Digital Signal Processing Lab Report

By-

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INTRODUCTION

What is Digital Signal Processing (DSP)

DSP controls various sorts of signs with the goal of separating, estimating, or compacting and delivering simple signs. Simple signs contrast by taking data and making an interpretation of it into electric beats of differing plentifulness, though computerized signal data is converted into double organization where each piece of information is spoken to by two recognizable amplitudes. Another recognizable contrast is that simple signs can be spoken to as sine waves and advanced signs are spoken to as square waves. DSP can be found in practically any field, regardless of whether it's oil handling, sound multiplication, radar and sonar, clinical picture preparing, or broadcast communications - basically any application in which signs are being compacted and duplicated.

All interchanges circuits contain some commotion. This is genuine whether the signs are simple or advanced, and paying little mind to the sort of data passed on. Clamour is the endless most despicable aspect of correspondences engineers, who are continually endeavouring to discover better approaches to improve the proportion in interchanges frameworks. Conventional strategies for advancing S/N proportion incorporate expanding the transmitted sign force and expanding the collector affectability. (In remote frameworks, particular receiving wire frameworks can likewise help.) Digital sign preparing significantly improves the affectability of a getting unit. The impact is most observable when commotion rivals an ideal sign. A decent DSP circuit can now and again appear to be an electronic supernatural occurrence specialist. Be that as it may, there are cut off points to what it can do. On the off chance that the clamour is solid to such an extent that all hints of the sign are wrecked, a DSP circuit can't discover any request in the tumult, and no sign can be received.

MATERIALS AND METHODS

About MATLAB

MATLAB is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation.
Typical uses include:

- Math and computation
- Algorithm development
- Modelling, simulation, and prototyping
- Data analysis, exploration, and visualization
- Scientific and engineering graphics
- Application development, including Graphical User Interface building

MATLAB is an interactive system whose basic data element is an array that does not require dimensioning. This allows you to solve many technical computing problems, especially those with matrix and vector formulations, in a fraction of the time it would take to write a program in a scalar non-interactive language such as C or FORTRAN. The name MATLAB stands for matrix laboratory. MATLAB was originally written to provide easy access to matrix software developed by the LINPACK and EISPACK projects, which together represent the state-of-the-art in software for matrix computation.
MATLAB has evolved over a period of years with input from many users. In university environments, it is the standard instructional tool for introductory and advanced courses in mathematics, engineering, and science. In industry, MATLAB is the tool of choice for high-productivity research, development, and analysis.
MATLAB features a family of application-specific solutions called toolboxes. Very important to most users of MATLAB, toolboxes allow you to learn and apply specialized technology. Toolboxes are comprehensive collections of MATLAB functions (M-files) that extend the MATLAB environment to solve classes of problems. Areas in which toolboxes are available include signal processing, control systems, neural networks, fuzzy logic, wavelets, simulation, and many others.

SOLUTIONS

Laboratory 1

1.1 Generation of Signals and Visualisation

Anything that carries information is defined as a signal. A signal is mathematically represented as a function of one or more independent variables.

Unit impulse signal: A signal which has infinite magnitude at time equal to zero only, which acts for a short duration with infinite magnitude of voltage.

Generally, it means zero width and infinite height.

1.1a). Generation of a unit step function and representing it using stem and plot.

A unit impulse signal of length 32 was generated by padding zeros on to all the 32 values.

Impulse peak at time k0 was set to 1.

Function x(k0) was made 1 to obtain a unit function

Step values are declared with unit time intervals.

Graph of stem was plotted with time vs unit impulse followed by the graph of stem is plotted with time vs unit time intervals.

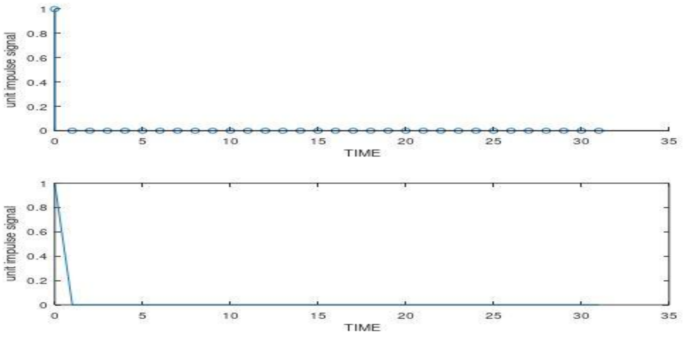


Fig 1.1a. Unit impulse signal

Unit impulse signals were generated were depicted using stem and plot.

Plot was displayed as continuous value for the curve where stem was displayed as discrete values of the points on the curve.

1.1b). Generation of a unit step function and representing it using stem and plot.

A unit impulse signal of length 16 was generated by padding zeros on to all the 16 values. Impulse peak at time k0 was set to 4. Function x(k0) is made 4 to obtain a unit function. Step values are declared with unit time intervals.

Graph of stem was plotted with time vs unit impulse followed by the graph of stem was plotted with time vs unit time intervals.

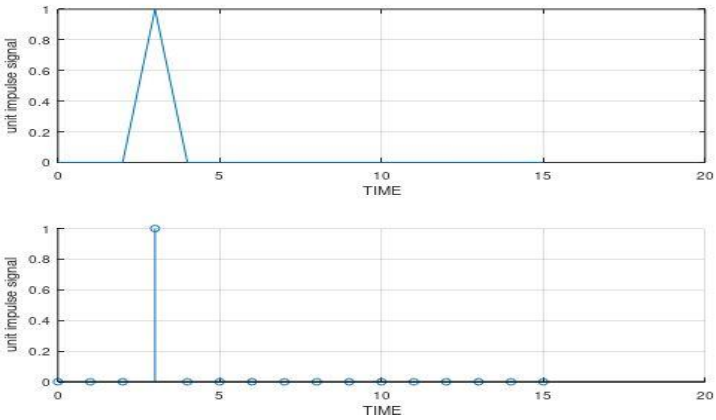


Fig 1.1b. Unit impulse signal of length 16

A pulse of arbitrary length L=16 was and a pulse peak at 4 was displayed.

Plot and stem used in graphical display of the unit impulse signal.

1.1c). generation of a sinusoidal wave

Sample length of 100 was considered with time interval of 2

Harmonic function was applied that follows the condition 2pi*f

A sinusoidal graph was plotted with time vs function of time.

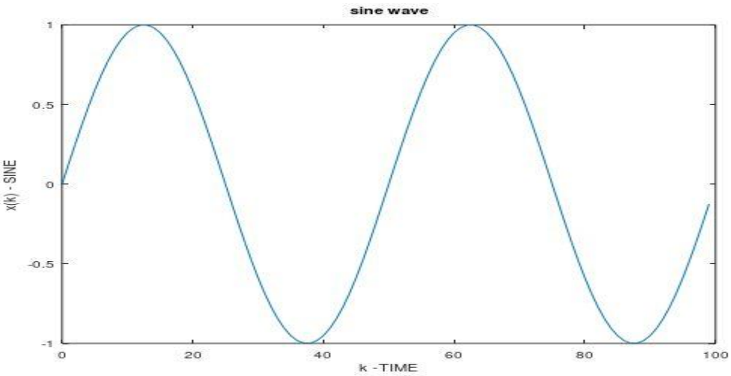


Fig 1.1c. Sine wave

Sinusoidal signal of 100 samples with exactly two periods was generated and displayed.

DISCRETE FOURIER TRANSFORM:

It establish relationship between the time domain and frequency domain.

The mathematical equation is given by

$$X(k+1) = \sum_{i=1}^{L} x(i+1)W_i^{ki}$$

Where, *X(k+1)* is the transformed version

$$W_{n= e^{-j2pi/N}}$$

FFT is an effective way of implementation to solve the DFT [1]

1.2a) Calculation of DFT for the generated rectangular signal.

A rectangular signal of length 64 having a width of 4 samples were generated.

DFT for the generated rectangular signal was calculated and the magnitude and phase graphs were plotted.

