$$\frac{(AB)}{2n} = -\frac{4}{4} = -1$$

$$\therefore \frac{1}{2}[\bar{y}_{11} + \bar{y}_{00}] - \frac{1}{2}[\bar{y}_{10} + \bar{y}_{01}] = \frac{(AB)}{2n}$$

(iv) Main effect of A $= \dfrac{\text{A contrast}}{2n} = \dfrac{28}{4} = 7$

Main effect of B $= \dfrac{\text{B contrast}}{2n} = \dfrac{12}{4} = 3$

Main effect of interaction AB $= \dfrac{\text{AB contrast}}{2n} = \dfrac{-4}{4} = -1$

Example 2. The following data represents the results of a 2^2 factorial design with 2 factors and 2 levels each with four replications. Analyse the data for response.

Treatment combination	**Replications**			
	I	**II**	**III**	**IV**
(1)	12	12.3	11.8	11.6
a	12.8	12.6	13.7	14
b	11.5	11.9	12.6	11.8
ab	14.2	14.5	14.4	15

Solution : Let A and B be the two factors.

Let high and low be the levels.

n = number of replications = 4

We code the data by subtracting 12 from each value. The coded data is

Treatment combination	**Replications**				**Total**	x_1^2	x_2^2	x_3^2	x_4^2
	x_1	x_2	x_3	x_4					
(1)	0	0.3	−0.2	−0.4	−0.3	0	0.09	0.04	0.16
a	0.8	0.6	1.7	2	5.1	0.64	0.36	2.89	4
b	−0.5	−0.1	0.6	−0.2	−0.2	0.25	0.01	0.36	0.04
ab	2.2	2.5	2.4	3	10.1	4.84	6.25	5.76	9
Total					14.7	5.73	6.71	9.05	13.2

$$N = 4 \times 4 = 16, \qquad T = 14.7$$

$$\text{Correction factor} = \frac{(14.7)^2}{16} = 13.5$$

$$\text{A contrast} = a + ab - b - (1)$$

$$= 5.1 + 10.1 - (-0.2) - (-0.3) = 15.7$$

$$\text{B contrast} = b + ab - a - (1)$$

$$= -0.2 + 10.1 - 5.1 - (-0.3) = 5.1$$

$$\text{AB contrast} = (1) + ab - a - b$$

$$= -0.3 + 10.1 - 5.1 - (-0.2) = 4.9$$

$$SSA = \frac{(\text{A contrast})^2}{4n} = \frac{(15.7)^2}{4 \times 4} = 15.41$$

$$SSB = \frac{(\text{B contrast})^2}{4n} = \frac{(5.1)^2}{4 \times 4} = 1.63$$

$$SSAB = \frac{(\text{AB contrast})^2}{4n} = \frac{(4.9)^2}{4 \times 4} = 1.5$$

$$SST = \Sigma x_i^2 - \frac{T^2}{N} = 5.73 + 6.71 + 9.05 + 13.2 - 13.5$$

$$= 34.7 - 13.5 = 21.2$$

$$SSE = SST - SSA - SSB - SSAB = 21.2 - 15.41 - 1.63 - 1.5 = 2.66$$

ANOVA TABLE

Source	SS	df	MS	Variation ratio F
Factor A	$SSA = 15.41$	1	$MSA = SSA = 15.41$	$F_A = \frac{MSA}{MSE} = \frac{15.41}{0.222} = 69.41$
Factor B	$SSB = 1.63$	1	$MSB = SSB = 1.63$	$F_B = \frac{MSB}{MSE} = \frac{1.63}{0.222} = 7.34$
Interaction AB	$SSAB = 1.5$	1	$MSAB = SSAB = 1.5$	$F_{AB} = \frac{MSAB}{MSE} = \frac{1.5}{0.222} = 6.76$
Error	$SSE = 2.66$	$4(n-1) = 12$	$MSE = \frac{SSE}{12} = \frac{2.66}{12} = 0.222$	
Total	$SST = 21.2$	$4n - 1 = 15$		

Null hypothesis : H_0 : All the mean effects are equal.

Alternative hypothesis : H_1 : Not all equal.

Factor A : The calculated value of $F_A = 69.41$

At 5% level of significance, table value of

$$F_A(1, 12) = 4.75$$

$\therefore$ the calculated value of $F_A >$ the table value of F_A

$\therefore$ H_0 is rejected at 5% level.

i.e., the effect of A is significant.

Factor B : At 5% level, the table value of $F_A(1, 12) = 4.75$

$\therefore$ the calculated value of $F_A >$ the table value of F_A

$\therefore$ H_0 is rejected. ie. the effect of B is significant.

Interaction AB : The calculated value of $F_{AB} = 6.76$

At 5% level the table value of $F_{AB}(1, 12) = 4.75$

$\therefore$ the calculated value of $F_{AB} >$ the table value of F_{AB}

$\therefore$ H_0 is rejected. i.e., the effect of interaction AB is significant.

