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Lista de Exercícios - 1°
PCD
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1 -
código de bloco linear sistemático → entrada
$(K+1, K) \to (5, 4)$ → saída
$\boxed{K=4}$

a) matriz geradora

$$G = \begin{array}{c} \left[\begin{array}{cccc|c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right] \end{array} \begin{array}{l} \to 2 \\ \to 2 \\ \to 2 \\ \to 2 \end{array} \quad \boxed{d_{min}=2}$$

b) código gerados - lista

m	c	m (continuação)	c
0000	00000	1000	10001
0001	00011	1001	10010
0010	00101	1010	10100
0011	00110	1011	10111
0100	01001	1100	11000
0101	01010	1101	11011
0110	01100	1110	11101
0111	01111	1111	11110

c) detecção:

$d_{min} \geq t_D + 1$

$2 \geq t_D + 1$

$t_D \leq 1$

$\boxed{t_D = 1}$

correção:

$d_{min} \geq 2t_c + 1$

$2 \geq 2t_c + 1$

$1 \geq 2t_c$

$t_c \leq \frac{1}{2}$

$\boxed{t_c = 0}$

2 - $G = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$

→ para forma sistemática

$L_1^N \Rightarrow L_1 + L_2 + L_3$
$\Rightarrow 1\ 0\ 0\ 1$

$L_2 \Rightarrow 0\ 1\ 0\ 1$

$L_3 \Rightarrow L_1^N + L_2 + L_3$
$\Rightarrow 0\ 0\ 1\ 1$

$G_S = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

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3-

$$G = [I_k \; P] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$$

vetor $c_i = [110101]$

$$H = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$cH_T = [110101] \cdot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} = \begin{matrix} 011 \\ 110 \\ 001 \\ 100 \\ \hline 000 \end{matrix}$$

→ válido, pertence ao código

4- $G = [P \; I_K] = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$

a) $H = [I \; P^T] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right]$ → matriz geradora de paridade

b) $m = [101]$

$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$ → $c_i = \begin{matrix} 101\,100 \\ 011\,001 \\ \hline \end{matrix}$

$\boxed{c_i = 110101}$ → código de [101]

c) condição de ortogonalidade

$$c \cdot H^T = (110101) \begin{bmatrix} 100 \\ 010 \\ 001 \\ 101 \\ 110 \\ 011 \end{bmatrix} = \begin{matrix} 100 \\ 010 \\ 101 \\ +011 \\ \hline 000 \end{matrix}$$

→ ortogonal !!!

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5 - vetor $c = 101011$ $\xrightarrow{\text{ruído}}$ $r = 101010$

do código (6,3,3) $\quad G = [P \mid I_k]$

correção

$d_{min} \geq 2t+1$

$3 \geq 2t+1$

$2t \leq 2$

$t \leq 1$

$\boxed{t = 1}$

$$G = \begin{bmatrix} 1\,0\,1 & 1\,0\,0 \\ 1\,1\,0 & 0\,1\,0 \\ 0\,1\,1 & 0\,0\,1 \end{bmatrix} \begin{matrix} \to 3 \\ \to 3 \\ \to 3 \end{matrix} \to H = \begin{bmatrix} 1\,0\,0 & 1\,1\,0 \\ 0\,1\,0 & 0\,1\,1 \\ 0\,0\,1 & 1\,0\,1 \end{bmatrix}$$

$$S = rH^T = (101011)\begin{bmatrix} 1\,0\,0 \\ 0\,1\,0 \\ 0\,0\,1 \\ 1\,0\,1 \\ 1\,1\,0 \\ 0\,1\,1 \end{bmatrix} = \begin{matrix} 100 \\ +001 \\ 110 \end{matrix}$$

$\boxed{S = 011}$ → com ruído

$\frac{6!}{5!\,1!} = 6$ erros possíveis

$$\begin{bmatrix} 100000 \\ 010000 \\ 001000 \\ 000100 \\ 000010 \\ \boxed{000001} \end{bmatrix} \begin{matrix} 100 \\ 010 \\ 001 \\ 101 \\ 110 \\ \to 011 \end{matrix}$$

→ $S = e \cdot H^T$

$c' = r + e$

$= 101010 + 000001$

$\boxed{c' = 101011}$

→ código com erro/ruído consertado

Questão 6.