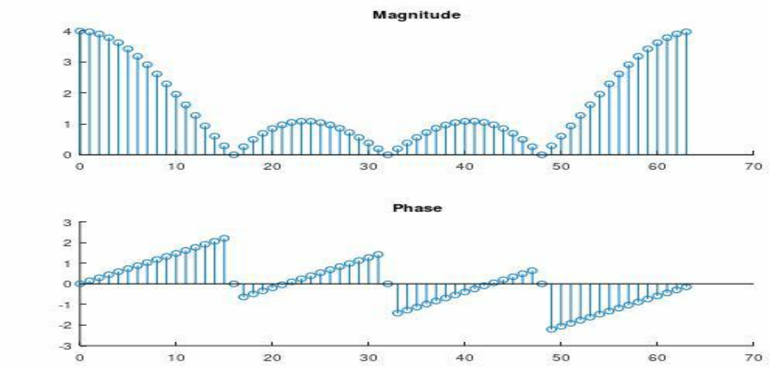


Fig 1.2a Phase and Magnitude of rectangular signal

The magnitude and phase plots for the FFT calculation was plotted.

These is a correspondence between the Fourier spectrum of the time function in the form of a square wave pulse.

1.2b) magnitude and phase potting for FFT

A rectangular signal of length 64 with a width of 8 samples was generated along with another rectangular signal of length 32 with a width of 4 samples was generated.

DFT for the generated rectangular signal was calculated and the magnitude and phase graphs were plotted.

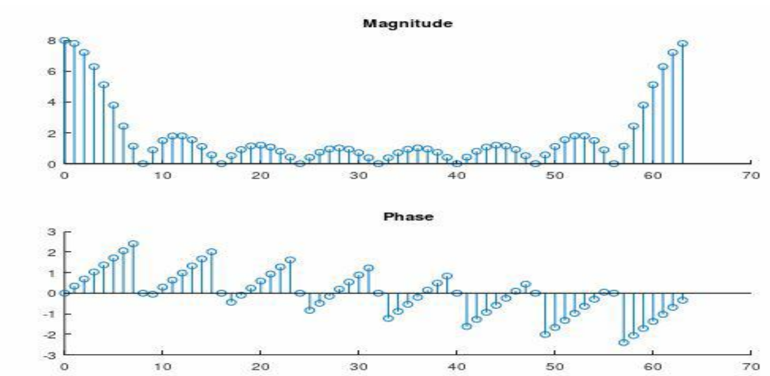


Fig 1.2b.a Magnitude and samples

The magnitude and phase plots for the length of 64 and width 8 was plotted.

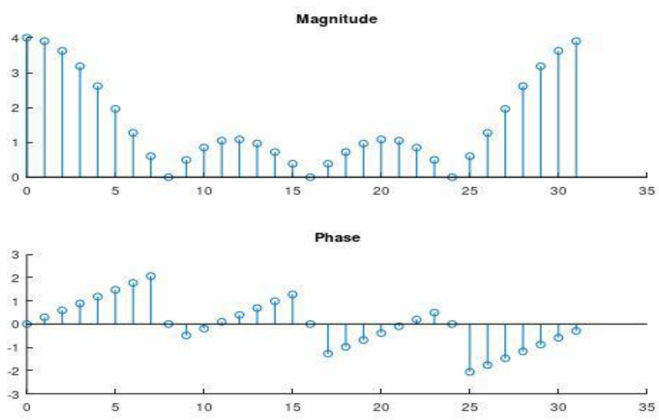


Fig 1.2b.b Magnitude and samples

The magnitude and phase plots for the length of 32 and width 4 was plotted.

DIGITAL FILTER DESIGN:

Digital filter which performs mathematical operations on a sampled and discrete-time signal to reduce or enhance particular aspects of respective signal.

Digital filter design: The process of design of signal processing filter which describes the typical requirements like frequency response, impulse response, time delay, phase shift dependent the design.[2]

The firpm function gives us the Park-McClellan filter in MATLAB and the syntax is firpm(n,f,a). where n is an order of filter, f is declaration of respective stop and passband and a is desired amplitude.
Further, we design an elliptic filter of order 5 with pass band at 0.5 and stop band at 40.
The above filters are used to find fine impulse response filter and to find optimal filter co-efficients.

1.3a) Park-McClellan FIR low pass filter

A Park-McClellan FIR low pass filter was generated.

A unit pulse sequence with 1024 samples was generated and the filter was applied to the generated unit pulse sequence.

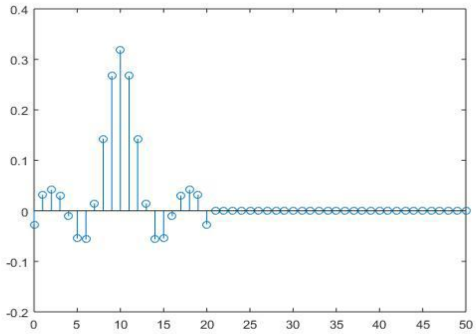


Fig 1.3a.a Time signal

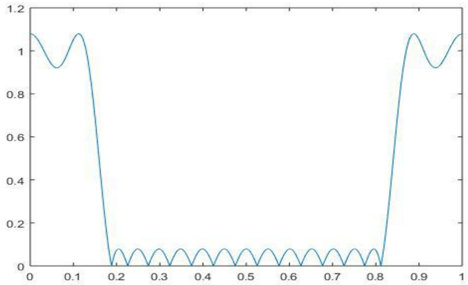


Fig 1.3a.b Magnitude in frequency domain

The above figure depicts Park-McClellan FIR low pass filter filtering the components with n=20

1.3b) Cauer low pass filter

A Cauer low pass filter was generated.

A unit pulse sequence with 1024 samples is generated and filter was applied to the generated pulse sequence.

A comparative study of cauer lowpass filter and Park-McClellan FIR low pass filter was carried out.

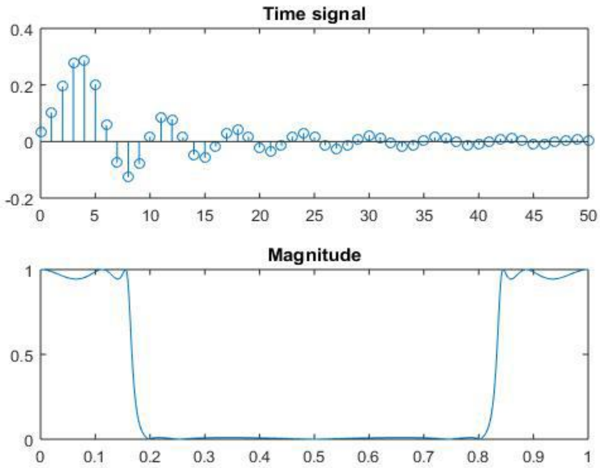


Fig1.3b Time signal and magnitude in frequency domain

The above figure depicts the low pass filtration using Cauer low pass filter.

LINEAR EQUATION

To solve linear equations is the most effective way is matrix method.
Given the linear equation system Ax = b with the known system matrix A and known vector b. To find the vector for X. We can determine by the solving equation systems, by $x = A^{-1} b$.
But still there is no exact and precise solution but still we can solve it by smallest least squares with e =Ax-b. Where $X= (A^H A)^{-1} A^H b$. Then Hermitian transpose method is used

1.4a) a solution vector and the sum of the squares is calculated

Hermitian transpose concept was used in the calculation of the sum of the squares.

Sum of square after simulation was 56.024

The vector X is 0.077657 + 2.3688i, -0.67258 + 2.0763i, 0.77443 + 1.6738i

1.4b) a matrix sum of squares is calculated

Matrix sum of squares was calculated using the concept of Hermitian transpose.

Sum of square after simulation was 713.05

The vector X is 0.14697 + 4.9095i, -0.90074 + 4.4524i, 0.691 + 4.2438i

ALIASING:

This occurs while sampling when the different signals to become indistinguishable. It acts as distortion for discrete signal. It can be avoided by applying proper low pass filter.

Here, to analyse Aliasing effect in three different scenarios with increasing frequency 0 to 8kHz frequency and with predefined function. This should be observed in sound with earphone for and analysis is done depending on the fall and raise in sound with respect to the signal.

1.5a)

Results : Here, the sound was tracked with summing the frequency of 40 kHz with 0 to 8khz frequency . After hearing to the audio there was no aliasing effect because of the summing of frequency to the original signal.

1.5 b)

Results : Here, the sound was tracked with summing the frequency of 8 kHz with 0 to 8khz frequency . After hearing to the audio there I can observe proper aliasing effect because of the summing with low frequency to the original signal.

