- f.g.m de $\mathcal{N}(m, \sigma^2)$

$$M_Z(t) = \mathbb{E}\left(t^Z\right) = \int_{\mathbb{R}} t^x f_Z(x)\,dx$$

$$= \exp\left(\mu t + \sigma^2 \frac{t^2}{2}\right)$$

- $C \sim \gamma(n, \alpha)$ $n \in \mathbb{N}^*$

Calcul de la f. de r. $f(x) = \dfrac{\alpha^n}{\Gamma(n)} x^{n-1} e^{-\alpha x}$

Soit $n \geq 2$

$$F(x) = \mathbb{P}(C \leq x) = \int_0^x f(y)\,dy$$

$$= \int_0^x \frac{\alpha^n}{\Gamma(n)} y^{n-1} e^{-\alpha y}\,dy$$

	D	I
	y^{n-1}	$e^{-\alpha y}$
+	y^{n-1}	$-\frac{1}{\alpha} e^{-\alpha y}$
−	$(n-1) y^{n-2}$	
+		

$$= \frac{\alpha^n}{\Gamma(n)} \times \left[-\frac{1}{\alpha} y^{n-1} e^{-\alpha y}\right]_0^x + \int_0^x (n-1) y^{n-2} \frac{1}{\alpha} e^{-\alpha y}\,dy$$

$$= -\frac{\alpha^{n-1}}{(n-1)!} x^{n-1} e^{-\alpha x} + \frac{\alpha^{n-1}}{(n-2)!} \int_0^x y^{n-2} e^{-\alpha y}\,dy$$

$\underline{n=1}$ $\quad I_1 = \alpha \int_0^x e^{-\alpha y} dy = \left[-e^{-\alpha y}\right]_0^x$

$$= 1 - e^{-\alpha x}$$

$$F(x) = I_n = I_{n-1} - \frac{\alpha^{n-1}}{(n-1)!} x^{n-1} e^{-\alpha x}$$

$$= I_1 - \sum_{k=1}^{n-1} \frac{(\alpha x)^k}{k!} e^{-\alpha x}$$

$$= 1 - e^{-\alpha x} \sum_{k=0}^{n-1} \frac{(\alpha x)^k}{k!} e^{-\alpha x}$$

f.g.m.

$$M_C(s) = \mathbb{E}\left[e^{sC}\right] = \int_{\mathbb{R}} e^{sx} f_C(x)\, dx = \left(\frac{\alpha}{\alpha - s}\right)^n$$

$$= \int_0^{+\infty} e^{sx} \frac{\alpha^n}{\Gamma(n)} x^{n-1} e^{-\alpha x} dx \qquad \left(\left(\begin{array}{l} \alpha - s > 0 \\ \Leftrightarrow \alpha > s \end{array}\right.\right.$$

$$= \frac{\alpha^n}{\Gamma(n)} \int_0^{+\infty} x^{n-1} e^{-(\alpha - s)x}$$

$$= \frac{\alpha^n}{\Gamma(n)} \frac{\Gamma(n)}{(\alpha - s)^n} \underbrace{\int_0^{+\infty} \frac{(\alpha - s)^n}{\Gamma(n)} x^{n-1} e^{-(\alpha - s)x} dx}_{= 1 \;\; \gamma(n, \alpha - s)}$$

3) $C \sim \gamma(r, \alpha)$

$$\mathbb{E}(C) = M_C'(0)$$

$$M_C'(s) = \frac{d}{dr}\left(\left(\frac{\alpha}{\alpha - s}\right)^r\right) = \frac{r\alpha^r}{(\alpha - s)^{r+1}}$$

$$\mathbb{E}(C) = M_C'(0) = \frac{r}{\alpha} \qquad M_C''(s) = \frac{r(r+1)\alpha^r}{(\alpha - s)^{r+2}}$$

$$\mathbb{E}(C^2) = M_C''(0) = \frac{r(r+1)}{\alpha^2}$$

$$\Rightarrow \operatorname{Var}(C) = \mathbb{E}(C^2) - \mathbb{E}(C)^2$$

$$= \frac{r(r+1)}{\alpha^2} - \left(\frac{r}{\alpha}\right)^2$$

$$= \frac{r}{\alpha^2}$$

$C \rightsquigarrow \log N(\mu, \sigma^2) \Leftrightarrow \ln(C) \sim N(\mu, \sigma^2)$

f.d.r ?

