UIT2504 Artificial Intelligence

Problem Solving Agents

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Introduction

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  - Given a map of cities, find a shortest Hamiltonian Circuit (Traveling Salesperson Problem)

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  - Given n objects of known weights $w_1, w_2, \cdots, w_n$ and values $v_1, v_2, \cdots, v_n$ and a knapsack of capacity W, find the most valuable subset of the items that fit into the knapsack (Knapsack Problem)

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- Is there any common approach to solve such problems? — general purpose problem solving approach

State-Space Approach

- "Formulate" the problem in "state space" and "search" for a "solution"

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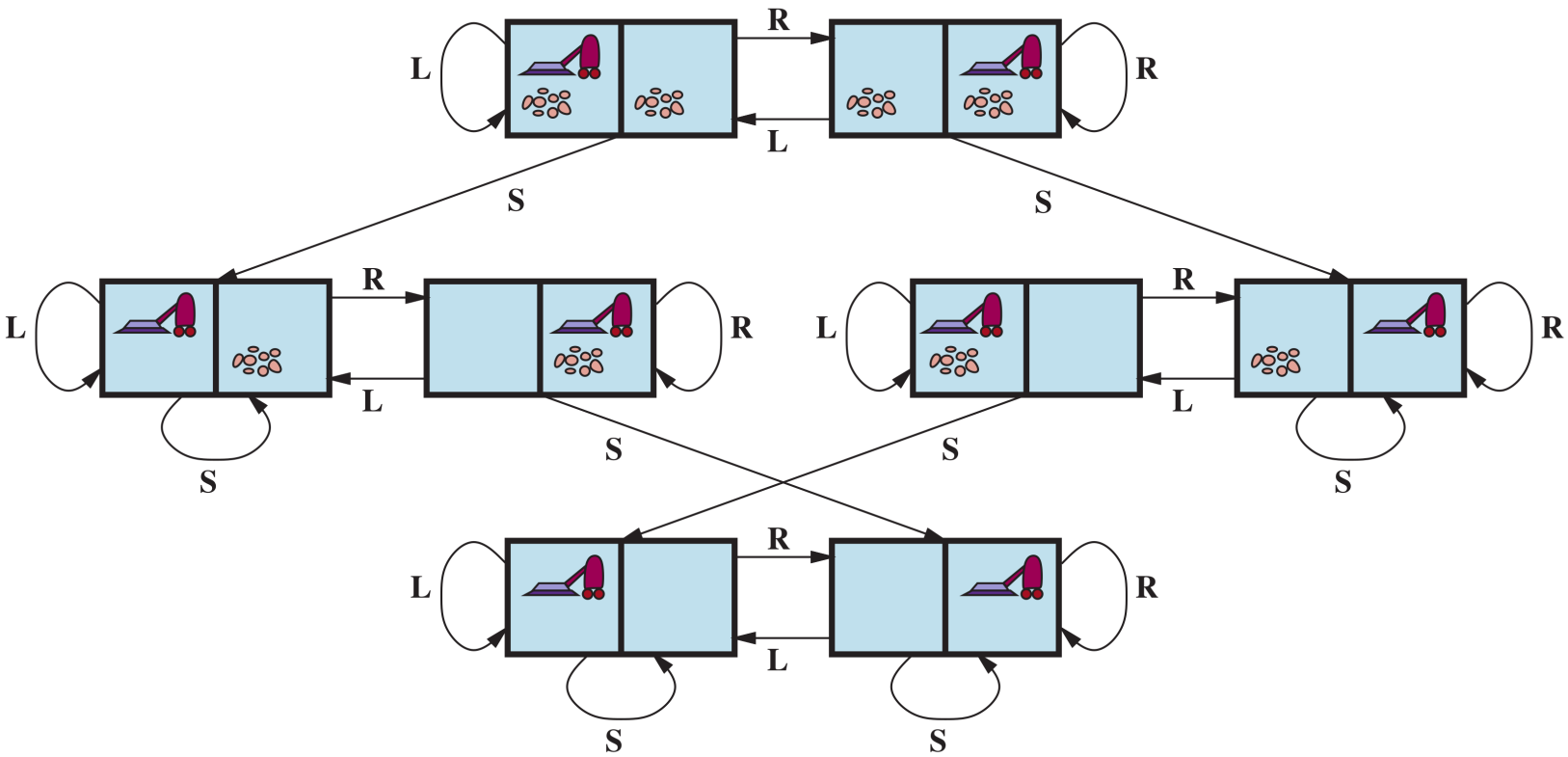
- States encapsulate certain properties of a system
- System changes from one state to another based on action / event
- Only certain actions may be permitted in a given state
- There is an "initial" state for the system and some "goal" states to terminate