1.5 c)

Results : Here, to avoid aliasing for above problem filter was applied and the sound was tracked with summing the frequency of 8 kHz with 0 to 8khz frequency . After hearing to the audio there I can observe there was no aliasing effect because of the summing of low frequency to the original signal. Hence Aliasing effect can be removed by applying filter.

Laboratory -2

RANDOM SIGNAL

“Signals can be isolated into two principle classifications - deterministic and random. The term random signal is utilized fundamentally to indicate signals, which have a random in its nature source. For instance, we can specify the warm clamour, which is made by the random development of electrons in an electric conduit. Aside from this, the term random signal is utilized additionally for signals falling into different classes, such as intermittent signals, which have one or a few boundaries that have proper random conduct. A model is an intermittent sinusoidal signal with a random stage or on the other hand sufficiency. Signals can be dealt with either as deterministic or random, depending on the application. Discourse, for instance, can be considered as a deterministic signal, if one explicit discourse waveform is thought of. It can likewise be a random process on the off chance that one considers the groups of all conceivable discourse waveforms so as to structure a framework that will ideally process discourse signals”. [10]

Measurement of characteristics of Probability Density Function.

Probability density function

PDF of uniform distributed signal is given by,

$$f(x) = \begin{cases} \frac{1}{b-a} & \text{for } a \le x \le b, \\ 0 & \text{for } x < a \text{ or } x > b \end{cases}$$

Where a and b are two boundaries of function f(x). Generally, PDF sketch as rectangle where (b-a) is base and (1 / b-a) is height. As the distance between a and b increases, the density at value start decreasing and vice versa. This we can observe in further solution. [4]

Uniform distributed value

The uniform distribution is the distribution for a uniform random variable. A continuous uniform random variable takes continuous values within a given interval with same probability. Therefore, the PDF of such a random variable is a constant over the given interval. [5]

2.1a) Generation of uniformly distributed variable.

100 uniformly distributed variables in the interval of [0,1] was generated. Mean and variance for the generated variables was calculated. Comparison between the theoretical and calculated value was carried out.

Results : After simulation, I obtained mean and variance as 0.4678 and 0.077035 respectively in [0, 1] interval and ideal theoretical values are 0.5 and 0.0850

2.1b) Generation of uniformly distributed variable several times.

Mean and variance of many distributed variables were calculated, and results were observed as shown below in table 2.1b..

Result: After several simulation, I recorded mean and variance values in below table 1 and the result was compared with theoretical value in [0, 1] interval. As we can observe the many difference in mean and variance.

Simulation	SIMULATION Mean	Variance	Theoretical Mean	Variance
1	0.49842	0.086953	0.5	0.0845
2	0.51069	0.091925	0.5	0.0845
3	0.46469	0.077040	0.5	0.0845

Table 2.1b.a. comparison of mean and variance to theoretical and simulation result

2.1c) Generation of uniformly distributed variable for N=10000

Mean and variance of all the 10000 variables are calculated and the results were observed**.** As I recorded in below table, not much difference in mean and variance value. So, I can conclude that as the number of observations increases the quality of estimation increases.

Simulation	SIMULATION Mean	Variance	Theoretical Mean	Variance
1	0.50371	0.083245	0.5	0.0845
2	0.50118	0.083364	0.5	0.0845
3	0.49957	0.082285	0.5	0.0845

Table 2.1c. comparison of mean and variance to theoretical and simulation result

2.1 d) **Probability density function for uniform distribution**

Histogram estimation for the nodes was done. Probability density function for the observation N =100 and 10000 was carried out.

As we see in fig2.1.d.a, the expected PDF was a uniform distribution. After simulation we can see and evident it was not uniformly distributed due to the smaller number of elements.

Conversely, in fig2.1.d.b, the PDF was uniform distribution. After simulation we can see and evident it was uniformly distributed due to the large number of random variable i.e N=10000. So we can conclude that for large number of observations generate the uniform distribution.

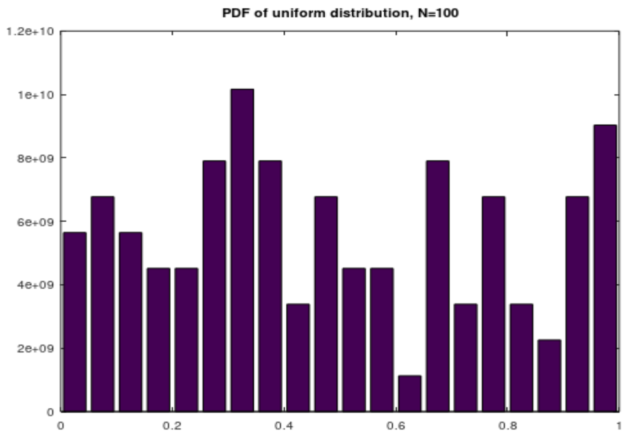


Fig 2.1.d.a. PDF for N=100

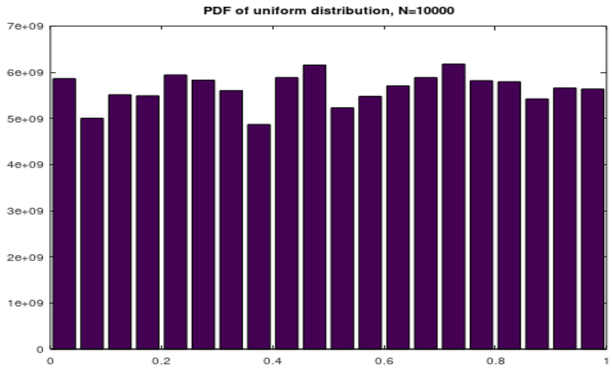


Fig 2.1.d.b. PDF for N=10000

2.1e) Probability density function for normal distribution

10000 observations of a normal distributed random variables were generated. Mean and variance of the random variables were calculated. Power density factor of the node point were calculated, and graph was plotted.

The below graph was plotted and observed the result for N=10000. As we see in fig 2.1.e the shape of the graph was bell shaped, it means distribution of data, mean, median, and mode are all the same value and coincide with the peak of the curve

The mean and variance were estimated 0.0076 and 0.97 respectively.

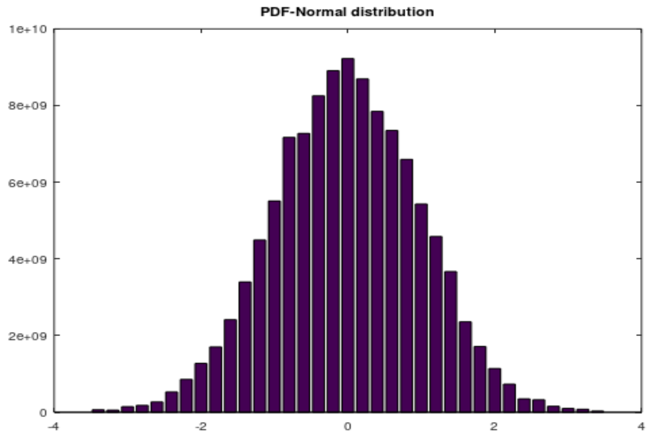


Fig2.1.e. PDF of normal distribution

2.1f) In this lab experiment, we are looking into the characteristics of 12 uniformly distributed random variables in conjunction. The PDF of sum of 12 uniformly distributed random variables is analysed. Also, the characteristics such as mean ad variance of 12 random variables is compared to the random variable generated by the sum of those 12 random variables

RESULT:

Random Variable	Mean	Variance
1	0.49906	0.083342
2	0.50266	0.08327
3	0.50049	0.083274
4	0.50315	0.083707
5	0.4993	0.082541
6	0.49596	0.084543
7	0.5011	0.083778
8	0.496	0.082877
9	0.49955	0.083712

10	0.50274	0.082588
11	0.49597	0.08379
12	0.49765	0.081942
Sum	**5.989**	**0.9818**

Table 2.1f: Mean and variance values of 12 random variables uniform distributed

After simulation, mean and variance of RV sum of 12 uniformly distributed random variables was 6.0086 and 0.97455 respectively. The calculation of individual mean and variance values of every random signal generated is very close to the calculated sum as shown in the above table. 2.1f.

