Department of Computer Science
COSC 4P80 - Artificial Neural Network

Deep Learning: Self-Organizing Maps

Prepared by: Hridoy Rahman
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1 Heat Map of Six Variations

The following section contains six different variations with descriptions: 5x5, 6x6, and 7x7 SOM neuron map space. Each dataset is mapped with all the neuron map space, and the results are provided below.

1.1 Variation 1: 5x5 with 16 features

SOM Configuration	Inputs
Grid	5x5
Dataset	16 bin data file
Learning Rate	0.5
Max Iterations	2000
Weight Matrix	grid $\times$ grid $\times$ feature_size: $5 \times 5 \times 16$
Sample Input Size	53
Normalization	Min Max Normalization (0,1)
Label Blue	Clusters of Good Motors
Label Red	Clusters of Bad Motors
Label White	Neutral

Table 1: 5x5 SOM Implementation with dataset 16 bin size

The following SOM topology from Table 1 consists of the 5x5 neuron space. The weight matrix is initialized with random values ranging from 0 to 1. The dataset defined for this topology is the file with 16 feature sizes. The learning rate is 0.5, and the SOM training algorithm is trained with 2000 epochs. The weight matrix looks like 5 x 5 x 16, where 5x5 is denoted as neurons, and for each neuron, there is an associated vector with 16 features. The base idea is that each index in the 2D matrix holds a weight vector that has 16-dimensional input features in it. After implementing the SOM training, the output of the SOM topology for 5x5 with 16 bin size looks like the following, as depicted in Figure 1.1. Figure 1.1 has a confidence level ranging from 0 to 1. A value less than 0.5 illustrates the red label, meaning the characteristic vector is less similar to the inputs. Furthermore, with greater than 0.5, there is more similarity between the neurons and associated input vectors. A quantization error graph is created with respect to 100 iterations until max iterations are covered. Figure 1.2 depicts the quantization error accumulated within each 100th iteration until the 2000 iteration in the case of 5x5 neuron map space or SOM topology. From the following Figure 1.2, it can be seen that the quantization error drops over time or, in this case, iterations. At 2000 or max iterations, the SOM topology performs well by decreasing the quantization error on seen or labelled data.

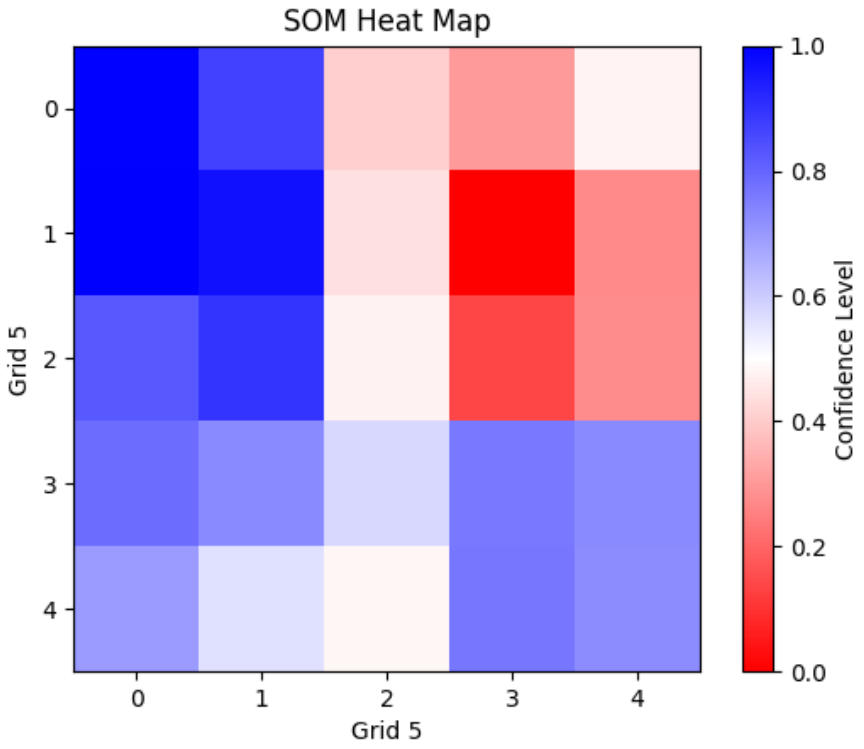

Figure 1.1: SOM Topology Heat Map with 5x5 Map Space and 16 Bin Size Dataset

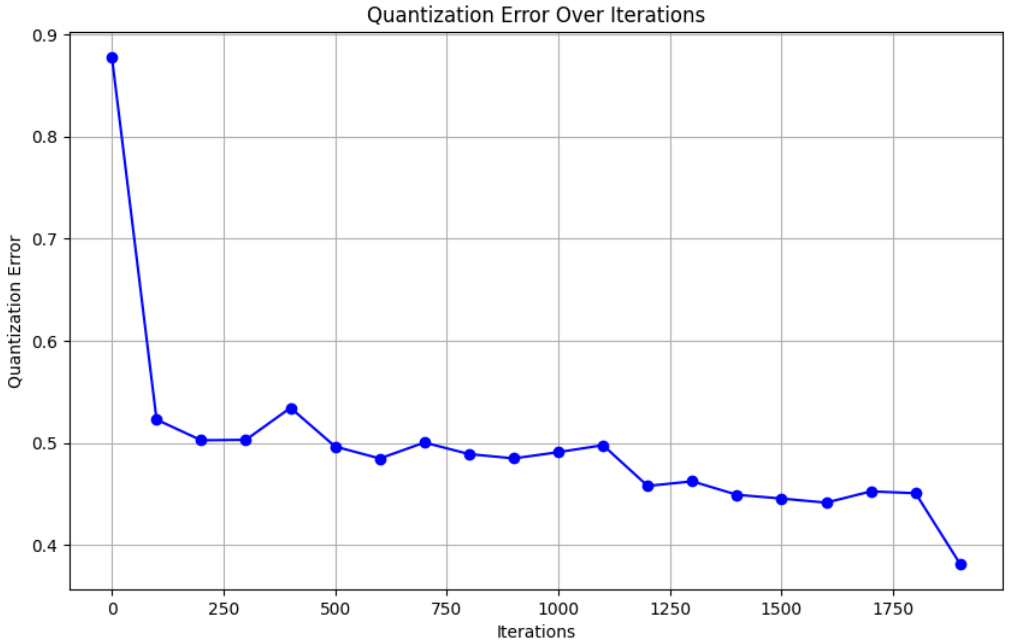

Figure 1.2: SOM Topology 5x5: Quantization Error Over 100 Iterations

Additionally, SOM is able to improve its clustering accuracy with time. The base idea is that the SOM topology clusters similar vectors together and not similar vectors together.

