Answer: 1

Strings are :-

- $A_1 = 11101111$
- $A_2 = 00010100$
- $A_3 = 01000011$

Schemata are :-

- $H_1 = 1{*}{*}{*}{*}{*}{*}{*}$
- $H_2 = 0{*}{*}{*}{*}{*}{*}{*}$
- $H_3 = {*}{*}{*}{*}{*}{*}11$
- $H_4 = {*}{*}{*}0{*}01{*}$
- $H_5 = 1{*}{*}{*}{*}{*}1{*}$

Schemata and String matching :-

- $H_1 = A_1$
- $H_2 = A_2, A_3$
- $H_3 = A_1, A_3$
- $H_4 = A_3$
- $H_5 = A_1$

Order and Length of Schemata :- $\delta(H)$ is length, and $o(H)$ is Order.

- $H_1 = \delta(H_1) = 0$, $o(H_1) = 1$
- $H_2 = \delta(H_2) = 0$, $o(H_2) = 1$
- $H_3 = \delta(H_3) = 1$, $o(H_3) = 2$
- $H_4 = \delta(H_4) = 3$, $o(H_4) = 3$
- $H_5 = \delta(H_5) = 6$, $o(H_5) = 2$

Survival of each Schemata under Mutation :-

Probability of single mutation is $p_m = 0.001$ For a schema H to survive, all fixed bits must remain unchanged.

The probability of a schema H survives under mutation is

$$S_m(H) = (1 - p_m)^{o(H)} \tag{1}$$

We can rewrite according to given p_m is

$$S_m(H) = (1 - 0.001)^{o(H)}$$

$$S_m(H) = (0.999)^{o(H)}$$

Taking $o(H)$ of schemeta from above calculated.

- $S_m(H_1) = (0.999)^1 = 0.999$
- $S_m(H_2) = (0.999)^1 = 0.999$
- $S_m(H_3) = (0.999)^2 = 0.998$
- $S_m(H_4) = (0.999)^3 = 0.997$
- $S_m(H_5) = (0.999)^2 = 0.998$

Survival of each Schemata under Cross Over :-

Probability of Crossover = $p_c = 0.85$
The probability of a schema H survives under cross over is

$$S_c(H) \geqslant 1 - p_c \frac{\delta(H)}{l - 1} \tag{2}$$

as l = 8, we can rewrite this as

$$S_c(H) \geqslant 1 - (0.85)\frac{\delta(H)}{7}$$

Taking $\delta(H)$ of schemeta from above calculated.

- $S_c(H_1) \geqslant 1 - (0.85)\frac{0}{7} = 1$
- $S_c(H_2) \geqslant 1 - (0.85)\frac{0}{7} = 1$
- $S_c(H_3) \geqslant 1 - (0.85)\frac{1}{7} = 0.878$
- $S_c(H_4) \geqslant 1 - (0.85)\frac{3}{7} = 0.635$
- $S_c(H_5) \geqslant 1 - (0.85)\frac{6}{7} = 0.271$

Answer: 2

#	String	Fitness
1.	10001	20
2.	11100	10
3.	00011	5
4.	01110	15

Schemata are :-

$$H_1 = 1****$$
$$H_2 = 0**1*$$

$p_m = 0.01$
$p_c = 0.7$

Expectation of Schema H in Next Generation is:-

$$E[m(H, k+1)] \geqslant m(H, k)\frac{f(H, k)}{\bar{f}}(1 - p_c\frac{\delta(H)}{l-1})(1 - p_m)^{o(H)} \tag{3}$$

Where :-

$$m(H, k) = \text{denotes the number of instances of H in the k-th generation}$$

$$f(H, k) = \text{denotes average fitness of H in the k-th generation}$$

$$\bar{f} = \text{denotes avergae fitness of population in k-th generation}$$

Expected number of Schema H_1 in generation 1 :-

$$H_1 = 1****$$

$$k + 1 = 1, k = 0$$

There are two string 1 and 2 belongs to schema H_1, and there fitness are 20 and 10

$$m(H_1, 0) = 2$$

$$f(H_1, 0) = (20 + 10)/2 = 15$$

$$\bar{f} = 50/4$$

$$l = 5$$

$$\delta(H_1) = 0$$

$$o(H_1) = 1$$

Using equation 3 for calculation of expectation and putting corresponding values :-

$$E[m(H_1, 1)] \geqslant 2 * \frac{15}{50/4} * (1 - 0.7 * \frac{0}{4})(1 - 0.01)^1$$

$$\boxed{E[m(H_1, 1)] \geqslant 2.376}$$

Expected number of Schema H_2 in generation 1 :-

$$H_2 = 0{*}{*}1{*}$$

$$k + 1 = 1, k = 0$$

There are two string 3 and 4 belongs to schema H_2, and there fitness are 5 and 15

$$m(H_2, 0) = 2$$

$$f(H_2, 0) = (5 + 15)/2 = 10$$

$$\bar{f} = 50/4$$

$$l = 5$$

$$\delta(H_2) = 3$$

$$o(H_2) = 2$$

Using equation 3 for calculation of expectation and putting corresponding values :-

$$E[m(H_2, 1)] \geqslant 2 * \frac{10}{50/4} * (1 - 0.7 * \frac{3}{4})(1 - 0.01)^2$$

$$\boxed{E[m(H_2, 1)] \geqslant 0.744}$$