Assignment: Project 3
Group: 6, CS325-400
Members: Kelsey Helms, Jay Steingold, Johannes Pikel
Date: 2016.11.11
Due Date: 2016.11.13

For each of the problems we used Lindo and have provided the code used inside each of the answers.

Problem 1 Transshipment Model

Part A.i Formulate the problem as a linear program with an object function and all constraints to minimize the cost.

Our objective is to minimize the cost of all possible shipping routes while still meeting the demands of the retailers and staying within the limits of the plants' production capacities. Then, the cost of shipping from a plant to a warehouse is denoted by cp(i,j) where i is a plant and j is a warehouse. Then based on the chart for allowable shipments and associated costs we need to minimize the following: $10cp_{11} + 15cp_{12} + 11cp_{21} + 8cp_{22} + 13cp_{31} + 8cp_{32} + 9cp_{33} + 14cp_{42} + 8cp_{43}$

However, we also need to consider the costs related to shipping from each warehouse to possible retailers so we also need to minimize the costs. Then, the cost of shipping from a warehouse to a retailer is denoted by cw(j,k) where j is a warehouse and k is retailer. Then based on the chart for allowable shipments and their associated costs we need to minimize:

$$5cw_{11} + 6cw_{12} + 7cw_{13} + 10cw_{14} + 12cw_{23} + 8cw_{24} + 10cw_{25} + 14cw_{26} + 14cw_{34} + 12cw_{35} + 12cw_{3}$$
$$+ 6cw_{37}$$

So then combining the two equations above we have.

Objective:

minimize cost c = $10cp_{11} + 15cp_{12} + 11cp_{21} + 8cp_{22} + 13cp_{31} + 8cp_{32} + 9cp_{33} + 14cp_{42} + 8cp_{43} +$

$$5cw_{11} + 6cw_{12} + 7cw_{13} + 10cw_{14} + 12cw_{23} + 8cw_{24} + 10cw_{25} + 14cw_{26} + 14cw_{34} + 12cw_{35} + 12cw_{36}$$
$$+ 6cw_{37}$$

There are a number of constraints for this problem based on the following:

Plant production capacity
Plant to warehouse shipping routes
Warehouse to Retail shipping routes
Retailer demands.

Then constraints are:

Production Capacity constraints for plants are as follows:

$$cp_{11} + cp_{12} \le 150 \qquad (plant\ 1\ supply\ for\ routes\ allowed)$$
$$cp_{21} + cp_{22} \le 450 \qquad (plant\ 2\ supply\ for\ routes\ allowed)$$
$$cp_{31} + cp_{32} + cp33 \le 250 \qquad (plant\ 3\ supply\ for\ routes\ allowed)$$
$$cp_{42} + cp_{43} \le 150 \qquad (plant\ 4\ supply\ for\ routes\ allowed)$$

Warehouse constraints for inputs from plants and outputs to retailers served by the warehouse are:

$$cp_{11} + cp_{21} + cp_{31} - cw_{11} - cw_{12} - cw_{13} - cw_{14} \ge 0 \qquad \text{(warehouse 1)}$$
$$cp_{22} + cp_{32} + cp_{42} - cw_{23} - cw_{24} - cw_{25} - cw_{26} \ge 0 \qquad \text{(warehouse 2)}$$

$cp_{33} + cp_{43} - cw_{34} - cw_{35} - cw_{36} - cw_{37} \geq 0$ (warehouse 3)

Retailers demands constraints need to meet a minimum demand from available shipment routes:

$cw_{11} \geq 100$ (retailer 1)

$cw_{12} \geq 150$ (retailer 2)

$cw_{13} + cw_{23} \geq 100$ (retailer 3)

$cw_{14} + cw_{24} + cw_{34} \geq 200$ (retailer 4)

$cw_{25} + cw_{35} \geq 200$ (retailer 5)

$cw_{26} + cw_{36} \geq 150$ (retailer 6)

$cw_{37} \geq 100$ (retailer 7)

We do need to observe some nonnegativity constraints as well, because production can't be less than 0 and there are no returns from retailer to warehouse or warehouse to plant:

$cp_{11}, cp_{12}, cp_{21}, cp_{22}, cp_{31}, cp_{32}, cp_{33}, cp_{42}, cp_{43} \geq 0$

$cw_{11},\ cw_{12}, cw_{13},\ cw_{14},\ cw_{23}, cw_{24},\ cw_{25},\ cw_{26},\ cw_{34},\ cw_{35},\ cw_{36},\ cw_{37} \geq 0$

Also to be explicit in our linear program we'll make sure the nonexistent routes are set to 0:

$cp_{13},\ cp_{23},\ cp_{41}\ =\ 0$

$cw_{15},\ cw_{16}, cw_{17},\ cw_{21}\ ,\ cw_{22}.\ cw_{27},\ cw_{31},\ cw_{32},\ cw_{33} =\ 0$

