Design of Machine Elements – Lecture 1

Course Instructor: Vikranth Racherla (Section 1, even roll numbers)

Course structure (LTP): 3-1-0, C Slot

Tutorial class: Monday 10 AM – 10:55 AM

Attendance requirement: Minimum 80% overall

Text books:

1. Mechanical design of machine elements and machines, Second Edition. Authors: Jack A Collins, Henry Busby, George Staab. ISBN-13: 978-0-470-41303-6
2. Shigley's Mechanical Engineering Design, Tenth edition. Authors: Richard G Budynas, J Keith Nisbett. ISBN-13: 978-9339221638

Course website: https://www.coursesites.com/s/_ME30602 (access code for self enrollment: NC433)

Engineered Products: Examples

Wet Grinders: Why do we need them?

For "smooth" texture, fluffiness, and retention of flavors (by not over heating the ingredients), while making dishes like idli, dosa, utappam, and chutneys, wet grinders are used in place of "Mixies". **Ref: www.wikipedia.org**

Wet Grinders: Varieties

Rubbu rolu (telugu), Attu Kallu (Tamil)

Table top wet grinder

Larger capacity wet grinder (of the kind used in halls)

Different designs, cater to different requirements (cost, size, weight, volume of batter, etc.).

Chapatti making machines: Need

Large number of chapatti's need to be made in short periods of time in halls (hostels), canteens, etc. Akshay Patra Foundation, e.g., prepares food in a centralized kitchen in Surat for about 1,50,000 kids at a time.

Chapatti making machines: Types

Different designs, cater to different requirements (cost, output, automatic/semi-automatic, LPG/Electricity based, etc.).

Excavators: Need

Large civil works projects require excavators to remove large volumes of material at a rapid pace. In many cases, projects would be infeasible without such machines.

Excavators: Types

Different models are designed keeping in mind the weight of material to be handled, reach, agility needed, terrain to work on, etc.

Engineering Design: Aspects to bother about

Things to keep in mind in machine design

- What features will the “machine” have (product specifications)?
- What materials will machine components be made of?
- Which manufacturing processes would be used to make the components?
- Would deflections and vibrations in the machine be low enough under typical working conditions?
- Would the components last long enough, i.e. would wear be relatively low, fracture and fatigue failure prevented, corrosion restricted, etc.?
- Would the machine be easy and comfortable to operate and maintain?

What will you learn from the course?

Topics to be covered in the course

- Steps involved in machine design explained through a case study
- Modes of failure (elastic deformation, plastic yielding, elastic instability, creep, wear, corrosion, etc.)
- Materials selection
- Mechanics of solids
- Failure theories
- Geometry determination (tailored shapes, role of tolerances, lazy material removal, etc.)
- Machine elements (shafts, cylinders, bearings, power screws, joints, springs, gears, belts, etc.): Design and analysis

Lecture 1 – Design of Machine Elements

30.11.2016

General considerations in machine design

Case study 1 Wet grinder

Step 1 : Decide the features of your design, i.e. lay down the technical specifications

Capacity 2 liters (volume of wet rice/pulses that can be put into the grinder at a time. In fact, it is preferable for a user to use the grinder at full capacity $\underbrace{\text{for best performance}}_{\text{e.g. for fluffy dosa/idly}}$)

Power supply 110 – 240 V , 50 – 60 Hz / Single phase AC supply, 5A (peak current drawn)

Motor ½ HP, single phase , AC induction motor ($\underbrace{\text{4 pole, ∼ 4 \% slip at rated load}}$)

90% motor efficiency at full load

Motor rotates at

$$\sim \frac{2 \times 50}{4} \times 60 \times 0.96 = 1440 \text{ rpm}$$

No. of seconds in a min

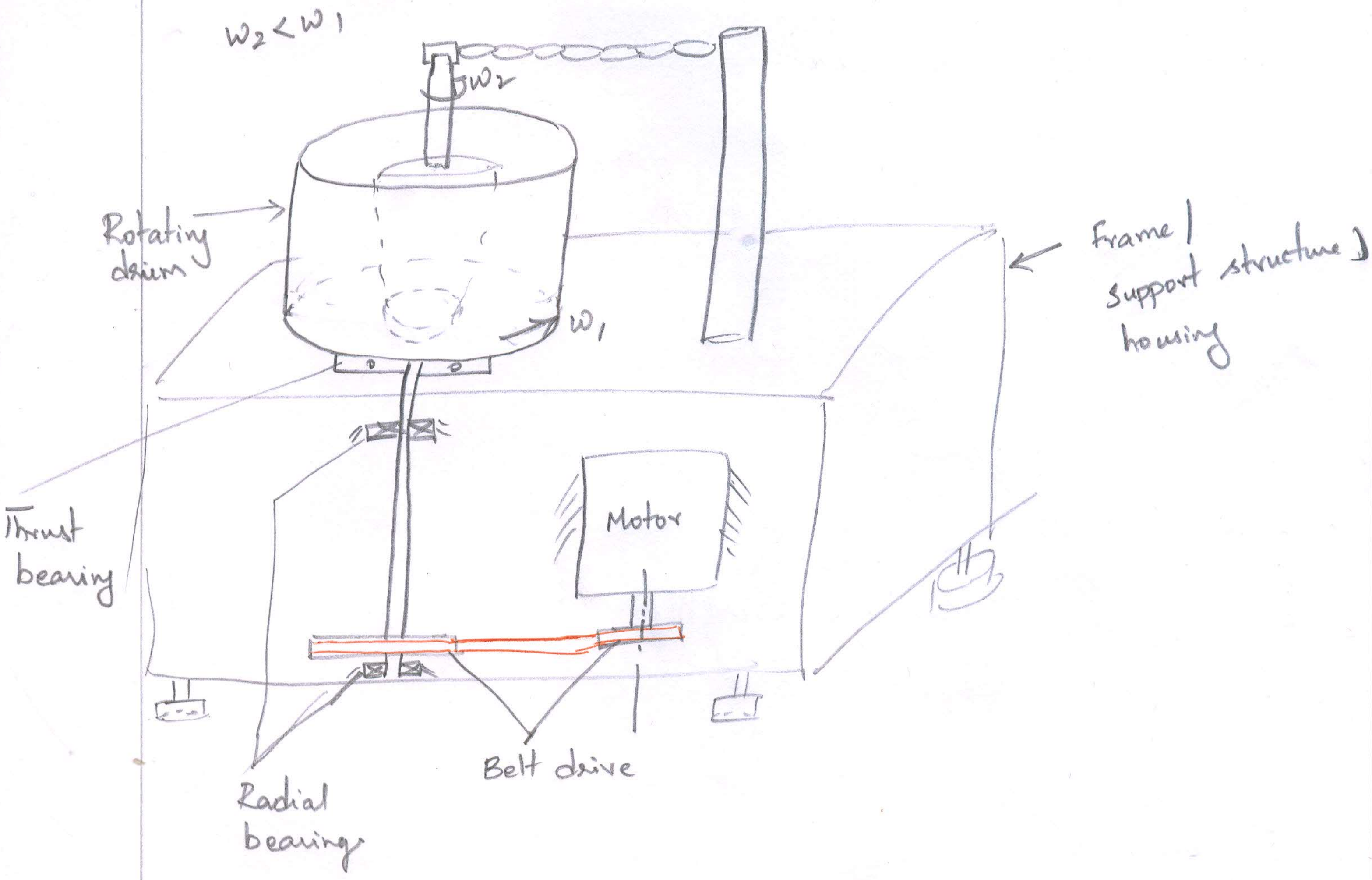
No of poles

frequency of supply

(1 - slip)

Materials used	SS 304L Stainless steel body, low alloy high strength, high toughness, surface hardened 4340 steel drive components, "high quality" grinding stone (higher hardness, wear resistance and fracture toughness)
Other features	① Over load protection, ② press fit panels on motor and drive unit for easy cleaning and maintenance, ③ effective sealing to protect motor, drives, and internal wiring from water, dust, humidity, rats, ants, etc, ④ heavy base for stability, ⑤ good grip on stone handle for easy handling, ⑥ table top
Size	0.5 m x 0.35 m x 0.35 m
Weight	15 kg

Step 2 Preliminary design



- Decide the basic working principle based on current designs and experience

- Identify important components that need to be "designed"

- Identify approximate size of important components

Wet grinder design — Important components

- Motor ~ 150 mm ϕ, 150 mm height
- Pulleys (1:3 ratio)
  Motor drum
- Belt (V belt, 1 belt)
- Bearings (radial, thrust / angular contact roller bearings)
- Steel drum with stone base
- Stone roller with steel handle
- Structure and housing
  On which parts are mounted
  Panels that cover important components
- Adjustable height, rubber mounts

Intermediate design

- In-depth design of engineering components, e.g. belt drive, pulleys, bearings, shaft, structure, roller, steel drum with stone base

- Material selection
  e.g. SS304 for steel drum, steel reinforced belts for less creep, low crack density and wear-resistant granite, etc.

- Fabrication, assembly, inspection, maintenance, safety and cost factors for the design are to be evaluated

- Protection from high humidity levels, dust, ants, rodents, etc to be evaluated

- Performance, reliability, life to be evaluated based on basic principles of heat transfer, dynamics, stress and deflection and failure analysis.

Things to keep in mind

- Waste heat generation.

  HOW HOT WILL COMPONENTS GET?

  WHAT TEMPERATURES ARE ACCEPTABLE?

- Vibration

  How much would the components vibrate?

  Will there be resonance under any operation conditions?

- Would the structure be rigid

  What will be the deflection from dead weight and dynamic loads?

- Which components undergo sliding/rolling?

  How quickly would these components wear out or how much fatigue life would the components offer?

Design of Machine Elements

by

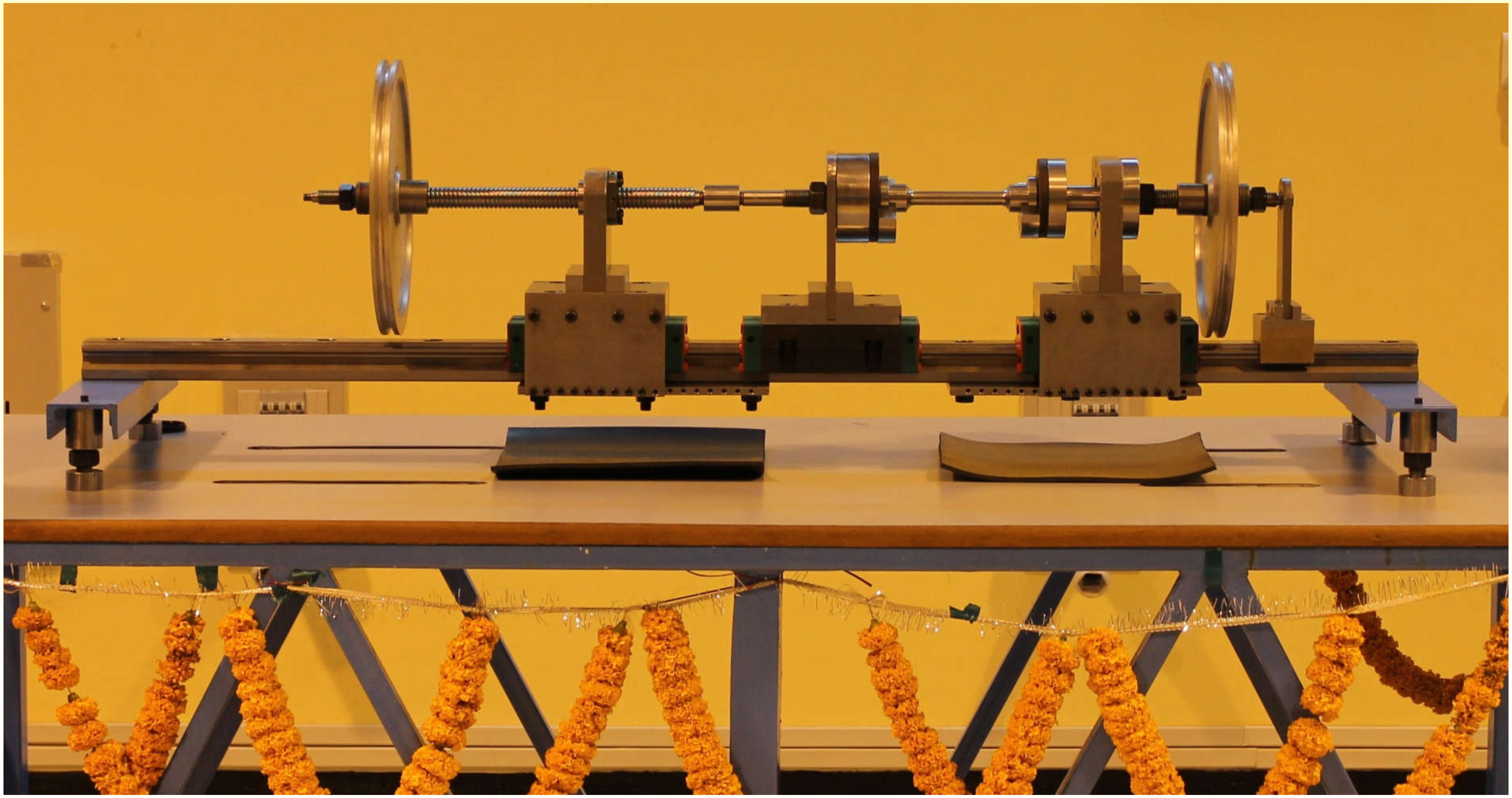
Prof. Vikranth Racherla

Lectures 5,6

Jan 12-16, 2017

Identify the Failure Mechanisms
in the Following Slides

Tension-Torsion Creep Set Up

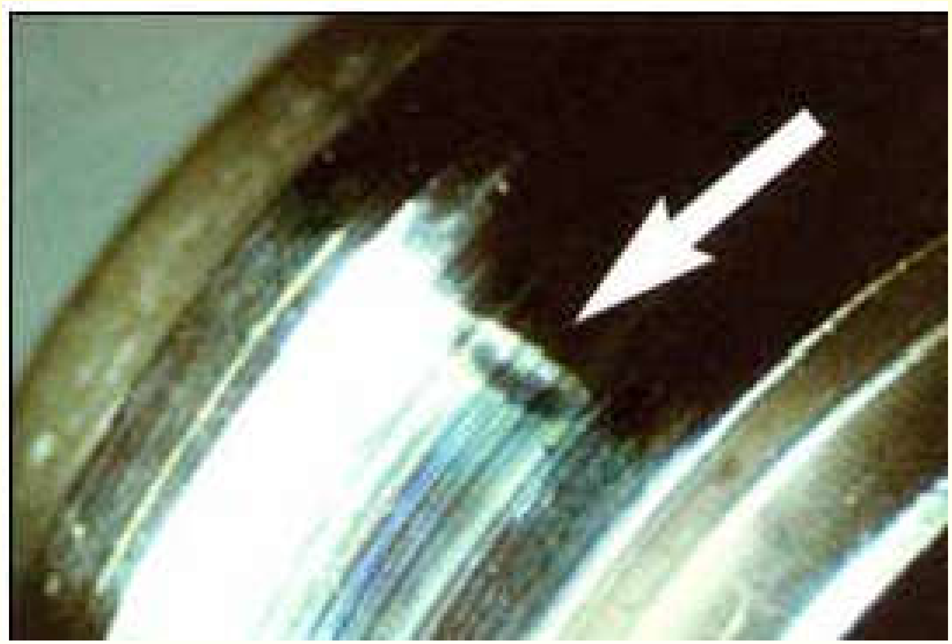


What is faulty in the design used? The central can be taken about 3 tons of tension/compression and about 100 Nm of torque.

Collapse of Thin Walled Structures

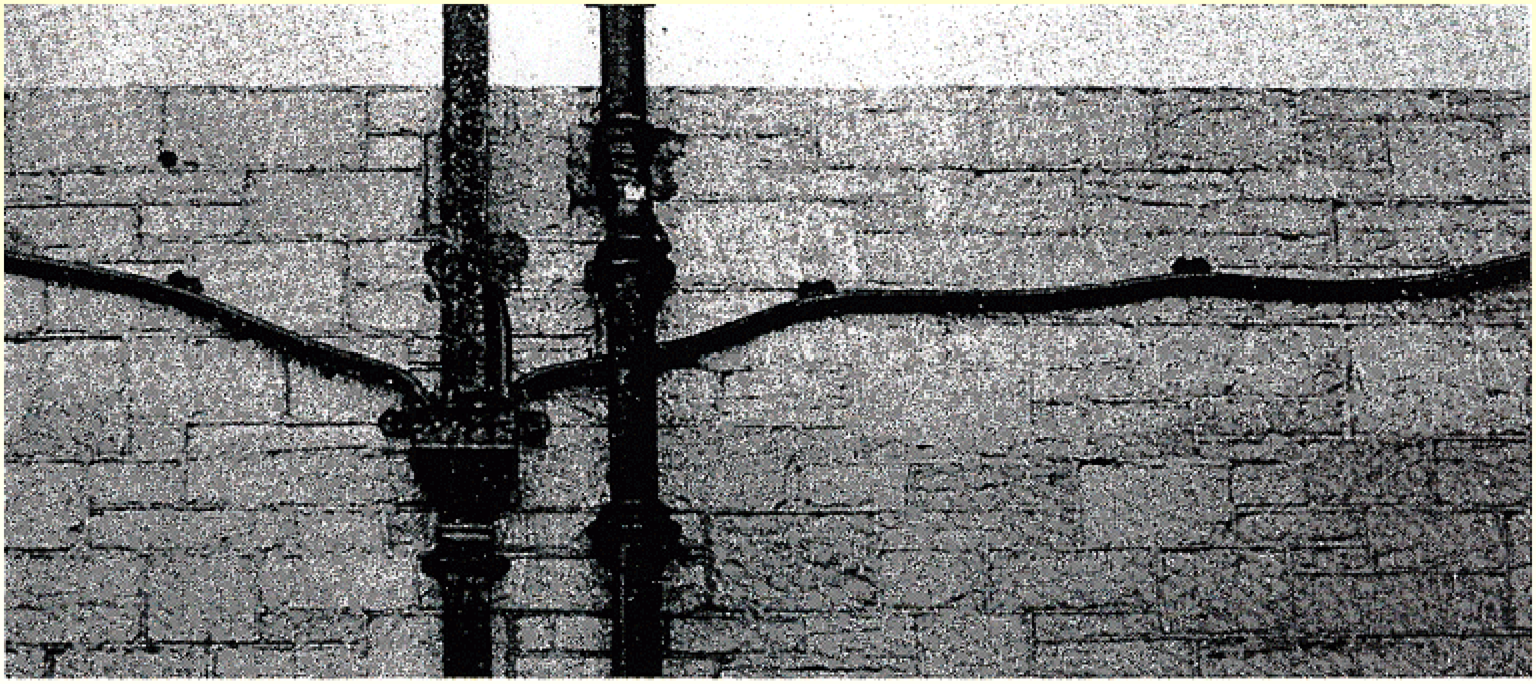
There was a condensation in the containers just before the collapse. Why do you think the structures here collapsed?

Roller Impression on Raceway Surfaces



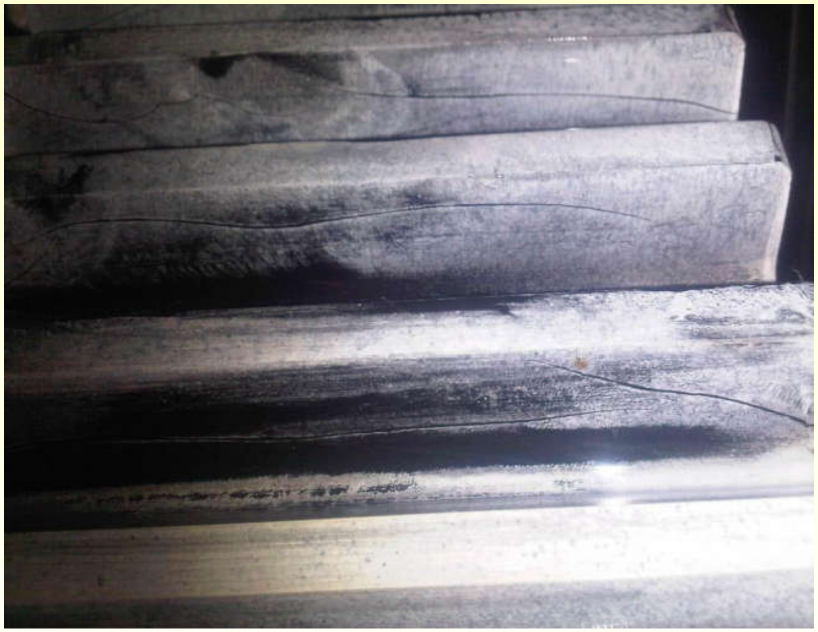
What could be causing these marks on the inner raceways of bearings? Why are such impressions bad?

Sagging of Lead Pipes



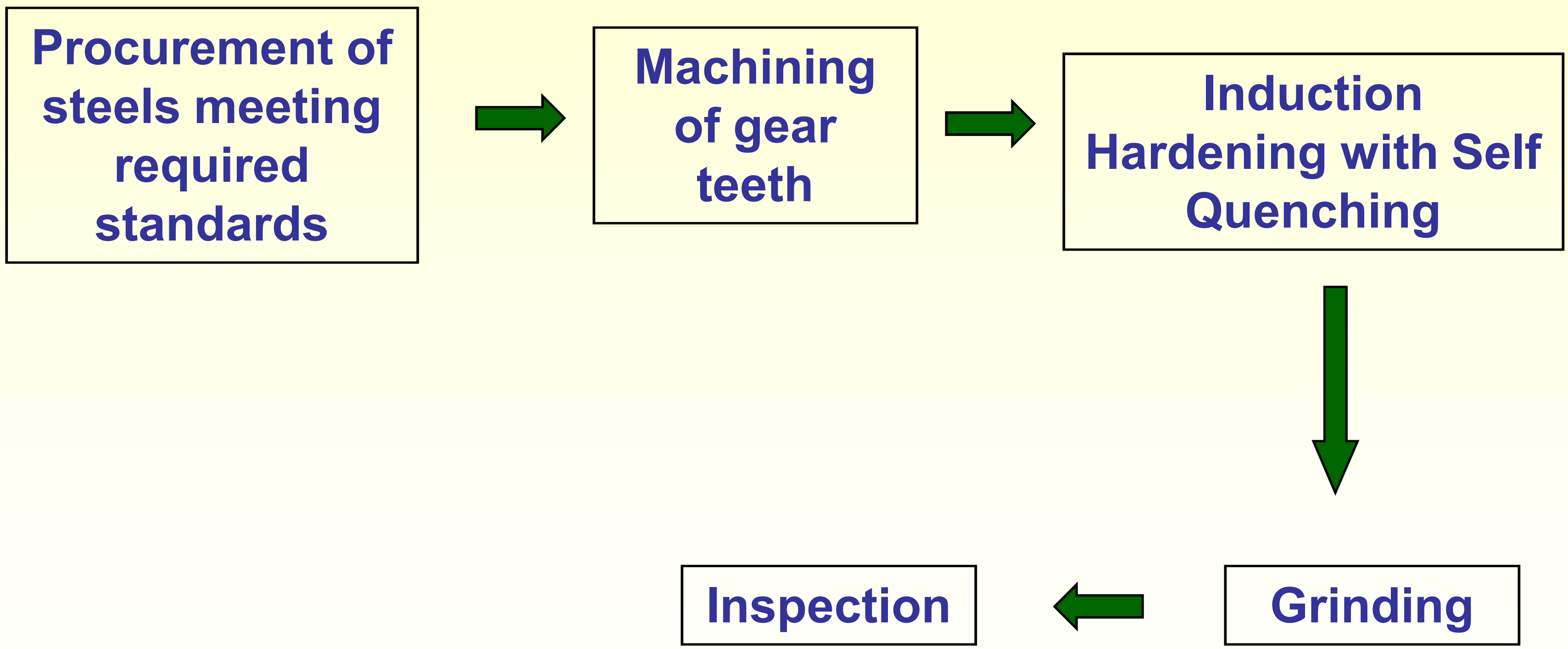
When initially installed the lead pipes were straight. However, after around 75 years after their installation, as shown in this picture, they SAG. WHY?

Fracture of Gear Teeth During Heat Treatment

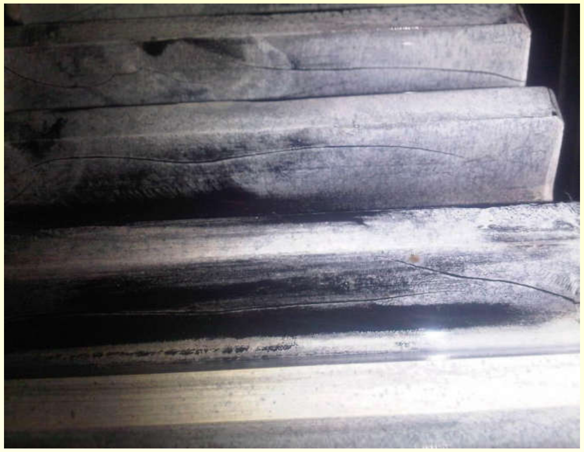


Gear teeth fractured during heat treatment. What could have gone wrong?

Steps Involved in Gear Teeth Fabrication



Gear Teeth Failure: Observations



Observations on cracks seen in the figures:

- They appear on most gear teeth
- Their points of origin are different for different teeth
- They run deep
- They have smooth profiles (not rough) similar to cracks seen in glass
- They appear during heat treatment

Implications of the Observations Made

- The fact that cracks do not always appear at same location (e.g. at the root) in each teeth implies that residual stresses are not uniform and that defects are playing an important role. As defect locations in a teeth are different for different teeth, points of origin are also different for each of them
- Deep cracks with smooth contours imply that these are not fatigue cracks (fatigue cracks grow slowly and have oscillatory contours)
- Crushing along with presence of deep fracture cracks suggest that the material is brittle and has low toughness

Catastrophic Failure of Liberty Ships

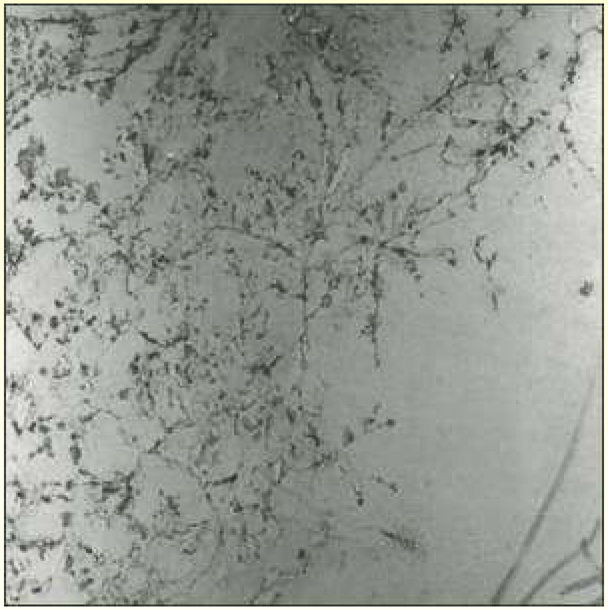
3 of around 2700 "liberty" ships made in US during second-world world war time broke in half during their use in North-Atlantic waters. Hull structures were all welded. What could have caused such catastrophic failure when they were in service?

Season Cracking of Brass Cartridges

Brass cartridges stored in horse stables, by British Army in India, in early 19th century underwent cracking as shown in the figure. The problem was observed to be more severe in summer and was termed as season cracking. CAN YOU GUESS AS TO WHAT IS GOING ON?

Ref: Wikipedia.org

Cracking of a Buried SS304L Pipeline



Piping exterior surface

Branched cracking in pipe cross-section (50X magnification with oxalic acid etch)

A buried pipeline carrying carbon dioxide feedstock experienced several failures during its short lifetime. The pipeline was buried four feet deep running parallel to a roadway which is frequently deiced with deicing salts during the winter months. Several weld repairs had already been made on the pipeline due to leaks at flanged joints and distorted sections of the pipeline caused by thermal expansion.

Ref: www.corrosionlab.com/

Failure of Axial Fan

All blades of one fan failed at approximately one hundred hours of operation. The blame game was on and was put on to construction for not removing some temporary material caught between the blades, damaging one blade and subsequently all others. After a few days, the second FD fan also failed exactly the similar way and exactly and at the same hours of operation.

What could be the root cause(s) of the failure?

Ref: http://www.brighthubengineering.com/

Failure of a Rail

Can you guess the failure mode here ?

Railway Wheels

Railway Wheels Need to be Periodically Remachined. Why?

Design of Machine Elements

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Prof. Vikranth Racherla

Lectures 7-13

Jan 17-30, 2017

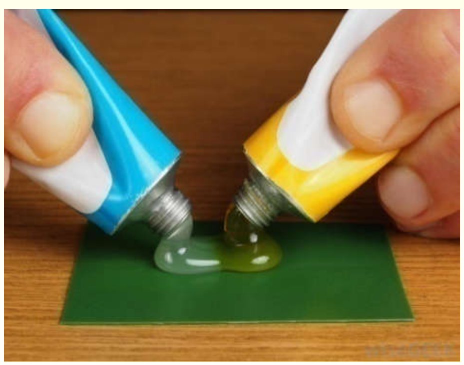
Engineering Materials: Broad Classification

Polymers

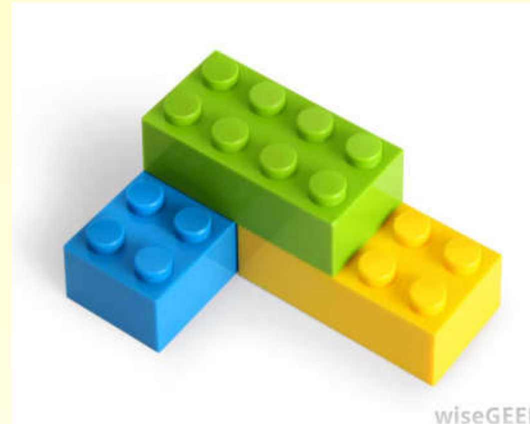
Metals

Ceramics

Polymers: Bonding based classification



Thermosets



Thermoplastics

Polymers: Mechanical Behavior Based Characterization

Vulcanized rubber

SBS (thermoplastic) rubber

Silicone

Polyurethane (thermoplastic)

Nitrile/Butyl rubber

Elastomers

Polymers: Mechanical Behavior Based Characterization

PMMA (Acrylic) glass (Used for their higher scratch resistance and transparency)

Polycarbonate (Relatively expensive, used for their higher impact resistance and transparency)

Plastics: Hard and Stiff

Polymers: Mechanical Behavior Based Characterization

Polyethylene Terephthalate (PET)

Polystyrene (hard but brittle)

Polypropylene (most widely used plastic in consumer products

Plastics: Hard and Stiff

Polymers: Mechanical Behavior Based Characterization

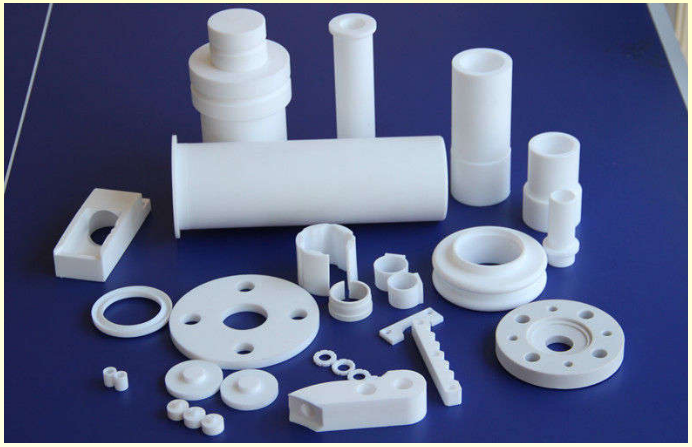
Acrylonitrile butadiene styrene (ABS), Hard and tough, degrades with exposure to sun right

LDPE (Compliant, tough)

HDPE (Offers good solvent resistance, tough)

Plastics: Hard and Stiff

Polymers: Mechanical Behavior Based Characterization



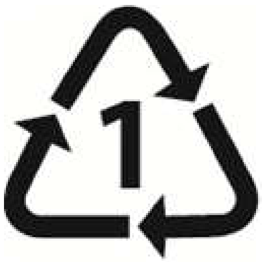
Polytetrafluoroethylene (Teflon, Popular for providing low friction coefficient)

Melamine

Nylon (Expensive, stiff, tough, wear resistant)

Plastics: Hard and Stiff

What do Recycling Symbols on Plastics Mean?

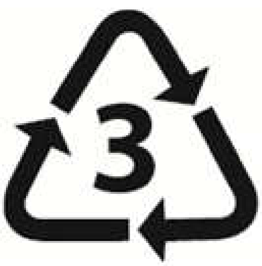


PET, PETE
(Polyethylene Terephthalate)

- Soft drink, water and salad dressing bottles; peanut butter and jam jars…
- Suitable to store cold or warm drinks. Bad idea for hot drinks.

HDPE
(High-density Polyethylene)

- Water pipes, milk, juice and water bottles; grocery bags, some shampoo / toiletry bottles…



PVC
(Polyvinyl Chloride)

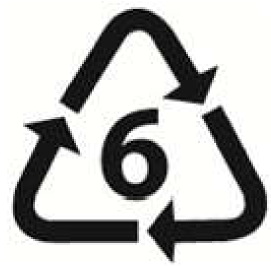
- Not used for food packaging.
- Pipes, cables, furniture, clothes, toys…

LDPE
(Low-density Polyethylene)

- Frozen food bags; squeezable bottles, e.g. honey, mustard; cling films; flexible container lids….

PP
(Polypropylene)

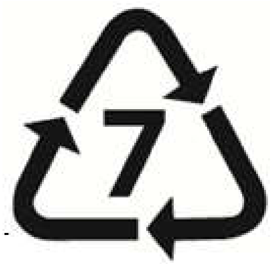
- Reusable microwaveable ware; kitchenware; yogurt containers; microwaveable disposable take-away containers; disposable cups; plates….



