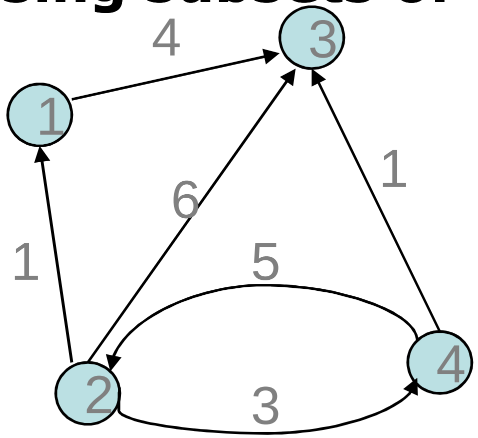
Unit III

Floyd's Algorithm

Floyd's Algorithm:All pairs shortest paths

- **Problem: In a weighted (di)graph, find shortest paths between every pair of vertices**

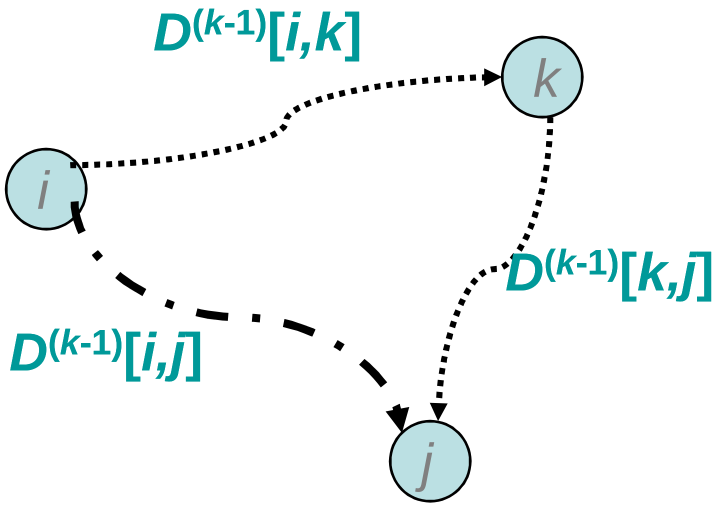
- **Same idea: construct solution through series of matrices $D^{(0)}$, …, $D^{(n)}$ using increasing subsets of the vertices allowed as intermediate**
- **Example:**



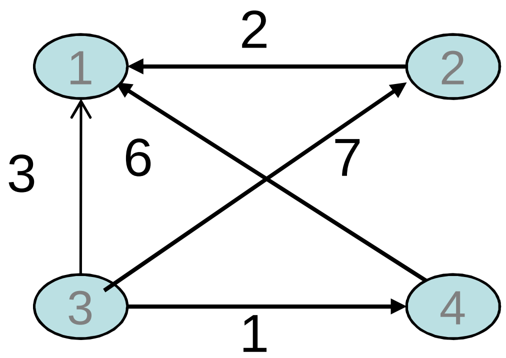
Floyd's Algorithm (matrix generation)

- **On the *k*-th iteration, the algorithm determines shortest paths between every pair of vertices *i, j* that use only vertices among 1,…,*k* as intermediate**

- $$D^{(k)}[i,j] = \min \{D^{(k-1)}[i,j],\ D^{(k-1)}[i,k] + D^{(k-1)}[k,j]\}$$



Floyd’s Algorithm (example)



$$D^{(0)} = \begin{matrix} 0 & \infty & 3 & \infty \\ 2 & 0 & \infty & \infty \\ \infty & 7 & 0 & 1 \\ 6 & \infty & \infty & 0 \end{matrix}$$

$$D^{(1)} = \begin{matrix} 0 & \infty & 3 & \infty \\ 2 & 0 & \mathbf{5} & \infty \\ \infty & 7 & 0 & 1 \\ 6 & \infty & \mathbf{9} & 0 \end{matrix}$$

$$D^{(2)} = \begin{matrix} 0 & \infty & 3 & \infty \\ 2 & 0 & 5 & \infty \\ \mathbf{9} & 7 & 0 & 1 \\ 6 & \infty & 9 & 0 \end{matrix}$$

$$D^{(3)} = \begin{matrix} 0 & \mathbf{10} & 3 & \mathbf{4} \\ 2 & 0 & 5 & \mathbf{6} \\ 9 & 7 & 0 & 1 \\ 6 & \mathbf{16} & 9 & 0 \end{matrix}$$

$$D^{(4)} = \begin{matrix} 0 & 10 & 3 & 4 \\ 2 & 0 & 5 & 6 \\ \mathbf{7} & 7 & 0 & 1 \\ 6 & 16 & 9 & 0 \end{matrix}$$

Floyd's Algorithm (pseudocode and analysis)

ALGORITHM *Floyd*$(W[1..n, 1..n])$

```
//Implements Floyd's algorithm for the all-pairs shortest-paths problem
//Input: The weight matrix W of a graph with no negative-length cycle
//Output: The distance matrix of the shortest paths' lengths
D ← W //is not necessary if W can be overwritten
for k ← 1 to n do
    for i ← 1 to n do
        for j ← 1 to n do
            D[i, j] ← min{D[i, j], D[i, k] + D[k, j]}
return D
```

Time efficiency: $\Theta(n^3)$

Space efficiency: Matrices can be written over their predecessors

Note: Shortest paths themselves can be found, too