UNIT-5 STATISTICAL QUALITY CONTROL

Controlling the variations of the quality of the product using statistical methods is called statistical quality control.

NOTE: The variations occur due to random causes (temperature, pressure, etc), or assignable causes (raw material, products)

The random causes are AKA chance variations.

Random Causes

Variations which may occur due to many minor causes but behave in a random manner

Assignable Causes

Variations which may occur due to special non-random causes such as fatigue of technicians, a change in the raw material, improper machine testing, etc

Process Control

Control of the quality of the goods while they are in the process of production

Control Chart

Control chart is a graphical device mainly used for the control of the manufacturing process
There are two types of control charts:

1. Control chart of variables
2. Control chart of attributes

Variables

The quality characteristics of a product that are measureable are called variables.

Eg: weights of items, lengths of rods

Attributes

The quality characteristics of a product that are not measureable are called attributes.

Eg: no. of defects in metal disk

$\bar{X}$ - Chart (mean chart)

It is used to show the quality means of the samples drawn during the manufacturing process.

Steps to constructing $\bar{x}$ - chart:

1. Find the mean of each sample, ie, $\bar{x}_1, \bar{x}_2, \ldots$ (where $\bar{x}_i = \frac{\Sigma x_i}{n}$)

2. Find the mean of sample means $\bar{\bar{x}}$ obtained by the formula $\bar{\bar{x}} = \frac{\Sigma \bar{x}}{\text{number of samples}}$

3. Set up the control limits as follows:

$$UCL = \bar{\bar{x}} + 3\frac{\sigma}{\sqrt{n}} \quad \text{or} \quad UCL = \bar{\bar{x}} + A_2\bar{R}$$

$$LCL = \bar{\bar{x}} - 3\frac{\sigma}{\sqrt{n}} \quad \text{or} \quad LCL = \bar{\bar{x}} - A_2\bar{R}$$

where σ is the standard deviation

$\bar{R}$ is the biased estimator of σ

found by $\bar{R} = \frac{\Sigma R}{n}$

R is the sample range

the value of A_2 can be obtained from the table

Construction of R - Chart

The general procedure for constructing the R chart is similar to that of $\bar{X}$ - chart:

① Range of each sample R is determined

② Mean of sample range $\bar{R}$ is calculated

③ using $UCL = D_4 \bar{R}$ $(\bar{R} + 3\sigma_R)$

$LCL = D_3 \bar{R}$ $(\bar{R} - 3\sigma_R)$

Problems

1) Samples of 4 items each are taken from a company's manufacturing process at regular intervals and their diameters are measured. After 25 samples it was noted that $\bar{\bar{x}} = 1.561$, and $\Sigma R = 41.1$. Construct $\bar{X}$ - chart and R - chart for the diameters of the items produced.

$\bar{X}$ - chart

Given, $\bar{\bar{x}} = 1.561$ (central line)

$$\bar{R} = \frac{\Sigma R}{n} = \frac{41.1}{25} = 1.644$$

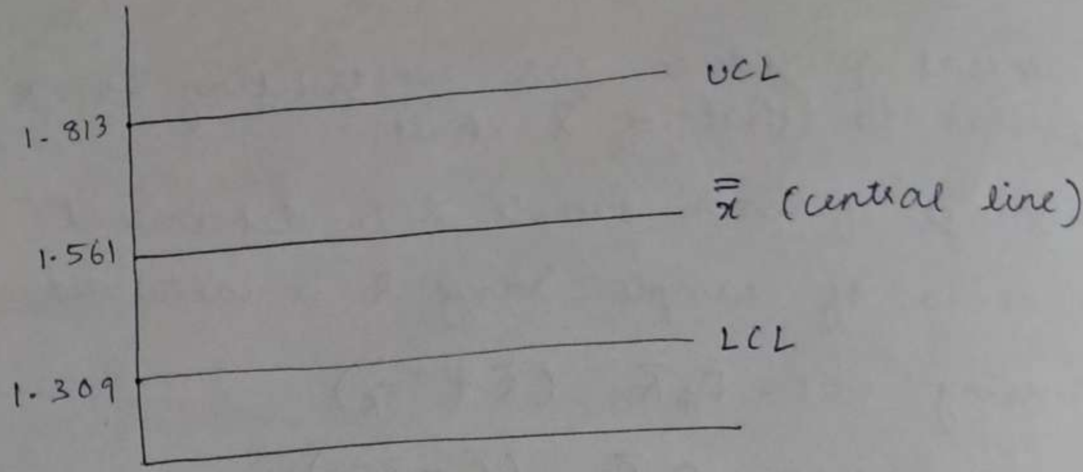
The value of A_2 corresponding to the sample size 25 is $A_2 = 0.153$.

