Resources Policy 55 (2018) 184–191

Contents lists available at ScienceDirect

Resources Policy

journal homepage: www.elsevier.com/locate/resourpol

Cutoff grade optimization in open pit mines using genetic algorithm

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ARTICLE INFO

Keywords:
Cutoff grade
Optimization
Net present value
Genetic algorithm

ABSTRACT

The fundamental objective of production planning is to create a mechanism for the implementation of the mining cutoff grades and short-term production planning. One of the most important parameters in open pit design is a determination of the optimal cutoff grade. Optimum cutoff grade results in maximizing profits or maximizing the net present value. Given that in determining the cutoff grade with the goal of maximizing profits, a constant value is obtained for the entire life of the mine. The annual income of the mine will be the same throughout its lifetime, and the time value of the money has not been neglected; which is the main disadvantage of this optimization process. While in optimization with the goal of maximizing net present value, the optimal value will be a function of the time and will be greater in the early years of the mine and will gradually decrease. Optimization of cutoff grades with the aim of maximizing the net present value over the life of the mine is important due to its dependence on the economic parameters, the design of the open pit mining and fundamental issues. Maximizing the net present value is a nonlinear programming problem. To determine the optimal cutoff grade Lane method is commonly used. Lane provided his method to determine the optimal cutoff grade by considering factors such as the capacity of the mine, concentrator and load capacity of the treatment plant, the time value of money and distributing grade. The procedure of the Lane method for cutoff grade calculation is complicated and time-consuming. Considering the widespread use of heuristic methods in optimizing parameters, In the present study genetic algorithm, which is a smart algorithm, is used to determine the optimal cutoff grade. In this paper, we compare the efficiency of the genetic algorithm and Lane's theory in optimizing the degree of limit based on maximizing the net present value. Using separate programming based on the genetic algorithm and considering the capacity limitations and the proportion between the parameters of mining to smelter and refining in the mine is done. For this purpose, consider precision of 0.001%, optimum cutoff grades, the amount of output per unit and the net present value are calculated. The optimum cutoff grade at the beginning of the life of the mine is equal to 0.506% and at the end of the life of the mine to 0.222% using the genetic algorithm. Using the Lane model at the beginning of the life of the mine the optimum cut-off grade from 0/503% to 0/220% reaches at the end of the mine's life. The net present value of earnings over a lifespan of 7 years mine in the genetic algorithm and Lane model is $ 93,467,914 and $ 94,408,000 respectively. Also, the amount of mining, the amount of processing and the amount of refinery obtained by the genetic algorithm method are compared to the Lane model. The results of the research indicate high speed and very low error of genetic algorithm and also a convergence of results with the Lane method.

1. Introduction

A grade of the ore and waste defines the boundary between simple means; it is a technical and economic scale which is determined by various parameters such as geological features (such as grade distribution), technical limitations of operations and financial parameters. The cutoff grade strategy for an open pit mine has an impact on annual liquidity flows and the net present value of the project. Therefore, in each given period, the allocation of materials sent to the processing unit and the production of the product in the refinery unit for sale to the cutoff grade depends (Cetin and Dowd, 2016). Due to the dependence of technical and economic parameters on the cutoff grade; determining this value is the fundamental issue at different periods over the life of mine planning, and it is the most difficult problems facing engineers (Rahimi and Ghasemzadeh, 2015; Azimi et al., 2013). Take into account that in calculating the break-even cutoff grade based on the break-even analysis; Time Value of Money, grade distribution and operational capacity of the various units are not considered. So, mining operation according to this grade will not lead to the optimization of the process. For this reason, since 1954 cutoff grade optimization has been gained more attention

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https://doi.org/10.1016/j.resourpol.2017.11.016
Received 24 August 2017; Received in revised form 13 November 2017; Accepted 28 November 2017
Available online 11 December 2017