Entering this linear program into Lindo :

```
<untitled>
min 10cp11+15cp12+11cp21+8cp22+13cp31+8cp32+9cp33+14cp42+8cp43
+5cw11+6cw12+7cw13+10cw14+12cw23+8cw24+10cw25+14cw26+14cw34+12cw35+12cw36+6cw37

ST
        cp11+cp12 <=150
        cp21+cp22 <=450
        cp31+cp32+cp33 <=250
        cp42+cp43 <=150

        cp11+cp21+cp31-cw11-cw12-cw13-cw14 >= 0
        cp12+cp22+cp32+cp42-cw23-cw24-cw25-cw26 >=0
        cp33+cp43-cw34-cw35-cw36-cw37 >=0

        cw11 >= 100
        cw12 >= 150
        cw13+cw23 >=100
        cw14+cw24+cw34>=200
        cw25+cw35>=200
        cw26+cw36>=150
        cw37>=100

        cp11 >=0
        cp12 >=0
        cp21 >=0
        cp22 >=0
        cp31 >=0
        cp32 >=0
        cp33 >=0
        cp42 >=0
        cp43 >=0

        cw11 >=0
        cw12 >=0
        cw13 >=0
        cw14 >=0
        cw23 >=0
        cw24 >=0
        cw25 >=0
        cw26 >=0
        cw34 >=0
        cw35 >=0
        cw36 >=0

        cp13 = 0
        cp23 = 0
        cp41 = 0
        cw15 = 0
        cw16 = 0
        cw17 = 0
        cw21 = 0
        cw22 = 0
        cw27 = 0
        cw31 = 0
        cw32 = 0
        cw33 = 0
END
```

min 10cp11+15cp12+11cp21+8cp22+13cp31+8cp32+9cp33+14cp42+8cp43
+5cw11+6cw12+7cw13+10cw14+12cw23+8cw24+10cw25+14cw26+14cw34+12cw35+12cw36
+6cw37

ST

cp11+cp12 <=150
cp21+cp22 <=450
cp31+cp32+cp33 <=250
cp42+cp43 <=150

cp11+cp21+cp31-cw11-cw12-cw13-cw14 >= 0
cp12+cp22+cp32+cp42-cw23-cw24-cw25-cw26 >=0
cp33+cp43-cw34-cw35-cw36-cw37 >=0

cw11 >= 100

```
    cw12 >= 150
    cw13+cw23 >=100
    cw14+cw24+cw34>=200
    cw25+cw35>=200
    cw26+cw36>=150
    cw37>=100

    cp11 >=0
    cp12 >=0
    cp21 >=0
    cp22 >=0
    cp31 >=0
    cp32 >=0
    cp33 >=0
    cp42 >=0
    cp43 >=0

    cw11 >=0
    cw12 >=0
    cw13 >=0
    cw14 >=0
    cw23 >=0
    cw24 >=0
    cw25 >=0
    cw26 >=0
    cw34 >=0
    cw35 >=0
    cw36 >=0
    cp13 = 0
    cp23 = 0
    cp41 = 0
    cw15 = 0
    cw16 = 0
    cw17 = 0
    cw21 = 0
    cw22 = 0
    cw27 = 0
    cw31 = 0
    cw32 = 0
    cw33 = 0
END
```

Gives the following result:

```
LP OPTIMUM FOUND AT STEP     15

       OBJECTIVE FUNCTION VALUE

       1)      17100.00

  VARIABLE        VALUE          REDUCED COST
      CP11       150.000000          0.000000
      CP12         0.000000          8.000000
      CP21       200.000000          0.000000
      CP22       250.000000          0.000000
      CP31         0.000000          2.000000
      CP32       150.000000          0.000000
      CP33       100.000000          0.000000
      CP42         0.000000          7.000000
      CP43       150.000000          0.000000
      CW11       100.000000          0.000000
      CW12       150.000000          0.000000
      CW13       100.000000          0.000000
      CW14         0.000000          5.000000
      CW23         0.000000          2.000000
      CW24       200.000000          0.000000
      CW25       200.000000          0.000000
      CW26         0.000000          1.000000
      CW34         0.000000          7.000000
      CW35         0.000000          3.000000
      CW36       150.000000          0.000000
      CW37       100.000000          0.000000
      CP13         0.000000          0.000000
      CP23         0.000000          0.000000
      CP41         0.000000          0.000000
      CW15         0.000000          0.000000
      CW16         0.000000          0.000000
      CW17         0.000000          0.000000
      CW21         0.000000          0.000000
      CW22         0.000000          0.000000
      CW27         0.000000          0.000000
      CW31         0.000000          0.000000
      CW32         0.000000          0.000000
      CW33         0.000000          0.000000
```

