2° Lista de Princípios de Comunicação Digital

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Questão 1.

Código Sistemático (7,3)

$x^n + 1 = x^7 + 1$

$x^7 + 1 \;|\; x^4 + x^3 + x^2 + 1$ ← É uma Boa Geradora [1011]

$x^7 + x^6 + x^5 + x^3 \;|\; x^3 + x^2 + 1$ ← H(x) → [

$x^6 + x^5 + x^3 + 1$

$x^6 + x^5 + x^4 + x^2$

$x^4 + x^3 + x^2 + 1$

$x^4 + x^3 + x^2 + 1$

(0)

$$G(x) = \left[\begin{array}{ccc|cccc} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right]$$

Colocando na forma sistemática:

$$G(x) = \left[\begin{array}{ccc:cccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right]$$

$$H(x) = \left[\begin{array}{ccccccc} 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

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Questão 2 a)

$x^6 \mid x^4+x^3+x^2+1$

$x^6+x^5+x^4+x^2 \quad x^2+x$

$x^5+x^4+x^2$

$x^5+x^4+x^3+x$

$x^3+x^2+x \rightarrow 0111$

$x^5 \mid x^4+x^3+x^2+1$

$x^5+x^4+x^3+x \quad x+1$

x^4+x^3+x

$x^4+x^3+x^2+1$

$x^2+x+1 \rightarrow 1110$

$x^4 \mid x^4+x^3+x^2+1$

$x^4+x^3+x^2+1 \quad 1$

$x^3+x^2+1 \rightarrow 1011$

$x^3 \mid x^4+x^3+x^2+1$

$\rightarrow 0001$

$x^2 \mid x^4+x^3+x^2+1$

$\rightarrow 0010$

$x \mid x^4+x^3+x^2+1$

$\rightarrow 0100$

$1 \mid x^4+x^3+x^2+1$

$\rightarrow 1000$

e	S	
1000000	0111	x^6
0100000	1110	x^5
0010000	1011	x^4
0001000	0001	x^3
0000100	0010	x^2
0000010	0100	x^1
0000001	1000	1

b)

$S = x^6+x^4+x \mid x^4+x^3+x^2+1$

$x^6+x^5+x^4+x^2 \mid x^2+x+1$

x^5+x^2+x

$x^5+x^4+x^3+x$

$x^4+x^3+x^2$

$x^4+x^3+x^2+1$

(1)

$b'(x) = 1+x+x^4+x^6$

$x^6+x^4+x+1 \mid x^4+x^3+x^2+1$

$x^6+x^5+x^4+x^2 \quad x^2+x+1$

x^5+x^2+x+1

$x^5+x^4+x^3+x$

$x^4+x^3+x^2+1$

$x^4+x^3+x^2+1$

(0)

mensagem = 11

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$$\theta(x) = x^6+x^5+x^4+x+1 \;\big|\; x^4+x^3+x^2+1$$

$$\underline{x^6+x^5+x^4+x^2} \qquad x^2$$

$$x^2+x+1$$

$$x^6+x^5+x^4+1 \;\big|\; x^4+x^3+x^2+1$$

$$\underline{x^6+x^5+x^4+x^2} \qquad x^2$$

$$x^2+1$$

não é possível decodificar.

$$x^6+x^4+x+1 \;\big|\; x^4+x^3+x^2+1$$

$$\underline{x^6+x^5+x^4+x^2} \qquad x^2+x+1$$

$$x^5+x^2+x+1$$

$$\underline{x^5+x^4+x^3+x}$$

$$x^4+x^3+x^2+1$$

$$\underline{x^4+x^3+x^2+1}$$

$$(0)$$

mensagem = 111

Questão 3.

a)
$$x^7+1 \;\big|\; x^3+x^2+1$$

$$\underline{x^7+x^6+x^4} \qquad x^4+x^3+x^2+1$$

$$x^6+x^4+1$$

$$\underline{x^6+x^5+x^3}$$

$$x^5+x^4+x^3+1$$

$$\underline{x^5+x^4+x^2}$$

$$x^3+x^2+1$$

$$\underline{x^3+x^2+1}$$

$$(0)$$

b)

mensagem	m(x)	código
0001	x^3	$x^6+x^5+x^3$
0010	x^2	$x^5+x^4+x^2$
0011	x^2+x^3	$x^6+x^4+x^3+x^2$
0100	x	x^4+x^3+x
0101	$x+x^3$	$x^6+x^5+x^4+x$
0110	$x+x^2$	$x^5+x^3+x^2+x$
0111	$x+x^2+x^3$	x^6+x^2+x
1000	1	x^3+x^2+1
1001	$1+x^3$	$x^6+x^5+x^2+1$
1010	$1+x^2$	x^5+x^4+1
1011	$1+x^2+x^3$	x^6+x^4+1
1100	$1+x$	x^4+x^2+x+1
1101	$1+x+x^3$	$x^6+x^5+x^4+x^3+x^2+x+1$
1110	$1+x+x^2$	x^6+x^4+1
1111	$1+x+x^2+x^3$	x^6+x^3+x+1

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c) $x^k \cdot h(x^{-1}) = x^4 \cdot [1 + x^{-2} + x^{-3} + x^{-4}]$

$h(x) = x^4 + x^2 + x + 1$

$$H(x) = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{bmatrix}$$

d) $r = 1101101$

$r = 1 + x + x^3 + x^4 + x^6$

$x^6 + x^4 + x^3 + x + 1 \quad | \; x^3 + x^2 + 1$

$x^6 + x^5 + x^3 \qquad\qquad x^3 + x^2$

$x^5 + x^4 + x + 1$

$x^5 + x^4 + x^2$

$(x^2 + x + 1)$

$\hat{r} = x^6 + x^4 + x^3 + x^2$

$x^6 + x^4 + x^3 + x^2 \quad | \; x^3 + x^2 + 1$

$x^6 + x^5 + x^3 \qquad\qquad x^3 + x^2$

$x^5 + x^4 + x^2$

$x^5 + x^4 + x^2$

0

Portanto: $m = 0011$

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Questão 4.

