Read Me File for *Is the Fed Behind the Curve?*

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1 Software Required

Matlab (R2023b).

2 Memory and Runtime Requirements

Last run on MacOS Sonoma 14.3.1, Apple M3 Max Processor, 64GB installed RAM. The entire code 'Main.m' takes about 70s to run.

3 Instructions for Analysis

- Program Counterfactual_inflation_rate_SEP_2019.m will generate Figure 2 and Table 3.
- Program Counterfactual_2019/Welfare.m will generate Figure 5.

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4 A Brief Introduction to Theory

The model employed to estimate the optimality of the Federal Reserve's monetary policy in this paper is based on the sufficient statistic approach proposed by Barnichon and Mesters (2023) for assessing macroeconomic policies. The model is grounded in the New Keynesian framework and incorporates Lucas critique robustness.

To estimate the impulse response, I follow the framework of Koop, Korobilis, et al. (2010). I use a Bayesian VAR with inflation, unemployment, the Fed funds rate and the monetary policy surprise. I estimate the reduced-form VAR coefficients using Bayesian methods following the default setup with an Independent Normal-Wishart Prior.

Let H represents the forecast horizon, and M_y represents the number of policy objectives in each period. Consider a loss function of the form $\mathcal{L}_t = \frac{1}{2} E_t \mathbf{Y}_t' \mathcal{W} \mathbf{Y}_t$, where $\mathcal{W} = diag(\beta \otimes \lambda)$ denotes a diagonal map of preferences with $\lambda = (\lambda_1, \lambda_2, \cdots, \lambda_{M_y})'$ capturing the weights on the different variables and $\beta = (\beta_0, \beta_1, \cdots)'$ the discount factors for the different horizon ,$\mathbf{Y}_t = (\mathbf{y}_t', \mathbf{y}_{t+1}', \ldots, \mathbf{y}_{t+H-1}')'$ represents the path of policy objectives, and $\mathbf{y}_t' = (y_{1,t}, y_{2,t}, \ldots, y_{M_y,t})'$ denotes the policy objectives in each period.

Now, consider imposing a perturbation δ_t on policy at period t. According to the impulse response function, this perturbation will have a certain impact on the forecast of current-period policy. To determine the optimal policy perturbation (OPP) δ_t^* that minimizes the loss function at period t,

$$\boldsymbol{\delta}_t^* = \arg\min_{\boldsymbol{\delta}_t} \mathcal{L}_t(\boldsymbol{\delta}_t) \quad \text{s.t.} \quad E_t \mathbf{Y}_t(\boldsymbol{\delta}_t) = E_t \mathbf{Y}_t^0 + \mathcal{R}_y^0 \boldsymbol{\delta}_t, \tag{1}$$

where $\mathcal{R}_y^0$ is the impulse response function capturing the impulse responses of the objectives to policy news shocks at different horizons–from horizon-0 to any horizon $h > 0$. For simplicity, I calculate the horizons of the impulse response function that match the forecast horizon H. The solution of optimal policy perturbation (OPP) is given by

$$\boldsymbol{\delta}_t^* = -\left(\mathcal{R}_y^{0\prime}\mathcal{W}\mathcal{R}_y^0\right)^{-1}\mathcal{R}_y^{0\prime}\mathcal{W}E_t\mathbf{Y}_t^0. \tag{2}$$

To estimate the impulse response function, consider a VAR(p) model, I define $A = (a_0\, A_1\, \cdots\, A_p)'$ where A_j is a $M \times M$ vector and $\alpha = vec(A)$ which is a $KM \times 1$ vector with $K = 1 + Mp$, and I can write the VAR either as

$$Y = XA + E, \tag{3}$$

or

$$y = (I_M \otimes X)\alpha + \varepsilon, \tag{4}$$

where $\varepsilon \sim N\left(0, \Sigma \otimes I_M\right)$, $\alpha|\Sigma, y \sim N\left(\widehat{\alpha}, \Sigma \otimes (X'X)^{-1}\right)$, and $\Sigma^{-1}|y \sim W\left(S^{-1}, T - K - M - 1\right)$. The prior has the form

$$\begin{aligned}
\alpha|\Sigma &\sim N\left(\underline{\alpha}, \Sigma \otimes \underline{V}\right), \\
\Sigma^{-1} &\sim W\left(\underline{S}^{-1}, \underline{\nu}\right),
\end{aligned}$$

where $\underline{\alpha}$, $\underline{V}$, $\underline{S}$ and $\underline{\nu}$ are prior hyperparameters.

