LET'S CLEAN UP SPACE JUNKS!

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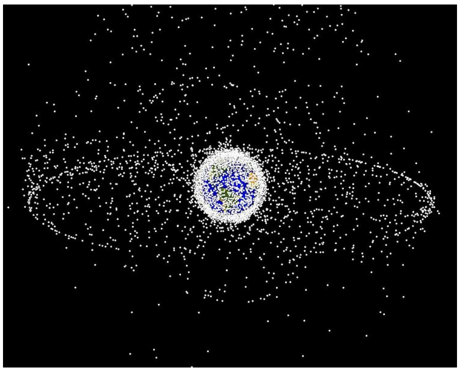
Contents

1 Introduction

“Boom! ”As more and more accidents caused by space bombards—space junk, the large amount—more than 500,000 pieces of space debris around earth has raised widely concern of people across the world. People first brought space debris to the agenda of the Scientific and Technical Subcommittee in February 1994.Later on, people started to try evolving appropriate and affordable strategies to minimize the potential impact of space debris on future space missions. Their efforts include developing debris measurement techniques, mathematical modeling the debris environment, characterizing the space environment, mitigating the risks of space debris, avoiding collision, as well as reducing the production of space debris and so on.[1]



The most interesting work among above is how to bring down these space debris. Creative ideas include water jets to build water wall in space, high energy lasers to drag down the debris, large satellites designed to sweep up the debris, space balloons to drag the debris, space pods to kick the debris out of orbit and so on,[2] but all of these methods have their limitations facing debris's high velocity and complex distributions. So it is significant to build up an innovative, flexible and reasonable time-dependent model to find the best way to allocate these methods. That's why we choose the problem, and we are glad that our model is accurate and reasonable enough to be useful.

The task asks us to build a time-dependent model to determine the best alternative or combination of alternatives for a private firm to adopt as a commercial opportunity to address the space debris problem and to carry out quantitative and/or qualitative estimates of costs, risks, benefits, as well as other important factors. The task also asks our model to be able to explore a variety of important abnormal emergencies.

What's more, we should also give our model the function of estimating the probability of a potential economic opportunity, providing comparison of the different options for removing debris, give specific recommendation among methods, as well as providing innovative alternatives for avoiding collisions if no such opportunity is possible.

Before we introduce the framework of our work, we will give some statement about specific concepts in the task:

- alternative: ways to clean up space debris.
- time-dependent model: a model whose parameters are functions of time, and the input and output of the model can be flexible with time change.
- "What if?"scenarios: scenarios under abnormal or urgent conditions.

Then we divide the task into several subtasks as follows:

- Constructing distribution model of space debris around earth at a certain time point, considering different latitudes and altitudes, and show in spatial density.
- Building time-changing distribution model of space debris, and influential factors including
  1. solar radiation
  2. atmosphere drag
  3. collision between different debris
  4. explosion of bigger debris
  5. replenish of newly generated debris from retired flying machine
  6. debris movement between different radius(or altitude)
- Summarize available methods in cleaning space debris, and build a model assessing each method by
  1. parameterizing the risk, cost, benefit and other important factors of each method
  2. linking the parameterized factor with the distribution model to build the time-distribution dependent assess model.
  3. comparing these methods using the assess model acting on the principle of shoot down most, mistaken least, cost lowest and efficiency highest[HAVE A CHANGE!!!!!!]

We can elaborate our model in the figure below:

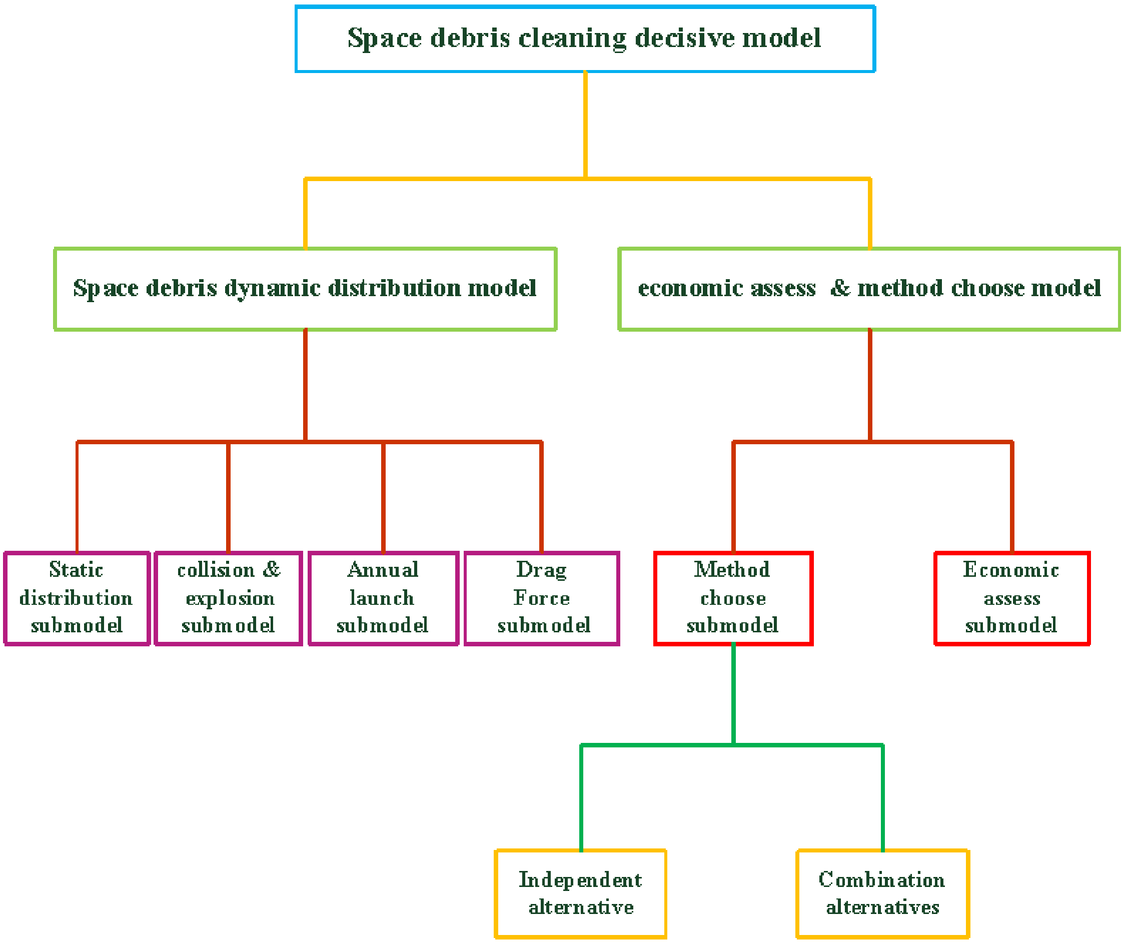


Fig 1: flow chart of our model.

2 Background

when the Russian satellite Kosmos-2251 and the USA satellite Iridium-33 collided on 10 February, 2009, the space debris issue became more widely discussed in the news media. However, it is not from then people started to work on the issue. People has started to throw rubbish in space as early as 1957, when the Soviet Union sent its first satellite into the space. The United States have also organized many organizations to measure and control space debris since 1990s.

In 1996, people started to measure space debris by understanding data and effects of environment on space systems by all kinds of ways they can find. They tried to describe the effect of this environment on space systems through ground- and space-based sensors.And in 1997,people started to model the debris environment and assess the risk. Those space debris model including the mathematical description of the current and future distribution, which is a function of debris size and other physical parameters. In 1998, people came out with many interesting space debris mitigation measures. Mitigation comprises reduction of the space debris population growth and protection against particulate impact. Measures for the reduction of space debris growth include methods for debris prevention and removal. Protection against space debris includes physical protection with shielding and protection through collision avoidance.[1]

So far, the United States has build series of models including NASA 90 model, NASA 96 model, ORDEM 96,2000, 2008 and so on. There has been lots of beneficial exploration about space debris.

Our model will therefore absorb the essence of these models, and create our own model by discretizing the change of space-time dependent distribution, then put more emphasis on the practical way to estimate different alternatives or combination of alternatives of different debris cleaning methods than these old models, as well as giving reasonable judges of different alternatives basing on several ”what if” scenarios.