$$\therefore UCL = \bar{\bar{x}} + A_2 \bar{R} = 1.561 + 0.153 \times 1.644$$

$$UCL = 1.813$$

$$LCL = \bar{\bar{x}} - A_2 \bar{R} = 1.561 - 0.153 \times 1.644$$

$$LCL = 1.309$$



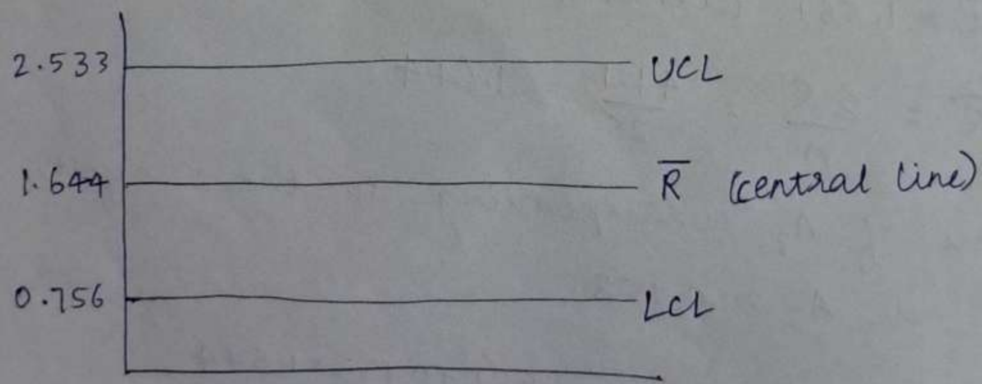
R-chart

We know that $\bar{R} = 1.644$

For the sample size 25, the values of D_3 and D_4 are given by $D_3 = 0.460$ and $D_4 = 1.540$ respectively.

We know that $UCL = D_4\bar{R} = 1.54 \times 1.644 = 2.533$

$LCL = D_3\bar{R} = 0.46 \times 1.644 = 0.756$



(2) Construct $\bar{X}$ chart and R chart for the following data.

Sample no	1	2	3	4	5	6	7	8	9	10
$\bar{X}$	14	15	14	13	12	10	16	17	18	20
R	3	1	2	1	1	1	2	2	3	4

Calculate $\bar{\bar{X}}$, $\bar{\bar{X}} = \dfrac{14+15+14+13+12+10+16+17+18+20}{10}$

$= 14.9$

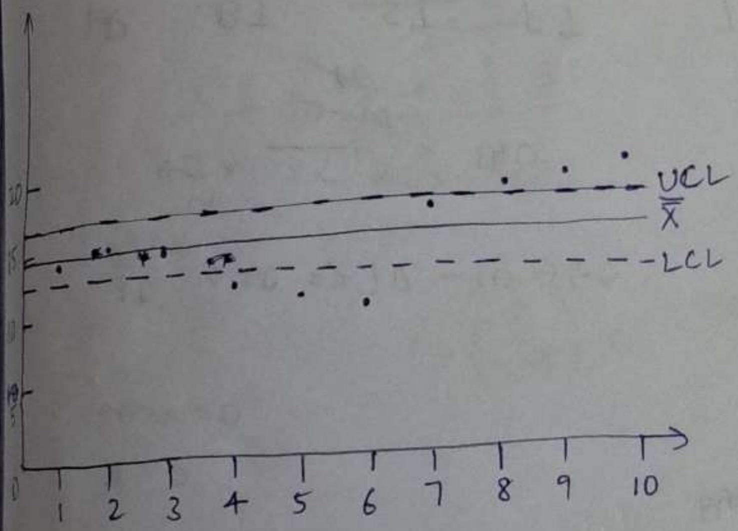
Calculate $\bar{R}$, $\bar{R} = \dfrac{3+1+2+1+1+1+2+2+3+4}{10} = 2$

$\bar{X}$-chart : $UCL = \bar{\bar{X}} + A_2\bar{R} = 14.9 + 0.577 \times 2 = 16.054$