$$r = x + x^2 + x^3 + x^4 + x^5$$

$$C^2 = x^2\left[x + x^2 + x^3 + x^4 + x^5\right] \bmod (x^7 + 1)$$

$$\begin{array}{l|l} x^7 + x^6 + x^5 + x^4 + x^3 & x^7 + 1 \\ \hline x^7 + 1 & 1 \\ \hline x^6 + x^5 + x^4 + x^3 + 1 & \end{array}$$

$$C^2 = 100\,1111$$

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03 | 10 | 24

5- código cíclico (7,4)

$m = 1110 \longrightarrow m(x) = 1 + x + x^2$

$g(x) = 1 + x + x^3$

↳ $x^{n-k} \cdot m(x)$

↳ $x^3 \cdot (1 + x + x^2) = x^3 + x^4 + x^5$

$x^{n-k} m(x) \mid g(x)$

$b(x)$

↳ $c(x) = x^{n-k} \cdot m(x) + b(x)$

$$\begin{array}{l|l} x^5 + x^4 + x^3 & x^3 + x + 1 \\ \pm x^5 + x^3 + x^2 & x^2 + x \\ \hline x^4 + x^2 & \\ \pm x^4 + x^2 + x & \\ \hline x \rightarrow b(x) & \end{array}$$

$\boxed{c(x) = x^5 + x^4 + x^3 + x}$

↳ $x = x^3 + x^4 + x^5 = (0101110)$

6- código cíclico (15,5) $g(x) = x^{10} + x^8 + x^5 + x^4 + x^2 + x + 1$

a) $m(x) = 1 + x^2 + x^4 \; (10101)$

↳ $c(x) = ?$

$x^{n-k} \cdot m(x) = x^{10}(x^4 + x^2 + 1)$

$= x^{14} + x^{12} + x^{10}$

$$\begin{array}{l|l} x^{14} + x^{12} + x^{10} & x^{10} + x^8 + x^5 + x^4 + x^2 + x + 1 \\ \pm x^{14} + x^{12} + x^9 + x^8 + x^6 + x^5 + x^4 & x^6 + 1 \\ \hline x^{10} + x^9 + x^8 + x^6 + x^5 + x^4 & \\ \pm x^{10} + x^8 + x^5 + x^4 + x^2 + x + 1 & \\ \hline x^9 + x^6 + x^2 + x + 1 & \end{array}$$

$c(x) = x^{14} + x^{12} + x^{10} + x^9 + x^6 + x^2 + x + 1$

↳ $1 + x + x^2 + x^6 + x^9 + x^{10} + x^{12} + x^{14}$

↳ (011000100 1 | 10101)

↳ bits de paridade ↳ mensagem

7- $h(x) = x^n + 1$ $\quad$ $x^{15} + 1 \mid x^8 + x^7 + x^6 + x^4 + 1$

$n = 15$ $\quad$ $g(x)$ $\quad$ $x^{14} + x^{13} + x^{11} + x^7 \mid x^7 + x^6 + x^4 + 1$

$x^9 + x^8 + x^{11} + x^7 + 1$

$\pm x^{14} + x^{13} + x^{12} + x^{10} + x^6$

$x^{12} + x^{11} + x^{10} + x^7 + x^6 + 1$

$\pm x^{12} + x^{11} + x^{10} + x^8 + x^4$

$x^8 + x^7 + x^6 + x^4 + 1$

$\pm x^8 + x^7 + x^6 + x^4 + 1$

(0)

↳ $h(x)$ - polinômio de verificação de paridade

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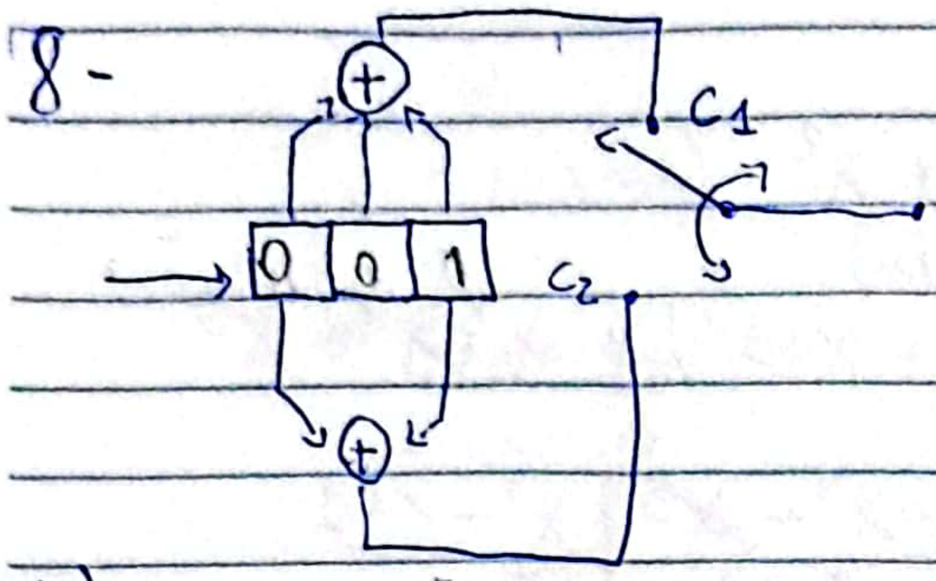
8-



a)