PS
(Polystyrene)

- Egg cartons; packing peanuts; disposable cups, plates, trays and cutlery; disposable take-away containers;…

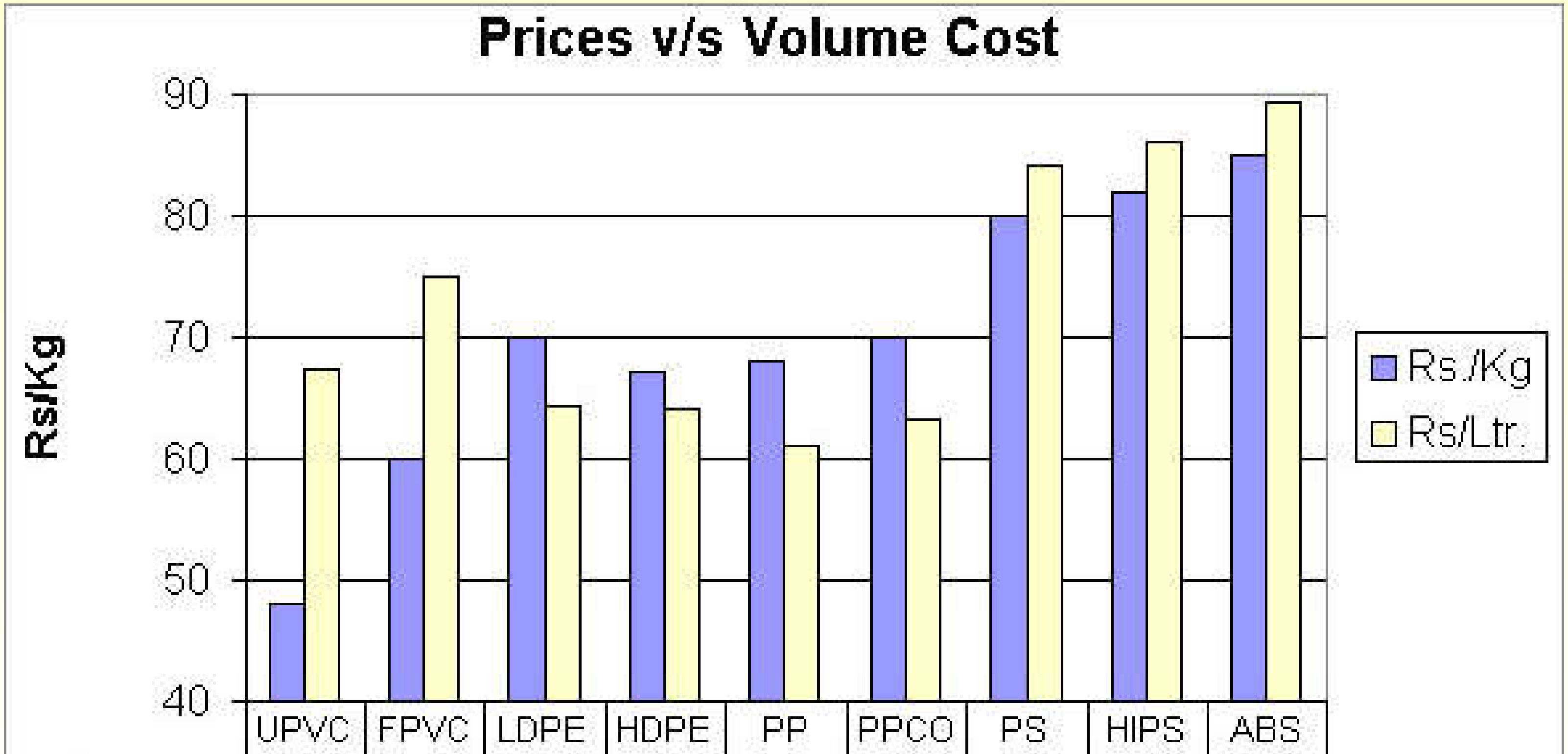
  Avoid for food storage!



Other
(often polycarbonate or ABS)

- Beverage bottles; baby milk bottles. compact discs; "unbreakable" glazing; lenses including sunglasses, prescription glasses, automotive headlamps, riot shields, instrument panels…

Price of a few commonly used plastics



	UPVC	FPVC	LDPE	HDPE	PP	PPCO	PS	HIPS	ABS
Rs./Kg	48	60	70	67	68	70	80	82	85
Rs/Ltr.	67.2	75	64.4	64.32	61.2	63.35	84	86.1	89.25

Metals: Popular ones

Steels

- Cost ~ Rs 50 Rs/kg
- Density 7.8 g/cc
- E = 200 GPa
- UTS = 300 MPa to 1750 MPa
- Thermal conductivity = 15 to 50 W/mK
- Electrical resistivity ~ 100 n Ohm/m
- Melting point = 1440 degrees C
- Annual usage in India ~ 60 million tonnes
- Iron electrochemical potential: -0.44

Aluminum

- Cost ~ Rs 240 Rs/kg
- Density 2.7 g/cc
- E = 70 GPa
- UTS = 80 MPa to 550 MPa
- Thermal conductivity = 237 W/mK
- Electrical resistivity ~ 30 n Ohm/m
- Melting point = 660 degrees C
- Annual usage in India ~ 1 million tonnes

Metals: Popular ones

Copper

- Cost ~ Rs 500 Rs/kg
- Density 8.96 g/cc
- E = 115 GPa
- Yield stress = 30 MPa to 550 MPa
- Thermal conductivity = 401 W/mK
- Electrical resistivity ~ 17 n Ohm/m
- Melting point = 1080 degrees C
- Annual usage in India ~ 0.7 million tonnes

Zinc

- Cost ~ Rs 300 Rs/kg
- Density 7.14 g/cc
- E = 108 GPa
- UTS = 100-150 MPa
- Thermal conductivity = 160 W/mK
- Electrical resistivity ~ 60 n Ohm/m
- Melting point = 420 degrees C
- Annual usage in India ~ 0.5 million tonnes
- Electrochemical potential: -0.76

Metals: Popular ones

Tin

- Cost ~ Rs 60 Rs/kg
- Density 7.2 g/cc
- E = 50 GPa
- UTS = 20 MPa
- Thermal conductivity = 67 W/m
- Electrical resistivity ~ 115 n Ohm/m
- Melting point = 230 degrees C
- Soft, corrosion resistant metal

Titanium

- Cost ~ Rs 2000 Rs/kg
- Density 4.5 g/cc
- E = 115 GPa
- UTS = 250 MPa to 1300 MPa
- Thermal conductivity = 22 W/mK
- Electrical resistivity ~ 420 n Ohm/m
- Melting point = 1670 degrees C
- Excellent corrosion resistance, bio-compatible
- Ti-6Al-4V most widely used alloy0

Metals: Popular ones

Nickel

- Cost ~ Rs 600 Rs/kg
- Density 8.91 g/cc
- E = 200 GPa
- UTS = 450-1200 MPa
- Thermal conductivity = 90 W/m
- Electrical resistivity ~ 70 n Ohm/m
- Melting point = 1455 degrees C
- Ni based supper alloys have excellent high temperature strength, corrosion resistance, oxidation resistance
- Inconel (~ Rs 2000/kg) most popular alloy

Tungsten

- Cost ~ Rs 3000-5000 Rs/kg
- Density 19.2 g/cc
- E = 411 GPa
- UTS = 2000 MPa
- Thermal conductivity = 173 W/m
- Electrical resistivity ~ 50 n Ohm/m
- Melting point = 3422 degrees C
- Refractory metal. Can be used up to around 1650 degrees C with protective coatings

Metals: Popular ones

Lead

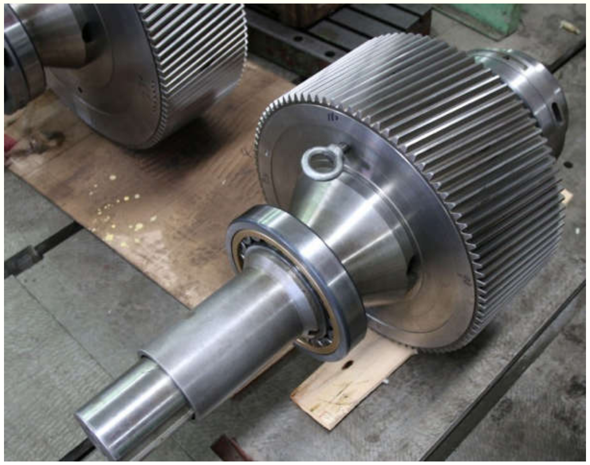
- Cost ~ Rs 150 Rs/kg
- Density 11.34 g/cc
- E = 16 GPa
- UTS = 18 MPa
- Thermal conductivity = 35 W/m
- Electrical resistivity ~ 208 n Ohm/m
- Melting point = 327 degrees C

Popular Steels: AISI 4340

Chemical analysis of AISI 4340 alloy steel

Elements	C	Si	Mn	Ni	Cr	Mo	P	S
wt.(%)	0.39	0.24	0.61	1.46	0.67	0.17	0.021	0.006

Properties	Metric
Tensile strength	745 MPa
Yield strength	470 MPa
Bulk modulus (typical for steel)	140 GPa
Shear modulus (typical for steel)	80 GPa
Elastic modulus	190-210 GPa
Poisson's ratio	0.27-0.30
Elongation at break	22%

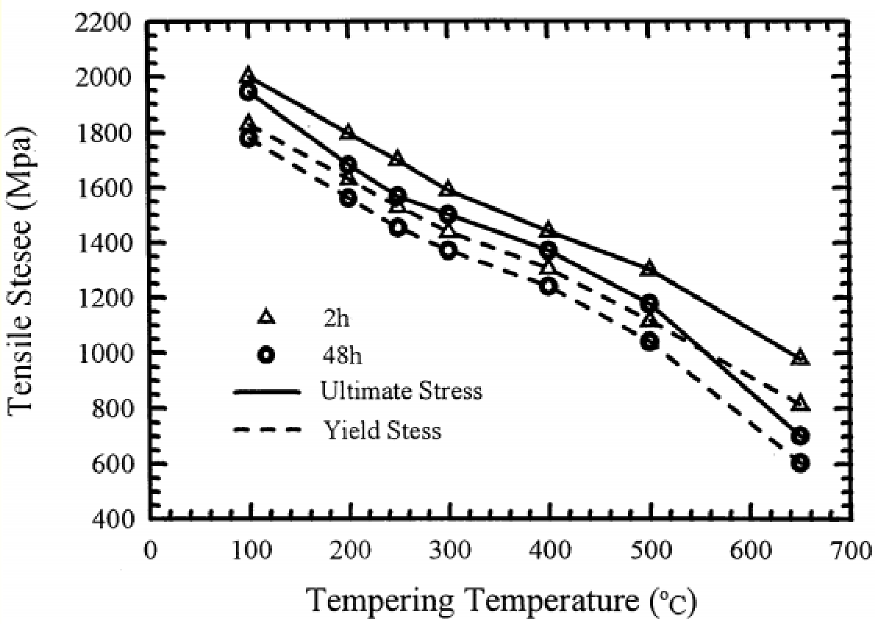


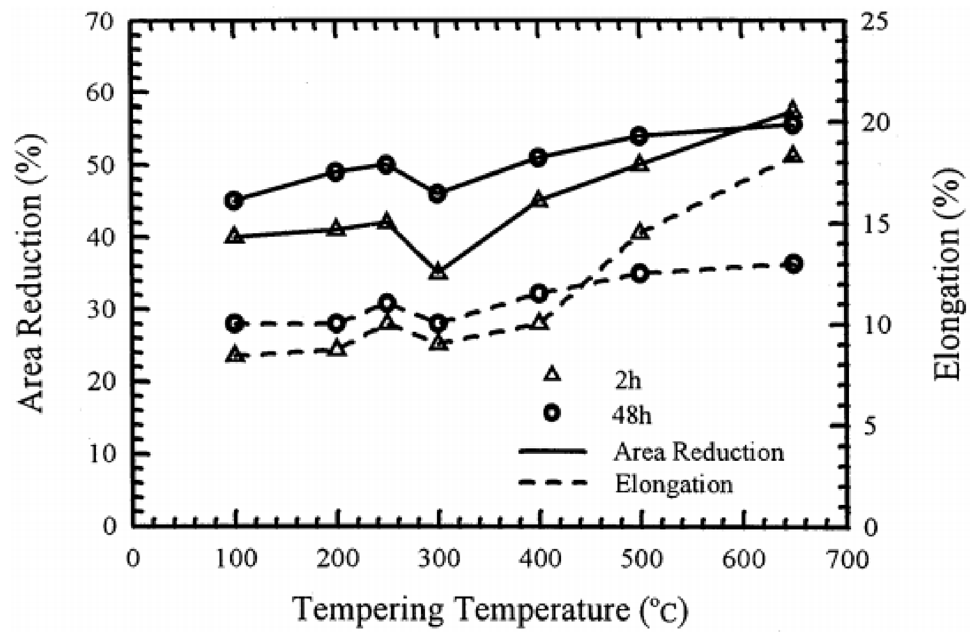
Properties in annealed condition

AISI 4340: Properties After Heat Treatment

The mechanical properties of AISI 4340 alloy steel after 48 h tempering

Tempering temperature (°C)	100	200	250	300	400	500	650	Quenching
σ_y (MPa)	1778	1557	1450	1367	1237	1037	600	2015
σ_T (MPa)	1940	1677	1564	1497	1366	1172	699	2214
Hv (62.5 kg)	597	512	470	457	430	379	660	660
A (%)	44	48	50	44	50	52	33.7	33.7
ε (%)	9.8	9.8	11	9.7	11.5	12.5	4.5	4.5
n	0.5	0.44	0.40	0.36	0.31	0.23	0.57	0.57



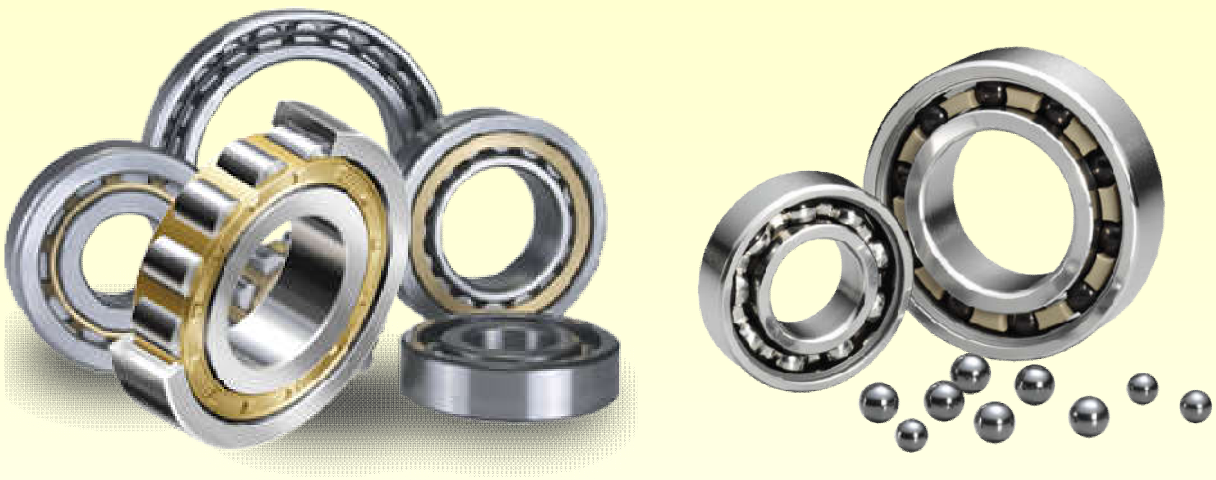


Ref: Lee and Su, 1999

Used after quenching and heat treatment wherever high hardness, wear resistance, and high toughness is desired, e.g. shafts, cams, gears, etc.

Popular Steels: AISI 52100

Element	Content (%)
Iron, Fe	96.5 - 97.32
Chromium, Cr	1.30 - 1.60
Carbon, C	0.980 - 1.10
Manganese, Mn	0.250 - 0.450
Silicon, Si	0.150 - 0.300
Sulfur, S	≤ 0.0250
Phosphorous, P	≤ 0.0250



Temperature (°C)	22	200	400	600	800	1000
Yield Strength (MPa) (0.2% offset except 22°C)	1410.17	1672.26	915.94	80.91	40.80	18.65
Tensile Strength (MPa)	NA#	2482.85	1221.36	221.46	84.06	33.14
Fracture Strength (MPa)	1866.85	2731.14	1343.50	243.61	92.47	36.45
Yield Strain (10^{-2})	0.70	1.09	1.09	0.30	0.30	0.20
Tensile Strain (10^{-2})	1.10	4.46	2.77	3.23	5.00	6.59
Fracture Strain (10^{-2})	1.10	6.97	74.35	252.18	128.37	42.64
Young's Modulus (GPa)	201.33	178.58	162.72	103.42	86.87	66.88*
Poisson's Ratio	0.277	0.269	0.255	0.342	0.396	0.490

Properties after quenching and tempering at 150 C for 1 hour.

Ref: Guo and Liu, 2002.

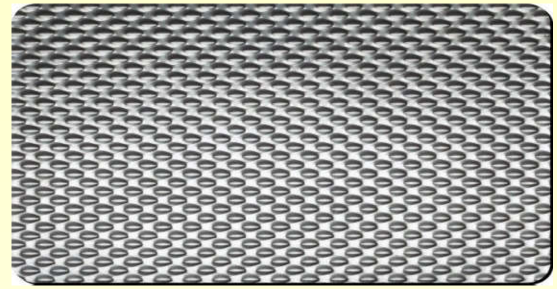
Used after quenching and heat treatment wherever high hardness and wear resistance at moderate temperatures (~200 C) is desired, e.g. for raceways and rollers in roller bearings.

Popular Steels: SS 304

MECHANICAL PROPERTIES

Typical Room Temperature Mechanical Properties

	UTS ksi (MPa)	0.2% YS ksi (MPa)	Elongation % in 2" (50.8 mm)	Hardness Rockwell
Type 304L	85 (586)	35 (241)	55	B80
Type 304	90 (621)	42 (290)	55	B82



COMPOSITION

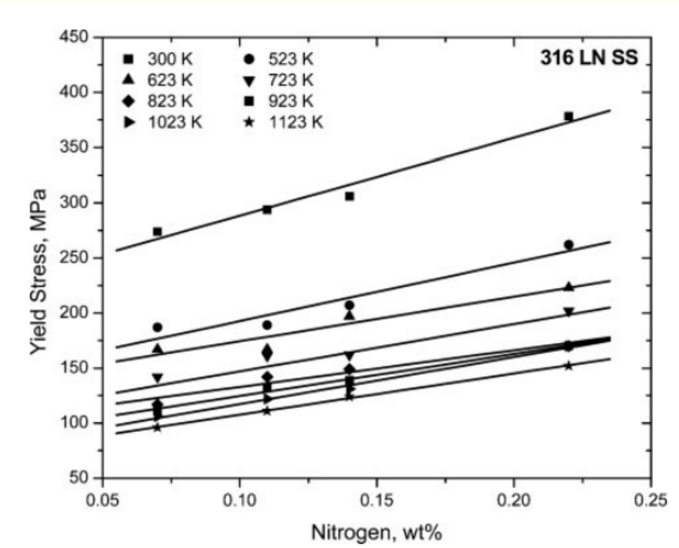
	Type 304 %	Type 304L %
Carbon	0.08 max.	0.03 max.
Manganese	2.00 max.	2.00 max.
Phosphorus	0.045 max.	0.045 max.
Sulfur	0.030 max.	0.030 max.
Silicon	0.75 max.	0.75 max.
Chromium	18.00-20.00	18.0-20.0
Nickel	8.00-12.00	8.0-12.0
Nitrogen	0.10 max.	0.10 max.
Iron	Balance	Balance

SS 304 has good corrosion resistance (expect in saline conditions or in presence of industrial solvents) and has good formability. It however work hardens easily and stress relieving is need in critical applications. It is used in structural panels, cooking utensils, expensive bathroom fitting, chemical plants, rust free wire meshes, etc.

Popular Steels: SS 316LN

Table 1 Chemical composition of 316LN SS, wt-%

Specification		N	C	Mn	Cr	Mo	Ni	Si	S	P	Fe	Grain size, μm
Heat no.	Desig-nation	0·06–0·22	0·02–0·03	1·6 – 2·0	17–18	2·30–2·50	12·0–12·5	0·50 max.	0·01 max.	0·03 max.	Bal.	<180
H8344	7N	0·07	0·027	1·7	17·53	2·49	12·2	0·22	0·0055	0·013	Bal.	87·3 ±8·7
H8335	11N	0·11	0·033	1·78	17·62	2·51	12·27	0·21	0·0055	0·015	Bal.	95·8 ±8·0
H8334	14N	0·14	0·025	1·74	17·57	2·53	12·15	0·20	0·0041	0·017	Bal.	77·7 ±7·7
H8345	22N	0·22	0·028	1·70	17·57	2·54	12·36	0·20	0·0055	0·018	Bal.	86·8 ±10·9



Yield stress as a function of nitrogen weight percentage and working temperature in K

(a)

(b)

(c)

(d)

Applications where SS 316 is used: (a) surgical instruments, (b) Food processing/Pharmaceutical industries, (c) Medical needles, (d) Pipes where salty conditions can exist.

SS 316 has greater corrosion (pitting) resistance than SS 304, particularly against chlorides and other industrial solvents, from presence of 2-3% Mo. It also has higher high temperature strength, creep resistance, corrosion resistance, and resistance to stress corrosion cracking even at weld locations.

Ref: Ganesan, Mathew, and Sankara Rao, 2013.

Ceramics

Alumina

- Cost ~ Rs 50 Rs/kg (powder cost)
- Density ~ 4 g/cc
- E ~ 410 GPa
- Compressive strength ~ 3000 MPa
- Flexural strength ~ 350 MPa
- Tensile strength ~ 250 MPa
- Thermal conductivity ~ 30 W/mK
- Melting point = 2072 degrees C

Silicon Carbide

- Cost ~ Rs 200 Rs/kg (powder cost)
- Density ~ 3.1 g/cc
- E ~ 410 GPa
- Compressive strength ~ 4000 MPa
- Flexural strength ~ 550 MPa
- Fracture toughness ~ 4 MPa Sqrt m
- Thermal conductivity ~ 120 W/mK
- Melting point = 2730 degrees C

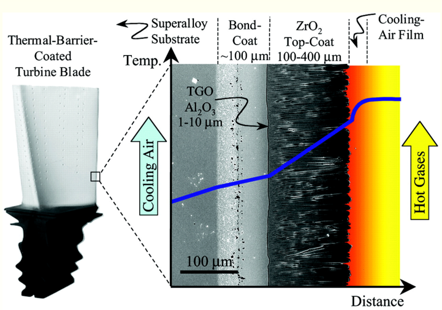
High hardness, abrasion resistance, electric insulation, corrosion resistance, heat impact resistance, bio inert capacity, high chemical resistance and high melting point are reasons for going with ceramics as opposed to metals. They are however, difficult to process (sintering is only way to make parts).

Ref: https://srdata.nist.gov/CeramicDataPortal/Pds/Scdaos

Ceramics

Tungsten carbide

- Density ~ 15.6 g/cc
- E ~ 600 GPa
- Compressive strength ~ 5100 MPa
- Fracture toughness ~ 3 MPa Sqrt m
- Thermal conductivity ~60 W/mK
- Melting point = 3100 degrees C

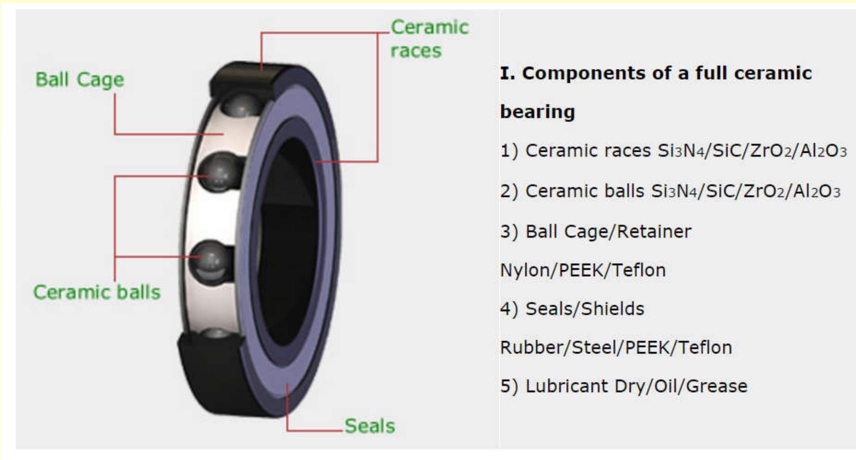


Cross-sectional scanning electron micrograph (SEM) of an electron-beam physical-vapor deposited (EB-PVD) TBC, superimposed onto a schematic diagram showing the temperature reduction provided by the TBC. The turbine blade contains internal hollow channels for air-cooling, whereas the outside hot-section surface is thermal barrier-coated, setting up a temperature gradient across the TBC.

(Yttria stabilized) Zirconia

- Density ~ 5.7 g/cc
- E ~ 205 GPa
- Compressive strength ~ 2000 MPa
- Flexural strength ~ 1000 MPa
- Fracture toughness ~ 10 MPa Sqrt m
- Thermal conductivity ~ 2 W/mK
- Melting point = 2715 degrees C

Ceramics



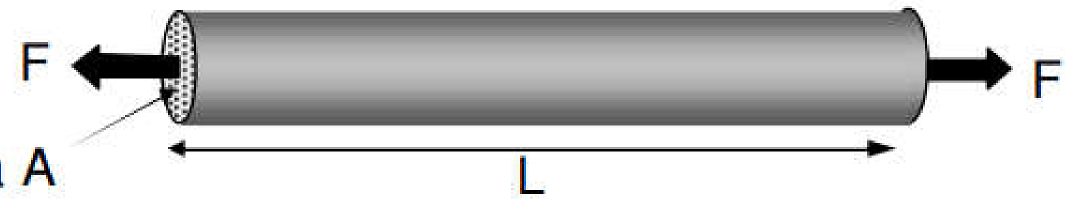
Silicon Nitride

- Density ~ 3.2 g/cc
- Thermal conductivity ~ 30 W/mK
- Melting point ~ 1900 degrees C

Materials Selection

Deriving Performance Indices: Light, stiff tie

Stiff tie of length L and minimum mass



Function — *Tie-rod*

Objective — *Minimise mass m:*

$$m = A L \rho \qquad (1)$$

Constraints — *Stiffness* of the tie S:

$$S = \frac{EA}{L} \qquad (2)$$

m = mass
A = area
L = length
ρ = density
S = stiffness
E = Youngs Modulus

Free variables

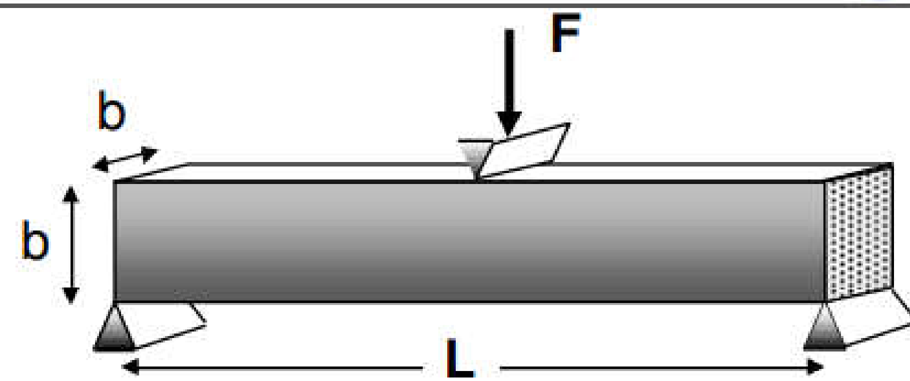
- *Material choice*
- *Section area A; eliminate in (1) using (2):*

$$m = SL^2\left(\frac{\rho}{E}\right)$$

Chose materials with smallest $\left(\frac{\rho}{E}\right)$

Ref: www.diim.unict.it/users/fgiudice/pdfs/SM_2.3.pdf

Deriving Performance Indices: Light, stiff beam



Function — *Beam* (solid square section).

Objective — *Minimise mass*, m, where:

$$m = AL\rho = b^2 L\rho$$

Constraint — *Stiffness* of the beam S:

$$S = \frac{CEI}{L^3}$$

ρ

I is the second moment of area:

$$I = \frac{b^4}{12}$$

m = mass
A = area
L = length
ρ = density
b = edge length
S = stiffness
I = second moment of area
E = Youngs Modulus

Free variables
- *Material choice.*
- *Edge length b.* Combining the equations gives:

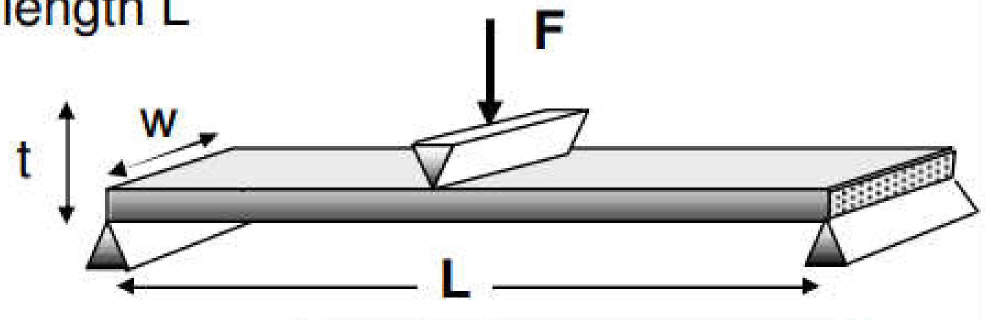
$$m = \left(\frac{12\, S\, L^5}{C}\right)^{1/2} \left(\frac{\rho}{E^{1/2}}\right)$$

Chose materials with smallest $\left(\frac{\rho}{E^{1/2}}\right)$

Ref: www.diim.unict.it/users/fgiudice/pdfs/SM_2.3.pdf

Deriving Performance Indices: Light, stiff panel

Function *Panel* with given width w and length L



Objective *Minimise mass*, m, where

$$m = A L \rho = w\ t\ L \rho$$

Constraint *Stiffness* of the panel S:

$$S = \frac{C E I}{L^3}$$

I is the second moment of area:

$$I = \frac{w\, t^3}{12}$$

m = mass
w = width
L = length
ρ = density
t = thickness
S = stiffness
I = second moment of area
E = Youngs Modulus

Free variables
- *Material choice.*
- *Panel thickness t.* Combining the equations gives:

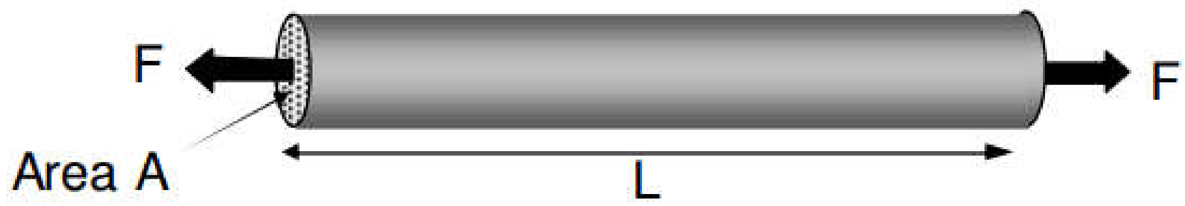
$$m = \left(\frac{12\ S\ w^2}{C} \right)^{1/3} L^2 \left(\frac{\rho}{E^{1/3}} \right)$$

Chose materials with smallest $\left(\frac{\rho}{E^{1/3}} \right)$

Ref: www.diim.unict.it/users/fgiudice/pdfs/SM_2.3.pdf

Deriving Performance Indices: Light, strong tie

Strong tie of length L and minimum mass



Function: *Tie-rod*

Objective: *Minimise mass m:*

$$m = A L \rho \qquad (1)$$

Constraints:

- *Length L is specified*
- *Must not fail under load F*
- *Adequate fracture toughness*

Equation for constraint on A:

$$F/A < \sigma_y \qquad (2)$$

Free variables:

- *Material choice*
- *Section area A;*

  eliminate in (1) using (2):

$$m = FL\left(\frac{\rho}{\sigma_y}\right)$$

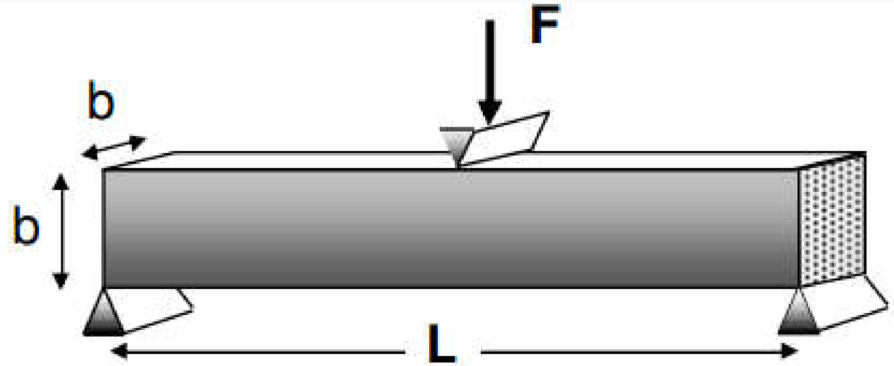
m = mass
A = area
L = length
ρ = density
σ_y = yield strength

STEP 4

Use this constraint to eliminate the free variable in performance equation

Ref: www.diim.unict.it/users/fgiudice/pdfs/SM_2.3.pdf

Deriving Performance Indices: Light, strong beam



Function *Beam* (solid square section).

Objective *Minimise mass*, m, where:

$$m = AL\rho = b^2 L\rho$$

Constraint *Must not fail under load F*

$$\sigma_y > \frac{M \cdot b/2}{I} \left(= \frac{3FL}{b^3} \right)$$

moment should be FL/4 here FL/2 is used

I is the second moment of area:

$$I = \frac{b^4}{12}$$

m = mass
A = area
L = length
ρ = density
b = edge length
I = second moment of area
σ_y = yield strength

Free variables
- *Material choice.*
- *Edge length b.* Combining the equations gives:

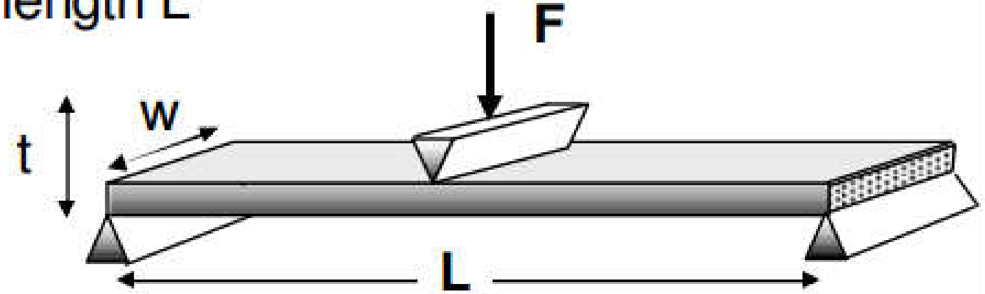
$$m = (L)^{5/3} (3F)^{2/3} \left(\frac{\rho}{\sigma_y^{2/3}} \right)$$

Chose materials with smallest $\left(\frac{\rho}{\sigma_y^{2/3}} \right)$

Ref: www.diim.unict.it/users/fgiudice/pdfs/SM_2.3.pdf

Deriving Performance Indices: Light, strong panel

Function: *Panel* with given width w and length L



Objective: *Minimise mass*, m, where

$$m = AL\rho = w\,t\,L\rho$$

Constraint: *Must not fail under load F*

$$\sigma_y > \frac{M \cdot t/2}{I} \left(= \frac{3FL}{wt^2} \right)$$

moment should be FL/4 here FL/2 is used

I is the second moment of area:

$$I = \frac{w\,t^3}{12}$$

m = mass
w = width
L = length
ρ = density
t = thickness
I = second moment of area
σ_y = yield strength

Free variables
- *Material choice.*
- *Panel thickness t.* Combining the equations gives:

$$m = (3Fw)^{1/2} (L)^{3/2} \left(\frac{\rho}{\sigma_y^{1/2}} \right)$$

Chose materials with smallest $\left(\frac{\rho}{\sigma_y^{1/2}} \right)$

Ref: www.diim.unict.it/users/fgiudice/pdfs/SM_2.3.pdf

Performance Indices for weight: Stiffness

Material properties --

the "Physicists" view of materials, e.g.

Cost,	C_m
Density,	ρ
Modulus,	E
Strength,	σ_y
Endurance limit,	σ_e
Thermal conductivity,	λ
T- expansion coefficient,	α

Material indices --

the "Engineers" view of materials

Objective: minimise mass

Function	**Stiffness**	**Strength**
Tension (tie)	ρ/E	ρ/σ_y
Bending (beam)	$\rho/E^{1/2}$	$\rho/\sigma_y^{2/3}$
Bending (panel)	$\rho/E^{1/3}$	$\rho/\sigma_y^{1/2}$

Minimise these!

Ref: www.diim.unict.it/users/fgiudice/pdfs/SM_2.3.pdf

1.2.17 Design of Machine Elements – Lecture 14

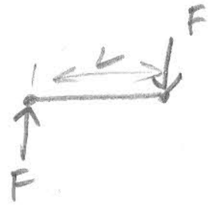
STRESS ANALYSIS

Properties of (Cauchy) stress : Stress defined in deformed/ current config.