Hence we can conclude that the sum of all the mean and variance values of 12 random variables which makes the new random variable

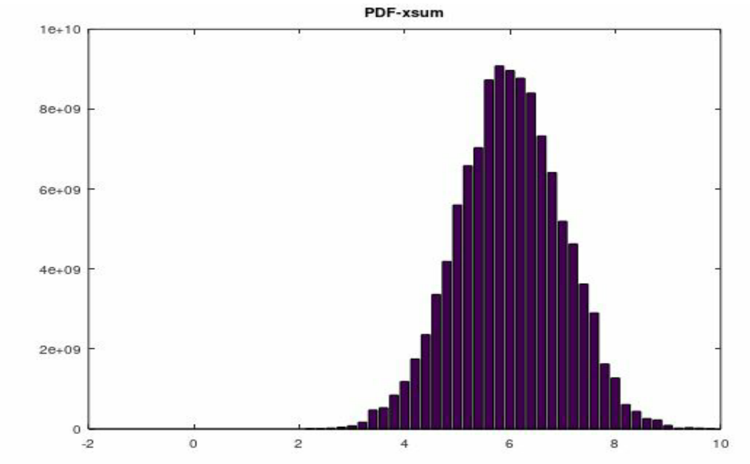


Fig2.1.f. PDF of normal distribution using sum of 12 RV

Stationery and Ergodicity:

Stationary property for random process:

The characteristics of the random variable depends on the time at which the random process is discrete (sampled). If the random process x(t) is said to be stationary if and only if the probability density function (PDF) of any set of samples do not vary with time. That mean it should be time invariant. Also, the combination of PDF of x(t1),….x(tn) is the same as the combination of the PDF of x(t1+ τ),….x(tn+ τ) for any time shift τ. Here mean and variance are constant throughout the period, but neither of them are dependent function of time.

Non-stationary process is just characterized by the combination of PDF is dependent on time instants t1, …, tn. In other words, it is time variant. [5]

Ergodicity property for random process:

If the random process is to be ergodic, then its statistical properties can be obtained from a single or whole random sample of the process. The entire random samples from a random process must represent the static average properties of each process and vice versa.

On contrast, a process that is not ergodic is a process that changes erratically at an inconsistent rate. [6]

2.2 a) Generate and analysis of 3 different random process

Here, we generated 3 different random process each contains 4 sampling samples and length of 100. For each random process we were drawing 4 sample sequences using subplot for individual random process.

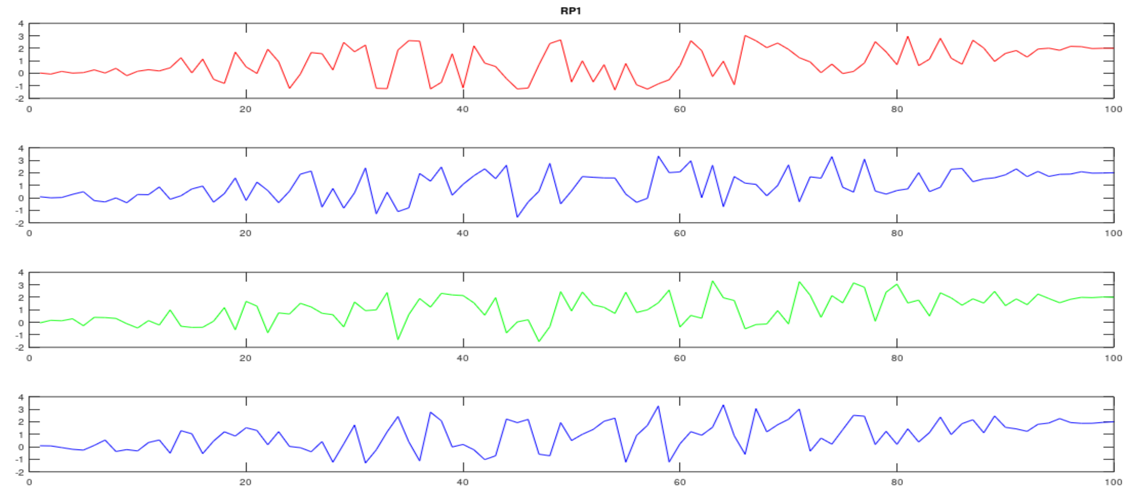


Fig 2.2.1.a: the sampling sequences for RP1

We can observe in the Fig 2.2.1.a that all the 4 sample sequences were random and value of each of the sample sequences were in the range between -2 and +4. Observing the sampling sequence in 10 sample and we can notice its average value of all 4 sequence is 0. It means the random process may be stationary at that time. But after 10 samples the significant variations in the values at the time interval. After comparing the mean values at 0^{th} and 100^{th} sample were 0 and 2 respectively.

Hence, the mean value is not constant and after observing the signal it obvious to evident that Random process RP1 was time variant. Therefore, it was not stationary, and it was not ergodic.

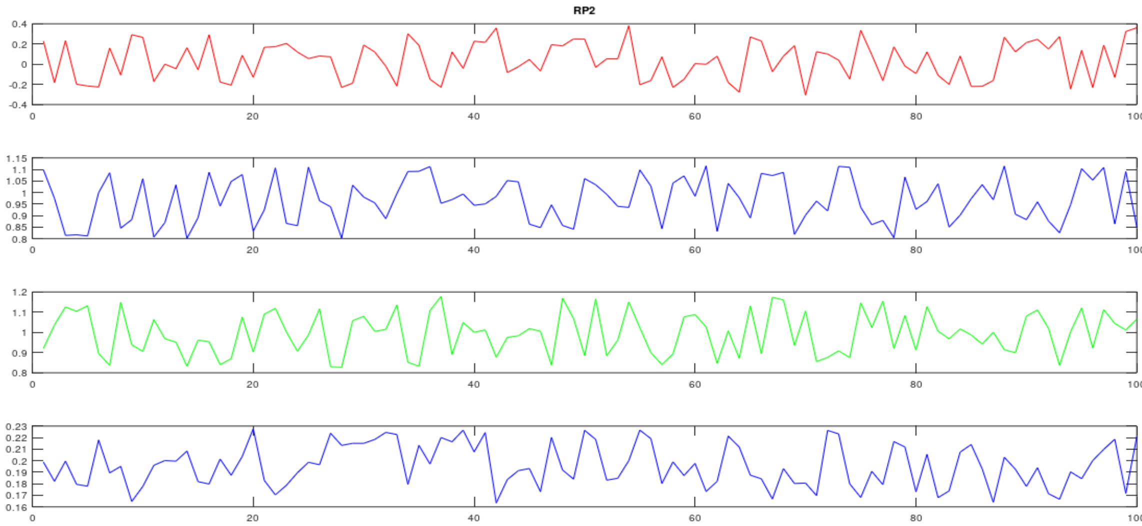


Fig 2.2.1.b: the sampling sequences for RP2

We can observe in the Fig 2.2.1.b that all the 4 sample sequences were random and value of each of the sample sequences were in the range between 0 and 1.2. Observing the sampling sequence in whole 0 to 100 sample and we can assume its average value for all 4 sequence is around 0.6. It means the random process may be stationary at every time After comparing the mean values at 0^{th} and 100^{th} sample were 0.6 and 0.65 respectively.

Hence, the mean value is constant throughout the sample and after observing the sequence it obvious to evident that Random process RP-2 is time invariant. Therefore, it was stationary.

Further, we need to check for ergodicity. After looking at figure 2.2.1.b, the mean value for 0^{th} to 100^{th} sample sequence 1, 2, 3 and 4 are approximately 0.3, 0.975, 1 and 0.21, respectively. The time variant mean values of the random process 2 ranges from 0.3 to 1. The time variant mean values and the linear time mean values were almost similar (i.e. 0.6) and statistical properties can be obtained from random sample of the random process 2. Hence it might be ergodic.

