Entropy Maximization

If there is no reason to discriminate between
two or several events
the best strategy is to consider them
as equally likely

Example

- How do you spend your week end?
- B (Going to Beach) – costs Rs. 1K
- C (Going to cinema) – costs Rs. 2K
- F (Visiting nearby water falls)–costs Rs. 3K
- Assume people only go to these places i.e. they do not have alternates
- People might have gone to multiple places
- On an average people spend Rs. 1.75K
- What is the probability they visit B, C and F?

Constraints

$$1 = P(B) + P(C) + P(F)$$

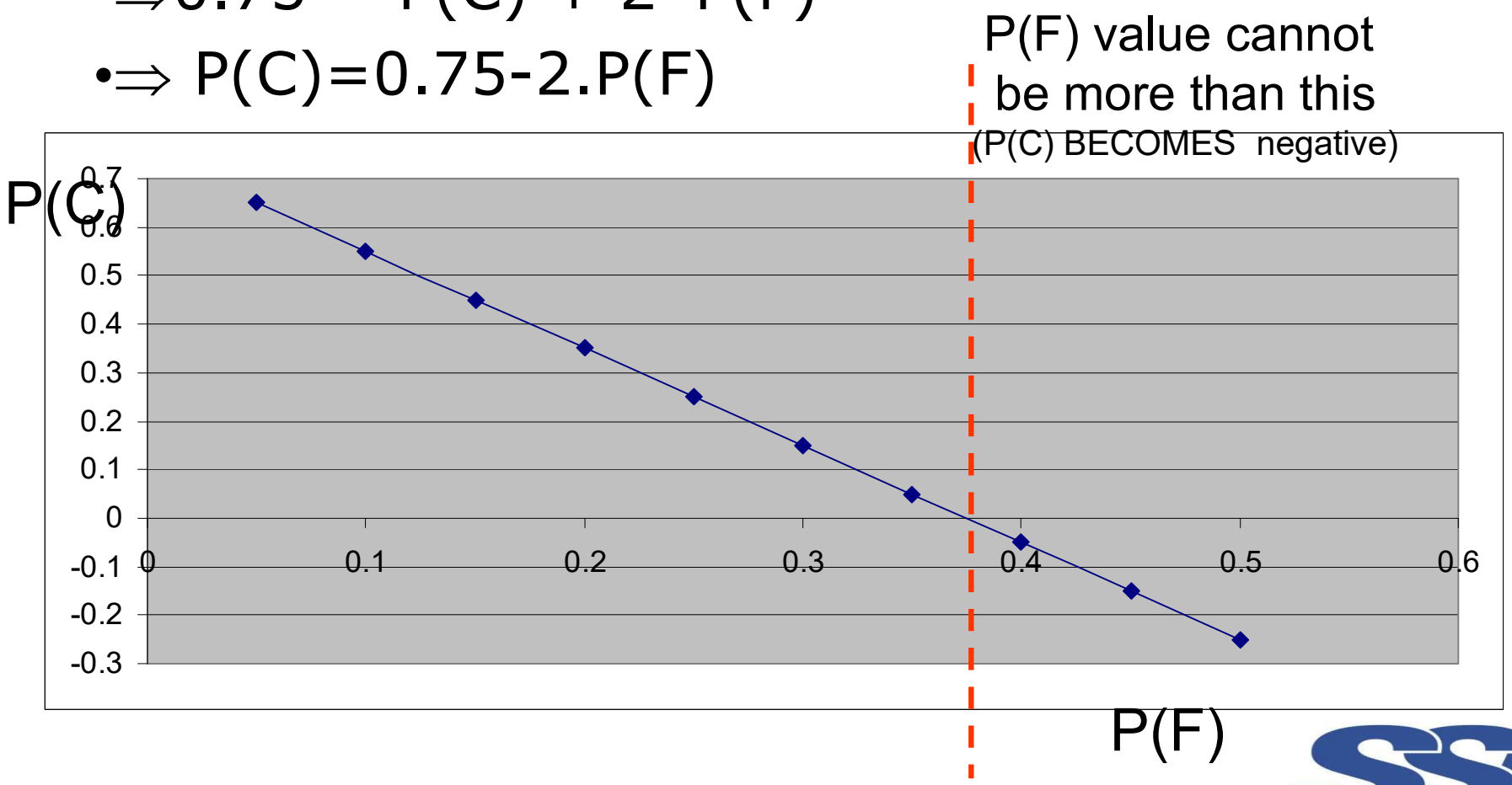
$$1.75 = 1*P(B) + 2*p(C) + 3*P(F)$$

Uncertainty

$$H= [-p(B)*Log(P(B))] + [-p(C)*Log(P(C))] + [-p(F)*Log(P(F))]$$

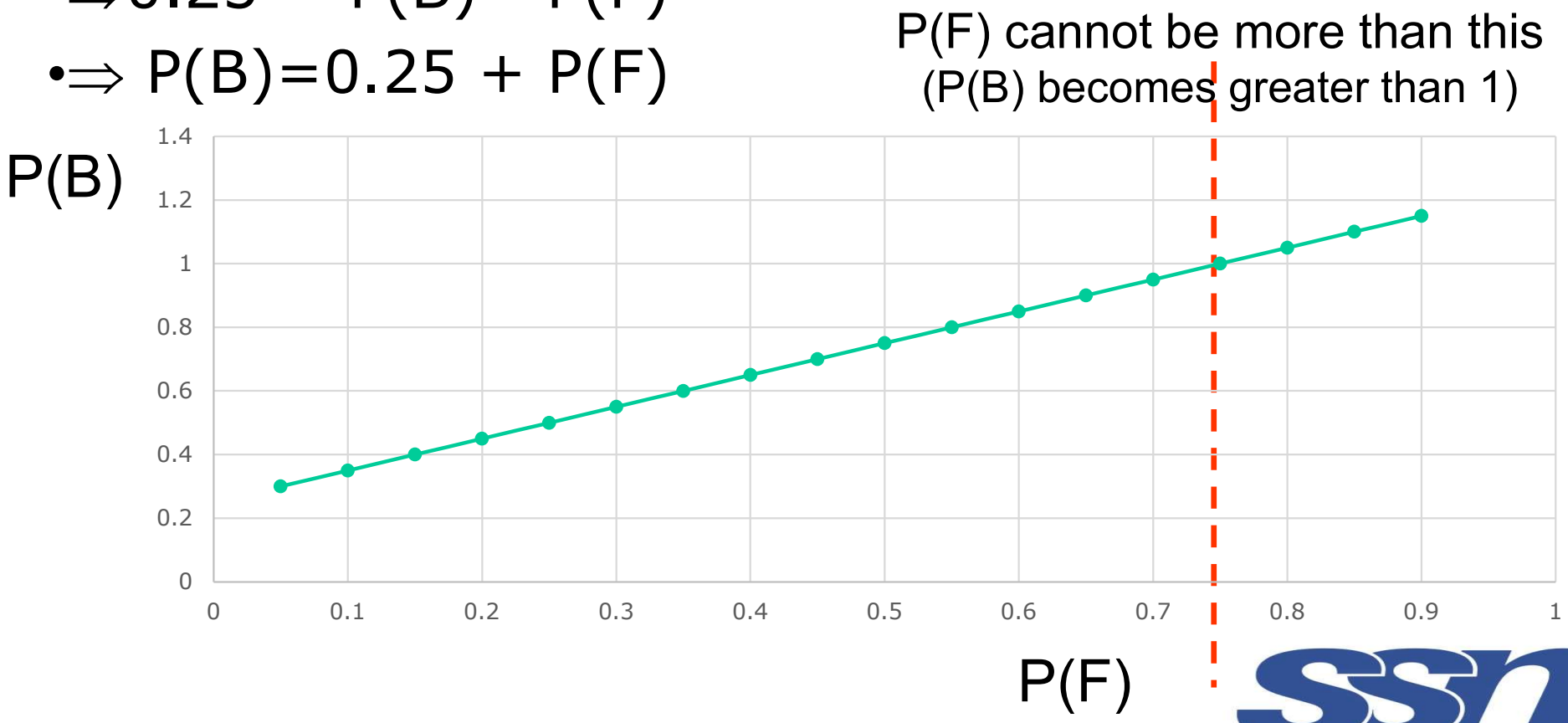
Write P(C) in terms of P(F)

- 1 = P(B) + P(C) + P(F)
- 1.75 = 1*P(B) + 2*p(C) + 3*P(F)
- Subtracting eq. 1 from eq. 2
- $\Rightarrow$0.75 = P(C) + 2*P(F)
- $\Rightarrow$ P(C)=0.75-2.P(F)



