Error Detection and Correction

Syndrome Decoding

Decoding involves parity-check information derived from the code's coefficient matrix, P.

Associated with any systematic linear (n,k) block code is a (n-k)-by-n matrix, H called the parity-check matrix.

H is defined as

$$\mathbf{H} = [\mathbf{I}_{n\text{-}k} \,\vdots\, \mathbf{P}^T]$$

Where $\mathbf{P}^T$ is the transpose of the coefficient matrix, P and is an (n-k)-by-k matrix.
$I_{n\text{-}k}$ is the (n-k)-by-(n-k) identity matrix.

For error detection purposes, the parity check matrix, H has the following property

$$c.H^T = (0\ \ 0\ \ \ldots..\ \ 0) \quad \text{(ie Null matrix)}$$

Syndrome Decoding

$$c.H^T = (0 \;\; 0 \;\; \ldots\ldots \;\; 0) \quad \text{(ie Null matrix)}$$

Since c=m.G, therefore

$$m.G.H^T = (0 \;\; 0 \;\; \ldots. \; 0)$$

This property is satisfied only when c is correctly received.
Errors are indicated by the presence of non-zero elements in the matrix.

Let **r** denotes the 1-by-n received vector that results from sending the code vector **c** over a noisy channel.
When there is an error, the decoding operation will give a syndrome vector, s whose elements contain at least 1 non-zero element.

Syndrome Decoding – Example for the (7,4) Hamming Code

A (7,4) Hamming code with the following parameters

n=7; k=4, m=7-4=3

The k-by-(n-k) (4-by-3) coefficient matrix, **P =**

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

The generator matrix, G is, G =

$$G = \left[\begin{array}{ccc:cccc} 1 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Syndrome Decoding –Example for (7,4) Hamming Code

Associated with the (7,4) Hamming Code is a 3-by-7 matrix, H called the parity-check matrix.

H is defined as

$$\mathbf{H} = [\mathbf{I}_{n-k} \vdots \mathbf{P}^T]$$

$$\left[\begin{array}{ccc:cccc} 1 & 0 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right]$$

When a codeword is correctly received, the $c.H^T$ will result in a null matrix, otherwise it will result in a syndrome vector, s.

Syndrome Decoding –Example for (7,4) Hamming Code

Example: The received code vector is [1110010], check whether this is a correct codeword

$$c.H^T = [1110010]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Syndrome Decoding –Example for (7,4) Hamming Code

Example: The received code vector is [1100010], check whether this is a correct codeword

$$c.H^T = [1100010]\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} = [0\ 0\ 1]$$

– this is called the error syndrome

Error pattern

Error pattern is an error vector E whose nonzero element mark the position of the transmission errors in the received codeword

We can work out all syndromes and find the corresponding error patterns and store them in a look up table for decoding purposes

For example the (7,4) Hamming code

Syndrome	*Error Pattern*
0 0 0	0 0 0 0 0 0 0
1 0 0	1 0 0 0 0 0 0
0 1 0	0 1 0 0 0 0 0
0 0 1	0 0 1 0 0 0 0
1 1 0	0 0 0 1 0 0 0
0 1 1	0 0 0 0 1 0 0
1 1 1	0 0 0 0 0 1 0
1 0 1	0 0 0 0 0 0 1

Error detection & correction

The error pattern, E is essentially the modulo-2 sum of the correct code vector and the erroneous received code vector. For example , c = 1110010 and r=1100010 (ie error in the 3rd bit)