(Ataei, 2003; Osanloo et al., 2008). In determining the optimum cutoff grade, the issues should be pointed out such as changing the cutoff grade over time for technological and economic reasons, and also the difference in the distribution of grade in different parts of the mineral deposit (Azimi and Osanloo, 2011). Considering the scope of mining activities, the price changes of minerals, the presence of valuable Secondary metals, the need to determine and optimize the cutoff grade and determining strategies mixing of minerals grade low and grade high in different scenarios to increase revenue and reducing tailing especially at high depths makes it inevitable (He et al., 2009; Bascetin and Nieto, 2007). In recent years, numerous attempts have been made to develop models and relationships that are capable of calculating cutoff grade optimization in order to maximize the net present value. In these relationships, in addition to economic factors, limitations such as mining capacity, processing capacity, smelter capacity, refinement, and the time value of money are also considered (Lane, 1988; Thompson and Barr, 2014). In order to calculate the optimum level, the minerals must be extracted so that the net present value of the operation is maximized. Thus, during the first years of the mine's life, extraction of high-grade materials will be achieved and the net present value will increase (Azimi and Osanloo, 2011; Ataei and Hosseini, 2011). In the Lane model, mining operations are divided into mining, processing and rifinery units. Then, using 6-grade candidates, based on the limited capacity of each of the three stages or they mutually balance, the optimum cutoff grades are calculated each year (Lane, 1988). After Lane theory, there is no independent method or algorithm done by other researchers; they focused on the use of other optimization methods based on the Lane method or investigating the role of various factors in this case based on Lane theory. Among these studies in metal deposits, in recent years, we can mention the following: The cutoff grade Optimization of single-metal ore deposits with the goal of maximizing the net present value by using the methods of knocking out and comparing its results with the Lane model (Ataei and Osanloo, 2013; Asad and Dimitrakopoulos, 2013). In 2005, during the study of Lane's algorithm, the various adjustment factors for the product price, fixed and operational costs combine in this algorithm, and proved the effectiveness of the adjustment of economic parameters in the target function. In this case, prices are monitored dynamically. (Asad, 2005). Were able to determine and optimize the optimum cutoff grades by applying an optimization factor based on the Generalized Reduced Gradient (GRG) algorithm for a metal mine (Bascetin and Nieto, 2007). To optimize cutoff grade by considering environmental issues based on minimizing acid leakages, they construct a model (Rashidinejad et al., 2008). The model is based on Lane's theory, with the difference that it takes into account the costs of waste accumulation and reduces incomes, and based on this model, determines the optimum cutoff grades (Gholamnejad, 2008). With the introduction of artificial intelligence technology in the field of mineral activity, Using an artificial neural network and a genetic algorithm, they developed a model for nonlinear simulation of mineral activity to optimize the cutoff grade (He et al., 2009). Determined a model based on the Lane algorithm, taking into account the combined stock mineral grade low, economic parameters, and modifications to optimize the cutoff grade (Asad and Topal, 2011). A model for determining the optimal cutoff grade of open pit mines by using the strategy of combining the genetic algorithm and nonlinear programming (Azimi and Osanloo, 2011). It was able to create a number of scheduled schedules for the cutoff grades and production rates, and then use dynamic planning to optimize and determine the cutoff grades that maximize the net present value (Barr, 2012). They developed a model for optimization cutoff grade based on Lane's theory that has three stages of mining, processing and market operations. In this model, the price of the mineral for the entire life of the mine is not fixed and variable. Using this model, the effect of mineral price changes on the optimum cutoff grade (Khodayari and Jafarnejad, 2012). In order to optimize the cutoff grade based on Lane's theory, with the goal of maximizing the net present value, multi-stage random planning was used for a metal mine (Li and Yang, 2012). To modify the Lane method, a model for optimizing the cutoff grade of a metal mine, taking into account the variable capacities of the units during the life of the mine, has improved the results (Abdolahisharif et al., 2012). The multi-criteria decision scoring method was used to optimize the cutoff grade and optimum cutoff grade under uncertain prices and to plan the production of a mine (Azimi et al., 2012). A model for optimizing the cutoff grade of minerals gold, lead, and zinc was determined using the genetic algorithm and compared the results with web search method and dynamic programming (Cetin and Dowd, 2016).

Lane (1988) modeled the process of operating a mine just sold its refined product for cutoff grade optimization, and accordingly define the objective function; as a result, his method is not usable in metal mines capable of producing and selling multiple product types (Mohammadi et al., 2015; Rafiee et al., 2016). The main aim of this study is to model the process of operating a mine, and use of the proposed model to determine the cost, revenue and profit can be achieved. Also, the objective function is defined based on the maximization of net present value. To optimize the objective function, the genetic algorithm as a meta-heuristics and intelligent method is used. The main advantage of the Intelligent methods is that they are free derivative methods. So, a solution of the problem is achieved easily and at high speed using such a method. The genetic algorithm coding is done in MATLAB R2012a software, and using this code, those values of optimum cutoff grades, mine production units, profit from operations and net present value of a hypothetical ore deposit are calculated.

2. Methodology

2.1. Objective function

According to Lane algorithm for cutoff grade optimization, mining operations including three stages of mining, concentrate production, smelter, and refining are taken into account. Each of these steps associated with costs and each one also has a limited capacity. Moreover, fixed costs are also included. Considering the expenditure and income in these operations, operating profit is calculated from the following equation (Hustrulid et al., 2013):

$$P = (s - r)Q_r - mQ_m - cQ_c - fT \tag{1}$$

Where **T** is the production period, $\mathbf{Q_m}$ is the amount of material that should be mining, $\mathbf{Q_c}$ denotes the amount of ore sent to the concentrator, $\mathbf{Q_r}$ represents the final product, **f** denotes the fixed costs per unit time, **S** is the selling price of the final product, **m** is the cost of mining each of material per tonne, **c** is the price of the condensed minerals per ton, and **r** is smelter costs per unit of final product. If **d** is the discount rate, the difference between the net present value of the remaining reserves at time t = 0 and t = T after the mining operation is (Hustrulid et al., 2013):