State-Graph



Recall: Granularity of state definition

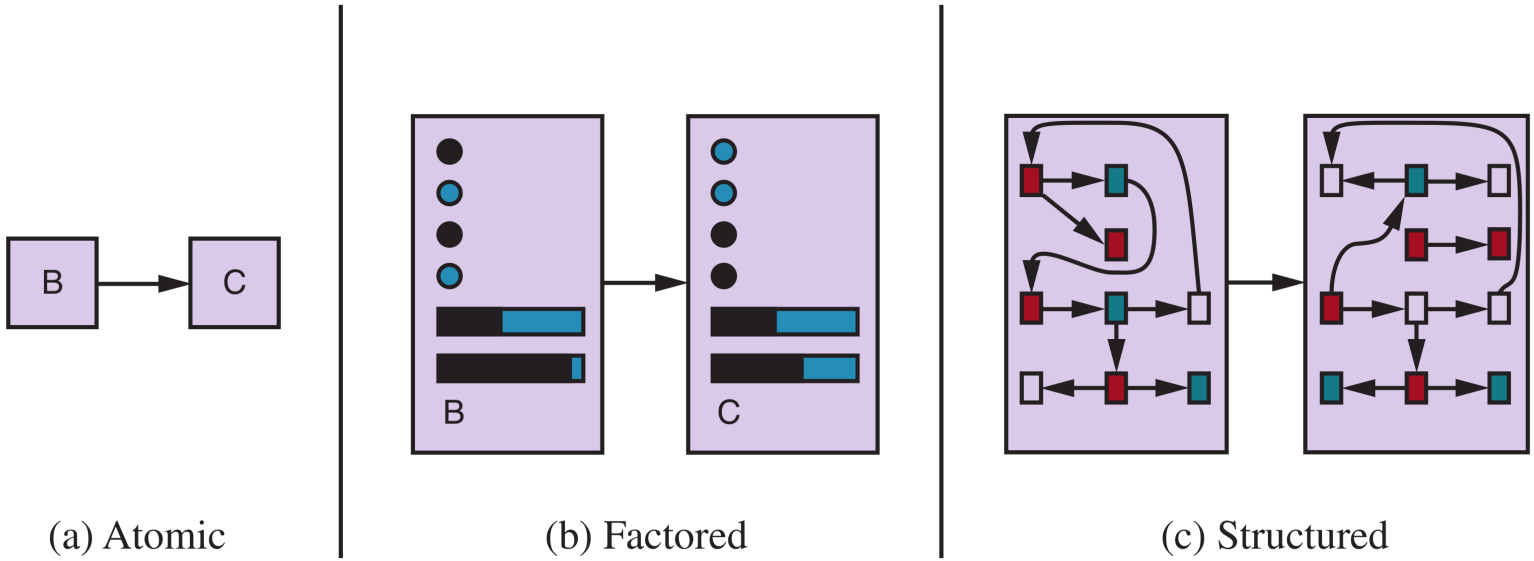


Illustration: 8-Puzzle

3	2	7
5	8	
1	4	6

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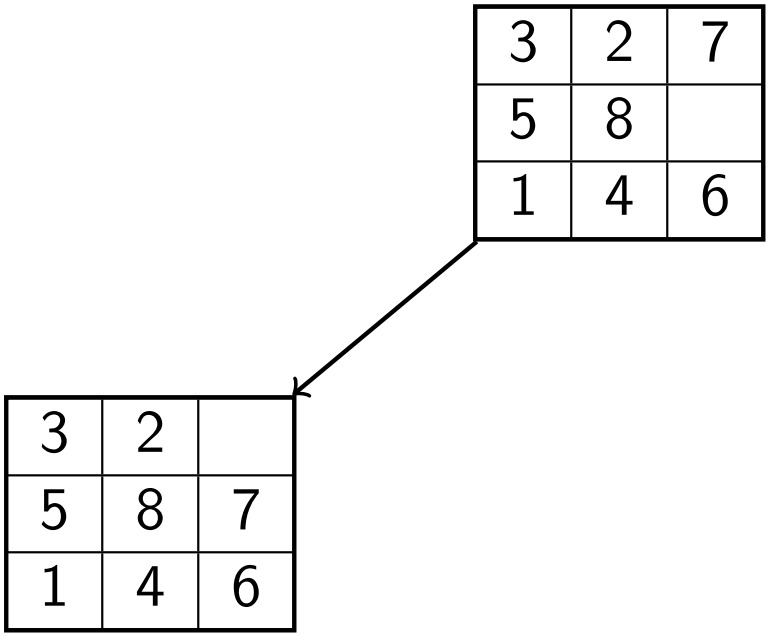