- It is a second-order tensor and has 9 components denoted by σ_{ij} $(i, j = 1, 2, 3)$

- The 9 components can be denoted in matrix form as

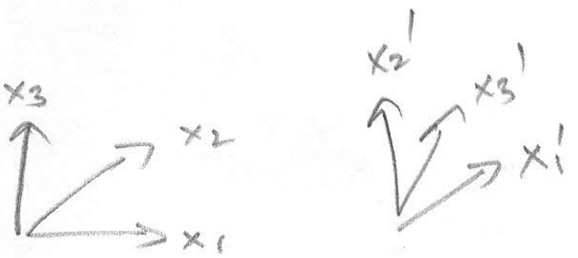
$$\begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}$$

- In absence of point couples (

$C = FL$, with $F \to \infty$ and $L \to 0$) – which is the case in essentially all engineering problems – angular momentum balance requires that stress tensor be symmetric and $\boxed{\sigma_{ij} = \sigma_{ji}}$

- Stress components depend on choice of co-ordinate system. However, components of stress in different co-ordinate systems are related through transformation relations.

More specifically,



if $x_1 - x_2 - x_3$ and $x_1' - x_2' - x_3'$ denote two rectangular cartesian co-ordinate systems with $\underset{\sim}{e}'_i$ and $\underset{\sim}{e}_j$ denoting unit vectors along x'_i and x_j $(i, j = 1,2,3)$ axes then

$$\begin{bmatrix} \sigma'_{11} & \sigma'_{12} & \sigma'_{13} \\ \sigma'_{21} & \sigma'_{22} & \sigma'_{23} \\ \sigma'_{31} & \sigma'_{32} & \sigma'_{33} \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{31} & Q_{32} \end{bmatrix} \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix} \begin{bmatrix} Q_{11} & Q_{12} & Q_{13} \\ Q_{21} & Q_{22} & Q_{23} \\ Q_{31} & Q_{32} & Q_{33} \end{bmatrix}^T$$

where σ_{ij}' are components of stress in x_i' co-ordinate system σ_{ij} are components of stress in x_i system and Q_{ij} are components of rotation matrix given by

$$\boxed{Q_{ij} = \underset{\sim}{e}_i' \cdot \underset{\sim}{e}_j}$$

with

$$[Q][Q]^T = [I] \equiv \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\det[Q] = \det[Q]^T = 1$$

- While components of stress depend on co-ordinate system certain "measures" of stress are independent of co-ordinate system. Such "measures" are called stress invariants

- Noting that for any real matrices $[A], [B], [C]$

$$\mathrm{tr}\left([A][B][C]\right) = \mathrm{tr}\left([C][A][B]\right) = \mathrm{tr}\left([B][C][A]\right)$$

and

$$\det\left([A][B][C]\right) = \det[A]\,\det[B]\,\det[C]$$

we have that

$$\mathrm{tr}([\sigma']) = \mathrm{tr}([Q][\sigma][Q]^T)$$

$$= \mathrm{tr}([Q]^T[Q][\sigma]) = \mathrm{tr}([I][\sigma])$$

$$\Rightarrow \boxed{\mathrm{tr}([\sigma']) = \mathrm{tr}([\sigma])}$$

Thus trace of stress $= \sigma_{11} + \sigma_{22} + \sigma_{33}$ $= -3p$ is a stress invariant

Similarly

$$\det([\sigma']) = \det[Q] \det[\sigma] \det[Q]^T$$

$$\Rightarrow \boxed{\det[\sigma'] = \det[\sigma]}$$

Thus "$\det[\sigma]$" is a stress invariant

$$\mathrm{tr}([\sigma']^2) = \mathrm{tr}([Q][\sigma][Q]^T[Q][\sigma][Q]^T)$$

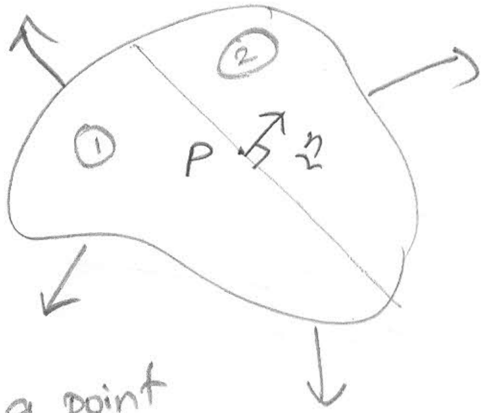
$$= \mathrm{tr}([Q][\sigma]^2[Q]^T) = \mathrm{tr}([Q]^T[Q][\sigma]^2)$$

$$\Rightarrow \boxed{\mathrm{tr}([\sigma']^2) = \mathrm{tr}([\sigma]^2)}$$

"$\mathrm{tr}[\sigma]^2$" is another stress invariant

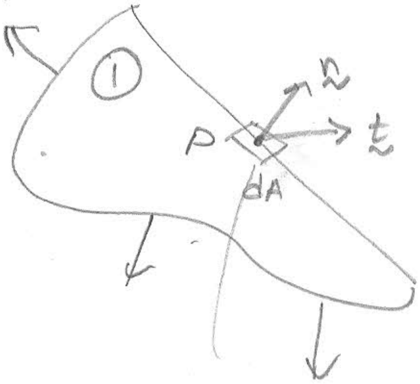
- Stress exists at all points within a body and is commonly non-uniform, i.e. different at different points.

- Consider a body subjected to external loads that results in "stress" at all points in the body.



Let $[\sigma]$ denote stress at a point "P" and let $\underset{\sim}{n}$ denote unit normal to a plane passing through P.

Force acting per unit area $\underset{\sim}{t}$ at P on plane with unit normal $\underset{\sim}{n}$ is given by



$$\underset{\sim}{t} = \underset{\approx}{\sigma}^T \underset{\sim}{n}$$

or

$$\boxed{\begin{Bmatrix} t_1 \\ t_2 \\ t_3 \end{Bmatrix} = \begin{bmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{bmatrix}^T \begin{Bmatrix} n_1 \\ n_2 \\ n_3 \end{Bmatrix}}$$

Force acting on area element dA on plane in question is

$$d\underset{\sim}{F} = \underset{\sim}{t}\, dA$$

passing through a point

- Planes on which ONLY normal traction exists and shear traction is zero, i.e.

$$\boxed{\underset{\sim}{t}(\underset{\sim}{n}) = \lambda \underset{\sim}{n} \quad \text{or} \quad \{t\} = \underbrace{\lambda}_{\text{principal stress}}\{n\}}$$

are called principal planes and normal tractions acting on these planes are called principal stresses

- Principal stresses are determined from characteristic eq. given by

$$[\sigma]\{n\} = \lambda\{n\}$$

$$\Rightarrow \quad ([\sigma] - \lambda[I])\{n\} = 0$$

$$\Rightarrow \quad \det\left([\sigma] - \lambda[I]\right) = 0$$

$$\Rightarrow \quad \begin{vmatrix} \sigma_{11}-\lambda & \sigma_{12} & \sigma_{13} \\ \sigma_{12} & \sigma_{22}-\lambda & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_{33}-\lambda \end{vmatrix} = 0$$

$$\boxed{\Rightarrow -\lambda^3 + (\underbrace{\sigma_{11}+\sigma_{22}+\sigma_{33}}_{\equiv tr[\sigma]})\lambda^2 - (\underbrace{\sigma_{11}\sigma_{22}+\sigma_{22}\sigma_{33}+\sigma_{33}\sigma_{11}-\sigma_{12}^2-\sigma_{13}^2-\sigma_{23}^2}_{\equiv \frac{1}{2}\left((tr[\sigma])^2 - tr[\sigma]^2\right)})\lambda + \det[\sigma] = 0}$$

Characteristic eq. for principal stresses

$$\Rightarrow \boxed{-\lambda^3 + tr[\sigma]\lambda^2 - \frac{1}{2}\left((tr[\sigma])^2 - tr[\sigma]^2\right)\lambda + \det[\sigma] = 0}$$

Coefficients of characteristic eq. are independent of co-ordinate system, thus, principal stresses are also co-ordinate invariants.

- In any problem at any point there is at least ONE set of mutually perpendicular principal planes

- If $\sigma_1 \geq \sigma_2 \geq \sigma_3$ denote three principal stresses at a point. "σ_1" is maximum normal stress that can act on any plane passing through the point "σ_3" is the minimum normal stresses that can act on planes passing through the point

- $\boxed{\dfrac{\sigma_1 - \sigma_3}{2} \text{ is the maximum shear stress that can act on any plane passing through the point}}$

- Von Mises stress

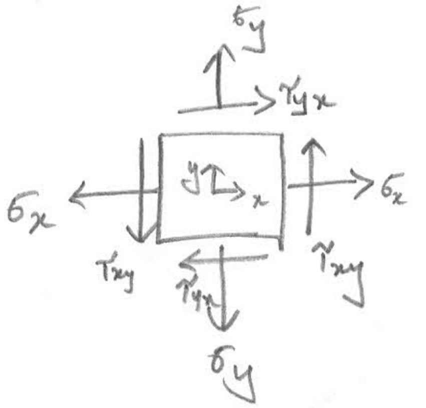
$$\underline{\sigma_{\text{von Mises}}} = \sqrt{\left\{\frac{1}{2}\left[(\sigma_{11}-\sigma_{22})^2 + (\sigma_{22}-\sigma_{33})^2 + (\sigma_{33}-\sigma_{11})^2\right] + 3\left(\sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2\right)\right\}}$$

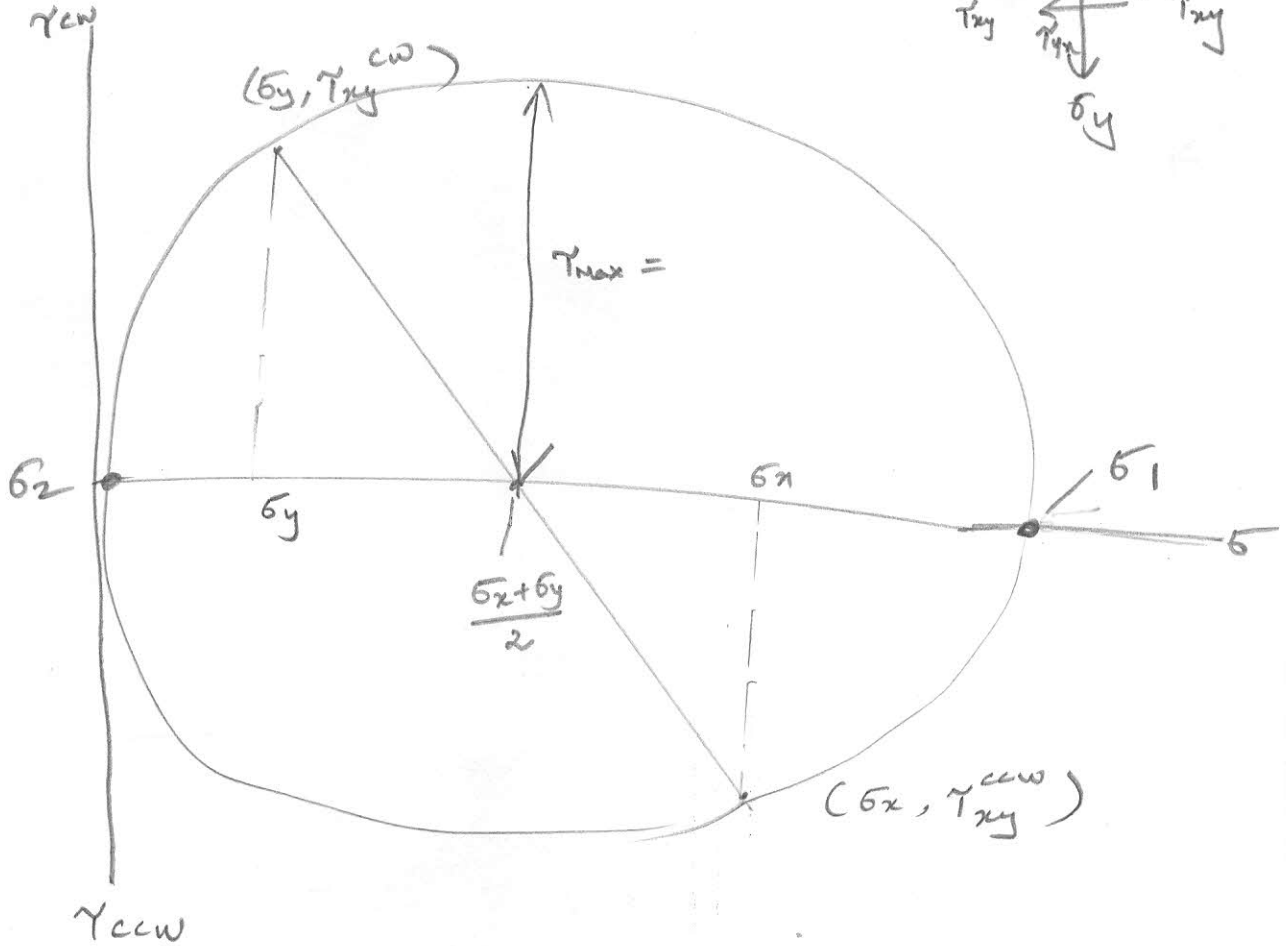
$$\underline{\sqrt{\frac{1}{2}\left\{3\,tr[\sigma]^2 - (tr[\sigma])^2\right\}}}$$

Thus $\sigma_{\text{von Mises}}$ is a stress invariant

Mohr circle for 2D state of stress

$$[\sigma] = \begin{bmatrix} \sigma_x & \tau_{xy} & 0 \\ \tau_{xy} & \sigma_y & 0 \\ 0 & 0 & 0 \end{bmatrix}$$





$$\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Generalized Hooke's Law for Isotropic, Linear-Elastic Material Behavior

$$\epsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\,\mathrm{tr}[\sigma]\,\delta_{ij}$$

$(i,j = 1,2,3)$

or

$$\boxed{[\epsilon] = \frac{1+\nu}{E}[\sigma] - \frac{\nu}{E}\,\mathrm{tr}[\sigma]\,[I]}$$

where

ν is Poisson's ratio

E is Young's modulus

I is identity matrix, i.e. $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Thus

$$\epsilon_{11} = \frac{\sigma_{11} - \nu\sigma_{22} - \nu\sigma_{33}}{E}$$

$$\epsilon_{22} = \frac{\sigma_{22} - \nu\sigma_{33} - \nu\sigma_{11}}{E}$$

$$\epsilon_{33} = \frac{\sigma_{33} - \nu\sigma_{11} - \nu\sigma_{22}}{E}$$

$$\epsilon_{12} = \frac{1+\nu}{E}\sigma_{12} \quad \text{or} \quad \boxed{\begin{aligned}\gamma_{12} &= 2\epsilon_{12} \\ &= G\,\sigma_{12}\end{aligned}}$$

$$\epsilon_{13} = \frac{1+\nu}{E}\sigma_{13} \quad \text{or} \quad \boxed{\gamma_{13} = G\,\sigma_{13}}$$

$$\epsilon_{23} = \frac{1+\nu}{E}\sigma_{23} \quad \text{or} \quad \boxed{\gamma_{23} = G\,\sigma_{23}}$$

Equivalently, it can be shown that

$$\sigma_{ij} = \frac{E}{1+\nu}\epsilon_{ij} + \frac{\nu E}{(1-2\nu)(1+\nu)} \operatorname{tr}[\epsilon]\,\delta_{ij}$$

or

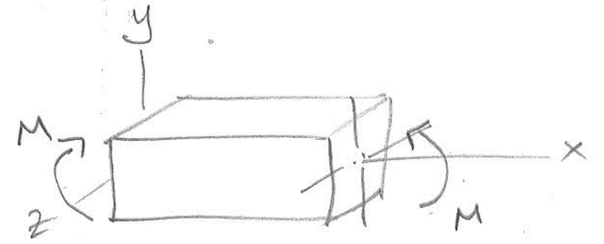
$$\boxed{[\sigma] = \frac{E}{1+\nu}[\epsilon] + \frac{\nu E}{(1-2\nu)(1+\nu)} \operatorname{tr}[\epsilon]\, I}$$

Normal stresses in bending

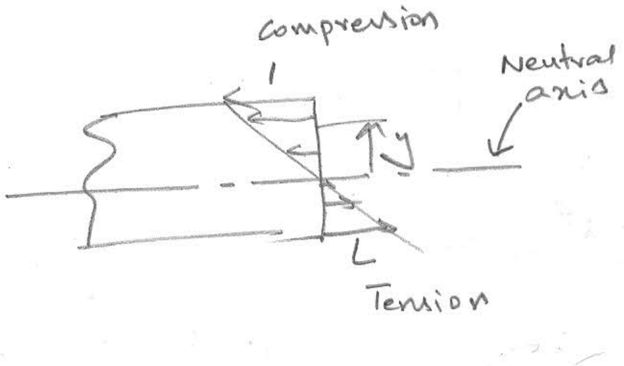
Assumptions

- Beam is subjected to pure bending
- Material is isotropic and homogenous
- Material obeys generalized Hooke's law
- Plane cross-sectoons of beam remain plane during bending (shear strain is zero ; Euler-Bernoulli beam theory is assumed)
- Beam is initially straight, i.e. initial curvature of beam is zero
- Beam has an axis of symmetry in plane of bending

$$\sigma_x = -\frac{My}{I}$$



$\boxed{Z = I/c}$ is called section modulus with $c = y_{max}$



Thus $\boxed{\sigma_{max} = \frac{M}{Z}}$

2.2.17

Design of Machine Elements — Lecture 16, 17

Uncertainity

There are several uncertainities in machine design. The uncertainities arise from several factors including

— Variation in material composition and propeties

— Variation in geometric features (e.g. diameter of a bar, thicknen of a sheet, etc.) across a parts and across different parts

— Validity of mathematical models

— Intensity of stress concentrations

— Effect of corrosion, wear

Uncertainities need to be accommodated. The primary methods are deterministic and stochastic methods. The deterministic method establishes a design factor based on absolute uncertainities of a loss-of-function parameter and a maximum-allowable-parameter.

The parameter could be load, stress, deflection, etc.

Design factor n_d is defined as

$$n_d = \frac{\text{loss-of-function parameter}}{\text{maximum allowable parameter}}$$

e.g. $$n_d = \frac{\text{Yield strength}}{\text{maximum von-Mises stress}}$$

Problem 1

Consider that maximum load on a structure is known with an uncertainty of ± 20 percent and load causing failure is known within ± 15 percent.

If load causing failure is 10 kN, determine the design factor and maximum load that can be applied.

Design factor ≈ 1.41

Sol

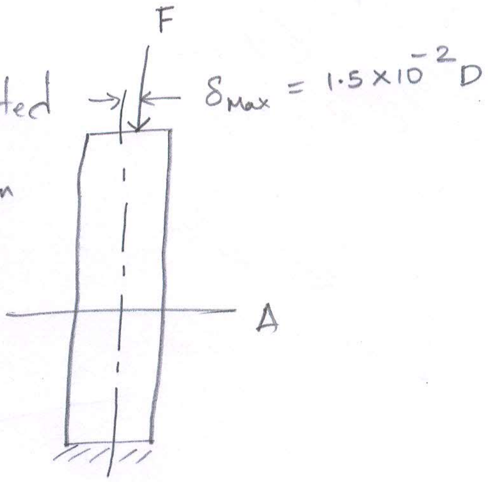
Worst case

Min. load for failure is 8.5 kN.

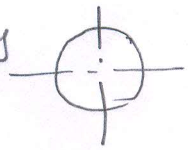
Load can be determined to 20 % uncertainity

thus, we can apply up to $x \cdot 1.2 = 8.5 \Rightarrow x = 7.1$ kN

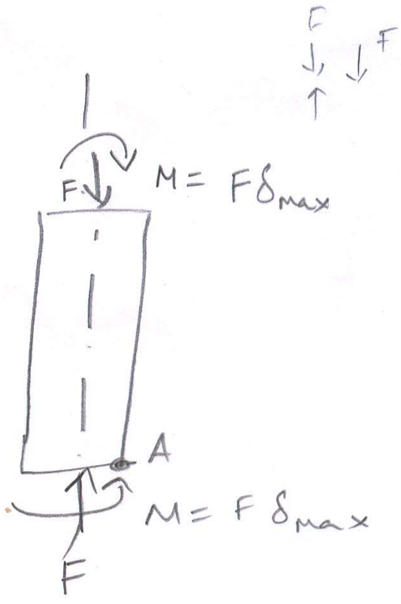
Problem 1



- Given that diameter can be fabricated to with $\pm 1\%$ of nominal dimension
- Support can be centered within $\pm 1.5\%$ of nominal diameter
- Weight is known to within 2% accuracy



- Strength to $\pm 3.5\%$.



Ans)

Peak stress would be at A

$$|\sigma_{peak}| = \frac{1.02\ F_{nominal}}{0.98\ A_{nominal}} + \frac{(1.02\ F_{nominal}) \times 0.015\ D_{nominal} \times 32}{\pi\ D_{nominal}^3 \times 0.97}$$

$$\Rightarrow |\sigma_{peak}| = \frac{F}{A_{nominal}} \times \left[\frac{1.02}{0.98} + \frac{1.02 \times 0.015 \times 8}{0.97} \right]$$

$$= F/A_{nominal} \times 1.167$$

If a weight is dropped from height h maximum load F_{max} experienced would be

$$F_{max} = \underbrace{W}_{\text{weight}} \left(1 + \sqrt{1 + \frac{2h\overbrace{k}^{\text{rod spring constant}}}{W}} \right)$$

For $h = 0$, which is the case here

$$F_{max} = 2\, m_{nominal} \times g =$$

Thus, for max stress not to exceed strength we have

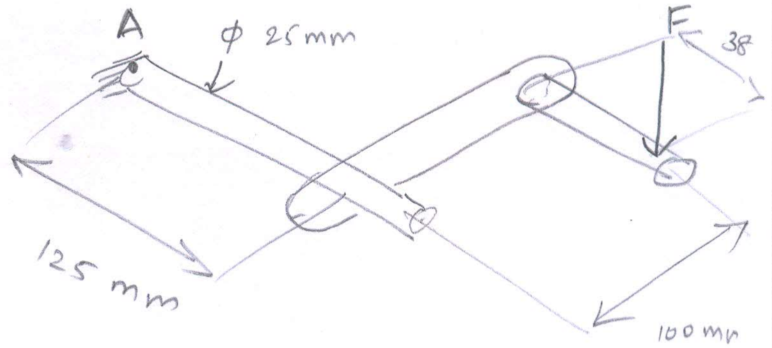
$$\underbrace{\frac{m_{nominal}\, g}{A_{nominal}}}_{\equiv\ \sigma_{nominal}} \times 2 \times 1.167 < 0.965\,(\sigma_s)_{nominal}$$

$\Rightarrow$ Design factor $n_d \approx \dfrac{2 \times 1.167}{0.965}$

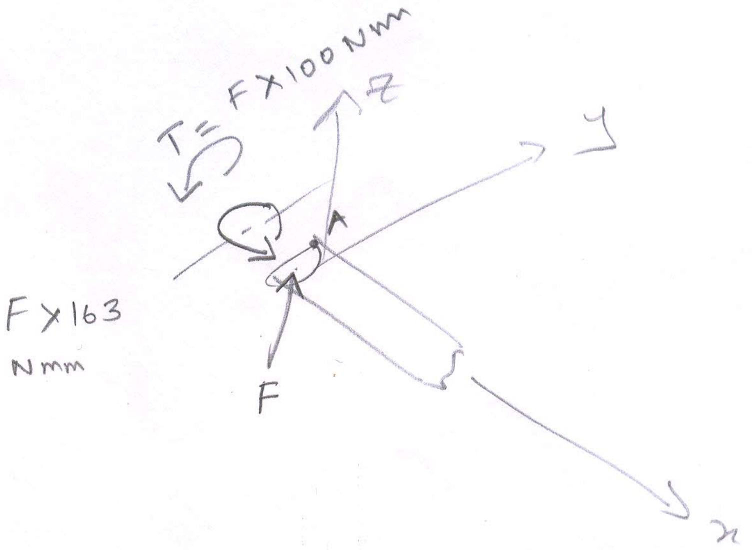
≈ 2.42

Problem

Determine maximum normal and shear stress at A



Sol



At A

$$\sigma_{xx} = \frac{My}{I} = \frac{F \times 163 \times 10 \times 64}{\pi \times 20^4} = 269.8 \frac{N}{mm^2}$$

$$\sigma_{xy} = \frac{Tr}{J} = \frac{F \times 100 \times 10 \times 32}{\pi \times 20^4} = 82.8 \frac{N}{mm^2}$$

$$\sigma_{xz} = \frac{F}{A} = \frac{F\,4}{\pi \times 20^2} = 4.14 \frac{N}{mm^2}$$

$$\Rightarrow [\sigma] \approx \begin{bmatrix} 270 & 82.8 & 0 \\ 82.8 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

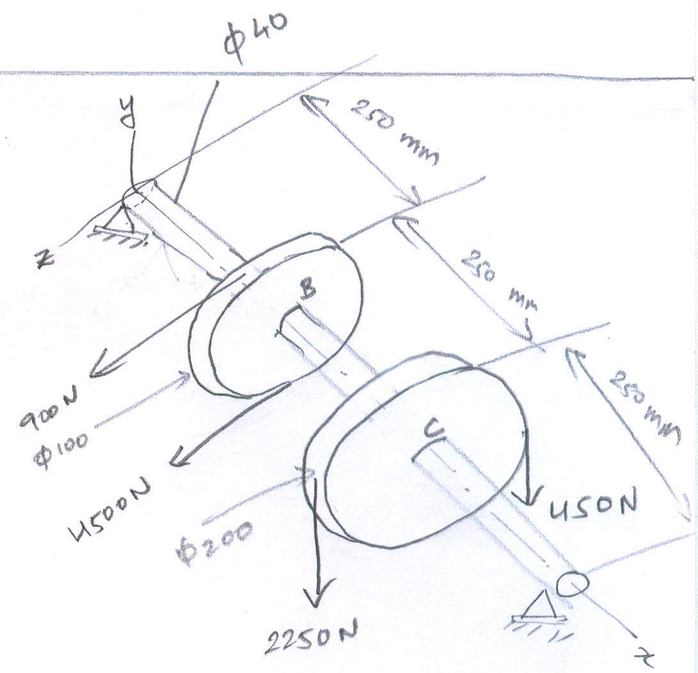
$$\Rightarrow \tau_{max} = \sqrt{\left(\frac{\sigma_{xx}}{2}\right)^2 + \sigma_{xy}^2}$$

$$= \sqrt{\left(\frac{270}{2}\right)^2 + 82.8^2}$$

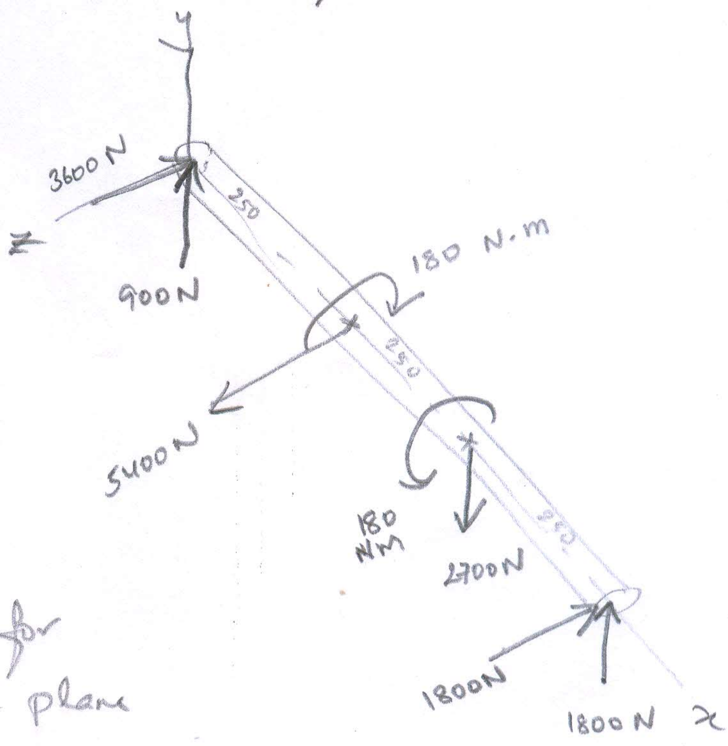
$$= 158.37 \text{ MPa}$$

Example 3-9

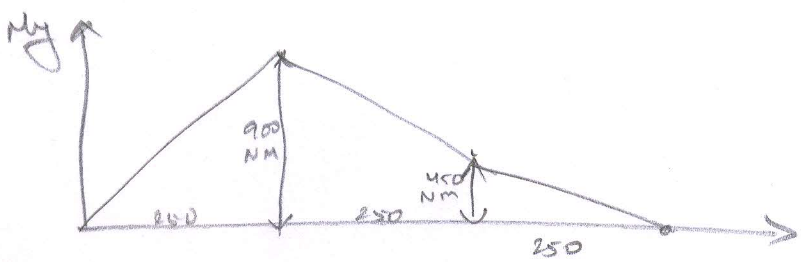
Considering bending and torsional stresses only determine greatest tensile, compressive, shear and von Mises stress in the shaft



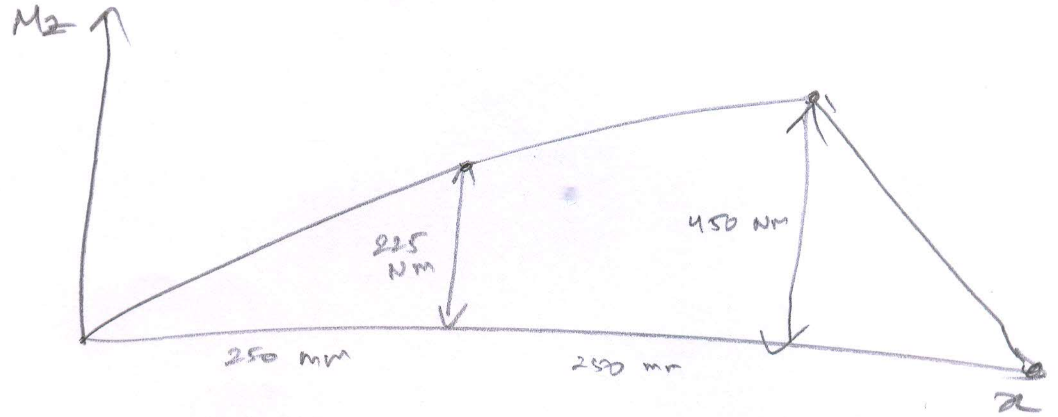
Free body diagram for the shaft



Bending moment diagram for My, i.e. bending in xz plane



Bending moment diagram for M_z, i.e. for bending in xy plane



Max tensile stress would occur at B

$$(\sigma_x)_{\text{Max at B}} = \frac{900 \times 0.02 \times 64}{\pi \times 0.04^4} = 143.24 \text{ MPa}$$

From torsion, max shear stress occurs ~~at~~ in between BC

$$\tau_{max} = \frac{Tr}{J} = \frac{180 \times 0.02 \times 32}{\pi \times 0.04^4} = 14.3 \text{ MPa}$$

7.2.17

Design of Machine Elements - Lectures 19,20

Stress concentration

Features such as threads, steps, slots, oil grooves

Locating bearing, transmitting axial loads, etc.

Connecting shafts to pulley's, gears

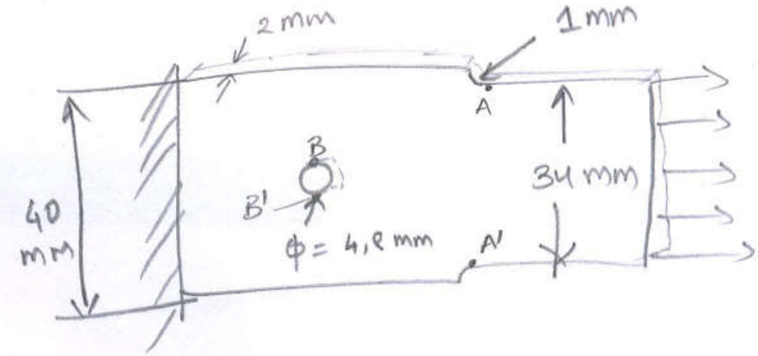
raise stress at that location. Regions where stress raises are called areas of stress concentration.

Theoretical stress concentration factor K_t or K_{ts} is used to relate actual maximum stress at the discontinuity to the nominal stress. The factors are defined as

$$K_t = \frac{\sigma_{max}}{\sigma_0}, \qquad K_{ts} = \frac{\tau_{max}}{\tau_0}$$

Note :

Effect of stress concentration is usually applied ONLY to brittle materials. For ductile materials stress redistribution occurs and $K_t \to 1$



A, B
A', B'
} critical locations

For 4 mm hole

$$\sigma_0 = \frac{F}{A} = \frac{F}{(w-d)t} = \frac{10000}{(40-4)2} = 139 \text{ MPa}$$

For $d/w = 4/40 = 0.1$, is $K_t = 2.7$

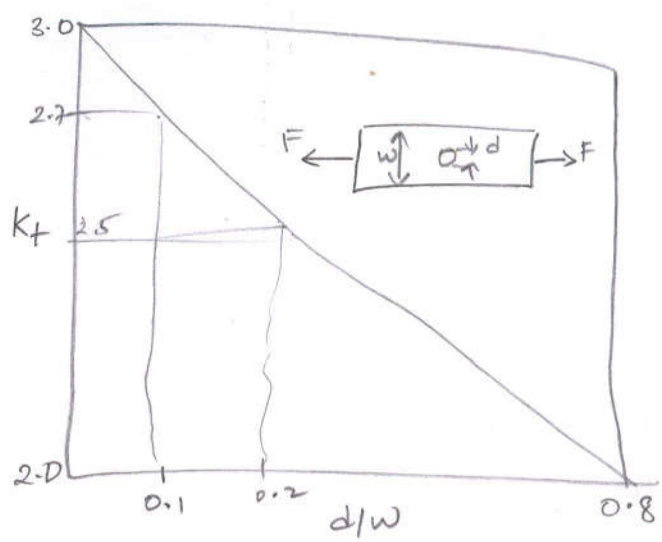
$$\sigma_{max} = K_t \sigma_0 = 2.7\,(139) = 375 \text{ MPa}$$

Similarly for an 8-mm hole

$$\sigma_0 = \frac{F}{A} = 156 \text{ MPa}$$

$$\sigma_{Max} = 2.5 \times \sigma_0$$

$$= 390 \text{ MPa}$$

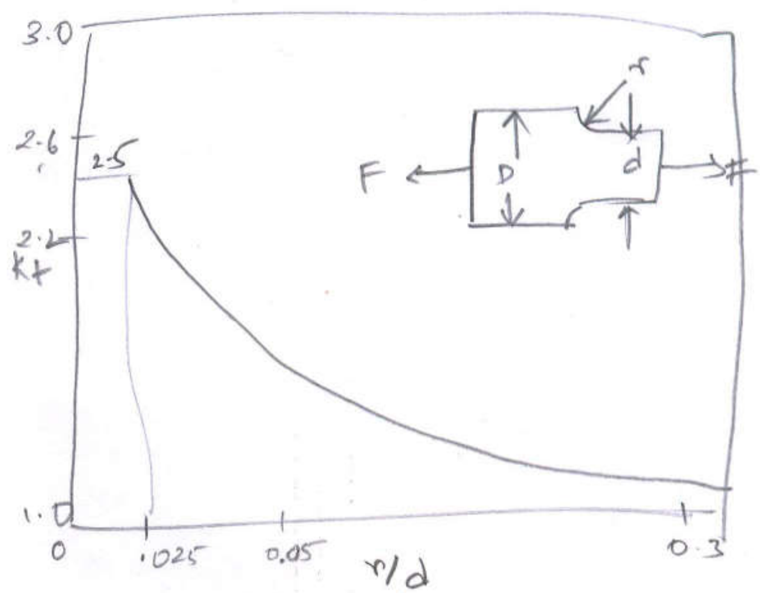


For the fillet

$$\sigma_o = \frac{F}{A} = \frac{10000}{(34)\,2} = 147 \text{ MPa}$$

$D/d = 1.18$, $\quad r/d = 1/34 = 0.026$ $\quad$ then $k_t = 2.5$

$$\sigma_{max} = K_t \sigma_o = 2.5\,(147) = 368 \text{ MPa}$$



7.2.17

Design of Machine Elements - Lectures 19,20 - 23

Stress concentration

Features such as threads, steps, slots, oil grooves

Locating bearing, transmitting axial loads, etc.