$$v = (s - r)Q_r - mQ_m - cQ_c(f + Vd)T \tag{2}$$

In which, **V** is the net present value of the remaining units in operation (time t = 0), and it can be achieved using replication process. The amount of refined material ($\mathbf{Q_r}$) which is sent to the mineral processing plant ($\mathbf{Q_c}$) depends on the amount of minerals. Considering the amount and average grade of minerals sent to the mineral processing facility (**g**), and percent recovery (**y**); the amount of refined material ($\mathbf{Q_r}$) is:

$$Q_r = \bar{g}.\, y.\, Q_c \tag{3}$$

$$v = [(s - r)\bar{g}y - c]Q_c - mQ_m - (f + Vd)T \tag{4}$$

To maximize the NPV, the value of **v** should be maximized. The capacity of each of the units, mining, concentrator, and a refining plant can be the limiting factor in the optimization process. Depending on which of the capacity restriction is in effect, the value of **T** in Eq. (4) changes. If mining capacity (**M**) is a decisive limitation, the amount of **T**

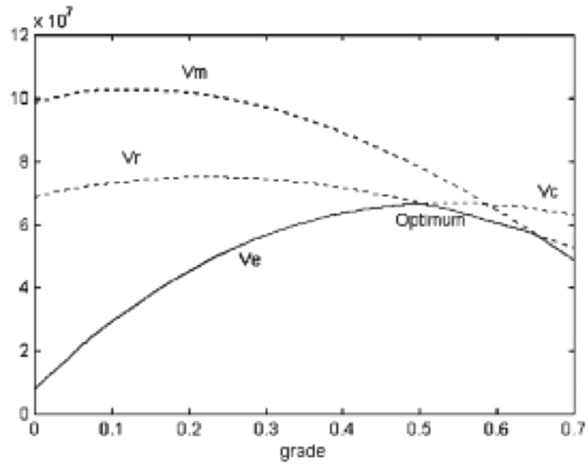

Fig. 1. Variation of V_m, V_c, V_r, And V_e vesus the corresponding grade (Rafiee et al., 2016).

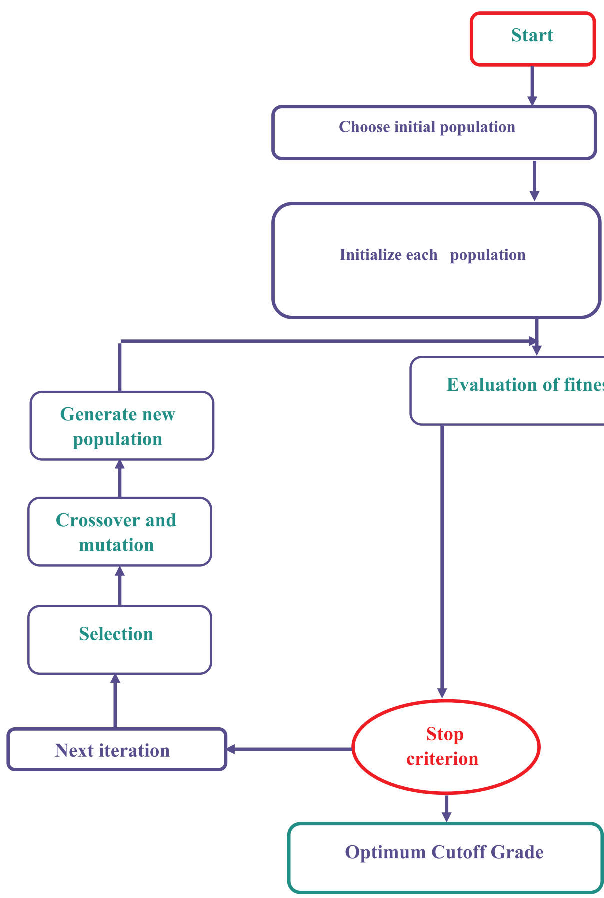

Fig. 2. Graphical illustration of genetic algorithm (GA).

is equal to Q_m / M, and if the capacity of the concentrator (**C**) is a decisive limitation, the amount of **T** is equal to Q_c / C. As a final case, if treatment capacity (**R**) is a decisive restriction, the amount of T is equal to Q_r / R. Therefore, for each of these restrictions, a value **v** is obtained:

$$v_m = [(s - r)\bar{g}y - c]Q_c - [m + \frac{f + Vd}{M}]Q_m \tag{5}$$

$$v_c = \left[(s - r)\bar{g}y - \left(c + \frac{f + Vd}{C}\right)\right]Q_c - mQ_m \tag{6}$$

$$v_r = \left[\left(s - r - \frac{f + Vd}{R}\right)\bar{g}y - c\right]Q_c - mQ_m \tag{7}$$

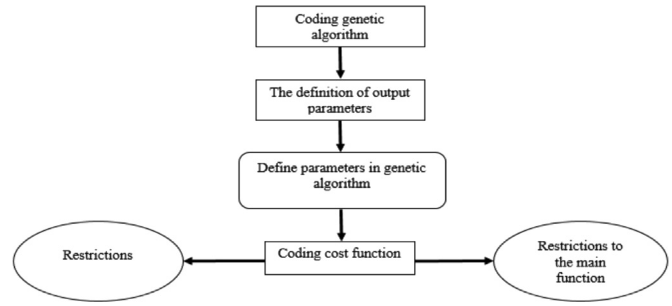

Fig. 3. The steps to implement a genetic algorithm.

