2) Si S et T sont 2 $(\mathcal{F}_n)$ - temps d'arrêt

$S \vee T = \sup\{S, T\}$

A-t-on ? $\{S \vee T \leq n\} \overset{?}{\in} \mathcal{F}_n$, $\forall n \in \mathbb{N}$

$$\{S \vee T \leq n\} = \{S \leq n \text{ et } T \leq n\}$$

$$= \underbrace{\underbrace{\{S \leq n\}}_{\in \mathcal{F}_n} \cap \underbrace{\{T \leq n\}}_{\in \mathcal{F}_n}}_{\mathcal{F}_n}$$

$S \wedge T = \inf\{S, T\}$

A-t-on $\{S \wedge T \leq n\} \overset{?}{\in} \mathcal{F}_n$

$$\{S + T \leq n\} = \bigcup_{\substack{(k,l) \in \mathbb{N}^2 \\ k+l \leq n}} \{\underbrace{S \leq k}_{\in \mathcal{F}_k \subset \mathcal{F}_n} \text{ et } \underbrace{S \leq l}_{\in \mathcal{F}_l \subset \mathcal{F}_n}\}$$

On a $\forall n \in \mathbb{N}$, $\{S \wedge T \leq n\} \subset \{S \leq n \text{ ou } T \leq n\}$

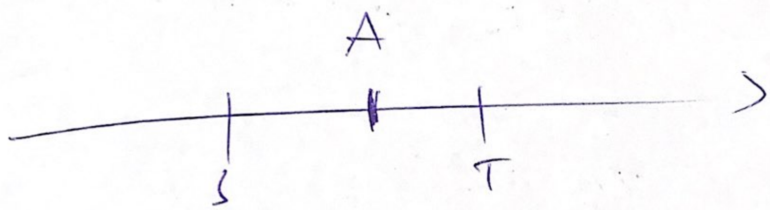
$$\supset \underbrace{\underbrace{\{S_n \leq n\}}_{\in \mathcal{F}_n} \cup \underbrace{\{T \leq n\}}_{\in \mathcal{F}_n}}_{\in \mathcal{F}_n}$$

3) $S \leq T \overset{?}{\Rightarrow} \mathcal{F}_S \subset \mathcal{F}_T$

Soit $A \in \mathcal{F}_S = \{ G \in \Omega \text{ tq } G \cap \{S \leq n\} \in \mathcal{F}_n \}$

On a $\forall n \in \mathbb{N}, \quad \underbrace{A \cap \{T \leq n\}}_{\updownarrow \text{ lien avec } \{S \leq n\}} \overset{??}{\in} \mathcal{F}_n$

$\{S \leq n\} \cancel{\subset} \{T \leq n\}$ ← faux



$\{T \leq n\} \subset \{S \leq n\}$

donc $A \cap \{T \leq n\} = A \cap \{T \leq n\} \cap \{S \leq n\}$

$= \underbrace{A \cap \{S \leq n\}}_{\in \mathcal{F}_n \text{ car } A \in \mathcal{F}_S} \cap \underbrace{\{T \leq n\}}_{\in \mathcal{F}_n \text{ car } T \text{ est un temps d'arrêt}}$