Connecting shafts to pulley's, gears

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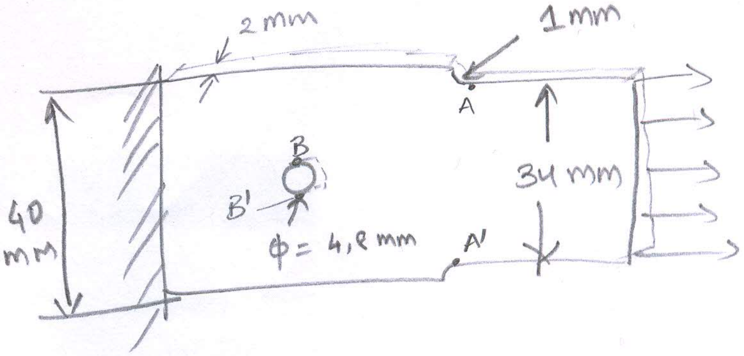
P2



A, B }
A', B' } critical locations

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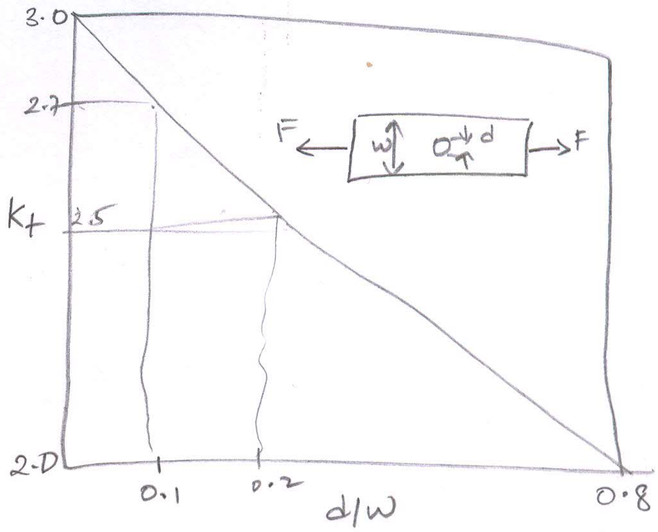

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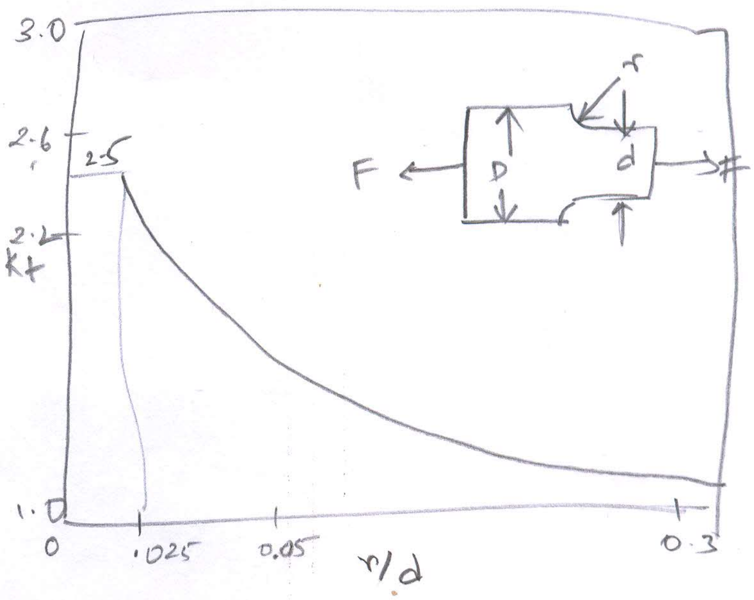
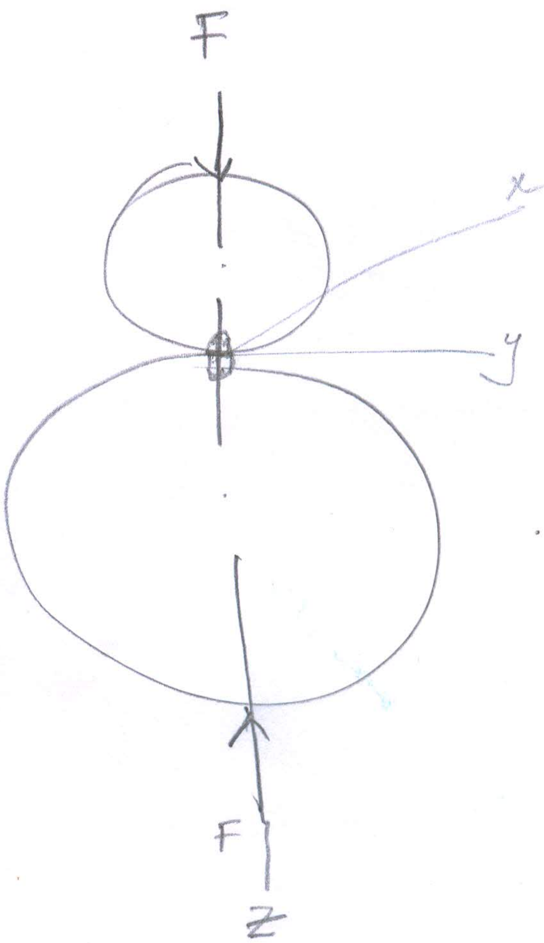


Contact stresses

Spherical contact



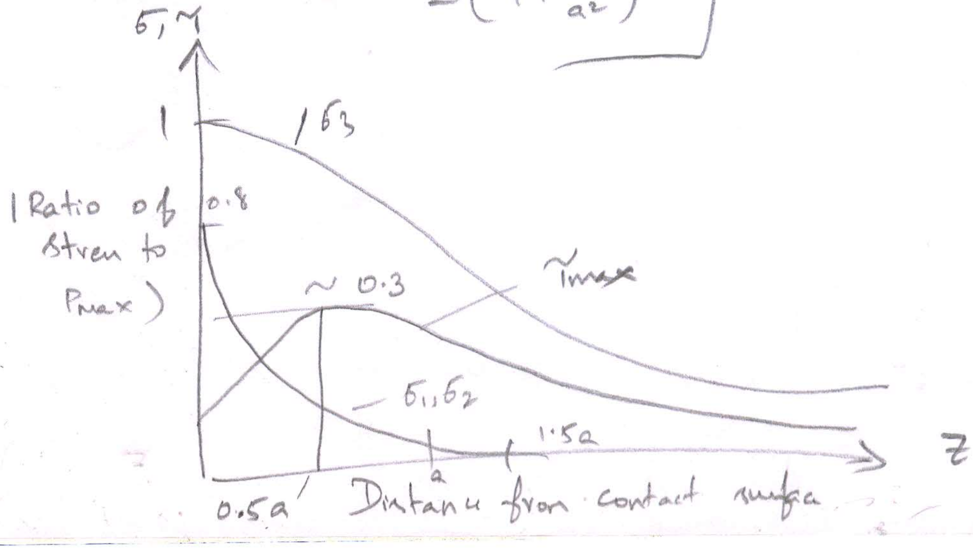
$$a = \sqrt[3]{\frac{3F}{8}\,\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

$$P_{max} = \frac{3F}{2\pi a^2}$$

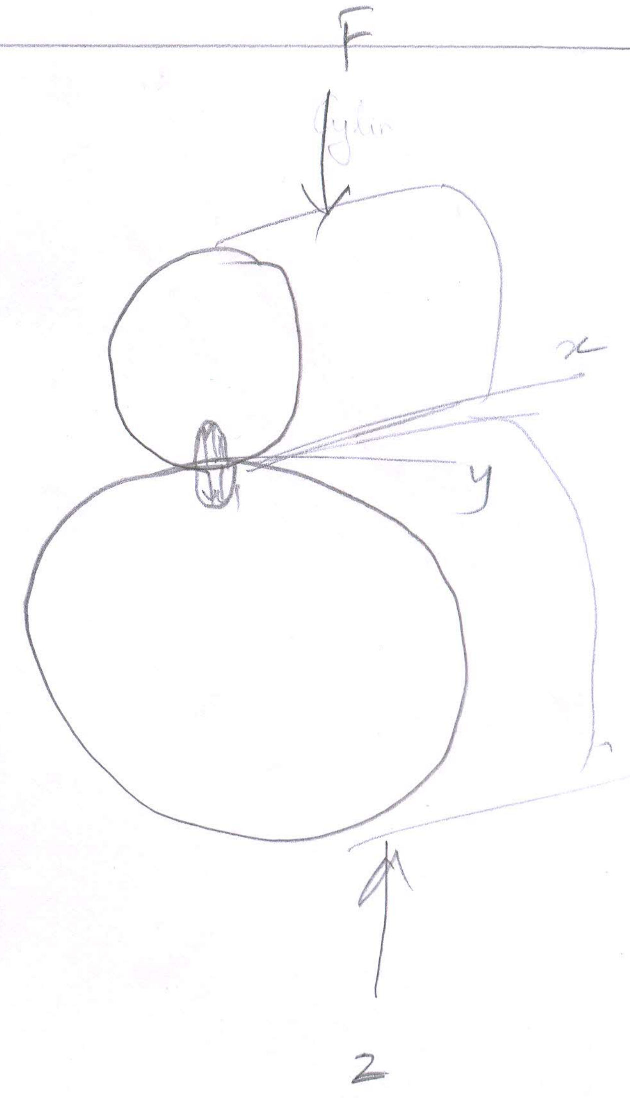
For a flat plane $\cancel{d} = \infty$

$$\sigma_3 = \frac{-P_{max}}{1 + z^2/a^2}$$

$$\sigma_1 = \sigma_2 = -P_{max}\left[\left\{1 - \left|\frac{z}{a}\right| \tan^{-1}\frac{1}{|z/a|}\right)(1+\nu) - \frac{1}{2\left(1+\frac{z^2}{a^2}\right)}\right]$$



Cylindrical contact



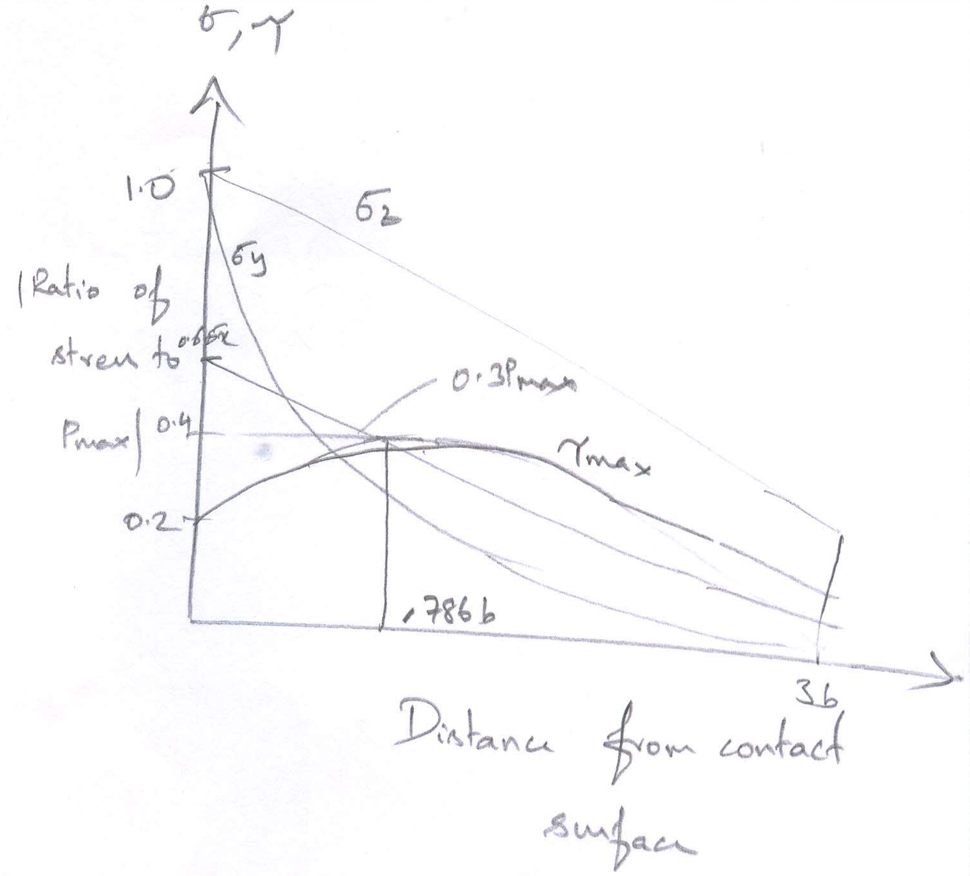
$$b = \sqrt{\frac{2F}{\pi l} \frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{1/d_1 + 1/d_2}}$$

$$p_{max} = \frac{2F}{\pi b l}$$

$$\sigma_3 = \frac{-p_{max}}{\sqrt{1+z^2/b^2}}$$

$$\sigma_x = -2\nu p_{max}\left(\sqrt{1+\frac{z^2}{b^2}} - \left|\frac{z}{b}\right|\right)$$

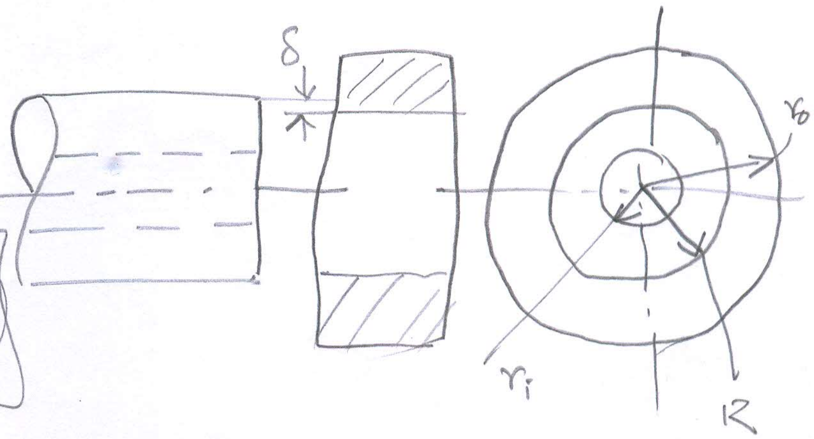
$$\sigma_y = -p_{max}\left(\frac{1+2\frac{z^2}{b^2}}{\sqrt{1+\frac{z^2}{b^2}}} - 2\left|\frac{z}{b}\right|\right)$$



σ, τ
1.0
σz
σy
Ratio of stress to Pmax
0.4
0.2
0.3Pmax
τmax
.786b
3b
Distance from contact surface

Press and shrink fits

$$P = \frac{\delta}{R\left[\frac{1}{E_o}\left(\frac{r_o^2+R^2}{r_o^2-R^2}+\nu_o\right)+\frac{1}{E_i}\left(\frac{R^2+r_i^2}{R^2-r_i^2}-\nu_i\right)\right]}$$



For $E_o = E_i = E$ & $\nu_o - \nu_i$

$$\boxed{P = \frac{E\delta}{2R^3}\left[\frac{(r_o^2-R^2)(R^2-r_i^2)}{r_o^2-r_i^2}\right]}$$

$$(\sigma_t)_i\Big|_{r=R} = -P\,\frac{R^2+r_i^2}{R^2-r_i^2}$$

& for outer member

$$(\sigma_t)_o\Big|_{r=R} = P\,\frac{r_o^2+R^2}{r_o^2-R^2}$$

For two spheres in contact, normal approach

$$\Delta_s = 1.04 \sqrt[3]{F^2\left(\frac{1}{d_1}+\frac{1}{d_2}\right)\left[\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)\right]^2}$$

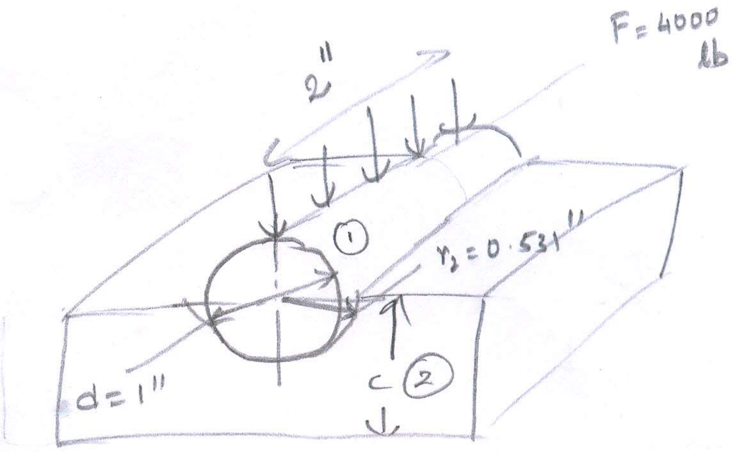
$$\Rightarrow k = \frac{F}{\Delta_s} = F^{1/3}\,\frac{1}{1.04\sqrt[3]{\left(\frac{1}{d_1}+\frac{1}{d_2}\right)\left[\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right]^2}}$$

Contact stiffness

$$E_t = \frac{dF}{d\Delta_s} = \frac{3}{2}\,\frac{\sqrt{\Delta_s}}{1.04^{1.5}}\left\{\left(\frac{1}{d_1}+\frac{1}{d_2}\right)\left[\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right]^2\right\}^{-1/2}$$

E_t is zero at zero Δ_s / F

$P_{max} = 250\,000$ psi



Q1) Determine whether surface contact pressure is below specified limiting value

(b) Determine the max. principal shearing stress and its depth from surface

$E_1 = E_2 = 30 \times 10^6$ psi
$\nu_1 = \nu_2 = 0.3$
$F = 4000$ lb

(c) Estimate change in dimension "c" as the joint goes from unloaded to fully loaded.

(a) Contact width

$$b = \sqrt{\frac{2F}{\pi l} \; \frac{\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2}}{1/d_1 + 1/d_2}}$$

$$= \sqrt{\frac{2 \times 4000}{\pi \times 2} \left(\frac{2 \times \frac{1-0.09}{30 \times 10^6}}{1 + 1/1.062} \right)} = \sqrt{1.323 \times 10^{-3}}$$

$$= 0.0364'' \simeq 0.924 \text{ mm}$$

$$P_{max} = \frac{2F}{\pi b l} = \frac{2 \times 4000}{\pi \times 2 \times 0.0364} = 34{,}979 \text{ psi}$$

$$< 250{,}000 \text{ psi}$$

(b) τ_{max} peaks at around $0.75b \simeq 0.0273'' = 0.7\,mm$

(c) Change in dimension c

$$\Delta C = 2F\,\frac{(1-\nu^2)}{\pi L E}\left(1 - 2\ln\frac{b}{2}\right)$$

$$+ \ln\frac{2d}{b}$$

$$= \frac{2 \times 4000 \times 0.91}{2 \times 30 \times 10^6}\left(1 - 2\ln\frac{0.0364}{2}\right)$$

$$\simeq 0.0011'' = 0.0278\ mm$$

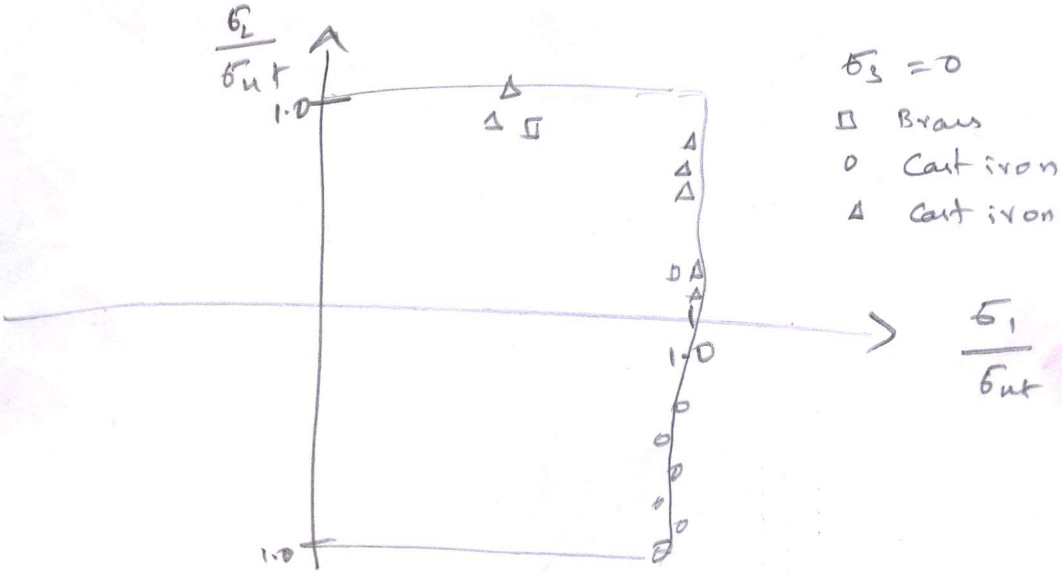
Failure theories

1. Maximum normal stress theory (Rankine's theory)

$\sigma_1 \geq \sigma_{fail-t}$ $\sigma_1 \leq \sigma_{fail-c}$

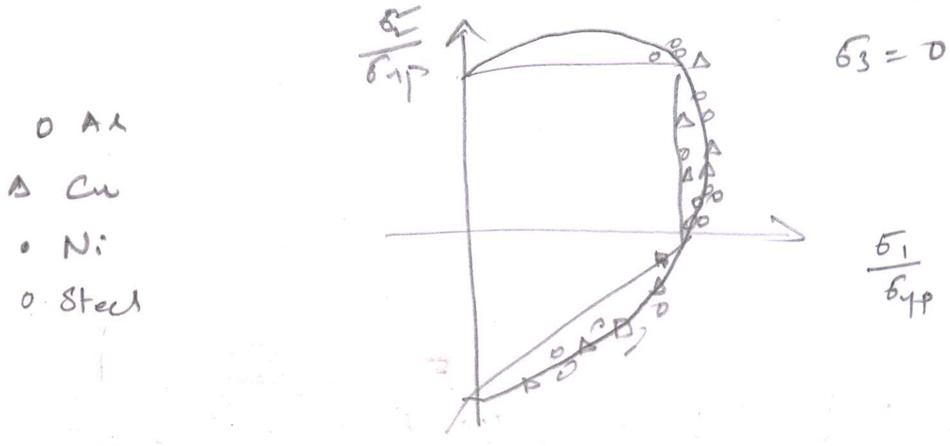
$\sigma_2 \geq \sigma_{fail-t}$ $\sigma_2 \leq \sigma_{fail-c}$

$\sigma_3 \geq \sigma_{fail-t}$ $\sigma_3 \leq \sigma_{fail-c}$



Brittle materials

2. Max shear (Tresca) / Distortion energy (von Mises)



Fluctuating loads, cumulative damage & fatigue life

Cyclic loading

Crack initiation

Crack propagation

Fracture

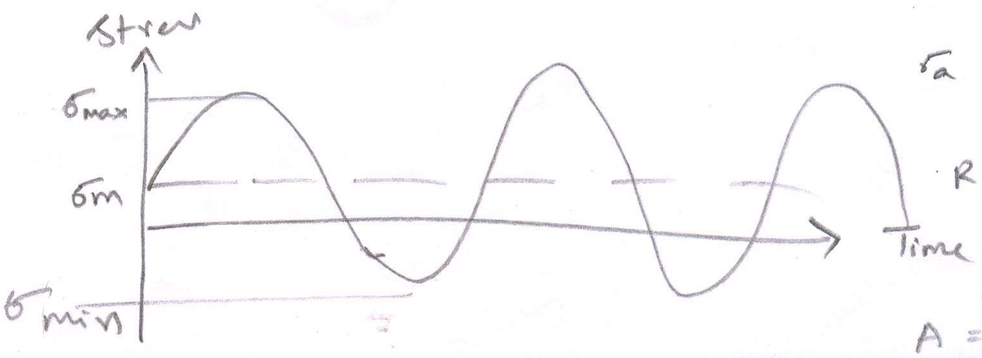
Approaches
- Stress life (S-N) approach
- Fracture mechanics (F-M) approach

Cycles to failure
- low cycle
- high cycle

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$



A = Amp Ratio

$$= \frac{\sigma_a}{\sigma_m}$$

Brittle Mohr-Coulomb

For plane stress conditions with $\sigma_1 = \sigma_A$ & $\sigma_2 = \sigma_B$ failure surface is given by

$$\sigma_A = \frac{S_{ut}}{n} \qquad \sigma_A \geq \sigma_B \geq 0$$

$$\frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \qquad \sigma_A \geq 0 \geq \sigma_B$$

$$\sigma_B = -\frac{S_{uc}}{n} \qquad 0 \geq \sigma_A \geq \sigma_B$$

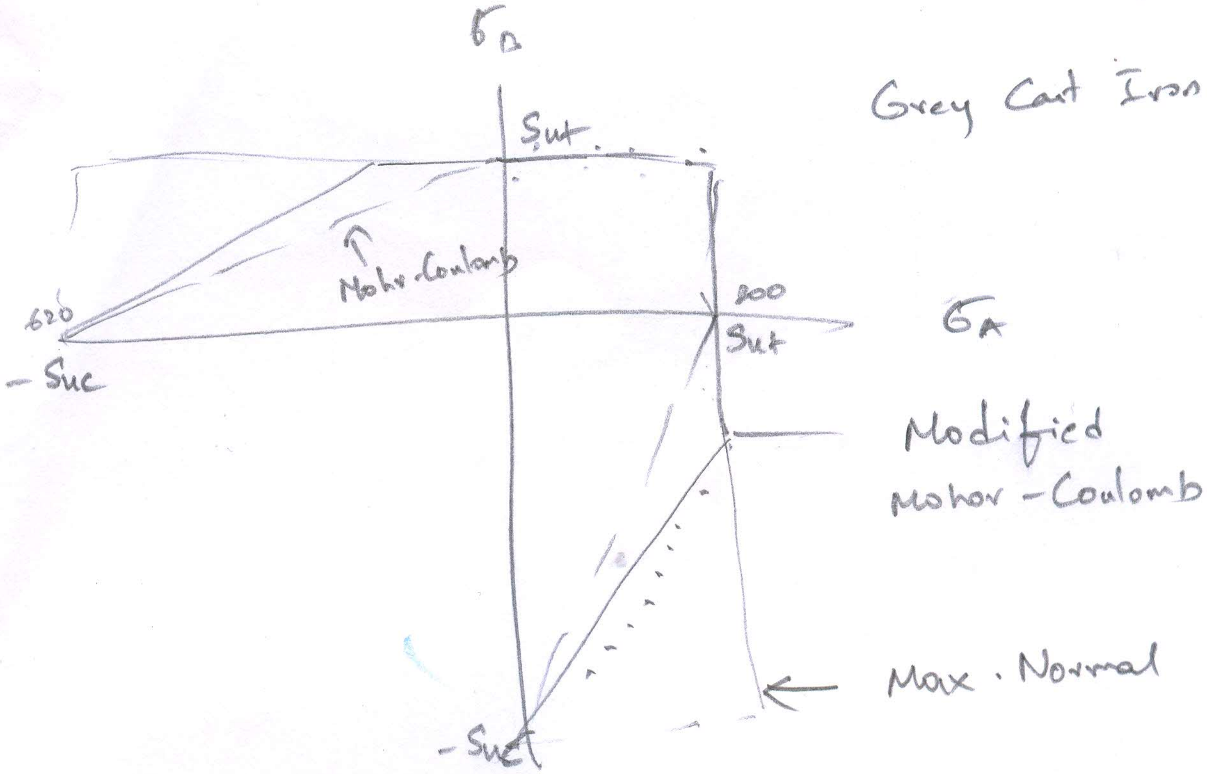
Modified Mohr

$$\sigma_A = \frac{S_{ut}}{n} \qquad \sigma_A \geq \sigma_B \geq 0$$

$$\sigma_A \geq 0 \geq \sigma_B \quad \& \quad \left|\frac{\sigma_B}{\sigma_A}\right| \leq 1$$

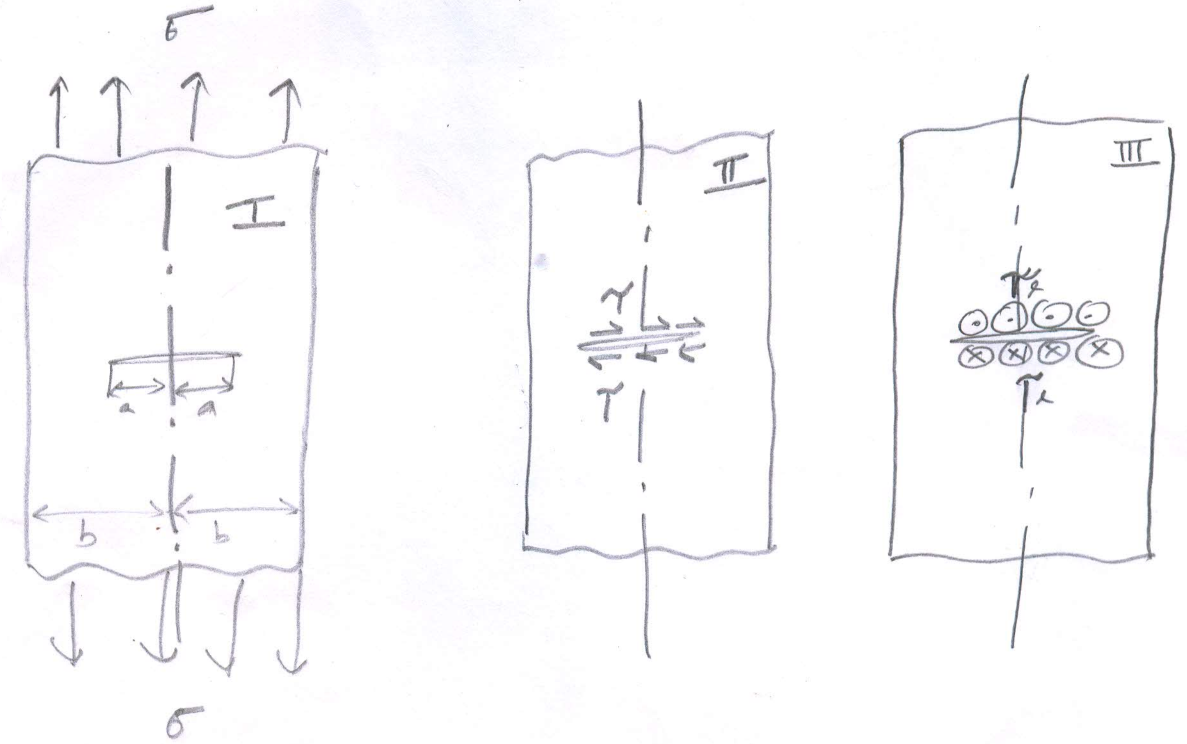
$$\frac{(S_{uc} - S_{ut})\,\sigma_A}{S_{uc}S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{1}{n} \qquad \sigma_A \geq 0 \geq \sigma_B \quad \left|\frac{\sigma_B}{\sigma_A}\right| > 1$$

$$\sigma_B = -\frac{S_{uc}}{n} \qquad 0 \geq \sigma_A \geq \sigma_B$$

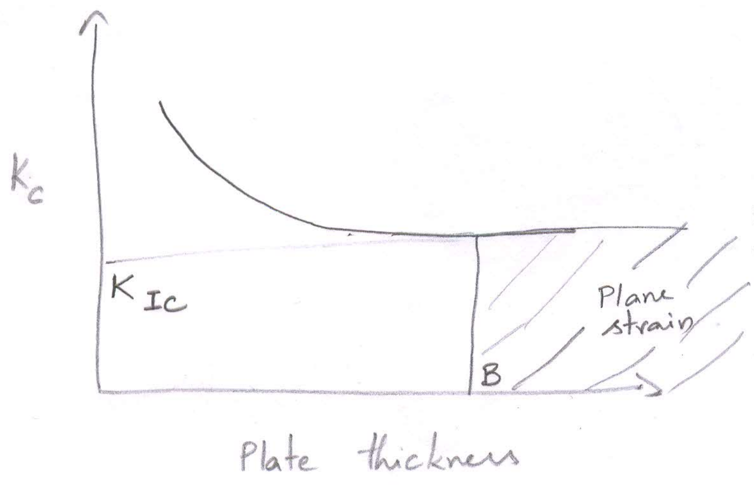

σ_B
Grey Cast Iron
S_{ut}
Mohr-Coulomb
620
$-S_{uc}$
800
σ_A
S_{ut}
Modified
Mohr-Coulomb
Max. Normal
$-S_{uc}$

Brittle Fracture - Linear Elastic Fracture Mechanics

Modes of fracture



$$\boxed{K_I = C_I\,\sigma\sqrt{\pi a}} \qquad \boxed{K_{II} = C_{II}\,\tau\sqrt{\pi a}} \qquad \boxed{K_{III} = C_{III}\,\tau_\ell\sqrt{\pi a}}$$



$$\boxed{B \geq 2.5\left(\frac{K_{Ic}}{\sigma_y}\right)^2}$$

for plane strain condition to be satisfied

$$K_c = K_{Ic}\left[1 + \frac{1.4}{B^2}\left(\frac{K_{Ic}}{\sigma_y}\right)^4\right]^{1/2} \text{ for}$$

plane stress condition

Plastic zone size must be much smaller than crack size for linear elastic fracture mechanics (LEFM) to be valid.

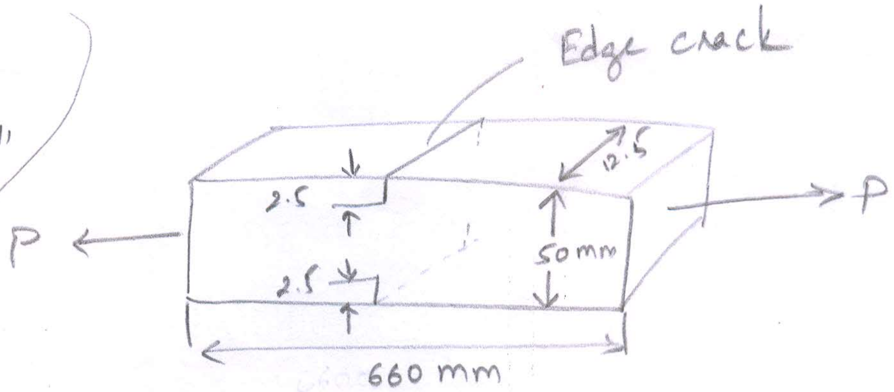
Yield strength and plane strain fracture toughness for selected alloys

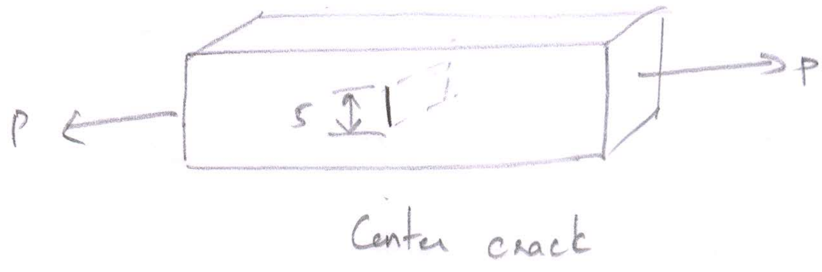
Alloy	Test Temperature	σ_y (MPa)	K_{IC} ($MPa\sqrt{m}$)
AISI 1045	−4°C	269	50
AISI 1045	−18°C	276	50
4340 steel (500°F temper)	21°C	1550	56
4340 steel (800°F temper)	21°C	1410	85
D6AC steel (1000°F temper)	21°C	1495	102
"	−54°C	1570	62
18 Ni Maraging steel	316	1627	87
"	21	1931	74
"	−73	2103	46

Alloy	Test temperature	σ_y (MPa)	K_{Ic}
A538 steel	—	1722	111
2014-T6 AA	24° C	440	31
6061-T651 AA	21° C	296	29
"	-80° C	310	33
7075-T6 AA	—	517	28
Ti-6Al-4V	23° C	820	106

Problem

Two "identical" support straps of forged 2014-T6 AA have been inspected and found to contain





through the thickness cracks. Here, P = 220 kN

a. What is the failure mode in each case ?

b. In each case is failure predicted to occur ?