3 Assumptions

3.1 Assumptions for spatial density dynamic distribution model

1. **We assume that space debris include three major group: diameter between 0.1cm and 1cm, 1cm and 10cm,and diameter bigger than 10cm, and all the debris are sphere-like.**
   Reasons for assumption 1:

   - Debris smaller than 1mm in diameter are hard to observe, so data is unavailable.
   - As we have assumed debris less than 1cm in diameter can't be divided into smaller debris, we also assume debris smaller than 1mm in diameter don't contribute much to damage, so we ignore them.
   - Sensors observing debris have different resolution, and boundaries between different-scale sensors are 1cm and 10cm.
   - According to main research conclusions of a 45-student study at the International Space University[4] they divide space debris into the same group as we do, and get convincing results about collision that the chance that small debris will strike a large LEO space asset is 45 times as high as the hazard from large objects.[4] Table 1 summarizes these conclusions
   - Many other researches about space debris divide debris into the same group, too.
   - the assumption of sphere-like debris can effectively simplify our model with preferable reliability.

2. **In the collision model, we ignore collisions happen between debris more than two at the same time, and assume that all collisions are catastrophic.**
   Reasons for assumption 2:

   - Debris whose diameter range between 0.1cm and 1cm have already been confirmed won't explode.[3] that means both of the debris in the collision break up except debris whose diameter between 0.1cm and 1cm.
   - It is nearly impossible for debris more than two bump into each other at the

same time, giving the low density of debris(nearly

$$10^{-8}$$

per$kilometer^(3)$).

- Considering no such things can happen at the same time absolutely, so as long as there is the former and the latter, we can transfer the situation into two collision model.

3. **The major tasks to address problem of space debris locates in LEO(Low Earth Orbit, space between 200km and 2000km)**
   Reasons for assumption 3:

   - the majority of space debris locates in LEO, so we think the model is still of great significance providing it can deal with space debris problem in LEO.[5]
   - data is really limited in GEO so it is of little use to build model in GEO at least facing the technical condition at present.

4. **In the rubbish-piling model, we assume the ratio of rubbish to operating equipment is a constant, so the amount of newly produced rubbish depends on the congestion degree, and the sum of historical annual launch amount.**

5. **In the rubbish-piling model, the annual increase process of debris's amount into two stages: First is growth basing on non-changed ratio between different sizes of debris's amount, and when the stage finishes, then the change(in ratios of different sizes causing by break-up events) starts.**

6. **In the atmosphere drag model,we assume only tangential component of atmosphere drag have impact on debris in LEO.** Reason for this assumption is clear. The atmosphere drag F can be divided into resistance force F_τ, lift force F_α and side force F_n. Among the three forces, F_τ is the most important force[7],and that's what we call atmosphere drag in the following part of paper.

7. **In the atmosphere drag model, we assume that atmosphere density is only the function of altitude.** This assumption will efficiently simplify our model, and is reasonable considering the rotate of atmospheric meridional circulation around the earth.

8. **We assume atmosphere drag force equation can apply to all altitude from 200km-2000km.** The assumption is supported by the common knowledge of the atmosphere drag force's control scope.(Atmosphere drag force equation mainly operates in free molecule layers, the tendency and mechanism is similar, though.)

9. **assumption in the Laser-based methods** The assumptions of this kind of method are shown below

    - Laser effect on objects whose Perigee is above 1000 km is ignored [12].
    - Laser can only aim at one target at one time.
    - Due to the extremely difficulty to detect Small Debris [13], we ignore the lasers' effect on Small Debris.
    - Target re-entry is achieved in one pass for any Middle debris within 1000 km range [16].
    - Single-pass re-entry of Large debris is not practical, it requires several passes to achieve re-entry for Large debris [12].

10. **assumption in the satellite-based method** The assumptions of this kind of method are shown below

    - Satellite-based methods cannot deal with "Small" debris [15]
    - The number of objects satellite-based methods are able to deal with in a certain period of time associates with the special density of its height.
    - Satellite-based methods could deal with debris in all heights inside LEO.

11. **assumption in the material-based method** The assumptions of this kind of method are shown below

    - Material-based methods have unignorable effects for all objects, no matter Small, Medium, Large debris, which pass through its area.
    - Material-based methods could only be used under 1000 km height.

12. **assumption in the economically attractive opportunity model**

    - We assume that the private firm have not entered the debris-removing industry before.

- The private firm has no difficulty in getting the industry-relative license from the government.
- The private firm has access to the capital, technology, raw materials and labor required in this industry.

4 Dynamic distribution model of Debris in LEO

4.1 Different-sizes debris's static distribution of altitude and latitude in 1999 in LEO

To build the time-dependent model of space debris distribution, we need to choose a point-in-time to set the origin distribution state, and get the distribution data of different sizes of debris, in different altitudes at that time. Because we focus on how to choose methods dealing with space debris problem from now on, we don't need to set the time point far before when the first "space debris" emerged. Providing the data we can find, we choose 1999 as a start point. First, considering both the availability and physical significance of data, we divide sizes of space debris into three groups: and the reasons

size	name	abbreviation
0.1-1cm	small debris	S
1-10cm	medium	M
10cm	large	L

for the category of size group are listed in assumptions, including physical significance, data availability and research convention, so we think it is reasonable to use this category system here.

Then we start to build distribution model of space debris in LEO. As we have assumed that the longitudinal distributions of space debris is uniform, then we can narrow the influential factors of distribution down to altitude and latitude. After that, according to Nickolay N.Smirnov's relevant work on space debris distribution, we find our ways to describe the statistic density $p(h,\varphi)$of space debris distribution over altitude h and latitudeφ, which represents the integral density of a stapling.

$$p(h,\varphi) = \frac{\partial^2 N(h,\varphi)}{\partial h \partial \varphi} \tag{1}$$

Here $N(h,\varphi)$is the number of space debris with current values of altitude and latitude, then we can easily define our spatial density of $N(h,\varphi)$ in a certain point in the space. We name the density $\rho(h,\varphi)$:

$$\rho(h,\varphi) = \frac{p(h,\varphi)}{2\pi(h+R)^2 \cos\varphi} \tag{2}$$

in which R represents the radius of earth. The equation showed that $\rho(h,\varphi)$is the average in stapling of $p(h,\varphi)$.
To move further, we need to discuss change of $\rho(h,\varphi)$ with altitude and φ, or in other approach, to find the accurate expression of $\rho(h,\varphi)$ directly.
Using Nickolay N.Smirnov's relevant work on similar topics for reference, we find initial factors of the spatial density function, including:
N_sum—-the total amount of space debris with perigee height h_p lower than some specified value h_max $p(h_p), p(e), p(i)$—-the densities of distribution of values of perigee height, eccentricity (e) and inclination (i) of space debris in the region under consideration.[6]
$p(h_p), p(e), p(i)$ meets equations below:

$$\int_{h_p} p(h_p)dh_p = 1 \int_e p(e)de = 1 \int_i p(i)di = 1 \tag{3}$$

We construct function $p(h,\varphi)$in two stages:

1. we determine the average number of objects

$$\Delta N(h,h+\Delta h)$$

, which are inside the mentioned spherical layer at some fixed moment of time.