Since the dataset only contains two classes. The neutral zone is the white grids, illustrated in Figure 1.1. From Figure 1.1, the map also shows the wrap-around mechanism implemented for the SOM. In Figure 1.1, let's say (4,0) coordinates calculate the influence based on the wrap-around functionality, where it shows the (4,4) neuron is influenced by the (4,0) neuron. Additionally, the SOM topology for the current configuration shows different outputs with similar quantization errors, depicting the fact that similar vectors are clustered in a similar class where less similar vectors are clustered into another class, namely the red label or the values less than (0.5). In other words, the quantization error reflects how well the input vectors are mapped to the closest neuron. In Figure 1.2, the quantization error is 0.3817 (Appendix: Quantization Error Data - Variation 1). In Figure 1.1, the heat map helps identify the clusters with accurate vector characteristics and areas where clusters are not so accurate. Since this SOM topology is built with 25 neurons, a 5x5 grid space is assigned to the SOM topology for 16 feature-size data samples. With 25 neurons and 53 samples, it is hard for the SOM topology to capture the granularity of the data samples with not many neurons. In the next sections, it will be shown that the quantization errors improve with increased neurons, capturing more characteristics of the data samples with increased neurons.

1.2 Variation 2: 6x6 with 16 features

SOM Configuration	Inputs
Grid	6x6
Dataset	16 bin data file
Learning Rate	0.5
Max Iterations	2000
Weight Matrix	6x6x16
Sample Input Size	53
Normalization	Min Max Normalization (0,1)
Label Blue	Clusters of Good Motors
Label Red	Clusters of Bad Motors
Label White	Neutral

Table 2: 6x6 SOM Implementation with dataset 16 bin size

The following SOM topology from Table 2 consists of the 6x6 neuron space. The weight matrix is initialized with random values ranging from 0 to 1. The rest of the configuration is similar to the configuration given in Table 1. Also, the following SOM space has 36 neurons. Where each coordinate represents a neuron within the SOM topology. Furthermore, each blue label is denoted as the positive class classification, meaning all the positive or good motors are clustered in a section with wrap-around ability. Compared with Figure 1.1, this SOM configuration has lower quantization error with respect to the one 5x5 or 25-neuron map space implementation. This SOM topology can capture more granular

data than the previous SOM. In Figure 1.4, it is shown that with increased neurons and having a similar learning rate between the two configurations, the SOM topology with 36 neurons learns pretty fast and has a quantization error of 0.3783 (Appendix: Quantization Error Data - Variation 2), which is slightly better than the implementation with 26 neurons. Again, similar to Figure 1.1, Figure 1.3 shows that similar characteristics vectors are clustered together, and non-similar characteristics vectors are clustered. And the neutral zone is automatically taken care of by the SOM clustering algorithm.

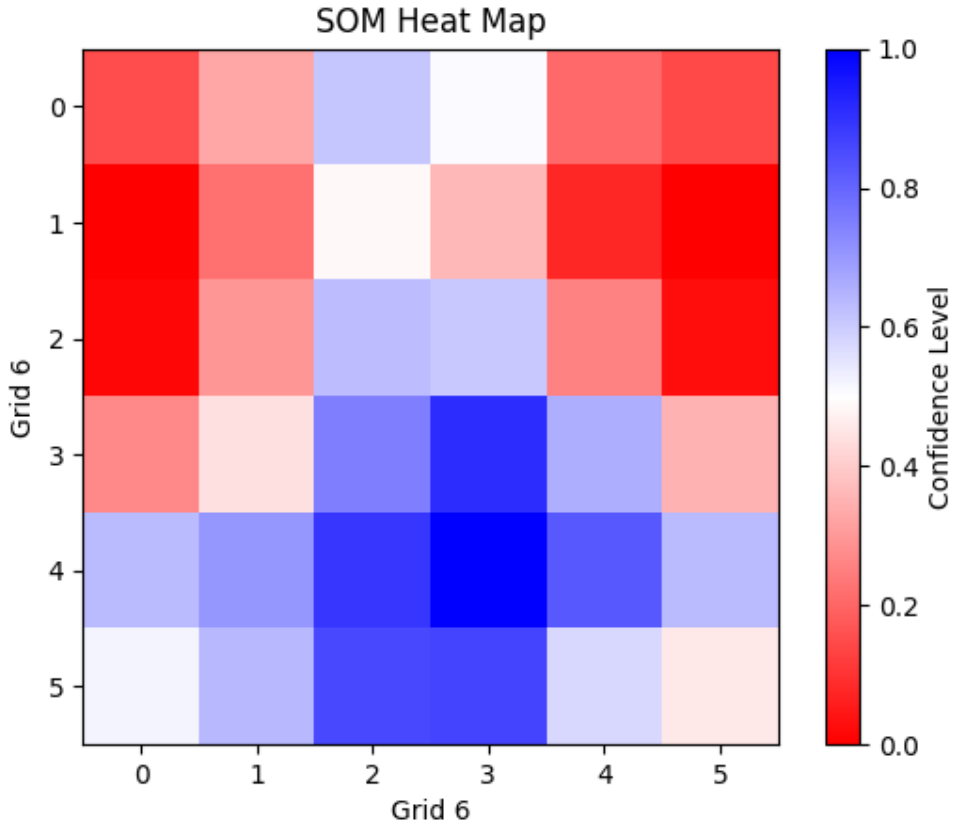

Figure 1.3: SOM Topology Heat Map with 6x6 Map Space and 16 Bin Size Dataset

This configuration also utilizes the wrap-around mechanism, meaning neurons on the edge have an influence on neurons on the other side, namely vertical wrap and horizontal wrap. Similar to Figure 1.1, Figure 1.3 has the good motors clustered in blue and red labels clustered together. Each cell/neuron in this SOM topology in the heat map shows how well the input vectors from dataset 16 bins match with the closest neuron, forming a cluster of good and bad motors with a neutral zone, reflecting no similarity between the two clusters (blue and red). Neurons on the edge also influence neurons on the opposite side in this configuration. For example, a neuron at (5,2) influences the neuron on the opposite side of the heat map (0,2) neuron cell. Lastly, increased neuron SOM enables finer clustering, reducing the quantization error and improving cluster accuracy to some degree. Also, increased neurons converge faster than the 25-neuron configuration SOM shown in Table 1.