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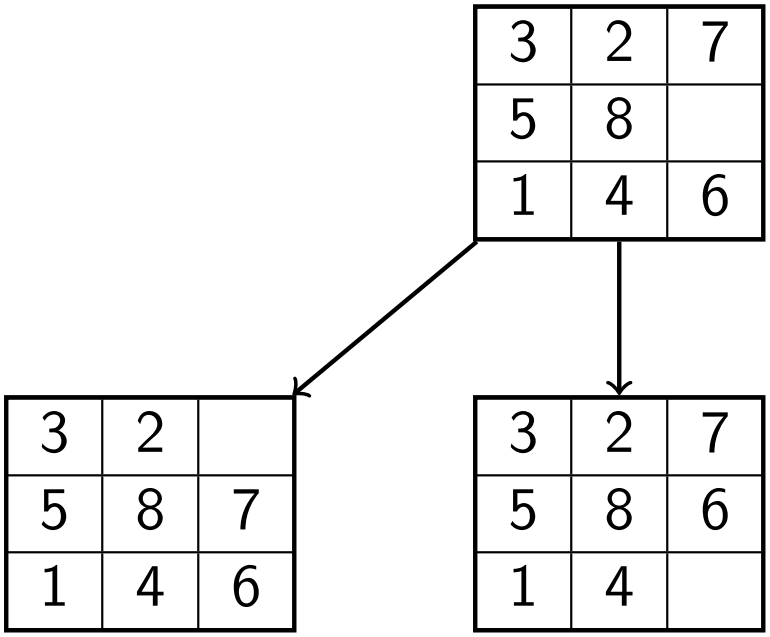

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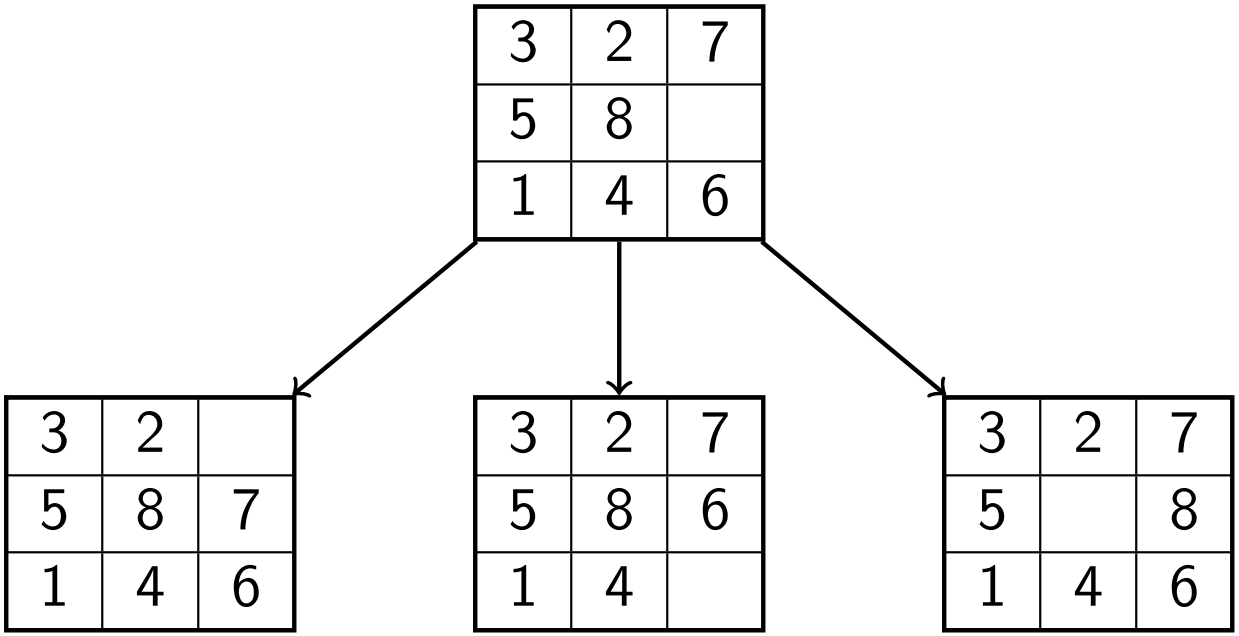