2. we construct the distribution $\Delta N(h,h+\Delta h,d\varphi)$ of objects in the given layer over the latitude, and this distribution satisfies the relationship

$$\int_\varphi \Delta N(h,h+\Delta h,d\varphi)d\varphi = \Delta N(h,h+\Delta h) \tag{4}$$

3. take into account, we obtain:

$$p(h,\varphi)\Delta h = N_sum\frac{\cos\varphi}{\pi}\int_i \frac{p(i)di}{i\sqrt{\sin^2 i - \sin^2\varphi}}\int_{h_p}\int_e \Delta\tau(h_p,e,h)\Phi(h_p,e,h)p(h_p)p(e)dh_pde \tag{5}$$

in which

$$\sin i \geq \sin\varphi$$

To explain more, quantity

$$\Delta\tau(h_p,e,h)$$

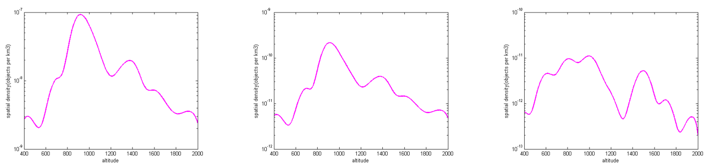
has a meaning of probability of finding space debris with orbital elements h_p, e, h in the spherical layer under consideration.
The function

$$\Phi(h_p, e, h)$$

is the following:

$$\Phi(h_p, e, h) = \frac{(1-e)^2}{\sqrt{1-e^2}}\left(\frac{h+R}{h_p+R}\right)^2 \tag{6}$$

In brief, in the relation **??** the integrals with respect to h_p and e arguments are taken throughout the region of their possible values. In accordance with dependence 2, based on the function$p(h,\varphi)$ we can easily calculate the unknown function $\rho(h,\varphi)$.that characterizes the altitude and latitude dependence of a number of space debris per unit of volume. Using our model introduced above and parameters data including N_sum,$p(e)$,$p(h_p)$,$p(i)$ in the CD attached to the book written by Nickolay N.Smirnov, we successfully rebuild the distribution of space debris of three sizes(small, medium, large) in 1999. We are glad to show our results in graphs as follows: The three figures



(a) small (b) medium (c) large

Fig 2: height-latitude dependent distribution of three sizes debris in LEO

above show the altitude distribution of debris of three sizes. In addition, we portray a 3-dimension figure to show our result on the latitude distribution of small, medium and large debris. Due to the lack of accurate data, we can only simulate latitude distribution of 100-mm debris between 800 and 850km altitude, and show it in figure below:

4.2 preview of different factors

After building the static distribution model above, we will move on to discuss factors or mechanical submodel in our dynamic distribution model. Before we go into details of different influential factors to discuss the change mechanisms of spatial density distributions, we would like to show the structure of our dynamic distribution spatial density distribution.png spatial density distribution.png

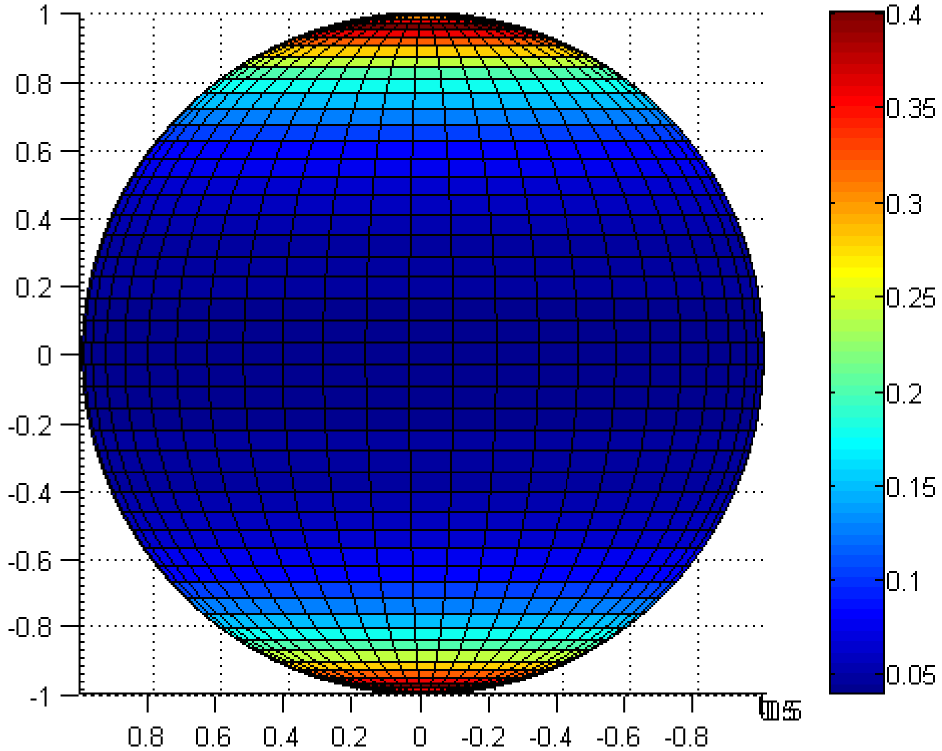


Fig 3: spatial density's altitude distribution of three sizes of debris spatial density distribution.png spatial density distribution.png

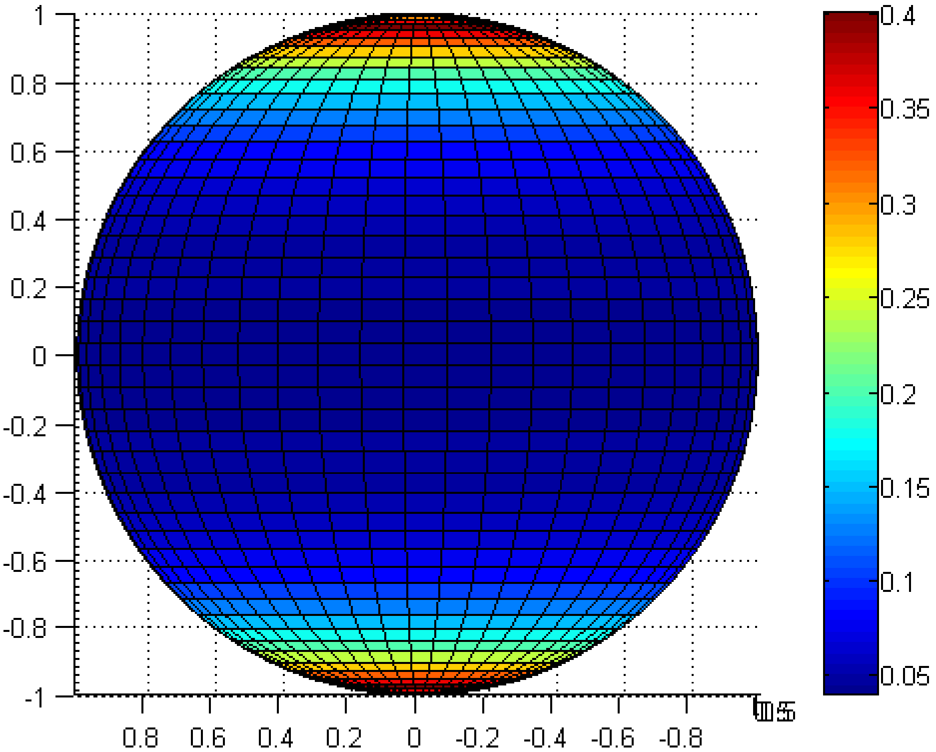


Fig 4: Color-coded spatial density distribution (no/km3) of 100-mm objects, between 800 and 850 km altitude, as viewed from the orbital of the Earth.

spatial density distribution2.png spatial density distribution2.png

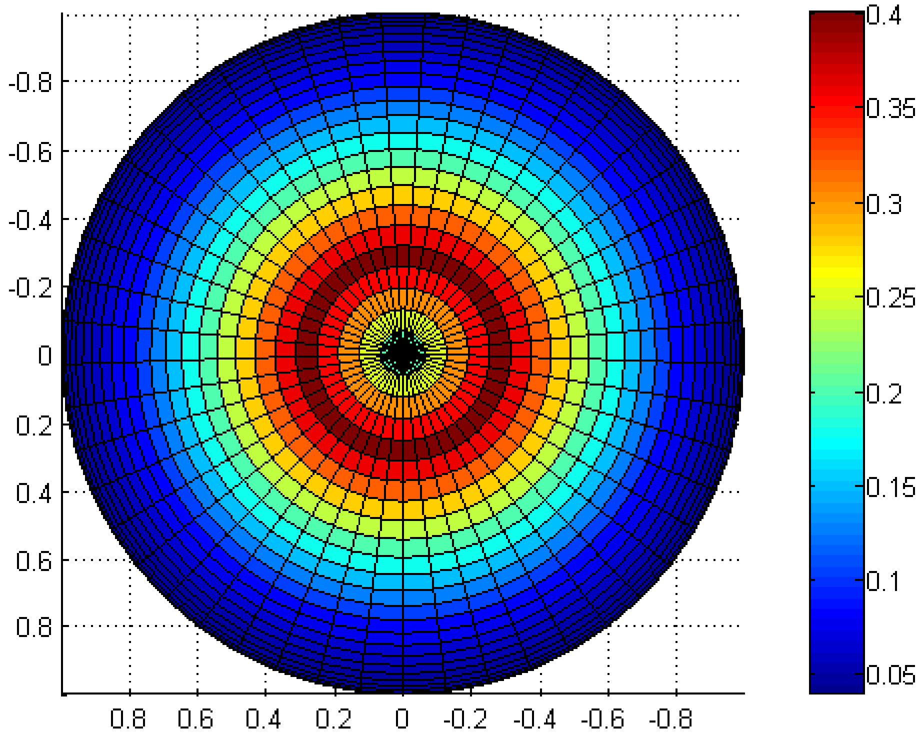


Fig 5: Color-coded spatial density distribution (no/km3) of 100-mm objects, between 800 and 850 km altitude, as viewed from the polar of the Earth.