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right] \xrightarrow{L_0 + L_1 \to L_0} \left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{array}\right]$$

TROCA-SE L_1 com L_2.

a)

$$G = (P|I) \left[\begin{array}{ccc|ccc} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

b)

msg	CODE
000	000000
001	110001
010	101010
011	011011
100	011100
101	101101
110	110110
111	000111

Questão 7.

$$H = (I|P^T) = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right] \Rightarrow H^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \cdot \boxed{100} = 000$$

$\boxed{3 \times 6}$

011100

É ortogonal.

Questão 8.

Código (5, 2)

a)

$$G_{(I|P)} = \left[\begin{array}{cc|ccc} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{array}\right] \rightarrow \begin{array}{c} d_m \\ 4 \\ 3 \end{array}$$

Maior distância Mínima.

b)

$$H_{(P|I)} = \left[\begin{array}{cc|ccc} 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$$H^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

c)

$e = d_m - 1$

$e = 3 - 1$

$e = 2$

$t = \left\lfloor \frac{d_{min} - 1}{2} \right\rfloor$

$t = \left\lfloor \frac{2}{2} \right\rfloor$

$t = 1$

d)

erro	S
10000	111
01000	101
00100	100
00010	010
00001	001

Scanné avec CamScanner

Questão 9

Cod (8,4)

a)

$$G_{(I|P)} = \begin{array}{c} \begin{matrix} m_0 & m_1 & m_2 & m_3 & p_0 & p_1 & p_2 & p_3 \end{matrix} \\ \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{array}\right] \end{array} \Rightarrow$$

b) d_m

4
4
4
4

- A distância mínima é o menor valor da soma dos 1's de cada linha.

$$H_{(P^T|I)} = \left[\begin{array}{cccc|cccc} 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \longrightarrow H^T = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

c)

e	s
1000.0000	0111
0100.0000	1110
0010.0000	1101
0001.0000	1011
0000.1000	1000
0000.0100	0100
0000.0010	0010
0000.0001	0001

$r_1 = 10101010$

$S = e \cdot H^T = 0000$ → está correto!

$r_2 = 01011100$

$S = e \cdot H^T = 1001$

↳ não é possível corrigir!!!

Scanné avec CamScanner

Questão 10.

$$G = \left[\begin{array}{ccccccccccc|cccc}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1
\end{array}\right]$$

$$\underset{4\times 15}{H} = \left[\begin{array}{ccccccccccc|cccc}
1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1
\end{array}\right]$$

(11)

I

											P1	P2	P3	P4
1	0	0	0	0	0	0	0	0	0	0	1	1	0	0
0	1	0	0	0	0	0	0	0	0	0	1	0	1	0
0	0	1	0	0	0	0	0	0	0	0	0	1	1	0
0	0	0	1	0	0	0	0	0	0	0	1	1	1	0
0	0	0	0	1	0	0	0	0	0	0	1	0	0	1
0	0	0	0	0	1	0	0	0	0	0	0	1	0	1
0	0	0	0	0	0	1	0	0	0	0	1	1	0	1
0	0	0	0	0	0	0	1	0	0	0	0	0	1	1
0	0	0	0	0	0	0	0	1	0	0	1	0	1	1
0	0	0	0	0	0	0	0	0	1	0	0	1	1	1
0	0	0	0	0	0	0	0	0	0	1	1	1	1	1

C = 11000101001 . G

= 100000000001100

010000000001010

\+ 000001000000101

000000010000011

000000000011111

110001010011111 //

(11)

$$
\begin{array}{c}
\begin{array}{ccc} \qquad\qquad P & \qquad\qquad\qquad\qquad\qquad & I \end{array} \\
H = \left[\begin{array}{ccccccccccc|cccc}
1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1
\end{array}\right]
\end{array}
$$

$$
H^T = \left[\begin{array}{cccc}
1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 \\
1 & 1 & 0 & 1 \\
0 & 0 & 1 & 1 \\
1 & 0 & 1 & 1 \\
0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
\begin{array}{cl}
1 & 1100 \\
1 & 1010 \\
0 & 0101 \\
0 & 0011 \\
0 & 1111 \\
1 & 1000 \\
0 & 0100 \\
1 & 0010 \\
0 & 0001 \\
0 & \overline{(0000)} \\
1 & \\
1 & \\
1 & \\
1 & \\
1 &
\end{array}
$$

