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Determination of the Optimum Cutoff Grade Considering Environmental Cost

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Abstract: Mining production planning is a very important subject of mine design process. One of the most important issues in mine production planning is the cutoff grade which is simply a grade used to distinguish between ore and waste. Waste materials may either be left in place or sent to waste dump. Ore is sent to the mill for further processing. Lower cutoff grade causes higher amounts of ore to be processed and subsequently lower amounts of waste materials to be dumped resulted in fluctuations in the cash flow of a mining project. Dumping waste is accompanied with the rehabilitation cost which will affect the overall cost of final production and also the optimum cutoff grade. Rehabilitation cost is the cost per tone of rehabilitating material of a particular type of rock after it has been dumped as waste. One of the most popular algorithms for determination of the optimum cutoff grade is Lane's method. Lane formulated the cutoff grade optimization, but he did not consider rehabilitation cost during optimization process. This cost item should be evaluated first, and then considered during cutoff grade optimization process. In this paper rehabilitation cost is inserted directly into cutoff grade optimization process using Lane's theory. The cutoff grades obtained using suggested method will be more realistic rather than ones by using the original form of the Lane's formulations.

Keywords: *Rehabilitation cost, Cutoff grade, Optimization, Production planning.*

1. Introduction

The production planning problem is related to the criterion that is used to optimize the open pit design. Ideally, the criterion should be the maximization of the NPV of the pit, but unfortunately, after four decades of continuing efforts, this goal could not be achieved. The reason for this problem has been simply paraphrased by Whittle (Whittle, 1989):

The pit outline with the highest value can not be determined until the block values are known. The block values are not known until the mining sequence is determined; and the mining sequence can not be determined unless the pit outline is available.

This is a large scale mathematical optimization problem which could not have been solved currently using commercial packages. The most common approach to the problem is dividing it into sub-problems similar to one shown in Figure 1.

The approach starts with the assumptions about initial production capacities in the mining system and estimates for the related costs and commodity prices. Then using the economic block values, each positive block is further checked to see whether its value can pay for the removal of overlaying waste blocks. This analysis is based on the breakeven cutoff grade that checks if undiscounted profits obtained from a given ore block can pay for the undiscounted cost of mining of waste blocks. Then, ultimate pit limit is determined using either a graph theory based algorithm (Lerchs & Grossman, 1965; Zhao & Kim, 1992) or a network flow one (Johnson & Barnes, 1988; Yegulalp & Arias, 1992) with the object of maximum (undiscounted) cash flow. Within ultimate pit, push backs are design so that the deposit is divided into nested pits going from the smallest pit with the highest value per ton of ore to the largest pit with the lowest value per ton of ore. These push backs act as a guide during the schedule of yearly based

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production planning. Before determination of the extraction scheduling, cutoff grade strategy should be defined to discriminate between ore and waste during scheduling process. Cutoff grade optimization is important because (Whittle & Vassiliev, 1998):

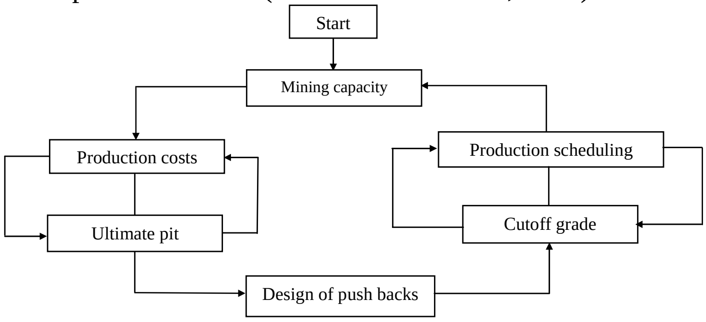

Figure 1. Open pit long-term production planning variables interacting in a circular fashion (Dagdelen, 2002).

- Cutoff grade optimization can improve both long-term and short-term cash- flows.
- Cutoff grade optimization is used for simulation of mining/ processing/ stockpiling configurations to determine which configuration yields the maximum economic benefit.