model here. First, we would like to give a list of variables and their meaning that we will use in this model: Another parameters with the subscript of S,M,or L, means the

variable	meaning
ρ	spatial density of total debris of three sizes in LEO
t	time(year)
h	altitude(height)
l	latitude
p	pressure of solar radiation
q	drag of atmosphere
K	spatial density change of dumping of retired space launcher
B	spatial density change brought by explosion and collision
E	spatial density change brought by explosion
C	spatial density change brought by collision

corresponding properties of "Small","Medium" and "Large" debris, and we don't make unnecessary statement about it here, and we will explain other parameters which only play a part in particular submodels later. First, we divide the spatial density of space debris in a certain place ρ into three parts:ρ_S,ρ_M and ρ_L,represents part density of different sizes of debris.

$$\rho = \rho_S + \rho_M + \rho_L \tag{7}$$

We also give equations to calculate the three partial densities.

$$\rho_S(t,h,l) = \rho_S(t-1,h,l) + q_S(t) + B_S(t) + N_S(t) \tag{8}$$

$$\rho_M(t,h,l) = \rho_M(t-1,h,l) + q_M(t) + B_M(t) + N_M(t) \tag{9}$$

$$\rho_L(t,h,l) = \rho_L(t-1,h,l) + q_L(t) + B_L(t) + N_L(t) \tag{10}$$

(11)

Then we will analysis the specific qualitive submodel of estimating the effect of each aspects seperately in the following sections.

4.3 break-up model

To better the dynamic distribution model, our paper describes the phenomenon of bigger debris breaking into small pieces by a Breakup Model. The model can simulate the influence of two kinds of break up effects, explosion and collision, on the numerical changing trend of three sizes of debris (Small, Medium, Large, the same as mentioned above). junk collision.jpg junk collision.jpg 

Thus, the break up influence factor B satisfies the relationship: B=E+C E—-the influence of explosion;
C—-the influence of collision.
We will discuss the effects of explosion and collision separately.

4.4 Explosion submodel

In order to stimulate the general characteristics of the influence caused by explosion, three assumptions need to be stated first:

1. Since we only have three sizes of debris, we assume the Small debris (¡1cm) would not explode, Medium debris would become Small debris after explosion, Large debris would become Medium debris and Small debris after explosion.

2. All debris are treated as sphere, the average diameter of Medium debris is 5 centimeters, the average diameter of Large debris is 1 meter.

According to NASA's EVOLVE model[3], the number of explosive fragments of size d, satisfies the following equation:

$$N(d) = 6d^{-}1.6 \tag{12}$$

As we have assumed that small debris cannot be divided any longer, so only debris of medium and large can exploded, and medium debris produce small debris by explosion, while large debris produce small and medium debris in a ratio of $\lambda_E$and $1 - \lambda_E$, we will show it in 1as follows: So, we can divide E into three equations below:

$$E_S(t) = \alpha_E \rho_M^{t-1} N_E(d_M) + \lambda_E \beta_E \rho_L^{t-1} N_E(d_L) \tag{13}$$
$$E_M(t) = -\alpha_E \rho_M^{t-1} + (1 - \lambda_E) \beta_E \rho_L^{t-1} N_E(d_L) \tag{14}$$
$$E_L(t) = -\beta_E \rho_L^{t-1} \tag{15}$$
$$\tag{16}$$

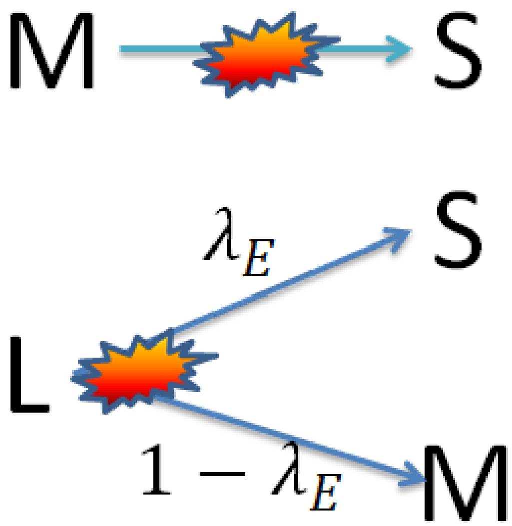


Fig 6: illustration of explosion model.

E_S, E_M, E_L are the effect of explosion on “Small”, “Medium” and “Large” debris.
ρ_S,ρ_M,ρ_L are the spatial density of “Small”, “Medium” and “Large” debris.
α_E and β_E represent the portion of exploded “Medium” and “Large” debris, $\lambda_E$represents the ratio of medium-sized debris to small-sized ones, both of which are produced by the explosion of “Large” debris.
$N_E(d_S), N_E(d_M)$ and $N_E(d_L)$represents the amount of space debris of three different sizes respectively.
Using the model above, we can figure out how can explosion change the amount of space debris between different time steps.

4.5 collision submodel

To better stimulate the impact of collision on the amount of space debris, we make several assumptions in this submodel:

1. We assume all the three sizes of debris would collide with each other. But after collision, small debris $(< 1cm)$ doesn't break up into smaller debris , “Medium” debris breaks up into small debris, “Large” debris becomes Medium debris and “Small” debris.

2. All debris are treated as sphere, the average diameter of "Medium" debris is 5 centimeters, the average diameter of "Large" debris is 1 meter.

3. Between the two types of collision: catastrophe collision and non-catastrophe collision [3], we choose to focus on catastrophe collision and ignore the non-catastrophe collision when first address this problem.

According to NASA's EVOLVE model[3], a power law distribution for the number of fragments of a given size and larger has been developed:

$$N_C(m,d) = 0.1(M)^0.75d^-1.71 \tag{17}$$

$N_C(m,d)$—-amount of debris produced by collision between two parent debris. m—-total mass of the two collided debris.
d—-the diameter of parent debris.
To illustrate more specifically, collided debris only includes debris of medium and large size, when parent debris bump into each other, medium debris produce small debris, while large debris produce small and medium debris in a ratio of $\lambda_C$and $1-\lambda_C$, we will show it in 1as follows: Let's call the ratio of collision in medium debris α_C, and

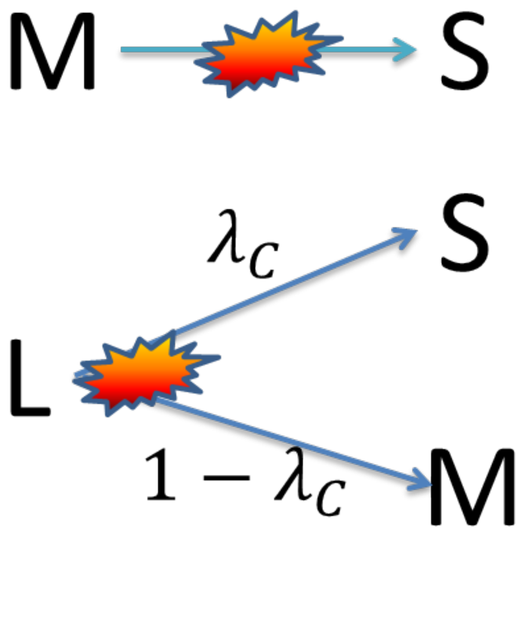


Fig 7: illustration of collision model.

