UIT2504 Artificial Intelligence

Markov Chain Simulations in Bayesian Networks

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MCMC Algorithms

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- In general, *Markov Chain* refers to a random process that generates a sequence of states

Markov Chain: Example

- In the sprinkler example, consider $Sprinkler = true$ and $WetGrass = true$ as evidences. A state space for Markov chain, where nonevidence variables $Cloudy$ and $Rain$ can be modified, is as shown below:

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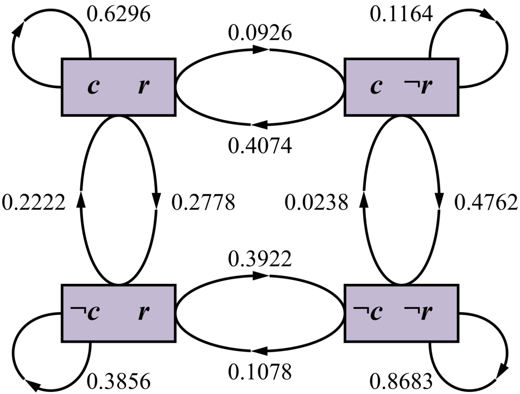

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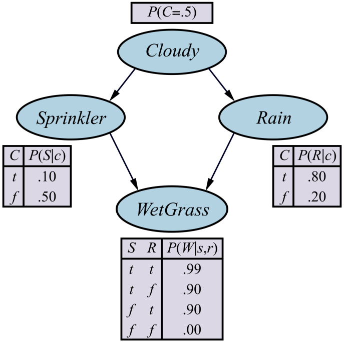
- The algorithm wanders randomly around the state space flipping one variable at a time (but keeping the evidence variables fixed)

Gibbs Sampling: Example

- Consider the query $\mathbf{P}(R|s, w)$



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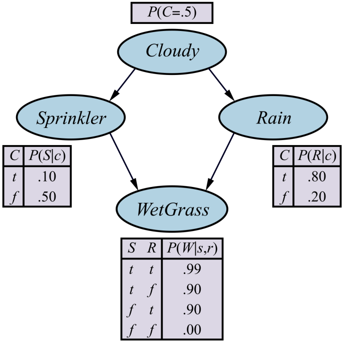

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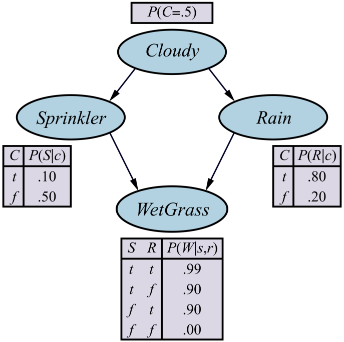

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- Choose a nonevidence variable randomly, say C. Markov blanket of C is S, R. So, sample from $\mathbf{P}(C|s, \neg r)$. Suppose we get $Cloudy = false$, the new state is $[false, \mathbf{true}, false, \mathbf{true}]$

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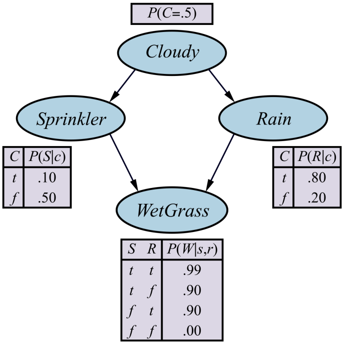

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- Suppose R is chosen next. Markov blanket of R is C, S, W. So, sample from $\mathbf{P}(R|\neg c, s, w)$. Suppose this yields $Rain = true$, the next state is

Gibbs Sampling Algorithm

function GIBBS-ASK($X, \mathbf{e}, bn, N$) **returns** an estimate of $\mathbf{P}(X \mid \mathbf{e})$
  local variables: $\mathbf{C}$, a vector of counts for each value of X, initially zero
    $\mathbf{Z}$, the nonevidence variables in bn
    $\mathbf{x}$, the current state of the network, initialized from $\mathbf{e}$

  initialize $\mathbf{x}$ with random values for the variables in $\mathbf{Z}$
  for $k = 1$ **to** N **do**
    choose any variable Z_i from $\mathbf{Z}$ according to any distribution $\rho(i)$
    set the value of Z_i in $\mathbf{x}$ by sampling from $\mathbf{P}(Z_i \mid mb(Z_i))$
    $\mathbf{C}[j] \leftarrow \mathbf{C}[j] + 1$ where x_j is the value of X in $\mathbf{x}$
  return NORMALIZE($\mathbf{C}$)

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$$P(c|s, \neg r) = \alpha P(c)P(s|c)P(\neg r|c) = \alpha \times 0.5 \times 0.1 \times 0.2$$
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- So, the sampling distribution is $\alpha\langle 0.001, 0.020\rangle \approx \langle 0.048, 0.952\rangle$

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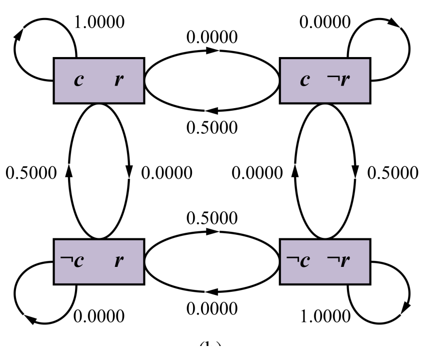

Metropolis–Hastings Sampling

- We saw that Gibbs sampling may get stuck to a part of the Markov Chain
- This may be solved by occassional random "restarts" where next state is generated from scratch
- For example, next state generation may be as follows:
    - With probability 0.95, perform a Gibbs sampling step to generate the next state x'
    - Otherwise, generate x' by running likelihood-weighted sampling algorithm
- This is an example of a *Metropolis–Hastings* algorithm, where the next state x' is generated from the current state x from a poposal distribution (an example is shown above)
- The proposed next state may be accepted or rejected according to the acceptance probability (depends on the ratio $\frac{\pi(x')}{\pi(x)}$ which can be computed by $\frac{P(x',e)}{P(x,e)}$

Summary

- We have discussed how knowledge under uncertainty can be represented using Bayesian Networks

What Next?

- Read chapter 13 of the text book!