t	0	1	2
c_1	1	1	1
c_2	1	0	1

↳ 11 10 11

↳ resposta ao impulso

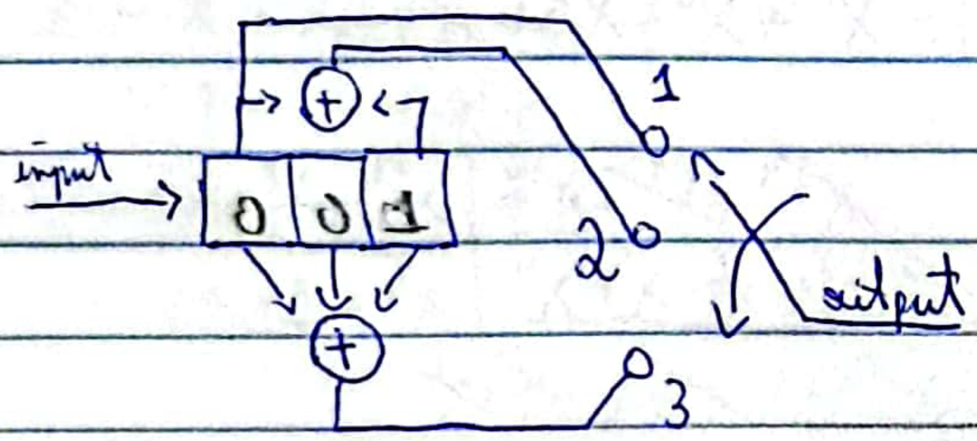
b)

m	saída
1	11 10 11
0	00 00 00
1	11 10 11

C = 11 10 00 10 11

↳ saída do decodificador da m=101

9-



t(s)	0	1	2
g_1	1	0	0
g_2	1	0	1
g_3	1	1	1

$g^{(1)}$ = 1 0 0
$g^{(2)}$ = 1 0 1
$g^{(3)}$ = 1 1 1

} sequências geradoras

G =
111	001	011	000	000
000	111	001	011	000
000	000	111	001	011

↳ matriz geradora

111 001 011

↳ resposta ao impulso

m	saída
1	111 001 011
0	000 000 000
1	111 001 011

C = 111 001 100 001 011

↳ saída do decodificador da m=101

Jandaia

Scanné avec CamScanner

10- (7,3)

$g(x) = x^4 + x^3 + x^2 + 1 \rightarrow g(D) = D^4 + D^3 + D^2 + 1$

$g_1(x) = x^3 + x^2 + 1$

$m(x) = 101 \rightarrow 1 + x^2 \rightarrow m(D) = D^2 + 1$

saída do codificador $= m(D) \cdot g(D) \rightarrow (D^2+1)(D^4 + D^3 + D^2 + 1)$

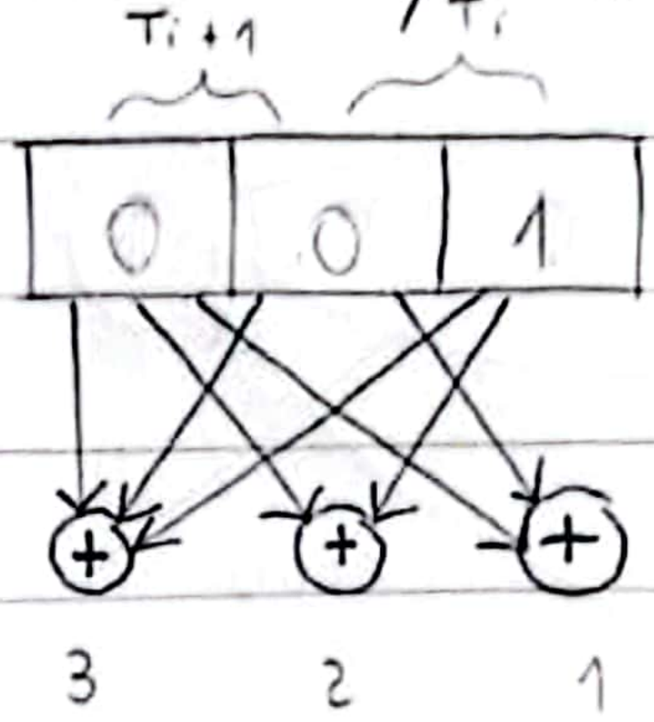
$\rightarrow D^6 + D^5 + D^4 + D^2 + D^4 + D^3 + D^2 + 1$

$\rightarrow D^6 + D^5 + D^3 + 1$

$\hookrightarrow 1 + D^3 + D^5 + D^6$

$c = 1001011$

(11) Esboce o diagrama de estados correspondente ao código convolucional abaixo representado



$2^3 = 8$

Considere a transmissão de 5 bits de informação 10110. Qual a sequência transmitida?

Deslocamento	Valores	Saída
1	1 0 0	1 1 1
2	0 1 0	1 0 1
3	0 0 1	0 1 1

Entrada	Reg	Ti	Ti+1	Saída
0	0 0 0	0 0	0 0	0 0 0
1	1 0 0	0 0	1 0	1 1 1
0	0 1 0	1 0	0 1	1 0 1
1	1 1 0	1 0	1 1	0 1 0
0	0 1 1	1 1	0 1	1 1 0
1	1 1 1	1 1	1 1	0 0 1
0	0 0 1	0 1	0 0	0 1 1
1	1 0 1	0 1	1 0	1 0 0