similarly, the ratio of large ones β_C. In this way, we get the amount of daughter debris of different sizes as follows:

$$C_S(t) = \alpha_C \rho_M^{t-1} N_C(d_M) + \lambda_C \beta_C \rho_L^{t-1} N_C(d_L) \tag{18}$$

$$C_M(t) = -\alpha_C \rho_M^{t-1} + (1-\lambda_C)\beta_C \rho_L^{t-1} N_C(d_L) \tag{19}$$

$$C_L(t) = -\beta_C \rho_L^{t-1} \tag{20}$$

C_S,C_M and C_L —-the collision effect on the amount of "Small", "Medium" and "Large" debris.
α_C and β_C —- the portion of exploded Medium and Large debris
$N_C(d_S)$,$N_C(d_M)$,$N_C(d_L)$—-amount of debris a small, medium or large parent debris can produce after collision. ρ_S,ρ_M,ρ_L—-spatial density of debris of different sizes. λ_C——the ratio of small debris to medium debris produced by large parent debris after collision. Using this two submodel, we can figure out how much does the ρ_S,ρ_M,ρ_L of a certain latitude and altitude change in a specific time t(yr) caused by explosion and collision of space debris, and apply the result to the final dynamic distribution model. Note:We attach the value of relevant parameter of break-up model to Appendix I(Break-up model).

4.6 Rubbish-piling submodel

The parameter N reflects the increasing number of orbital debris due to the natural change from space equipment to space junk in LEO space. To portray a picture of K, we choose the proportion of the total launches from 1957, when the first satellites were ejected into space, to present and the total launches from 1957 to last year to stand for the changing rate. The launches in the world within each year since 1957 is shown as follows: In addition, we make three assumptions about the process of debris's number to increase:

1. The growth of debris's amount depends on the congestion degree(measured by the until-present sum of launches since 1957, when people launched the first material into the space.)

2. We divide the annual increase process of debris's amount into two stages: First is growth basing on non-changed ratio between different sizes of debris's amount, and when the stage finishes, then the change(in ratios of different sizes causing by break-up events) starts.

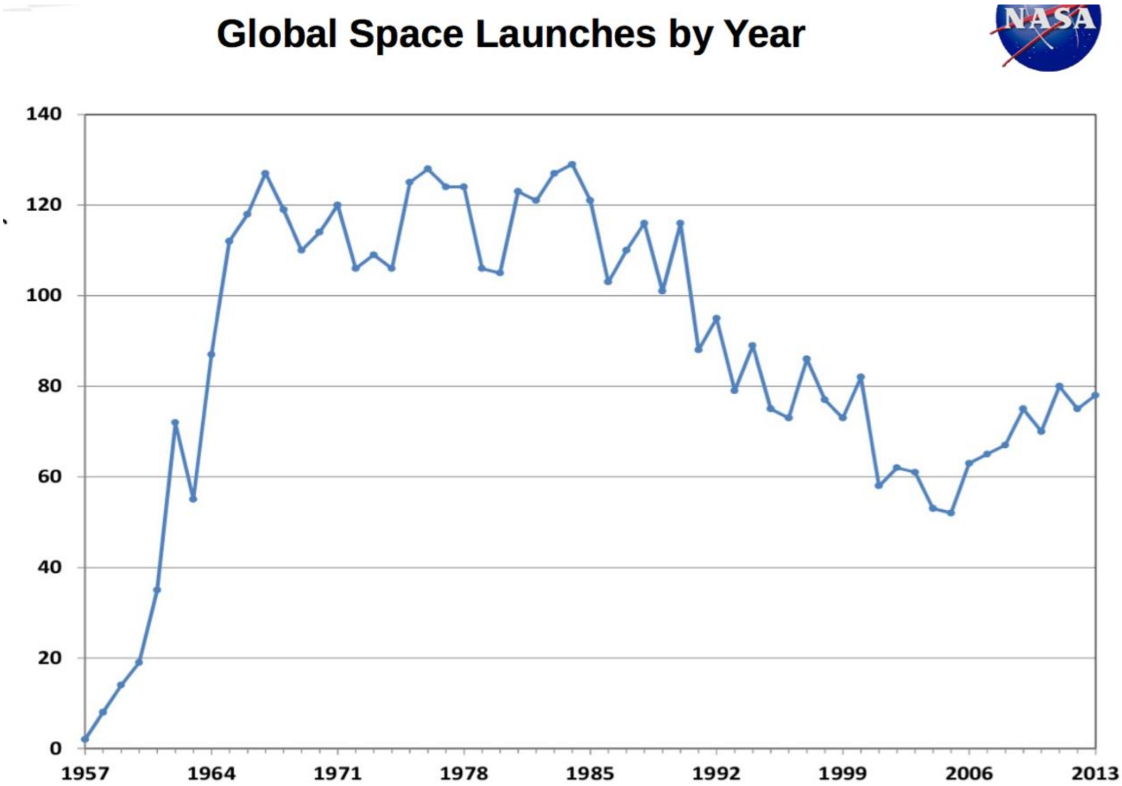


Fig 8: Global space launches by year

According to the two assumptions, we think that the first stage of growth(rubbish-piling growth) can be calculated by multiplying a relative growth rate of historical total launches to spatial density of all debris in the last year. Because of the second assume, so it is reasonable for us to use ρ of all sizes to calculate. So N could be indicated by the following equation:

$$N(t) = \rho(t-1) * (\frac{\sigma(1957- > t)\sigma(1957- > t-1)}{-}1) \tag{21}$$

Using this model, we can calculate how much the growth of debris's amount caused by launches activities annually and can use the result into the final dynamic model.

4.7 Atmosphere drag submodel

The parameter q reflects the drag effect of atmosphere in high altitude(200km-2000km), which plays an important role in the decrease of velocity and fall in height of space debris. So we use this model to calculate how much height a debris will drop in height according to atmosphere drag, and thus to better simulate the change in spatial density distribution. Before start to introduce our model, we would like to point out several basic assumptions:

1. We assume only tangential component of atmosphere drag have impact on debris in LEO.
   Reason for the the atmosphere drag F can be divided into resistance force F_τ, lift force F_α and side force F_n. As shown in **??**. Among the three forces, F_τ is the most important force[7],and that's what we call atmosphere drag in the following part of paper.

2. We assume that atmosphere density is only the function of altitude. The assume will efficiently simplify our model, and is reasonable considering the rotate of atmospheric meridional circulation around the earth.

3. We assume atmosphere drag force equation can apply to all altitude from 200km-2000km. Which is reasonable considering the widely accept of the atmosphere drag force.(Atmosphere drag force equation mainly operates in free molecule layers, the tendency and mechanism is similar, though.)

4. To simplify the calculation of area of thrust surface, we assume all space debris in this model are sphere at their origin diameters.

There has been abundant relevant studies about atmosphere drag in LEO, and we manage to give our innovative model to calculate **our** result. As a widely accepted equation, the atmosphere drag force in LEO has the form of:

$$\overrightarrow{F} = -\frac{1}{2}C_D\rho S\left|\overrightarrow{V}\right| \tag{22}$$

F—-the atmosphere drag force, in the opposite direction of tangential velocity. V—-the velocity of debris in LEO realtive to atmosphere movement. C_D—-the drag coefficient of atmosphere. Our major task is to deduce the radial speed of debris:

$$\frac{F}{m} = \frac{-\frac{1}{2}C_D\rho S|\vec{v}|^2\hat{e_\theta}}{m} + \frac{GM}{R^2}\hat{e_\rho} = (\ddot{R} - R\omega^2)\hat{\rho} + (R\dot{\omega} + 2\dot{R}\omega)\hat{\theta}$$

Because direction of $\hat{e_\theta}$ and $\hat{e_\rho}$ is vertical to each other,we compare the component in the direction of $\hat{e_\theta}$ and find the two below equations:

$$\begin{cases} \dfrac{-\frac{1}{2}C_D\rho S|\vec{v}|^2\hat{e_\theta}}{m} = R\dot{\omega} + 2\dot{R}\omega \\ \dfrac{GM}{R^2} = \ddot{R} - R\omega^2 \end{cases} \tag{23}$$

We just need to use the first one.What's more,we have$|\vec{v}| = \sqrt{\frac{GM}{R}}, \omega = \sqrt{\frac{GM}{R^3}}$and put the two formulas into the equation and generate the differential equation below:

$$\frac{dR}{dt} = -\frac{C_D\rho S\sqrt{GM}}{mR^{\frac{1}{2}}}$$

To put the result above into practice, we need more data for other parameters in the Atmosphere Drag Force Equation: **First, we cited data of the drag coefficient of atmosphere from the work of Xia C Y.[8]** According to Xia, for spheric debris, their C_D under the altitude of 500km is 2.1, and their C_D depends on the number density and ratio of different gases.Schamberg[9] not only takes gases and mass of atoms on the surface of debris into consideration, but also considers the reflection type on debris surface with atoms in the atmosphere, figure out different C_D of different gases. According to the relative ratio of gases and their corresponding C_D, we make a reasonable estimation of C_D in different altitude, mainly by weighted average basing on the relative ratio and the C_D respectively. The table of C_D of different gases and of different altitudes are listed as follows:

Second, according to the atmosphere density data we find in a related work[10],we can get a reasonable estimate of atmosphere density from 200km to 2000km.