- **The optimal shipping routes from plant to warehouses (WH) and then warehouse to plants is at a minimum cost of $17,100.00:**
- Plant 1 -> WH 1 = 150 fridges
- Plant 2 -> WH 1 = 200 fridges
- Plant 2 -> WH 2 = 250 fridges
- Plant 3 -> WH 2 = 150 fridges
- Plant 3 -> WH 3 = 100 fridges
- Plant 4 -> WH 3 = 150 fridges

- WH 1 contains 350 fridges
- WH 2 contains 400 fridges
- WH 3 contains 200 fridges

- WH 1 -> retailer 1 = 100 fridges
- WH 1 -> retailer 2 = 150 fridges
- WH 1 -> retailer 3 =100 fridges
- WH 2 -> retailer 4 = 200 fridges
- WH 2 -> retailer 5 = 200 fridges
- WH 3 -> retailer 6 = 150 fridges
- WH 3 -> retailer 7 = 100 fridges

- Warehouses only distribute the number of fridges they receive and retailers all have their minimum demand met.

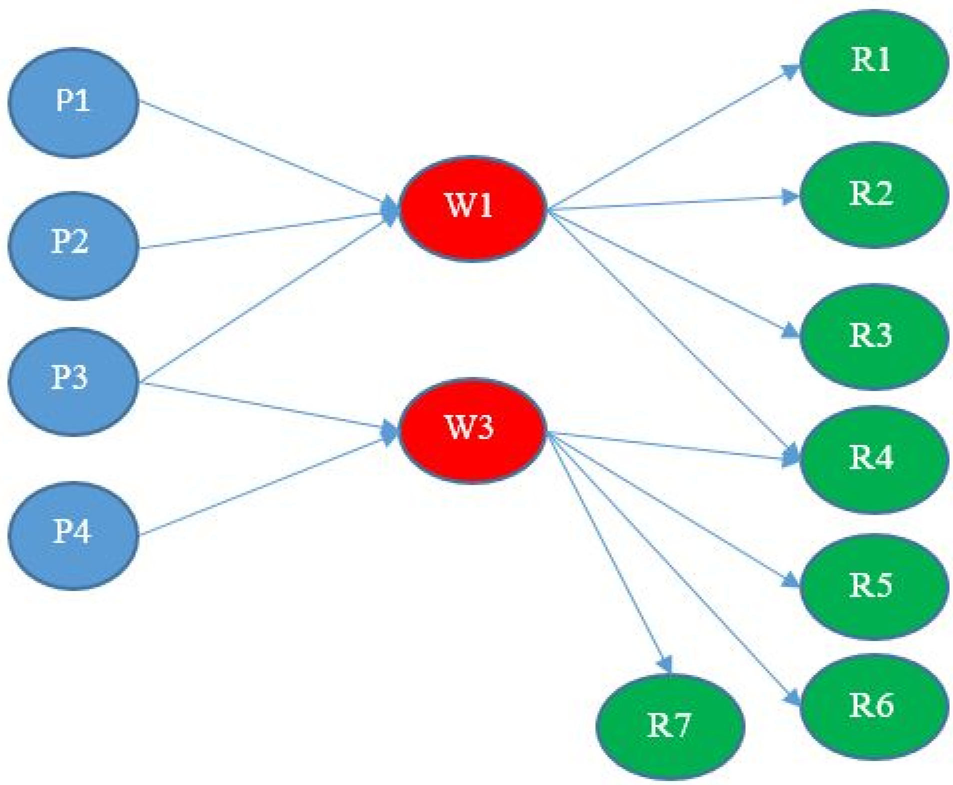
- In this case the plants do not have a surplus of production because total plant production = 1000 units and retailer demand = 1000 units

Problem 1 Part B:

There is not a feasible solution to this problem. The reason that warehouse 3 now has to serve a minimum demand of 450 units of fridges to retailers 5, 6 and 7 not including 4. But, warehouse 3 only receives at most 400 units of fridges from Plants 3 and 4. Warehouse is short 50 units.

Warehouse 1 would have demands from retailers 1 through 4 for a total demand of 550 units of fridges and since Warehouse 3 already takes the entire production from Plant 3. Warehouse 1 gets the units from Plants 1 and 2 for a total of 600, which is an excess of 50 units.

This could be solved by allowing Plant 1 or Plant 2 to ship to Warehouse 3, and choosing the route that is least expensive. The graph for this problem would be as follows.