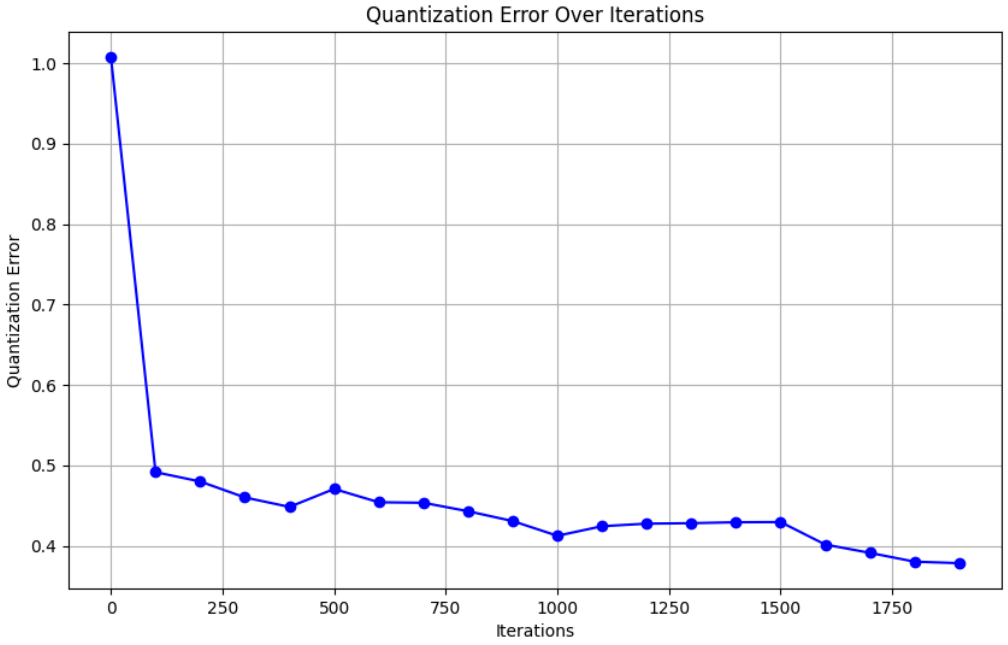

Figure 1.4: SOM Topology 6x6: Quantization Error Over 100 Iterations

1.3 Variation 3: 7x7 with 16 features

SOM Configuration	Inputs
Grid	7x7
Dataset	16 bin data file
Learning Rate	0.5
Max Iterations	2000
Weight Matrix	7x7x16
Sample Input Size	53
Normalization	Min Max Normalization (0,1)
Label Blue	Clusters of Good Motors
Label Red	Clusters of Bad Motors
Label White	Neutral

Table 3: 7x7 SOM Implementation with dataset 16 bin size

In this SOM Topology, 7x7 neuron space is used to train the SOM model. It has the same configuration as previous SOMs, except that the neuron space has increased. An increased number of neurons this time helps the SOM to capture more granularity of the input dataset, converging SOM faster than the previous models with lower quantization than the previous two SOM configurations shown in Tables 1 and 2.

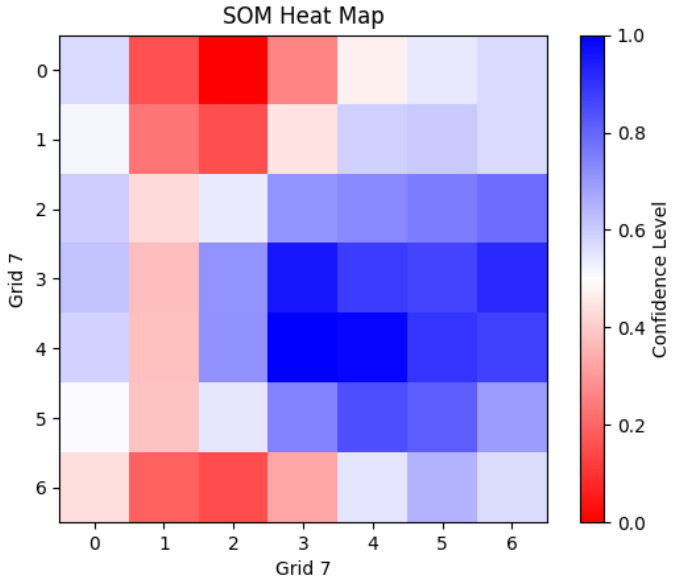

Figure 1.5: SOM Topology Heat Map with 7x7 Map Space and 16 Bin Size Dataset

Figure 1.5 shows a better clustering of data of good and bad motors with low quantization error with respect to its previous smaller neuron space topologies. It can also be seen from Figure 1.5 that there is some vertical and horizontal influence from the right to the left side of the heat map and from the top to the bottom side of the SOM grid space. As before, blue labels depict the cluster of matched characteristics, whereas the red label is the opposite of the blue label, depicting the cluster of non-similar characteristics of vectors. The quantization error over 100 iterations is depicted in Figure 1.6. With increased neuron space, this SOM configuration improves in capturing the granularity of the 16 bis dataset with a quantization error of 0.3596 (Appendix: Quantization Error Data: Variation 3). It is shown here that this SOM configuration converges faster than any of the previous ones, with improved accuracy in clustering the dataset 16 bins.

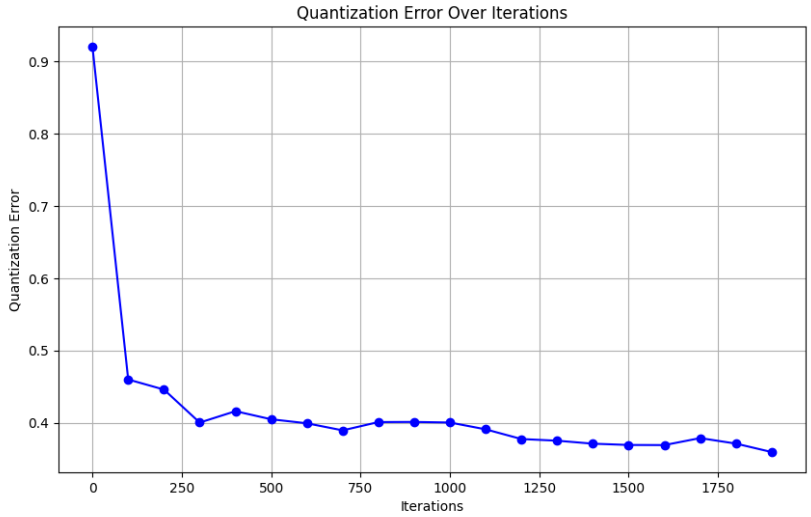

Figure 1.6: SOM Topology 7x7: Quantization Error Over 100 Iterations

1.4 Variation 4: 5x5 with 32 features

SOM Configuration	Inputs
Grid	5x5
Dataset	32 bin data file
Learning Rate	0.5
Max Iterations	2000
Weight Matrix	5x5x32
Sample Input Size	53
Normalization	Min Max Normalization (0,1)
Label Blue	Clusters of Good Motors
Label Red	Clusters of Bad Motors
Label White	Neutral