Lane (Lane, 1964) proposed an algorithm to determine cutoff grade strategy that maximizes the profit/net present value of a project subjected to mine, mill and refinery capacity constraints. In the present form of Lane's formulation, rehabilitation cost is not considered during optimization process. In this paper the effects of rehabilitation cost on the economic cutoff grade is surveyed. Also in order to show this effect, an illustration example is discussed.

2- Cutoff grade optimization

The objective of cutoff grade optimization is to determine the long-term ore/waste discrimination strategy that will maximize profits. In large open pit mines, there are typically three stages of operation: the mining stage in which various grades are mined up to some capacity. Material with the grade bellow cutoff is sent to stockpile and the amount above cutoff is sent to treatment stage, where ore is milled and concentrated with a capacity. In the third stage, concentrate is smelted and refined in order to produce sellable product. Each stage has its own capacity constraints and costs. Definition of the maximum capacities, unit costs and quantities of the model are presented bellow:

- ***Maximum capacity***

M: Maximum mining capacity in terms of tons per year.
C: Maximum concentrator capacity in terms of tons per year.
R: Maximum refinery and/or marketing capacity in terms of lbs per year.

- ***Costs***

m: Mining cost in terms of $ per ton of material removed.
c: Concentrating cost in terms of $ per ton of material milled.
r: All costs incurred at the product and selling stage. These costs are reported in terms of $ per unit product.
h: The rehabilitation cost which is the cost per tone of rehabilitating material of a particular type of rock after it has been dumped as waste. It also includes some haulage if waste is trucked farther than ore.
f: Fixed costs over the production period (for example 1 year)

- ***Selling price(s)****:* in terms of selling price per unit of product.
- ***Recovery (y)****:* The percentage of mineral that cab be recovered in the final product.
- ***Quantities****:* Q_m is the amount of material to be mined, Q_c is the amount of ore that is going to be sent to the concentrator and Q_r is the amount of product produced over

the production period of *T*. Therefore $Q_m - Q_c$ is the quantity of material that is sent to the waste dump.

With regard to the above definitions the profit equation is:

$$P = sQ_r - \left[mQ_m + cQ_c + rQ_r + fT + (Q_m - Q_c)h\right] \quad (1)$$

Combining terms yields:

$$P = (s-r)Q_r - (m+h)Q_m - (c-h)Q_c - fT \quad (2)$$

Economic cutoff grade may be limited by mining, processing and/or marketing capacities; consequently, six cases arise depending upon which of the constraint(s) is (are) limiting factor:

- ✓ If the mining capacity is the governing constraint, then the time needed to mine material Q_m is given by:

$$T = \frac{Q_m}{M} \quad (3)$$

Substituting Eq. (3) into Eq. (2) yields:

$$P = (s-r)Q_r - (m+h+f/M)Q_m - (c-h)Q_c \quad (4)$$

The amount of product (Q_r) is related to the quantity of ore sent to the concentrator (Q_c) by the following relation:

$$Q_r = \bar{g}.y.Q_c \quad (5)$$

Where $\bar{g}$ is the average grade sent for concentrator. Combining Eqs. (4) and (5) results in:

$$P = [(s-r).\bar{g}.y - (c-h)]Q_c - (m+h+f/M)Q_m \quad (6)$$

In order to find the grade that maximizes the profit under mining capacity constraint, the derivative of Eq. (6) must be taken with respect to grade (g):

$$\frac{dP}{dg} = [(s-r).\bar{g}.y - (c-h)]\frac{dQ_c}{dg} - (m+h+f/M)\frac{dQ_m}{dg} \quad (7)$$