Entering this into Lindo results in the response No Feasible Solution

<untitled>

```
min 10cp11+11cp21+13cp31+9cp33+8cp43
+5cw11+6cw12+7cw13+10cw14+14cw34+12cw35+12cw36+6cw37

ST
        cp11 <=150
        cp21 <=450
        cp31+cp33 <=250
        cp43 <=150

        cp11+cp21+cp31-cw11-cw12-cw13-cw14 >= 0
        cp33+cp43-cw34-cw35-cw36-cw37 >=0


        cw11 >= 100
        cw12 >= 150
        cw13 >=100
        cw14+cw34>=200
        cw35>=200
        cw36>=150
        cw37>=100

        cp11 >=0
        cp21 >=0
        cp31 >=0
        cp33 >=0
        cp43 >=0

        cw11 >=0
        cw12 >=0
        cw13 >=0
        cw14 >=0
        cw34 >=0
        cw35 >=0
        cw36 >=0

        cp13 = 0
        cp23 = 0
        cp41 = 0
        cw15 = 0
        cw16 = 0
        cw17 = 0
        cw21 = 0
        cw22 = 0
        cw27 = 0
        cw31 = 0
        cw32 = 0
        cw33 = 0
END
```

LINDO Error Message

Error code: 54

Help | OK

Error text:

```
NO FEASIBLE SOLUTION AT STEP          11.
SUM OF INFEASIBILITIES=       50.000000000000000
VIOLATED ROWS HAVE  NEGATIVE SLACK,  OR
(EQUALITY ROWS) NONZERO  SLACKS.   ROWS
CONTRIBUTING TO  INFEASIBILITY  HAVE  A
NONZERO  DUAL  PRICE.   USE THE "DEBUG"
COMMAND FOR MORE INFORMATION.
```

Problem 1 Part C:

Yes, this is feasible but significantly increases the cost of meeting all the retailers' demands, two of the routes to warehouse 2 from Plants 2 and 3 were the among the least expensive routes of the entire transportation graph. Thus the requiring that more expensive routes be used to meet the extra demand. From problem A, 350 units of fridges will need to be redirected if only 100 units are able to be shipped through Warehouse 2.

We can use all the same formulas entered into Lindo from Problem A, with one additional constraint that will limit the throughput of Warehouse 2 (all the routes leaving warehouse 2 can be at most 100 units):

$cw_{23} + cw_{24} + cw_{25} + cw_{26} \leq 100$ Will limit the throughput of warehouse 2.

So then the output from Lindo is an optimal solution:

```
 LP OPTIMUM FOUND AT STEP      15

        OBJECTIVE FUNCTION VALUE

        1)      18300.00

  VARIABLE        VALUE          REDUCED COST
      CP11       150.000000          0.000000
      CP12         0.000000          8.000000
      CP21       350.000000          0.000000
      CP22       100.000000          0.000000
      CP31         0.000000          4.000000
      CP32         0.000000          2.000000
      CP33       250.000000          0.000000
      CP42         0.000000          9.000000
      CP43       150.000000          0.000000
      CW11       100.000000          0.000000
      CW12       150.000000          0.000000
      CW13       100.000000          0.000000
      CW14       150.000000          0.000000
      CW23         0.000000          7.000000
      CW24        50.000000          0.000000
      CW25        50.000000          0.000000
      CW26         0.000000          4.000000
      CW34         0.000000          4.000000
      CW35       150.000000          0.000000
      CW36       150.000000          0.000000
      CW37       100.000000          0.000000
      CP13         0.000000          0.000000
      CP23         0.000000          0.000000
      CP41         0.000000          0.000000
      CW15         0.000000          0.000000
      CW16         0.000000          0.000000
      CW17         0.000000          0.000000
      CW21         0.000000          0.000000
      CW22         0.000000          0.000000
      CW27         0.000000          0.000000
      CW31         0.000000          0.000000
      CW32         0.000000          0.000000
      CW33         0.000000          0.000000
```

Now the minimum cost has increased to $18,300.00

- Plant 1 -> Warehouse 1 = 150
- Plant 2 -> Warehouse 1 = 350
- Plant 2 -> Warehouse 2 = 100
- Plant 3 -> Warehouse 3 = 250
- Plant 4 -> Warehouse 3 = 150

- WH 1 contains 500
- WH 2 contains 100
- WH 3 contains 400

- WH 1 -> Retailer 1 = 100
- WH 1 -> Retailer 2 = 150
- WH 1 -> Retailer 3 = 100
- WH 1 -> Retailer 4 = 150
- WH 2 - > Retailer 4 = 50
- WH 2 - > Retailer 5 = 50
- WH 3 -> Retailer 5 = 150
- WH 3 -> Retailer 6 = 150
- WH 3 -> Retailer 7 = 100

So now Warehouse 2 is limited to only 100 units.

