Optimization 2020 - Fifth compulsory assignment

The weighted MAXCUT problem is the following optimization problem. Given an undirected graph $G = (V, E)$ with non-negative edge weights, w_{ij} for every $ij \in E$, find a partition $V = S \cup T$, $S \cap T = \emptyset$ of the vertices, such that the weight $w(S, T)$ of the cut is maximized.

Here, for sets $A, B \subseteq V$ we have used the shorthand notation

$$w(A, B) = \sum_{\substack{ij \in E \\ i \in A, j \in B}} w_{ij}$$

Consider the following algorithm due to Johnson:

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1: S := ∅, T := ∅.
2: for v ∈ V do
3:     if w({v}, S) > w({v}, T) then
4:         T := T ∪ {v}
5:     else
6:         S := S ∪ {v}
7:     end if
8: end for
9: return (S, T)
```

Figure 1: JohnsonCut.

Show that JohnsonCut is an approximation algorithm for MAXCUT with an approximation ratio of 2. What can you say about the running time of the algorithm?