Write P(B) in terms of P(F)

- 1 = P(B) + P(C) + P(F)
- 1.75 = 1*P(B) + 2*p(C) + 3*P(F)
- Multiplying 1st eq. by 2 and subtracting 2nd eq. from it
- $\Rightarrow$0.25 = P(B) - P(F)
- $\Rightarrow$ P(B)=0.25 + P(F)

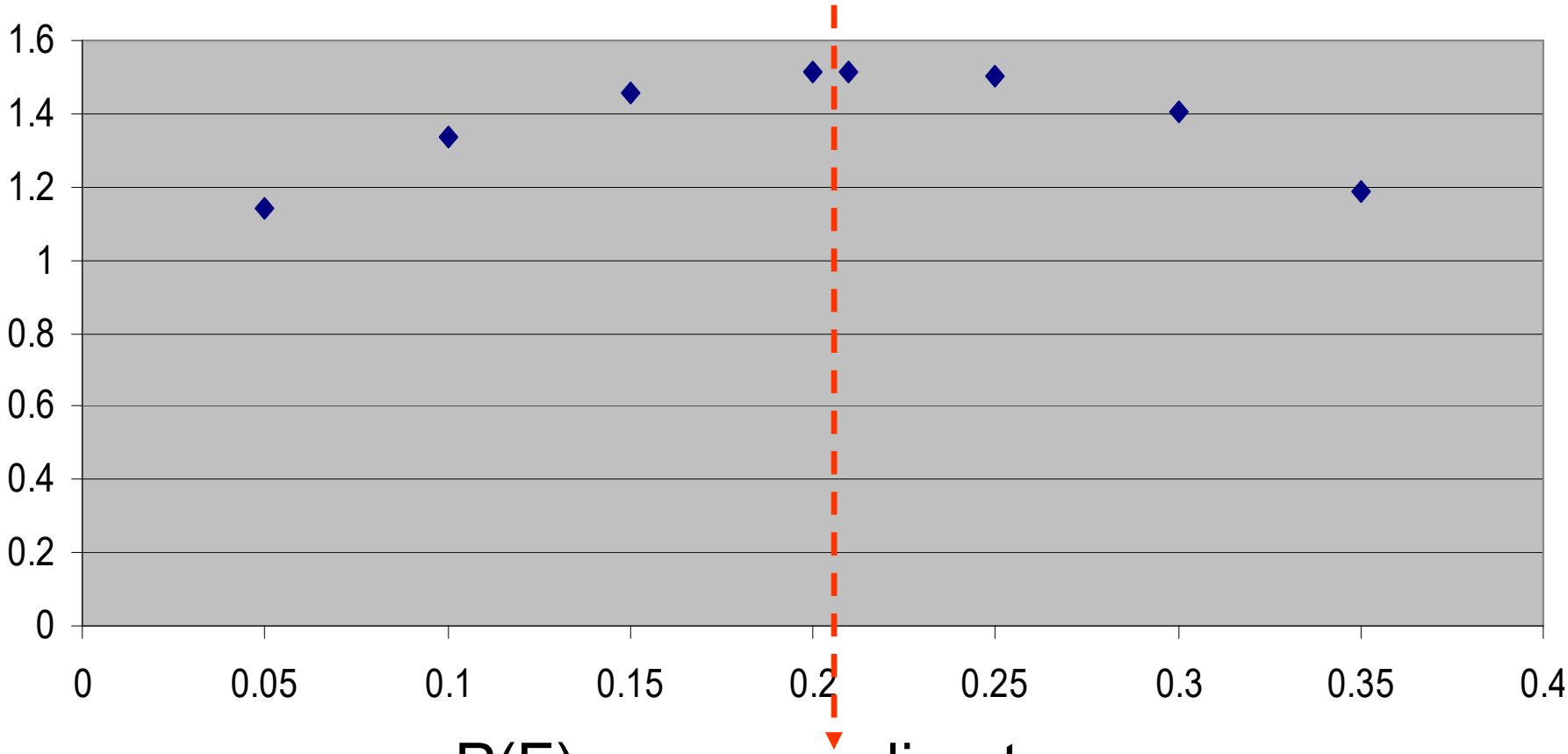


Entropy

$$S = (0.25+p(F))\log_2\left(\frac{1}{(0.25+p(F))}\right)+(0.75-2p(F))\log_2\left(\frac{1}{(0.75-2p(F))}\right)+p(F)\log_2\left(\frac{1}{p(F)}\right)$$