Problem 1 Part D:

A generalized linear programming model as a mathematical formula is as follows where we need to minimize the costs along all routes from plant to warehouse and warehouse to retailer:

min $\sum_{i \in I} \sum_{j \in J} \sum_{k \in K} cp_{ij} cw_{jk}$

s.t.

$$\sum_{i \in I} p_i w_j \leq s_i \qquad \forall\, j \in J$$

$$\sum_{i \in I} p_i w_j - \sum_{j \in J} w_j r_k \geq 0 \qquad \forall\, j \in J,\ \forall k \in K$$

$$\sum_{j \in J} w_j r_k \geq d_k \qquad \forall\, k \in K$$

$$\sum_{i \in I} p_i w_j \geq 0 \qquad \forall\, j \in J$$

$$\sum_{j \in J} w_j r_k \geq 0 \qquad \forall\, k \in K$$

Constraint 1 sum of all routes for each plant must be less than or equal to that plant's output.
Constraint 2 is that the outputs from each warehouse may not be larger than the inputs
As in, a warehouse may not ship more goods than it receives.
Constraint 3 the sum of all routes for each retailer must meet at least the minimum demand
Constraint 4 is that all routes from a plant to warehouse cannot be negative meaning no returns
Constraint 5 all routes from a warehouse to retailer cannot be negative, meaning no returns.
We could add the explicit constraints that

$$\sum_{i \in I} p_i w_j = 0 \qquad \forall\, j \notin J$$

$$\sum_{j \in J} w_j r_k = 0 \qquad \forall\, k \notin K$$

Meaning that routes that do not exist must be equal to 0 as not goods may be transferred along that route.

I rewrote these equations in terms as given in Problem 1, whereas when writing the linear program for input into Lindo they were all written using similar terms but different indices.

Problem 2: A mixture problem

Assignment: Project 3
Group: 6, CS325-400
Members: Kelsey Helms, Jay Steingold, Johannes Pikel
Date: 2016.11.11
Due Date: 2016.11.13

Veronica the owner of Very Veggie Vegeria is creating a new healthy salad that is low in calories but meets certain nutritional requirements. A salad is any combination of the following ingredients: Tomato, Lettuce, Spinach, Carrot, Smoked Tofu, Sunflower Seeds, Chickpeas, Oil

Each salad must contain:

- At least 15 grams of protein
- At least 2 and at most 8 grams of fat
- At least 4 grams of carbohydrates
- At most 200 milligrams of sodium
- At least 40% leafy greens by mass.

The nutritional contents of these ingredients (per 100 grams) and cost are:

Ingredient	Energy (kcal)	Protein (grams)	Fat (grams)	Carbohydrate (grams)	Sodium (mg)	Cost (100g)
Tomato	21	0.85	0.33	4.64	9.00	$1.00
Lettuce	16	1.62	0.20	2.37	28.00	$0.75
Spinach	40	2.86	0.39	3.63	65.00	$0.50
Carrot	41	0.93	0.24	9.58	69.00	$0.50
Sunflower Seeds	585	23.4	48.7	15.00	3.80	$0.45
Smoked Tofu	120	16.00	5.00	3.00	120.00	$2.15
Chickpeas	164	9.00	2.6	27.0	78.00	$0.95
Oil	884	0	100.00	0	0	$2.00

Part A: Determine the combination of ingredients that minimizes calories but meets all nutritional requirements.

I. Formulate the problem as a linear program with an objective function and all constraints.

Variables:
X_1 = 100g of Tomato
X_2 = 100g of Lettuce
X_3 = 100g of Spinach
X_4 = 100g of Carrot
X_5 = 100g of Sunflower Seeds
X_6 = 100g of Smoked Tofu
X_7 = 100g of Chickpeas
X_8 = 100g of Oil

<u>Objective Function:</u>
Minimize $Z = 21x_1 + 16x_2 + 40x_3 + 41x_4 + 585x_5 + 120x_6 + 164x_7 + 884x_8$

where Z = number of calories

<u>Constraints:</u>

Protein:	$0.85x_1 + 1.63x_2 + 2.86x_3 + 0.93x_4 + 23.4x_5 + 16x_6 + 9x_7 \geq 15$
Fat:	$2 \leq 0.33x_1 + 0.2x_2 + 0.39x_3 + 0.24x_4 + 48.7x_5 + 5x_6 + 2.6x_7 + 100x_8 \leq 8$
Carbohydrates:	$4.64x_1 + 2.37x_2 + 3.63x_3 + 9.58x_4 + 15x_5 + 3x_6 + 27x_7 \geq 4$
Sodium:	$9x_1 + 28x_2 + 65x_3 + 69x_4 + 3.8x_5 + 120x_6 + 78x_7 \leq 200$
Leafy Greens:	$(x_2 + x_3) / (x_1 + x_4 + x_5 + x_6 + x_7 + x_8) \geq 0.4$

I. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

<u>Optimal Solution:</u>

Tomato:	0.00000 g
Lettuce:	57.4713 g
Spinach:	0.00000 g
Carrot:	0.00000 g
Sunflower Seeds:	0.00000 g
Smoked Tofu:	87.9310 g
Chickpeas:	0.00000 g
Oil:	0.00000 g