$LCL = \bar{\bar{X}} - A_2\bar{R} = 14.9 - 0.577 \times 2 = 13.746$

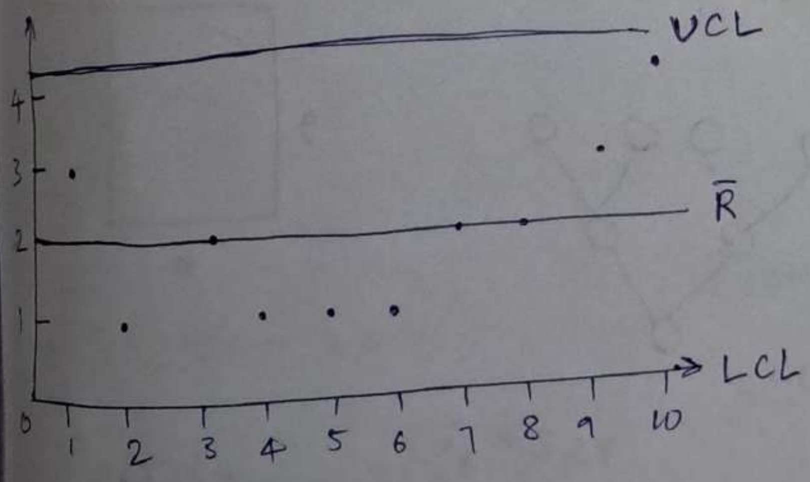
R-chart : $UCL = D_4\bar{R} = 2.114 \times 2 = 4.228$

$LCL = D_3\bar{R} = 0 \times 2 = 0$



Hence, the process is not in control, regarding the process mean.

$\bar{X}$ - chart



Hence, the process is under control regarding the process variability.

R - chart

P-chart

It is the control chart for fraction defective of or attributes.

The steps in constructing the control chart are as follows:

Q① Find the average fraction defective $\bar{P}$ by dividing the no. of defectives by total no. of units inspected.

② Central line corresponds to $\bar{P}$

③ Set up control limits as follows: $UCL = \bar{P} + 3 \times \sqrt{\frac{\bar{P}(1-\bar{P})}{n}}$

$LCL = \bar{P} - 3 \times \sqrt{\frac{\bar{P}(1-\bar{P})}{n}}$

where n = sample size

Problems

① 35 successive samples of 100 castings, each taken from the production line, contain 3, 3, 5, 3, 5, 0, 3, 2, 3, 5, 6, 5, 9, 1, 2, 4, 5, 2, 0, 10, 3, 6, 3, 2, 5, 6, 3, 3, 2, 5, 1, 0, 7, 4 and 3 rejectable castings respectively. Construct a p-chart and state whether the process is under control or not.

Sample no	No. of rejectable	F.D	Sample no	No. of rejectable	F.D	Sample no	No. of rejectable	F.D
1	3	0.03	16	4	0.04	31	[illegible]	0.0[illegible]
2	3	0.03	17	5	0.05	32	[illegible]	0
3	5	0.05	18	2	0.02	33	[illegible]	0.0[illegible]
4	3	0.03	19	0	0	34	[illegible]	0.0[illegible]
5	5	0.05	20	10	0.10	35	[illegible]	0.0[illegible]
6	0	0	21	3	0.03			
7	3	0.03	22	6	0.06			
8	2	0.02	23	3	0.03			
9	3	0.03	24	2	0.02			
10	5	0.05	25	5	0.05			
11	6	0.06	26	6	0.06			
12	5	0.05	27	3	0.03			
13	9	0.09	28	3	0.03			
14	1	0.01	29	2	0.02			
15	2	0.02	30	5	0.05			

Total = 1.2[illegible]

∴ average ~~defective~~ fraction defective $= \frac{1.29}{35} = 0.04 = \bar{p}$

$\Rightarrow UCL = \bar{p} + 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.04 + 3 \times \sqrt{\frac{0.04 \times 0.96}{100}} = 0.10$

$LCL = \bar{p} - 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.04 - 3 \times \sqrt{\frac{0.04 \times 0.96}{100}} = -0.02$ (taken 0)

HW

sample no	No of defectives	Fraction Defective
1	2	0.02
2	2	0.02
3	3	0.03
4	6	0.06
5	1	0.01
6	3	0.03
7	6	0.06
8	7	0.07
9	4	0.04
10	2	0.02
11	5	0.05
12	0	0
13	3	0.03
14	2	0.02
15	4	0.04
16	5	0.05
17	3	0.03
18	8	0.08
19	1	0.01
20	4	0.04
		0.71