Table 1
Distribution of grade deposit.

Grade classes		Quantity (tons)	Grade classes		Quantity (tons)
From	Until		From	Until	
0.0	0.15	14,400,000	0.45	0.5	3,800,000
0.15	0.2	4,600,000	0.5	0.55	3,700,000
0.2	0.25	4,400,000	0.55	0.6	3,600,000
0.25	0.3	4,300,000	0.6	0.65	3,400,000
0.3	0.35	4,200,000	0.65	0.7	3,300,000
0.35	0.4	4,100,000	0.7	1.5	42,300,000
0.4	0.45	3,900,000	Total tonnage		100,000,000

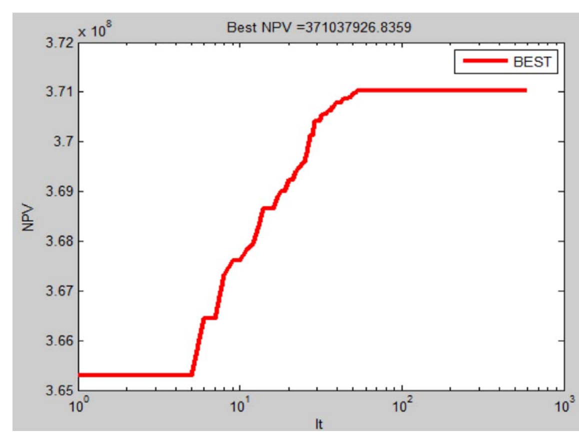

Fig. 4. Optimized net present value using a genetic algorithm versus number of iterations.

In the above three modes, $\mathbf{v_m}$, $\mathbf{v_c}$, $\mathbf{v_r}$ can be plotted as a function of scale. It should be noted that, in all the plots, they have upward convexity as shown in Fig. 1. As mentioned above, the purpose of obtaining optimal cutoff grade is a determination of grade at which v is maximum. Since the there are three functions **v**, One should try to optimize the three objective functions as much as possible. The common part of three functions should be considered and then the maximum value of the common part of the three functions to be determined. The common part of these curves in different ranges is the minimum value of $\mathbf{v_m}$, $\mathbf{v_c}$, $\mathbf{v_r}$. Therefore, to find out an optimal cutoff grade, first, the common section of three curves should be determined ($\mathbf{v_e}$), then the grade that maximizes this section should be calculated. In other words, the goal is to gain the grade that will satisfy the following function.

$$\max v_e = \max\,[\min(v_m, v_c, v_r)] \tag{8}$$

2.2. Genetic algorithm

Genetic Algorithm is a mathematical algorithm in which a set of mathematical objects, often referred to as strings of constant length characters, which call these strings a chromosome. Based on the Darwinian evolution theory, the survival of the superior generation with respect to the compatibility factor was first introduced by Darwin in 1852 and generates a new generation using various genetic actions. Among the inspirational optimization techniques of nature, The genetic algorithm is the evolution of them.

This algorithm is included in the class of random optimization algorithms. This algorithm is particularly suited to optimizing complex issues with an unknown search space. Genetic algorithms have been very successful for classical optimization methods for solving linear problems, convex, and some similar difficulties, but genetic algorithms are much more efficient for solving discrete and nonlinear problems (He et al., 2009).

Genetic Algorithms (GA), to gain a robust search engine and optimization approach, implements the origin of natural evolution with the genetic proliferation of characteristics which called the principle of "survival of the fittest". The important characteristic of a genetic algorithm (GA) is that it specifies simultaneously many feasible answers and explores various areas in the desired space chosen by the researcher (Ahmadi and Golshadi, 2012; Ahmadi et al., 2015; Holland, 1975; Hassan et al., 2005). GA employs a straightforward answer to genetics in biological systems, and Darwinian natural selection is a robust further to the classical methods.

The main idea of the genetic algorithm is taken from Darwin's theory of evolution (1859). Those natural traits that are more compatible with natural laws, Have a greater chance of survival. Darwin's evolutionary theory has no analytical and definitive proof. But it has been empirically and statistically confirmed. Among other people, human society, animal, and plant community create a new generation through mating. The chance for a person to survive in a new generation depends on the specific chromosomal composition of that person in the new generation. Usually, new generation people are more compatible with nature. In rare cases it is possible, an inventory with excellent characteristics and high compatibility may be produced. In short, in each generation, reproductive opportunities are better represented, and species with unfavorable characteristics are gradually eliminated. As a result, by the passing of time, people of different generations evolve In nature, the combination of better chromosomes, Better generations will emerge. Sometimes there are mutations in the chromosomes, which may lead to better next generation (Cetin and Dowd, 2016). According to the Darwinian theory of 'survival of the fittest', genetic algorithm (GA) can gain the best answer after a series of loop calculations. Artificial selection, crossover, and mutation operators constitute the search procedure above. The genetic algorithm also solves the problem using this idea.

