HARARE INSTITUTE OF TECHNOLOGY

SCHOOL OF INFORMATION SCIENCE AND TECHNOLOGY

DEPARTMENT OF SOFTWARE ENGINEERING

B.TECH DEGREE SOFTWARE ENGINEERING

ISE 125: DISCRETE MATHEMATICS

TIME: 3 HOURS **DATE: MAY/JUNE 2018**

INSTRUCTIONS TO CANDIDATES:

- Answer **ALL** questions from Section A.
- Answer **THREE** questions from section B.

ADDITIONAL MATERIALS

- *None*

SECTION A (40 marks)

Candidates may attempt ALL questions being careful to number them A1 to A5.

A1. (a) Let α be the permutation on $\{1,2,3,4,5\}$ with $\alpha : 1 \mapsto 4, 2 \mapsto 3, 3 \mapsto 1, 4 \mapsto 5, 5 \mapsto 2$, and β be the permutation on $\{1,2,3,4,5\}$ with $\beta : 1 \mapsto 4, 2 \mapsto 2, 3 \mapsto 5, 4 \mapsto 1, 5 \mapsto 3$.
Express $\alpha\beta$ in the arrow form. [3]

(b) Find the rotational symmetries of regular pentagon. [5]

A2. (a) Given that $A = \{a, b, c, 0, c\}$ and $B = \{0, 0, a, a\}$. Find

(i) $A \times B$, [2]

(ii) A^2. [2]

(b) Show that if $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$ [4]

A3. (a) Using the sets $\{1,2,3,4\}$ and $\{a,b,c\}$ construct a function that is

(i) onto. [2]

(ii) one-to-one. [3]

(b) Sketch the graph of $f(x) = \lfloor 3x - 1 \rfloor$, for $0 \leq x \leq 1$. [3]

A4. (a) Prove that

$$p(E \cup F) = p(E) + p(F) - p(E \cap F).$$

[4]

(b) In the game of lotto a player wins a prize if she selects 4 to 6 numbers(in any order) picked by rolling the first 6 balls numbered 1 to 49 out of a ball holder. What is the probability that she wins a prize? [4]

A5. (a) How many integers between 1000 and 10 000 are divisible by 6 and 9? [5]

(b) How many number plates are possible in the Zimbabwe system where three letters and four digits are used? The letters I, O and U are prohibited. [3]

SECTION B (60 marks)

Candidates may attempt THREE questions being careful to number them B6 to B12.

B6. Let $\varepsilon = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix}$; $\rho = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$, $\phi = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 3 & 2 \end{pmatrix}$,

(a) Show that $\{\varepsilon, \rho, \rho^2, \phi, \phi\rho, \rho^2\phi\}$ is the set S_3 of all permutations on $\{1,2,3\}$. [8]

(b) Complete the following multiplication table for S_3

$\circ$	ε	ρ	ρ^2	ϕ	$\rho\phi$	$\rho^2\phi$
ε	ε	ρ	ρ^2	ϕ	$\rho\phi$	$\rho^2\phi$
ρ	ρ	ρ^2	ε	$\rho\phi$	$\rho^2\phi$	ϕ
ρ^2	ρ^2			$\rho^2\phi$		
ϕ						
$\rho\phi$						
$\rho^2\phi$						

[12]

B7. (a) Suppose that E and F are events such that $p(E) = \frac{2}{3}, p(F) = \frac{1}{5}$. What are the largest and smallest values of $p(E \cap F)$? Give examples to show that both extremes are possible. [6]

(b) State **Bayes' Theorem** and use it to find $p(F|E)$ if $p(E|F) = \frac{1}{4}, p(E|F^C) = \frac{1}{5}$, and $p(F) = \frac{1}{3}$ where E and F are events from a sample space S. [7]

(c) An **octahedral** die has eight faces that are numbered 1 though 8. One such die is biased such that the odds are twice likely to appear than evens when it is rolled. What is the

(i) expected value

(ii) variance

of the number of outcomes, when the biased die is rolled? [7]

B8. Let $\mathbf{e} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\mathbf{I} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $\mathbf{J} = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix}$ and $\mathbf{K} = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$ where $i^2 = -1$.
By constructing the multiplication table, or otherwise, show that

$$Q_8 = \{\pm\mathbf{e}, \pm\mathbf{I}, \pm\mathbf{J}, \pm\mathbf{K}\}$$

is a group under the usual matrix multiplication. [20]

B9. A computer repair technician can use 25 different types of keyboards, 12 types of mice and 6 types of LCD screens.

(a) in how many ways can five computers be repaired so that each type of keyboard is used more than once and the order does not matter. [4]

(b) How many different computers can be fitted so that it has each type of keyboard, mouse and LCD screen? [2]

(c) How many different computers can be fitted for a client so that there are three batches, where each keyboard can be used more than once and the order does not matter; two kinds of mice where each mouse can be used only once and the order does not matter; and three LCD screens are where each can be used only once and the order does not matter? [7]

(d) Given that $A_k = (-k, k]$, find $\bigcup_{k=1}^{\infty} A_k$ and $\bigcap_{k=1}^{\infty} A_k$. [7]

B10. Let A, B and C be sets

(a) Show that $(A \setminus B) \setminus C \subseteq (A \setminus C)$. [4]

(b) Show that $A \oplus B = B \oplus A$. [5]

(c) Prove that $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$. [11]

B11. (a) Determine whether the function $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$

$$f(m, n) = m^2 + n^2 - 2mn + 3$$

is onto. [4]

(b) Let $f(x) = \lceil \frac{x^3}{4} \rceil$, find $f(S)$ is $S = \{-2, -1, 3, 5, 10\}$. [3]

(c) Data are transmitted over a particular Ethernet network in blocks of 956 octets (blocks of 8 bits). How many blocks are required to transmit 2,516 megabytes of data? [3]

(d) Using the same sets X and Y and the function f

(i) Let f be a function from X to Y, and S, T be subsets of X. Show that $f(S \cap T) \subseteq f(S) \cap f(T)$. [5]

(ii) Let f be a function from X to Y, and S, T be subsets of Y. Show that $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$. [5]

END OF QUESTION PAPER