

Optimization 2020 - Second compulsory assignment

In the game of soccer the original scoring rule of a game was that a team would receive two points for a win, one point for a draw, and zero points for a loss. We will refer to this rule as the “two-point rule”. The current scoring rule, which we will refer to as the “three-point rule”, was introduced gradually over the years in many countries, starting with England in 1981, and formally adopted by FIFA in 1995. The difference to the two-point rule is that a team now receives three points for a win. A draw and a loss is still awarded one and zero points, respectively.

During the tournament season, a popular pastime amongst sports journalist is to determine whether a given team may still become champions. Suppose we have a tournament with N teams, and we are interested in whether it is possible for team 1 to win the tournament. We are given a list of the current number of points p_1, \dots, p_N of each team and a list of remaining matches of the tournament $(a_1, b_1), \dots, (a_m, b_m)$. Here a pair (a, b) represents that team a must play team b . The winner of the tournament will be the team with the most points. Note that, without loss of generality we may assume that team 1 wins all its remaining matches, so we assume these have already been removed from the list of remaining matches.

Possible tie-breaking is handled by a long list of criteria which we will not consider here, but just note that scoring enough goals will make a team win. In other words we say that team 1 can win the tournament if after all matches have been played, team 1 has the maximum number of points of any team.

1. Show that the question of whether team 1 can win, *using the two-point rule*, can be formulated as a maximum flow problem.
2. Explain why your maximum flow formulation can not easily be adapted to work using the three-point rule.
(Actually, no worst-case efficient algorithm is known for solving the problem using the three-point rule, thus making the life of sports journalists harder).
3. Formulate the question of whether team 1 can win, *using the three-point rule*, as an integer linear programming problem.