$$c + r = E$$

$$1110010 + 1100010 = 0010000$$

This error pattern corresponds to a syndrome vector in the look up table, 001

Recall that the syndrome vector, $s = rH^T$

$$s = (c + E)H^T$$

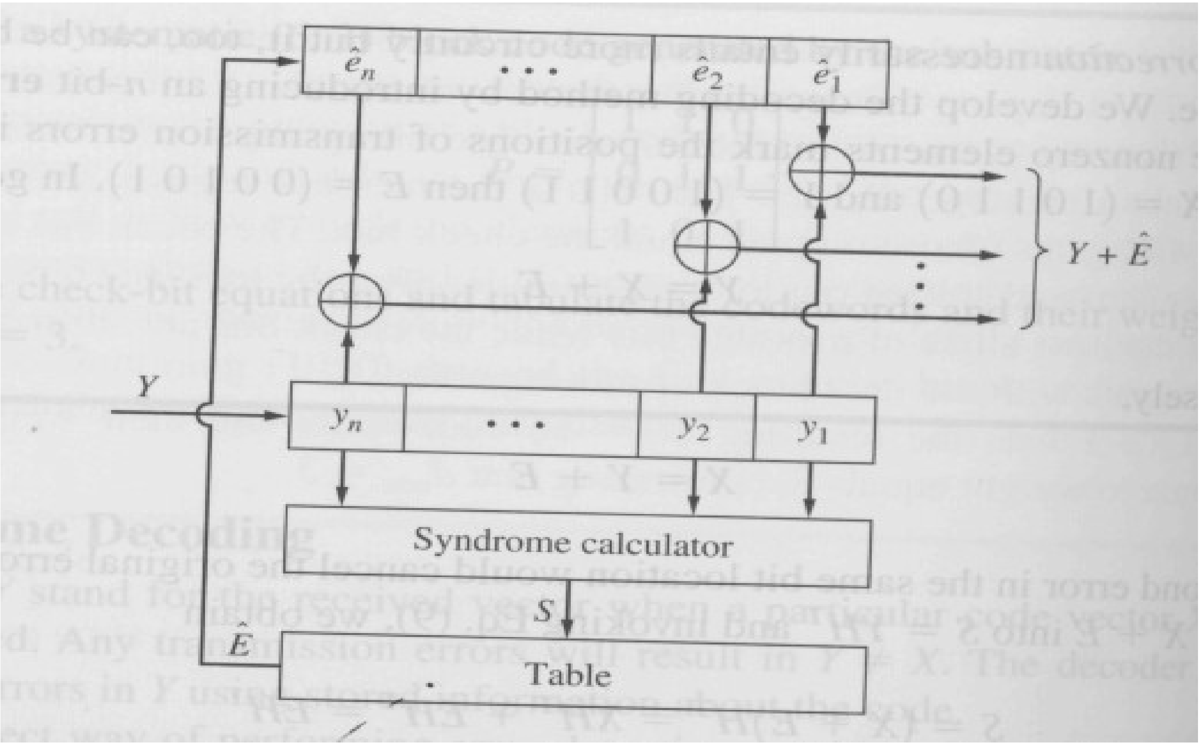
$$= cH^T + EH^T$$

$$= EH^T$$

Error detection and correction

Therefore, the decoding procedure involves working out the syndrome for the received code vector and look up for the corresponding error pattern.

Then, modulo-2 sum the error pattern, E and the received vector, r , so that c = r + E, and the correct codeword can be recovered.



Error detection and correction Example

For message word 0010, the correctly encoded codeword is c = 1110010. Due to channel noise, the received code vector is r = [1100010]. Show how the decoder recover the correct codeword.

1) The decoder uses r and the H^T to find the error syndrome, s

$S = r.H^T \quad = 001$

2) Using the resulting syndrome, refer the look up table for the corresponding assumed error vector, E.

S=001 corresponds to assumed error vector, E = 0010000

3) Then ex-OR E and r to recover the correct codeword

$E+r = 0010000 + 1100010 = 1110010$

Error detection and correction Exercise

i) For message word 0110, the correctly encoded codeword is c = 1000110. Due to channel noise, the received code vector is r = [1100110]. Show how the decoder recover the correct codeword.

ii) For message word 0110, the correctly encoded codeword is c = 1000110. Due to channel noise, the received code vector is r = [1100100]. Show how the decoder performs its decoding operation. What is your observation and explain it.