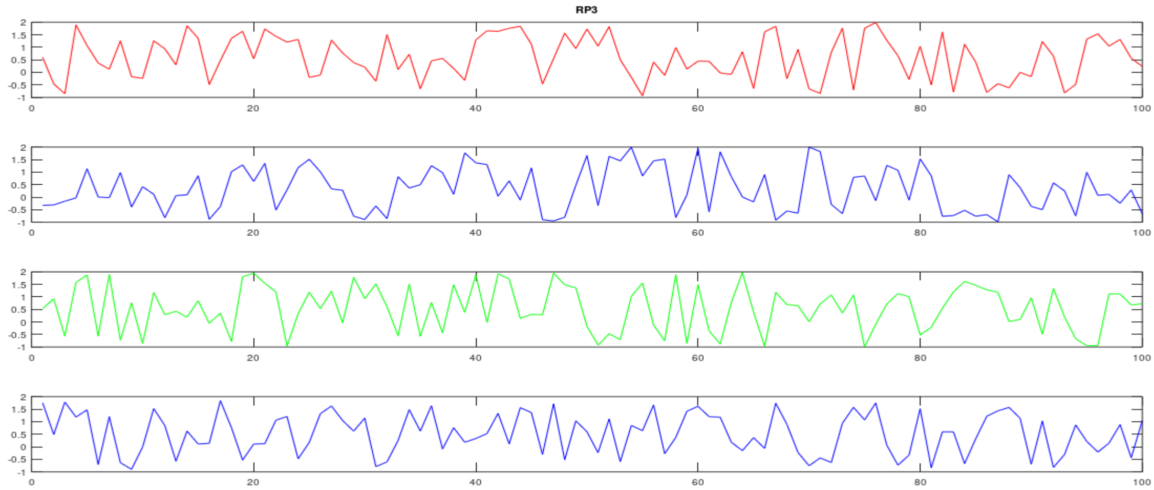


Fig 2.2.1.c: the sampling sequences for RP3

We can observe in the Fig 2.2.1.c that all the 4 sample sequences were random and value of each of the sample sequences were in the range between -1 and 2. Observing the sampling sequence in 0 to 100 sample and we can assume its average value all 4 sequence is varying a lot. The significant variations at all sample interval, so it was time invariant. After comparing the mean values at 0^{th} and 100^{th} sample and obtain mean value as 0.525 and 0.65 respectively.

Hence, the mean value is constant and after observing the signal it obvious to evident that Random process RP3 is time invariant. Therefore, it was stationary,

Further, we need to check for ergodicity. After looking at figure 2.2.1.c, the mean value for 0^{th} to 100^{th} sample sequence 1, 2, 3 and 4 are 0.20, -0.25, 0.65 and 1.25, respectively. The linear time variant mean values of the random process 2 ranges from -0.35 to 1.25. The time variant mean values and the linear time mean values are almost similar and statistical properties can be obtained from random sample of the random process 3. Hence it might be ergodic

2.2b) Are the RP's stationary?

In this lab experiment, we generated 3 random processes which consists of 1000 sample sequences each of length 100 using 3 identity noise generators.

As per the stationary property of a random process, if the mean and variance values of the random process are constants then the random process is termed as stationary.

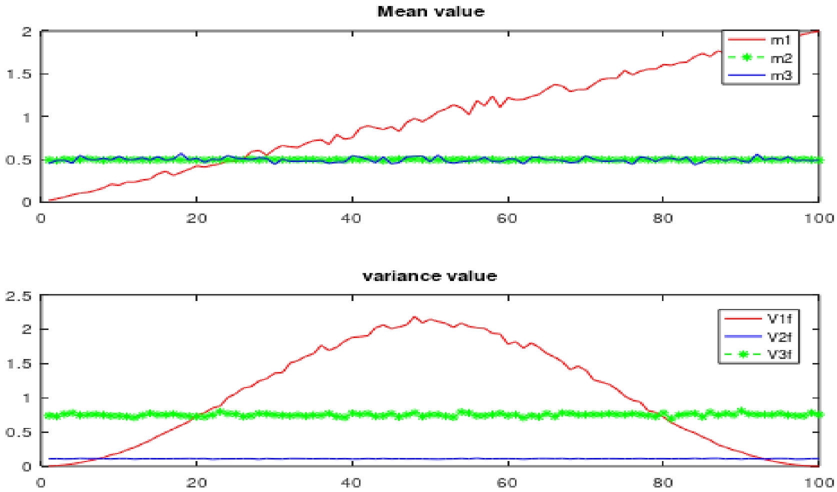


Fig2.2b: Mean and variance of all Random process

Here in the above figure, the mean and variance of RP1 is changing. So, it is time variant. Hence it is not stationary. On other hand, RP2 and RP3 are time invariant, we can observe the mean and variance of RP1 and RP2 are constant throughout the sequence. Hence it is stationary.

2.2c) Are the RP's ergodic?

In this lab experiment, we generate 3 random processes which consists of 10 sample sequences each of length 1000 using 3 identity noise generators

As per the ergodic property of a random process, if a small sample set can derive the properties of the whole random process then the random process is termed as stationary. We need to check the by comparing the linear time mean values and time variant mean values of each random process.

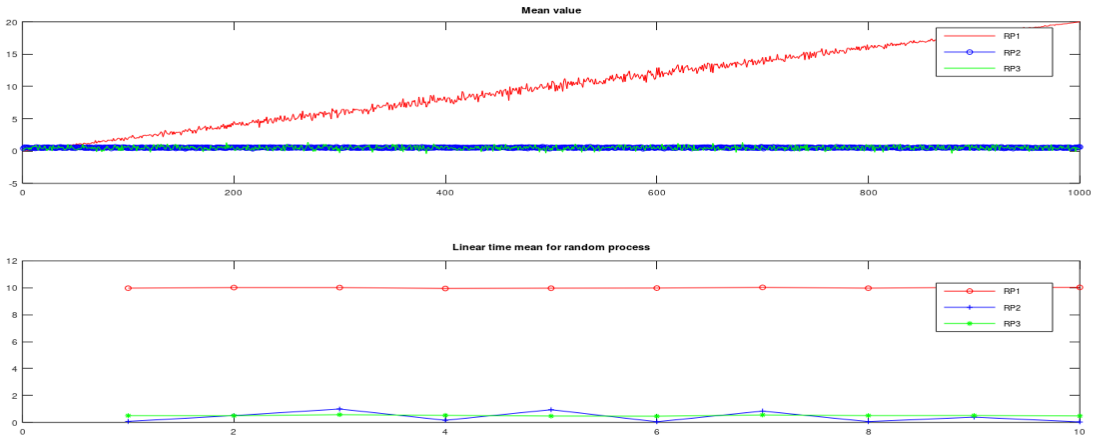


Fig2.2c: The mean and linear time mean value for RP's

Here in the above figure, the mean of RP1 is gradually increasing. So, it is time variant. Hence it was not stationary, then it was not ergodic. On other hand, RP2 and RP3 are time invariant, we can observe and compare with fig2.2b the mean and linear time mean of RP2 and RP3 were constant throughout the sequence in both time variant mean and linear time mean. Hence it was ergodic.

Passing Random Signals through Systems

In this section, we need to check how the mean value, the autocorrelation sequence and the PDF are changed by filtering while passing through the LTI system.
The following steps should consider for better understanding for further solutions:

1. The output signal is formed after convolution with impulse response.

$$y[k] = h[k] * x[k] = \sum_{\kappa=-\infty}^{\infty} h[\kappa]\, x[l-\kappa].$$

…………………(1)

Where y[k]= output signal
x[k]= input signal, h[k] =impulse signal

2. The transfer function for impulse signal in z-transform is given by

$$H(z) = \sum_{\kappa=-\infty}^{\infty} h[k]\, z^{-k}.$$

…………………(2)

The period of the above function is 2π.

3. The mean value my of the filter output process is obtained by multiplying the mean value m_x of the RP x[k] by the DC transfer factor H(ej0) as shown below

$$m_y = m_x\, H(e^{j0}).$$

…………………(3)

4. The relationship between Auto correlation function (ACF) of input and output signal is given in below equation.

$$\varphi_{yy}[\lambda] = \rho[\lambda] * \varphi_{xx}[\lambda],$$

…………………..(4)

5. The system ACF is obtained considering all above equations

$$\rho[\lambda] = h[\lambda] * h^*[-\lambda] = \sum_{\kappa=-\infty}^{\infty} h[\lambda-\kappa]\, h^*[-\kappa] = \sum_{\kappa=-\infty}^{\infty} h[\lambda+\kappa]\, h^*[\kappa].$$

All above equations and explanations are described in []

2.3a) Calculation of ACF

Noise generator rand and randn uses MATLAB in order to generate a sequence x1 or x2 of length N= 20,000 which is of a uniformly and normally distributed white RP. Simultaneously the mean values mx1 and mx2 was measured. 128 values of ACF (* E {1,2}) was obtained by using

[phi*, lambda] = acf (x*,128). The ACF is calculated using the second output vector lambda that indicates the time index lambda (). Using plot (lambda, real (phi*)) the ACF was plotted

RESULT:

As we can observe two plots in figure 2.3a, which represents the uniform distributed and normal distributed signal . Here both the signals are uncorrelated. We can observe at first and second plot, that just an lean peak in 0 without any correlation .