To apply the genetic algorithm, first, the problem parameters are coded as binary strings. Each answer corresponds to a matching fitness that identifies the quality of the response to other members of the population. The more fitness the answer is, the chance for more survival and reproduction and it will appear in later generations. A simple algorithm of three operators selection, crossover, and mutation. Each strand of the zero ones consists of a pink code encoded by the answer to the optimization problem. The genetic algorithm generates the next generation from the current generation using a crossover and mutation operators. This cycle continues until the condition is met (Azimi and Osanloo, 2011).

A) **Functional function**: In solving all optimization problems, it is necessary to determine an appropriate optimality criterion. The objective function is a function that is supposed to be optimized and provides the necessary tools for evaluating each discipline. This function assigns to each string a numerical value that determines its quality in comparison to other strings. The higher the response thread quality is, the more amount of fitness is the answer.
B) **Selection**: Select a process in which some strings are matched to their fitness function. The easiest way is to use a roulette wheel, in which each strand of the population, in proportion to the amount of self-discipline, comes from the wheel. With each wheel rotation you want, a candidate is selected.
C) **Crossover**: Intersection is performed in two steps; First, members are randomly selected for mating, and then each pair of strings is randomly cut, and pieces are cut off after cutting.
D) **Mutation**: After the crossover operator, the strings are exposed to the mutation operator. The operation mutation is random, that is, the locale of the string is randomly selected and the mutation applied to it.
E) **Apply limitations**: In many practical issues, one or more limitations should be considered. The genetic algorithm produces a string of parameters that are tested using the system model, target function, and limitations. With the system model and the evaluation of the target function, violations of the constraints are investigated.
F) **Stop condition of the algorithm**: The stop conditions are of two types, which are the condition of the inactive and sequential. The inactive condition acts by the passage of a fixed number of generations, but the sequel condition requirement is that the population becomes more uniform than a certain amount. For example, if 25% of the genes have the same value, then that convergent gene is considered (Kia, 2015; Fatahi, 2011).

A correct route for an issue is commonly named a chromosome or an individual in genetic algorithm (GA) approach. In the first stage of the algorithm, an initial population, demonstrating deputies of the possible route, is generated to commence the search procedure. The components of the population are coded into chromosomes that are bit-strings. With the aim of another index that named fitness value, the robustness of the strings above is assessed that illustrating the restrictions of the issue. Chromosomes are chosen for further genetic manipulation by their own fitness value. It is worth mentioning that the process of selection is chiefly dependable on guaranteeing the survival of the best-fit chromosomes. Further choosing of the chromosomes strings the genetic treatment procedure including two stages is implemented. In the previous stage, the crossover operation which recombines the genes of each two chosen chromosomes is utilized. Different kinds of crossover operators are carried out in previous works. The points for the crossover of each two chromosomes are collected casually. The next stage in the genetic utilization procedure is called mutation process, which the bits of the chromosomes are changed at one or more randomly chosen positions. The operation of mutation aids to defeat issue of trapping at a local optimum. The off-springs generated by the genetic utilization procedure are the following population to be assessed (Ahmadi and Golshadi, 2012; Ahmadi et al., 2015; Holland, 1975; Hassan et al., 2005). The graphical demonstration of the GA- algorithm is illustrated in Fig. 2.

2.3. Optimization procedure

In this study, a real-coded genetic algorithm was used to optimize the objective functions. To implement genetic algorithm optimization, limitations of the parameters should be defined. Therefore, three restrictions including limitations of extraction, processing, and smelter were employed and considering these limitations several penalties for improving the precision were defined.

In the coding section of the genetic algorithm, you must split the window opened in MATLAB software into five sections, which include: problem definition, parameters settings, initialization, main loop, and outputs. In the definition section of the problem, for the implementation of the genetic algorithm, the target function is called, and the number of variables in the problem and the size of the variables matrix are determined. In the parameters setting section, respectively, the upper and lower limits of the variables, population numbers, maximum repetitions, crossover percentage and crossover number, the percentage of mutations, and the number of mutations and the matrix of upper and lower limit variables are written. To begin, the genetic algorithm needs an initial value and the number of the target population, this value can be randomly generated. In the main ring section, for the best answer in each replication, a zero-dimensional matrix is defined as the maximum generation production and one, and the number of generations (from one to the maximum number of generations production) is applied to the crossover and mutation operators to get the population from solutions. To apply any of the mutation and crossover operators, Separately, a function is defined to run them, and the required parameters are referred to as input data to the function, the following is for the implementation of the intersection function: the number of crossovers, the main population and its number, and the space generated for storing generations produced from the crossover, as well as for the mutation operator, include, the main population and their number, the number of mutations, the number of variables and their upper and lower limits, and the space created to store generations produced by the mutation operator. The total population, include, the main population, the produced population results from the crossover operator, and the produced population results from the mutation operator, and this population is arranged according to their value, which is achieved by their target function, and separated by the size of the main population, and considered as a better population. The first member of the new population is the worst member of the population because the goal is to maximize the objective function.