K = 3 //

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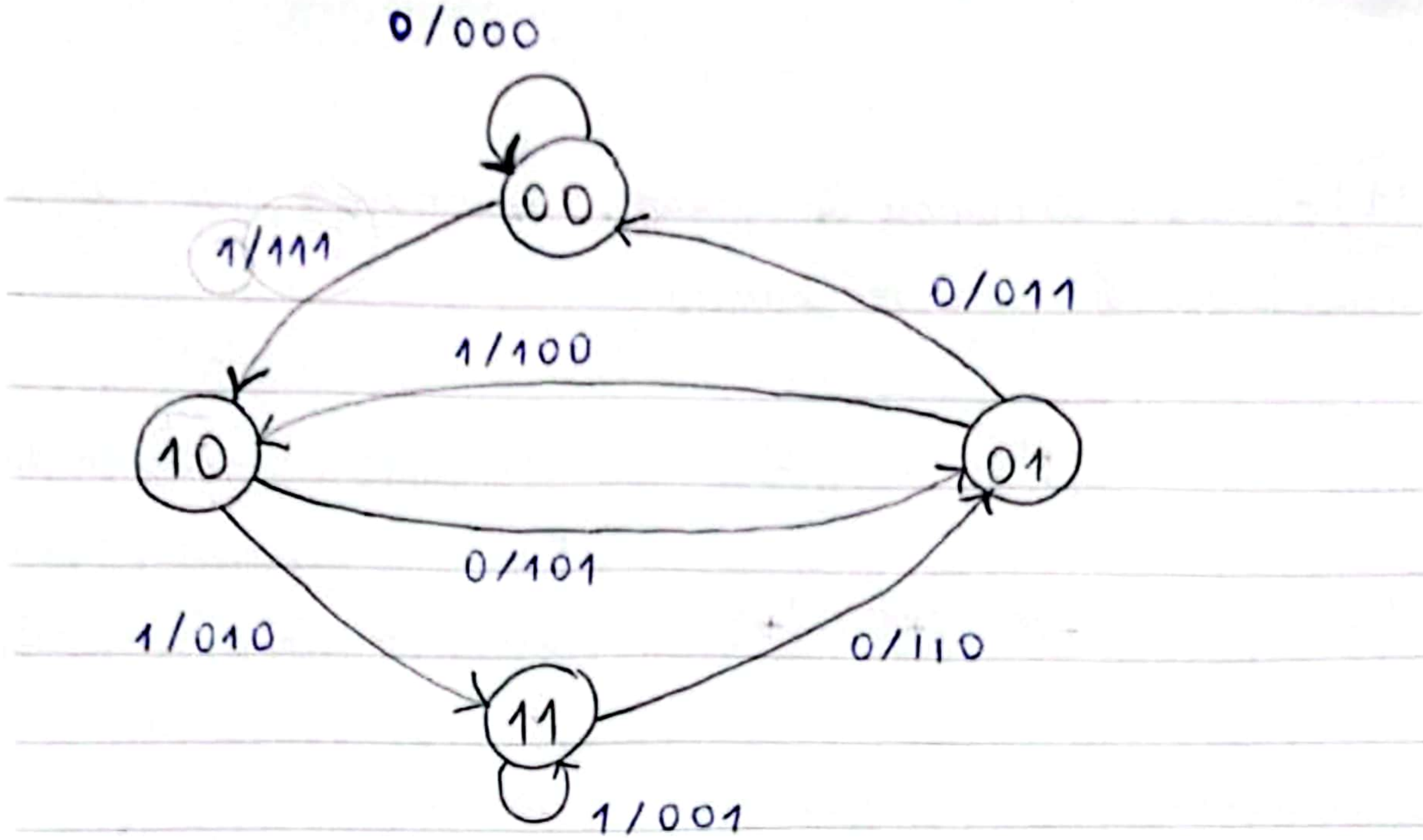
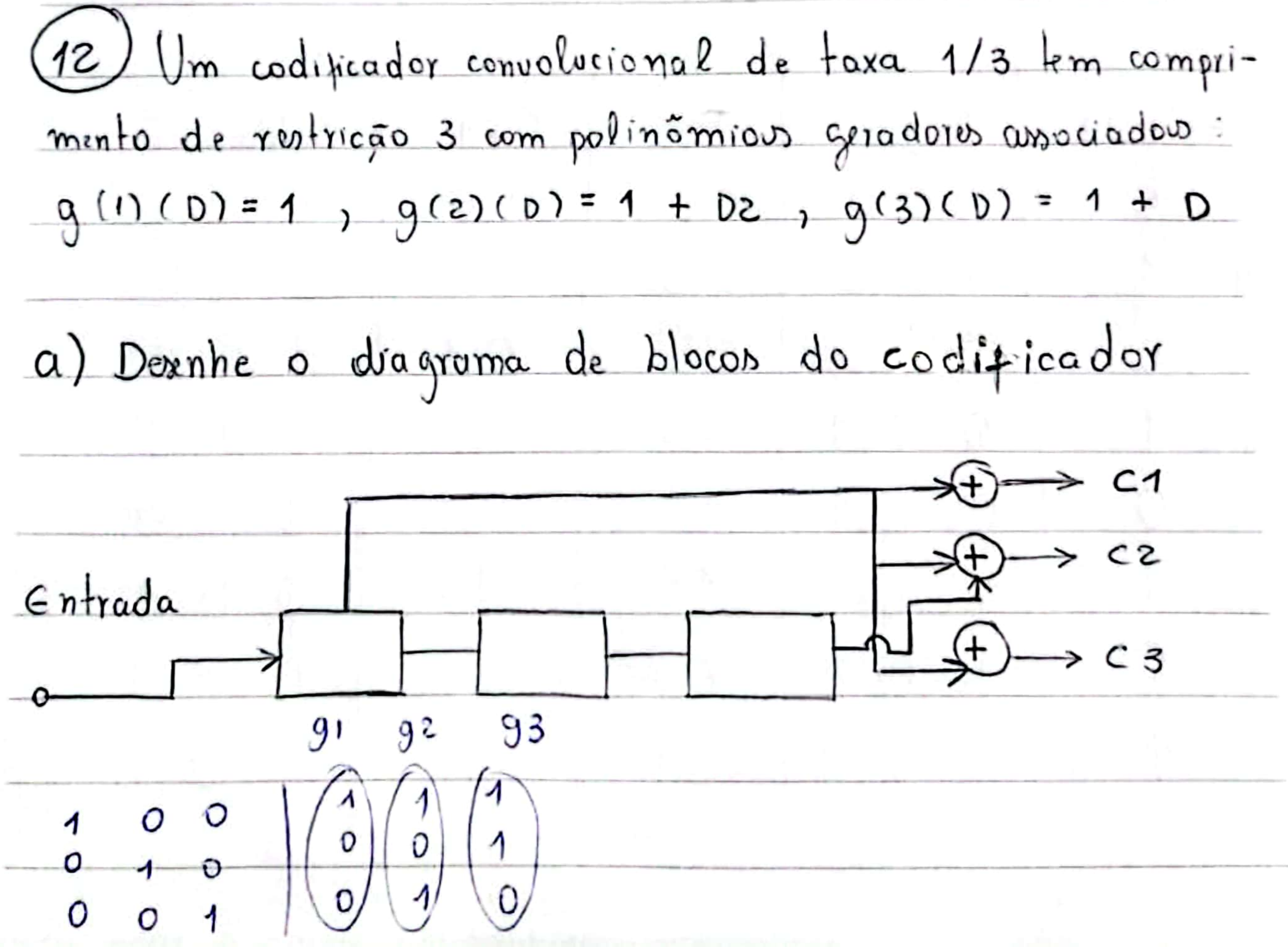


