UIT2504 Artificial Intelligence

Playing Games

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September 04, 2024

Problem Solving

- Problem formulation in state space
- Basic Search Strategies
- Heuristics — best-first search
- A^* Search
- Local Search Strategies — Hill climbing and its variants, simulated annealing, genetic algorithms
- Searching in continuous domains

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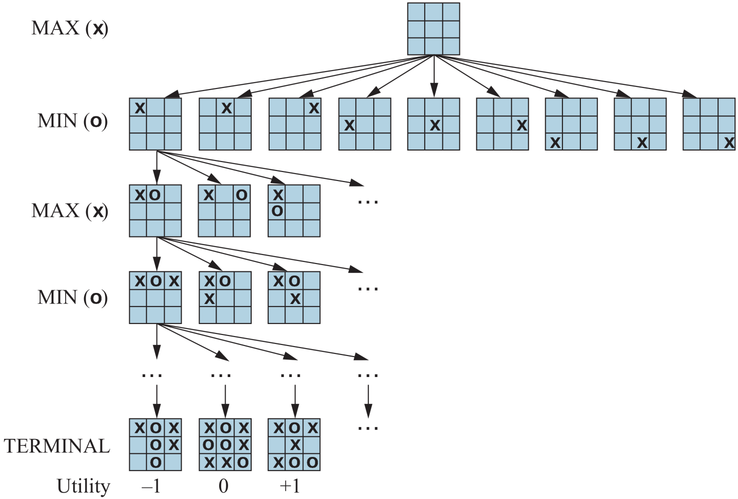
- Will these ideas be useful for taking decisions in multi-player games?

Search Strategies for Playing Games



Game Theory

- Search tree is huge! — in chess, average branching factor is 35 and a game may go to a depth of 100 (50 moves by each player) — 35^{100}

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- We will consider two-player Zero-sum games of perfect information
- We will call our two players *MAX* and *MIN*

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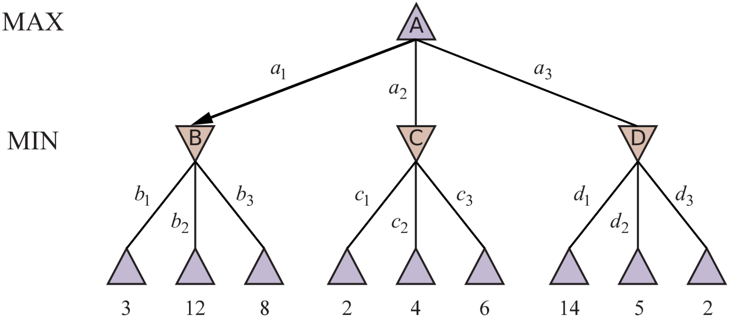
- IS_TERMINAL(s): Terminal test — 'true' when the game is over — terminal states
- UTILITY(s,p): Utility function — an objective function that defines the final numeric value to a player p when the game ends in a terminal state s — in chess, outcome is a win, loss, or draw, with values $+1, 0, \frac{1}{2}$

Optimal Decisions



Minimax Decision

- Optimal strategy for deterministic games
- Utility values percolate up from terminal states
- At MAX level, choose successor with highest utility
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MINIMAX(s) $=$

$$\begin{cases} Utility(s, MAX) & \text{if IS\_TERMINAL}(s) \\ max_{a \in ACTIONS(s)} MINIMAX(RESULT(s, a)) & \text{if TO\_MOVE}(s) = \text{MAX} \\ min_{a \in ACTIONS(s)} MINIMAX(RESULT(s, a)) & \text{if TO\_MOVE}(s) = \text{MIN} \end{cases}$$