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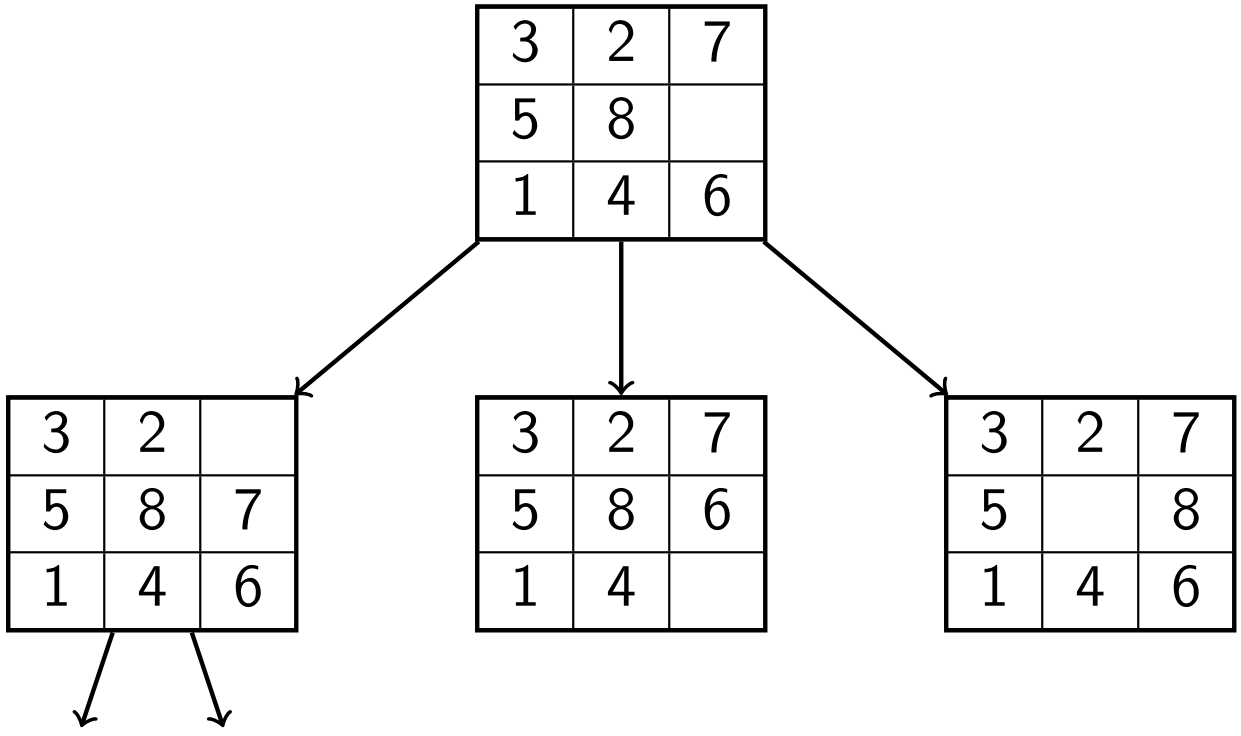