Table 4: 5x5 SOM Implementation with dataset 32 bin size

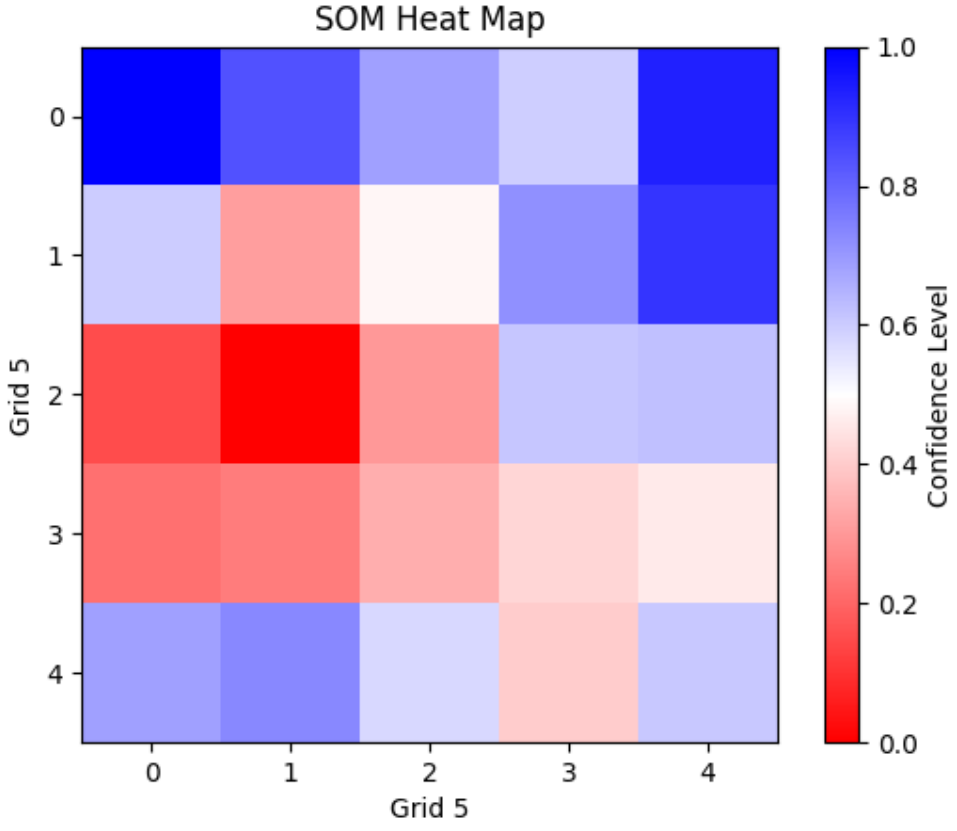

Figure 1.7: SOM Topology Heat Map with 5x5 Map Space and 32 Bin Size Dataset

This SOM configuration changes the dataset from 16 bin size to 32 bin size. And reiterates the configurations from variations 1 to 3. In comparison with the previous three variations, the next three variations will have similar neuron spaces, 5x5, 6x6, and 7x7, but with increased numbers of features.

In this section, variation 4 is trained with 5x5 neuron space, where the dataset has 32 feature sizes. This SOM configuration keeps the same learning rate and max iterations as with previous variations. The weight matrix is as follows: $5 \times 5 \times 32$. Where 5x5 is the neuron space or SOM topology, each neuron in SOM topology has an associated weight matrix with 32 dimensions. Which corresponds to the 32 features in the input data. These weights are adjusted later during the training session. The heat map of the following configuration is given in Figure 1.7. This heat map in Figure 1.7 shows the confidence level of neurons for clustering the input data into good, bad and neutral categories based on the activation function and neighbourhood influence. The red clusters indicate neurons with bad motors; these neurons are activated more strongly by the input vectors from the dataset. The blue cluster region indicates good motors, meaning input vectors with strong characteristics correlate to this blue region. Also, there is some wrap-around influence regarding the first and last row from Figure 1.7. The quantization error over 100th iterations is shown in Figure 1.8.

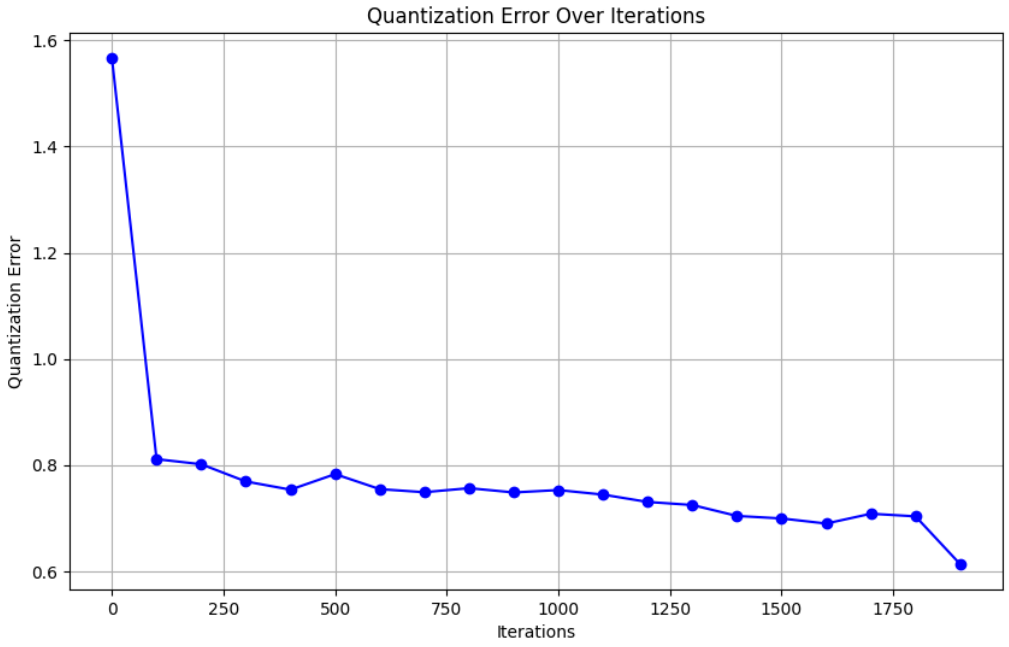

Figure 1.8: SOM Topology 5x5: Quantization Error Over 100 Iterations

From Figure 1.8, it is visible that with 5x5 neuron space, SOM with 32 feature sizes performs poorly. The quantization error is higher than that of the implementation with 16 feature size SOM configuration. The quantization error for variation 4 or this configuration is 0.6140 (Appendix: Quantization Error Data: Variation 4). From the Appendix: Quantization Error Data, it is visible that the quantization error does not go below 0.6140. With multiple runs of the SOM, the quantization error will differ. However, it stays around 0.6140 values for this particular setting of SOM topology. The reason

behind this is that fewer neurons and more data features add more noise to the weight matrix, giving the SOM less opportunity to capture data to represent the input dataset with clusters perfectly. Fewer neurons, such as 25 neurons for 32 input datasets, which have an increased feature, enabling the SOM to capture more data granularity and hinder the clustering ability. Furthermore, this SOM topology also utilizes the wrap-around influence vertically and horizontally.