C = 111 101 100 010 110 011

(12) Um codificador convolucional de taxa 1/3 km comprimento de restrição 3 com polinômios geradores associados:

$g^{(1)}(D) = 1$, $g^{(2)}(D) = 1 + D2$, $g^{(3)}(D) = 1 + D$

a) Desenhe o diagrama de blocos do codificador



Scanné avec CamScanner

b) 0 0 1 0 1 1

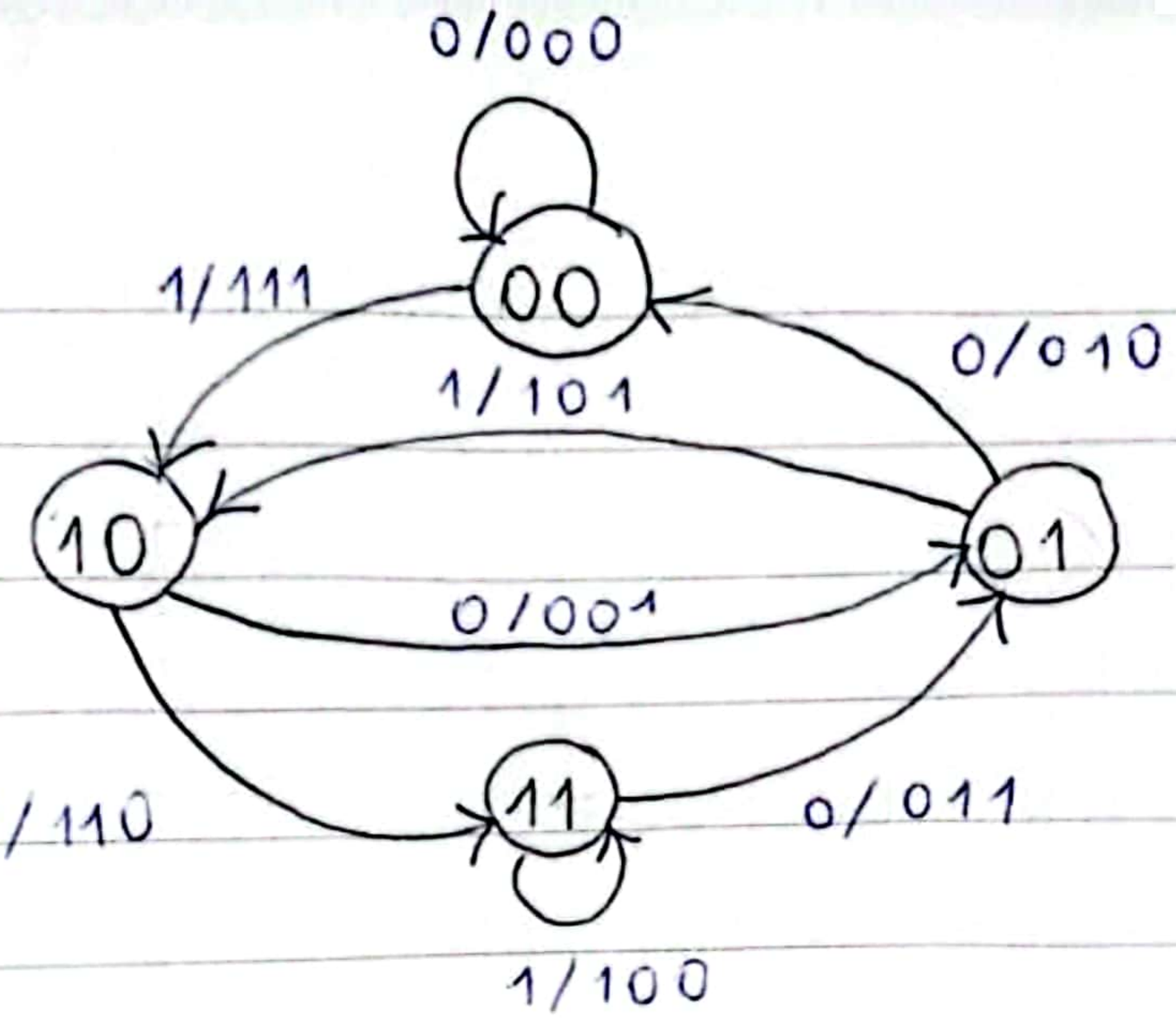
0	1	1	1	0	0	1	0	1	0		
0			1	1	1	0	0	1	0	1	0
1				1	1	1	0	0	1	0	1	0
0					1	1	1	0	0	1	0	1	0
1						1	1	1	0	0	1	0	1	0
1							1	1	1	0	0	1	0	1	0

