CSCE 421
HW 1
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1.1. The gradient of a multi-variate function is a vector of the respective partials of the function. Our given function is shown below:

$$f(x, y) = x^2 + ln(x) + xy + y^3$$

The gradient vector is as shown below:

$$\begin{bmatrix} f_x \\ f_y \end{bmatrix}$$

The gradient values are shown below:

$$f_x = 2x + x^{-1} + xy + y$$

$$f_y = x + 3y^2$$

$$\begin{bmatrix} 2x + x^{-1} + xy + y \\ x + 3y^2 \end{bmatrix}$$

The gradient value for $(1, -1)$ can be computed by plugging those values into the gradient.

$$\bigtriangledown f(1, -1) = \begin{bmatrix} 2(1) + 1^{-1} + 1(-1) + (-1) \\ 1 + 3(-1)^2 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

1.2. Given the definition of the gradient function, we need to calculate the gradient for the given function below:

$$f(x, y, z) = tanh(x^3y^3) + sin(z)$$

The partials of the function are shown below:

$$f_x = 3y^3x^2sech^2(x^3y^3)$$

$$f_y = 3x^3y^2sech^2(x^3y^3)$$

$$f_z = cos(z)$$

$$\triangledown f = \begin{bmatrix} 3y^3x^2sech^2(x^3y^3) \\ 3x^3y^2sech^2(x^3y^3) \\ cos(z) \end{bmatrix}$$

The gradient value at $(-1, 0, \pi/2)$ is shown below:

$$\triangledown f(-1, 0, \pi/2) = \begin{bmatrix} (-1)^2(0)^3sech^2((-1)^3(0)^3) \\ (-1)^2(0)^3sech^2((-1)^3(0)^3) \\ cos(\pi/2) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

2.1. The following matrix multiplication computation is shown below:

$$\begin{bmatrix} 1 & -1 & 6 & 7 \\ 9 & 0 & 8 & 1 \\ -8 & 5 & 2 & 3 \\ 10 & 4 & 0 & 1 \end{bmatrix} \begin{bmatrix} 6 & 2 \\ 0 & -1 \\ -3 & 0 \\ 11 & 4 \end{bmatrix} =$$

$$\begin{bmatrix} 1(6) - 1(0) - 3(6) + 7(11) & 1(2) - 1(-1) + 6(0) + 7(4) \\ 6(9) + 0(0) - 3(8) + 11 & 9(2) - 1(0) + 8(0) + 4 \\ -8(6) + 5(0) - 3(2) + 3(11) & -8(2) - 5 + 2(0) + 3(4) \\ 10(6) + 4(0) - 3(0) + 11 & 2(10) - 4 + 4 \end{bmatrix} = \begin{bmatrix} 65 & 31 \\ 41 & 22 \\ -21 & -9 \\ 71 & 20 \end{bmatrix}$$

2.2.

$$\begin{bmatrix} 10 \\ 4 \\ -1 \\ 8 \end{bmatrix} \begin{bmatrix} 7 & 3 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 10(7) & 3(10) & 0(10) & 1(10) \\ 4(7) & 3(4) & 0(4) & 1(4) \\ -7 & -3 & 0 & -1 \\ 8(7) & 3(8) & 0(8) & 1(8) \end{bmatrix} \begin{bmatrix} 70 & 30 & 0 & 10 \\ 28 & 12 & 0 & 4 \\ -7 & -3 & 0 & -1 \\ 56 & 24 & 0 & 8 \end{bmatrix}$$

2.3

$$\begin{bmatrix} 9 & -3 & 1 & 6 \end{bmatrix} \begin{bmatrix} -3 \\ 4 \\ -9 \\ 0 \end{bmatrix} = 9(-3) - 3(4) + 1(-9) + 6(0) = -48$$

3.1. We want to calculate the l_0 norm for each vector given. The two vectors are shown below:

$$\mathbf{a} = \begin{bmatrix} 5 \\ 0 \\ -1 \\ 4 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 7 \\ 9 \\ 5 \\ 2 \end{bmatrix}$$

The l_0 norm counts the number of non-zero entries in the vector. For **a**, we can see $l_0 = 3$ and for **b**, $l_0 = 4$.

3.2. The l_1 norm is the sum of the absolute values of each element in the vector:

$$l_1 = \sum_{i=1}^{n} |a_n|$$

For **a**, $l_1 = |5| + |0| + |-1| + |4| = 10$ and for **b**, $l_1 = |7| + |9| + |5| + |2| = 23$.