1.5 Variation 5: 6x6 with 32 features

SOM Configuration	Inputs
Grid	6x6
Dataset	32 bin data file
Learning Rate	0.5
Max Iterations	2000
Weight Matrix	6x6x32
Sample Input Size	53
Normalization	Min Max Normalization (0,1)
Label Blue	Clusters of Good Motors
Label Red	Clusters of Bad Motors
Label White	Neutral

Table 5: 6x6 SOM Implementation with dataset 32 bin size

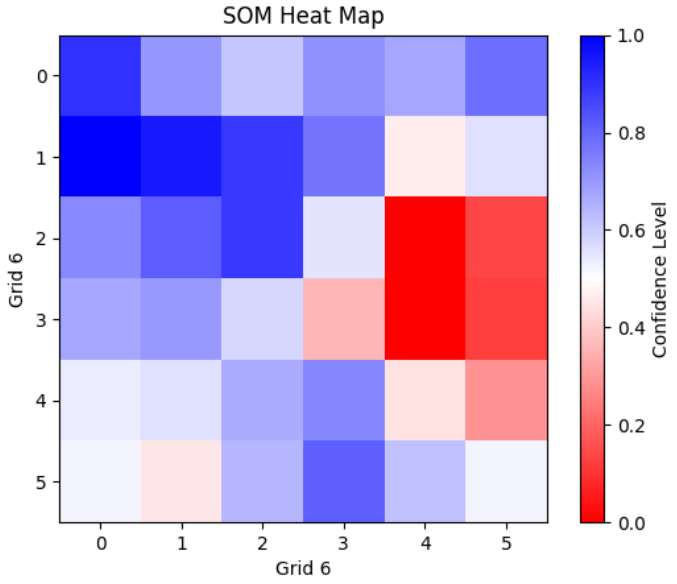

Figure 1.9: SOM Topology Heat Map with 6x6 Map Space and 32 Bin Size Dataset

In this SOM topology, the neuron space is increased, and this topology has 36 neurons in total to represent the input dataset with 32 feature sizes. The rest of the configuration stays the same as other variations shown previously. The heat map of this SOM is given in Figure 1.9. The heat map, like with other variations, shows the data clusters of good, bad, and neutral with blue, red, and white, respectively. The quantization error for this SOM implementation is 0.6132 with 2000 max iterations (Appendix: Quantization Error Data - Variation 5). From this, it is likely that, with increased neurons, it performs slightly better and can represent input data than SOM in variation 4. With increased iterations and learning rate, it is possible to improve this SOM configuration to cluster that input dataset better than shown in this configuration. The quantization graph is shown in Figure 1.10.

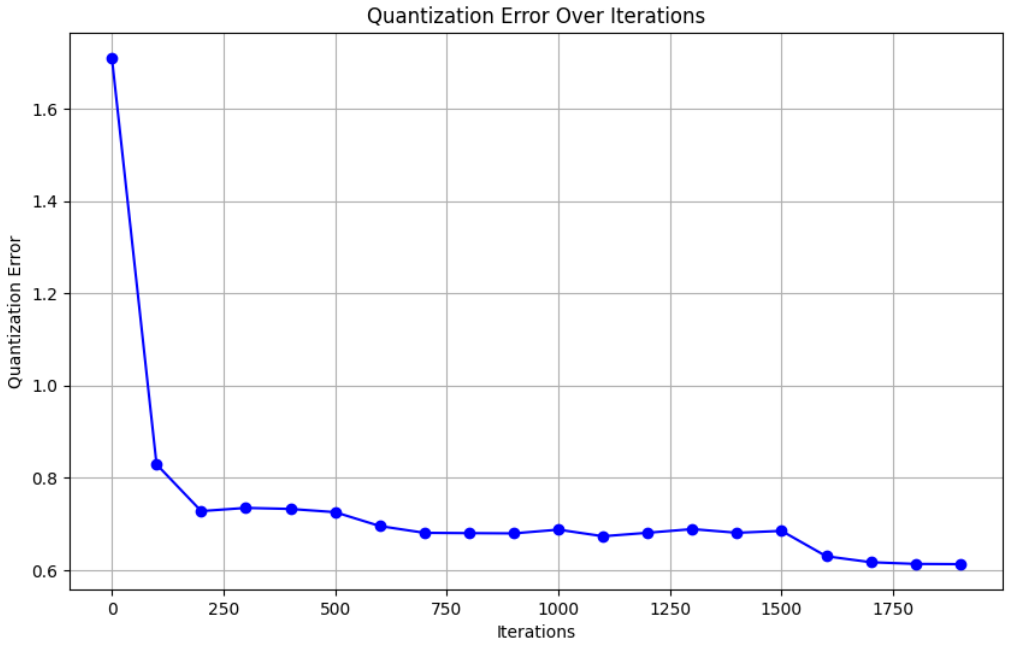

Figure 1.10: SOM Topology 6x6: Quantization Error Over 100 Iterations

1.6 Variation 5: 6x6 with 32 features

The topology defined in Table 6 is a similar configuration with other variations except for increased SOM map space. This configuration uses 7x7 neuron map space. With increased map space, the heat map looks as illustrated in Figure 2.1. Figure 2.1 depicts the normalized activation data of the input dataset. For the input dataset 32 bin size, the 7x7 SOM model performs better than the previous SOM variations shown in this assignment. With increased neuron space, 49 or 7x7, in this case, captures more data. Previous SOM variations from 1 to 3 had less dimensionality than the dataset 32 bin size. Therefore, the quantization error with respect to this dataset was greater; however, in this configuration, SOM has more neurons to capture the granularity of the input dataset, with a better ability to cluster the good and bad motors. However, it still doesn't perform at the level where variation 3 performed. In comparison with variation 3 in Table 3, this configuration performs poorly. By tweaking the parameters such as learning rate, increased max iterations and increased neuron map space, it is possible for

SOM to improve the clustering ability of this 32-bin dataset.