Gas Categories	H	He	N	O	Ar
C_D	3.002	2.701	2.210	2.161	2.107

Tab 1: $C_D$of different categories of gases

altitude(km)	< 500	500-600	600-700	700-800	800-900	900-2000
C_D	2.1	2.4	2.5	2.7	2.8	2.9

Taking all the factors above we can calculate the atmosphere drag force at every height, as well as the radial velocity of debris at different height. Once knowing the radial velocity of different height, we know the radial migration characteristic of debris—- that's just what we need for parameter q.

4.8 explanations on solar radiation pressure

For particles smaller than 1 mm in diameter, the effects of solar radiation pressure can become important. Solar radiation pressure typically causes the eccentricity of an orbiting object to oscillate. This usually shortens the lifetime of small debris by lowering their perigee into the atmosphere faster than would be expected from atmospheric decay alone. According to our assumption, although solar radiation pressure is important in altering the altitude distributions of debris smaller than 1mm in diameter, but these debris isn't in our study range. So we decide to ignore the impact of solar radiation, and use atmosphere drag model to make up for the altitude distribution altering.
The decision is reasonable because atmosphere drag is more influential than solar pressure on debris in LEO, especially debris in our study range.

5 Merge of all distribution-relevant submodels

Let's give a second view of our major model of distribution:

$$\rho_S(t,h,l) = \rho_S(t-1,h,l) + q_S(t) + B_S(t) + N_S(t) \tag{24}$$

$$\rho_M(t,h,l) = \rho_M(t-1,h,l) + q_M(t) + B_M(t) + N_M(t) \tag{25}$$

$$\rho_L(t,h,l) = \rho_L(t-1,h,l) + q_L(t) + B_L(t) + N_L(t) \tag{26}$$

(27)

Until now, we have got all parameters estimated or calculated, and all of them are time-dependent, it is easy to find out the three equations are recurrence formulas, so we use MATLAB to simulate the change of the model with time, and finally, we can successfully get the spatial density of space debris of different altitudes, different latitudes and different time.

6 Find the best alternative model

There are many creative methods of addressing the space debris problem, but some can only function in limited altitudes, and some have dear cost, in **??**, we list some of the typical methods and their brief description before we determine which is the best.

To better define which method is the best one, three factors are considered in our model:

- the average cost on a debris, which we call p;
- the average speed to clean a debris, which we call v;
- the risk of each method, which we call r;

First, we set up the assessment function F considering the three target factors(p,v and r) for each size of debris.

$$F = w1 * p + w2 * v + w3 * rp = C/Nv = N/t \sum(wi) = 1 \tag{28}$$

in the formula group, p,v,$r$is of the same concept as mentioned above, and wi represents the weight of factor i(i=1,2,3) in finding the best alternative. To help

results.png results.png

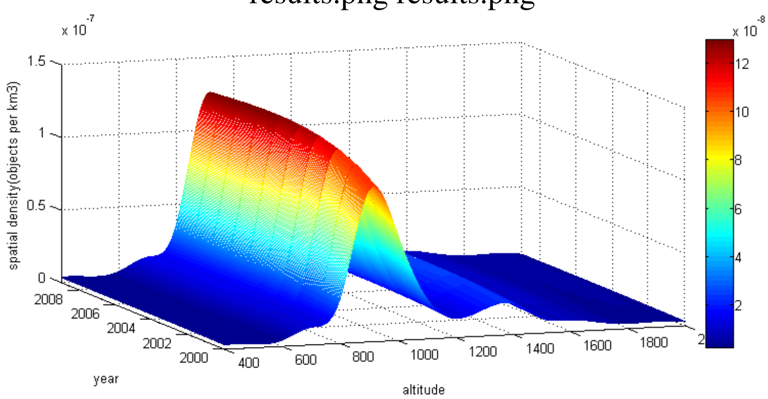


Fig 9: distribution results of dynamic model simplify the problem, we divide the common methods to clean orbital debris into three categories: **laser-based methods, satellite-based methods and material-based methods.** Statements of these methods are as follows:

1. **Laser-based methods** The first kind of method to introduce in our paper is a laser system to re-enter or lower LEO orbital debris. Laser-Based methods can be divided into three general categories distinguished by their goals and laser beam parameters. First, at low intensities below the ablation threshold, the effect of laser is mainly about diverting debris through light pressure [11]. However, the intensity of the light pressure is even less than the sunlight since the debris could pass above the laser site only during a few minutes' time. Second, at higher laser intensity, we can consider continuous (CW) laser ablation, but the slow heating and decay characteristic of CW thrust on tumbling debris will normally give an ablation jet whose average momentum contribution cancels itself [12]. CW heating causes messy melt ejection rather than clean jet formation, possibly adding to the debris problem. Last, since the effectiveness of pulsed lasers has been testified [12], we choose pulsed lasers in our method. The assumptions of this kind of method are shown below

    - Laser effect on objects whose Perigee is above 1000 km is ignored [12].
    - Laser can only aim at one target at one time.
    - Due to the extremely difficulty to detect Small Debris [13], we ignore the lasers' effect on Small Debris.
    - Target re-entry is achieved in one pass for any Middle debris within 1000 km range [16].
    - Single-pass re-entry of Large debris is not practical, it requires several passes to achieve re-entry for Large debris [12].

2. **Satellite-based methods** Satellite-based methods include methods which requires sending a satellite into space, such as Electrodynamic Tether or EDT (A removal vehicle will remove an object by capturing it using a robot arm and then de-orbiting, taking the debris by attaching an EDT unit to the object [5]). These methods directly "touch" the debris to capture or re-entry them. The assumptions of satellite-based methods are:

    - Satellite-based methods cannot deal with "Small" debris [15]

- The number of objects satellite-based methods are able to deal with in a certain period of time associates with the special density of its height.
- Satellite-based methods could deal with debris in all heights inside LEO.

3. **Material-based methods** Material-based methods are methods which release some kind of material such as mist and blocks of aerogel in order to increase the drag force on the debris. Common methods are:

   - GOLD balloon in space to drag the space debris taking advantage of its big volume, soft surface and light weight;
   - launch rockets full of water into space and release their payload to create a wall of water that orbiting junk would bump into, slow down, and fall out of orbit, or by launching water on decommissioned missiles to reduce cost;
   - send up tons of tungsten microdust and spread into low earth orbit, and on a trajectory opposite that of the targeted space junk, to slow smaller space debris (with dimensions under 10 cm), and it can accelerate the speed of debris dropping down into earth by tens of times.

   Beneath represents the assumption of this kind of methods in our model:

   - Material-based methods have unignorable effects for all objects, no matter Small, Medium, Large debris, which pass through its area.
   - Material-based methods could only be used under 1000 km height.

Now let's look back at our equation: F = w1*p + w2 *v + w3*r. We will now analysis the factors p, v, r one by one.

6.1 p - The average cost for one debris

Due to that different methods contain totally different cost structures meanwhile the cost structures of methods in the same type of methods are nearly the same, we will address this problem according to the type of methods.

1. **Laser-based methods** $p = C \div N = (CC + RC \times N) \div N = CC \div N + RC$ CC —-the construction cost of a laser system;
   RC —-the average cost to achieve a re-entry for a certain size of debris;
   N —-the number of debris a laser system could deal within its life period.
   Thus, CC/Nmeans depreciation of the system per debris.