Sol

For 2014 – T6 AA

$\sigma_y = 440$ MPa

$K_{IC} = 31$ MPa $\sqrt{m}$

Edge crack

For failure to occur by yielding

$P \geq \sigma_y \times 12.5 \times 45$

$\Rightarrow$ $P \geq 440 \times 12.5 \times 45$ N

$\Rightarrow$ $P \geq 247.5$ kN

For failure to occur by fracture

Check for plane strain condition

$$B \geq 2.5 \left(\frac{K_{Ic}}{\sigma_y} \right)^2$$

$$\Rightarrow B \geq 2.5 \left(\frac{31}{440} \right)^2 m$$

$$\Rightarrow B \geq 0.01241\ m \quad \text{or} \quad \boxed{B \geq 12.41\ mm}$$

Here, plate thickness, is $12.5\ mm > 12.41\ mm$

thus plane strain condition is valid.

P_{cr} for brittle fracture to occur is

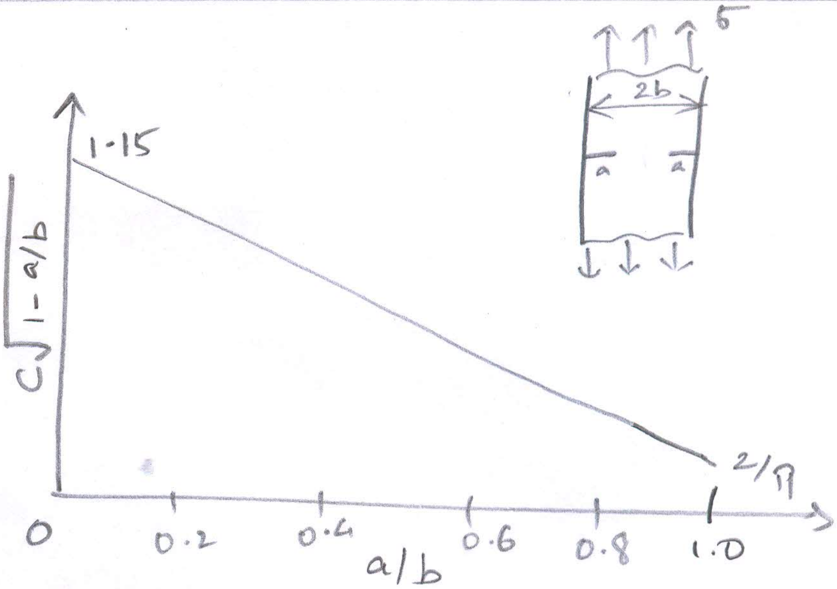
$$K_I \geq K_{Ic}$$

in N

$$\Rightarrow C_I \frac{P_{cr}}{12.5 \times 50} \sqrt{\pi \times 2.5 \times 10^{-3}} \geq 31$$

Here

$$a/b = \frac{2.5}{25} = 0.1$$



$$\Rightarrow C_I\sqrt{1-0.1} = 1.07$$

$$\Rightarrow \boxed{C_I = 1.13}$$

$$\Rightarrow 1.13 \times \frac{P_{cr}}{12.5 \times 50}\sqrt{\pi \times 0.0025} = 31$$

$$\Rightarrow P_{cr} = \frac{31 \times 12.5 \times 50}{1.13} \times \frac{1}{\sqrt{\pi \times 0.0025}}$$

$$\Rightarrow \boxed{P_{cr} = 193.472 \text{ kN}}$$

Here, $P = 220$ kN, hence failure from brittle fracture would occur for the edge crack case.

(b) Here too failure from plastic yielding would not occur

For center crack here $C = 1.01$, thus P_{cr} for the case of center crack is

$$1.01 \times \frac{P_{cr}}{50 \times 12.5} \sqrt{\pi \times 0.0025} = 31$$

$$\Rightarrow \quad P_{cr} = \frac{31 \times 50 \times 12.5}{1.01 \times \sqrt{\pi \times 0.0025}} = 216.5 \text{ kN}$$

Thus, failure by brittle fracture is also predicted for center crack

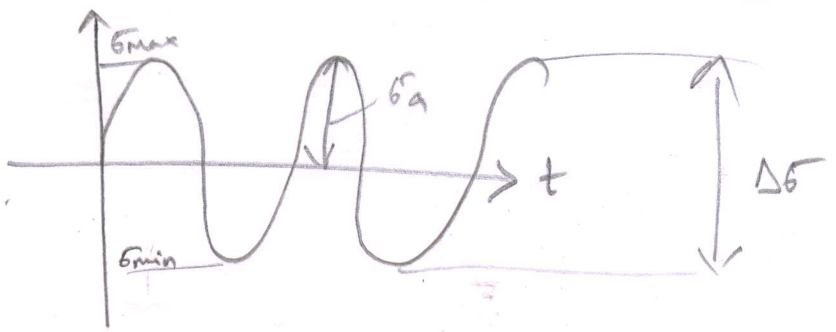
Lectures 29 – 35

Fluctuating loads, cumulative damage and fatigue life

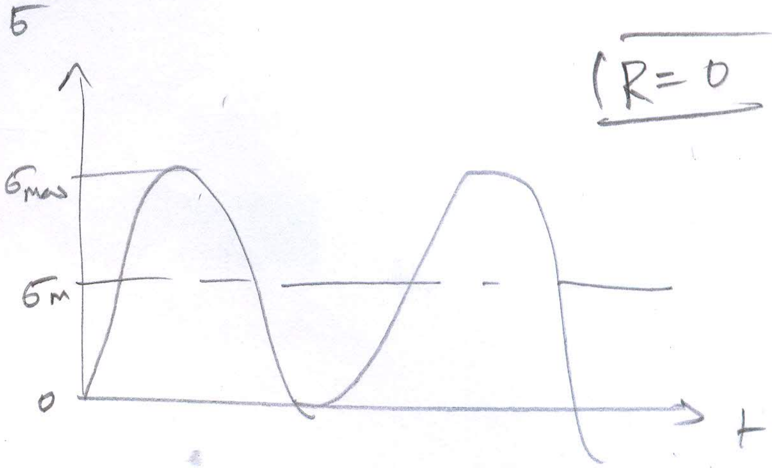
Approach

- Stress-life
  - High-cycle
    - Use S-N curves
- Strain-life
  - low-cycle
- Fracture mechanics approach

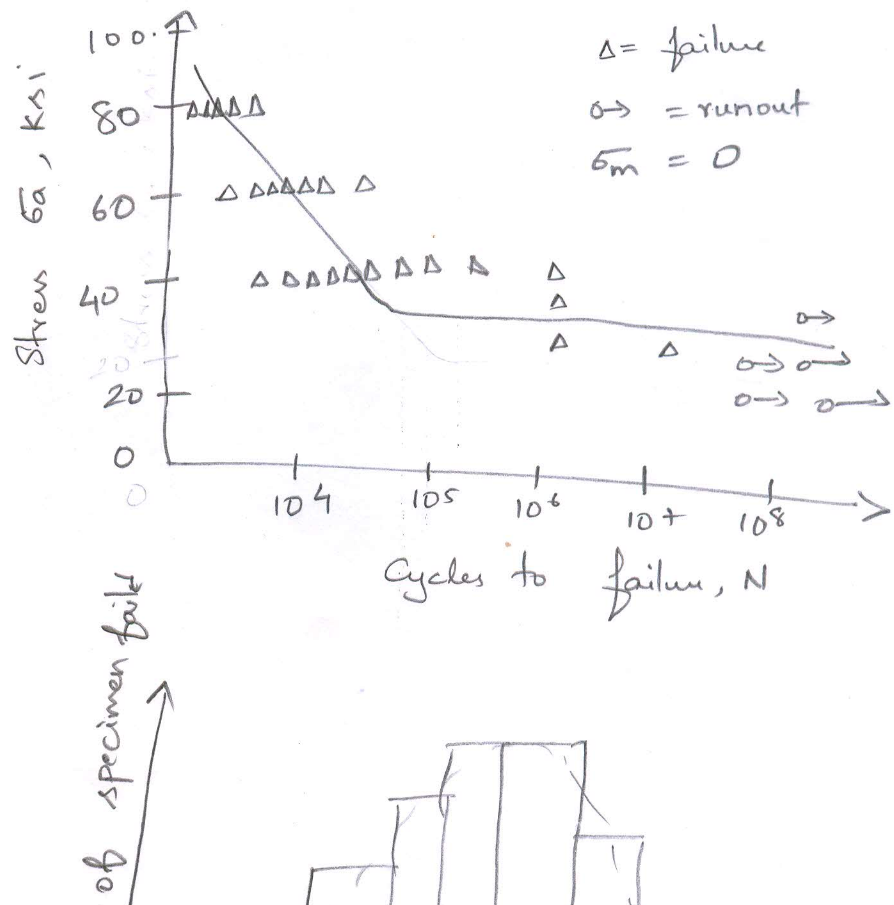
Stress-life (S-N) approach



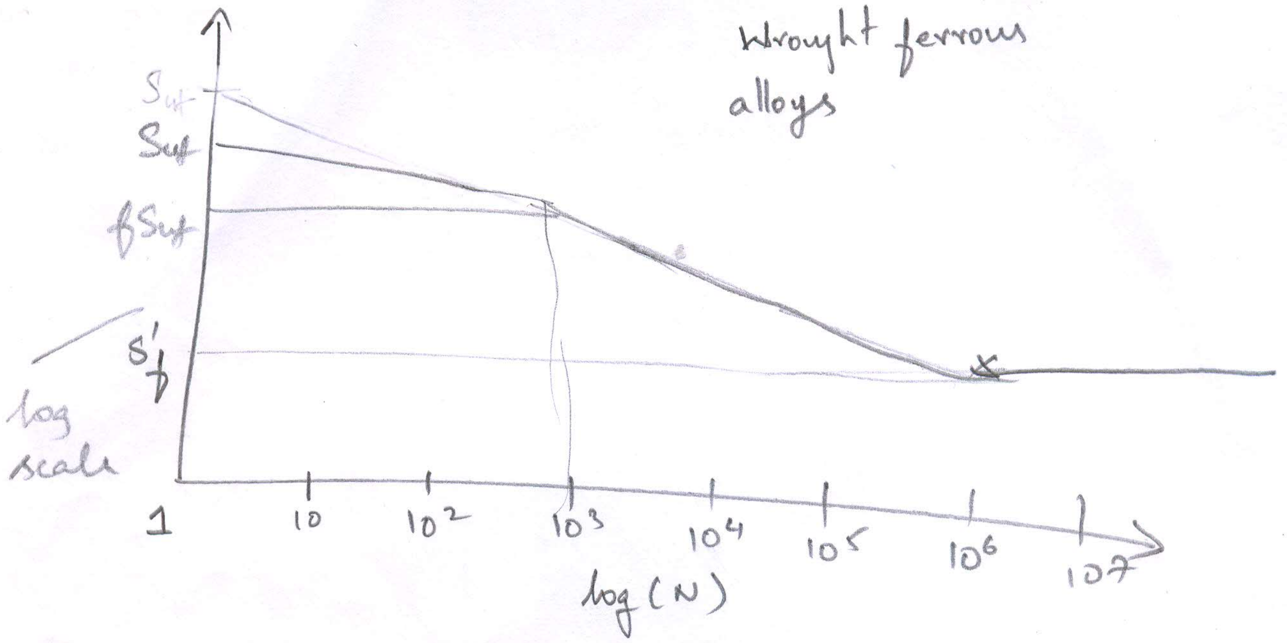
$$\boxed{R = \frac{\sigma_{min}}{\sigma_{max}} = 1}$$



Fatigue strength and fatigue limit



Estimating S-N curves



$S'_f = 0.5\ S_{ut}$ at $N = 10^6$ cycles if

$S_{ut} \leq 200$ ksi

or

$S'_f = 100$ ksi at $N = 10^6$ cycles if

$S_{ut} > 200$ ksi

Cast irons & cast steels

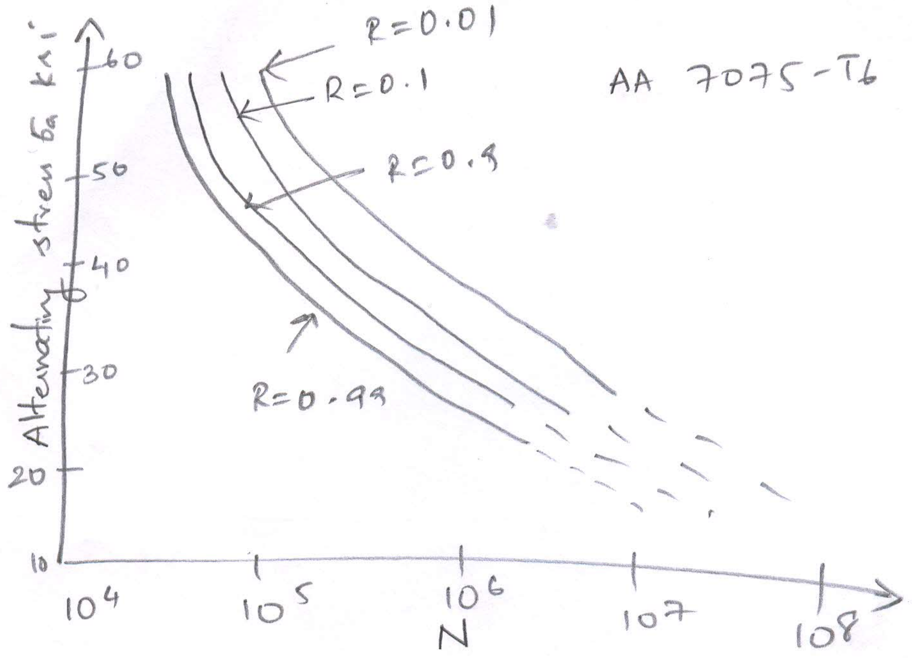
$S'_f = 0.4\ S_{ut}$ at $N = 10^6$ cycles if $S_{ut} \leq 88$ ksi

$S'_f = 40$ ksi at " if $S_{ut} > 88$ ksi

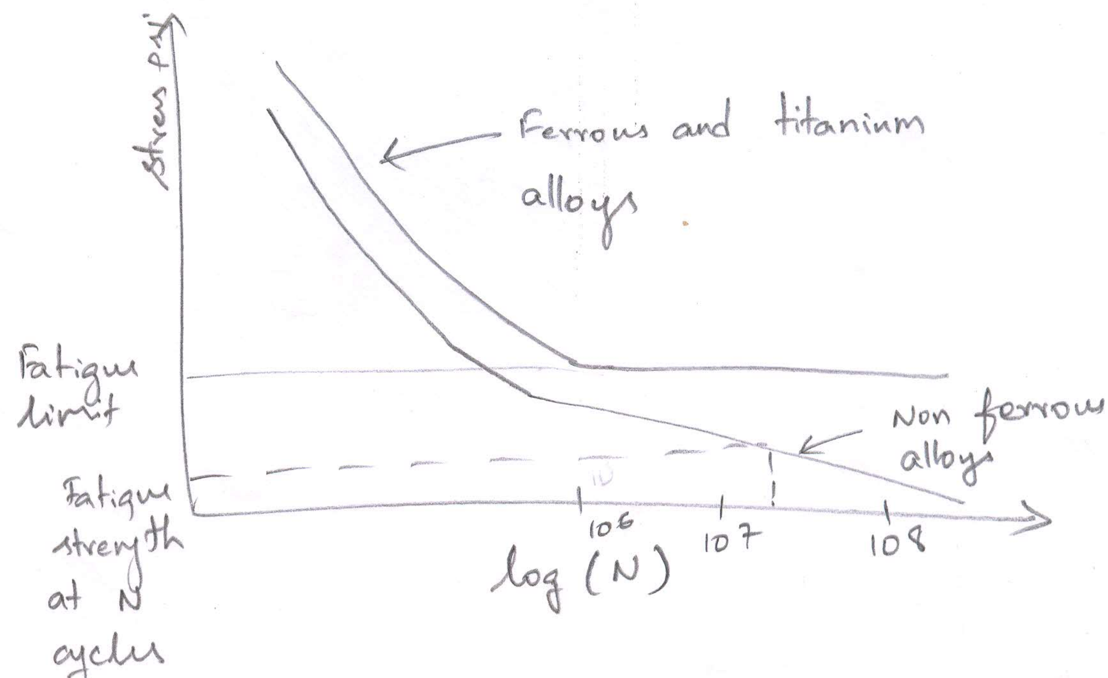
10

Reliability

R – S – N curves



Types of material response to cyclic loading



Other alloys

Alloy	S'_f / S'_N	
Ti Alloys	0.45 to 0.65 Sut	at $N = 10^6$ cycles
Al alloys	0.4 Sut	at $N = 5\times10^8$ cycles
Mg alloys	0.35 Sut	at $N = 10^8$ cycle
Cu alloys	0.2 Sut to 0.5 Sut	"
Ni alloys	0.3 Sut to 0.5 Sut	"

Factors affecting S-N curves

infinite cycles

$$S_f = K_\infty S'_f \quad \text{or} \quad S_N = K_N S'_N$$

with

$$K_\infty = (k_{we}\, k_r\, k_{cr}\, k_{fr} \cdots)_\infty$$

$$K_N = (k_{we}\, k_r\, k_{cr}\, k_{fr} \cdots)_N$$

Endurance limit modifying factors

$$S_e = \left(K_{sr}\ K_{sz}\ K_{\ell}\ K_T\ K_r\ K_{fr}\ K_{cr} \cdots \right) S_e'$$

Rotary beam test specimen endurance limit

Surface roughness factor K_{sr}

$$\boxed{K_{sr} = a\, S_{ut}^{\,b}}$$

with

Surface Finish	Factor 'a' for S_{ut} (in MPa)	Exponent b
Ground	1.58	−0.085
Machined or cold rolled	4.51	−0.265
Hot rolled	57.7	−0.718
As-forged	272	−0.995

Size factor K_{sz}

For axial loading

$$K_{sz} = 1$$

For bending and torsion

in inches

$$K_{sz} = \begin{cases} 0.879\, d^{-0.107} & \text{for } 0.11 \le d \le 2 \text{ in} \\ 0.91\, d^{-0.157} & \text{for } 2 < d \le 10 \text{ in} \end{cases}$$

Loading factor k_c

$$k_c = \begin{cases} 1 & \text{Bending} \\ 0.85 & \text{Axial} \\ 0.59 & \text{Torsion} \end{cases}$$

Temperature factor for carbon steels & alloy steels

Temp (°C)	K_T
20	1.0
50	1.01
100	1.02
150	1.025
200	1.02
250	1.0
300	0.975
350	0.943
400	0.9
450	0.843
500	0.763
550	0.672
600	0.549

Reliability factor

SD of $S'_\infty / S'_e \sim 0.08\ S'_e$ (i.e $\sim$ 8 %)

Reliability	K_r
50	1
90	0.897
99	0.814
99.99	0.702

Reliability	K_r
99.999	0.652
99.9999	0.62

Electroplating

k_{EP} — 0.39 to 1 — Galvanizing

Electroplating $\longrightarrow$ $k_{EP} \sim 0.5$

Fretting corrosion

k_{fr} = 0.24 to 0.95 (typical value in case of fretting ~ 0.35)

Stress concentration & Notch sensitivity

Notch sensitivity

$$q = \frac{K_f - 1}{K_t - 1}$$

K_f — fatigue stress concentration factor

K_t — Static stress concentration factor

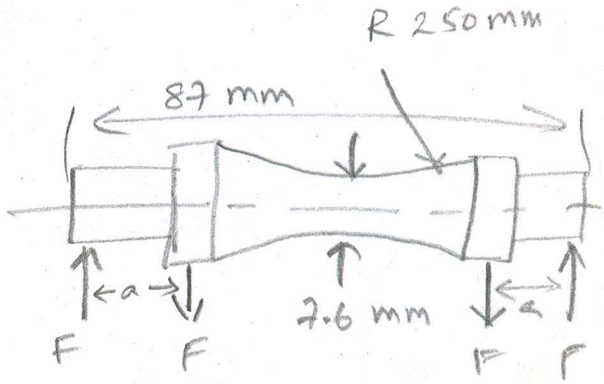
We take

$$q = \frac{1}{1 + \sqrt{a/r}}$$

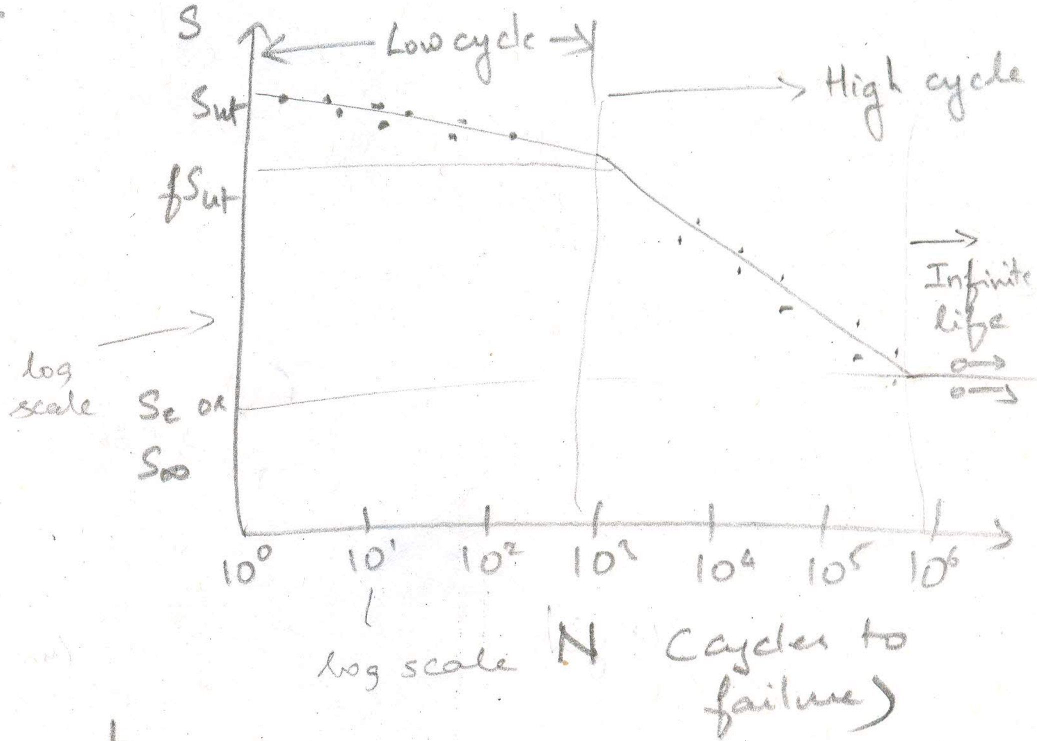
r — Notch radius

a — Neuber const.

Specimen for RR Moore rotating bending machine



S-N diagram



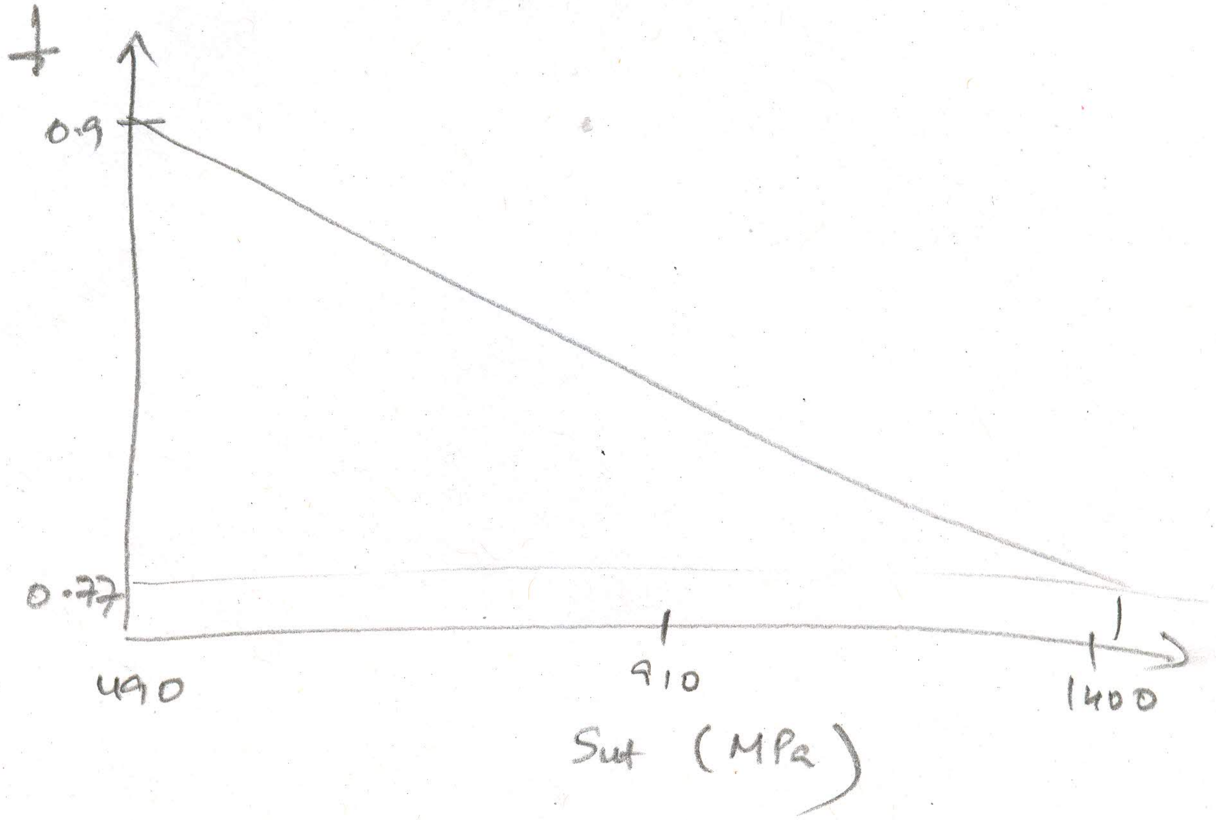
$$S_f = a N^b$$

with

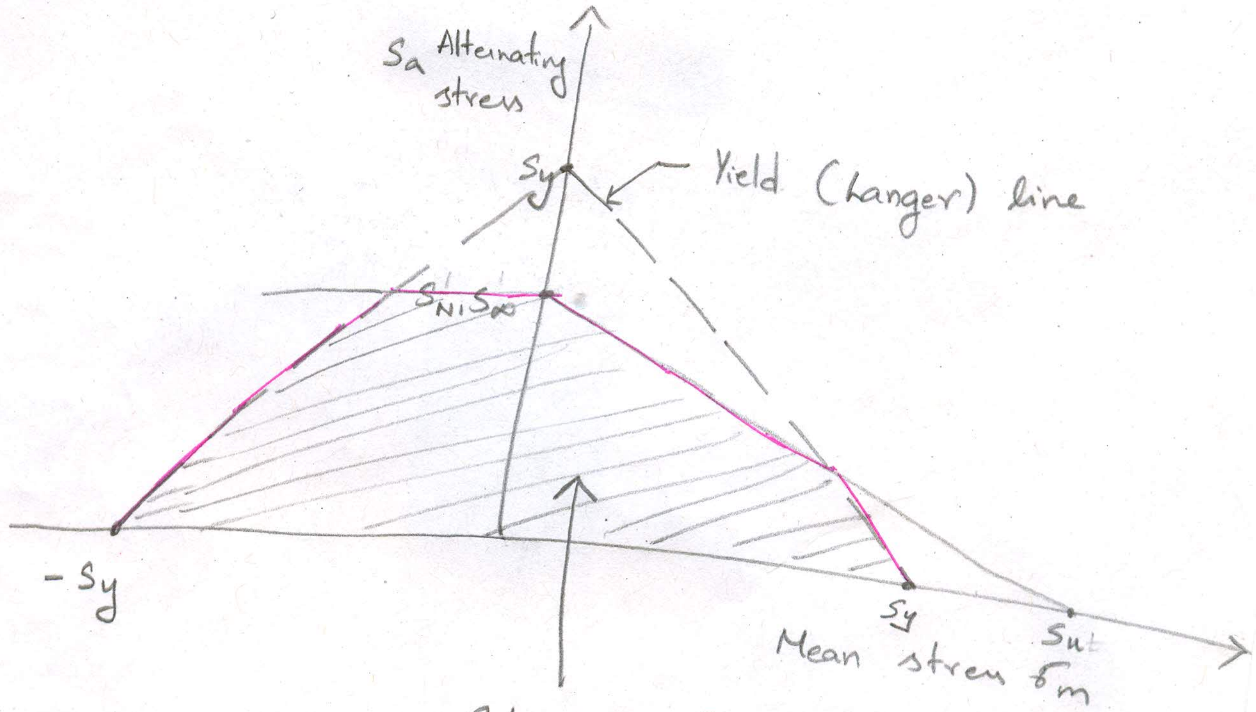
$$a = \frac{(f S_{ut})^2}{S_e} \quad \text{and} \quad b = -\frac{1}{3} \log \left(\frac{f S_{ut}}{S_e} \right)$$

with

$f = 0.9$ for $S_{ut} < 350$ MPa



Fatigue failure criteria



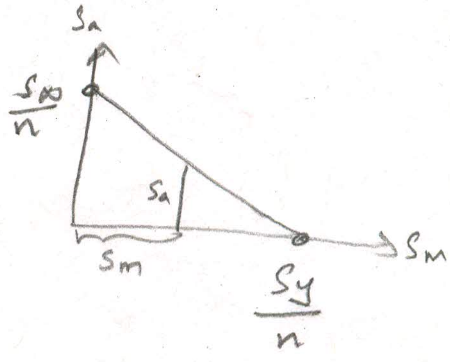
Safe region as per modified Goodman diagram

For $S_m > 0$ & $S_m + S_a \le S_y$

Soderberg line

$$\boxed{\frac{S_a}{S_\infty} + \frac{S_m}{S_y} = \frac{1}{n}}$$

Design factor / Factor of safety



we have that

$$\frac{S_\infty}{S_y} = \frac{S_a}{S_y/n - S_m}$$

Mod- Goodman line

$$\boxed{\frac{S_a}{S_\infty} + \frac{S_m}{S_{ut}} = \frac{1}{n}}$$

Gerber-line

$$\boxed{\frac{n S_a}{S_\infty} + \left(\frac{n S_m}{S_{ut}}\right)^2 = 1}$$

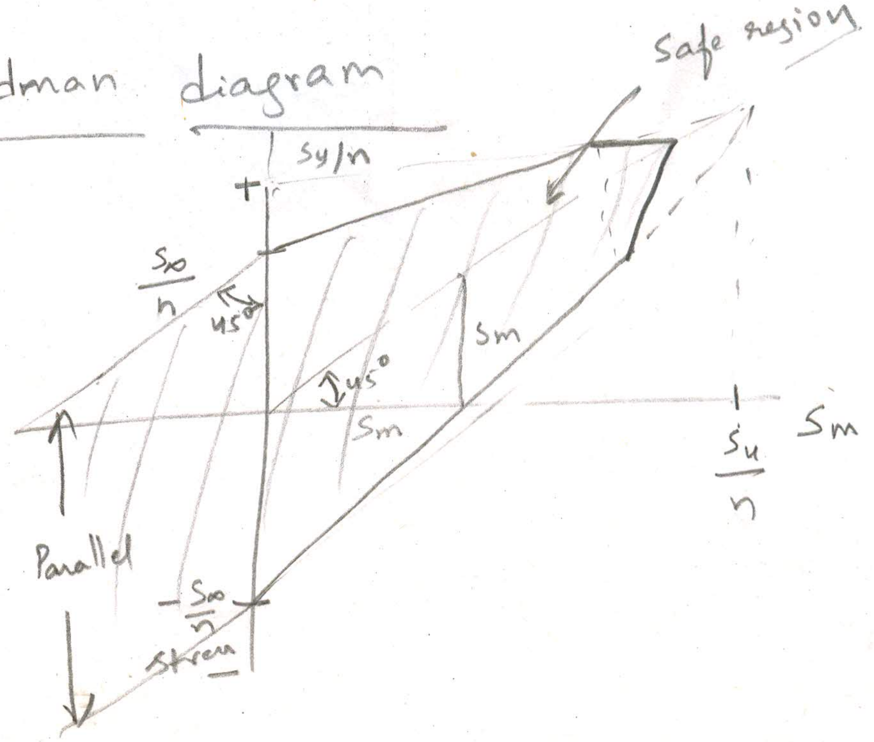
ASME-
Elliptic line

$$\boxed{\left(\frac{n S_a}{S_\infty}\right)^2 + \left(\frac{n S_m}{S_y}\right)^2 = 1}$$

Langer (first-cycle yielding) line

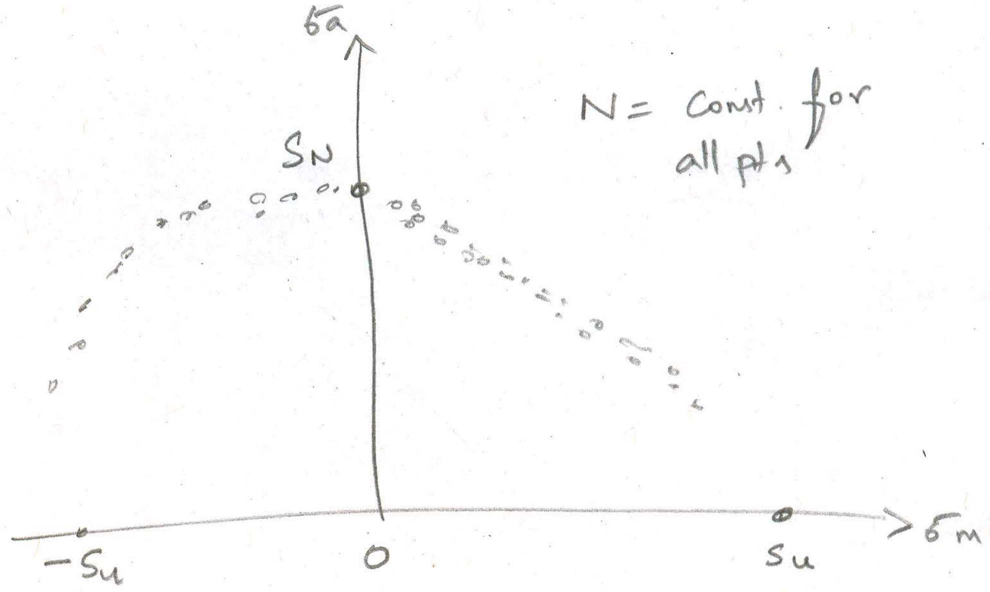
$$\boxed{S_a + S_m = \frac{S_y}{n}}$$

Modified Goodman diagram



$$\boxed{S_f = 0.44\,(0.5 \times S_{ut}) = 115.3 \text{ MPa}}$$

Non-zero mean stress



Problem

A wrought-carbon steel alloy with $S_u = 524$ MPa

$S_{yp} = 290$ MPa and $e(50\ mm) = 18\ \%$.

is a candidate material for proposed machine part to be used under the following conditions:

- Part is to be turned in lathe
- Part is uniform in shape at cr point.
- Very long life is desired
- 99.9 % reliability is desired

Solution

$S_f = K_\infty S'_f$ (as infinite life is desired)

Here

$K_r = 0.75$

$K_{sr} = 0.65$

$K_{sz} = 0.9$

$\Rightarrow K_\infty = 0.75 \times 0.65 \times 0.9$
$= 0.44$

$S_e = K_{sr} K_r S'_e$
$= 0.65 \times 0.75 \times 0.5 \times 524$
$= 126.5$ MPa

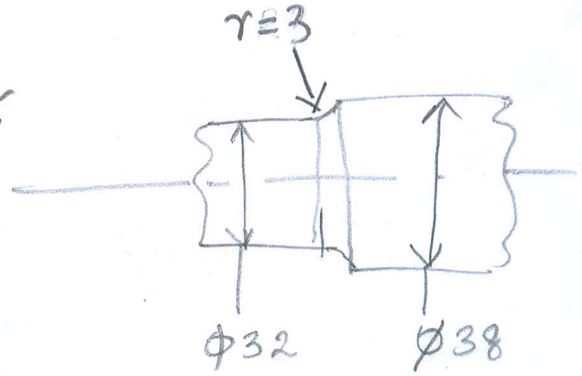
Example 6-6

A steel shaft in bending has an ultimate strength of 690 MPa and a shoulder with a fillet radius of 3 mm connecting a 32 mm diameter with a 38-mm diameter. Estimate K_f for this situation

Sol.)

