Ratio Objectives

Source: Model Building in Mathematical Programming, 5th Edition (https://www.wiley.com/en-us/Model+Building+in+Mathematical+Programming%2C+5th+Edition-p-9781118443330)

Section 3.2.4

In some applications, the following non-linear objective arises:

$$\text{Maximize (or Minimize)} \quad \frac{\sum_j a_j x_j}{\sum_j b_j x_j}.$$

Rather surprisingly, the resultant model can be converted into a linear programming form by the following transformations.

1. . Replace the expression $\frac{1}{\sum_j b_j x_j}$ by a variable t.
2. . Represent the products $x_j t$ by variables w_j The objective now becomes $\text{Maximize} \quad \sum_j a_j w_j$
3. . Introduce a constraint $\sum_j b_j w_j = 1$ in order to satisfy condition 1. Convert the original constraints of the form $\sum_j d_j x_j <=> e$ to $\sum_j d_j w_j - et <=> 0.$

It must be pointed out that this transformation is only valid if the denominator $\sum_j b_j x_j$ is always of the same sign and non-zero. If necessary (and it is valid), an extra constraint must be introduced to ensure this. If $\sum_j b_j x_j$ always be negative the directions of the inequalities in the constraints above must, of course, be reversed.

Once the transformed model is solved, the values of the x_j variables can be found by dividing w_j by t.