2. **Satellite-based methods** $p = C \div N(\rho) = (CC + LC(h) + EC \times N(\rho)) \div N(\rho)$ CC —-the construction cost of a laser system;
   LC —-the launch cost of a satellite;
   EC —-the average energy cost for the satellite to deal with a debris in space; N —-the number of debris such satellite could deal within its life period which associates with the spatial density of its height.

3. **Material-based methods** $p = C \div N(\rho) = LC(h) \div N(\rho)$ The only expense of material-based methods lies in sending the material into space. N means the number of debris such material could deal within its life period which associates with the spatial density of its height.

6.2 v - The average time spent on a single debris

It is clear that expect for laser-based methods, v of both satellite-based methods and material-based methods associates with the spatial density of its height. For that the higher spatial density is, the easier the satellite or the material meet a debris. As for laser, since its effect is targeted at a debris at a time, v remains stable regardless of spatial density. The equation of v is as follows: Laser-based methods:
$v = N/t$
Satellite-based methods and Material-based methods:
$v = N/t = \theta * \rho$

6.3 r - Risk assessment model for three kinds of methods

To evaluate the risk for each method and give a certain result for the relative risk between the three method, we should first select the factors that can present the complete potential bad results of all kinds of methods, then establish an appropriate mechanism to score each factors in each method and make sure both the possible outcome of risk and the effort it requires to make up for the bad outcome. r can be represented by the following equation: $r = \theta(p1 \times s1 + p2 \times s2 + p3 \times s3)$
$\sum(pi)1$
Then comes our concrete process in risk assessment:

1. **Factor choosing process** To guarantee the comprehensiveness of the factors, we discuss all the possible bad out comings of each methods and summarize as follows:

   - **Breaking down**: Breaking down could be one of the most common bad results for any machine.
   - **Interfering with objects that are not supposed to influence**: There is a chance that the methods meant for cleaning the orbital debris influences objects that are not space junk.
   - **Additional financial risks:**Other potential costs that the methods may bring, expect the cost mentioned in section.(csection)

2. **Build a score mechanism** Take both the severity of the bad results and the difficulty to recover it into consideration, 4 stages represented by score 1-4 is shown as follows:

Score	Explanation
4	Severe and difficult to control $\div$ prevent
3	Not that severe but hard to control $\div$ prevent
2	Severe but easy to control $\div$ prevent
1	Not that severe and easy to control $\div$ prevent

   We use the below chart to show score of different methods

Methods	Breaking Down	Interfering with other objects	Additional financial risks	$r \div$
Laser-based	1	2	4	2
Satellite-based	4	2	1	2.
Material-based	1	4	1	2.

3. **Reasons for scoring: Breaking down:** if a laser system stops to work or the material is not effective, it only costs money to re-do them. But if a satellite breaks down, it become a huge orbital debris that may bring about plenty of small debris in orbits around earth. **Interfering with other objects:** If the three methods aim at the wrong target, there is huge possibility that the objects that they aimed at wrongly break down. Ways to prevent this error is easy for laser and satellite. But all objects moving through the material would be effected. The material-methods cannot discern ”good” from ”bad”. The only thing we could do is to send it to places that with less possibility for normal satellite or other things to pass. **Additional financial risks:** The laser station requires an additional maintenance

and repair cost. Besides, the field of view of a laser station is limited, which may lead to additional construction of laser stations if the firm is hired to clean debris out of that area. Let p1 = p2 = 0.4, p3 = 0.2, $r \div \theta$ is shown above in the chart.

7 Whether an economically attractive opportunity exists

To evaluate whether the result of our model could become an economically attractive opportunity, we create a new model based on Entering a New Market Model [17]. Before we explain how the model works, several assumptions should be restated.

1. We assume that the private firm have not entered the debris-removing industry before.
2. The private firm has no difficulty in getting the industry-relative license from the government.
3. The private firm has access to the capital, technology, raw materials and labor required in this industry.

The model consists of three major step:

1. **Assessment of the private firm's ability** According to assumption 3, the firm 's ability in capital, technology, raw materials and labor, which means this firm has enough ability to enter
2. **Assessment of the debris-removing market**
   - **Market size:** The urgency that we should deal with the increasing orbital debris in space has become a common sense for the world. The thought that debris-removing industry has a large market size is warranted.
   - **Market trend:** A NASA headquarters concept validation study [13] concluded that the idea of using pulsed lasers to remove essentially all dangerous orbital debris in the 1-10 cm range between 400 and 1100 km altitude within 2 years was feasible. Due to the urgency of this problem, a heat in the industry can be foreseen in such an emerging industry. With time passing by, the number of debris need to be removed becomes less and less,

the demand of this industry may decline. The trend of this industry in our model is like the graphic below:industry demand

- **Competitors:** There are two kinds of competitors that should be taken into consideration:
  1)**companies that produce the same product as this firm.** In this case, this kind of competitor means companies whose business includes removing orbital debris.
  2) **companies that provide alternative product.** In this case, these are the companies that doesn't provide debris-removing business, but they can do something else to protect the satellites or other space machines from the collision caused by orbital debris, such as building a protection shield around the spacecraft etc.
  After searching from the internet, we found that the main business for space companies is space transportation. Companies that aiming at removing debris or building shields cannot be easily found. Thus, we can infer the business of removing debris is at emerging stage and the competition is not severe.

- **Profit:**According to the equation $Profit = Revenue - Cost$, if we want to analysis profit, we must first see the revenue and cost in this industry.
  Cost: The cost of three kinds of methods has been analyzed in the previous section. The average cost is

  $$\$$$

  20k/kg. **Revenue:** Since the pricing strategy and operation model of this firm is unknown, we cannot make accurate estimation. However, no matter what kind of business strategy, it requires as highest efficiency as possible, which is analyzed in previous section as well. Take Laser-based methods as an example, the average number of Medium debris one laser system can deal with in one year is 20,000 [18], which is a relative large quantity.
  All in all, we can at least see the potential space for profit.

- **Barriers to exit:** if this industry stops growing and starts to decline, the resources in this industry can be used in other space business. For example, the rockets to send satellite could be used in space transportation business.

Factors in the debris-removing method	status
Market size	Large
Market trend	First grow, then decline
Competition	Not severe
Profit	Have potential profit
Barriers to exit	Relative small

Seen from the analysis above, the market is worth entering. Thus an economically attractive opportunity exists.

3. **Possible ways to start debris-moving business in this firm**

- Start from scratch and grow organicallyNot recommend, because it takes a lot of time to catch up with the top space companies.
- Acquire an existing player from within the industry: Recommend, because it helps the firm to quickly develop its own business.
- Form a joint venture/strategic alliance with another player with a similar interest: Recommend, a corporation between companies may allow the firm skip several hard steps in the business line. For example, the firm could cooperate with SpaceX, SpaceX provide rocket techniques while this firm provide other necessary parts.

8 Comparison between different methods

To briefly illustrate this, we use a table below, in which "+" represents advantages while "-" represents disadvantages.

Laser-based methods	satellite-based methods
+ Laser system could aim at a certain debris	the satellite can deal with a
+Laser would not cause additional debris	-retired satellites may beco
-One laser station can only have effect on debris within its field of view	+satellite can be ejected int
-laser cannot effect debris in high altitude	+satellite-based methods ca
+laser has cost and efficiency advantage when dealing with Medium debris	-satellite-based methods ha

9 The multi-objective optimal model

In this part, size i and method j $(i, j = 1 \cdots 3)$ can be interpreted as follows:

- size 1: small (less than 1cm)
- size 2: medium(between 1cm and 10cm)
- size 3: large(more than 10cm)
- method 1: satellite-based method
- method 2: material-based method
- method 3: laser-based method

Also there are some variables:

- x_{ij} represents the number of space objects sized i that we clean up with method j.
- Q_{ij} represents the unit profit that we use method j to clean up space objects of size i and it is the difference of revenue and cost ,which is related to the altitude and spatial density.
- T_{ij} represents the unit time that we use method j to clean up space objects of size i.It is also related to the altitude and spatial density.
- R_{ij} represents the unit risk that we use method j to clean up space objects of size i which can be estimated by the risk model above.