The first step in the implementation of genetic algorithm optimization is defining all the inputs, variables and parameters in the form of a matrix. In this algorithm, the number of variables (n_{var}) is equal to the number of cutoff grades, the mining rate, the amount of processing, and the amount of refinery unit in each year. Accurate determination of the number of population is one of the most important factors in the achievement of the optimal solution at the right time. In this research, the number of population is equal to 1000 and this value gained by a try and error method. Also, the termination condition of genetic algorithm optimization was set based on a maximum number of repetitions. It should be noted that practical limitations were considered in the genetic algorithm optimization to improve the convergence speed as well as reasonable accuracy. The reasonable restrictions are Mining constraints, processing restrictions, and refinery unit limitations. In the case of any of these limitations being violated, the value of the objective function for this string of specific solutions is attributed to a minimal amount. Therefore, the possibility of choosing this solution is very small and using the penalty function, if we come up with a final answer, our answer is definitely a practical answer. Moreover, the restrictions apply to the objective function; at this stage, the objective function equation is implemented using the penalty method. Then each of the constraints is added to the objective function separately. Values (α_1), (α_2), (α_3) factors are subject to the penalties. The number of penalties for each limitation have been limited by the multiplication of the factors of the penalty imposed on the amount of the violation. Finally, the objective function value is achieved by applying the code along with all restrictions.

Three penalty functions are considered for the genetic algorithm, Relationships 9–11 are the penalty functions of the genetic algorithm.

$$Z_1(n) = \alpha_1 \times \left(\frac{Q_r(n)}{Q_c(n)} - \frac{R}{C}\right) \tag{9}$$

$$Z_2(n) = \alpha_2 \times \left(\frac{Q_c(n)}{Q_m(n)} - \frac{C}{M}\right) \tag{10}$$

$$Z_3(n) = \alpha_3 \times \left(\frac{Q_r(n)}{Q_m(n)} - \frac{R}{M}\right) \tag{11}$$

In these relationships:

$Z_1(n)$: The amount of penalty related to the limitation between the processing plant and the refinery unit per year
$Z_2(n)$: The amount of penalty related to the limitation between the mining capacity and the processing plant per year
$Z_3(n)$: The amount of penalty related to the limitation between the mining capacity and the refinery unit per year
$Q_m(n)$: The amount of minerals per year
$Q_c(n)$: Mineral amount sent to processing plant per year
$Q_r(n)$: The amount of refined matter per year
M : Capacity Mining
C : Capacity of the processing plant
R : The capacity of the refinery unit

Finally, the objective function is coded, and the value of the objective function is achieved by applying all the limitations. In Eq. (12), the objective function of the genetic algorithm is given.

$$V(n) = \left(\left(\frac{P(n)}{(1+\ d)^n}\right) + Z_1(n) + Z_2(n) + Z_3(n)\right) \tag{12}$$

In this regard:

$V(n)$: net present value per year
$P(n)$: Profit per year
d: discount rate

After executing the program, the value of the best answer for each repetition and other parameters that are required is displayed, and the final value of the best answer is determined. In Fig. 3, the steps of the implementation of the genetic algorithm are shown.

3. Results and discussion

To apply the genetic algorithm, the optimal grade of the deposit is calculated hypothetically. Table 1 reports a grade distribution for the range of 100 million tons of ore. Mining capacity (extraction of ore and waste removal) 20 million tons per year, concentrator capacity of 10 million tons per year and 90 thousand tons per year refinery capacity, mining costs of $ 0.5 per ton, condensed cost of $ 0.6 per ton, smelter costs $ 50 per ton, the fixed cost of $ 4 million per year, mineral sales price of $ 550 per ton, recovery 0.9(90%) and discount rate is equal to 15%.

In solving this problem, we are going to find a combination of mining capacity, concentrator and a refining unit that will provide the maximum benefit and net present value. The advantage of this program is finding out the optimum cut-off grade based on the maximum net present simultaneously. To solve this problem 300 randomly, populations are generated as initial population, and they used in the first optimization loop. In this algorithm, the number of variables needed to solve, according to the mine life, is equal to 28. To produce a new generation in the genetic algorithm, the crossover rate is equal to 0.8, and the mutation rate is equal to 0.6. The uniform selection function is also used to provide a better result. Maximum iteration number in the genetic algorithm is considered as 600. To consider the effect of the limitations on NPV, different restrictions are considered. These limitations including the capacity of the concentrator and refinery, the capacity of mining and concentrator, and the capacity of the refinery and mining. It should be noted that in a case of contravention, the violation has taken place. Running the genetic algorithm for more than 20 times results in a decrease in error for both of the cutoff grade and the net present value as close as possible to zero. But, there is no reduction was observed in the objective functions including the capacity of the mine, concentrator, a refining unit, and grade. Thus, after optimization, the net present value of hypothetical deposits increased to $ 371,037,926. Fig. 4 depicts the variation of the NPV versus the corresponding iteration number. As depicted in Fig. 4, after 40 iterations the NPV increased to the maximum value.