Minimax Algorithm

```
function MINIMAX-SEARCH(game, state) returns an action
  player ← game.TO-MOVE(state)
  value, move ← MAX-VALUE(game, state)
  return move

function MAX-VALUE(game, state) returns a (utility, move) pair
  if game.IS-TERMINAL(state) then return game.UTILITY(state, player), null
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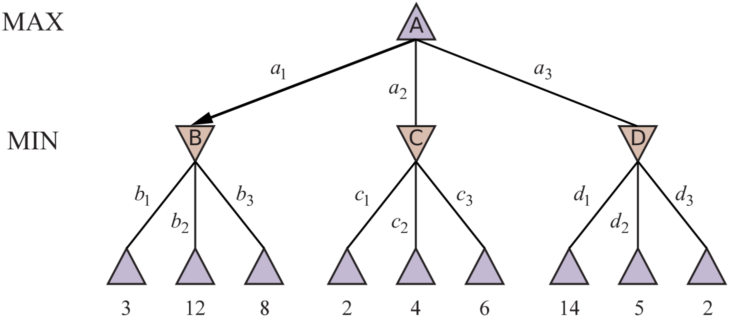

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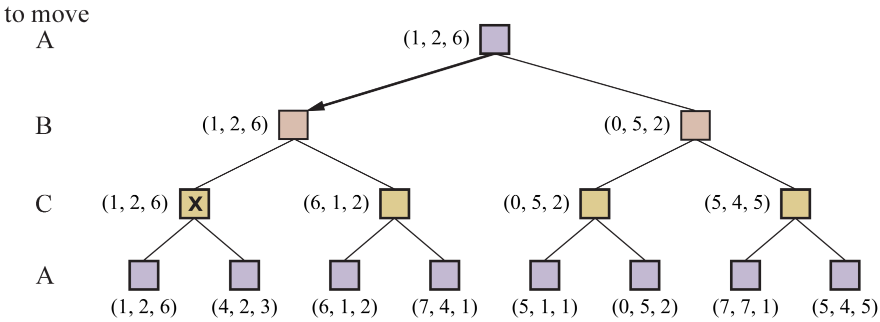
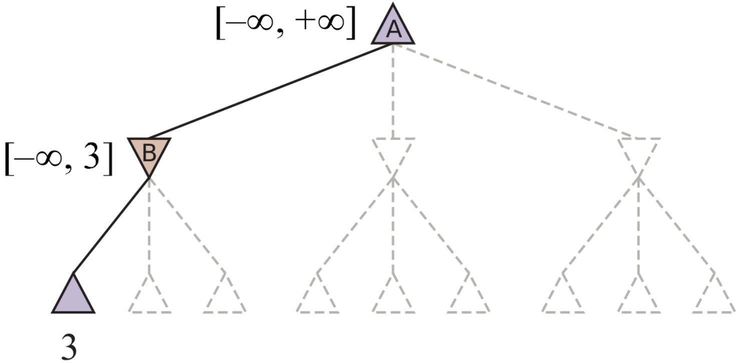
- Optimal? — Yes, if the opponent is optimal
- Space Complexity? — $O(bm)$ — may be reduced to $O(m)$ if successors are generated one at a time
- Time Complexity? — $O(b^m)$
- Impractical for non-trivial games such as chess — 35^{100}

Multiplayer Games



Questions?

Pruning



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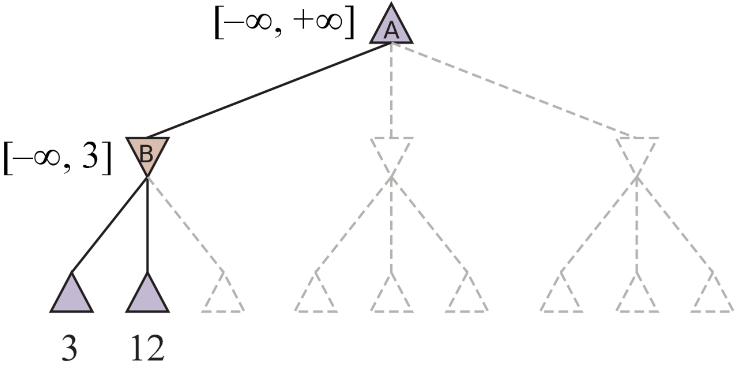

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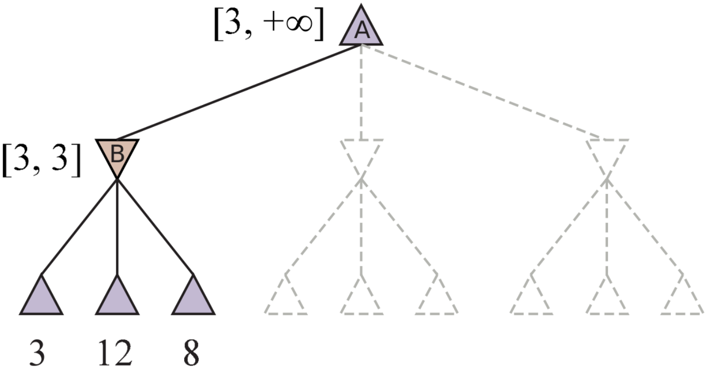

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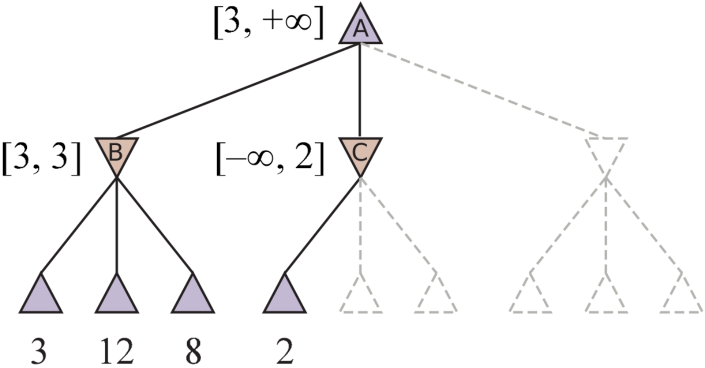

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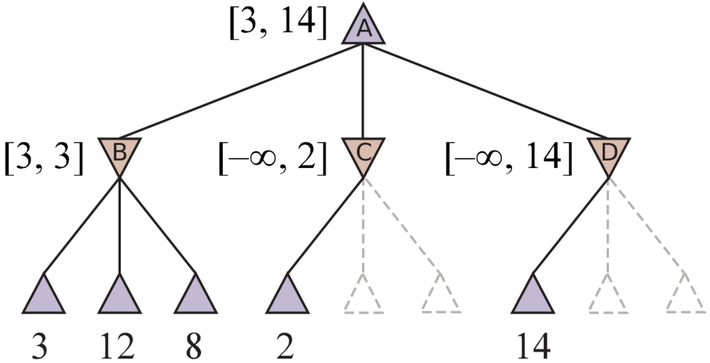

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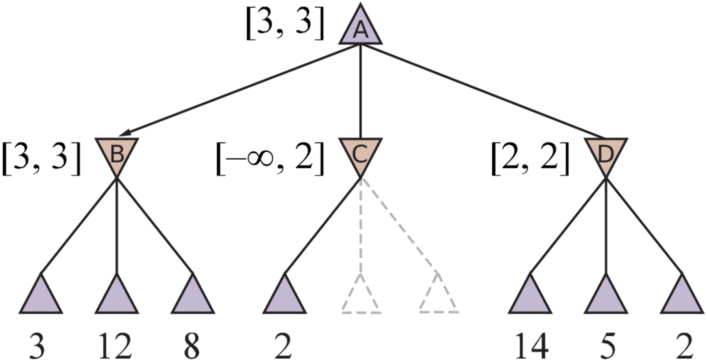

Alpha-Beta Pruning

- α : Value of the best choice we have found so far along a path for MAX — α = "at least" — lower bound for MAX
- β : Value of the best choice we have found so far along a path for MIN — β = "at most" — upper bound for MAX

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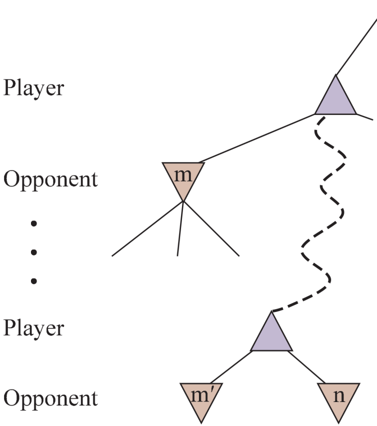

Alpha-Beta Search Algorithm

