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Lecture 11

In this lecture, we will do a brief survey of topics concerning asymmetric information. This, as the name implies, is simply a situation in which two or more parties have differing information regarding a certain environment.

Obviously such a thing is quite common in real life. However, it frequently makes solving economic models much more difficult, so it is often assumed away.

Lemons: A common instance of asymmetric information is the buying and selling of goods. If I offer to sell you a car, you may have a limited idea of how well the car is functioning, while I, having been driving it for some time, am the know.

Let's consider a concrete example. There are 100 buyers and 100 sellers in a market. Each seller has one car. There are two types of cars: high quality cars ("plums") and low quality cars ("lemons"). Of the sellers, 50 have plums and 50 have lemons, but only the sellers know which car is which!

We'll use a very simple utility form.

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Agents have linear utility in money and some fixed value of owning a plum V_P and a fixed value of owning a lemon V_L .

For sellers, let $V_L^S = 1000$ and $V_P^S = 2000$.

For buyers, let $V_L^B = 1200$ and $V_P^B = 2400$.

Assume that everyone gets no additional utility from owning a second car.

First, consider an ideal world where everyone knows the quality of every car. In this case, all cars will be sold.

The price of a lemon will be between 1000 and 1200, and the price of a plum will be between 2000 and 2400.

This is in fact the Pareto efficient allocation since for each type of car, buyers value them more than sellers, so it is efficient to sell every car.

Now consider the true asymmetric information case where the quality of the car is known only to the seller. Suppose all sellers sell their car, then a buyer should expect a 50% chance of a lemon and a 50% chance of a plum. His expected gain is then:

$$\frac{1}{2} \cdot 2400 + \frac{1}{2} \cdot 1200 = 1800$$

So he will pay at most 1800 for a car. But a seller with a plum will sell

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his car for no less than 2000. Thus it cannot be that both types sell.

Suppose only plums sell. Then the price would be between 2000 and 2400. In this case, lemons would also want to sell.

Thus the last option we are left with is that only lemons sell. In this case the price would be between 1000 and 1200, and indeed plums would not like to sell for those prices.

Is this Pareto efficient? Well, it's not clear. Obviously, we do not do as well as if we knew what types of cars were which. But if we didn't, then we would have to compensate both types of sellers the same amount, in which case we can do no better than the market outcome.

(This "lemons" model was introduced by George Akerloff. He got a Nobel prize for it.)

Is there any way out of this situation? One is trustworthy mechanics. This costs money and maybe is manipulable. Another is a warranty. Suppose the sale price was 2100, but if the car turns out to be a lemon, then the seller pays the buyer 1000. In this case, the effective price is 1100 for the lemon. Thus 2100 is between 2000 and 2400 and 1100 is between 1000 and 1200.

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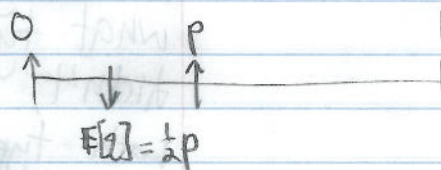
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Ex.: Lemons with a continuum of qualities

Suppose now the quality instead of being high or low is some $q \in [0, 1]$ and that q is uniformly distributed.

The sellers value a car of quality q at simply q , while buyers value a quality q car at $aq+b$.

We seek a market price p . If the market price is p , any seller with $q < p$ will sell, so that the expected quality is $\frac{1}{2}p$.



So the expected value to a buyer is

$$E[v_b] = \frac{1}{2}ap + b$$

Thus a buyer wants to buy if

$$\frac{1}{2}ap + b > p$$

To find an equilibrium, we make the buyers indifferent between buying and selling, so that

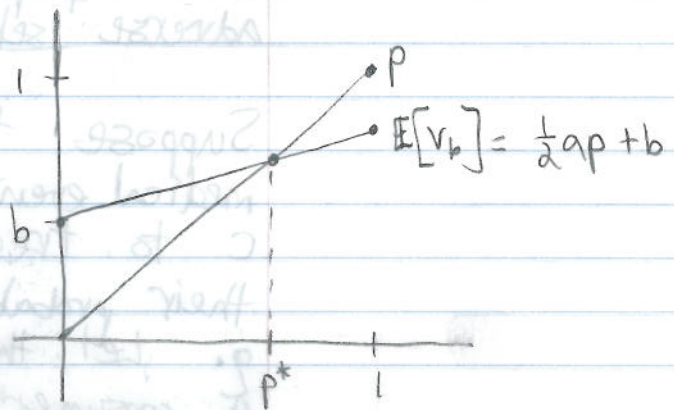
$$p = \frac{b}{1 - \frac{1}{2}a}$$

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Consider the case where $b=0$. Then if $a < 2$, for any p , buyers do not wish to buy, so the market totally collapses. If $a \geq 2$, then buyers always want to buy and we get the efficient outcome.

