QOSF Task1

November 11, 2024

1 Statevector simulation of quantum circuits

To write a statevector simulation of the quantum circuit having gates $CNOT$, X and H gates, we first construct a quantum circuit of specified qubits having these gates. A large number of randomizations strategies can be employed to create such a circuit. For the purpose of this task we chose to implement CNOT gates on successive states. We also chose to keep the depth of the circuit as 1 while varying the number of qubits and gates. Also, the number of gates with 2 qubits and 1 qubit are randomized.

We simulated the calcluation of statevector when circuit gates are applied. Kronecker product was used to calculate the final transformaion matrix. If each gate is represented by M_i then the resultant transformation matrix after this operation can be represented as $M = M_1 \otimes M_2 \otimes M_3 ... M_n$. The size of the matrix M was found to grow exponentially. Then combined gate M was then applied to the state vector $M|\psi>$

Experimentally it was observed that for an AMD 5800 Processor 3.2GHz machine, maximum number of qubits that can be simulated in this circuit arrangement were 14 within an upper time limit of 10seconds.

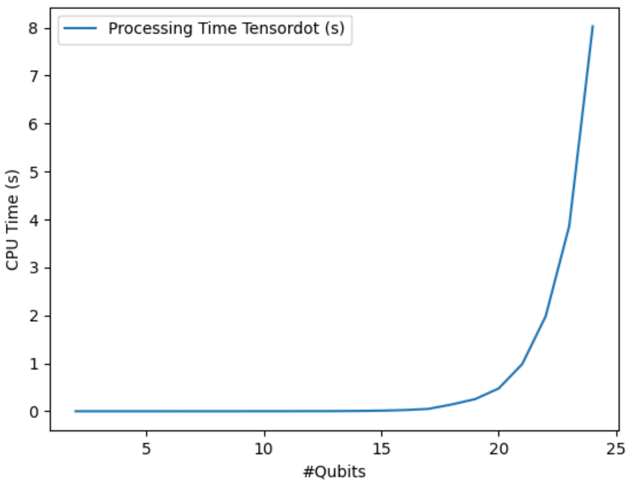

Figure 1: Processing time for simulation of qubits increases exponentially when using Kronecker product of gate matrices. Within a time limit of 10 seconds, upto 14 qubits could be simulated. For more than 15 qubits the computation takes prohibitively large amount of virtual memory and processing time, hence 10second limit was chosen for comparison.

2 Advanced simulation using tensor multiplication

To experiment with a faster simulation method for quantum circuits, the state tensor was represented in multidimensional tensor form. Each gate was applied sequentially without computing the combined gate transformation. This trick was found to increase the number of qubits that can be simulated with 10 seconds to 24. Tensordot function from numpy was used for contracting tensors.

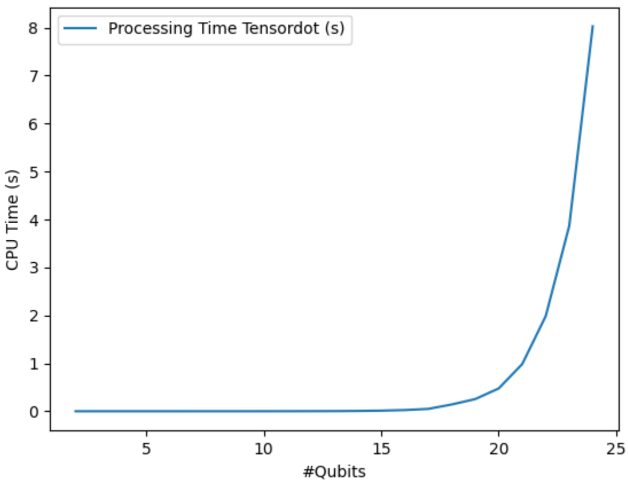

Figure 2: When numpy tensordot was used instead of Kronecker product, the complexity remains exponential however a significant increase in number of qubits that can be simulated is observed. 24 qubits were simulated within 10 seconds. The virtual memory consumption was found to have decreased as compared to method where Kronecker product was used.

3 Bonus question

The components of the state vector represent the amplitudes of the corresponding computational basis states. To sample from the the statevector, we computed the inner product of conjugate transpose of statevector and the statevector itself. The resultant vector was normalized to unit magnitude. The resultant normalized vector was used to sample from a proabability distribution with density corresponding to magnitude of elements of the vector. The samples were converted to binary representation for better interpretability.

More formally, given the quantum state of an n-qubit system represented as a statevector, we can sample from the state by performing a measurement in the computational basis. The process involves the following steps:

The quantum state of an n-qubit system can be written as:

$$|\Psi\rangle = \sum_{i=0}^{2^n-1} \alpha_i |i\rangle$$

where α_i are the complex probability amplitudes, and $|i\rangle$ represents the computational basis states.

The probability $P(i)$ of measuring the state $|i\rangle$ is given by the squared modulus of the corresponding amplitude:

$$P(i) = |\alpha_i|^2$$

where i runs over all computational basis states from 0 to $2^n - 1$.

To sample from the quantum state, generate a random number $r \in [0, 1]$ and choose the state corresponding to $P(i)$ based on the cumulative distribution of the probabilities.

The statevector $|\psi>$ represents the quantum state of the system at a given point in time. To compute the exact expectation $< \psi|Op|\psi >$, the operator Op was computed. While a number of observable operators can be construcuted X gate was used for constructing the operator in this task. The statevector computed after tensor product with circuit gates was used as ψ. The state $|\psi>$ contains the probability amplitudes for the system's possible states. The representation encodes the full quantum information including all possible quantum superpositions, therefore the calculation of the expectation value is exact. Due to the representation limits floating point arithmetic, there may be numerical error in the computation.