21} = -\begin{vmatrix} -2 & 3 \\ -2 & 1 \end{vmatrix} = -(-2 \cdot 1 - 3 \cdot (-2)) = -4$

$A_{22} = \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} = 1$

$A_{23} = -\begin{vmatrix} 1 & -2 \\ 0 & -2 \end{vmatrix} = 2$

$A_{22} = |0 \quad 1|$

$$A_{33} = \begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix} = 3 + 4 = 7$$

$$A_{32} = -\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -(-1-6) = 7$$

$$A_{31} = \begin{vmatrix} -2 & 3 \\ 3 & -1 \end{vmatrix} = 2 - 9 = -7$$

2. $\tilde{A} = \begin{pmatrix} 1 & -4 & -7 \\ -2 & 1 & 7 \\ -4 & 2 & 7 \end{pmatrix}$

3. $A^{-1} = \frac{1}{-7} \begin{pmatrix} 1 & -4 & -7 \\ -2 & 1 & 7 \\ -4 & 2 & 7 \end{pmatrix} =$

$$= \begin{pmatrix} -\frac{1}{7} & \frac{4}{7} & 1 \\ \frac{2}{7} & -\frac{1}{7} & -1 \\ \frac{4}{7} & -\frac{2}{7} & -1 \end{pmatrix}$$

3) $X = \begin{pmatrix} -\frac{1}{7} & \frac{4}{7} & 1 \\ \frac{2}{7} & -\frac{1}{7} & -1 \\ \frac{4}{7} & -\frac{2}{7} & -1 \end{pmatrix} \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix} =$

$3\times3 \ 3\times1$
$= 3\times1$

$$\begin{pmatrix} -\frac{1}{7}\cdot 2 + \frac{4}{7}\cdot(-1) + 1\cdot 3 \\ \frac{2}{7}\cdot 2 - \frac{1}{7}\cdot(-1) - 1\cdot 3 \\ \frac{4}{7}\cdot 2 - \frac{2}{7}\cdot(-1) - 1\cdot 3 \end{pmatrix} = \begin{pmatrix} \frac{15}{7} \\ -\frac{16}{7} \\ -\frac{11}{7} \end{pmatrix}$$

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$$A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 3 & -1 \\ 0 & -2 & 1 \end{pmatrix} X \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 0 \end{pmatrix}$$

(над второй матрицей: $C =$, над третьей: $B =$)

$X = A^{-1} B C^{-1}$

1) $\det A = \overset{-12}{-2\cdot 2\cdot 3} + \overset{3}{1\cdot 3\cdot 1} + 0 - \overset{-2}{1\cdot(-1)\cdot(-2)} - 0 + \overset{4}{2\cdot 2\cdot 1} =$

$= -7 \neq 0 \Rightarrow A^{-1}$ сущ-ет.

$\det C = \overset{96}{4\cdot 8\cdot 3} + 0 + \overset{84}{2\cdot 6\cdot 7} - \overset{48}{8\cdot 6\cdot 1} - \overset{105}{7\cdot 5\cdot 3} - 0 =$

$= 27 \neq 0 \Rightarrow C^{-1}$ сущ-ет.

2) $A^{-1} = \frac{1}{\det A} \tilde{A}$

1. $A_{11} = \begin{vmatrix} 3 & -1 \\ -2 & 1 \end{vmatrix} = 3 - 2 = 1$

$A_{12} = -\begin{vmatrix} 2 & -1 \\ 0 & 1 \end{vmatrix} = -(2-0) = -2$

$A_{13} = \begin{vmatrix} 2 & 3 \\ 0 & -2 \end{vmatrix} = -4$

$A_{21} = -\begin{vmatrix} -2 & 3 \\ -2 & 1 \end{vmatrix} = -(-2+6) = -4$

$A_{22} = \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} = 1$

$$A_{23} = -\begin{vmatrix} 1 & -2 \\ 0 & -2 \end{vmatrix} = -(-2) = 2$$

$$A_{33} = \begin{vmatrix} 1 & -2 \\ 2 & 3 \end{vmatrix} = 3 + 4 = 7$$

$$A_{32} = -\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -(-1-6) = 7$$

$$A_{31} = \begin{vmatrix} -2 & 3 \\ 3 & -1 \end{vmatrix} = 2 - 9 = -7$$

2. $$\tilde{A} = \begin{pmatrix} 1 & -4 & -7 \\ -2 & 1 & 7 \\ -4 & 2 & 7 \end{pmatrix}$$

3. $$A^{-1} = \frac{1}{-7}\begin{pmatrix} 1 & -4 & -7 \\ -2 & 1 & 7 \\ -4 & 2 & 7 \end{pmatrix} =$$

$$= \begin{pmatrix} -\frac{1}{7} & \frac{4}{7} & 1 \\ \frac{2}{7} & -\frac{1}{7} & -1 \\ \frac{4}{7} & -\frac{2}{7} & -1 \end{pmatrix}$$

3) $$C^{-1} = \frac{1}{\det C}\tilde{C}$$

1. $$C_{11} = \begin{vmatrix} 5 & 6 \\ 8 & 0 \end{vmatrix} = -48$$

$C_{12} = -\begin{vmatrix} 4 & 6 \\ 7 & 0 \end{vmatrix} = +42$

$C_{13} = \begin{vmatrix} 4 & 5 \\ 7 & 8 \end{vmatrix} = 32 - 35 = -3$

$C_{21} = -\begin{vmatrix} 2 & 3 \\ 8 & 0 \end{vmatrix} = -(-24) = 24$

$C_{22} = \begin{vmatrix} 1 & 3 \\ 7 & 0 \end{vmatrix} = -21$

$C_{23} = -\begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = -(8-14) = 6$

$C_{31} = \begin{vmatrix} 2 & 3 \\ 5 & 6 \end{vmatrix} = 12 - 15 = -3$

$C_{32} = -\begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} = -(6-12) = 6$

$C_{33} = \begin{vmatrix} 1 & 2 \\ 4 & 5 \end{vmatrix} = 5 - 8 = -3$

2. $\tilde{C} = \begin{pmatrix} -48 & 24 & -3 \\ 42 & -21 & 6 \\ -3 & 6 & -3 \end{pmatrix}$

3. $C^{-1} = \frac{1}{27}\begin{pmatrix} -48 & 24 & -3 \\ 42 & -21 & 6 \\ -3 & 6 & -3 \end{pmatrix} =$

$= \begin{pmatrix} -\frac{16}{9} & \frac{8}{9} & -\frac{1}{9} \\ \frac{14}{9} & -\frac{7}{9} & \frac{2}{9} \\ -\frac{1}{9} & \frac{2}{9} & -\frac{1}{9} \end{pmatrix}$

4) $\cancel{A^{-1}BC}$ т.к. $B = C$, то $C^{-1} = B^{-1} \Rightarrow$

$\Rightarrow BB^{-1} = E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

5) $A^{-1}BC^{-1} = A^{-1}$, т.к. $BC^{-1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\Rightarrow X = A^{-1} = \begin{pmatrix} -\frac{1}{7} & \frac{4}{7} & 1 \\ \frac{2}{7} & -\frac{1}{7} & -1 \\ \frac{4}{7} & -\frac{2}{7} & -1 \end{pmatrix}$