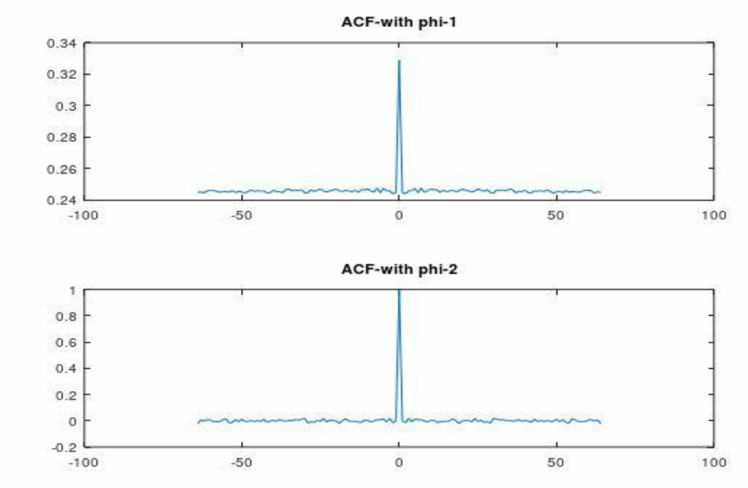


Fig2.3a. ACF with x1 and x2 sequence

2.3b) Random signal with averaging filter

Use plot (lambda, real (phi *)) in order to plot the ACF. On the basis of the respective sequences the PDF of the RV y1[k] and y2[k] was measured under ergodic assumption. With respect to the node points ybin1= [0.0:01:1] for Y1 or ybin2= [-1:0.05:1] for Y2 histogram estimation was utilized.

RESULT:

Here, after passing through the digital filter with infinite impulse response. The frequency response has been calculated for filter co-efficient. As we can see in the below figure magnitude of frequency response has been plotted. From the below graph we can identify DC transfer factor H(Ø) = $H(1)_i$= 1.

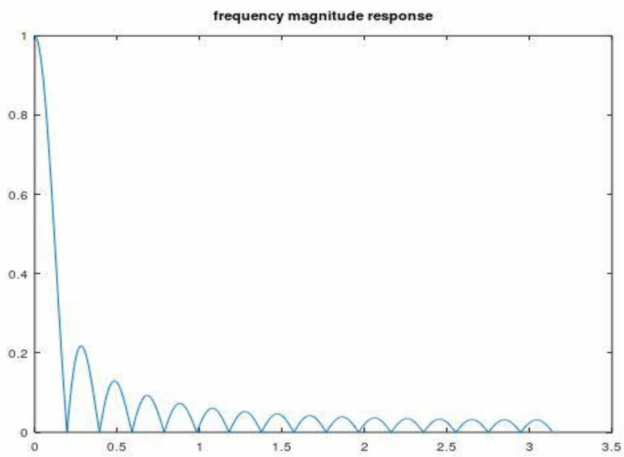


Fig2.3b Frequency magnitude response

2.3c)

RESULT:

As we can observe two plots in figure2.3c.a, which represents the uniform distributed and normal distributed signal . Here both the signals are correlation because of filtering effect. We can observe at first and second plot, that just an mountain shaped peak which is ranging from -32 to +32 with correlation.

Here, the average values and ACF match our expectation and we can observe the difference after comparing with figure2.3a and figure2.3c.a.

Figure 2 is just histogram estimation for the same function, it returns the 32(n) interval hits.

Under the ergodic assumption, the PDF has been estimated. In figure 2.3c.b , at plot 1 has higher density since distance between a and b is less. On contrast at plot 2, more distance between a and b so the density is less compared to plot 1.

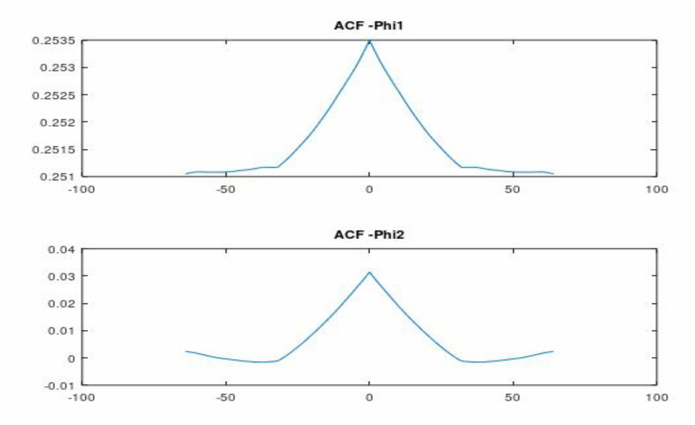


Fig2.3c.a ACF after normalization with x1 and X2 sequence

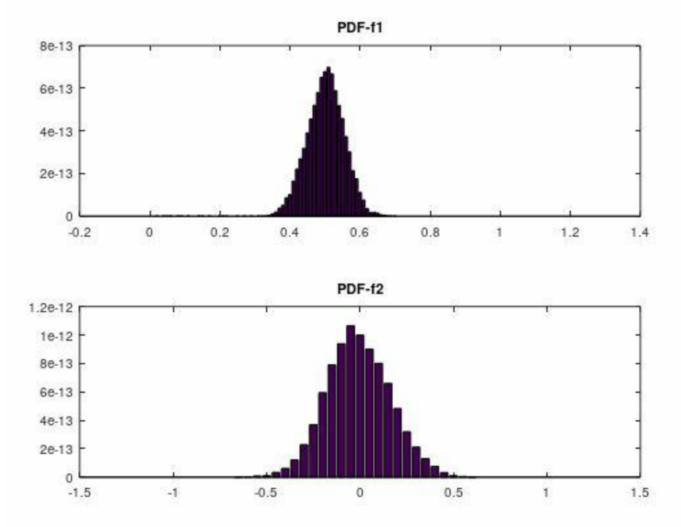


Fig2.3c.b PDF after normalization

Laboratory 3

Digital Communication

Performance of M-QAM/M-PSK over an AWGN Channel

1. AWGN Channel: Additive white Gaussian noise is a basic noise model used in transmission and information theory. This mimic the random process that occur in nature. It's basic characteristics are:
1) Added to any noise and intrinsic in nature.
2) Uniform power across the frequency band.
3) It has a normal distribution in the time domain.

Generally, It used as channel model in the communication system which are linear addition of wideband, white noise. It is good model for satellite and space communication but is not good for terrestrial path because of multipath, interference, ground clutter, mountain, and many others.

The channel capacity C for the AWGN channel is given by:

$$C = \frac{1}{2}\log\left(1 + \frac{P}{N}\right)$$

Where P is power
N is Noise.

2 SER and BER over Gaussian channel

Let BPSK received signal can be written as:

$$y = x + n \qquad \text{……………. (1)}$$

SNR (γb)is defined as follows:

$$\gamma_b := \frac{E_b}{N_0} \models \frac{A^2}{N_0} = \frac{d_{min}^2}{4N_0} \qquad \text{…………………. (2)}$$

The Q- function is defined as:

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-\frac{x^2}{2}}\,dx \qquad \text{………………………… (3)}$$

2.1 BER for BPSK modulation

Binary Phase shift keying which uses two phases which are separated by 180^0. Particularly in this constellation points don't matter. Since it lies in real axit with angle 0^0 and 180^0. The constellation figure is showed as below

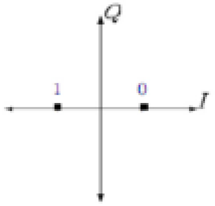


Fig 2.1.1: BPSK constellation

Bit error probability P_b can be identified by using above equation 1 and 2

$$P_b = P\{n > A\} = \int_A^\infty \frac{1}{\sqrt{2\pi\sigma^2/2}} e^{-\frac{x^2}{2\sigma^2/2}}$$ ………… (4)

After using Equation 3 in 4 we obtain

$$P_b = Q\left(\sqrt{\frac{d_{min}^2}{2N_0}}\right) = Q\left(\frac{d_{min}}{\sqrt{2N_0}}\right) = Q\left(\sqrt{2\gamma_b}\right)$$ ………… (5)

Here Symbol error rate (P_s) can be identified using P_b . This equation for BPSK

$P_s \approx P_b * 2$

2.2 BER for 8-PSK modulation

Eight phase shift keying is a model to transmit information on a carrier by changing its phase of the carrier. In this, there are 8 different phases and at each phase change represents the transmission of 3 unique bits.

