Workshop 6 – Week 7 – Worksheet 7

Question 6.1 You are asked to show the operation of quicksort on the following keys.

For simplicity, use the rightmost element as the partition element:

2 3 97 23 15 21 4 23 29 37 5 23

For brevity, only the first and final result of each quicksort step will be shown. The pivot is marked in red.

2	3	97	23 ¹	15	21	4	23 ²	29	37	5	23 ³
2	3	5	23 ²	15	21	4	23 ³	29	37	97	23 ¹
2	3	5	23 ²	15	21	4					
2	3	4	23 ²	15	21	5					
2	3										
2	3										
2											
			23 ²	15	21	5					
			5	15	21	23 ²					
				15	21	23 ²					
				15	21	23 ²					
				15	21						
				15	21						
				15							
								29	37	97	23 ¹
								23 ¹	37	97	29
									37	97	29
									29	97	37
										97	37
										37	97
											97
2	3	4	5	15	21	23 ²	23 ³	23 ¹	29	37	97

Comment on the stability of quicksort and its behaviour on almost sorted inputs.

As can be seen by 23^3 appearing before 23^1 , Quicksort is not stable. As can also be seen, lists which are almost sorted call quicksort with n-1 elements, which means quicksort must be called O(n) times, each of those quicksort calls operate on O(n) elements (rather than the O(log n) elements we see when we pick a pivot that is approximately in the middle of all the given values).

Question 6.2 Trace the action of bottom-up mergesort on the same keys:

2 3 97 23 15 21 4 23 29 37 5 23

For brevity, merges are done in a single step. The first list in the merge is marked in Green and the second in Red. Merged lists are marked in Blue.

2	3	97	23 ¹	15	21	4	23 ²	29	37	5	23 ³
2	3										
2	3										
		97	23 ¹								
		23 ¹	97								

15 21 15 21 15 21 15 21 15 21 15 21 15 22 15 15 21 15 21 15 22 15 15 15 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 <td< th=""><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th><th></th></td<>												
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2 3 23¹ 97											5	23 ³
15 21 4 23² 15 21 23² 15 21 23² 15 22 15 22 23² 15 23³ 23³ 23³ 23³ 23³ 29 37 5 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 37 23³ 29 <td< td=""><td>2</td><td>3</td><td>23¹</td><td>97</td><td></td><td></td><td></td><td></td><td></td><td></td><td></td><td></td></td<>	2	3	23 ¹	97								
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									5	23 ³	29	37
2 3 4 5 15 21 23 ¹ 23 ² 23 ³ 29 37 97	2	3	4	15	21	23 ¹	23 ²	97	5	23 ³	29	37
	2	3	4	5	15	21	23 ¹	23 ²	23 ³	29	37	97

Is mergesort stable?

Yes.