c= 0 0 0 0 0 0 0 1 1 1 0 0 1 1 0 1 1 1 0 0 1 1 0 1 0

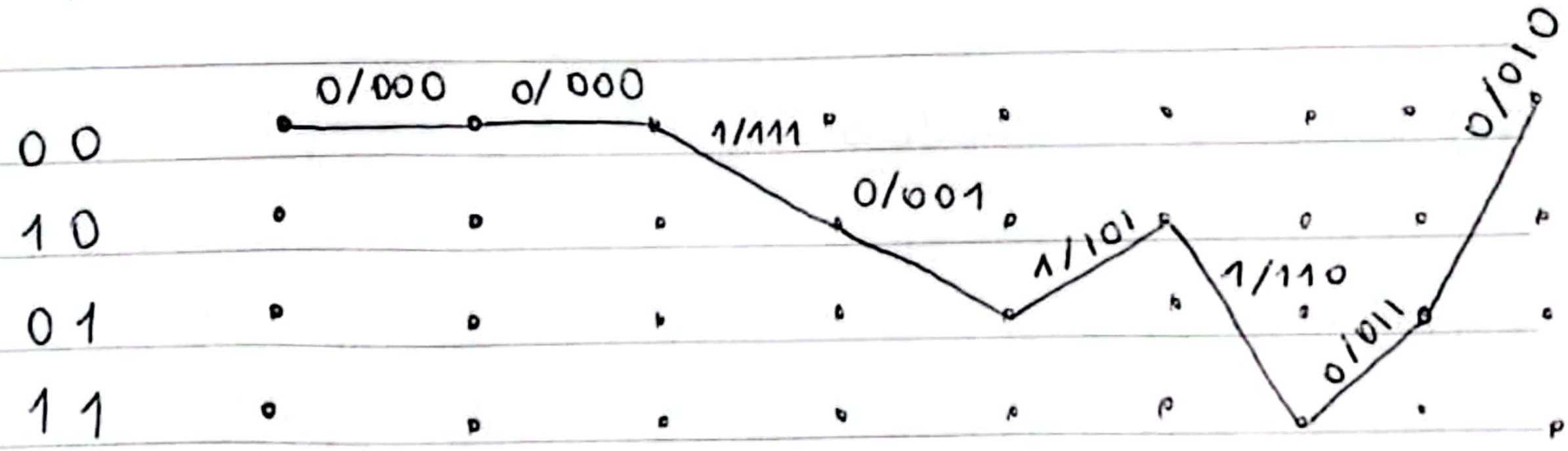
c) Diagrama de estados

Entrada	Reg	Ti	Ti+1	Saída
0	0 0 0	0 0	0 0	0 0 0
1	1 0 0	0 0	1 0	1 1 1
0	0 1 0	1 0	0 1	0 0 1
1	1 1 0	1 0	1 1	1 1 0
0	0 1 1	1 1	0 1	0 1 1
1	1 1 1	1 1	1 1	1 0 0
0	0 0 1	0 1	0 0	0 1 0
1	1 0 1	0 1	1 0	1 0 1

Scanné avec CamScanner



d) 00 10 11



C = 000 000 111 001 101 110 011 010

Scanné avec CamScanner

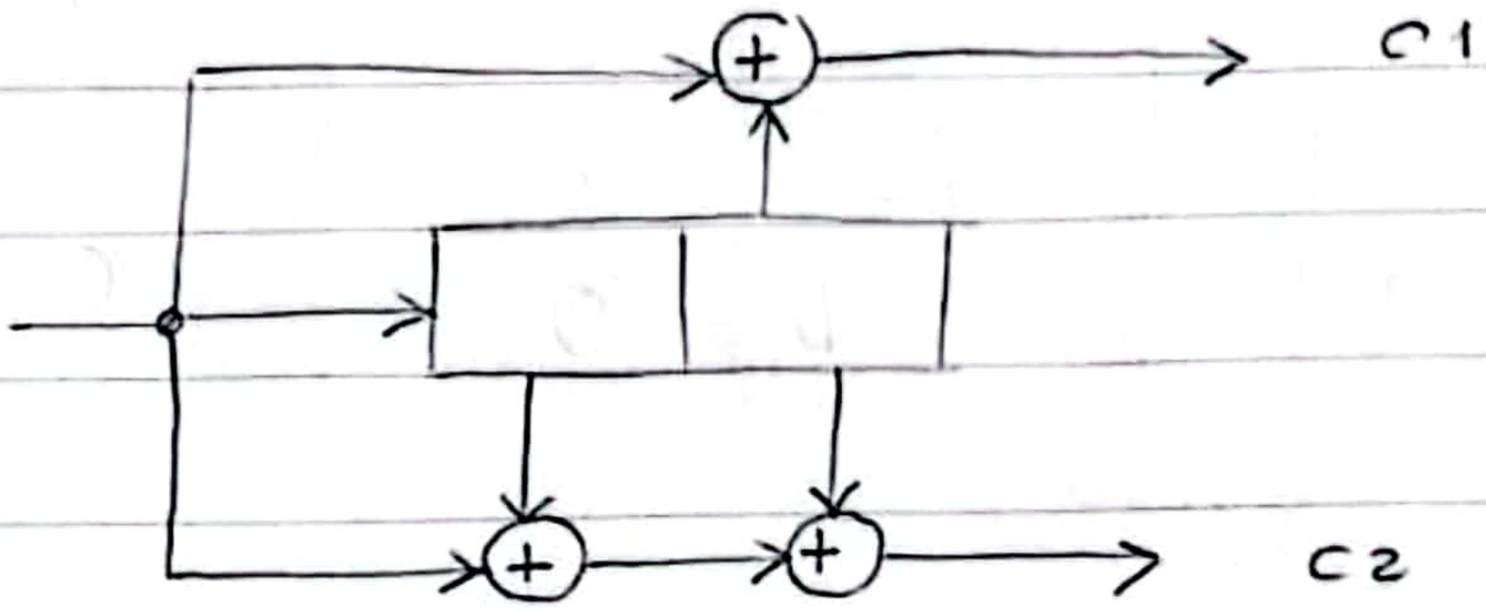
(13) Para o codificador abaixo determine:

a) a resposta ao impulso

b) saída do codificador para a mensagem 1101

c) esboce o diagroma de estados

d) esboce o diagroma de treliça



(A)