The constellation figure is showed as below

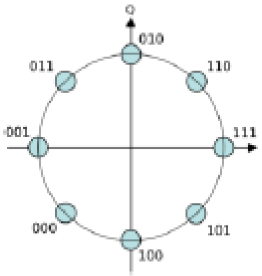


Fig2.2.1: Constellation figure for 8-PSK.

Bit error probability P_b can be identified by using above equation 1 and 2

$$P_s = 1 - \left(1 - \frac{2\,(sqrtM - 1)}{sqrtM} Q\left(\sqrt{\frac{3\overline{\gamma}_s}{M-1}}\right)\right)^2$$

Where M is integer which is greater than 4
Here M = 4.

2.3 BER for 4-QAM and 16-QAM (M-QAM) modulation

M-Quadrature amplitude modulation is popularly used in digital communication (Modulation) to transmit information. It sends 2 message signals (or 2 bits) by changing its amplitude using ASK (Amplitude shift keying).

QAM which two carriers named sine and cosine shifted in phase by 90 are modulated and combined. There are two phases, often called as In-phase and other signal called as Quadrature phase.

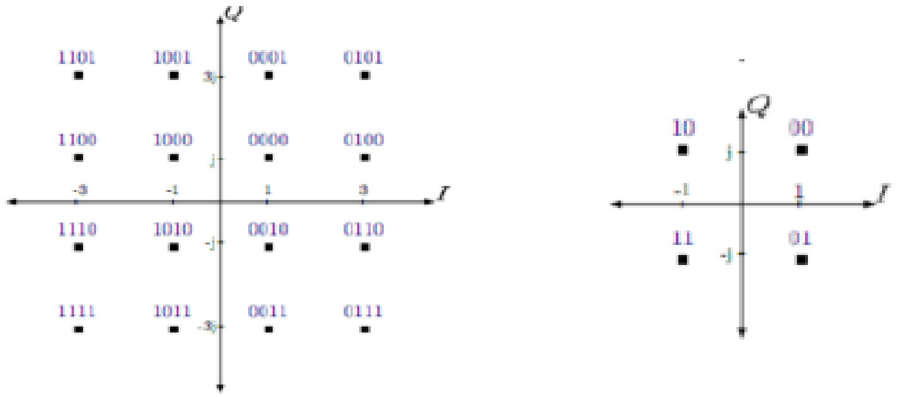


Fig2.2.2: The constellation figure for 16-QAM and 4-QAM

Bit error probability P_b can be identified by using above equation 1 and 2

$$P_s = 1 - \left(1 - \frac{2\,(sqrtM - 1)}{sqrtM} Q\left(\sqrt{\frac{3\gamma_s}{M-1}}\right)\right)^2$$

Note: The above all equations and diagrams are obtained from DSP_lab_v1.0.pdf and [8]

3.1 Solutions

1) Generation of random binary sequence b of N= 10000 values

For BPSK, M-QAM, and 8-PSK b declaration is same.

b= randi([0, L],1 , N);

where L= 1 for BPSK and 8-PSK,

L = 3 for 4-QAM, L = 15 for 16-QAM

2) Modulate b to a

a = symbol_Conste(b+1);

for BPSK Symbol Constellation is [-1,1], for 8PSK Symbol Constellation is [000,001,010,011,100,101,110,111].

for 16-QAM and 4-QAM Symbol Constellation is square root of average value of real and imaginary value.

3) Generation of Gaussian Noise n[k] by varying SNR from 0 to 14dB with 2dB step. It should be generated with the help of randn() predefined function.

n = sqrt(1/10 ^ (SNR_db/10)/2) * (randn(size(a)) + j * randn(size(a)));

4) Obtain the signal (Receiver)

r = a + n;

4.1 Demodulation:

Obtain Â[k] by taking minimum of Ã(k)

Â[k] = avgmin(r(k)-Ã(k))^2;

4.2 Euclidean distance:

This should be obtain by calculating distance between the two symbol constellation which is given by

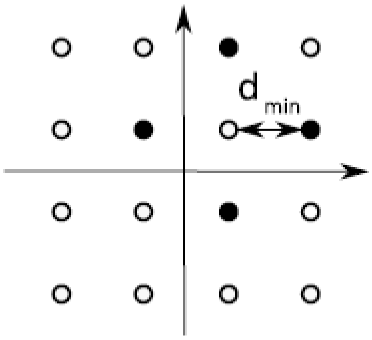


Fig2.2.3: The d_{min} for further steps

d(k)= abs(r(i) - symbol_conste(k)).^2;

obtain minimum of d for every SNR step and the minimum of d is Ã(k)

5) Demapping:

Now, to move on to further steps we need to demap Â[k] with B_hat[k]. It is nothing but recovering the signal from Â[k].

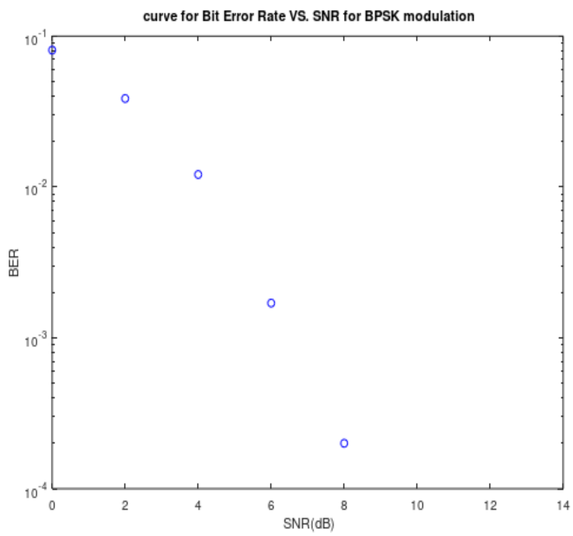
Â[k] → B_hat[k]

6) Identifying the Bit error and symbol error just by comparing B_hat[k] with b(k).

BER is defined as number of bit error to the number of transmitted bits. SER is defined as number of symbol error to the number of transmitted bits.

For BPSK, BER is equal to SER.

7) Plotting BER and SER using semiology() and comparing the result with theoretical value.



Theoretical calculation for BPSK

Using equation 5, We can determine BER after calculating Q and P_b.

X=0, P_b = 2.645

X=2, P_b=1*10^-1

X=4, P_b = 1.67 * 10^-3

X=6, P_b= 5*10^-7

X=8, P_b= 3*10^-15

X=10, P_b= 5*10^-33

X=12, P_b= 10^-42

Here in the above section we are comparing with simulation values and theoretical value which I obtained after solving with the help of equation 5 and 3.

As the SNR value increases the BER value decreases here in both simulation and at theoretical calculation. But, in theoretical calculation the BER value is very low compared to simulated value.