```
function ALPHA-BETA-SEARCH(game, state) returns an action
  player ← game.TO-MOVE(state)
  value, move ← MAX-VALUE(game, state, −∞, +∞)
  return move

function MAX-VALUE(game, state, α, β) returns a (utility, move) pair
  if game.IS-TERMINAL(state) then return game.UTILITY(state, player), null
  v ← −∞
  for each a in game.ACTIONS(state) do
    v2, a2 ← MIN-VALUE(game, game.RESULT(state, a), α, β)
    if v2 > v then
      v, move ← v2, a
      α ← MAX(α, v)
    if v ≥ β then return v, move
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    if v2 < v then
      v, move ← v2, a
      β ← MIN(β, v)
    if v ≤ α then return v, move
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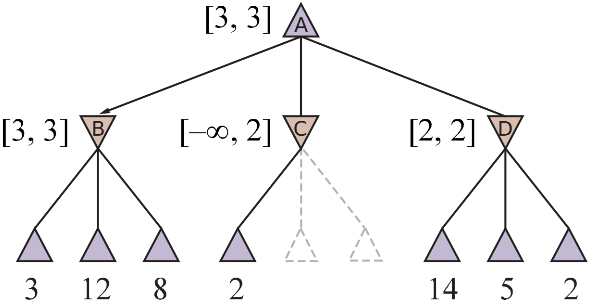

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- Transposition table — hash the α–β values for future use

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- Type A strategy: consider all possible moves to certain depth and then use a heuristic evaluation function to estimate the utilities of the states at that depth (wide but shallow strategy)
- Type B Strategy: ignore moves that look bad, and follow promising paths as far as possible (deep but narrow strategy)

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