ELEC-E9111 - Mathematical computing

Optimization

26 September 2024

When submitting the homework, please include all your code as `.m` or `mlx` files. You do not need to include any files that were provided for the exercises.

1 Linear optimization, (5 pts)

Solve the following linear optimization problem:

$$
\begin{aligned}
\text{minimize } & 3x_1 - x_2 + 4x_3 \\
\text{so that } & x_1 + x_2 = x_3 \\
& x_1 + x_3 \leq 2 \\
& x_2 + 2x_3 \geq 1
\end{aligned}
$$

$$-10 \leq x_1 \leq 10, \quad -10 \leq x_2 \leq 10, \quad -10 \leq x_3 \leq 10$$

Solve the same problem with the additional constraint that x_1, x_2 and x_3 are required to be integers. Finally, round the optimized values you found in the first part and compare with the values calculated in the second part. Are the results identical?

Check out `linprog` and `intlinprog`

2 Quadratic optimization (5 pts)

A small boat factory produces 3 types of products: i) rowboats, ii) canoes, and iii) kayaks. The positive cash flows p_i (eur/day) produced by selling these products are linearly dependent on the production rates x_i (units/day):

$$p = a - b \cdot x,\ a_i, b_i > 0,\ i = 1, 2, 3.$$

Consequently, the total revenue $T(x)$ can be written as

$$T(x) = p(x)^T x = \frac{1}{2} x^T \begin{bmatrix} -2b_1 & 0 & 0 \\ 0 & -2b_2 & 0 \\ 0 & 0 & -2b_3 \end{bmatrix} x + ax$$

where $a = [750, 300, 400]$ and $b = [12, 5, 17]$. The total cost C_{tot} for the productions of the three boats is

$$C_{tot}(x) = c_p x + c_f + c_m(x)$$

where $c_p = [22, 35, 40]$ (eur/unit), $c_f = 1000$ (eur/day) and c_m denote variable production costs, fixed production costs and material costs, respectively.

In particular, the factory utilizes i) steel, ii) aluminum, and iii) plastic, and one can approximate that the consumption rates y_i of these parts are linearly related to the production rates as $y = Rx$, where

$$R = \begin{bmatrix} 5 & 1 & 0 \\ 4 & 2 & 6 \\ 1 & 2 & 4 \end{bmatrix}$$

and the availability of the three materials is limited by $y \leq [35, 100, 70]^T$. Finally, the material cost can be written as a function of the production rate as

$$c_m = [12, 20, 7]y + \frac{1}{2}y^T \begin{bmatrix} -0.05 & 0 & 0 \\ 0 & -0.01 & 0 \\ 0 & 0 & -0.08 \end{bmatrix} y$$

and the total rate $x_1 + x_2 + x_3$ at which the the factory produces different products has to be less than or equal to 24 units/day. What are then the optimal production rates for rowboats, canoes, and kayaks? What is the optimum profit per day?

Hint: First find the functions for revenue $T(x)$ and costs $C_{tot}(x)$ separately. Then calculate the coefficients H, f and c of the profit function

$$P(x) = T(x) - C_{tot}(x) = \frac{1}{2}x^T Hx + fx + c$$

and find the maximum. Read the documentation of `quadprog`.

3 Global optimization (10 pts)

a)

`fmincon` and other gradient based methods work nicely on smooth problems, but they will only find the local minimum of the attraction basin in which the starting point is located. Minimize the function `z = hills(x)` defined in `hills.m` using genetic algorithm, `ga`. Use the options:

- `PopulationSize = 10` and `MaxGenerations = 400`

Run the solver 100 times with the same parameters. Plot the function `z = hills(x)` using `contour`, and the 100 solutions found by `ga` using `scatter` into the same figure. Set the color of the scatter plot to red to see the markers better. The final figure should look similar to fig. 1.

b)

By default `ga` uses a scattered crossover function which creates the child solution by simply mixing elements of the parent solutions. A better approach might be to take the mean values of the x- and y-coordinates of the parents.

Repeat part a) using three different built-in crossover functions: the default crossover function (`crossoverscattered`), `crossoverintermediate` and `crossoverarithmetic`. The crossover function can be changed by setting the following option:

- `options.CrossoverFcn=@functionName`,

where `functionName` is the name of the crossover function.
For each crossover function, you should get 100 solutions. Plot the fitness function values of these three solution groups using `boxchart`. Is there any difference between the different crossover funcitons?

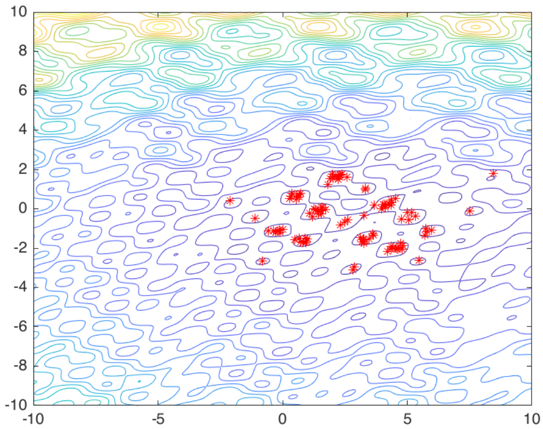

Figure 1: Example solution to ex. 3 a)