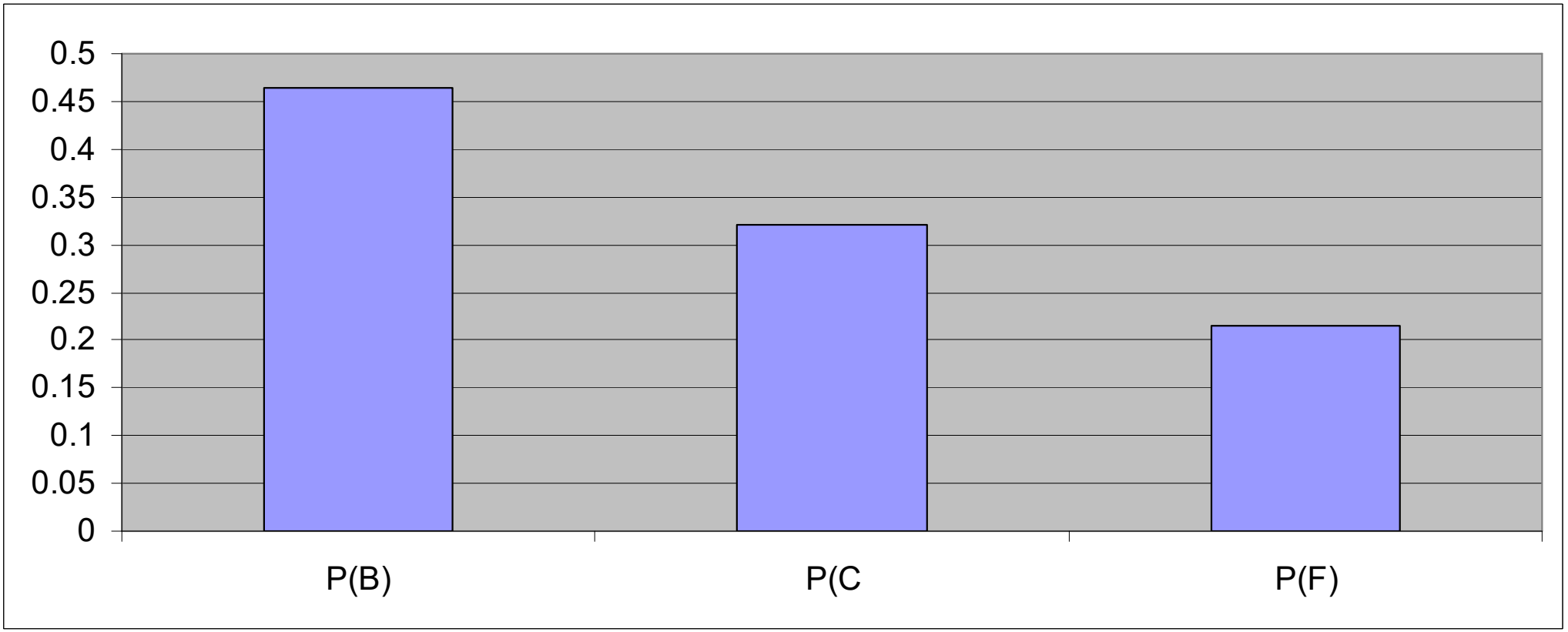
Solve for P(F)

P(F) = 0.215, P(B)=0.25+0.215=0.465, P(C)=0.75-2*0.215=0.320



P(F) corresponding to Maximum entropy point

PDF corresponding to H_{MAX}



Maximum entropy principle

- Find PDF along with the constraints
- PDF should maximize entropy

1st constraint

$$\sum_i p(A_i) = 1$$

2nd constraint

$$\sum_i p(A_i).g(A_i) = K$$

Maximize entropy

$$\sum_i p(A_i).\log\left(\frac{1}{p(A_i)}\right)$$

PME for ill conditioned problems

- Three properties of solution
  - Existence
  - Uniqueness
  - Stability
- A problem for which at least one of the three above requirements is not met is called an *ill-posed (ill-conditioned) problem*
- Caused by incomplete and/or noisy data