Here $D/d = \frac{38}{32} = 1.1875$

$r/d = 0.09375$



Dimensions are in mm

From Figure A-15-7 in Shigley's book

$K_t \approx 1.65$

$$K_f = 1 + \frac{K_t - 1}{1 + \sqrt{a/r}}$$

with

$$\sqrt{a} = \left[0.246 - 0.00308\, S_{ut} + \overset{1.51}{\cancel{1.15}} \times 10^{-5}\, S_{ut}^{2} - 2.67 \times 10^{-8}\, S_{ut}^{3}\right]^{2}$$

with S_{ut} in ksi

Here

$$S_{ut} = 690 \text{ MPa} = 100 \text{ ksi}$$

$$\Rightarrow \sqrt{a} = 0.0623 \sqrt{in}$$

$$\Rightarrow a = 3.88129 \times 10^{-3} \text{ in}$$

$$= 0.0986 \text{ mm}$$

$$\Rightarrow q = \frac{1}{1+\sqrt{a/r}} = 0.847$$

&

$$K_f = q \times (K_t - 1) + 1$$

$$\Rightarrow \boxed{K_f = 1.55025}$$

Example 6-7

For the step shaft of Ex. 6-6 it is determined that fully corrected endurance limit in $S_e = 280$ MPa. Consider the shaft undergoes a fully reversed nominal stress of

$(\sigma_{rev})_{nom} = 260$ MPa.. Estimate the number of cycles to failure.

Sol.

Peak stress in bending from stress concentration is

$$(\sigma_a)_{peak} = K_f \, (\sigma_{rev})_{nom}$$

$$= 1.55 \times 260 = 403 \text{ MPa}.$$

Cycles to failure

We have that

$$S = a N^b \quad \text{or}$$

$$N = (S/a)^{1/b} \quad \text{with}$$

$$a = \frac{(f \, S_{ut})^2}{S_e}$$

Factors affecting S-N curves (fatigue life)

$K_{sr} = a\, S_{ut}^{\,b} = 4.51 \times 340^{-0.265}$

Surface roughness modifying factor

$= 0.9623$

Load factor

$K_l = 0.85$ (Axial loading

Temp factor

$K_T = 0.975$

Reliability factor

$K_r = 0.814$

$\Rightarrow \quad K_\infty = K_{sr} \; K_l \; K_T \; K_r \; , \quad S_e = K_\infty S_e'$

$\Rightarrow \quad S_e = 0.9623 \times 0.85 \times 0.975 \times 0.814 \times 170$

$\Rightarrow \quad S_e = 110.36$ MPa

Fatigue strength for 70,000 cycles is

$S = a N^b, \quad a = \frac{(f S_{ut})^2}{S_e}, \quad b = -\frac{1}{3} \log \frac{f S_{ut}}{S_e}$

Here $S_{ut} = 690$ MPa

$\Rightarrow \quad f = 0.845$

Thus

$$a = \frac{(0.845 \times 690)^2}{280} = 1214.1 \text{ MPa}$$

$$b = -\frac{1}{3} \log \frac{f S_{ut}}{S_e} = -\frac{1}{3} \log 2.08232$$

$$= -0.1062$$

$$\Rightarrow N = \left(\frac{403}{1214.1}\right)^{-\frac{1}{0.1062}} = \left(\frac{1214.1}{403}\right)^{\frac{1}{0.1062}}$$

$$\Rightarrow \boxed{N = 32.35 \times 10^3 \text{ cycles}}$$

cycles to failure

Example 6-8

A 1015 hot-rolled steel bar is machined to a diameter of 25 mm. It is to be placed in reversed axial loading for 70000 cycles at 300°C. Using ASTM minimum properties and a reliability of 99%. estimate endurance limit and fatigue strength at 70,000 cycles.

Sol.)

ASTM minimum strength for AISI 1015 hot rolled steel is

$$S_{ut} = 340 \text{ MPa}$$

Endurance limit (estimated)

$$S_e' = 340 \times 0.5 = 170 \text{ MPa}$$

$S_{ut}\Big|_{300°C}$

$$\Rightarrow \quad a = \frac{(0.9 \times 331.5)^2}{110.36} = 801.36 \text{ MPa}$$

$$b = -\frac{1}{3} \log \frac{0.9 \times 331.5}{110.36} = -0.1431$$

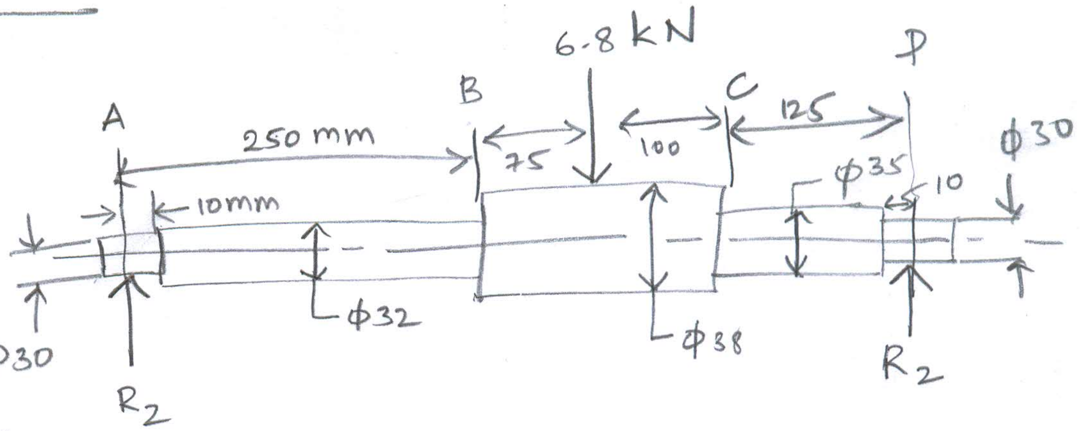
$$\Rightarrow \quad S_N = 801.3 \times (70000)^{-0.1431}$$

for $N = 70{,}000$

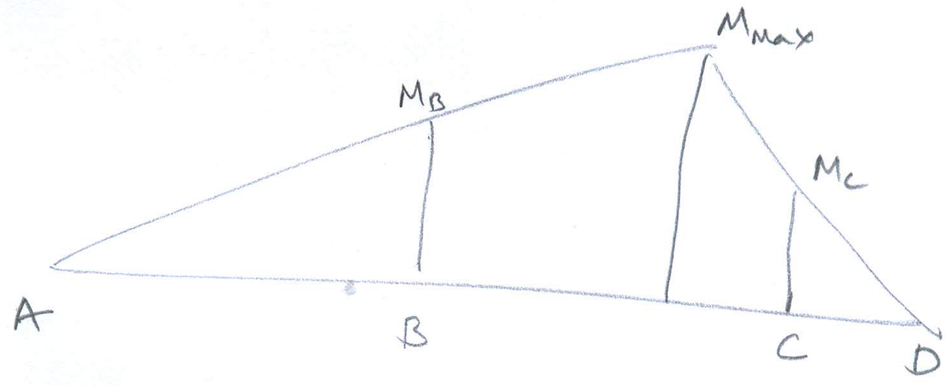
$$\Rightarrow \quad \boxed{S_N = 162.514 \text{ MPa}}$$

Example 6-9

All fillets have 3 mm radius.
Material is machined AISI 1050 cold drawn steel.



Bending moment diagram



$$\frac{R_2}{R_1} = \frac{325}{225} = 1.4444$$

$\Rightarrow$ $2.4444 \; R_1 = 6.8 \text{ kN}$

$\Rightarrow$ $R_1 = 2.782 \text{ kN}$ & $R_2 = 4.02 \text{ kN}$

$M_B = 695.5 \text{ Nm}$

$M_{Max} = 904.15 \text{ Nm}$

$M_C = 502.5 \text{ Nm}$

Minimum S_{ut} for AISI 1050 Cold rolled steel is

$$S_{ut} = 690 \text{ MPa}$$

Theoretical endurance limit for wrought steels is

$$S'_e = 0.5 \times S_{ut} = 345 \text{ MPa}$$

Modifying factors

$$\underset{\text{surface}}{K_{sr}} = a\, S_{ut}^{\,b}$$

with (for machined surface)

$$a = 4.51 \quad \& \quad b = -0.265$$

$$\Rightarrow \boxed{K_{sr} = 4.51 \times (690)^{-0.265} = 0.798}$$

$$\underset{\text{size}}{K_{sz}} = 0.879 \times \underset{\text{in inch}}{d}^{\,-0.107}$$

At B

$$K_{sz} = 0.8575$$

At C

$$K_{sz} = 0.8494$$

At M_{max}

$$K_{sz} = 0.842$$

For 99% reliability

$$K_r = 0.814$$

Neuber const associated with notch sensitivity is

$$\sqrt{a} = 0.246 - 3.08 \times 10^{-3}\, S_{ut} + 1.51 \times 10^{-5} \times S_{ut}^2 - 2.67 \times 10^{-8} \times S_{ut}^3$$

in inch

$$\Rightarrow \sqrt{a} = 0.0623 \sqrt{in}$$

$$\Rightarrow \sqrt{a/r} = 0.1813 \quad \& \quad q = \frac{1}{1+\sqrt{a/r}} = 0.8466$$

Thus, for this problem

$$S_e\Big|_B = K_{sr}\, K_{sz}\, K_\ell\, K_r\, S'_e$$

$$= 0.798 \times 0.8575 \times 0.814 \times 345$$

$$\Rightarrow \boxed{S_e|_B = 192.2 \text{ MPa}}$$

$$\boxed{S_e|_C = 190.35 \text{ MPa}}$$

$$\boxed{S_e|_{M_{max}} = 188.7 \text{ MPa}}$$

$K_t = 1.65$ & $q = 0.8466$

$$\Rightarrow \boxed{K_f = 1.5503}$$

fatigue
stress concentration
factor

Max. reversed stress at B, C, M_{max} are

$$\left(S_B\right)_{nom} = \frac{M_B \; d_B/2}{\pi \, d_B^4/64} = \frac{695.5 \times 0.032 \times 32}{\pi \times 0.032^4}$$

$$\Rightarrow \boxed{\left(S_B\right)_{nom} = 216.2 \text{ MPa}}$$

$$\boxed{\left(S_B\right)_{max} = K_f \left(S_B\right)_{nom} = 335.17 \text{ MPa}}$$

Max stress

$$\boxed{\begin{aligned}\left(S_{M_{max}}\right)_{peak} &= \frac{904.15 \times 0.038 \times 32}{\pi \times 0.038^4} \\ &= 167.8 \text{ MPa}\end{aligned}}$$

life in cycles is thus

$$N = \left(\frac{S}{a}\right)^{1/b}, \quad a = \frac{\left(f \, S_{ut}\right)^2}{S_e}$$

$$b = -\frac{1}{3}\log\left(\frac{f S_{ut}}{S_e}\right)$$

$$\Rightarrow a = \frac{(0.844 \times 690)^2}{192.2} = 1764.5 \text{ MPa}$$

$$b = -\frac{1}{3} \log\left(\frac{0.844 \times 690}{192.2}\right) = -0.16048$$

$$\Rightarrow \quad N = \left(\frac{335.17}{1764.5}\right)^{-\frac{1}{0.16048}}$$

$$= \left(\frac{1764.5}{335.17}\right)^{6.23133}$$

$$\Rightarrow \quad \boxed{N = 31.26 \times 10^3 \text{ cycles}}$$

Example 6-10

A 40 mm diameter bar has been machined from an AISI 1050 cold-drawn bar. This bar is to within stand a fluctuating tensile load of 0 to 70 kN. Because of ends & fillet radius, a fatigue stress concentration factor K_f is 1.85 for 10^6 or larger life. Find S_a and S_m and the factor of safety against fatigue & first cycle yielding using (a) Gerber fatigue line and (b) ASME-elliptic fatigue line

---

Sol) $S_{ut} = 690$ MPa and $S_y = 580$ MPa for AISI 1050 cold rolled bar.

Theoretical fatigue limit $S'_e = 0.5 \times 690$

$\Rightarrow S'_e = 345$ MPa.

Fatigue modifying factors

$$K_{sr} = a\, S_{ut}^{\,b} = 4.51\ 690^{-0.265}$$

$$\Rightarrow \quad \boxed{K_{sr} = 0.798}$$

Loading factor

$\boxed{K_L = 0.85}$ (for axial loading)

Reliability factor (99 % reliability)

$$\boxed{K_r = 0.814}$$

$$\Rightarrow \quad S_e = 0.798 \times 0.85 \times 0.814 \times 345$$

$$\Rightarrow \quad \boxed{S_e = 190.5 \text{ MPa}}$$

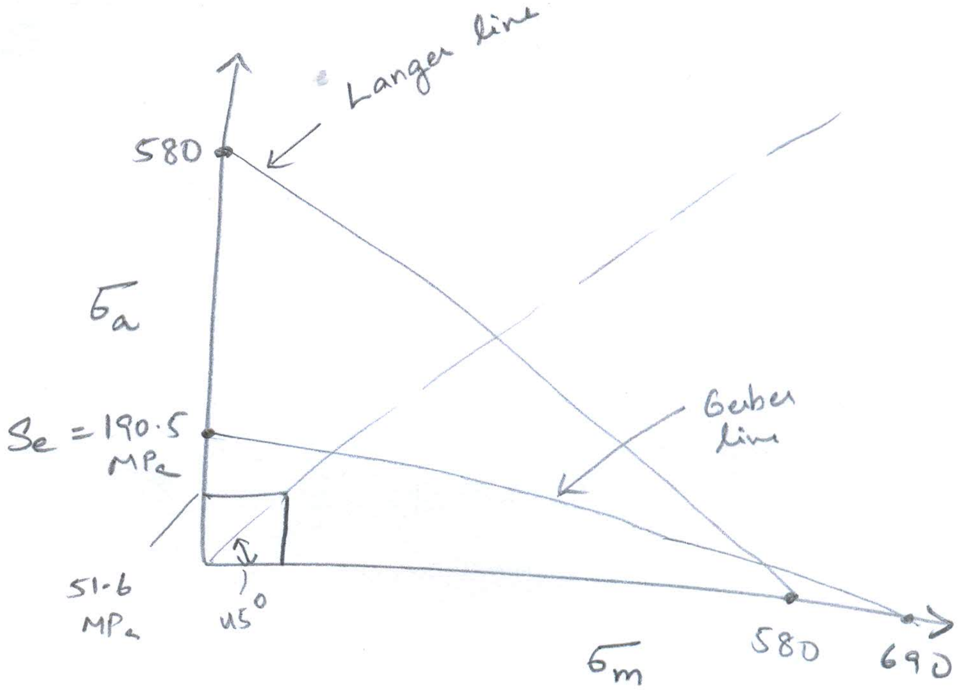
Nominal mean & amplitude of stresses are

$$(\sigma_m)_{nom} = \frac{35 \times 10^3}{\pi \times 0.02^2} = 27.85 \text{ MPa}$$

$$(\sigma_a)_{nom} = 27.85 \text{ MPa}$$

$$(\sigma_m)_{peak} = (\sigma_m)_{nom} \times K_f = 51.5225 \text{ MPa}$$

$$(\sigma_a)_{peak} = 51.5225 \text{ MPa}$$



Gerber line

Factor of safety

$$\left(\frac{n\sigma_a}{S_e}\right) + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1$$

For $\sigma_a = \sigma_m = 51.6$ MPa

$S_e = 190.5$ MPa

$S_{ut} = 690$ MPa

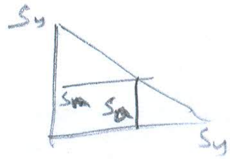
$\Rightarrow \boxed{n = 3.445}$

$\Rightarrow n^2 \, 5.6 \times 10^{-3} + n \times 0.271 = 1$

$$\Rightarrow n = \frac{-0.271 + \sqrt{0.271^2 + 4 \times 5.6 \times 10^{-3}}}{2 \times 5.6 \times 10^{-3}}$$

Factor of safety for ASME - elliptic fatigue line

$$\left(\frac{n\,\sigma_a}{S_e}\right)^2 + \left(\frac{n\,\sigma_m}{S_y}\right)^2 = 1$$



$$\Rightarrow n^2 = \frac{1}{\left[\left(\frac{51.6}{190.5}\right)^2 + \left(\frac{51.6}{580}\right)^2\right]}$$

$$\Rightarrow \boxed{n = 3.5075}$$

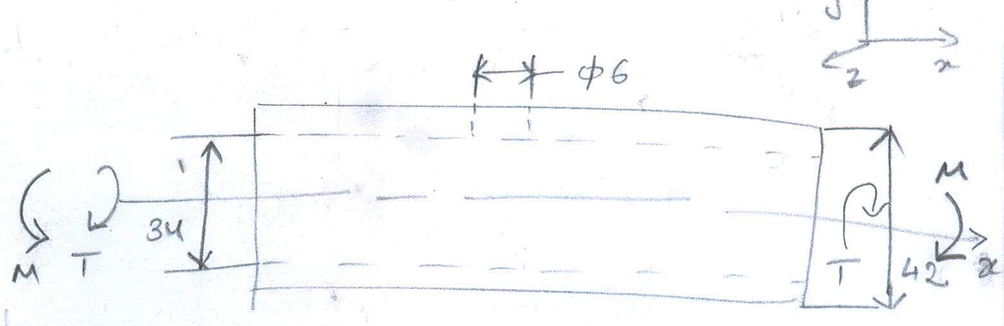
Factor of safety for 1st cycle yield is

$$n = \frac{S_y}{\sigma_a + \sigma_m} = \frac{580}{103.2}$$

$$\Rightarrow \boxed{n = 5.62}$$

Example 6-14

All dimensions are in mm



Consider the tube shown in the figure with a 6 mm diameter through hole. The tube is made from AISI 1018 cold drawn steel.

Case A

$M_z = 150 \sin \omega t \quad (N.m)$

$M_x = 120 \sin \omega t \quad (N.m)$

From table A-20, minimum yield and tensile strengths for AISI 1018 steel are

$S_y = 370 \text{ MPa}, \quad S_{ut} = 440 \text{ MPa}$

Correction factors for this case for high cycle fatigue are

$K_{ar} = a\, S_{ut}^{b} = 4.51 \times 440^{-0.265} = 0.899$

$K_{az} = 0.832$

$K_{r} = 0.814$ (99% reliability)

Thus, corrected fatigue strength for infinite life is

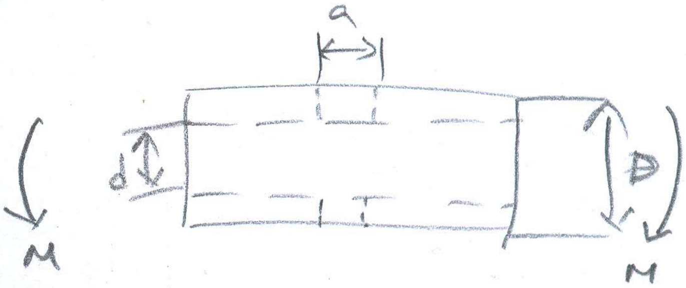
$$S_e = K_{sr}\, K_{sz}\, K_r\, S'_e$$

$$\Rightarrow S_e = 0.899 \times 0.832 \times 0.814 \times 0.5 \times 440$$

$$\Rightarrow \boxed{S_e \simeq 134 \text{ MPa}}$$

Stress concentration factor due to bending is estimated from Table A-16

Nominal bending stress



$$\sigma_b = \frac{M}{Z_{net}}$$ ← Reduced section modulus

Here $a = 6$ mm

$D = 42$ mm

$d = 34$ mm

$$Z_{net} = \frac{\pi A}{32D}(D^4 - d^4)$$

$$\Rightarrow \frac{a}{D} = 0.143$$

$$\frac{d}{D} = 0.81$$

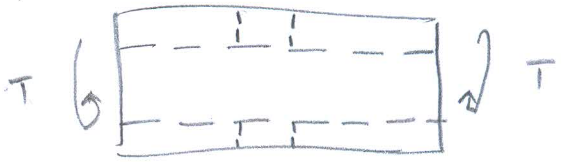
with

$A \simeq 0.798$

$K_t = 2.366$

From table A-16

Nominal shear stress



$$\tau_o = \frac{TD}{2J_{net}}$$

with

$$J_{net} = \frac{\pi A (D^4 - d^4)}{32}$$

with

$A = 0.89$ and $K_t = 1.75$

Notch sensitivity facts for bending and torsion can be estimated from Figs. 6-20 and 6-21 with notch radius 3 mm as

$q_b = 0.78$ and $q_t = 0.96$

$$\Rightarrow \quad K_f = 1 + q_b (K_t - 1) = 2.066$$

$$K_{fs} = 1 + q_s (K_{ts} - 1) = 1.72$$

Max. mises stress from bending & torsion can thus be estimated as

$$\sqrt{(K_f \sigma_0)^2 + 3\,(K_{fs}\,\tau_0)^2}$$

Here

$$\sigma_0 = \frac{150}{Z_{net}}, \quad Z_{net} = \frac{\pi A}{32 D}(D^4 - d^4)$$

$$= \frac{\pi \times 0.798}{32 \times 42} \times (42^4 - 34^4)\, mm^3$$

$$= 3.311 \times 10^3\ mm^3$$

$$\Rightarrow \sigma_0 = \frac{150 \times 10^3}{3.311 \times 10^3}\ \frac{N}{mm^2}$$

$$\Rightarrow \boxed{\sigma_0 \approx 45.3\ MPa}$$

$$\tau_0 = \frac{120 \times 10^3 \times 42}{2 \times J_{net}}, \quad J_{net} = \frac{\pi A\,(D^4 - d^4)}{32}$$

$$= \frac{\pi \times 0.89\,(42^4 - 34^4)}{32}$$

$$\boxed{\tau_0 = 16.25\ MPa}$$

$$\sigma_a = \sqrt{3 \times (1.72 \times 16.25 \times 70/120)^2} = 28.2 \text{ MPa}$$

Gerber failure line

$$\frac{n\sigma_a}{S_e} + \left(\frac{n\sigma_m}{S_{ut}}\right)^2 = 1 \quad , \; n \equiv \text{Factor of safety}$$

$$\Rightarrow \quad n^2 \times 0.05206 + n \times 0.278 = 1$$

$$\Rightarrow \quad n = \frac{-0.278 + \sqrt{0.278^2 + 4 \times 0.05206}}{2 \times 0.05206}$$

$$\boxed{n = 2.462}$$

Langer line

$$\boxed{n = \frac{S_y}{\sigma_m + \sigma_a} = \frac{370}{(100.39 + 28.2)} = 2.88}$$

$\Rightarrow$ Max mises stress

$$= \sqrt{(2.066 \times 45.3)^2 + 3\,(16.25 \times 1.72)^2}$$

$$= 105.37 \text{ MPa}$$

$\Rightarrow$ Factor of safety in fatigue $= \frac{134}{105.37} = 1.27$

Factor of safety in yield $= \frac{370}{105.37} = 3.51$

Case B

$M_z = 150$ Nm

$M_x = 90 + 70 \sin \omega t$ Nm

Thus

$$\sigma_m = \sqrt{(2.066 \times 45.3)^2 + 3\,(1.72 \times 16.25 \times 90/120)^2}$$

$$= 100.39 \text{ MPa}$$

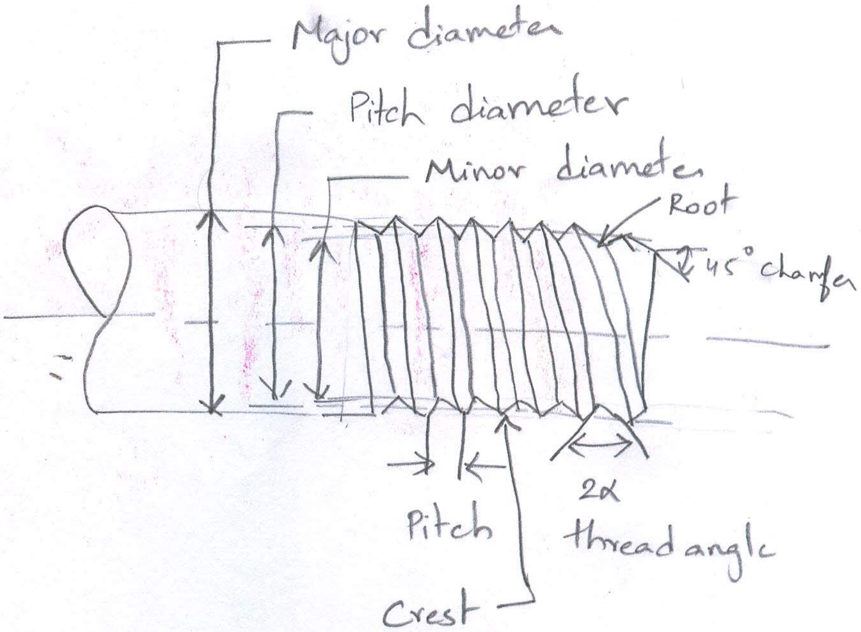
18-3-17

Screws, fasteners, and non-permanent joints

Metric threads

$2\alpha = 60^\circ$

MJ profile has rounded fillet at root of fillet. Better for higher fatigue strength.



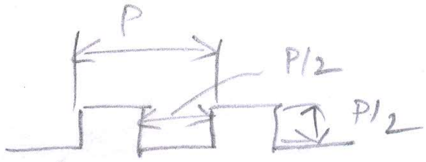
Terminology of screw threads. Sharp vee threads are actually flattened or rounded during forming.

Metric threads are specified by writing the diameter and pitch in mm in that order. Thus

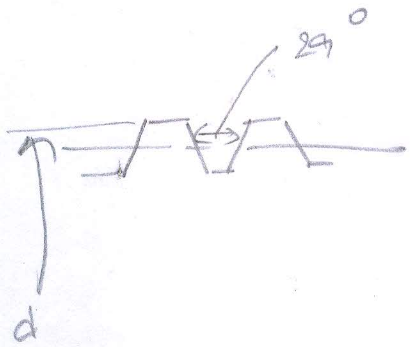
M12 x 1.75

is a thread having major diameter of 12 mm and a pitch of 1.75 mm. Note that letter M which precedes the diameter is the clue to the metric designation.

Square thread

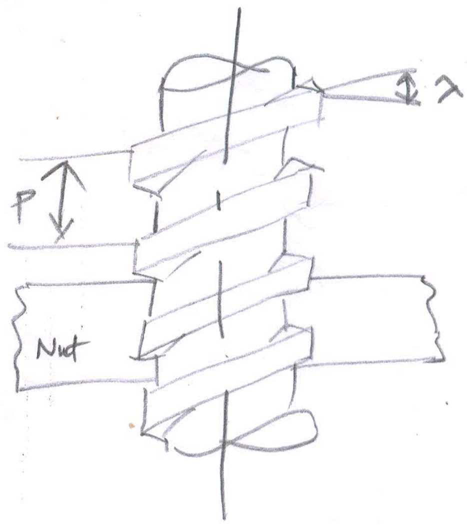


Acme thread



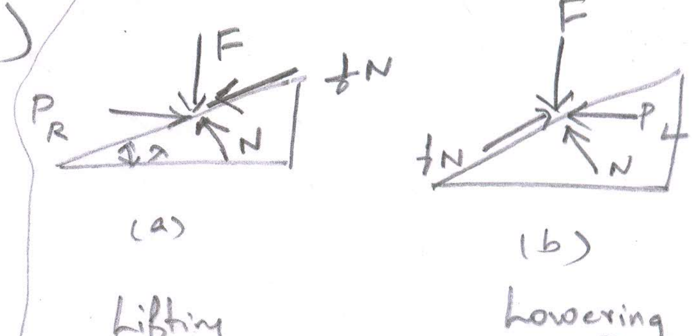
Mechanics of power screw

For square power screws we get

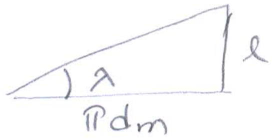


$$P_R = \frac{F(\sin\lambda + f\cos\lambda)}{\cos\lambda - f\sin\lambda}$$

$$P_L = \frac{F(f\cos\lambda - \sin\lambda)}{\cos\lambda + f\sin\lambda}$$



Noting that torque is the product of force Product of force P and mean radius dm/2 for raising the load we can write



$$T_R = F\,dm/2 \left(\frac{l + f\pi dm}{\pi dm - fl} \right)$$

Torque needed to lower the load is

$$T_L = \frac{F dm}{2} \left(\frac{\pi f dm - l}{\pi dm + fl} \right)$$

Condition for self-locking

$$\pi f dm > l$$

or $\boxed{f > \tan\lambda}$

If friction is absent then torque needed to raise the load is $T_0 = \frac{Fl}{2\pi}$

Thus, thread efficiency for square power screws is

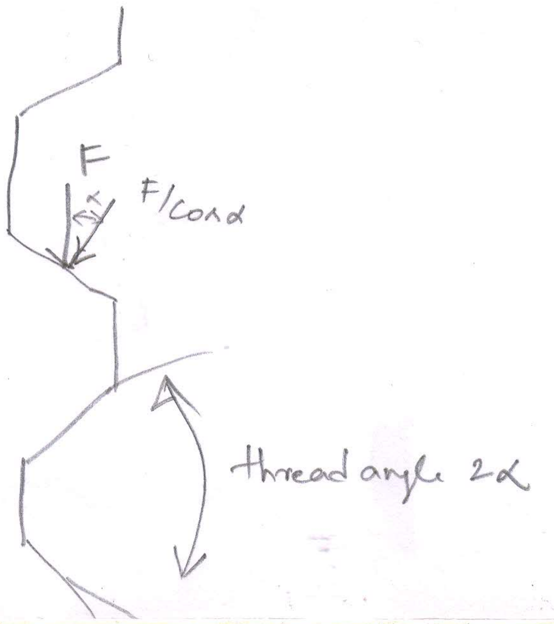
$$e = \frac{T_0}{T_R} = \frac{Fl}{2\pi \times T_R}$$

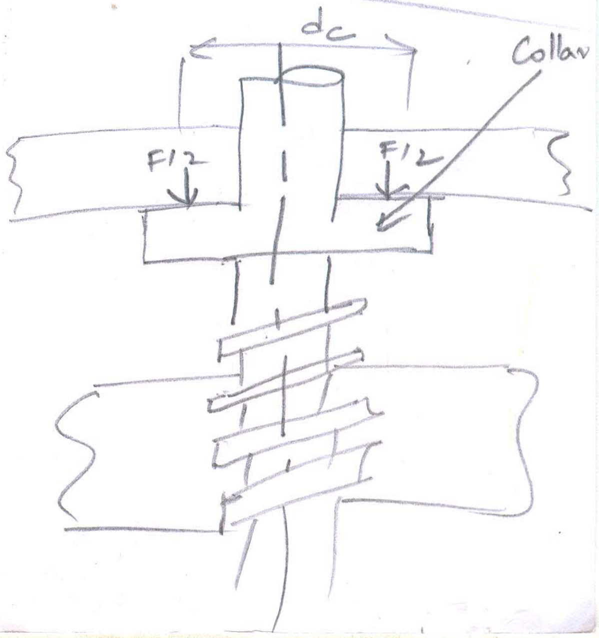
$$\Rightarrow \quad e = \frac{l}{2\pi} \times \frac{\pi d_m - fl}{l + \pi f d_m} \times \frac{2}{d_m}$$

$$\Rightarrow \quad e = \tan\lambda \times \frac{1 - f\tan\lambda}{\tan\lambda + f}$$

$$\Rightarrow \quad \boxed{e = \frac{1 - f\tan\lambda}{1 + f\cot\lambda}}$$

Normal force increases because of angle α





In case of Acme threads or any other normal thread, neglecting lead angles

$$T_R = \frac{F d_m}{2} \left(\frac{l + \pi f d_m \sec\alpha}{\pi d_m - f l \sec\alpha} \right)$$

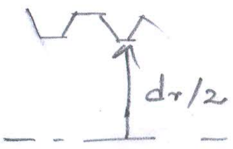
<u>Collar torque</u>

$$T_c = \frac{F f_c d_c}{2}$$

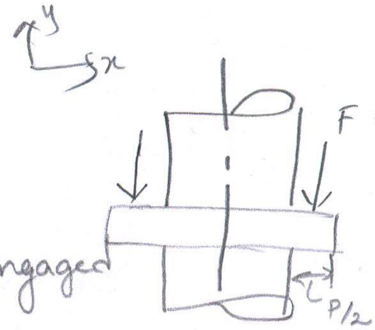
<u>Stresses in a power screw</u>

<u>Axial</u>

$$\sigma = \frac{F}{A} = \frac{4F}{\pi d_r^2}$$ — Minor diameter



Thread stresses :



Bearing stress $\sigma_B = \dfrac{-F}{\pi d_m n_t P/2}$

└ No of engaged threads

Bending stress at root of teeth

$$\sigma_b = \frac{My}{I} = \frac{F \times P/\cancel{4} \times P/\cancel{4} \times \cancel{8}^{2} \times \cancel{12}^{3}}{n_t \pi d_r P^3}$$

$$\Rightarrow \boxed{\sigma_b = \frac{6F}{\pi n_t d_r P}}$$

Transverse shear stress τ at center of the root of the thread due to load F is

$$\tau = \frac{3V}{2A} = \frac{3F}{2n_t \pi d_r P/2} = \frac{3F}{2n_t \pi d_r P}$$

and shear stress at top of root is zero.

Thus at top of root of thread

$$\sigma_x = \frac{6F}{\pi n_t d_r P}, \quad \sigma_y = -\frac{2F}{\pi d_m n_t P}$$

$$\tau_{xz} = \frac{16T}{d_r^3}, \quad \sigma_z = 0, \quad \tau_{xy} = 0, \quad \tau_{yz} =$$

A power screw lifting the load is in compression & nut in tension. Engaged threads cannot share the load equally. Some exps. show first engaged thread carries 38% of load, second 25%, 3rd 18% & 7th none.

Thus we take "0.38 F" in place of "F" & take "$n_F = 1$" for getting largest stresses.

