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Proposition 1: If π_1 and π_2 are good representations that are polynomially equivalent, then $L_1 \in P \Leftrightarrow L_2 \in P$.

Proof

Given an instance π_1 in L_1 , we have an algorithm A that can translate it into an instance in L_2 in polynomial time. This means there is a function r which evaluates an instance π_1 in polynomial time. E.g $r : \{0, 1\} * \rightarrow \{0, 1\} *$

If we know that $L_1 \in P$ and there is an algorithm A that can translate it into an instance in L_2 in polynomial time, then we can show that $L_2 \in P$ by mapping instances of L_1 to L_2 . Thus we have

$\exists r(\pi_1 \in L_1 \wedge \pi_2 \in L_2 \wedge r(L_1) = L_2) \Rightarrow r(\pi_1) = \pi_2$

The proof is symmetric, so the reverse implication holds as well.