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- A homogeneous equation is always consistent. TRUE - The trivial solution is always a solution.
- The equation $A\mathbf{x} = \mathbf{0}$ gives an explicit descriptions of its solution set. FALSE - The equation gives an implicit description of the solution set.
- The homogeneous equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution if and only if the equation has at least one free variable. FALSE - The trivial solution is always a solution to the equation $A\mathbf{x} = \mathbf{0}$.
- The equation $\mathbf{x} = \mathbf{p} + t\mathbf{v}$ describes a line through $\mathbf{v}$ parallel to $\mathbf{p}$. False. The line goes through $\mathbf{p}$ and is parallel to $\mathbf{v}$.
- The solution set of $A\mathbf{x} = \mathbf{b}$ is the set of all vectors of the form $\mathbf{w} = \mathbf{p} + \mathbf{v}_h$ where $\mathbf{v}_h$ is any solution of the equation $A\mathbf{x} = \mathbf{0}$ FALSE This is only true when there exists some vector $\mathbf{p}$ such that $A\mathbf{p} = \mathbf{b}$.

- If $\mathbf{x}$ is a nontrivial solution of $A\mathbf{x} = \mathbf{0}$, then every entry in $\mathbf{x}$ is nonzero. FALSE. At least one entry in $\mathbf{x}$ is nonzero.
- The equation $\mathbf{x} = x_2\mathbf{u} + x_3\mathbf{v}$, with x_2 and x_3 free (and neither $\mathbf{u}$ or $\mathbf{v}$ a multiple of the other), describes a plane through the origin. TRUE
- The equation $A\mathbf{x} = \mathbf{b}$ is homogeneous if the zero vector is a solution. TRUE. If the zero vector is a solution then $\mathbf{b} = A\mathbf{x} = A\mathbf{0} = \mathbf{0}$. So the equation is $A\mathbf{x} = \mathbf{0}$, thus homogenous.
- The effect of adding $\mathbf{p}$ to a vector is to move the vector in the direction parallel to $\mathbf{p}$. TRUE. We can also think of adding $\mathbf{p}$ as sliding the vector along $\mathbf{p}$.
- The solution set of $A\mathbf{x} = \mathbf{b}$ is obtained by translating the solution set of $A\mathbf{x} = \mathbf{0}$. FALSE. This only applies to a consistent system.

- The columns of the matrix A are linearly independent if the equation $A\mathbf{x} = \mathbf{0}$ has the trivial solution. FALSE. The trivial solution is always a solution.
- If S is a linearly dependent set, then each vector is a linear combination of the other vectors in S. FALSE- For example, $[1, 1]$, $[2, 2]$ and $[5, 4]$ are linearly dependent but the last is not a linear combination of the first two.
- The columns of any 4×5 matrix are linearly dependent. TRUE. There are five columns each with four entries, thus by Thm 8 they are linearly dependent.
- If $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, and if $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent, then $\mathbf{z}$ is in Span$\{\mathbf{x}, \mathbf{y}\}$. TRUE Since $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, and $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent, it must be that $\mathbf{z}$ can be written as a linear combination of the other two, thus in in their span.

Section 1.7 22

- Two vectors are linearly dependent if and only if they lie on a line through the origin. TRUE. If they lie on a line through the origin then the origin, the zero vector, is in their span thus they are linearly dependent.
- If a set contains fewer vectors then there are entries in the vectors, then the set is linearly independent. FALSE For example, $[1, 2, 3]$ and $[2, 4, 6]$ are linearly dependent.
- If $\mathbf{x}$ and $\mathbf{y}$ are linearly independent, and if $\mathbf{z}$ is in the Span$\{\mathbf{x}, \mathbf{y}\}$ then $\{\mathbf{x}, \mathbf{y}, \mathbf{z}\}$ is linearly dependent. TRUE If $\mathbf{z}$ is in the Span$\{\mathbf{x}, \mathbf{y}\}$ then $\mathbf{z}$ is a linear combination of the other two, which can be rearranged to show linear dependence.

Section 1.7 22 Continued

- If a set in $\mathbb{R}^n$ is linearly dependent, then the set contains more vectors than there are entries in each vector. False. For example, in $\mathbb{R}^3$ $[1, 2, 3]$ and $[3, 6, 9]$ are linearly dependent.

- A linear transformation is a special type of function. TRUE The properties are (i) $T(\mathbf{u}+\mathbf{v}) = T(\mathbf{u}) + T(\mathbf{v})$ and (ii) $T(c\mathbf{u}) = cT(\mathbf{u})$.
- If A is a 3×5 matrix and T is a transformation defined by $T(\mathbf{x}) = A\mathbf{x}$, then the domain of T is $\mathbb{R}^3$. FALSE The domain is $\mathbb{R}^5$.
- If A is an $m \times n$ matrix, then the range of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is $\mathbb{R}^m$ FALSE $\mathbb{R}^m$ is the codomain, the range is where we actually land.
- Every linear transformation is a matrix transformation. FALSE. The converse (every matrix transformation is a linear transformation) is true, however. We (probably) will see examples of when the original statement is false later.

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- A transformation T is linear if and only if $T(c_1\mathbf{v}_1 + c_2\mathbf{v}_2) = c_1T(\mathbf{v}1) + c_2T(\mathbf{v}_2)$ for all $\mathbf{v}_1$ and $\mathbf{v}_2$ in the domain of T and for all scalars c_1 and c_2. TRUE If we take the definition of linear transformation we can derive these and if these are true then they are true for $c_1, c_2 = 1$ so the first part of the definition is true, and if $\mathbf{v} = 0$, then the second part if true.

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- Every matrix transformation is a linear transformation. TRUE To actually show this, we would have to show all matrix transformations satisfy the two criterion of linear transformations.
- The codomain of the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is the set of all linear combinations of the columns of A. FALSE The If A is $m \times n$ codomain is $\mathbb{R}^m$. The original statement in describing the range.
- If $T : \mathbb{R}^n \to \mathbb{R}^m$ is a linear transformation and if $\mathbf{c}$ is in $\mathbb{R}^m$, then a uniqueness question is "Is $\mathbf{c}$ is the range of T." FALSE This is an existence question.
- A linear transformation preserves the operations of vector addition and scalar multiplication. TRUE This is part of the definition of a linear transformation.
- The superposition principle is a physical description of a linear transformation. TRUE The book says so. (page 77)