Example 8-1

A square-thread power screw has a major diameter of 32 mm, pitch of 4 mm with double threads. Given that $f = f_c = 0.08$, $d_c = 40$ mm, and F = 6.4 kN per screw

(a) Find thread depth, thread width, pitch diameter, minor diameter, and lead,

Thread depth $= P/2 = 2$ mm

Thread width $= P/2 = 2$ mm

Pitch diameter $= d_m = 32 - P/2 = 30$ mm

Minor diameter $d_r = 32 - P = 28$ mm

lead $= l = np = 2 \times 4 = 8$ mm

(b)

$$T_R = \frac{F\,d_m}{2}\,\frac{[f + l/\pi d_m]}{1 - f l/\pi d_m} + \frac{F\,f_c d_c}{2}$$

$$= 15.94 + 10.24 = 26.18 \text{ Nm}$$

$$T_L = -0.466 + 10.24 = 9.77 \text{ Nm}$$

(c) Overall efficiency

$$e = \frac{F l}{2\pi T_R} = \frac{6.4 \times 8}{2\pi \times 26.18} = 0.311$$

(d) Body shear stress

$$\tau = \frac{16\,T_R}{\pi d_r^3} = 6.07 \text{ MPa}$$

Axial stress

$$\sigma = -\frac{4F}{\pi d_r^2} = -10.39 \text{ MPa}$$

(e) Bearing stress

$$\sigma_B = -12.9 \text{ MPa} \quad (\text{with } 0.38 F)$$

(f) Thread root bend stress

$$\sigma_b = \frac{6\ (0.38\ F)}{\pi\, d_r\, (1)\, P} = 41.5\ \text{MPa}$$

(g) Von Mises stress

$$\sigma' = 48.7\ \text{MPa}$$

Screw bearing pressure P_b

Screw material	Nut material	Safe P_b (MPa)	Notes
Steel	Bronze	17 – 24	Low speed
Steel	Bronze	11 – 17	≤ 50 mm/s
	Cast iron	7 – 17	≤ 40 mm/s
Steel	Bronze	5.5 – 9.7	100–200 m/s
	Cast iron	4.1 – 6.9	"
Steel	Bronze	1.0 – 1.7	≥ 250 mm/s

Metric threads – Mechanical properties

Property class	Size range	Min proof stress (MPa)	Sut (MPa)	Mat	Marking
4.6	M5 – M36	225	400	low-medium carbon steel	4.6
4.8	M.16 – M16	310	420	//	4.8
5.8	M5 – M24	380	520	//	5.8
8.8	M16 – M36	600	830	Medium carbon, Q&T	8.8
9.8	M1.6 – M16	650	900	//	9.8
10.9	M5 – M36	830	1040	low carbon martensitic Q&T	10.9
12.9	M1.6 – M36	970	1220	Alloy Q&T	12.9

Fatigue loading of tension joints

Distribution of typical bolt failures

15 % under the head

20 % at end of thread

65 % in thread at nut face

Fatigue stress concentration factors K_f

SAE Grade	Metric grade	Rolled threads	Cut	Fillet
0 to 2	3.6 to 5.8	2.2	2.8	2.1
4 to 8	6.6 to 10.9	3.0	3.8	2.3

Fully corrected endurance strength.

Grade	Size	Se (MPa)
ISO 8.8	M16 - M36	129
" 9.8	M1.6 - M16	140
" 10.9	M5 - M36	162
" 12.9	M1.6 - M36	190

Relating bolt torque to bolt tension

Preload

Average of major & minor diameter

Friction coeff. of collar

$$T = \frac{F_i d_m}{2}\left(\frac{\tan\lambda + f\sec\alpha}{1 - f\tan\lambda\sec\alpha}\right) + \frac{F_i f_c d_c}{2}$$

$$\boxed{d_c = 1.25d}$$

Torque coefficient is defined as

$$K = \frac{d_m}{2d}\left(\frac{\tan\lambda + f\sec\alpha}{1 - f\tan\lambda\sec\alpha}\right) + 0.625 f_c$$

$$\Rightarrow \boxed{T = K F_i d}$$

$$\boxed{K \approx 0.2 \text{ for } f = f_c = 0.15}$$ - For bolts of different sizes with fine or coarse threads

Gasketed joints

Dia. of bolt circle

$$3 \le \frac{\pi D_b}{N d} \le 6$$

No of bolts

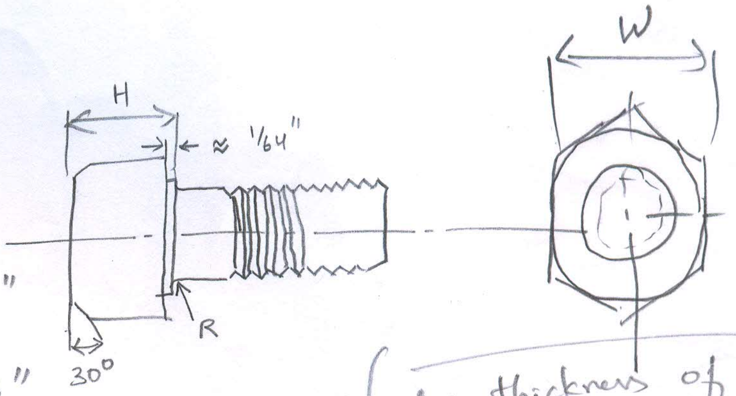
diameter of bolt

Good material combination

Hard steel on bronze or Bronze on bronze

Threaded fasteners

For inch-series bolts thread length is



$$L_T = \begin{cases} 2d + 1/4 \text{ in} & L \le 6'' \\ 2d + 1/2 \text{ in} & L > 6'' \end{cases}$$

l = thickness of all material squeezed between face of bolt & face of nut

$$l = \begin{cases} h + t_2/2 & t_2 < d \\ h + d/2 & t_2 \ge d \end{cases}$$

and for metric bolts is

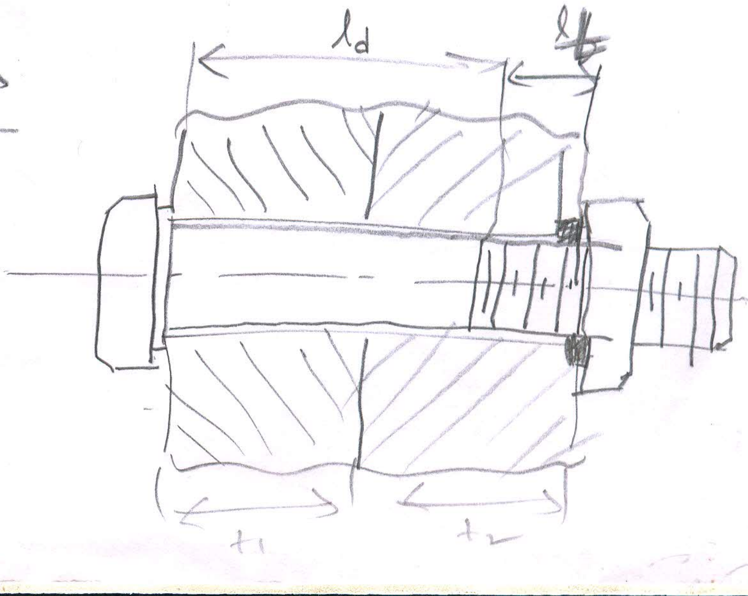
$$L_T = \begin{cases} 2d + 6 & L \le 125 \quad d \le 48 \\ 2d + 12 & 125 \le L \le 200 \\ 2d + 25 & L > 200 \end{cases}$$

where dimensions are in mm.

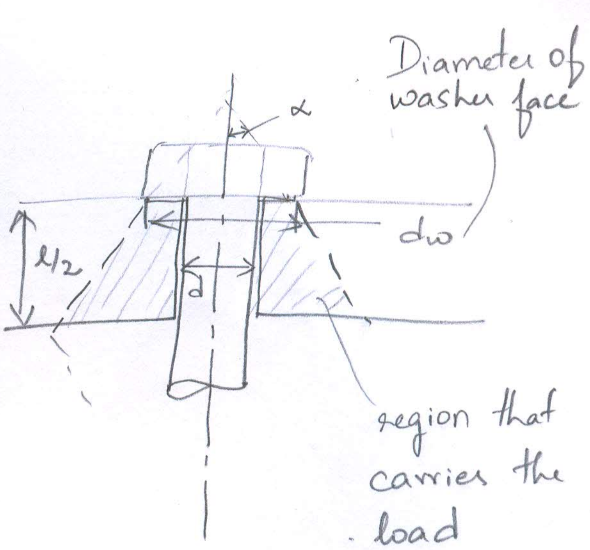
Joints - Fastener Stiffness

Fastener stiffness

$$K_b = \frac{E}{l_t/A_t + l_d/A_d}$$



Joints – Member stiffness



When top and bottom plates / sheets that are being joined by bolts are made of identical materials then joint stiffness

$$\boxed{d_w = 1.5\,d}$$

$$K_m = \frac{\pi E d \tan\alpha}{2 \ln\left[\dfrac{(l\tan\alpha + d_w - d)(d_w + d)}{(l\tan\alpha + d_w + d)(d_w - d)}\right]}$$

with $\alpha = 30^\circ$ (as seen from FEA simulations) for hardened steel, cast iron or aluminum members.

Tension joints – External load

F_i = Preload

P_{total} = Total external tensile load applied to joint

P = External tensile load per bolt

P_b = Portion of P taken by bolt

P_m = Portion of P taken by joint members

$F_b = P_b + F_i$ = resultant bolt load

$F_m = P_m - F_i$ = resultant member load

C = Fraction of load carried by bolt

N = No of bolts in the joint

If all bolts share load equally then

$P = P_{Total} / N$

$\delta = P_b / K_b$ & $\delta = \dfrac{P_m}{K_m}$

$\Rightarrow \quad P_m = \dfrac{P_b \, K_m}{K_b}$

Bolt load

$$F_{b\,min} = C P_{min} + F_i$$

$$F_{b\,max} = C P_{max} + F_i$$

$$\sigma_a = \frac{F_{b\,max} - F_{b\,min}}{2 A_t}$$

$$\Rightarrow \boxed{\sigma_a = \frac{C (P_{max} - P_{min})}{2 A_t}}$$

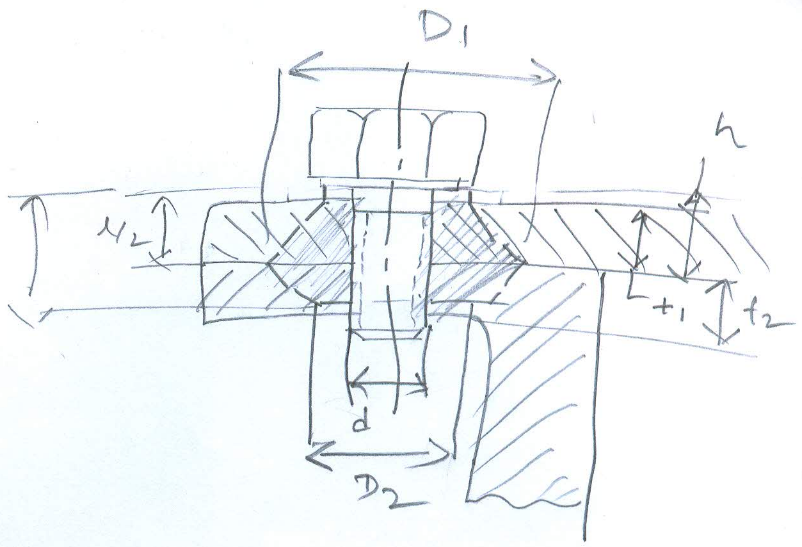
$$\sigma_m = \frac{F_i}{A_t} + \frac{C (P_{max} + P_{min})}{2 A_t}$$

Example 8-5

$D_1 = d_w + l \tan\alpha$

effective grip

$D_2 = d_w = 1.5\,d$



$\alpha = 30^\circ$ (Metric screw)

$d_w = 1.5\,d$

$l = \begin{cases} h + t/2 & t_2 \le d \\ h + d/2 & t_2 \ge d \end{cases}$

The joint is subjected to a fluctuating force whose maximum is 5 kip per screw. The required data are

Cap screw, 5/8 in 11 UNC, SAE 5, hardened steel washer, $t_w = 1/16''$, $t_1 = 5/8''$,

$E_s = 30$ Mpsi, cast-iron base $t_2 = \frac{5}{8}''$

$E_{ci} = 16$ Mpsi.

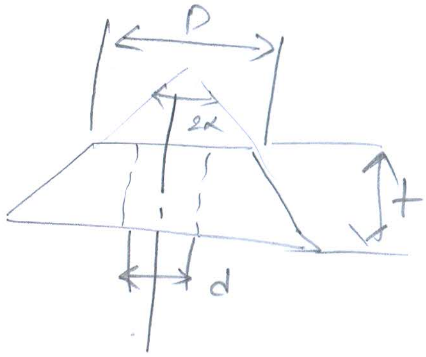
(a) Find k_b, k_m and C

(b) Find factors of safety.

$h = t_1 + t_w = 0.6875''$

$l = h + d/2 = 1''$

$D_2 = 1.5d = 0.9375$



For frustrum shown in the figure stiffness

$$k = \frac{\pi E d \tan\alpha}{\ln\left[\frac{(2t\tan\alpha + D - d)(D+d)}{(2t\tan\alpha + D + d)(D-d)}\right]}$$

Thus for top frustrum stiffness

$k_1 = 46.46$ Mlbf/in

For middle frustrum

$k_2 = 197.43$ Mlbf/in

For bottom frustrum

$k_3 = 32.39$ Mlbf/im

Thus $k_m = \left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right)^{-1} = 17.41$ Mlbf/in

$$K_b = \frac{A_t E}{l} = 6.78 \ \frac{\text{Mlbf}}{\text{in}}$$

Thus

$$C = \frac{K_b}{K_b + K_m} = \frac{6.78}{6.78 + 17.40} = 0.280$$

(b)

$$F_i = \begin{cases} 0.75\,F_p & \text{for non-permanent conn} \\ 0.9\,F_p & \text{for permanent connect.} \end{cases}$$

Proof strength $\equiv S_p \times A_t$

Here thus

$$F_i = 0.75 \times 0.226 \times 85 = 14.4 \text{ kip}$$

$$n_p = \frac{S_p A_t}{CP + F_i} = \frac{85 \times 0.226}{0.28 \times 5 + 14.4}$$

$$= 1.22$$

Overload that can be applied w.o. causing bolt failure is

$$n_L = \frac{S_p A_t - F_i}{CP} = 3.44.$$

Load factor which two plates seperate is

$$n_0 = \frac{F_i}{P(1-C)} = 4.0$$

(b)

Here

$S_p = 85$ kpsi

$S_e = 18.6$ kpsi

$S_{ut} = 120$ kpsi

$\sigma_m = 66.82$ kpsi

$\sigma_a = 3.08$ kpsi

From modified goodman we get

$n_f = 2.44$

from proof stress by yield

$n_p = 3.43$

For shank

$$\sigma_{all} = 0.6\ S_y = 148.8\ MPa$$

Here

$$\frac{F}{A} = \frac{107\,000}{0.02 \times 0.05 + 0.05 \times 0.01} = 71.3\,MPa$$

For shank

$$\sigma_{all} = 0.6\ S_y = 148.8\ MPa$$

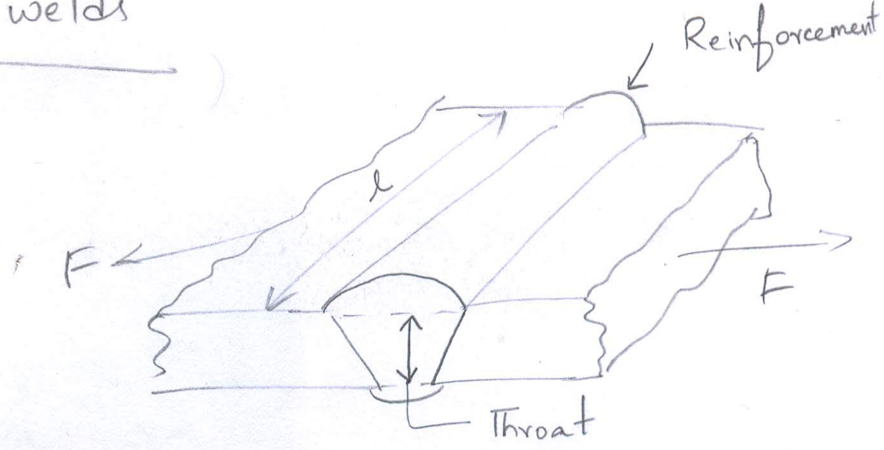
Here

$$\frac{F}{A} = \frac{107000}{0.02 \times 0.05 + 0.05 \times 0.01} = 71.3 MPa$$

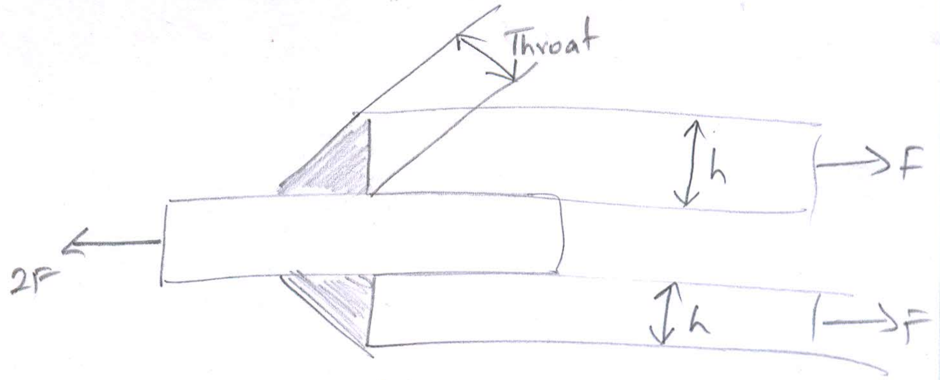
Lectures 38-42

Welding, Bonding and Design of Permanent Joints

Butt and fillet welds



Butt load



Tranverse Fillet weld

Butt welds

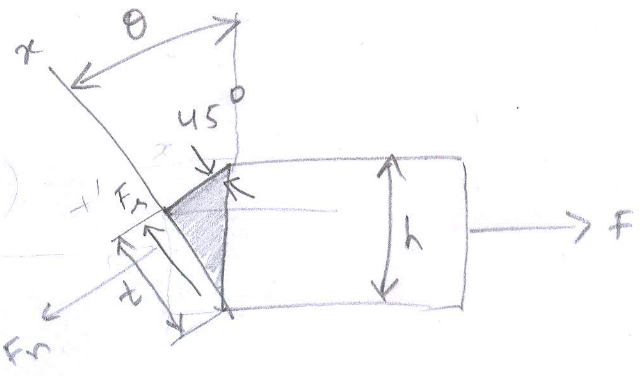
For tension or compression average normal stress is

$$\sigma = \frac{F}{hl}, \quad h \equiv \text{weld throad}$$

Ave. stress in butt weld due to shear loading is

$$\tau = \frac{F}{hl}$$

Fillet weld



$F_n = F\cos\theta$

$F_s = F\sin\theta$

$$\boxed{t = \frac{h}{\cos\theta + \sin\theta}}$$

$$\Rightarrow \tau = \frac{F_s}{t l}, \quad \sigma = \frac{F_n}{t l}$$

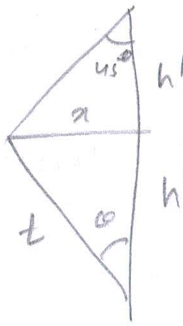
$$\Rightarrow \tau = \frac{F\sin\theta\,(\sin\theta + \cos\theta)}{l h}$$

$$\sigma = \frac{F\cos\theta\,(\cos\theta + \sin\theta)}{l h}$$

$$\sigma_{von\ Mises} = \sqrt{\sigma^2 + 3\tau^2}$$

$$\sigma_{von\ Mises}\Big|_{Max} \text{ @ } \theta = 62.5^\circ$$

$$= 2.16\, F/hl$$



$h' + h'' = h$

$$\frac{x}{t} = \sin\theta$$

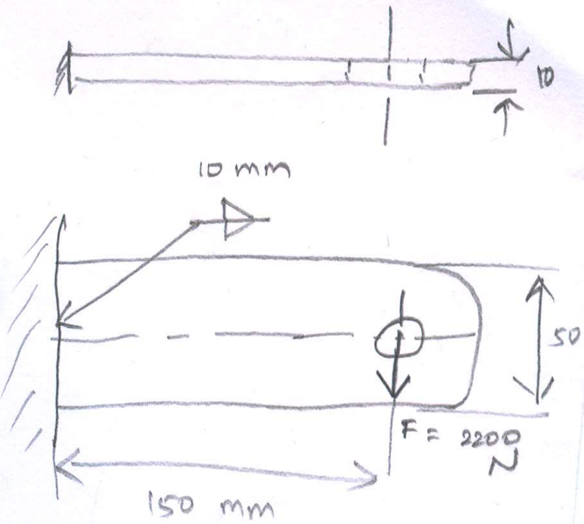
$$\Rightarrow x = t\sin\theta$$

$h' = x$

$h'' = x/\tan\theta$

$$\Rightarrow h = t\sin\theta + \frac{t\sin\theta\cos\theta}{\cancel{\sin\theta}}$$

Example 9-4



Cantilever is made of AISI 1018 HR steel and welded with E6010 electrode. Design factor was 3.0

Check adequacy accessment of the statically loaded structure.

All dimensions are in mm

Sol

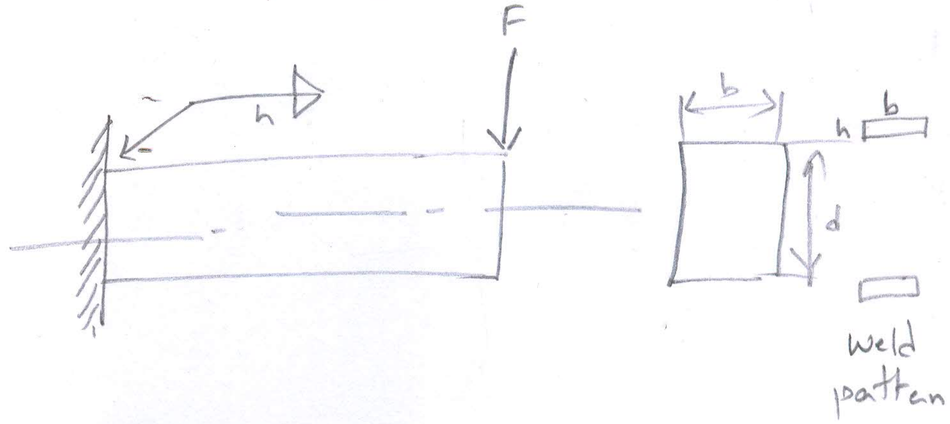
Allowable shear in weld metal with design factor 3 is

$$\boxed{S_{\substack{shear\\allow}}\Big|_{weld} = \frac{1}{3} \times \frac{S_y}{\sqrt{3}} = \frac{1}{3\sqrt{3}} \times 345 = 66.39 \text{ MPa}}$$

$$S_{\substack{shear\\allow}}\Big|_{BM} = \frac{1}{3} \times \frac{1}{\sqrt{3}} \times S_y = 0.19245 \times 220$$

$$= 42.34 \text{ MPa}$$

Stresses in welded joints in bending



FBD of the beam would show shear reaction force V and moment reaction M.

Shear forces produces primary shear stress in welds of magnitude

$$\tau' = \frac{V}{A} \text{ — Throat area}$$

Moment M produces horizontal shear stress

Nominal secondary shear stress

$$\tau'' = \frac{Mc}{I} = \frac{Md/2 \quad 2}{h\,0.707\,bd^2}$$

$$I = 0.707\,h\,I_u$$

$$\Rightarrow \boxed{\tau'' = \frac{1.414\,M}{bdh}} \qquad \tau = \sqrt{\tau'^2 + \tau''^2}$$

Stresses in weld

Primary shear $= \frac{F}{A} = \frac{2200}{1.414\, h d} = \frac{2000}{1.414 \times 10 \times 50}$

$\tau' \simeq \cancel{7.78}\ 3.1$ MPa

Secondary shear $= \frac{M\, d/2}{I}$, $I = \frac{1}{\sqrt{2}} h\, I_u$

$$\tau'' = \frac{2200 \times 150 \times 25 \times 6}{0.707 \times 10 \times 50^3} = 56 \text{ MPa}$$

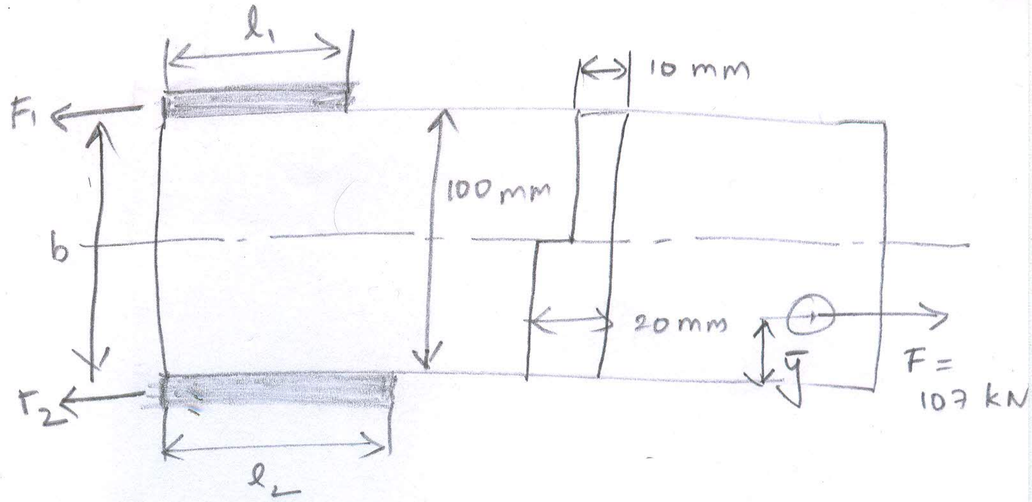
$\Rightarrow$ Net shear stress $= \sqrt{\tau'^2 + \tau''^2} = 56.1$ MPa

Thus weld strength is adequate

Max Shear in base metal is

$= 39.6$ MPa

Thus base metal strength is also adequate



$$\bar{y} = \frac{\Sigma y_i A_i}{\Sigma A_i} = \frac{10 \times 50 \times 75 + 20 \times 50 \times 25}{10 \times 50 + 20 \times 50} = 41.7 \text{mm}$$

$$\Rightarrow \quad F_1 \times 58.3 = F_2 \times 41.7$$

$$= \quad \& \quad F_1 + F_2 = 107$$

$$\Rightarrow \quad F_1 \left(1 + \frac{58.3}{41.7}\right) = 107$$

$$\Rightarrow F_1 = 44.62 \text{ kN}, \quad F_2 = 62.38 \text{ kN}$$

Base metal $S_y = 248$ MPa $\quad S_{ut} = 400$ MPa

E70xx electrode is being used. Thus, weld metal $S_y^w = 393$ MPa, $S_{ut} = 482$ MPa

Permissible stress in shear for welt is

$= 0.3\ S_{ut} = 0.3 \times 482 = 144.6$ MPa

Thus weld length needed is

$$\frac{F_1}{0.707 \times 8 \times l_1} = 144.6$$

$\Rightarrow l_1 = 54.55$ mm

$$\frac{F_2}{0.707 \times 8 \times l_2} = 144.6 \Rightarrow l_2 = 76.27 \text{ mm}$$

Base metal next to weld is allowed to experience shear stress of $\leq 0.4\ S_y$

$\Rightarrow \tau_{BM} \leq 0.4 \times 248 \leq 99.2$

$$\Rightarrow \frac{F_1}{h l_1} \leq 99.2 \Rightarrow \boxed{l_1 \geq 56.224}$$

$$\frac{F_2}{h l_2} \leq 99.2 \Rightarrow \boxed{l_2 \geq 78.6 \text{ mm}}$$

Bearings

Bearing life

If a bearing is clean and properly lubricated is mounted and sealed against the entrance of dust and dirt, is maintained in this condition and is operated at reasonable temperatures then metal fatigue will be the only cause of failure.

Life
- No. of revolutions of inner raceway until tangible evidence of fatigue
- Hrs of use at standard angular speed until first tangible evidence of fatigue

Spalling or pitting area of 0.01 in^2

Commonly used life rating:

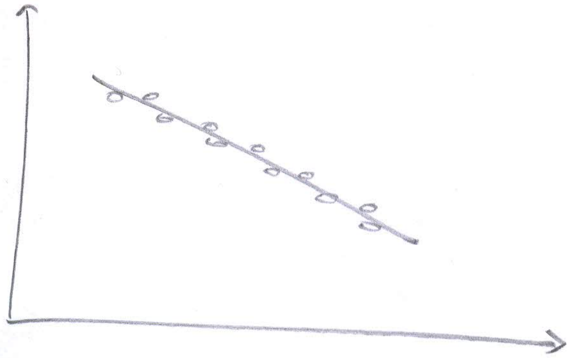
L_{10} life — Minimum life of 90 % of bearings in a large lot

L_{50} life — Median life / Ave life is min. life of 50 % of bearings in a large lot

$$\boxed{L_{50} \approx 4-5\, L_{10}}$$

Bearing life at rated reliability

$$F^a L = \text{Constant}$$



$a = 3$ for ball bearings

$a = 10/3$ for roller (cylindrical and taper roller) bearings

Catalog / Basic dynamic load rating is denoted by C_{10} — Rated life of 10^6 revolutions

$$x_D = \frac{L_D}{L_R} \quad \begin{matrix} L_D \text{ — Desired life} \\ L_R \text{ — Rated life} \end{matrix}$$

$$\mathcal{L} = \frac{L}{60 \times n}$$

$\mathcal{L}$ — Life in hrs; L — Life in rev.; n — rev/min

Example 1-1

Desired life of 5000 hrs at 1725 rev/min with load of 2 kN with 90% reliability. C_{10} life needed = ? Consider ball bearings.

$$C_{10} = F_D \left(\frac{\mathcal{L}_D}{\mathcal{L}_R} \right)^{1/a}, \quad \mathcal{L}_D = 5000, \quad \mathcal{L}_R = \frac{10^6}{60 \times 1725} = 9.66 \text{ hrs}$$

$$\Rightarrow C_{10} = 2 \times 8.03 = 16.06 \text{ kN}$$

Reliability versus life

Three-parameter Weibull distribution

$$R = \exp\left[-\left(\frac{(x-x_0)}{(\theta-x_0)}\right)^b\right]$$

Reliability

x_0 = Garanteed or "minimum" value of x

x = L/L_{10}

θ = Characteristic parameter

b = Shape parameter. For rolling contact bearings $b \approx 1.5$.

Probability function

$$f(x) = \begin{cases} \dfrac{b}{\theta-x_0}\left(\dfrac{x-x_0}{\theta-x_0}\right)^{b-1} e\left[-\left(\dfrac{x-x_0}{\theta-x_0}\right)^b\right] & x \geq x_0 \\ 0 & x < x_0 \end{cases}$$

Mean & standard deviation of f are

$$\mu_x = x_0 + (\theta - x_0)\,\Gamma(1 + 1/b)$$

$$\hat{\sigma}_x = (\theta - x_0)\sqrt{\Gamma(1+2/b) - \Gamma^2(1+1/b)}$$

Gamma function

$$\Gamma(n) = (n-1)!$$

$$\Gamma(z) = \int_0^\infty x^{z-1} e^{-x} \, dx$$

Example 11-2

For a 02-30 mm deep groove ball bearing

$x_0 = 0.02$, $\theta = 4.46$, $b = 1.483$. Find

μ_x, $x_{0.5}$, $x_{0.1}$, $\hat{\sigma}_x$, Coefficient

of variation $C_x = \dfrac{\hat{\sigma}_x}{\mu_x}$

$$\mu_x = x_0 + (\theta - x_0) \, \overbrace{\Gamma(1 + 1/b)}^{= 0.9040}$$

$$\boxed{\mu_x = 4.033}$$

$$x_{0.5} = x_0 + (\theta - x_0) \left(\ln 1/R\right)^{1/b}_{R = 0.5} = 3.487$$

Median life

$$\hat{\sigma}_x = (\theta - x_0) \sqrt{\underbrace{\Gamma(1+2/b)}_{= 1.2023} - \Gamma^2(1+1/b)}$$

$$= 2.755$$

$$\Rightarrow C_x = 0.683$$

Example 11-3

Design load on a ball bearing is 1840 N

Design factor is 1.2. Shaft speed on which bearing is mounted is 400 rev/min. $L_D = 30000$ hr

Reliability needed is 0.99. $C_{10} = ??$

Weibull parameters are $x_0 = 0.02$,

$\theta - x_0 = 4.439$ & $b = 1.483$.

$$\Rightarrow \quad x_D = (\theta - x_0) \ln\left(\frac{1}{R}\right)^{1/b} + x_0$$

$$= 0.2246$$

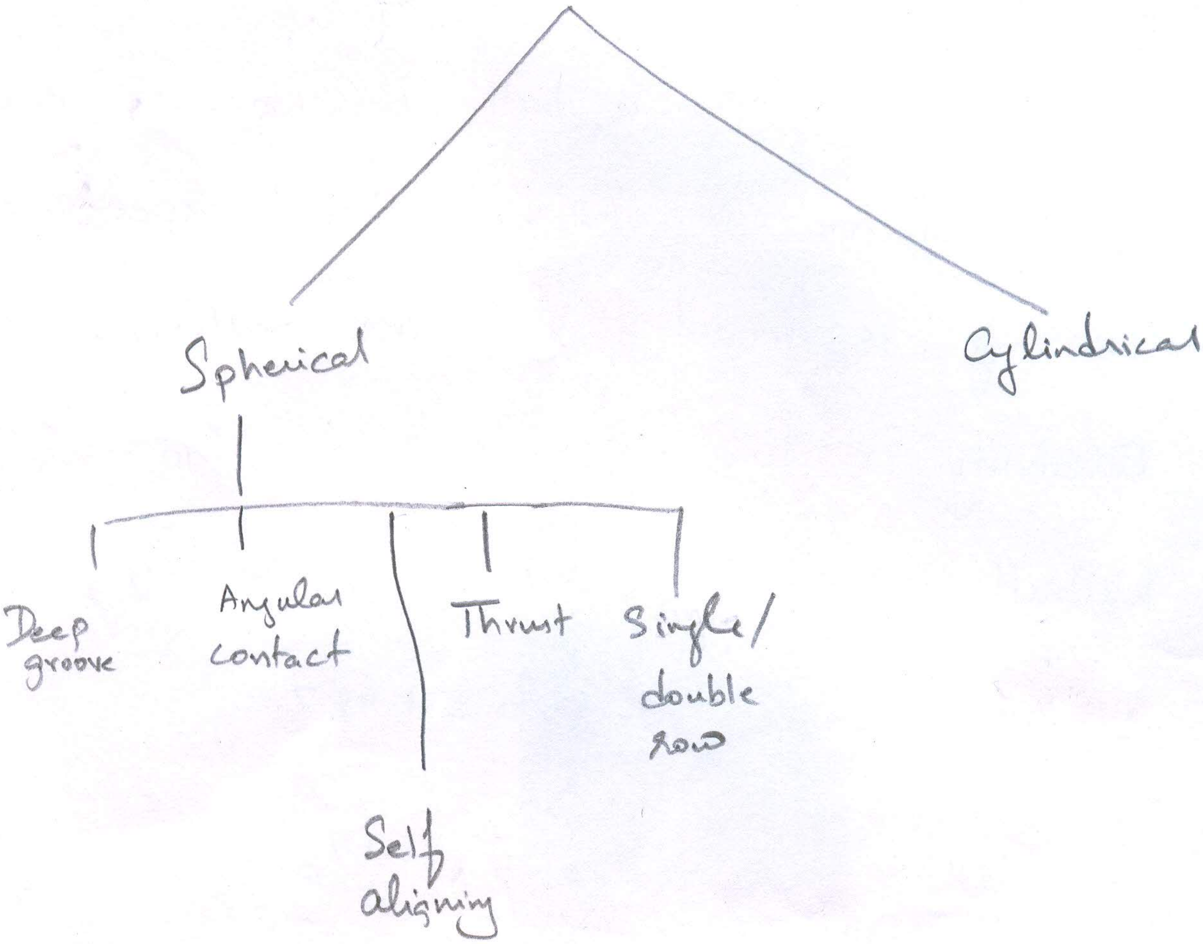
$$\Rightarrow \quad \boxed{L_{0.99} = 0.2246 \, L_{10}}$$

$$C_{10} \times (10^6)^{1/3} = 1.84 \times 1.2 \times \left(\frac{1}{0.2246} \times 30000 \times 60 \times 400\right)^{1/3}$$

$$\Rightarrow \quad C_{10} = 1.84 \times 1.2 \times \left(\frac{3 \times 60 \times 4}{0.2246}\right)^{1/3}$$

$$= 1.84 \times 1.2 \times 3205.7^{1/3} = 32.55 \text{ kN}$$

Rolling contact bearings



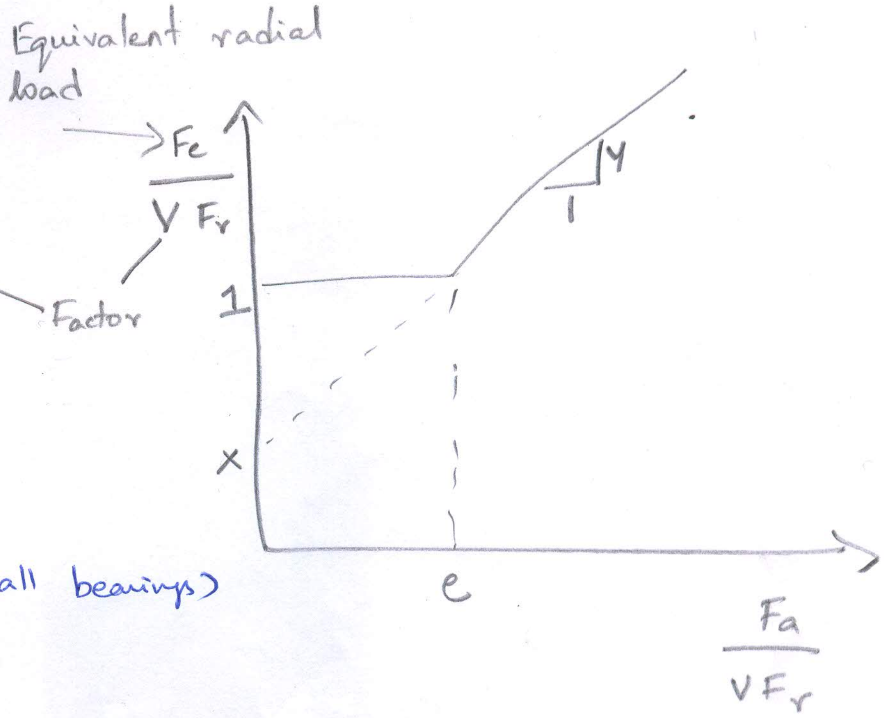
5.4.17

Combined radial and thrust loading

V = 1 if inner raceway rotates

and

V = 1.2 if outer raceway rotates (for ball bearings)



$$F_e = VF_r \quad \text{if} \quad F_a \le e\,(VF_r)$$

$$\frac{F_e}{VF_r} = X + Y\frac{F_a}{VF_r} \quad \text{if} \quad \frac{F_a}{VF_r} > e$$

Example 11-4

SKF 6210 angular contact ball bearing has an axial load F_a of 1780 N and a radial load F_r of 2225 N with outraceway sationary. Static load rating $C_0 = 19800$ N

&

$C_{10} = 35150$ N

Estimate L_{10} life at 720 rev/min.