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$$G_{7x4} = \begin{array}{c} \begin{array}{cccc} & I & & \end{array} \quad \begin{array}{ccc} & P & \end{array} \\ \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{array}\right] \end{array} \begin{array}{l} \\ \rightarrow 3 \\ 3 \rightarrow dm \\ 3 \\ 4 \end{array}$$

a) $\bar{m} = [0100] \times G = 0100101$ //

$$H^T = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$0100101 \times H^T$

$$\begin{array}{r} 101 \\ +\ 100 \\ 001 \\ \hline (000) \end{array}$$ //

b) $\bar{m} = [0101] \times G = 0100101$

$$\begin{array}{r} 0100101 \\ \oplus\ 0001111 \\ \hline 0101010 \end{array}$$ //

$0101010 \times H^T$

$$\begin{array}{r} 101 \\ +\ 111 \\ 010 \\ \hline (000) \end{array}$$ //

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c) $\bar{m} = [1110] \times G =$

$$\begin{array}{r} 110 \\ 101 \\ 011 \\ \hline (000) \end{array}$$

$\bar{m} = 1110000$ //

d) $\bar{m} = [1001] \times G =$

$$\begin{array}{r} 1000110 \\ +\ 0001111 \\ \hline 1001001 \end{array}$$ //

$$\begin{array}{r} 110 \\ +\ 111 \\ 001 \\ \hline (000) \end{array}$$ //

(13)

$$H = \begin{array}{cc} P & I \\ \left[\begin{array}{cccc|ccc} 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 1 \end{array}\right] \end{array} \qquad H^T = \left[\begin{array}{ccc} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] \begin{array}{cc} 1 & 1 \\ 1 & 0 \\ 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{array}$$

$$t = \frac{(3-1)}{2} = 1$$

a) $\bar{v} = [1101001] \cdot H^T$

$$\begin{array}{r} 110 \\ 101 \\ +\ 111 \\ 001 \\ \hline 101 \end{array}$$

e110

aqui

n H

e (t = 1)	3
1 0 0 0 0 0 0	1 1 0
0 1 0 0 0 0 0	1 0 1
0 0 1 0 0 0 0	0 1 1
0 0 0 1 0 0 0	1 1 1
0 0 0 0 1 0 0	1 0 0
0 0 0 0 0 1 0	0 1 0
0 0 0 0 0 0 1	0 0 1

a) 101 → 0 erro

```
  1 1 0 1 0 0 1
  0 1 0 0 0 0 0
  -------------
  1 0 0 1 0 0 1 //
```

```
  1 1 0
  1 1 1
  0 0 1 / (0 0 0) //
```

b) $\bar{v} = [1000111] \cdot H^T$

```
  1 1 0
  1 0 0
  0 1 0
  0 0 1
  -----
  0 0 1  → erro
```

```
  1 0 0 0 1 1 1
+ 0 0 0 0 0 0 1
  -------------
  1 0 0 0 1 1 0 //
```

```
  1 1 0
  1 0 0
  0 1 0
  -----
  (0 0 0) //
```

c) $\bar{v} = [1111100] \cdot H^T$

```
  1 1 0
  1 0 1
+ 0 1 1
  1 1 1
  1 0 0 / 0 1 1
```

```
  1 1 1 1 1 0 0
+ 0 0 1 0 0 0 0 / 1 1 0 1 1 0 0 //
```

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$$\begin{array}{r} 110 \\ 101 \\ +\ 111 \\ \hline \cancel{100}\ (000) // \end{array}$$

d) $\bar{v} = [1011001] \cdot H^T$

$$\begin{array}{r} 110 \\ 011 \\ 111 \\ 001 \\ \hline 011 \end{array} \qquad \begin{array}{r} 1011001 \\ +\ 0010000 \\ \hline 1001001 \ // \end{array}$$

$$\begin{array}{r} 110 \\ +\ 111 \\ \hline \cancel{001}\ (000) // \end{array}$$

14 $G = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \rightarrow G = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} \begin{matrix} L \\ L \\ L \end{matrix}$

$L0 + L1 + L2 \Rightarrow L0 \rightarrow$
$$\begin{array}{r} 1111 \\ 0101 \\ 0011 \\ \hline 1001 \end{array} \qquad G = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix} //$$

(15) $G = [i \mid P] = \begin{array}{c} \begin{array}{cccc} & I & & P \end{array} \\ \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{array}\right] \end{array}$

$G_{3\times4}$

$H = [P \mid I]$

$H = [P^T_{1\times3} : I_1]$

$$\begin{array}{r} 100 \\ \oplus\ 010 \\ 001 \\ \hline 111 \end{array}$$

$H = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}$ //

e a $H^T = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ //

Scanné avec CamScanner