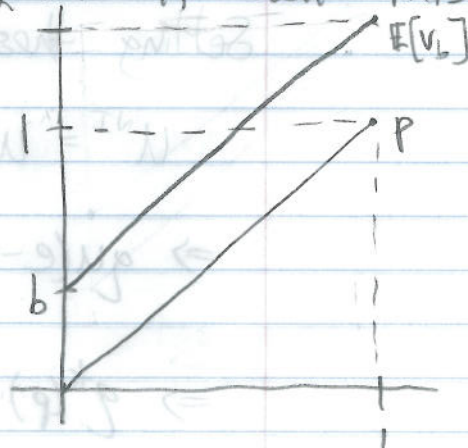
Now let $b > 0$. If $a < 2$, we get



(this) is $p^* = \frac{b}{1 - \frac{1}{2}a}$. Given this

price, a fraction p^* of the cars will get sold. This is inefficient.

If $\frac{1}{2}a + b \geq 1$, then this becomes



So that $p^* \geq 1$ and all cars sell.

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Adverse Selection: A very similar dynamic to the lemons market occurs in the insurance market as well. If insurance companies (say health) cannot observe the health of their customers (or are legally prevented from using it as a factor), then by setting a single price for insurance coverage, only the sickest people will buy it. This is called adverse selection.

Suppose there is only one type of medical event that can occur and it costs c to treat. People are characterized by their probability of this event occurring, q . Let the price of insurance be p . A consumer's utility without insurance is

$$U^{NI} = qu(e-c) + (1-q)u(e)$$

while utility with insurance is

$$U^I = u(e-p)$$

Setting these equal

$$U^{NI} = U^I$$

$$\Rightarrow qu(e-c) + (1-q)u(e) = u(e-p)$$

$$\Rightarrow q^*(p) = \frac{u(e) - u(e-p)}{u(e) - u(e-c)}$$

So given a price p , any consumer

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with $q > q^*(p)$ will buy insurance, while those with $q < q^*(p)$ will not. Notice that $q^*(p)$ is increasing since

$$\frac{\partial q^*}{\partial p} = \frac{u'(e-p)}{u(e) - u(e-c)} > 0$$

Furthermore, $q^*(0) = 0$ and $q^*(c) = 1$, that is, everyone will buy fire insurance and no one will buy insurance that costs as much as the procedure.

We assume a competitive insurance market so that profits are equal to zero. Given a price p and a distribution over q , $f(q)$, firm profits are:

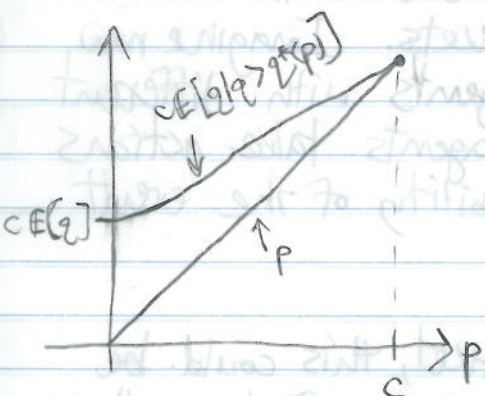
$$\pi = \int_{q^*(p)}^1 [p - qc] f(q) dq$$

Setting profits to zero yields

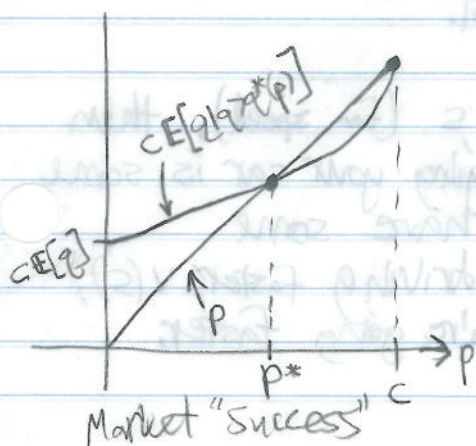
$$p = c \cdot E[q | q > q^*(p)]$$

The price is equal to the firm's expected outlay. One solution to the above equation is $p = c$, since $q^*(c) = 1$, so that $E[q | q > q^*(c)] = 1$. This means no one buys insurance, a total market breakdown.

This example is a little cooked up but there is certainly some truth to it. How can we get around this?



Market Failure



Market "Success"

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One possibility is to mandate that everyone buy insurance. We essentially enforce $q^* = 0$. Firm profits will then be:

$$\pi = p - c \cdot E[q]$$

In a competitive setting then $p = c \cdot E[q]$.

In the insurance industry, the competitiveness assumption may break down. Furthermore, in the US, health insurance is often provided through employers, which could change the dynamic. Even then, small employers are at risk of being dropped.

Moral Hazard: A very similar dynamic occurs in other types of insurance markets. Imagine now that instead of various agents with different probabilities of an event, agents take actions that can affect the probability of the event occurring.