SOM Configuration	Inputs
Grid	7x7
Dataset	32 bin data file
Learning Rate	0.5
Max Iterations	2000
Weight Matrix	7x7x32
Sample Input Size	53
Normalization	Min Max Normalization (0,1)
Label Blue	Clusters of Good Motors
Label Red	Clusters of Bad Motors
Label White	Neutral

Table 6: 7x7 SOM Implementation with dataset 32 bin size

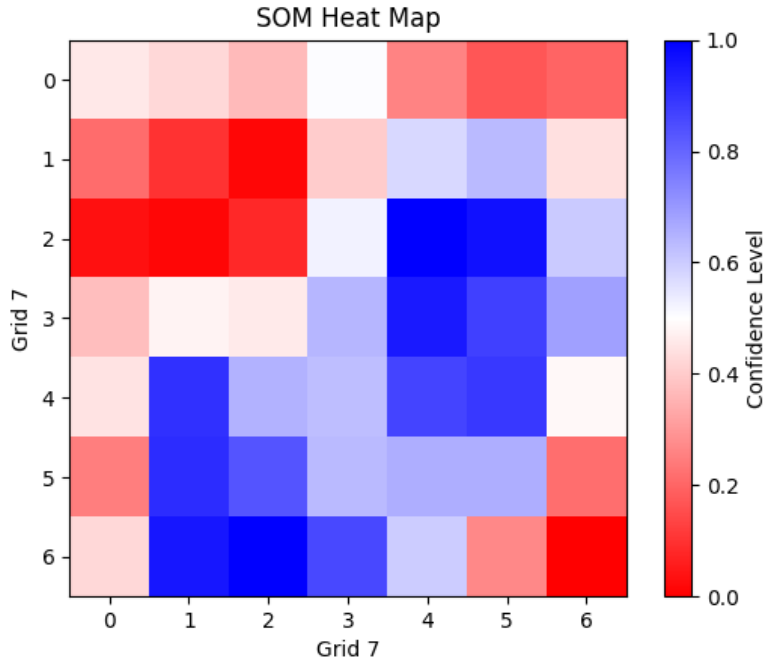

Figure 1.11: SOM Topology Heat Map with 7x7 Map Space and 32 Bin Size Dataset

The heat map depicts the blue, red, and white labels, where blue is associated with good and red with bad motor clusters. It is visible that SOM performs better for the 32 bins dataset with increased neuron space. Furthermore, the lowest quantization error for this SOM is 0.5671 (Appendix: Quantization Error Data - Variation 6). The following graph

of quantization error in Figure 1.12 depicts that the converging is faster than the previous variations in 4 and 5. The quantization error improves a little over 0.4.

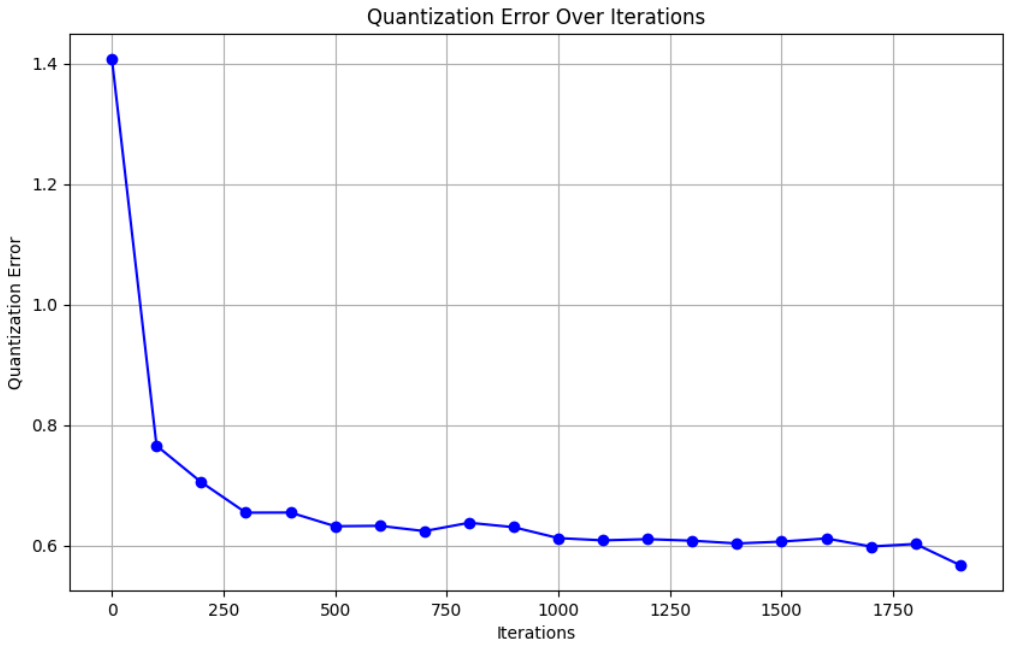

Figure 1.12: SOM Topology 7x7: Quantization Error Over 100 Iterations

2 Conclusion

Section 1 elaborates on the six different variations of required SOM topology with different neuron spaces and two different datasets. The quantization error shows how well each SOM topology performed in clustering the input datasets provided in the SOM implementation. With sufficient neurons, Self Organizing Maps perform relatively better than SOMs with fewer neurons in them. The rule of thumb for implementing SOM would $5 \times \sqrt{\text{number\_of\_samples\_in\_dataset}}$. The SOM implementation with 5x5, 6x6, and 7x7 generally performs better with 16 feature sizes. However, the SOM with 5x5, 6x6, and 7x7 performs poorly with respect to variations 1 to 3 with 32 feature sizes or 32 bin sizes of the dataset. It is possible to improve variations 4 to 6 with increased neuron space and learning rate with 0.7 and increasing the max iterations to around 4000 to 5000. In Figure 1.13, an improvement is shown, and quantization error data in Figure 2.2 and SOM configuration shown in the appendix (Appendix: SOM Improved). Furthermore, this assignment successfully implements six different heat maps with different configurations of self-organizing maps with wrap-around capability where side wraps and top wraps to bottom. In conclusion, SOM is able to cluster good and bad motors. The heat maps show the good motor with blue labels and the bad motor with red labels, where in between is the neutral zone where input data neither fall into the good nor bad characteristic vectors.