<u>Code:</u>

```
MIN 21 X1 + 16 X2 + 40 X3 + 41 X4 + 585 X5 + 120 X6 + 164 X7 + 884 X8
ST
        .85 X1 + 1.62 X2 + 2.86 X3 + .93 X4 + 23.4 X5 + 16 X6 + 9 X7 > 15
        .33 X1 + .2 X2 + .39 X3 + .24 X4 + 48.7 X5 + 5 X6 + 2.6 X7 + 100 X8 > 2
        .33 X1 + .2 X2 + .39 X3 + .24 X4 + 48.7 X5 + 5 X6 + 2.6 X7 + 100 X8 < 8
        4.64 X1 + 2.37 X2 + 3.63 X3 + 9.58 X4 + 15 X5 + 3 X6 + 27 X7 > 4
        9 X1 + 28 X2 + 65 X3 + 69 X4 + 3.8 X5 + 120 X6 + 78 X7 < 200
        X2 + X3 - .4 X1 - .4 X4 - .4 X5 - .4 X6 - .4 X7 - .4 X8 > 0
        X1 > 0
        X2 > 0
        X3 > 0
        X4 > 0
        X5 > 0
        X6 > 0
        X7 > 0
        X8 > 0
END
```

III. What is the cost of the low calorie salad?

Cost: 0.574713 * $0.75 + 0.879310 * $2.15 = $2.32

Part B: Veronica realizes that it is also important to minimize the cost associated with the new salad. Unfortunately, some of the ingredients can be expensive. Determine the combination of ingredients that minimizes cost.

I. Formulate the problem as a linear program with an objective function and all constraints.

Variables:
X_1 = 100g of Tomato
X_2 = 100g of Lettuce
X_3 = 100g of Spinach
X_4 = 100g of Carrot
X_5 = 100g of Sunflower Seeds
X_6 = 100g of Smoked Tofu
X_7 = 100g of Chickpeas
X_8 = 100g of Oil

Objective Function:
Minimize $Z = x_1 + 0.75x_2 + 0.5x_3 + 0.5x_4 + 0.45x_5 + 2.15x_6 + 0.95x_7 + 2x_8$
where Z = cost

Constraints:

Protein:	$0.85x_1 + 1.63x_2 + 2.86x_3 + 0.93x_4 + 23.4x_5 + 16x_6 + 9x_7 \geq 15$
Fat:	$2 \leq 0.33x_1 + 0.2x_2 + 0.39x_3 + 0.24x_4 + 48.7x_5 + 5x_6 + 2.6x_7 + 100x_8 \leq 8$
Carbohydrates:	$4.64x_1 + 2.37x_2 + 3.63x_3 + 9.58x_4 + 15x_5 + 3x_6 + 27x_7 \geq 4$
Sodium:	$9x_1 + 28x_2 + 65x_3 + 69x_4 + 3.8x_5 + 120x_6 + 78x_7 \leq 200$
Leafy Greens:	$(x_2 + x_3) / (x_1 + x_4 + x_5 + x_6 + x_7 + x_8) \geq 0.4$

II. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

Optimal Solution:

Tomato:	0.000000 g
Lettuce:	0.000000 g
Spinach:	53.86590 g
Carrot:	0.000000 g
Sunflower Seeds:	9.302900 g
Smoked Tofu:	0.000000 g
Chickpeas:	125.3617 g
Oil:	0.000000 g

<u>Code:</u>

```
MIN X1 + .75 X2 + .5 X3 + .5 X4 + .45 X5 + 2.15 X6 + .95 X7 + 2 X8
ST
        .85 X1 + 1.62 X2 + 2.86 X3 + .93 X4 + 23.4 X5 + 16 X6 + 9 X7 > 15
        .33 X1 + .2 X2 + .39 X3 + .24 X4 + 48.7 X5 + 5 X6 + 2.6 X7 + 100 X8 > 2
        .33 X1 + .2 X2 + .39 X3 + .24 X4 + 48.7 X5 + 5 X6 + 2.6 X7 + 100 X8 < 8
        4.64 X1 + 2.37 X2 + 3.63 X3 + 9.58 X4 + 15 X5 + 3 X6 + 27 X7 > 4
        9 X1 + 28 X2 + 65 X3 + 69 X4 + 3.8 X5 + 120 X6 + 78 X7 < 200
        X2 + X3 - .4 X1 - .4 X4 - .4 X5 - .4 X6 - .4 X7 - .4 X8 > 0
        X1 > 0
        X2 > 0
        X3 > 0
        X4 > 0
        X5 > 0
        X6 > 0
        X7 > 0
        X8 > 0
END
```

III. How many calories are in the low cost salad?

<u>Calories:</u>
0.538659 * 40 + 0.093029 * 585 + 1.253617 * 164 = 281.561513 calories

Part C: Compare the results from part A and B. Veronica's goal is to create a Very Veggie Salad that is both low calorie and low cost. She would like to sell the salad for $5.00 and still have a profit of at least $3.00. However, if she can advertise that the salad has under 250 calories then she may be able to sell more.