∴ avg FD $= \frac{0.71}{20} = 0.0355 = \bar{p}$

$UCL = \bar{p} + 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.041$

$LCL = \bar{p} - 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.029$

21.05.24

~~Pchart~~

P-chart (all samples have same size)

n = sample size

$$\bar{p} = \frac{\text{total no. of defectives}}{\text{total no. of units inspected}} = \frac{\text{total fraction defective}}{\text{no. of samples}}$$

P-chart (all samples have different sizes)

$$n = \frac{\text{total no. of units}}{\text{no. of samples}} \qquad \bar{p} = \frac{\text{total no. of defectives}}{\text{total no. of units}}$$

③ Construct a control chart for the defectives for the following data:

S no	Inspected	Defective	Proportion of defective
1	90	9	9/90 = 0.1
2	65	7	7/65 = 0.11
3	85	3	3/85 = 0.04
4	70	2	2/70 = 0.03
5	80	9	9/80 = 0.11
6	80	5	5/80 = 0.06
7	70	3	3/70 = 0.04
8	95	9	9/95 = 0.09
9	90	6	6/90 = 0.07
10	75	7	7/75 = 0.09
	800	60	0.74

$$n = \frac{800}{10} = 80$$

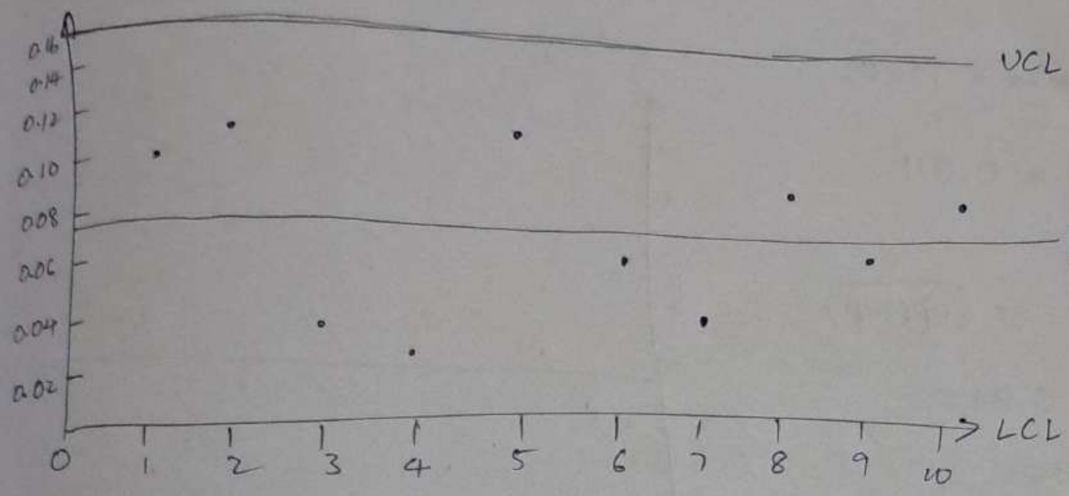
$$\bar{p} = \frac{0.74}{10} = 0.074 \quad (\text{total FD / no. of samples})$$

(or)

$$\bar{p} = \frac{60}{800} = 0.075 \quad (\text{total def / total inspected})$$

$\Rightarrow LCL = \bar{p} - 3 \times \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.075 - 0.087 = -0.012 \simeq 0$

$UCL = 0.075 + 0.087 = 0.162$



CAT-2 syllabus

unit 2 → transformation, correlation, regression

Unit 3 → full

Unit 4 → one-way, two-way, latin square

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np-chart

one can plot the numbers of defectives rather than the fractions of defective if the sample size is constant throughout, and construct a control chart. Such a chart is called np-chart.

$$CL = \bar{p} = \frac{\text{no. of defective items}}{\text{no. of samples} \times \text{sample size}}$$

$UCL = n\bar{p} + 3\sqrt{n\bar{p}(1-\bar{p})}$ where n = sample size

$LCL = n\bar{p} - 3\sqrt{n\bar{p}(1-\bar{p})}$

Problems

① In a factory, 1000 bolts are examined daily for defects in 15 days. The following are the no. of defects

9, 10, 12, 8, 7, 15, 10, 12, 10, 8, 7, 13, 14, 15, 16

Draw a np-chart and give your findings.

n = sample size = 1000

$\bar{p} = \dfrac{166}{15 \times 1000} = 0.011$

$CL = n\bar{p} = 11$

$UCL = n\bar{p} + 3 \times \sqrt{n\bar{p}(1-\bar{p})}$

$= 11 + 9.895$

$= 20.895$

$LCL = 11 - 9.895$

$= 1.105$