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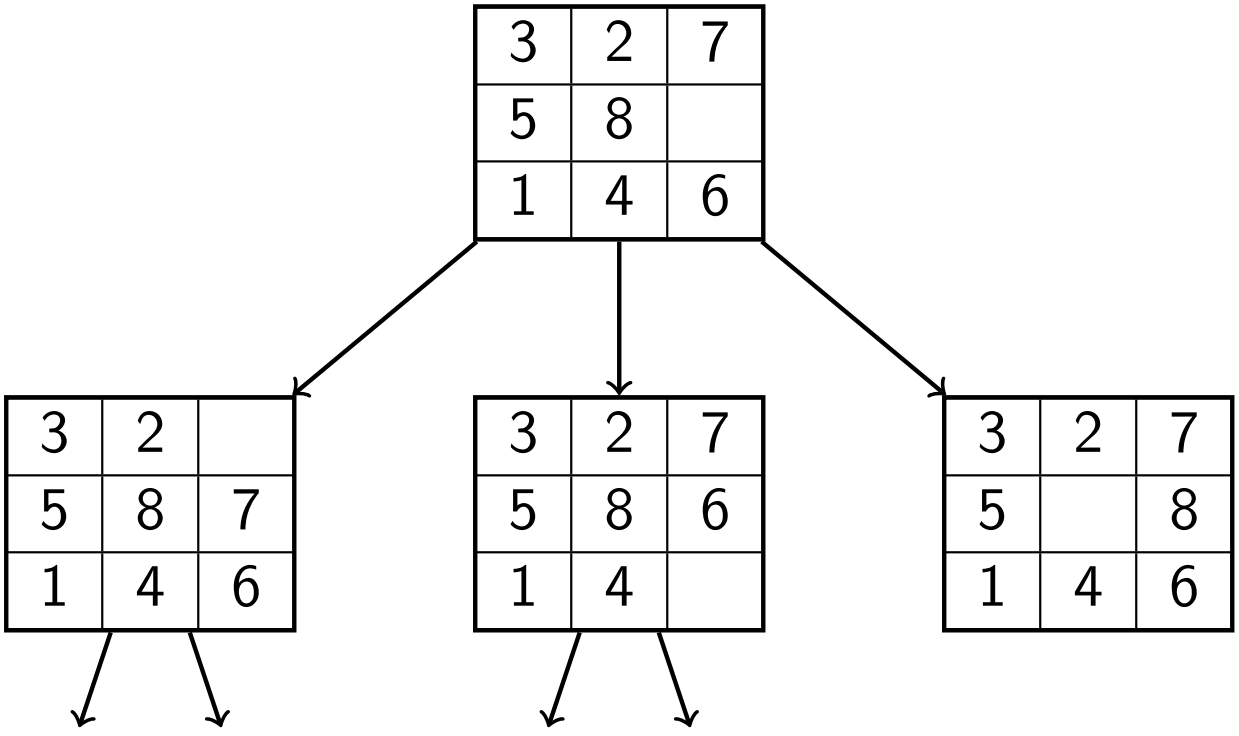