I. Suggest some possible ways that she select a combination of ingredients that is "near optimal" for both objectives. This is a type of multi-objective optimization.

The solution depends on whether Veronica would rather minimize the calories or minimize the cost. If she wants to minimize the calories, add one more constraint for the cost to be under $2.00 (so she can have a profit of $3.00 while selling it for $5.00). If she wants to minimize the cost, add one more constraint for the calories to be under 250 (so she can advertise the salads as low-calorie and sell more).

II. What combination of ingredient would you suggest and what is the associated cost and calorie.

<u>Optimal Solution:</u>

Tomato:	0.00000 g
Lettuce:	0.00000 g
Spinach:	48.8706 g
Carrot:	0.00000 g
Sunflower Seeds:	9.08390 g
Smoked Tofu:	18.5476 g
Chickpeas:	94.5451 g
Oil:	0.00000 g

<u>Calories:</u>

0.488706 * 40 + 0.090838 * 585 + 0.185476 * 120 + 0.945451 * 164 = 249.9995

<u>Cost:</u>

0.488706 * $0.50 + 0.090838 * $0.45 + 0.185476 * $2.15 + 0.945451 * $0.95 = $1.58

III. Note: There is not one “right” answer. Discuss how you derived your solution.

I chose the solution to minimize cost. This way, Veronica is able to maximize her profit while still advertising her salads as low-calorie. She can now have a profit of per salad.

<u>Code:</u>

```
MIN X1 + .75 X2 + .5 X3 + .5 X4 + .45 X5 + 2.15 X6 + .95 X7 + 2 X8
ST
	.85 X1 + 1.62 X2 + 2.86 X3 + .93 X4 + 23.4 X5 + 16 X6 + 9 X7 > 15
	.33 X1 + .2 X2 + .39 X3 + .24 X4 + 48.7 X5 + 5 X6 + 2.6 X7 + 100 X8 > 2
	.33 X1 + .2 X2 + .39 X3 + .24 X4 + 48.7 X5 + 5 X6 + 2.6 X7 + 100 X8 < 8
	4.64 X1 + 2.37 X2 + 3.63 X3 + 9.58 X4 + 15 X5 + 3 X6 + 27 X7 > 4
	9 X1 + 28 X2 + 65 X3 + 69 X4 + 3.8 X5 + 120 X6 + 78 X7 < 200
	X2 + X3 - .4 X1 - .4 X4 - .4 X5 - .4 X6 - .4 X7 - .4 X8 > 0
	21 X1 + 16 X2 + 40 X3 + 41 X4 + 585 X5 + 120 X6 + 164 X7 + 884 X8 < 250
	X1 > 0
	X2 > 0
	X3 > 0
	X4 > 0
	X5 > 0
	X6 > 0
	X7 > 0
```

```
    X8 > 0
END
```

Problem 3. Solving Shortest path problem

A. What are the lengths of the shortest paths from vertex a to all other vertices.

NOTE: D + Edge indicates the distance from vertex A to the edge

DB	2.000000
DC	3.000000
DD	8.000000
DE	9.000000
DF	6.000000
DG	8.000000
DH	9.000000
DI	8.000000
DJ	10.000000
DK	14.000000
DL	15.000000
DM	17.000000
DA	0.000000

<u>Code used:</u>

```
max db + dc + dd + de + df + dg + dh + di + dj + dk + dl + dm
ST
    da = 0
    db - da <= 2
    dc - da <= 3
    dd - da <= 8
    dh - da <= 9
    da - db <= 4
    dc - db <= 5
    de - db <= 7
    df - db <= 4
    dd - dc <= 10
    db - dc <= 5
    dg - dc <= 9
    di - dc <= 11
    df - dc <= 4
    da - dd <= 8
```

dg - dd <= 2
dj - dd <= 5
df - dd <= 1
dh - de <= 5
dc - de <= 4
di - de <= 10
di - df <= 2
dg - df <= 2
dd - dg <= 2
dj - dg <= 8
dk - dg <= 12
di - dh <= 5
dk - dh <= 10
da - di <= 20
dk - di <= 6
dj - di <= 2
dm - di <= 12
di - dj <= 2
dk - dj <= 4
dl - dj <= 5
dh - dk <= 10
dm - dk <= 10
dm - dl <= 2

END

B. If a vertex z is added to the graph for which there is no path from vertex a to vertex z, what will be the result when you attempt to find the lengths of shortest paths as in part a).

The function will become unbounded since there are no paths and therefore no constraint for the new vertex.

C. What are the lengths of the shortest paths from each vertex to vertex m. How can you solve this problem with just one linear program?