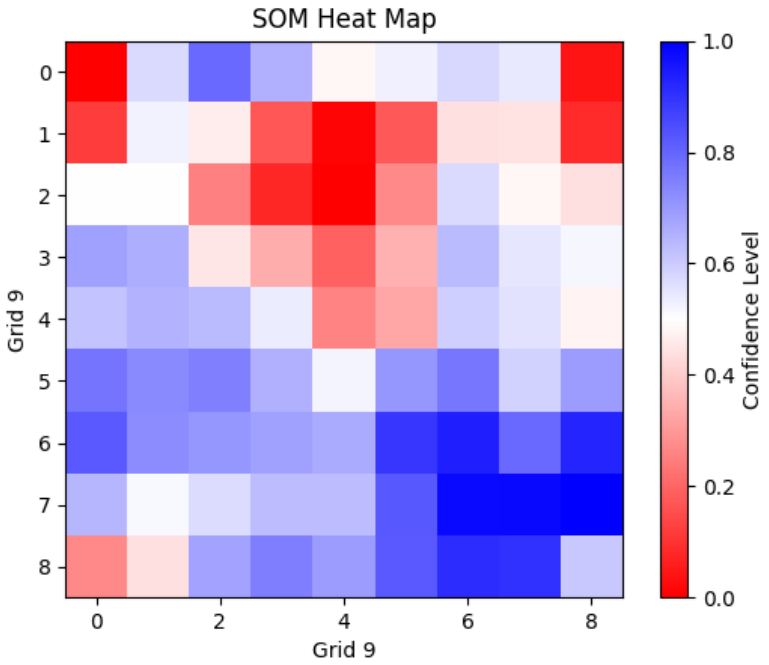

Figure 2.1: SOM Topology Heat Map with 9x9 Map Space and 32 Bin Size Dataset

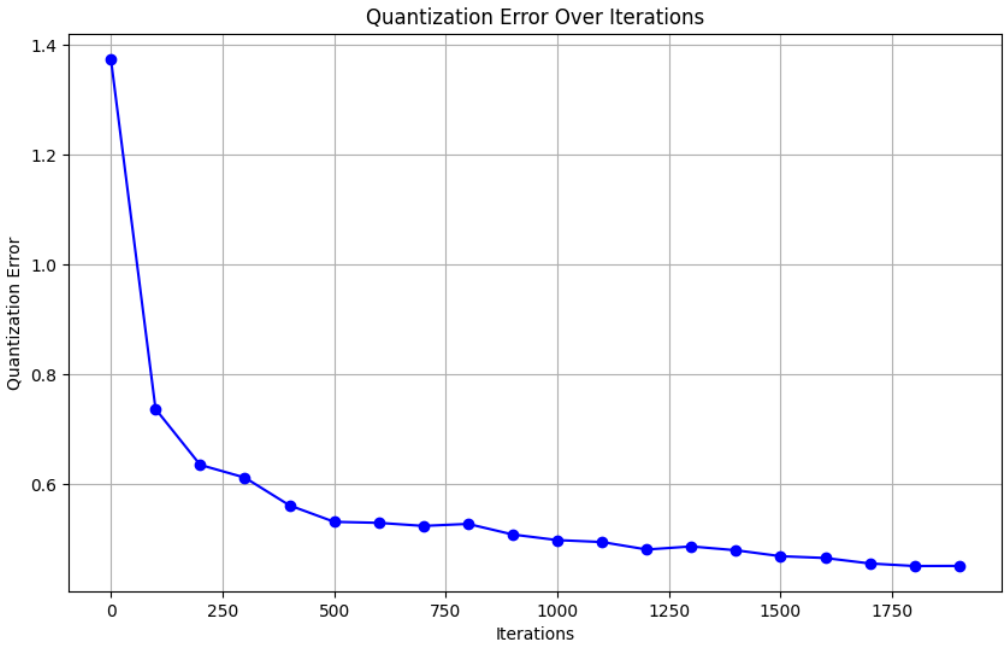

Figure 2.2: SOM Topology 9x9: Quantization Error Over 100 Iterations

Appendix

Quantization Error Data - Variation 1

```
Grid Size: 5x5
Dataset: inputs/L30fft16.out
Learning Rate: 0.5
Max Iterations: 2000
Input Samples X Feature Size: (53, 16)
Quantization Error:
Iteration 0/2000, Quantization Error: 0.8777
Iteration 100/2000, Quantization Error: 0.5231
Iteration 200/2000, Quantization Error: 0.5028
Iteration 300/2000, Quantization Error: 0.5032
Iteration 400/2000, Quantization Error: 0.5346
Iteration 500/2000, Quantization Error: 0.4967
Iteration 600/2000, Quantization Error: 0.4850
Iteration 700/2000, Quantization Error: 0.5006
Iteration 800/2000, Quantization Error: 0.4893
Iteration 900/2000, Quantization Error: 0.4850
Iteration 1000/2000, Quantization Error: 0.4912
Iteration 1100/2000, Quantization Error: 0.4980
Iteration 1200/2000, Quantization Error: 0.4581
Iteration 1300/2000, Quantization Error: 0.4626
Iteration 1400/2000, Quantization Error: 0.4494
Iteration 1500/2000, Quantization Error: 0.4457
Iteration 1600/2000, Quantization Error: 0.4417
Iteration 1700/2000, Quantization Error: 0.4528
Iteration 1800/2000, Quantization Error: 0.4510
Iteration 1900/2000, Quantization Error: 0.3817
```

Quantization Error Data - Variation 2

```
Grid Size: 6x6
Dataset: inputs/L30fft16.out
Learning Rate: 0.5
Max Iterations: 2000
Input Samples X Feature Size: (53, 16)
Quantization Error:
Iteration 0/2000, Quantization Error: 1.0077
Iteration 100/2000, Quantization Error: 0.4915
Iteration 200/2000, Quantization Error: 0.4799
Iteration 300/2000, Quantization Error: 0.4601
Iteration 400/2000, Quantization Error: 0.4483
Iteration 500/2000, Quantization Error: 0.4706
Iteration 600/2000, Quantization Error: 0.4541
Iteration 700/2000, Quantization Error: 0.4535
Iteration 800/2000, Quantization Error: 0.4429
Iteration 900/2000, Quantization Error: 0.4307
```

```
Iteration 1000/2000, Quantization Error: 0.4124
Iteration 1100/2000, Quantization Error: 0.4243
Iteration 1200/2000, Quantization Error: 0.4275
Iteration 1300/2000, Quantization Error: 0.4281
Iteration 1400/2000, Quantization Error: 0.4292
Iteration 1500/2000, Quantization Error: 0.4295
Iteration 1600/2000, Quantization Error: 0.4015
Iteration 1700/2000, Quantization Error: 0.3912
Iteration 1800/2000, Quantization Error: 0.3802
Iteration 1900/2000, Quantization Error: 0.3783
```