Because the amount of material to be mined is independent of the grade, we have:

$$\frac{dQ_m}{dg} = 0 \quad (8)$$

Therefore, Eq. (7) becomes:

$$\frac{dP}{dg} = [(s-r).\bar{g}.y - (c-h)]\frac{dQ_c}{dg} \quad (9)$$

The lowest acceptable value of *g* is that which makes:

$$\frac{dP}{dg} = 0 \quad (10)$$

Thus, the cutoff grade (g_m) based on mining constraint is the value of $\bar{g}$ which makes:

$$[(s-r).\bar{g}.y - (c-h)] = 0 \quad (11)$$

Thus:

$$g_m = \bar{g} = \frac{c-h}{(s-r).y} \tag{12}$$

- ✓ If the concentrating rate is the governing constraint, then the time (T) is controlled by the concentrator:

$$T = \frac{Q_c}{C} \tag{13}$$

Hence, the profit function can be written as:

$$P = (s-r)Q_r - mQ_m - (c - {}^{f}\!/_{C})Q_c - (Q_m - Q_c)h \tag{14}$$

Following the same procedure which is done on the previous case, the cutoff grade when the concentrator is the limiting constraint (g_c) is:

$$g_c = \bar{g} = \frac{c + {}^{f}\!/_{C} - h}{(s-r).y} \tag{15}$$

- ✓ If the refining rate is the governing constraint, then the time is controlled by the refining as follows:

$$T = \frac{Q_r}{R} \tag{16}$$

Hence, the profit function can be written as:

$$P = \left[(s-r) - {}^{f}\!/_{R}\right]Q_r - mQ_m - cQ_c - (Q_m - Q_c)h \tag{17}$$

As before, the cutoff grade when the refining is the limiting constraint can be achieved as:

$$g_r = \bar{g} = \frac{c-h}{\left[(s-r) - {}^{f}\!/_{R}\right].y} \tag{18}$$

- ✓ If both mining and concentrating capacities are the limiting factors. In this case the Eqs. (4) and (14) are equal, ie.:

$$\begin{aligned}(s-r)Q_r - (m + h + {}^{f}\!/_{M})Q_m - (c-h)Q_c &= (s-r)Q_r \\ - mQ_m - (c - {}^{f}\!/_{C})Q_c - (Q_m - Q_c)h\end{aligned} \tag{19}$$

Equation (19) becomes:

$$\frac{Q_m}{M} = \frac{Q_c}{C} \tag{20}$$

Therefore, the balancing cutoff grade between mine and concentrator (g_{mc}) is the cutoff grade that results in satisfaction of Eq. (20).

- ✓ If both mining and refining capacities are the limiting factors. In this case the Eqs. (4) and (16) are equal. This gives:

$$\frac{Q_m}{M} = \frac{Q_r}{R} \tag{21}$$

As before, the balancing cutoff grade between mine and refinery (g_{mr}) is the grade that results in satisfying Eq. (21).

✓ If both refining and concentrator capacities are the limiting factor. In this case Eqs. (14) and (16) are equal. This gives:

$$\frac{Q_c}{C} = \frac{Q_r}{R} \tag{22}$$

And the balancing cutoff grade between concentrator and refinery (g_{cr}) is the cutoff grade that satisfies Eq. (22). Until now, six possible cutoff grades are achieved. Three (g_m, g_c, g_r) are based on capacities, costs and price; the other three (g_{mr}, g_{cr}, g_{mc}) are only based the grade distribution of the mined material and capacities. In order to find the optimum cutoff grade among these six cutoffs, the Lane's method can be applied (Lane, 1964).

Example

In this section we follow an example to show the effect of rehabilitation cost on the optimum cutoff grade.
In an open pit mine there is 1000 tons of material within the pit outline. The grade distribution of this material is shown in Table 1.
The associated costs, price, capacities and recovery are:

- *Maximum capacity*

M=100 tons/year
C=50 tons/year
R=40 lbs/year

Table 1. Initial material inventory.

Grade (lbs/ton)	**Quantity (tons)**
0.0-0.1	100
0.1-0.2	100
0.2-0.3	100
0.3-0.4	100
0.4-0.5	100
0.5-0.6	100
0.6-0.7	100
0.7-0.8	100
0.8-0.9	100
0.9-1.0	100
Total	1000

- *Costs*

M=\$1/ton
c=\$2/ton
r=\$5/ton
h=\$0.5/ton
f=\$300/year

- *Selling price(s)* is equal to \$25/lb

- *Recovery (y)* is equal to 1.0 (100% recovery is assumed).