In the auto insurance market, this could be people choosing how fast to drive affecting their probability of an accident. Alternatively, in the health insurance market, it be people choosing whether to smoke or not.

Let this parameter be s (for speed), then the probability of damaging your car is some function $q(s)$, and you have some intrinsic utility from driving faster $v(s)$, i.e., you get to where you're going faster.

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Without insurance, your utility is

$$U(s) = q(s)u(e-c) + (1-q(s))u(e) + v(s)$$

We can solve for the optimal s :

$$\frac{\partial U}{\partial s} = q'(s) \cdot [u(e-c) - u(e)] + v'(s) = 0$$

$$\Rightarrow v'(s) = q'(s) \cdot [u(e) - u(e-c)]$$

Now imagine you can buy full insurance for price p . Your utility is simply

$$U(s) = u(e-p) + v(s)$$

So you will drive as fast as possible. In that case, the insurance company would charge a price of c , in which case you would not want to buy insurance.

What if the insurance company offered partial insurance. That is, if you damage your car, they pay you some amount $x < c$. This is equivalent to a deductible, where you pay some portion of the damages.

Now utility is

$$U(s) = q(s)u(e-c+x-p) + (1-q(s))u(e-p) + v(s)$$

Solving for the optimal speed:

$$\frac{\partial U}{\partial s} = q(s) \cdot [u(e-c+x-p) + u(e-p)] + v'(s) = 0$$

$$\Rightarrow v'(s) = q'(s) \cdot [u(e-p) - u(e-p-(c-x))]$$

Similar to before. The profits of the insurance firm will be

$$\pi = p - q(s(p, x)) \cdot x$$

Zero profit implies

$$p = q(s(p, x)) \cdot x$$

This can be solved for fixed x . We might also be interested in the firm's optimal choice of x .

$$\frac{\partial \pi}{\partial x} = -q(s(p, x)) - q'(s(p, x)) \cdot s_x(p, x) \cdot x < 0$$

So firms will always want to provide a bit less coverage. In some sense though, any x is sustainable, except for $x=c$, so long as the consumer prefers to buy it.

Ex: Let $s \in [0, 1]$ and $q(s) = s$

$$\text{Use } v(s) = \alpha [1 - \exp(-s)]$$

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The first order condition for the optimal s becomes

$$\alpha \exp(-s) = \exp(-(e-p-(c-x))) - \exp(-(e-p)) \\ = \exp(-(e-p)) \cdot [\exp(c-x) - 1]$$

The zero profit condition is $p = s \cdot x$.
Plugging this in:

$$\alpha \exp(-s) = \exp(-(e-sx)) \cdot [\exp(c-x) - 1]$$

$$\Rightarrow \alpha \exp(e - (1+x)s) = \exp(c-x) - 1$$

$$\Rightarrow e - (1+x)s = \log\left[\frac{\exp(c-x) - 1}{\alpha}\right]$$

$$\Rightarrow s = \left(\frac{1}{1+x}\right) \left[e - \log\left[\frac{\exp(c-x) - 1}{\alpha}\right]\right]$$

Notice that s increases with your preference for speed (α), decreases with cost (c) and increases with the coverage level (x).

The price will be:

$$p = sx = \left(\frac{x}{1+x}\right) \left[e - \log\left[\frac{\exp(c-x) - 1}{\alpha}\right]\right]$$

This is increasing in x , as expected.

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Let's put in some specific numbers:

$$c = 8$$

$$x = 3$$

$$e = 10$$

$$\alpha = \frac{1}{100}$$

$$S = \left(\frac{1}{4}\right) \cdot \left[10 - \log\left(\frac{\exp(S) - 1}{\frac{1}{100}}\right)\right] = 0.10$$

$$p = SX = 0.10 \cdot 3 = 0.30$$

What happens without insurance? We can find this by simply looking at when $x = 0$. Then

$$S = e - \log\left(\frac{\exp(c) - 1}{\alpha}\right)$$

Using the above numbers, we actually just get $S = 0$. However, the agent is better off buying insurance.

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Signaling: This is a situation where an agent takes an action that, though not inherently beneficial to themselves, reveals to others some piece of information.

The classic example described by Michael Spence is that of educational signalling.

Suppose we have a large number of agents and each agent has a certain skill level, which is unobservable. Only the agent knows his or her skill level, and there are no reliable tests to determine skill.

This skill manifests itself in two ways. First, higher skill workers are more productive and would be paid more in a full information world. Second, education is less costly for higher skilled agents, perhaps they don't have to study as much.

In this setting, higher skilled workers will get more education, and employers, knowing that high skill workers get more educated will pay higher wages. We just need to ensure that low skill agents do not have an incentive to imitate high skill workers and get highly educated.

Notice that this logic holds even if getting educated does not increase your productivity at all.