Quantization Error Data - Variation 3

```
Grid Size: 7x7
Dataset: inputs/L30fft16.out
Learning Rate: 0.5
Max Iterations: 2000
Input Samples X Feature Size: (53, 16)
Quantization Error:
Iteration 0/2000, Quantization Error: 0.9207
Iteration 100/2000, Quantization Error: 0.4601
Iteration 200/2000, Quantization Error: 0.4461
Iteration 300/2000, Quantization Error: 0.4004
Iteration 400/2000, Quantization Error: 0.4162
Iteration 500/2000, Quantization Error: 0.4050
Iteration 600/2000, Quantization Error: 0.3994
Iteration 700/2000, Quantization Error: 0.3896
Iteration 800/2000, Quantization Error: 0.4011
Iteration 900/2000, Quantization Error: 0.4013
Iteration 1000/2000, Quantization Error: 0.4005
Iteration 1100/2000, Quantization Error: 0.3911
Iteration 1200/2000, Quantization Error: 0.3776
Iteration 1300/2000, Quantization Error: 0.3753
Iteration 1400/2000, Quantization Error: 0.3712
Iteration 1500/2000, Quantization Error: 0.3694
Iteration 1600/2000, Quantization Error: 0.3692
Iteration 1700/2000, Quantization Error: 0.3789
Iteration 1800/2000, Quantization Error: 0.3713
Iteration 1900/2000, Quantization Error: 0.3596
```

Quantization Error Data - Variation 4

```
Grid Size: 5x5
Dataset: inputs/L30fft_32.out
Learning Rate: 0.5
Max Iterations: 2000
Input Samples X Feature Size: (53, 32)
```

```
Quantization Error:
Iteration 0/2000, Quantization Error: 1.5665
Iteration 100/2000, Quantization Error: 0.8117
Iteration 200/2000, Quantization Error: 0.8021
Iteration 300/2000, Quantization Error: 0.7696
Iteration 400/2000, Quantization Error: 0.7542
Iteration 500/2000, Quantization Error: 0.7835
Iteration 600/2000, Quantization Error: 0.7552
Iteration 700/2000, Quantization Error: 0.7494
Iteration 800/2000, Quantization Error: 0.7570
Iteration 900/2000, Quantization Error: 0.7490
Iteration 1000/2000, Quantization Error: 0.7535
Iteration 1100/2000, Quantization Error: 0.7449
Iteration 1200/2000, Quantization Error: 0.7311
Iteration 1300/2000, Quantization Error: 0.7254
Iteration 1400/2000, Quantization Error: 0.7048
Iteration 1500/2000, Quantization Error: 0.7000
Iteration 1600/2000, Quantization Error: 0.6904
Iteration 1700/2000, Quantization Error: 0.7089
Iteration 1800/2000, Quantization Error: 0.7040
Iteration 1900/2000, Quantization Error: 0.6140
```

Quantization Error Data - Variation 5

```
Grid Size: 6x6
Dataset: inputs/L30fft_32.out
Learning Rate: 0.5
Max Iterations: 2000
Input Samples X Feature Size: (53, 32)
Quantization Error:
Iteration 0/2000, Quantization Error: 1.7103
Iteration 100/2000, Quantization Error: 0.8296
Iteration 200/2000, Quantization Error: 0.7283
Iteration 300/2000, Quantization Error: 0.7352
Iteration 400/2000, Quantization Error: 0.7329
Iteration 500/2000, Quantization Error: 0.7260
Iteration 600/2000, Quantization Error: 0.6959
Iteration 700/2000, Quantization Error: 0.6811
Iteration 800/2000, Quantization Error: 0.6805
Iteration 900/2000, Quantization Error: 0.6799
Iteration 1000/2000, Quantization Error: 0.6881
Iteration 1100/2000, Quantization Error: 0.6738
Iteration 1200/2000, Quantization Error: 0.6813
Iteration 1300/2000, Quantization Error: 0.6893
Iteration 1400/2000, Quantization Error: 0.6813
Iteration 1500/2000, Quantization Error: 0.6853
Iteration 1600/2000, Quantization Error: 0.6304
Iteration 1700/2000, Quantization Error: 0.6175
```

```
Iteration 1800/2000, Quantization Error: 0.6138
Iteration 1900/2000, Quantization Error: 0.6132
```

Quantization Error Data - Variation 5

```
Grid Size: 7x7
Dataset: inputs/L30fft_32.out
Learning Rate: 0.5
Max Iterations: 2000
Input Samples X Feature Size: (53, 32)
Quantization Error:
Iteration 0/2000, Quantization Error: 1.4075
Iteration 100/2000, Quantization Error: 0.7655
Iteration 200/2000, Quantization Error: 0.7049
Iteration 300/2000, Quantization Error: 0.6542
Iteration 400/2000, Quantization Error: 0.6545
Iteration 500/2000, Quantization Error: 0.6317
Iteration 600/2000, Quantization Error: 0.6324
Iteration 700/2000, Quantization Error: 0.6236
Iteration 800/2000, Quantization Error: 0.6375
Iteration 900/2000, Quantization Error: 0.6301
Iteration 1000/2000, Quantization Error: 0.6120
Iteration 1100/2000, Quantization Error: 0.6083
Iteration 1200/2000, Quantization Error: 0.6103
Iteration 1300/2000, Quantization Error: 0.6077
Iteration 1400/2000, Quantization Error: 0.6031
Iteration 1500/2000, Quantization Error: 0.6062
Iteration 1600/2000, Quantization Error: 0.6115
Iteration 1700/2000, Quantization Error: 0.5981
Iteration 1800/2000, Quantization Error: 0.6023
Iteration 1900/2000, Quantization Error: 0.5671
```

SOM Improved

```
Grid Size: 9x9
Dataset: inputs/L30fft_32.out
Learning Rate: 0.7
Max Iterations: 4000
Input Samples X Feature Size: (53, 32)
Quantization Error:
Iteration 0/2000, Quantization Error: 1.3737
Iteration 100/2000, Quantization Error: 0.7365
Iteration 200/2000, Quantization Error: 0.6348
Iteration 300/2000, Quantization Error: 0.6116
Iteration 400/2000, Quantization Error: 0.5611
Iteration 500/2000, Quantization Error: 0.5311
Iteration 600/2000, Quantization Error: 0.5292
Iteration 700/2000, Quantization Error: 0.5236
```

```
Iteration 800/2000, Quantization Error: 0.5273
Iteration 900/2000, Quantization Error: 0.5080
Iteration 1000/2000, Quantization Error: 0.4978
Iteration 1100/2000, Quantization Error: 0.4943
Iteration 1200/2000, Quantization Error: 0.4806
Iteration 1300/2000, Quantization Error: 0.4862
Iteration 1400/2000, Quantization Error: 0.4792
Iteration 1500/2000, Quantization Error: 0.4684
Iteration 1600/2000, Quantization Error: 0.4652
Iteration 1700/2000, Quantization Error: 0.4552
Iteration 1800/2000, Quantization Error: 0.4506
Iteration 1900/2000, Quantization Error: 0.4508
```