$$F(x) = \mathbb{P}(C \leq x) = \mathbb{P}\left(\ln(C) \leq \ln(x)\right)$$

$$= \mathbb{P}\left(\frac{\ln(C) - \mu}{\sigma} \leq \frac{\ln(x) - \mu}{\sigma}\right)$$

$$\Rightarrow F(x) = \Phi\left(\frac{\ln(x) - \mu}{\sigma}\right)$$

où Φ est la f.d.r. de la loi normale centrée réduite.

$$f(x) = F'(x) = \frac{d}{dx}\left(\Phi\left(\frac{\ln(x) - \mu}{\sigma}\right)\right)$$

$$= \frac{1}{\sigma x} \Phi'\left(\frac{\ln(x) - \mu}{\sigma}\right)$$

$$= \frac{1}{\sigma x} \cdot \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\ln(x) - \mu}{\sigma}\right)^2\right]$$

$C \sim \log\text{Normale}(\mu, \sigma^2) \iff \ln(C) \sim N(\mu, \sigma^2)$

$$\mathbb{E}(C) = \mathbb{E}\left(e^{\ln(C)}\right) = M_C(\ln(C))$$

$$= \exp\left(\mu + \frac{\sigma^2}{2}\right)$$

$$\mathbb{E}(C^2) = \mathbb{E}\left(e^{\ln(C^2)}\right) = M_C(2\ln(C))$$

$$V(C) = e^{2\mu + 2\sigma^2} - \left(e^{\mu + \frac{\sigma^2}{2}}\right)^2$$

$$= e^{2\mu + \sigma^2}\left(e^{\sigma^2} - 1\right)$$

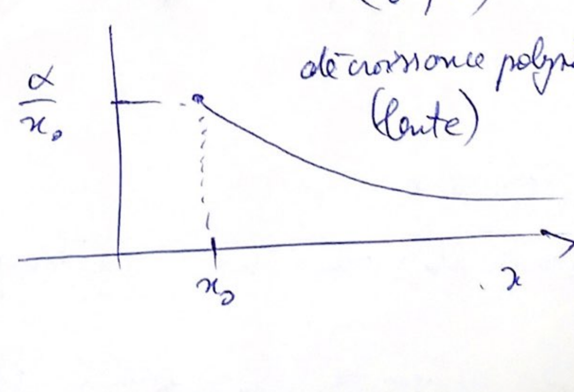
$C \sim \text{Pareto}(x_0, \alpha) \qquad x_0 > 0$

$$F(x) = 1 - \left(\frac{x_0}{x}\right)^\alpha \quad (x > x_0)$$

$$f(x) = F'(x) = \frac{\alpha x_0^\alpha}{x^{\alpha+1}}$$

densité de Pareto (x_0, α)

décroissance polynômiale (lente)



$$\mathbb{E}(C) = \int_{x_0}^{+\infty} \frac{\alpha x_0^{\alpha+1}}{x^{\alpha}} dx = \alpha x_0^{\alpha} \left[-\frac{1}{(\alpha-1)} \frac{1}{x^{\alpha-1}} \right]_{x_0}^{+\infty}$$

$$= \alpha x_0^{\alpha} \times \frac{1}{\alpha-1} \times \frac{1}{x_0^{\alpha-1}} \qquad \alpha > 1$$

$$\mathbb{E}(C) = \begin{cases} \frac{\alpha x_0}{\alpha-1} & \text{si } \alpha > 1 \\ +\infty & \text{si } \alpha < 1 \end{cases}$$

Pareto modelise les sinistres extremes

$$\mathbb{E}(C^2) = \int_{x_0}^{+\infty} x^2 \frac{\alpha x_0^{\alpha}}{x^{\alpha+1}} dx = \alpha x_0^{\alpha} \int_{x_0}^{+\infty} \frac{1}{x^{\alpha-1}} dx$$

~~les distribution des queues~~

les queues des distribution de sinistres extrêmes

$$= \alpha x_0^{\alpha} \left[-\frac{1}{\alpha-2} \cdot \frac{1}{x^{\alpha-2}} \right]_{x_0}^{+\infty}$$

$$= \frac{\alpha x_0^2}{\alpha-2}$$

$$V(C) = \frac{\alpha x_0^2}{(\alpha-2)^2} - \left(\frac{\alpha x_0}{(\alpha-1)} \right)^2 = \begin{cases} \frac{\alpha x_0^2}{(\alpha-1)^2(\alpha-2)} & \text{si } \alpha > 2 \\ +\infty & \text{sinon} \end{cases}$$

densité avec une queue de distribution épaisse

Actuaire :

Base de données $\Rightarrow$ calibrage des paramètres x_0 et α

pour representer au mieu les sinistres

Modelisation du coût des sinistres (differentes lois) TARIFICATION