Deslocamento	Valor	Saída
1	1 0 0	1 1
2	0 1 0	0 1
3	0 0 1	1 1

(B)

1	1	1		0	1		1	1					
1			1	1		0	1		1	1			
0					1	1		0	1		1	1	
1							1	1		0	1		1	1
	1	1		1	0		1	0		0	0		0	1		1	1

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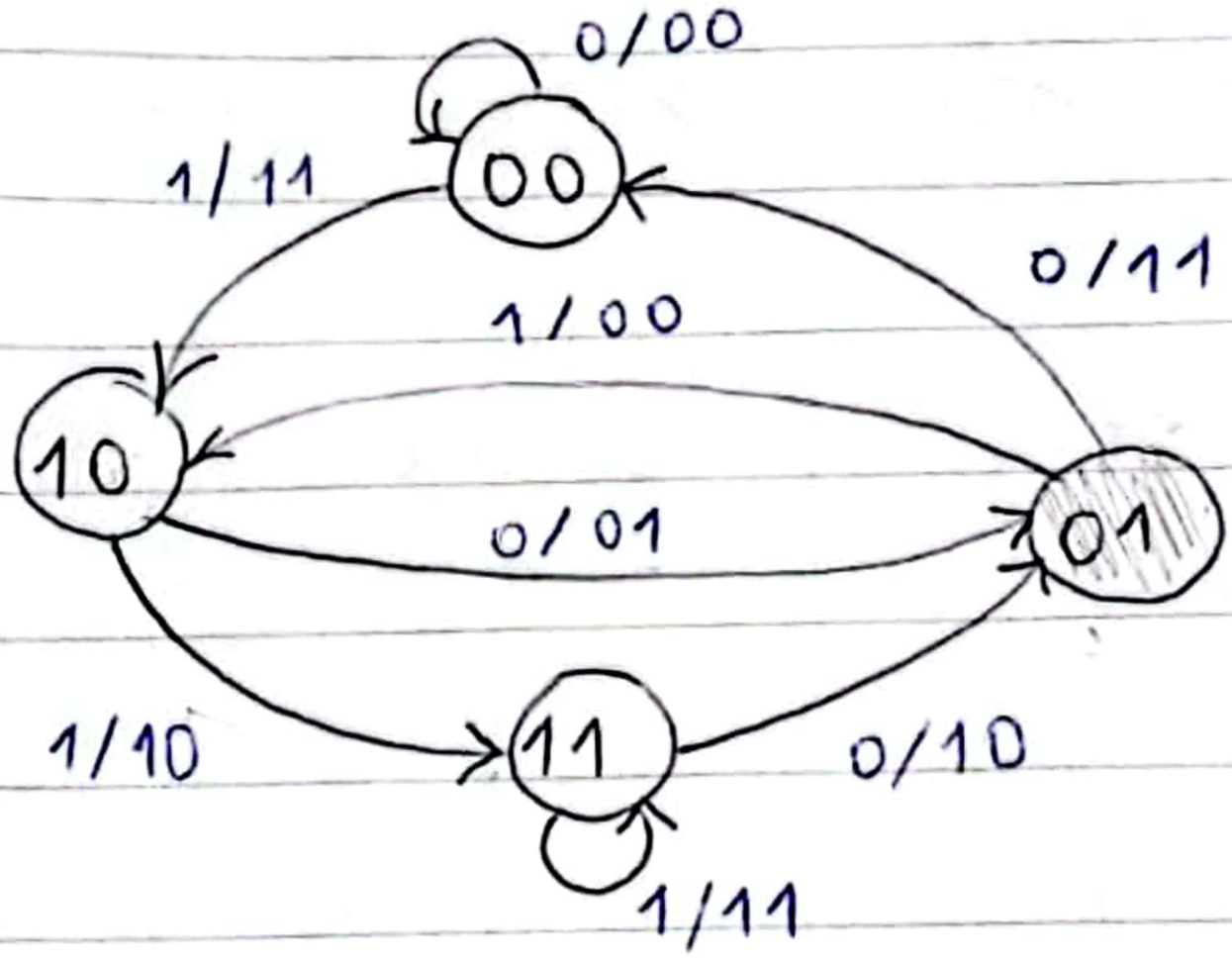
(c)

Entrada	Reg	Ti	Ti+1	saída
0	00	00	00	00
1	00	00	10	11
0	01	01	00	11
1	01	01	10	00
0	10	10	01	01
1	10	10	11	10
0	11	11	01	10
1	11	11	11	01



(d)

C = 11 10 10 00 01 11

(14) Determine:

a) a resposta ao impulso

b) a saída do decodificador para 11101

c) diagroma de estados

d) diagroma de treliça

Deslocamento	Valor	saída
1	1 0 0	1 1
2	0 1 0	1 0
3	0 0 1	1 1

(A)

(B)

