Problem #1

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$$T_1 = \begin{bmatrix} -0.5 & \frac{\sqrt{3}}{2} & 0 & 0 \\ -\frac{\sqrt{3}}{2} & 0.5 & 0 & 3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_2 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 2 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_3 = \begin{bmatrix} 0 & \frac{1}{2} & \frac{\sqrt{3}}{2} & \sqrt{3} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} & 4 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = T_1 \cdot T_2 .$$

$$\begin{bmatrix} 0.5 & R_{12} & R_{13} \\ R_{21} & R_{22} & -0.5 \\ R_{31} & R_{23} & 0.5 \end{bmatrix}$$

Problem #2.

Using Python to solve the problem, one answer is

$$\begin{bmatrix} 0.5 & -0.5 & -\frac{\sqrt{2}}{2} \\ 0.146 & 0.8536 & -0.5 \\ 0.8536 & 0.146 & 0.5 \end{bmatrix}$$

There are four answers in total

It is not unique, another solution is

$$v_b^2 = v_b^3 \times v_b^1 = [-0.5,\ 0.146,\ 0.8535]$$

$$R_{21} = \frac{\sqrt{2} \pm 2}{4} \qquad \begin{bmatrix} 0.5 & -0.5 & \frac{\sqrt{2}}{2} \\ 0.8535 & 0.146 & -0.5 \\ 0.146 & 0.8535 & 0.5 \end{bmatrix}$$

$$R_{21} = R_{31} + \frac{\sqrt{2}}{2}.$$

$$R_{13} = \frac{\sqrt{2}}{2}. \qquad \frac{\sqrt{2}}{4} - \frac{1}{2} R_{21} + \frac{1}{2} R_{31} = 0. \qquad \frac{\sqrt{2}}{2} - R_{21} + R_{31} = 0.$$

$$\frac{1}{4} + R_{21}^2 + R_{31}^2 = 1. \qquad R_{31} = \frac{-\sqrt{2} \pm 2}{4}.$$

Problem #3

$$\frac{1}{4} + R_{31}^2 + \sqrt{2} R_{31} + \frac{1}{2} + R_{31}^2 = 1.$$

$$d(t) = \begin{bmatrix} \cos(t) \\ \sin(t) \\ \sin(t) \end{bmatrix} \qquad R(t) = \begin{bmatrix} \cos(10t) & -\sin(10t) & 0 \\ \sin(10t) & \cos(10t) & 0. \\ 0 & 0 & 1 \end{bmatrix}$$

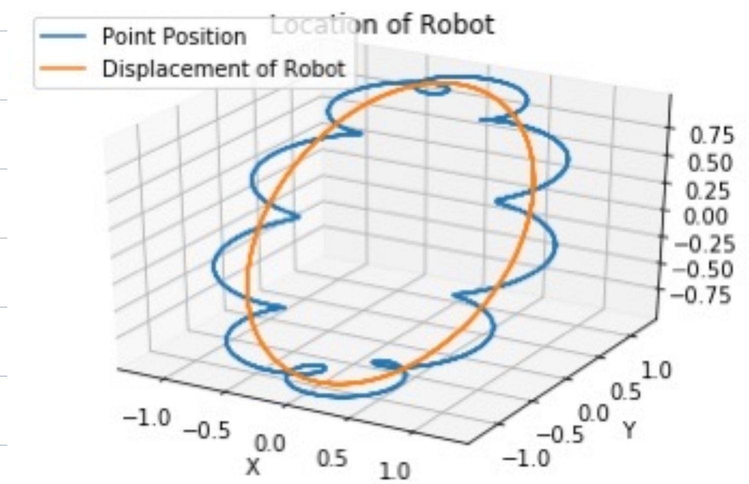
$$\text{a)}\ {}^{A}v(t) = \frac{d\,{}^{A}d(t)}{dt} = \begin{bmatrix} -\sin(t) \\ \cos(t) \\ \cos(t) \end{bmatrix}.$$

$$\text{(b)}\ {}^{b}\hat{\omega}(t) = R^{T} \cdot \dot{R} = \begin{bmatrix} 0 & -10 & 0 \\ 10 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \qquad {}^{b}\omega(t) = \begin{bmatrix} 0 \\ 0. \\ 10 \end{bmatrix}.$$

$$\text{(c)}.\ {}^{A}p(t) = A\ {}^{B}p(t) = \begin{bmatrix} \cos(10t) & -\sin(10t) & 0 & \cos t \\ \sin(10t) & \cos(10t) & 0 & \sin t \\ 0 & 0. & 1 & \sin t. \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos t + c\cos 10t \\ \sin t + c\sin 10t \\ \sin t \\ 1 \end{bmatrix} . \quad {}^{A}P(t) = \begin{bmatrix} \cos t + c\cos 10t \\ \sin t + c\sin 10t \\ \sin t \end{bmatrix}$$

d).



e). $${}^{A}\dot{P} = \frac{d\,{}^{A}P}{dt} = \begin{bmatrix} -\sin t - 10c\sin 10t \\ \cos t + 10c\cos 10t \\ \cos t \\ \end{bmatrix}$$

If $c = 0$, $${}^{A}\dot{P} = \begin{bmatrix} -\sin t \\ \cos t \\ \cos t \end{bmatrix}$$ which is equal to $\dot{d}(t)$.

f). $^{A}\ddot{p} = \frac{d\,^{A}\dot{p}}{dt} = \begin{bmatrix} -\cos t - 100c\cos 10t \\ -\sin t - 100c\sin 10t \\ -\sin t \end{bmatrix}$.