I solved it by reversing all of the edges and finding the shortest path from m to all of the other vertices.

DA	17.000000
DB	15.000000
DC	15.000000
DD	12.000000
DE	19.000000
DF	11.000000
DG	14.000000
DH	14.000000
DI	9.000000
DJ	7.000000
DK	10.000000

<u>Code:</u>

```
max da + db + dc + dd + de + df + dg + dh + di + dj + dk + dl + dm
ST
	dm = 0
	da - db <= 2
	da - dc <= 3
	da - dd <= 8
	da - dh <= 9
	db - da <= 4
	db - dc <= 5
	db - de <= 7
	db - df <= 4
	dc - dd <= 10
	dc - db <= 5
	dc - dg <= 9
	dc - di <= 11
	dc - df <= 4
	dd - da <= 8
	dd - dg <= 2
	dd - dj <= 5
	dd - df <= 1
	de - dh <= 5
	de - dc <= 4
	de - di <= 10
	df - di <= 2
	df - dg <= 2
	dg - dd <= 2
	dg - dj <= 8
	dg - dk <= 12
```

```
    dh - di <= 5
    dh - dk <= 10
    di - da <= 20
    di - dk <= 6
    di - dj <= 2
    di - dm <= 12
    dj - di <= 2
    dj - dk <= 4
    dj - dl <= 5
    dk - dh <= 10
    dk - dm <= 10
    dl - dm <= 2
END
```

Part D.Suppose that all paths must pass through vertex i. How can you calculate the length of the shortest path from any vertex x to vertex y that pass through vertex i (for all x,y Î V)? Calculate the lengths of these paths for the given graph. (Note for some vertices x and y it may be impossible to pass through vertex i).

Calculate length of all to i.
No path to vertices l and m to i since the result was unbounded

DA	8.000000
DB	6.000000
DC	6.000000
DD	3.000000
DE	10.000000
DF	2.000000
DG	5.000000
DH	5.000000
DI	0.000000
DJ	2.000000
DK	15.000000

<u>Code:</u>

```
max da + db + dc + dd + de + df + dg + dh + di + dj + dk
ST
    di = 0
    da - db <= 2
    da - dc <= 3
    da - dd <= 8
    da - dh <= 9
```

```
    db - da <= 4
    db - dc <= 5
    db - de <= 7
    db - df <= 4
    dc - dd <= 10
    dc - db <= 5
    dc - dg <= 9
    dc - di <= 11
    dc - df <= 4
    dd - da <= 8
    dd - dg <= 2
    dd - dj <= 5
    dd - df <= 1
    de - dh <= 5
    de - dc <= 4
    de - di <= 10
    df - di <= 2
    df - dg <= 2
    dg - dd <= 2
    dg - dj <= 8
    dg - dk <= 12
    dh - di <= 5
    dh - dk <= 10
    di - da <= 20
    di - dk <= 6
    di - dj <= 2
    di - dm <= 12
    dj - di <= 2
    dj - dk <= 4
    dj - dl <= 5
    dk - dh <= 10
    dk - dm <= 10
    dl - dm <= 2
END
```

Calculate length of i to all
No path to vertices l and m from i since the results were unbounded to i

```
DA    20.000000
DB    22.000000
DC    23.000000
DD    28.000000
DE    29.000000
```

DF	26.000000
DG	28.000000
DH	16.000000
DJ	2.000000
DK	6.000000
DL	7.000000
DM	9.000000
DI	0.000000

<u>Code:</u>

```
max da + db + dc + dd + de + df + dg + dh + dj + dk + dl + dm
ST
        di = 0
        db - da <= 2
        dc - da <= 3
        dd - da <= 8
        dh - da <= 9
        da - db <= 4
        dc - db <= 5
        de - db <= 7
        df - db <= 4
        dd - dc <= 10
        db - dc <= 5
        dg - dc <= 9
        di - dc <= 11
        df - dc <= 4
        da - dd <= 8
        dg - dd <= 2
        dj - dd <= 5
        df - dd <= 1
        dh - de <= 5
        dc - de <= 4
        di - de <= 10
        di - df <= 2
        dg - df <= 2
        dd - dg <= 2
        dj - dg <= 8
        dk - dg <= 12
        di - dh <= 5
```

```
    dk - dh <= 10
    da - di <= 20
    dk - di <= 6
    dj - di <= 2
    dm - di <= 12
    di - dj <= 2
    dk - dj <= 4
    dl - dj <= 5
    dh - dk <= 10
    dm - dk <= 10
    dm - dl <= 2
END
```

NOTE: DL and DM are not present since there was no path
Total Length
DA = 28
DB = 28
DC = 29
DD = 31
DE = 39
DF = 28
DG = 33
DH = 21
DI = 0
DJ = 4
DK = 21