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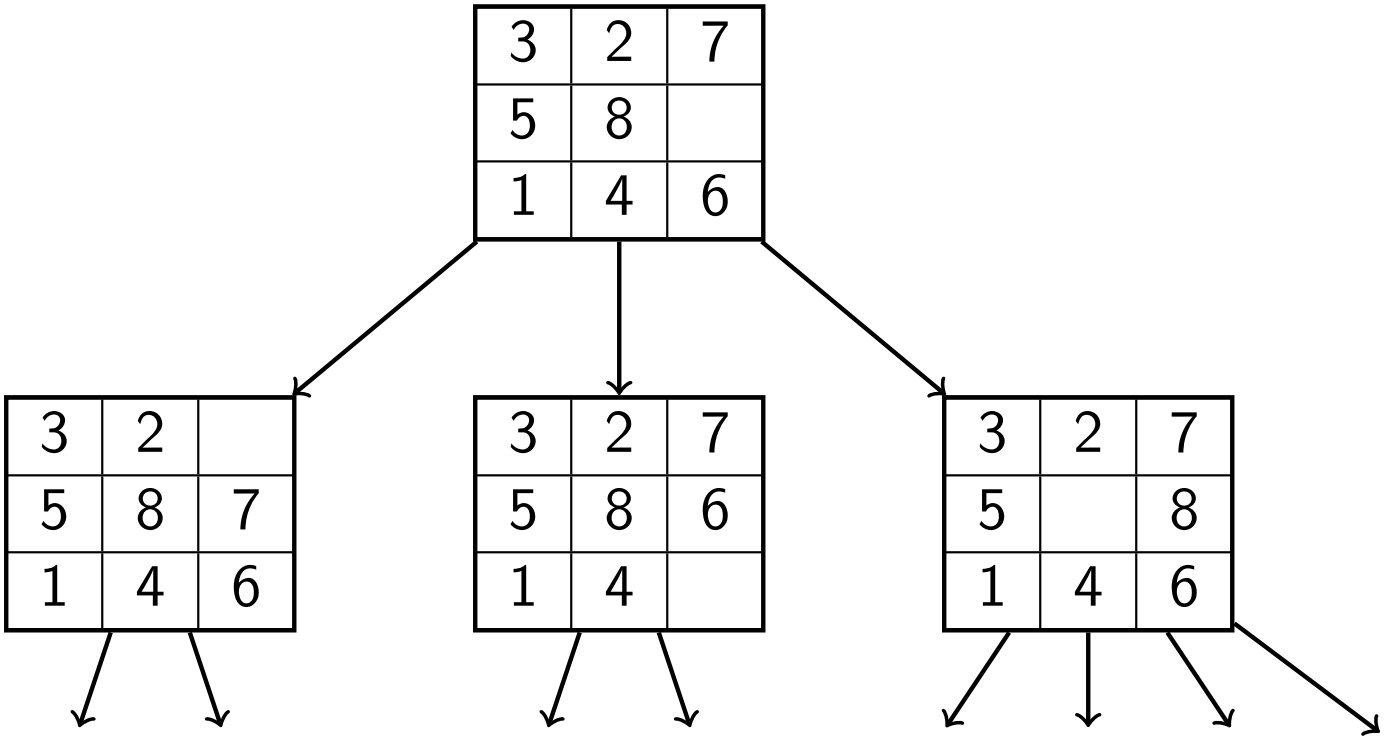