With this prior the posterior becomes

$$\begin{aligned}
\alpha|\Sigma, y &\sim N\left(\overline{\alpha}, \Sigma \otimes \overline{V}\right), \\
\Sigma^{-1}|y &\sim W\left(\overline{S}^{-1}, \overline{\nu}\right),
\end{aligned}$$

where

$$
\begin{aligned}
\overline{V} &= \left[\underline{V}^{-1} + X'X\right]^{-1}, \\
\overline{A} &= \overline{V}\left[\underline{V}^{-1}\underline{A} + X'X\widehat{A}\right], \\
\overline{S} &= S + \underline{S} + \widehat{A}'X'X\widehat{A} + \underline{A}'\underline{V}^{-1}\underline{A} - \overline{A}'\left(\underline{V}^{-1} + X'X\right)\overline{A}, \\
\overline{\nu} &= T + \underline{\nu}.
\end{aligned}
$$

Using a Gibbs sampling algorithm, I can obtain the posterior distribution.

I further developed an iterative model incorporating policy objectives, policy instruments, and forecasts using the impulse response function to create counterfactual paths.

Now, I assume that starting from period t_0, the Federal Reserve adopts the OPP as its policy guideline. Utilizing the impulse response function $\mathcal{R}_y^0$ and the optimal policy perturbation $\boldsymbol{\delta}_t^*$, I can iteratively compute the counterfactual paths for policy objectives,

$$
\begin{cases}
\mathbf{Y}_{t_0}^{counter,\,1} = \mathbf{Y}_{t_0}^0 + \mathcal{R}_y^0\boldsymbol{\delta}_{t_0}^* \\
\mathbf{Y}_t^{counter,\,I+1} = \mathbf{Y}_t^{counter,\,I} + \mathcal{R}_y^0\boldsymbol{\delta}_t^*,
\end{cases}
\tag{5}
$$

where $I \leq H$ represents the number of iterations.

Similarly, I can iteratively compute the counterfactual paths for policy instruments,

$$
\begin{cases}
\mathbf{P}_{t_0}^{counter,\,1} = \mathbf{P}_{t_0}^0 + \mathcal{R}_p^0\boldsymbol{\delta}_{t_0}^* \\
\mathbf{P}_t^{counter,\,I+1} = \mathbf{P}_t^{counter,\,I} + \mathcal{R}_p^0\boldsymbol{\delta}_t^*.
\end{cases}
\tag{6}
$$

Furthermore, I estimated the impact of interest rate shocks on forecasts by examining the relationship between changes in current policy objectives and changes in forecasts of policy objectives for the current period. I depict this relationship using the following equation,

$$
\Delta\mathbb{E}_t\mathbf{Y}_{t+i} = \lambda_i\Delta\mathbf{Y}_t, \tag{7}
$$

where $\lambda_i \Delta \mathbf{Y}_t$ represents the cumulative change in policy objectives or policy instrument at period t, $\Delta \mathbb{E}_t \mathbf{Y}_{t+i}$ denotes the change in forecasts of policy objectives or policy instrument for period $t+i$, $i = \{0, 4, 8\}$, representing the current year (t), year $t+1$, and year $t+2$, and λ_i represents the coefficient capturing the relationship between changes in forecasts of policy objectives and changes in policy objectives. This coefficient is estimated through reduced-form regression.

Choosing different starting points for iteration can generate different counterfactual paths, reflecting the impact of the timing of the Federal Reserve's adoption of the OPP strategy on policy objectives and policy instruments within my theoretical framework.