What we want is to maximize the profits which should be no less than C_1, to minimize the cleaning time which should be no more than T_1 and to minimize the risks which should be no more than R_1.

$$\max \sum_{i,j=1}^{3} x_{ij} Q_{ij}$$
$$\min \sum_{i,j=1}^{3} x_{ij} T_{ij}$$
$$\min \sum_{i,j=1}^{3} x_{ij} R_{ij}$$

s.t.

$$\sum_{i,j=1}^{3} x_{ij} Q_{ij} \geq C_1$$
$$\sum_{i,j=1}^{3} x_{ij} T_{ij} \leq T_1$$
$$\sum_{i,j=1}^{3} x_{ij} R_{ij} \leq R_1$$
$$x_{ij} \geq 0 \quad i,j = 1 \cdots 3$$

In order to get the standard multi-objective optimal model, we define d as a function of decision variable and d_0 as the goal value of d. Let $d^+ = \max\{d - d_0, 0\}$ and $d^- = -\min\{d - d_0, 0\}$ be the positive and negative deviation variable and d_1, d_2, d_3 represent the profits, time and risk, respectively. Then we have to decide the priority of the three goals. If we want the most profits, just put d_1 in the first place. If we want the least time, d_2 comes in the first place. The last priority is always the risks. Now we have two different goal functions:

$$\min z = P_1 d_1^- + P_2 d_2^+ + P_3 d_3^+ \tag{29}$$

or

$$\min z = P_1 d_2^+ + P_2 d_1^- + P_3 d_3^+ \tag{30}$$

$$(P_1 \gg P_2 \gg P_3) \tag{31}$$

such that

$$
\begin{cases}
\sum_{i,j=1}^{3} x_{ij}Q_{ij} + d_1^- - d_1^+ = C_1 \\
\sum_{i,j=1}^{3} x_{ij}T_{ij} + d_2^- - d_2^+ = T_1 \\
\sum_{i,j=1}^{3} x_{ij}R_{ij} + d_3^- - d_3^+ = R_1 \\
x_{ij} \geq 0 \quad i,j = 1\cdots 3 \\
d_i^+ \geq 0, d_i^- \geq 0 \quad i = 1\cdots 3
\end{cases}
\tag{32}
$$

For an altitude h, the spatial density and the coefficients in the constraints above can be determined. Given the parameters C_1, T_1 and R_1, we can solve the multi-objective optimization problem and obtain the best combination of the three methods.

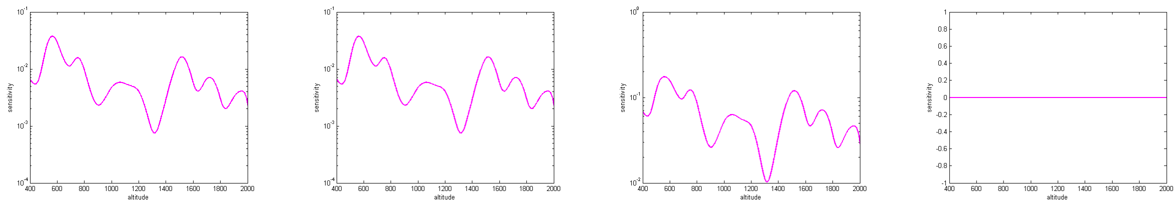
10 Sensitivity and Stability Analysis

10.1 A sensitivity analysis of the spatial density ρ to proportion λ.

In our first model, λ defines the proportion that large space debris explodes or collides into small space debris. The change of λ have different effects on the spatial density of different size.

Let λ_E and λ_C increase by 10%. We use subscript 0 as a symbol of the origin density distribution and 1 as the changed distribution. The sensitivity can be estimated by the index $\frac{\rho_0-\rho_1}{\rho_0}$. Using our model, the results are:

From the chart, while λ_E and λ_C increase by 10%, the increase of ρ and ρ_S is less than



(a) 1001 (b) 1002 (c) 1003 (d) 1004

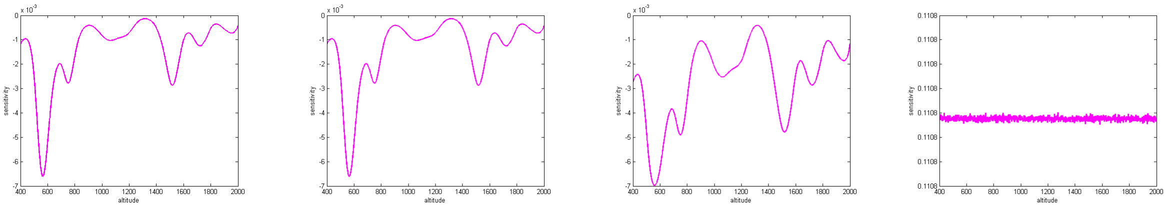
Fig 10: Example probability density functions

10%. ρ_M increases around 10% and ρ_L has no change. It implies that ρ_M is more sensitive to λ than ρ and ρ_S and the value of ρ_L is not related to λ at all.

10.2 A sensitivity analysis of the spatial density ρ to parameter β_E

The subscript and the index is the same as the previous one. Let β_E increase by 10% and the results of our model are:

Thus, ρ and ρ_S decrease by about 0.1% and ρ_M decreases by around 0.3%. They are



(a) 2001 (b) 2002 (c) 2003 (d) 2004

Fig 11: Example probability density functions

neither sensitive to the change of β_E. However, ρ_L has a positive correlation with β_E.

10.3 A stability analysis of P and F to cost

With the development of science and technology, the costs of emission and construction will decrease a lot. The value of CC,HC,EC,LC is a function of time. We assume that the average cost decreases by 20%. The indexes of stability are $\frac{P_0-P_1}{P_0}$ and $\frac{F_0-F_1}{F_0}$.

By calculation, we obtain $\frac{P_0-P_1}{P_0} = 1.6622 \times 10^{-16}$ and $\frac{F_0-F_1}{F_0} = 1.3553 \times 10^{-20}$.

As a result, P and F are hardly effected by the change of cost. The assessment model is stable while the cost doesn't change a lot.

If we increase N by 20% and keep other factors fixed, $\frac{P_0-P_1}{P_0} = 6.91\%$. By contrast, P or F is more sensitive to N.

10.4 A sensitivity analysis of the multi-objective optimal model

Consider the goal function

$$\min z = -P_1 \sum_{i,j=1}^{3} x_{ij}Q_{ij} + P_2 \sum_{i,j=1}^{3} x_{ij}T_{ij} + P_3 \sum_{i,j=1}^{3} x_{ij}R_{ij} \tag{33}$$

we define

$$\Delta z = \sum_{i,j=1}^{3} \frac{\partial z}{\partial x_{ij}} \Delta x_{ij} \quad , \quad \frac{\partial z}{\partial x_{ij}} = P_1 Q_{ij} + P_2 T_{ij} + P_3 T_{ij} \tag{34}$$

And the index of sensitivity is

$$H_{ij} = \frac{\frac{\Delta z_{ij}}{z}}{\frac{\Delta x_{ij}}{x_{ij}}} \tag{35}$$

The larger H_{ij} is, the more sensitive z is to x_{ij}. Since $P_1 \gg P_2 \gg P_3$, Δz_{ij} is mainly decided by the coefficients of the first priority. In this case, Q_{ij} plays an important role.

a definite conclusion or discuss your answer,but make it clear what part of the discussion is your opinion and what part is your conclusion. provide evidence to support your solution as good,if not the best answer. check the sensitivity and stability of your model.beaware of using the real data. describe the algorithm show them that you are astute enough to see the weaknesses yourselves. mention things you could do should more time and resources be available. According to NASA's EVOLVE model[3], the number of explosive fragments of size d, satisfies the following equation:

10.5 discussion of the strength and weaknesses

aaaaaaaaaabbbbbbb

1. ITEM1
2. ITEM2
3. ITEM3......

$$\kappa \equiv \frac{resource\ allocation\ at\ crime\ point,\ model\ 1}{resource\ allocation\ at\ crime\ point,\ model\ 2} \tag{36}$$

Offender	11	12
C	1	2
B	1	2
A	1	2

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