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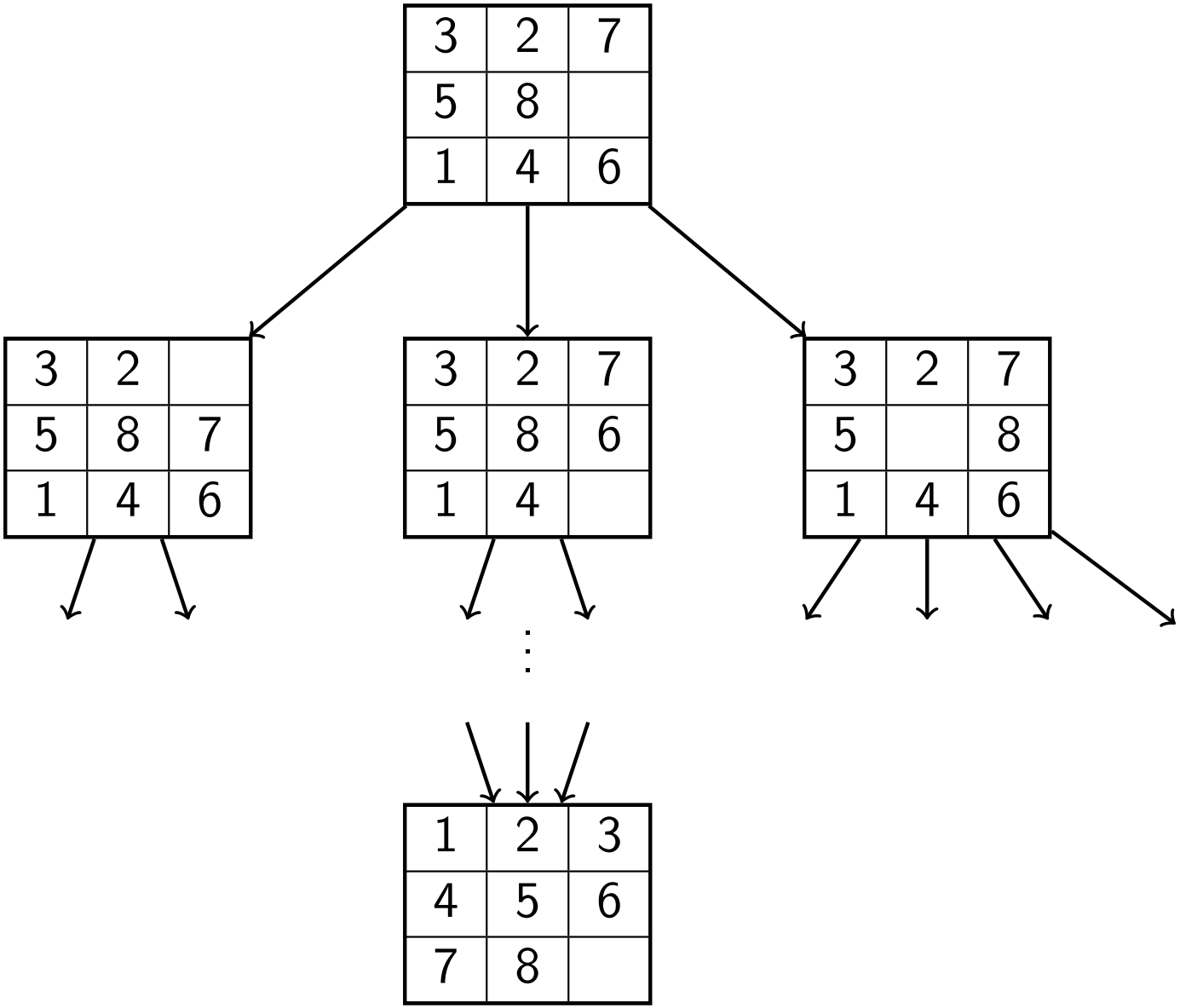

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- In some cases, goal state itself is a solution!

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  - An action cost function that gives cost of applying an action in a given state Path cost — sum of all the costs of actions from initial state to a goal state

Example: 8-Puzzle

- *State*:

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- *State*: Arrangement of eight tiles on a 3×3 grid

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- *Action cost*: unit cost for each action

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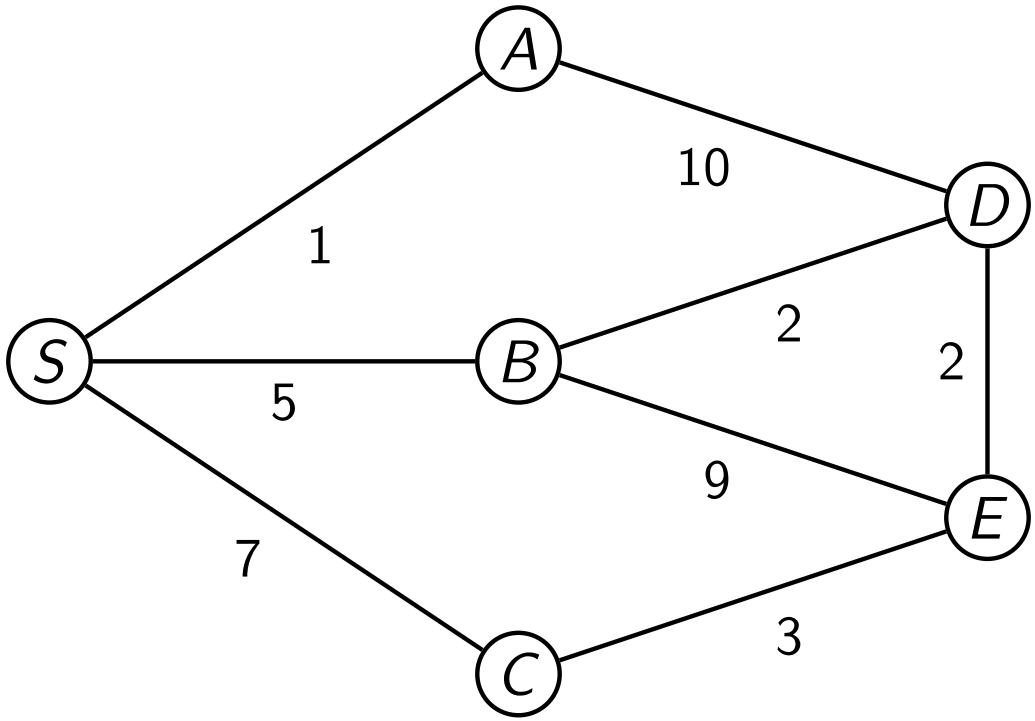