Substituting the above data into Eqs. (12), (15) and (18) yields:

$$g_m = \bar{g} = \frac{2-0.5}{(25-5).1} = 0.075 \text{ lbs/ton}$$

$$g_c = \bar{g} = \frac{2+{}^{300}\!/_{50}-0.5}{(25-5).1} = 0.375 \text{ lbs/ton}$$

$$g_r = \bar{g} = \frac{2-0.5}{\left[(25-5)-{}^{300}\!/_{40}\right]1} = 0.12 \text{ lbs/ton}$$

The total profits assuming only one of the constraints is a limiting factor as a function of cutoff grade are given in Table 2 and shown in Figure 2. The balancing cutoff grades are:

$g_{mc} = 0.5$ lbs/ton

$g_{cr} = 0.6$ lbs/ton

$g_{mr} = 0.456$ lbs/ton

Now six cutoffs are candidate for overall optimum cutoff grade. All these cutoff grades have shown in Figure 2. Lane proposed a method to determine the overall optimum cutoff grade among all of these six cutoffs (Lane, 1964). His method was applied on this example and optimum cutoff grade is achieved as:

$g_{op} = g_c = 0.375$ lbs/ton

From the Table 2, the average grade of material sent to the mill for a cutoff 0.375 *lbs/ton* is:

Table 2. Average grade, quantities (Q_m, Q_c and Q_r) and the total profits as a function of cutoff grade considering rehabilitation cost.

Cut off (lbs/ton)	Average grade(lbs/ton)	Qm (tons)	Qc (tons)	Qr (lbs)	Pm ($)	Pc ($)	Pr ($)
0	0.5	1000	1000	500	4000	1000	3250
0.1	0.55	1000	900	495	4050	1650	3337.5
0.2	0.6	1000	800	480	3900	2100	3300
0.3	0.65	1000	700	455	3550	2350	3137.5
0.4	0.7	1000	600	420	3000	2400	2850
0.5	0.75	1000	500	375	2250	2250	2437.5
0.6	0.8	1000	400	320	1300	1900	1900
0.7	0.85	1000	300	255	150	1350	1237.5
0.8	0.9	1000	200	180	-1200	600	450
0.9	0.95	1000	100	95	-2750	-350	-462.5
1	1	1000	0	0	-4500	-1500	-1500

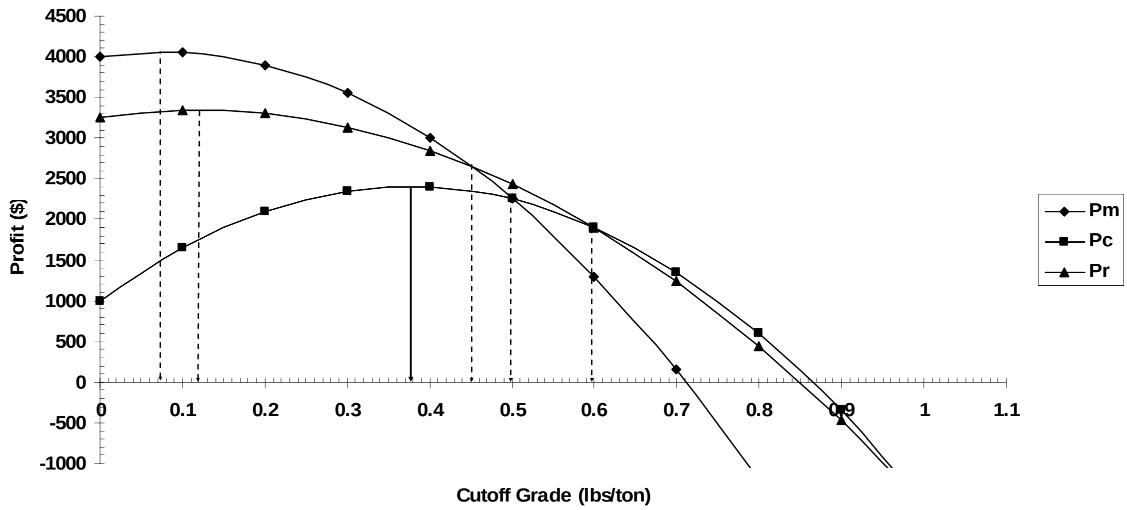

Figure 1. Total profit as a function of cutoff grade under different constraints.