8) The BER calculation and plotting graph for 4-QAM, 16QAM ,8-PSK and BPSK was done and shown in below

a) 4 and 16-Qam with 0 to 14dB with 1 step:

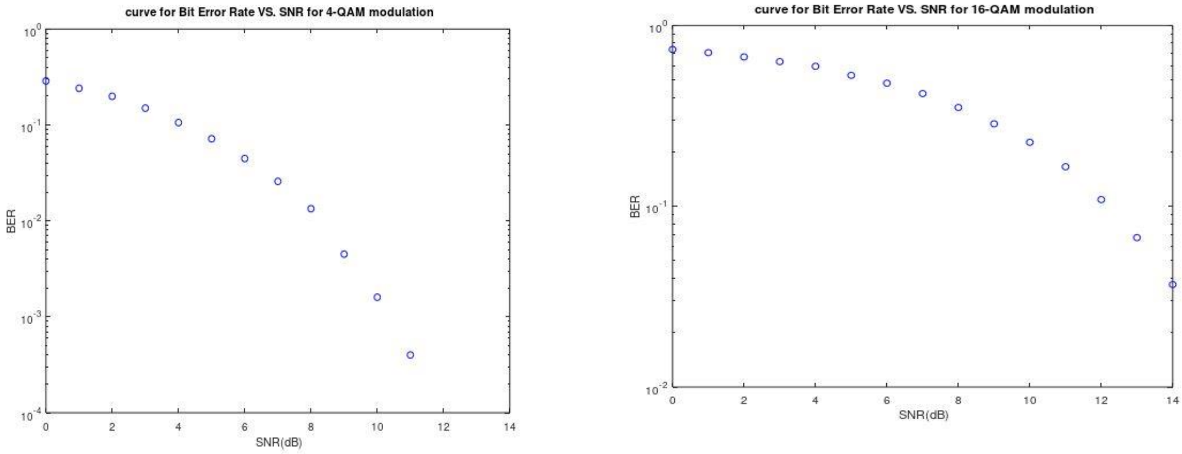


Fig3.1.a: BER plot for 4QAM and 16-QAM

b) The BER calculation and plotting graph for 8-PSK and BPSK with 0 to 14dB with 1 step:

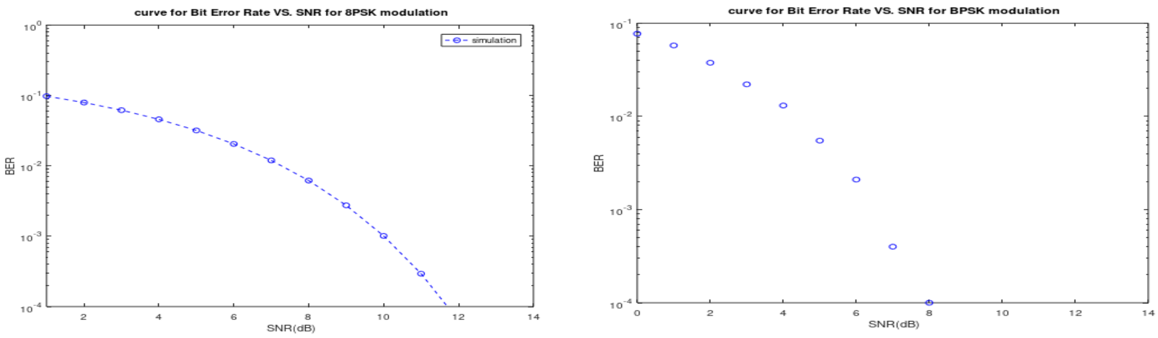


Fig3.1.b: BER plot for 8-PSK and 16 -QAM

Performance of 4-QAM over a Rayleigh fading Channel

1. Rayleigh fading Channel:

"It defines as the magnitude of a signal that has passed through such a transmission medium or communication channel will vary randomly, or fade, according to a Rayleigh distribution. The radial component of the sum of two uncorrelated Gaussian random variables."[9]

This model is used in situation where the signal may scatter between the transmitter and receiver because of terrestrial bodies. The most obvious path of the signal is the direct or line of sight path. Then its obvious in the form of fading, the multipath propagation exists.

At the receiver side, all received signal reached through the multiple paths are considered as variable and these signals are sum up together by considering the phase. It means if the signal are in phase with each other then they would sum up.

BER over Rayleigh fading Channel

To understand this, we need to start with coherent detection in a flat fading channel. This Here, the decision variable u[k] can obtain after sampling, phase rotation and fading the signal is given by

$$u[k] = \sqrt{\frac{E_g}{T}}\,|h[k]|\,a[k] + \tilde{n}[k].$$

Where h[k] is channel gain.
This coherent detection performs symbol by symbol.

BER for slow fading is given by

$$\overline{p}(\overline{\gamma}) = E\{p_b(\gamma)\} = \int_0^\infty p_b(\gamma)\, f_\gamma(\gamma)\, \mathrm{d}\gamma,$$

………………….. (1)

Where

$\gamma = E_b/N_0$: instantaneous E_b/N_0

$\overline{\gamma} = E\{E_b/N_0\}$: mean E_b/N_0

If h[k] is exponentially distributed and the noise power is constant then probability density function is given by

$$f_\gamma(\gamma) = \frac{1}{\overline{\gamma}} e^{-\frac{\gamma}{\overline{\gamma}}}.$$

………………………… (2)

The SNR for above equation is exponentially distributed.

Using equation 1 and 2 we can obtain BER as given below.

$$\overline{p}_b(\overline{\gamma}) = \frac{1}{2}\left(1 - \sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}\right)$$

3.2 Solutions:

1. Here we need to follow the same procedure as we did in 3.1. But here we need to add channel coefficient h[k] which realizes of the random process. So that we can draw a complex valued random number that is normal distributed with respect to real and imaginary part.

h = sqrt(1/2) *(randn() + j* randn());

At the receiver we need to bit multiply with h and sum of symbol and noise.

r = h .* a + n ;

Also we have to do derotation u[k]. It was done by multiplying the received signal r with exp(-j angle (h)).

i.e u= r .* (exp(-j * angle(h)));

And the same method carries out further as we did in 3.1, while carrying out demodulation we just need to change the Euclidean distance.

d(k)= abs(u(i)-abs(h(i))*Â(k)).^2;

2. Determining the Bit Error Probability (BER) for QAM using MATLAB simulation after carrying out the earlier discussed procedure, we obtain as shown bellow

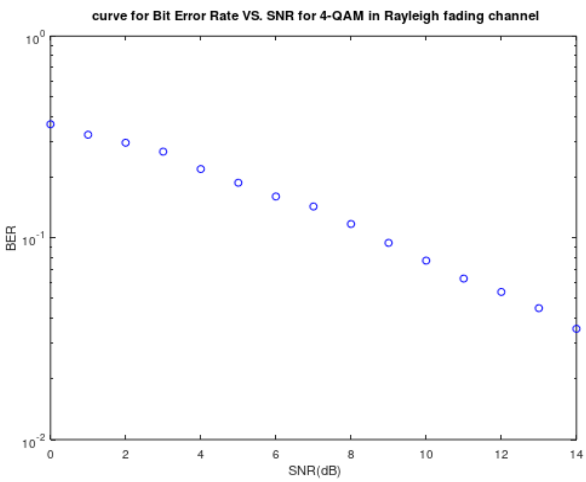


Fig3.2.a : BER for 4-QAM in Rayleigh fading channel

3) Plotted and compared Rayleigh with AWGN channel using 4-QAM. Simulation was done and plotted in the same figure. Also theoretical AWGN is calculated using the formula:

theoryBerAWGN = 1/2*erfc(sqrt(10.^(SNR_db/10)));

where SNR_db was 0 to 14dB with 1 step.

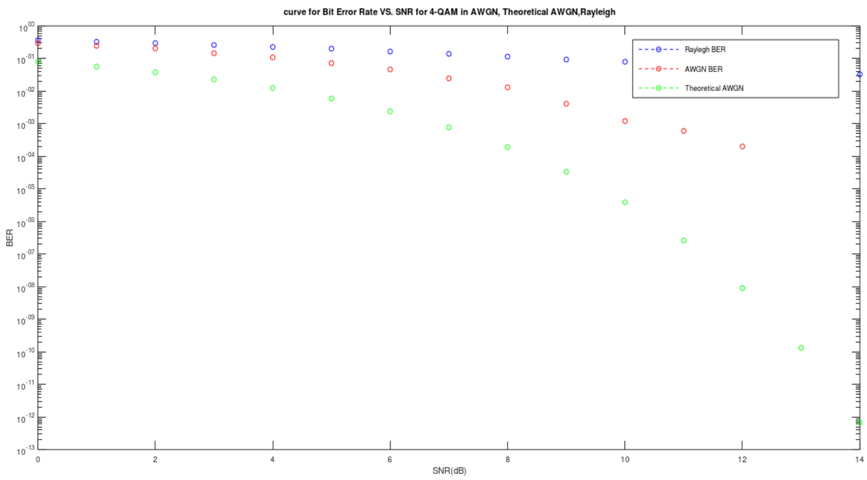


Fig 3.2.b: BER for Rayleigh , AWGN and theoretical AWGN

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