1 | 11 10 11

1 | 11 10 11

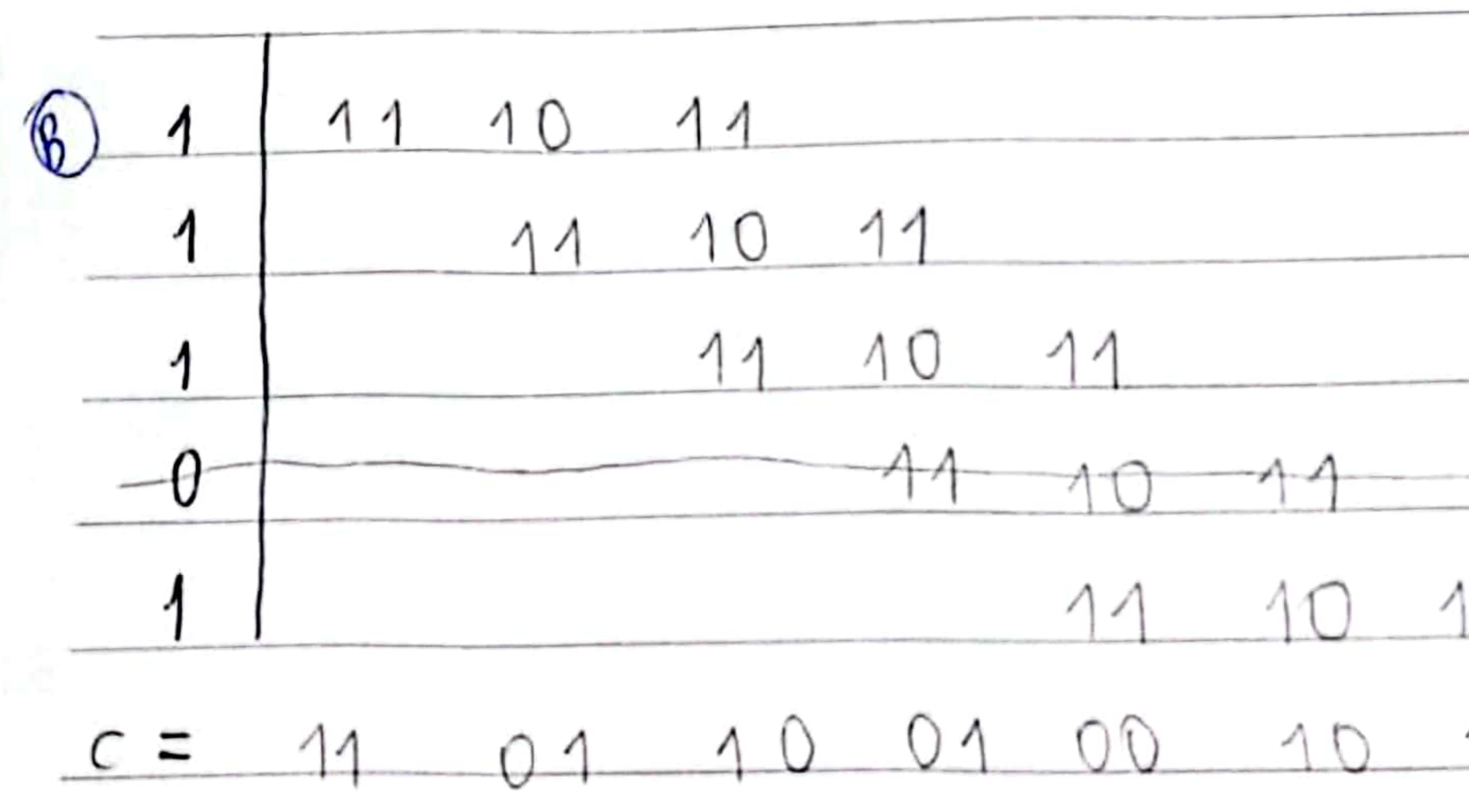
1 | 11 10 11

0 | 11 10 11

1 | 11 10 11

c = 11 01 10 01 00 10 11

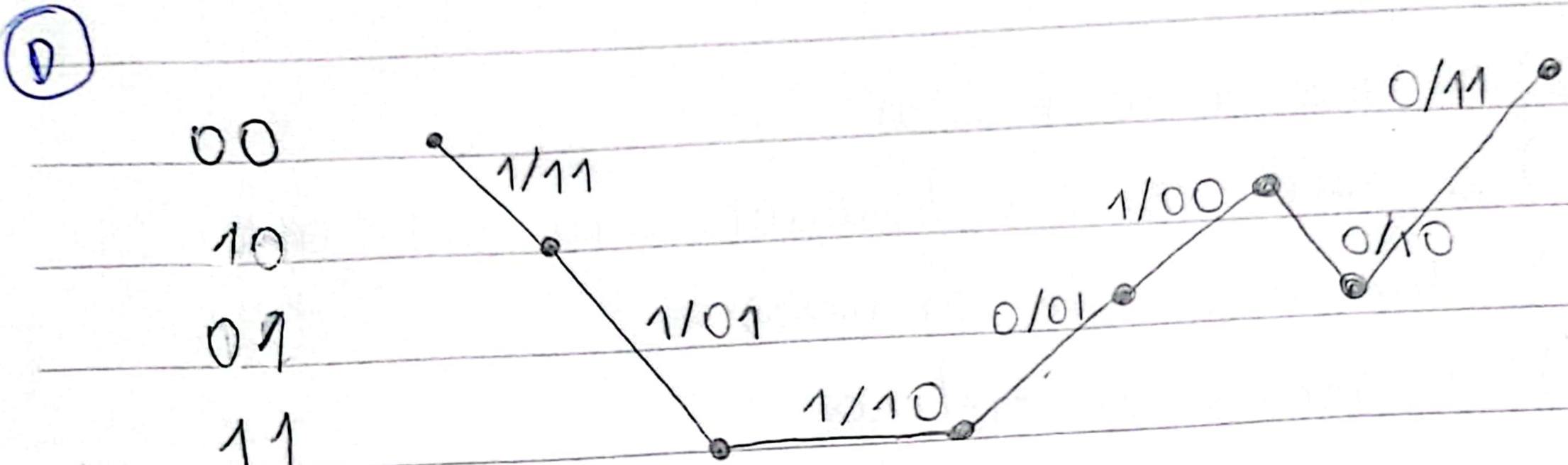
(C)



Scanné avec CamScanner

1/1 0/1 1/0 0/1 0/0 1/0 1 1

(1)



Scanné avec CamScanner