Sol)

Here $V = 1$, $F_a / C_o \simeq 0.09$

$e \simeq 0.285$ (from table 11-1)

$$\frac{F_a}{V F_r} = \frac{1780}{2225} = 0.8 > e$$

$\Rightarrow X_2 = 0.56, \; Y_2 = 1.527$

Thus $F_e = F_r \left(0.56 + 1.527 \times \frac{1780}{2225} \right)$

$= 2225 \, (1.7816) = 3964 \text{ N}$

$$\Rightarrow L_{10} = \left(\frac{35150}{3964} \right)^3 C_{10} \simeq 698 \times 10^6 \text{ rev}$$

$$\Rightarrow \mathcal{L}_{10} = \frac{698 \times 10^6}{720 \times 60} \text{ hr} = 16157 \text{ hr}$$

Variable loading

$$F_e = \left[\sum_i \frac{F_{ei}^{a} l_i}{\sum_i l_i} \right]^{1/a}$$

Example 11-7

Conjugate action ⟶ Const. angular velocity ratio

March 27, 2017

Lecture - 37

Gears

1. Types of gears —
   (1) Spur gears — teeth parallel to axis of rotation (transmit motion between parallel shafts)
   (2) Helical gears — teeth inclined to axis of rotation (transmit motion betn parallel & nonparallel shafts, less noisy)
   (3) Bevel gears — teeth formed on a conical surface. (transmit motion between intersecting shafts)
   (4) Worm and worm gears — used for large speed ratio.

Nomenclature : Spur gears : (show picture from projector 13.5

Pitch circle : Theoretical circle
Pitch circles of a pair of mating gears are parallel to each other.
— small mating gear — pinion
— larger ——— — gear.

Circular pitch — (p) 'distance between a point on one tooth to a corresponding point on an adjacent tooth along pitch circle

$$\boxed{p = (\text{tooth thickness} + \text{width of space})}$$

Module : $m = \frac{\text{pitch circle dia}}{\text{No. of teeth}} = \frac{d}{n} \Rightarrow \boxed{p = \frac{\pi d}{N} = \pi m}$

Diametral pitch : $P = \frac{N}{D} = \frac{1}{m}$ (no. of teeth / unit dia)

Addendum : a = radial distance betn topland and pitch circle

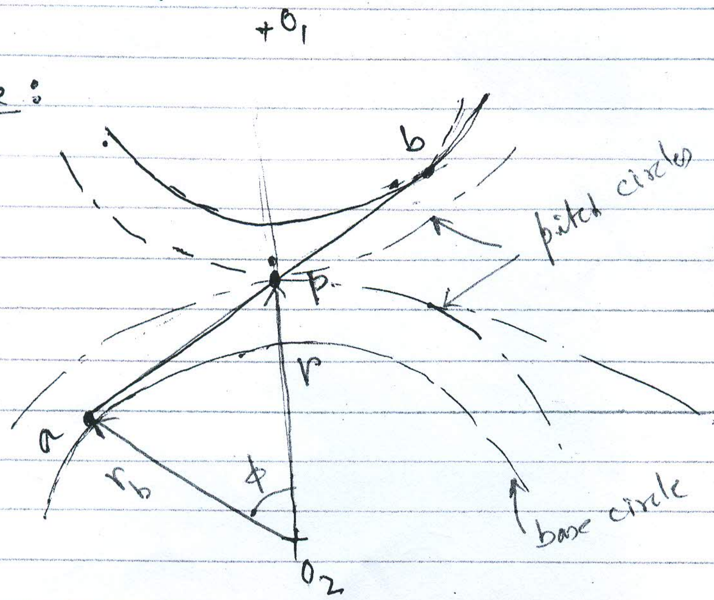
Dedendum : b = radial distance between bottom land and pitch circle

Clearance circle : Circle tangent to the addendum circle of mating gear.

Clearance $\boxed{c = b - a}$

Back lash : Difference betn width of tooth space and thickness of the engaging tooth on mating gear. on pitch circle

Pressure angle :



- ab : contact line. (locus of the contact point betn two mating teeth.)
- ab is always normal to the tooth profile engaged
- ϕ := pressure angle
  Normal force acts along this line.

r := pitch circle radius.

$\Rightarrow$ Base circle radius $\boxed{r_b = r\cos\phi}$

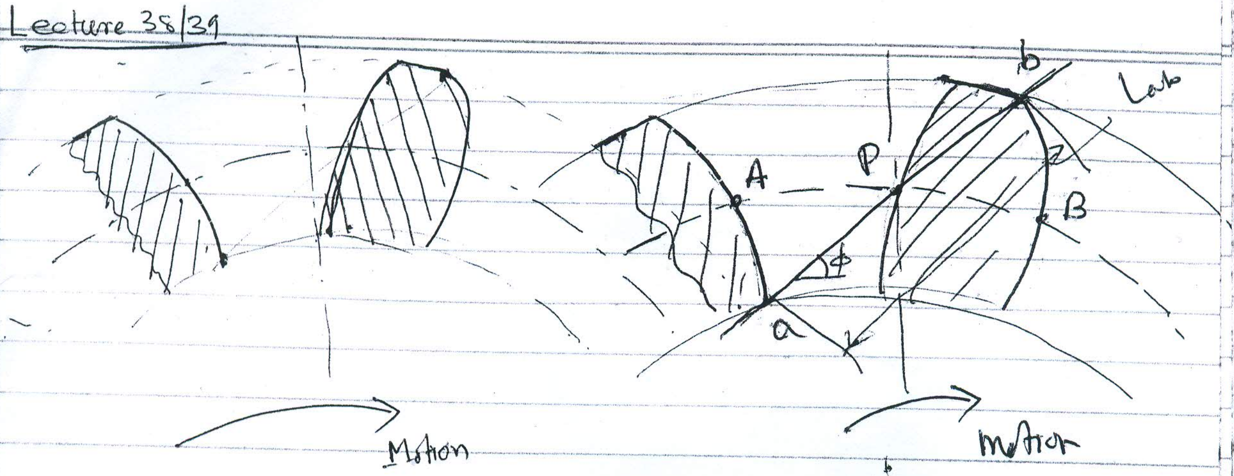
Base pitch $\boxed{p_b = p\cos\phi}$

a := point of first contact between two mating teeth

b := final contact point :

March 29, 2017

Lecture 38/39



a := first contact when teeth of meshing gears engage
b := final contact
q_a = length of ~~approach~~ AP = arc of approach
q_b = ——————— PB = ——— recess } (along pitch circle)
$q_t = q_a + q_b$ = arc of action

If $q_t = p$ (circular pitch)
at any point one tooth in action

If $q_t > p$: a new pair gets into contact before a already engaged pair reach final contact point

Contact ratio:

$$m_c = q_t/p = \frac{L_{ab}}{p\cos\phi} = \frac{L_{ab}}{p_b}$$

Recommended:

$m_c \geq 1.2$ to avoid mounting inaccuracies

Interference: Happens due to contact bet^n non-conjugate portion of teeth.

eg. contact occurs in the clearance region.
$\Rightarrow$ Causes removal of flank.

To avoid ~~undercut~~ interference, undercutting is done (involute profile below base circle).

$\Rightarrow$ Makes tooth weaker.

N_p = no. of teeth on pinion
N_G = ______ gear
$m_G = N_G/N_p$ = gear ratio

min. no. of teeth:

$$N_p \geq \frac{2k}{(1+2m_G)\sin^2\phi}\left(m_G + \sqrt{m_G^2 + (1+2m_G)\sin^2\phi}\right)$$

$k = 1$ for full depth teeth
$k = 0.8$ ______ stub teeth (shorter height)

See table 13-1 ~~for~~ 13-4 for tooth systems.
— values for pressure angle
" addendum etc.

Largest gear with a specified pinion that is interference free is

$$N_G = \frac{N_p^2 \sin^2\phi - 4k^2}{4k - 2N_p \sin^2\phi}$$

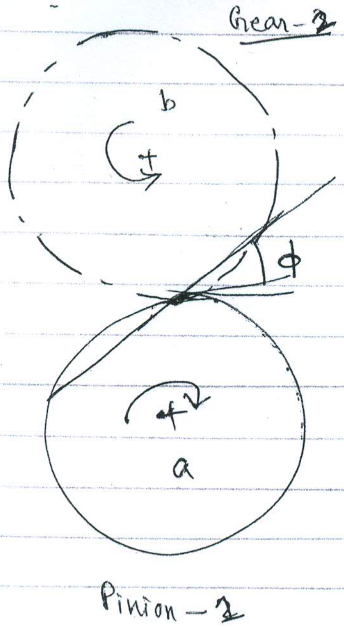
Smallest spur pinion that will operate with a rack without interference is

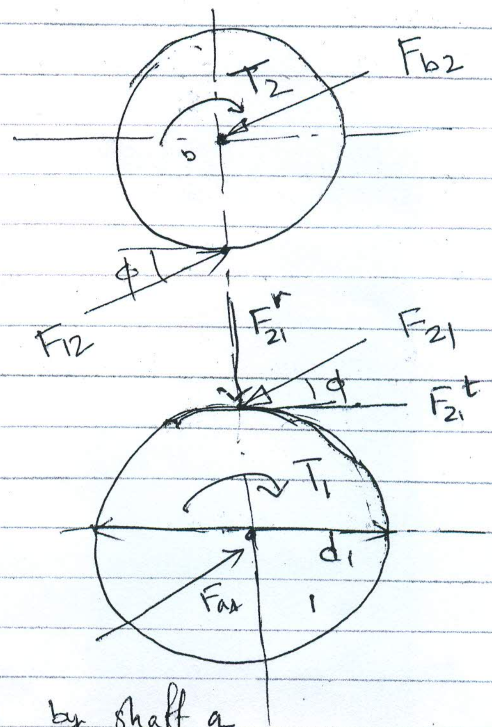
$$N_p = \frac{2k}{\sin^2\phi}$$

$m = \frac{D}{N}$

$m_G = \frac{N_G}{N_P} = \frac{N_G}{D_G} \times \frac{D_P}{N_P} \times \frac{D_G}{D_P}$

Force Analysis – Spur gear :





F_{a1} = force on gear 1 by shaft a

F_{b2} = ——— 2 ——— b

F_{12} = ——— by gear 1

Transmitted load: $W_t = F_{21}^t = F_{21}\cos\phi$

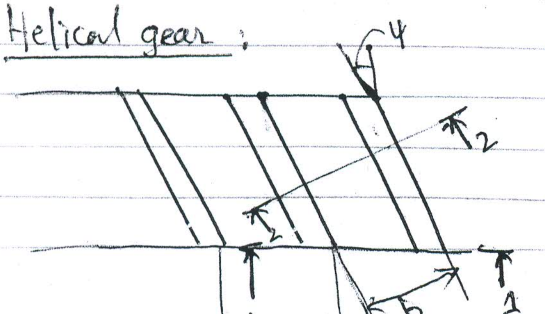
Applied Torque: $T_1 = \frac{W_t d_1}{2}$ or $T = \frac{W_t d}{2}$

Power transmitted: $H = T\omega = \frac{W_t d\omega}{2}$

Pitch line velocity: $V = \frac{d}{2}\omega = \frac{\pi d n}{2}$

$\rightarrow \quad W_t = \frac{H}{V} = \frac{60000H}{\pi d n}$ $\begin{pmatrix} H \text{ in } KW \\ d \text{ in } mm \\ n \text{ in } rpm \\ W_t \text{ in } KN \end{pmatrix}$

Helical gear :



ψ: Helix angle

ψ = Helix angle.

Normal pitch $p_n = p_t \cos\psi$ transverse pitch.

W = total force

$W_t = W \cos\phi_n \cos\psi$, $W_a = W \cos\phi_n \sin\psi$

$W_r = W \sin\phi_n$

ϕ_n = normal pressure angle

ϕ_t = transverse ———

(show picture 13-37)

Some relations :

$$P_n = P_t \cos\psi$$

$$P_x = \frac{P_t}{\tan\psi}$$

Normal diametrical pitch

$$P_n = \frac{P_t}{\cos\psi}$$

$$\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}$$

For a given gear ratio m_G, smallest tooth count is . . .

$$N_P = \frac{2k\cos\psi}{(1+2m)\sin^2\phi_t}\left[m + \sqrt{m^2 + (1+2m)\sin^2\phi_t}\right]$$

Smallest pinion to run on a rack $\Rightarrow$

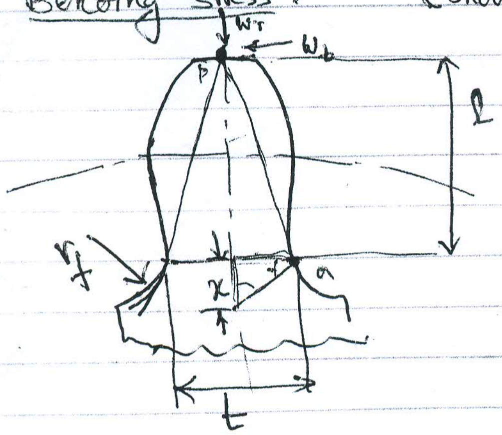
$$N_P = \frac{2k\cos\psi}{\sin^2\phi_t}$$

Largest gear with specified pinion is

$$N_G = \frac{N_P^2\sin^2\phi_t - 4k^2\cos^2\psi}{4k\cos\psi - 2N_P\sin^2\phi_t}$$

Design of Spur gear.

Bending stress: (show video)



a := point of max. bending stress (at root fillet)

t = tooth thickness at a

l := depth of a

Lewis Bending eqn:

Considers a cantilever: ~~of thickness~~

as length $= l$

thickness $= t$

width $= b$ (face width)

$$\sigma_{bending} = \frac{Mc}{I} = \frac{M}{I/c}$$

Section modulus: $$I/c = \frac{\frac{1}{12} b t^3}{t/2} = \frac{bt^2}{6}$$

$$M = W_t \cdot l$$

$$\Rightarrow \sigma_{bending} = \frac{6 W_t}{b} \cdot \left(\frac{l}{t^2}\right)$$

Also: $$\frac{x}{t/2} = \frac{t/2}{l} \Rightarrow \left(\frac{t^2}{l} = 4x\right)$$

$$\Rightarrow \sigma_{bending} = \frac{3W_t}{b} \cdot \frac{1}{2x} = \frac{W_t}{pb} \cdot \left(\frac{3p}{2x}\right)$$

$$\Rightarrow \boxed{\sigma_{bending} = \frac{W_t}{bpy}}$$

$y := \frac{2x}{3p}$ = Lewis form factor, obtained from tooth profile.

Note: Not considered

1. Compression due to radial load
2. Dynamic effects.
3. Non uniformity of load distribution due to size and
4. Stress concentration at root fillet.
5. Rim as elastic support not rigid.
6. Overloading

Lecture-40 :   March 30, 2012.

AGMA stress equation for bending :

$$\sigma = K_o K_v K_s \frac{W_t}{b\, m_t} \frac{K_H K_B}{Y_J}$$

face width → b ; ↑ transverse module / module

K_o = overload factor
K_v = Dynamic factor (Eq. 14-27, 14-28).
K_s = size factor (sec. 14-10)
K_H = load distribution factor (Section 14-11)
K_B = rim thickness factor (sec. 14-16)
Y_J = geometry factor for bending. Similar to Lewis form factor. Includes stress concentration at root fillet. (Fig 14-6 for spurgear, 14-7, 14-8 for helical gear)

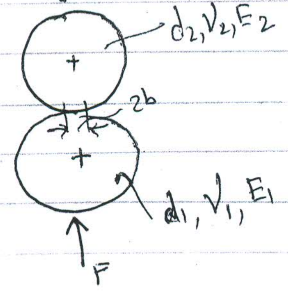
Generally: $b = 3p$ to $5p$.

Failure at the surface :

- wear
- pitting
- scoring (related to lubrication)
- abrasion,

⇒ Failure will be due to contact stress at the contact point.

Hertzian Theory of Two cylinders pressing against each other :



(show Fig 3-38)

Maximum surface pressure:

$$p_{max} = \frac{2F}{\pi b l} = \sqrt{\frac{2F}{\pi l} \cdot \left\{ \frac{\frac{1}{d_1} + \frac{1}{d_2}}{(1-\nu_1^2)/E_1 + \frac{1-\nu_2^2}{E_2}} \right\}}$$

l = length of cyl.

b = width of contact area.

$$= \left\{ \frac{2F}{\pi l} \left[\frac{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{\frac{1}{d_1} + \frac{1}{d_2}} \right] \right\}^{1/2}$$

For gears: Normal load on tooth: $W = W_t/\cos\phi$

p_{max} = max. contact stress = $-\sigma_c$.

radius of curvature ~~at~~ of tooth profiles at pitch line:

$$r_1 = \frac{d_P \sin\phi}{2} \Rightarrow d_1 = d_P \sin\phi$$

$$r_2 = \frac{d_G \sin\phi}{2} \Rightarrow d_2 = d_G \sin\phi$$

$l \rightarrow$ face width

$$\cancel{\sigma_c = \frac{2W_t/\cos\phi}{\pi b}}$$

$$\sigma_c = -\sqrt{\frac{W_t}{\pi b \cos\phi}} \cdot \left\{ \frac{\frac{1}{r_1} + \frac{1}{r_2}}{(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2} \right\}^{1/2}$$

$$= -C_p \cdot \left[\frac{W_t}{b\cos\phi} \left(\frac{1}{r_1} + \frac{1}{r_2} \right) \right]^{1/2}$$

where C_p = elastic coefficient

$$= \left[\frac{1}{\pi\left(\frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} \right)} \right]^{1/2}$$

Again, AGMA stress equation for pitting (contact stress)

$$\sigma_c = Z_E \cdot \left\{ W_t \cdot K_o K_v K_s \frac{K_H \cdot Z_R}{d_{w1}\, b\, Z_I} \right\}^{1/2}$$

Z_E = Elastic Coefficient (same as C_p) — Table 14-8 or Eq for C_p

Z_R = surface condition factor (table 5)

Z_I = geometry factor for pitting resistance. (Eq - 14-23)

AGMA strength equations (for allowable stress)

Allowable bending stress:

$$\sigma_{all} = \frac{S_t}{S_F} \cdot \frac{Y_N}{Y_\theta Y_Z}$$

based on 10^7 cycles → S_t := strength (Table 14-2 to 14-4)
Figures 14-2 - 14-4

S_F = AGMA factor of safety (See 14-17)

(Fig 14-14) Y_N = stress cycle factor — See 14-13

Y_θ = temperature factor See - 14-15

Y_Z = reliability factor See - 14-19

Allowable contact stress:

$$\sigma_{c,all} = \frac{S_c}{S_H} \cdot \frac{Z_N Z_W}{Y_\theta Y_Z}$$

based on 10^7 cycles → S_c = Contact strength (Depends on hardness)
Fig 14-5, Table 14-6.

S_H = AGMA factor of safety

(Fig 14-15) Z_N = stress cycle life factor — see 14-13

Z_W = hardness ratio factor for pitting resistance
(To adjust surface strength of gear See - 14-12.
wrt. pinion — pinion undergoes more cycles of contact stress).

April-3, 2017.

Lecture / Tutorial — 41

Solve problems 13.(10–14)

13.10 : Spur gear set : pressure angle $\phi = 20°$

(a) Smallest pinion tooth count – run with it self.

$$N_p \geq \frac{2K}{(1+2m_G)\sin^2\phi}\left(m_G + \sqrt{m_G^2 + (1+2m_G)\sin^2\phi}\right)$$

$K = 1$, (full depth tooth)

$m_G = 1$, $\phi = 20°$

$$\Rightarrow N_p \geq \frac{2K}{3\sin^2\phi}\left(1 + \sqrt{1 + 3\sin^2\phi}\right) = 12.3$$

$$\Rightarrow \quad N_p = 13 \text{ teeth.}$$

(b) When the gear ratio is $m_G = 2.5$

$$\Rightarrow N_p \geq \frac{2}{(1+5)\sin^2\phi}\left(2.5 + \sqrt{6.25 + 6\sin^2\phi}\right) = 14.64$$

$$\Rightarrow \quad N_p = 15 \text{ teeth.}$$

Largest gear tooth count possible:

$$N_G \leq \frac{N_p^2 \sin^2\phi - 4K^2}{4K - 2N_p\sin^2\phi} = \frac{15^2\sin^2 20 - 4}{4 - 2\times 15\sin^2 20} = 45.49$$

$$\Rightarrow \boxed{N_G = 45}$$

$\Rightarrow$ Maximum possible gear ratio with

m_G, $N_p = 15$, $\phi = 20°$: $\frac{N_{G\,max}}{N_{p\,min}} = \frac{45}{15} = 3.$

(c) Smallest pinion that will mesh with a rack :

$m_G \approx \infty$

$$N_p \geq \frac{2K}{\sin^2\phi} = 17.1$$

$$\Rightarrow \quad N_{p\,min} = 18 \text{ teeth.}$$

13.11: Helical gear: $\phi_n = 20°$, $\psi = 30°$ (helix angle)

ϕ_n normal pressure angle

For Helical gears:

$$N_P \geq \frac{2k\cos\psi}{(1+2m_G)\sin^2\phi_t} \cdot \left[m_G + \sqrt{m_G^2 + (1+2m_G)\sin^2\phi_t}\right]$$

$$N_G \leq \frac{N_P^2 \sin^2\phi_t - 4k^2\cos^2\psi}{4k\cos\psi - 2N_P\sin^2\phi_t}$$

For rack: $N_P \leq \frac{2k\cos\psi}{\sin^2\phi_t}$

(a) $m_G = 1$, $k = 1$, $\tan\phi_t = \frac{\tan\phi_n}{\cos\psi}$ $\left(\cos\psi = \frac{\tan\phi_n}{\tan\phi_t}\right)$

$\Rightarrow \phi_t = 22.80°$

$$\Rightarrow N_P \geq \frac{2k\cos 30°}{(1+2)\sin^2 22.8}\left[1 + \sqrt{1+3\sin^2 22.8}\right] = 8.48$$

$\Rightarrow N_P|_{min} = 9$ teeth

(b) $m_G = 2.5$

$\Rightarrow N_P \geq 9.95$

$\Rightarrow N_P|_{min} = 10$ teeth

$$N_G \leq \frac{10^2\sin^2\phi_t - 4\cos^2 30}{4\cos 30 - 2\times 10\sin^2 22.8} = \frac{12.018}{0.461} = 26.1$$

$\Rightarrow N_G|_{max} = 26$ teeth

(Max. possible gear ratio with 10 teeth pinion and $\phi_n = 20°$, $\psi = 30°$ is 2.6).

(c) $N_P \geq \frac{2k\cos\psi}{\sin^2\phi_t} = 11.53$

$\Rightarrow N_P = 12$ teeth

13:12: $\phi_n = 20°$; $\psi = 30°$, $m_G = 2$.

$\Rightarrow \phi_t = 22.80°$.

$\Rightarrow N_p \geq \dfrac{2 \cdot \cos 30°}{5 \sin^2 22.8°}\left(2 + 5 \sin^2 22.8°\right)$

$= 9.65$

$\Rightarrow N_p|_{min} = 10$ teeth

$N_G|_{max} = 26.01 \Rightarrow N_G = 25$ teeth

$\Rightarrow$ Can use: 10:20 (9:18 not allowed as $N_p \geq 10$)

11:22

12:24

(13:26) Not allowed as $N_G \leq 25.74$

13:13: Choose $\psi = 45°$, $m_G = 2$, $\phi_n = 20°$

$\Rightarrow N_p \geq 5.74$

$\Rightarrow$ ~~$N_G = 6$~~ $N_p = 6$

$N_G \leq 17.2 \Rightarrow N_G|_{max} = 17$

Possibilities: 6:12, 7:14, 8:16

13:14: $N_p = \dfrac{2k \cos\psi}{\sin^2\phi_t}$

$N_p \geq \dfrac{2k}{\sin^2\phi_t} \Rightarrow \sin^2\phi_t \geq \dfrac{2k}{9} = \dfrac{2}{9}$

$\Rightarrow \sin\phi_t \geq \dfrac{\sqrt{2}}{9} \Rightarrow \phi_t \geq 28.13°$.

Lecture 42/43 April-5, 2017

Solved problem from tutorial 7

Given: Gear ratio $m_G = \frac{N_G}{N_P} = 4$

Pressure angle $\phi = 20°$

(a) To avoid interference:

$$N_P \geq \frac{2k}{(1+2m_G)\sin^2\phi}\left(m_G + \sqrt{m_G^2 + (2m_G+1)\sin^2\phi}\right)$$

$$= 15.46$$

$$\Rightarrow \boxed{N_P = 16, \quad N_G = 4 \times 16 = 64}$$

Module:

Center to center distance:

$$C = \frac{(N_P + N_G)}{2} m = 200 \text{ mm}$$

$$\Rightarrow \boxed{m = 5 \text{ mm}}$$

Diameters: $\boxed{\begin{array}{l} d_P = N_P \cdot m = 80 \text{ mm} \\ d_G = N_G \cdot m = 320 \text{ mm} \end{array}}$

(b) Power rating $H = 25$ kW, pinion speed $n = 2000$ rpm

$\Rightarrow$ Transmitted load: $W_t = \frac{60000 H}{\pi d n}$

$$= \frac{60{,}000 \times 25}{\pi \times 80 \times 2000} = 2.984 \text{ kN}$$

$$\Rightarrow \boxed{W_t = 2.984 \text{ kN}}$$, Pitch line vel. $V = 8.378$ m/s

Factors for bending stress and strength calculation

Over load factor: $K_o = 2.0$ (L.S. Power, H.H. driven) (Fig 14-17)

Surface factor: $K_s = 1$

Dynamic factor: (Eq 14-27 & 28)

$$K_v = \left(\frac{A+200\sqrt{V}}{A}\right)^B, \quad A = 50+56(1-B)$$
$$B = 0.25(12-Q_v)^{2/3}$$ ← quality no.

$Q_v = 10$

$\Rightarrow \boxed{B = 0.3969 \quad A = 83.776}$

$\Rightarrow \boxed{K_v = 1.171}$

Load distribution factor : $b = 50\,mm$

Accurate mounting : $K_H = 1.3$ (table)

Rim thickness factor : Rim thickness to tooth height ratio $\doteq 1.5 > 1.2$

$\Rightarrow K_B = 1$ (Fig 14-16)

Geometry factor :

$(Y_J)_p = 0.27$ (pinion – 16 teeth)

$(Y_J)_G = 0.41$ (gear – 64 teeth)

(Fig 14-6)

Stress cycle factor :

$H_B = 250$, Desired pinion life $= 10^8$ cycles $= N_p$

$H_B = 200$, $N_G = \frac{10^8}{4} = 2.5 \times 10^7$ cycles

$\Rightarrow Y_N = 1.3558 \times N^{-0.0178}$

$\Rightarrow \boxed{(Y_N)_p = 0.977 \text{ (pinion, } N = 10^8) \quad (Y_N)_G = 1.009 \text{ (gear, } N = 2.5\times10^7 \text{ cycle)}}$ (Fig. 14-14)

Temperature factor : $\boxed{Y_\theta = 1}$

Reliability factor : $Y_Z = 1$ (Reliability $= 0.99$, Table 14-10)

Bending strength: $S_t = 0.7255\, H_B + 153.63$

Nitrided, 2.5% Chrome steel, Grade 2, HB $\frac{250}{200}$ — P / G

$(S_t)_P = 335\,\text{MPa}$ (pinion HB = 250)
$(S_t)_G = 298.73\,\text{MPa}$ (Gear HB = 200)
(Fig - 14 - 4)

Bending failure:

(1) Pinion: $\sigma = \frac{W_t}{bm} K_o K_v K_s \frac{K_H K_B}{Y_J}$

$= \frac{2984}{50\times5} \times 2\times1.171\times1\times\frac{1.3\times1}{0.27}$

$(\sigma)_P = 134.6\,\text{MPa}$

Allowable stress: $\sigma_{all} = \frac{S_t \cdot Y_N}{S_F\, Y_\theta\, Y_Z}$

$(\sigma_{all})_P = \frac{335\times0.977}{S_F} = \frac{327.3}{S_F}$

Factor of safety: $\boxed{(S_F)_P = \frac{327.3}{134.6} = 2.43}$

2. Gear:

$(\sigma)_G = \frac{W_t}{bm}\times 2\times1.171\times\frac{1.3\times1}{0.41} = 88.6\,\text{MPa}$

$(\sigma_{all})_G = \frac{298.73\times1.009}{S_F} = \frac{299}{S_F}$

$\Rightarrow$ Factor of Safety: $\boxed{(S_F)_G = 3.373}$

Factors for pitting resistance and strength:

Elasticity coefficient : $Z_E = 191\ \sqrt{MPa}$ (table 14-8)

Surface condition factor: $Z_R = 1$

Geometry factor: $Z_I = \frac{\cos\phi_t \sin\phi_t}{2 m_N} \cdot \frac{m_G}{m_G+1}$ (Eq 14-23)

$\phi_t = \phi = 20°$, $m_N = 1$ (spur gear)

$m_G = 4$

$\Rightarrow \boxed{Z_I = 0.1285}$

Stress cycle factor: $Z_N = 1.4488\ N^{-0.023}$ (Fig-14-15)

$$\Rightarrow \begin{aligned} (Z_N)_P &= 0.948 \quad \text{(pinion)} \\ (Z_N)_G &= 0.979 \quad \text{(gear)} \end{aligned}$$

Hardness ratio factor:

$$1.7 \geq \frac{H_{BP}}{H_{BG}} = \frac{250}{200} = 1.25 \geq 1.2$$

$$\Rightarrow \begin{aligned} (Z_W)_P &= 1 \quad \text{(pinion)} \\ (Z_W)_G &= 1.009 \quad \text{(gear)} \end{aligned} \quad \text{(fig-14-12)}$$

Contact strength:

$S_c = 1350$ MPa for both gear & pinion

Pitting failure:

Pinion
$$(\sigma_c)_P = Z_E \cdot \left\{ \frac{W_t}{d_p b} \cdot K_o K_v K_s \cdot \frac{K_H Z_R}{Z_I} \right\}$$

$$= 191 \left\{ \frac{2984}{80 \times 50} \times 2 \times 1.171 \times 1 \times \frac{1.3 \times 1}{0.1285} \right\}^{1/2}$$

$$= 803.\ \text{MPa}$$

Allowable contact stress:

$$(\sigma_{c,all})_P = \frac{S_c}{S_H} \cdot \frac{Z_N Z_W}{Y_\theta Y_Z} = \frac{1350 \times 0.948 \times 1}{S_H}$$

$$= \frac{1279.8}{S_H}\ \text{MPa}$$

Factor of safety

$\Rightarrow (S_H)_p = \frac{1279.8}{803} = 1.594$

Gear:

$(\sigma_c)_G = 803 \text{ MPa}$

Allowable contact stress: $(\sigma_{c,all})_G = \frac{S_c \times 0.979 \times 1.009}{S_H}$

$= \frac{1333.54}{S_H} \text{ MPa}$

$\Rightarrow$ Factor of safety:

$(S_H)_G = \frac{1333.54}{803} = 1.66$

(c) Likely failure mode:

Pinion: $(S_F)_p = 2.43, \quad (S_H)_p^2 = (1.594)^2 = 2.54$

Gear: $(S_F)_G = 3.373, \quad (S_H)_G^2 = (1.66)^2 = 2.756$

$\Rightarrow$ Most threat to failure is from bending of pinion tooth.

(d) Power to failure

$(\sigma_{all})_p = (\sigma)_p$ for pinion chose $(S_F)_p = 1$

$\Rightarrow \sigma_p = 327.3 \text{ MPa}$

$\Rightarrow W_t = \frac{(\sigma)_p \times b \times m \times Y_J}{K_o K_v K_s K_H K_B} = \frac{327 \times 80 \times 5 \times 0.27}{2 \times 1.171 \times 1.3 \times 1}$

$= 7.25 \text{ kN}$ at failure

Also: $W_T = \left(\frac{\sigma_{all}}{\sigma}\right)_p \times W_{t\,old} = \frac{327.3}{134.6} \times 2.984 \text{ kN}$

$= 7.25 \text{ kN}$

$\Rightarrow H = 60.7$ KN at failure

$\Rightarrow$ This is 143 % overload.