ELEC-E9111 - Mathematical computing

The Basics

September 12, 2024

Please include all your code as `.m` or `.mlx` files in your submission.

1 Exercises, (20 pts)

1. (5p) **MATLAB Grader**
   Some of the exercises will be submitted through MATLAB Grader. By now you should be enrolled on the MATLAB Grader platform which you can access from https://grader.mathworks.com/courses/146176-mathematical-computing-2024 by logging in with your Aalto account. If you encounter problems, email the TA (tom.rindell@aalto.fi).

2. (2p) Plot the following functions in one figure using `subplot()` to generate separate axes for each function:

$$\begin{aligned} &a) \quad f(x) = \sin^2(x) + 3 * \cos(x), && x \in [-2, 3] \\ &b) \quad g(x, y) = x^3 - 6y^2 + x - 1, && x, y \in [-5, 6] \\ &c) \quad r(\varphi) = 2 + \cos(2\varphi), && \varphi \in [0, 2\pi] \end{aligned}$$

3. (2p) Generate $N = 20000$ random points in the xy-plane with $x, y \in [-1, 1]$ and for each point, compute the distance from origin r. Plot the points using `gscatter()` so that points with $r \leq 1$ have a different color than points with $r > 1$. Finally, compute the value

$$p = 4\frac{N_{r\leq 1}}{N},$$

   where $N_{r\leq 1}$ is the number of points with $r \leq 1$. The value of p should approach π with large enough N. How good is the approximation with $N = 2000$?

4. (3p) Plot

$$r(\varphi) = 3 - 3\sin(\varphi) + \frac{\sin(\varphi)\sqrt{|\cos(\varphi)|}}{\sin(\varphi) + 1.4}$$

   where $\varphi \in [0, 2\pi]$ using red color and linewidth of 2 in polar coordinates.

   The surface area between a curve defined in polar coordinates and the origin can be calculated with the integral

$$A = \frac{1}{2}\int_a^b [r(\varphi)]^2 d\varphi.$$

   Numerically compute the total area inside the curve.

5. (4p) Compute the values of x, y and z defined by

$$\begin{cases} x(u, v) = 2(1 - e^{u/6\pi})\cos(u)\cos^2(v/2) \\ y(u, v) = 2(-1 + e^{u/6\pi})\sin(u)\cos^2(v/2) \\ z(u, v) = 1 - e^{u/3\pi} - \sin(v) + e^{u/6\pi}\sin(v) \end{cases}$$

   where $u \in [0, 6\pi]$ and $v \in [0, 2\pi]$. Plot the surface defined by x, y and z using e.g. `surf()`.

6. (4p) Smoothstep functions are widely used in graphics processing to achieve a gradual transition between two values. Implement the function `y = smoothstep(left,right,x)`, where `left` and `right` are scalar values, and `x` a vector of linearly spaced values created with e.g. `linspace()`. The output y is a vector with same size as the input `x`, with the following definition:

$$\mathtt{y} = \begin{cases} 0 & x < \mathtt{left} \\ 3x_{sc}^2 - 2x_{sc}^3 & \mathtt{left} \leq x \leq \mathtt{right} \\ 1 & \mathtt{right} < x \end{cases}$$

$$x_{sc} = \frac{x - \mathtt{left}}{\mathtt{right} - \mathtt{left}}$$

Plot the output of the function in the interval $x \in [-5, 5]$, with the boundaries $\mathtt{left} = -3$ and $\mathtt{right} = 2$.