$g_{av} = 0.688$ lbs/ton

Hence, the quantities are:

$Q_m = 1000$ tons

$Q_c = 640$ tons

$Q_r = 426$ lbs

And the amount of waste that should be dumped and rehabilitated is equal to:

1000-640=360 tons

According to the above quantities the total life of mine is equal to:

$$T = \frac{640}{50} = 12.8 \text{ years}$$

The total profit is equal to $2400.

In this example if *h* is set to zero then we have:

- Optimum cutoff grade =0.4 lbs/ton
- Average grade of material sent to the mill= 0.7 lbs/ton
- $Q_m = 1000$ tons
- $Q_c = 600$ tons
- $Q_r = 420$ lbs
- Total amount of waste that should be dumped and rehabilitated= 400 tons.
- Total life of mine =12 years
- Total profit =$2600

As can be seen in the above example, considering rehabilitation cost can results in decreasing optimum cutoff grade, decreasing the average grade of material sent to the mill, increasing the amount of ore sent to the mill and decreasing the amount of waste that should be sent to the waste dump. Inserting rehabilitation cost into the optimization process will force the model to decrease the cutoff grade as much as possible (in the light of economic and capacities considerations) in order to decrease the amount of material that is sent to the waste dump.

3- Net present value maximization

In the previous section, the optimum cutoff grade is obtained with objective of profit maximization. Nowadays, most mining companies attempt to maximize the Net Present Value

(NPV) rather than profit. The objective function of the problem with regard to NPV maximization is proposed by Lane and has the form of (Lane, 1964):

$$\nu = (s-r)Q_r - mQ_m - cQ_c - T(f + Vd) \tag{23}$$

Where:
ν : The difference between the present values of the remaining reserves at time *t=0* and *t=T*.
V: present value at time *t=0*.
d: The discount rate.
Now considering the rehabilitation cost into the optimization process results in converting the Eq. (23) as follows:

$$\nu = (s-r)Q_r - mQ_m - cQ_c - h(Q_m - Q_c) - T(f + Vd) \tag{24}$$

Thus,

$$\nu = (s-r)Q_r - (m+h)Q_m - (c-h)Q_c - T(f + Vd) \tag{25}$$

The above function should be maximized. As in the profit maximization case, we first take the derivative of ν with respect to grade and then set the derivative equal to zero. Finally the resulting equation is solved for the appropriate cutoff grades subject to mining, concentrating and refining constraints. This results in:

$$g_m = \overline{g} = \frac{c-h}{(s-r).y} \tag{26}$$

$$g_c = \overline{g} = \frac{c + \dfrac{f+dV}{C} - h}{(s-r).y} \tag{27}$$

$$g_r = \overline{g} = \frac{c-h}{\left[(s-r) - \dfrac{f+dV}{R}\right].y} \tag{28}$$

In Eqs. (26)-(27) *V* is unknown value, because it depends on the cutoff grade. Thus, the iterative process must be used in order to find the best cutoffs. This iterative process is the same as proposed by Lane (Lane, 1964).

4- Conclusion

Cutoff grade optimization is one of the major steps in mine planning and design of the open pits. Cutoff determine the destination of mined material, material above the cutoff is sent to concentrator and material bellow it is sent to waste dump. Lane's method is one the widespread algorithm for determination of the cutoff grade. In the previous forms of this algorithm rehabilitation cost has not been considered. In this paper this cost item is inserted into optimization process. Results show that considering rehabilitation cost can decrease the cutoff grade. As a result, the amount of ore that is sent to concentrator is decreased and the amount of material that should be sent to the waste dump is decreased also. Hence the total amount of rehabilitation cost during and after ore extraction is decreased and the total achievable NPV of the project will be increased.

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