To check the validity of the proposed model, the optimal cutoff

Table 2
Optimum cut off grade, production of various units, profit and net present value in the years of the project life using genetic algorithm.

Year	Cutoff Grade optimization (%)	The mining (tons)	The concentrate (tons)	Filtration rate (tons)	Profit ($)	Net present value ($)
1	0.506	17,834,634	9,932,519	89,411	25,828,672	93,467,914
2	0.493	17,551,431	9,987,398	89,292	25,877,846	82,823,407
3	0.455	16,786,703	9,926,552	87,427	25,364,217	70,366,059
4	0.411	15,870,685	10,000,000	84,820	24,474,568	56,030,989
5	0.357	14,695,600	10,000,000	81,777	23,540,700	41,056,582
6	0.296	13,729,677	10,000,000	77,595	21,932,662	23,437,702
7	0.222	3,300,000	2,600,000	18,030	1,805,000	1,671,437

Table 3
Optimum cut off grade, production of various units, profit and the net present value of the project life years using lane method.

Year	Cutoff grade optimization (%)	The mining (tons)	The concentrate (tons)	Filtration rate (tons)	Profit ($)	Net present value ($)
1	0.503	17,835,000	10,000,000	89,968	26,067,000	94,408,000
2	0.493	17,586,000	10,000,000	89,342	25,878,000	82,725,000
3	0.460	16,860,000	10,000,000	87,425	25,283,000	69,869,000
4	0.412	15,857,000	10,000,000	84,561	24,352,000	55,740,000
5	0.361	14,879,000	10,000,000	81,489	23,305,000	40,186,000
6	0.294	13,727,000	10,000,000	77,455	21,864,000	23,146,000
7	0.220	3,257,000	2,568,000	18,834	2,248,000	528,000

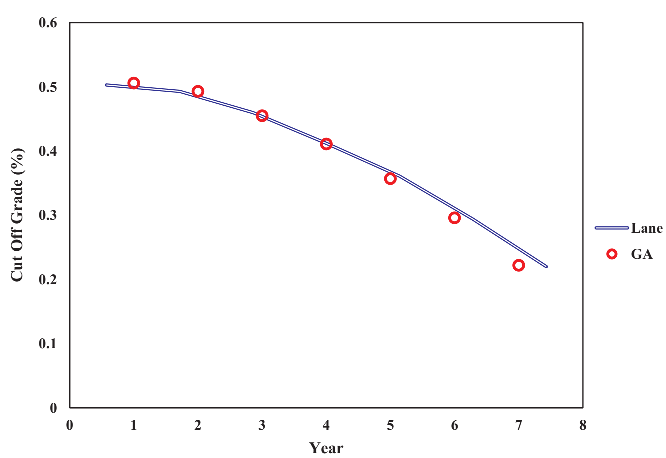

Fig. 5. Comparison of two methods to optimize cut off grade.

grade of ore was calculated theoretically using a method based on the Lane theory. The results of this method are shown in Table 3. At the beginning of the project the optimum cutoff grade was 0.503%, and at the end of the project life is equal to 0.22%. Mine, concentrator and a refining plant are not working at full capacity, as well as the net present value obtained from the proposed method are equal to 94,408 thousand dollars. The results gained from genetic algorithm are shown in Table 2. As reported in Table 2, the optimal cutoff grade at the beginning of the project is equal to 0.506% and at the end of the project is equal to 0.222%, and the net present value is 93,467,914$. As can be seen from Table 2, the results of the genetic algorithm have a good agreement with the outputs of the Lane method. The results of both methods were compared graphically in Figs. 5–10. According to Fig. 5, the optimal cutoff grade percentage is close together in both methods. Table 4 shows the optimal cutoff grade errors rate by genetic algorithm. Also, the average error in the genetic algorithm model for an optimal cutoff grade percentage of 0.64%. The results of the genetic algorithm are found to be faster than the results of the Lane model. So that the time Come to the answer at the genetic algorithm is about 47 s, but the time to reach the answer in the Lane model is high, and it takes about 22 min. In terms of time, the genetic algorithm is about 28 times faster than To the answer the Lane model.

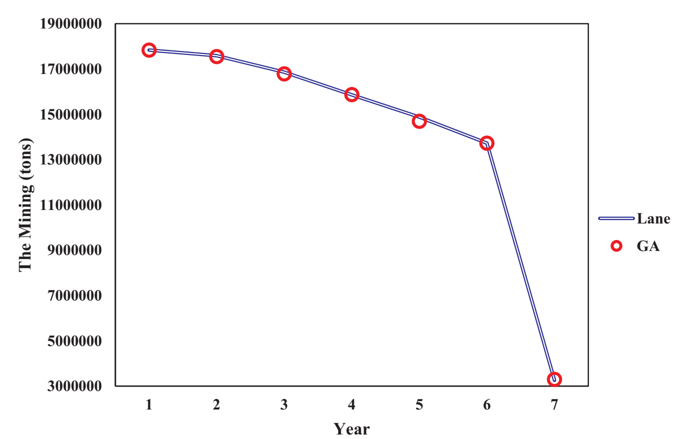

Fig. 6. Comparison of two methods of mining.