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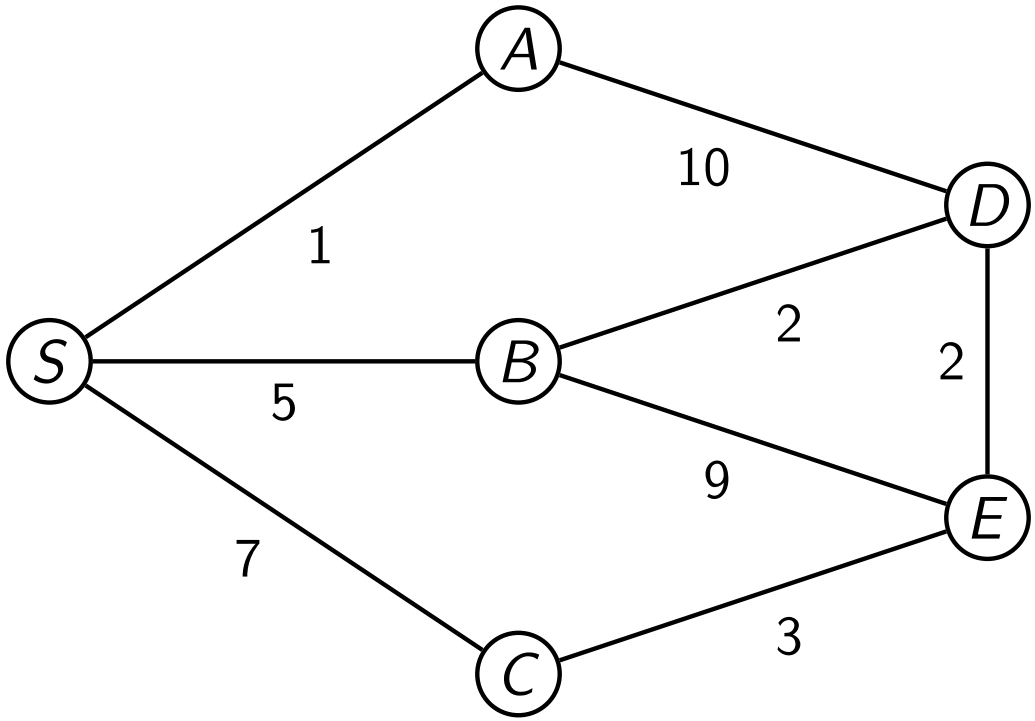

Find a route from S to E

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- *Action cost*: Cost associated with the road taken

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- Only 2057 possible sequences!!!

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- Complete state formulation — only one state is kept in memory — and it is iteratively improved

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  - If it is a goal state, return solution
  - Else, expand x and add the successor states $S(x)$ to the working set

Search Strategies

- Uninformed:
  - Depth-First
  - Breadth-First
  - Depth-Limited
  - Bi-Directional Search
- Informed (Heuristics):
  - Best-first Greedy
  - A^*
  - Local Search Strategies
- Constraint Satisfaction

Performance Measures

- Completeness
- Time Complexity
- Space Complexity
- Optimality

Questions?

- Read Chapter 3