Example 3. The following data are obtained from a 2^2 factorial experiment replicated three times. Evaluate the sum of the squares for all factorial effect by the contrast method. Draw conclusions.

Treatment combination	Replicate 1	Replicate 2	Replicate 3
(1)	12	19	10
a	15	20	16
b	24	16	17
ab	24	17	29

Solution: First we code the data by substracting 20 from every value.

$\therefore$ The coded data is

Treatment combination	Replication			Total	x_1^2	x_2^2	x_3^2
	x_1	x_2	x_3				
(1)	–8	–1	–10	–19	64	1	100
a	–5	0	–4	–9	25	0	16
b	4	–4	–3	–3	16	16	9
ab	4	–3	9	10	16	9	49
Total				–21	121	26	174

$$n = \text{number of replications} = 3$$

$$N = 4 \times 3 = 12$$

$$\text{Correction factor} = \frac{T^2}{N} = \frac{(-21)^2}{12} = 36.75$$

$$\text{A contrast} = a + ab - b - (1)$$

$$= -9 + 10 - (-3) - (-19) = 23$$

$$\text{B contrast} = b + ab - a - (1)$$

$$= -3 + 10 - (-9) - (-19) = 35$$

$$\text{AB contrast} = (1) + ab - a - b$$

$$= -19 + 10 - (-9) - (-3) = 3$$

$$SST = \Sigma x_i^2 - \frac{T^2}{N} = 121 + 26 + 174 - 36.75 = 284.25$$

$$SSA = \frac{(\text{A contrast})^2}{4n} = \frac{(23)^2}{12} = 44.08$$

$$SSB = \frac{(\text{B contrast})^2}{4n} = \frac{35^2}{12} = 102.08$$

$$SSAB = \frac{(\text{AB contrast})^2}{4n} = \frac{3^2}{12} = 0.75$$

$$\therefore SSE = SST - SSA - SSB - SSAB$$

$$= 284.25 - 40.08 - 102.08 - 0.75 = 141.34$$

ANOVA TABLE

Source	SS	df	MS	Variation ratio F
Factor A	SSA = 44.08	1	MSA = 44.08	$F_A = \frac{MSA}{MSE} = \frac{44.08}{17.66} = 2.49$
Factor B	SSB = 102.08	1	MSB = 102.08	$F_B = \frac{MSB}{MSE} = \frac{102.08}{17.66} = 5.78$
Factor AB	SSAB = 0.75	1	$MSAB = \frac{SSAB}{1} = 0.75$	$F_{AB} = \frac{MSE}{MSAB} = \frac{17.66}{0.75} = 23.54$
Error	SSE = 141.34	$4(n-1)$ $= 4 \times 2$ $= 8$	$MSE = \frac{SSE}{8}$ $= \frac{141.34}{8} = 17.66$	
Total	SST = 284.25	$4n - 1$ $= 11$		

Null hypothesis : H_0 : All the mean effects are equal.

Alternative hypothesis : H_1 : Not all mean effects equal.

Factor A : The calculated value of $F_A = 2.49$

At 5% level of significance, the table value of $F_A(1, 8) = 5.32$

$\therefore$ the calculated value of $F_A <$ the table value of F_A

Hence H_0 is accepted at 5% of level of significance.

i.e., the mean effect of A is not significant.

Factor B : The calculated value of $F_B = 5.78$

At 5% level, the table value of $F_B(1, 8) = 5.32$

$\therefore$ the calculated value of $F_B >$ the table value of F_B

$\therefore$ H_0 is rejected at 5% level of significance.

i.e., the mean effect of B is significant.

Interaction AB : The calculated value of $F_{AB} = 23.54$

At 5% level, the table value of F_{AB} (8, 1) = 161

$\therefore$ the calculated value of F_{AB} < the table value of F_{AB}.

$\therefore$ H_0 is accepted at 5% level of significance.

i.e., the mean effect of interaction AB is not significant.

Example 4. In an experiment conducted by a mining department, to study a particular filtering system for coal, a coagulant was added to a solution in a tank containing coal and sludge, which was then placed in a recirculation system in order that the coal could be washed. Two factors were varied in the experimental process :

Factor A : Percent solids circulated initially in the overflow.

Factor B : Flow rate of the polymer.

The amount of solids in the under flow of cleaning system determines how clean the coal has become. Two levels of each factor were used and two experimental run were made for each of the $2^2 = 4$ combinations. The responses, percent solids by weight, in the underflow of the circulation system are as specified in the following table.

Treatment combination	**Response**	
	1	**2**
(1)	4.65	5.81
a	21.42	21.35
b	12.66	12.56
ab	18.27	16.62

Do a complete analysis of the data. Use 5% level of significance to find the effects :

Solution : Let A and B be two factors.

First we code the data by subtracting 20 from every value.

n = number of replications = 2