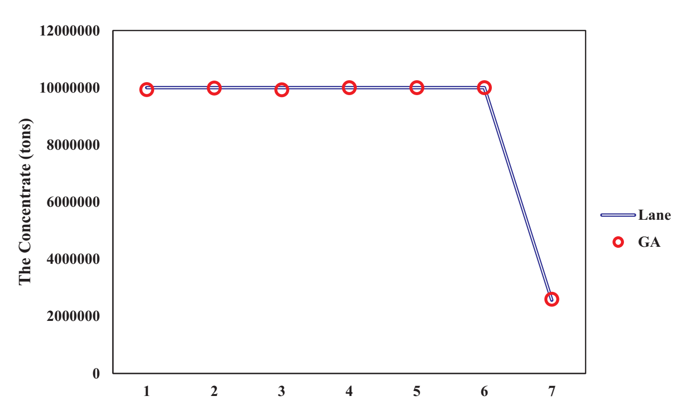

Fig. 7. Comparison of two methods of concentrate.

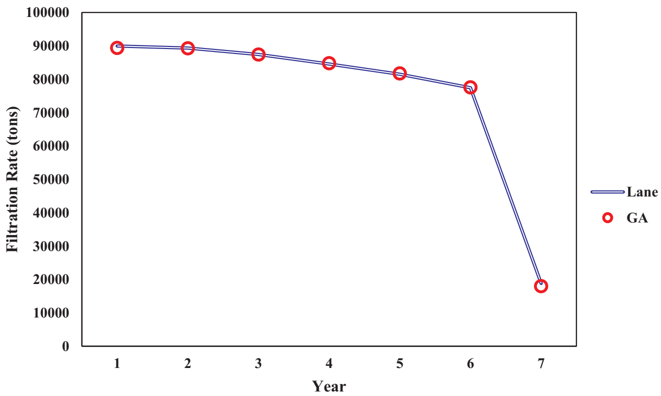

Fig. 8. Comparison of two methods of filtration rate.

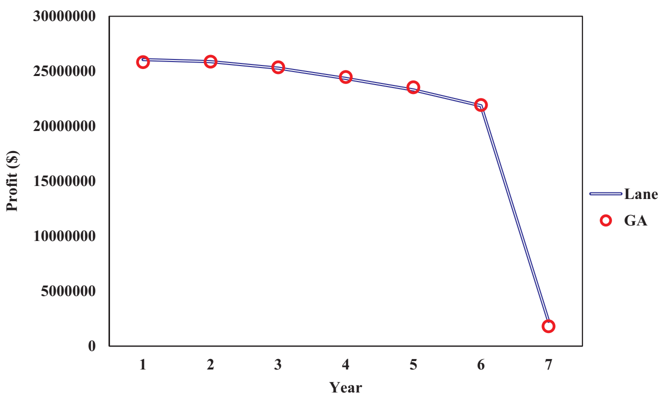

Fig. 9. Comparison the profit of the two methods.

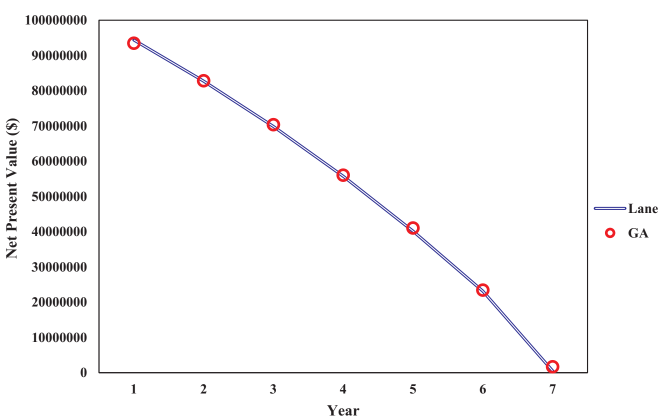

Fig. 10. Comparison the net present value of the two methods.

Table 4
The error rate of the optimum cutoff grade of the genetic model.

Year	Lane method (%)	Genetic algorithm (%)	Relative error (%)
1	0.503	0.506	0.5
2	0.493	0.493	0
3	0.460	0.455	1.08
4	0.412	0.411	0.21
5	0.361	0.357	1.1
6	0.294	0.296	0.68
7	0.220	0.222	0.9
Average Error		0.64	

4. Conclusions

In this paper, a genetic algorithm was used to determine the optimal cut-off grade to maximize the net present value. One of the most important parameters in open pit production planning is a determination of the optimal cut-off grade. For this purpose, the cut-off grade should be optimized for different years of the project life, to optimize profits or net present value. For this purpose, different values for hypothetical deposit were considered. The genetic algorithm implemented and in each year due to the limits, the maximum net present value and the optimal cut-off grade was calculated. In the genetic algorithm, the correct set of parameters (mutation and crossover rates) are effective in achieving the optimum solution, as well as genetic algorithm parameters are critical to achieving fast and accurate results. So the genetic algorithm converges to the optimal solution more quickly, and in most applications also provided better results.

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