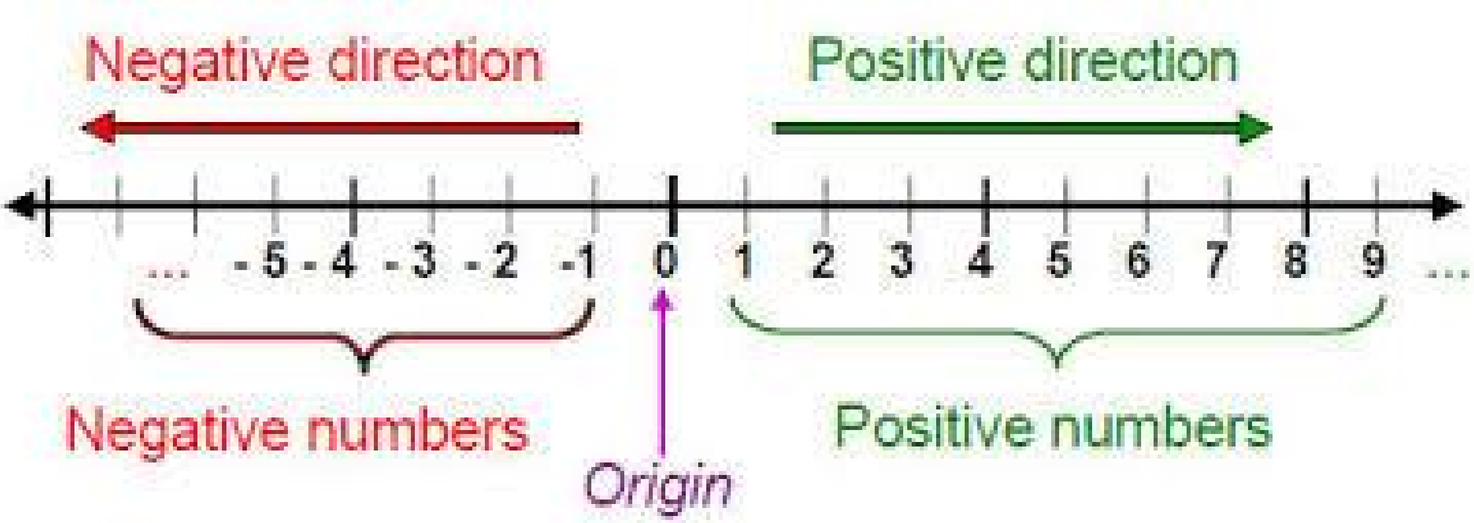
Image comparison based on pixels

Distance in 1 D

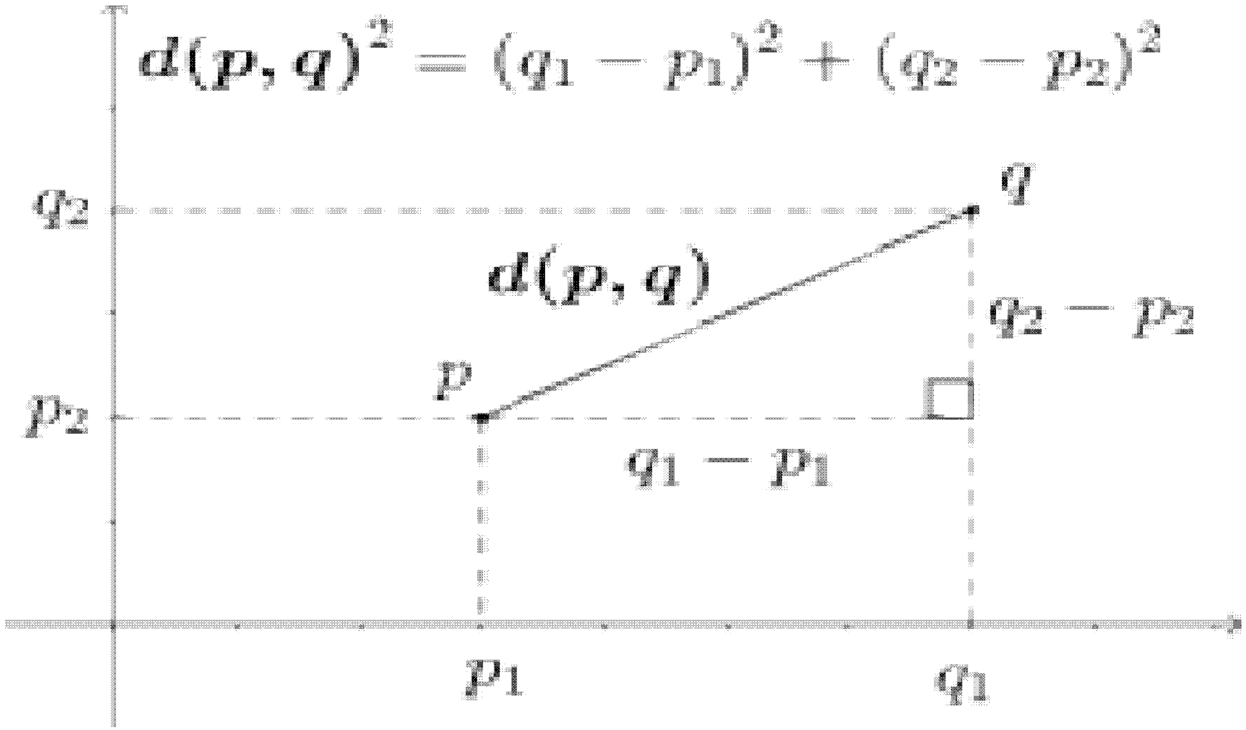


$$p = (p_1), \ \& \ q = (q_1)$$

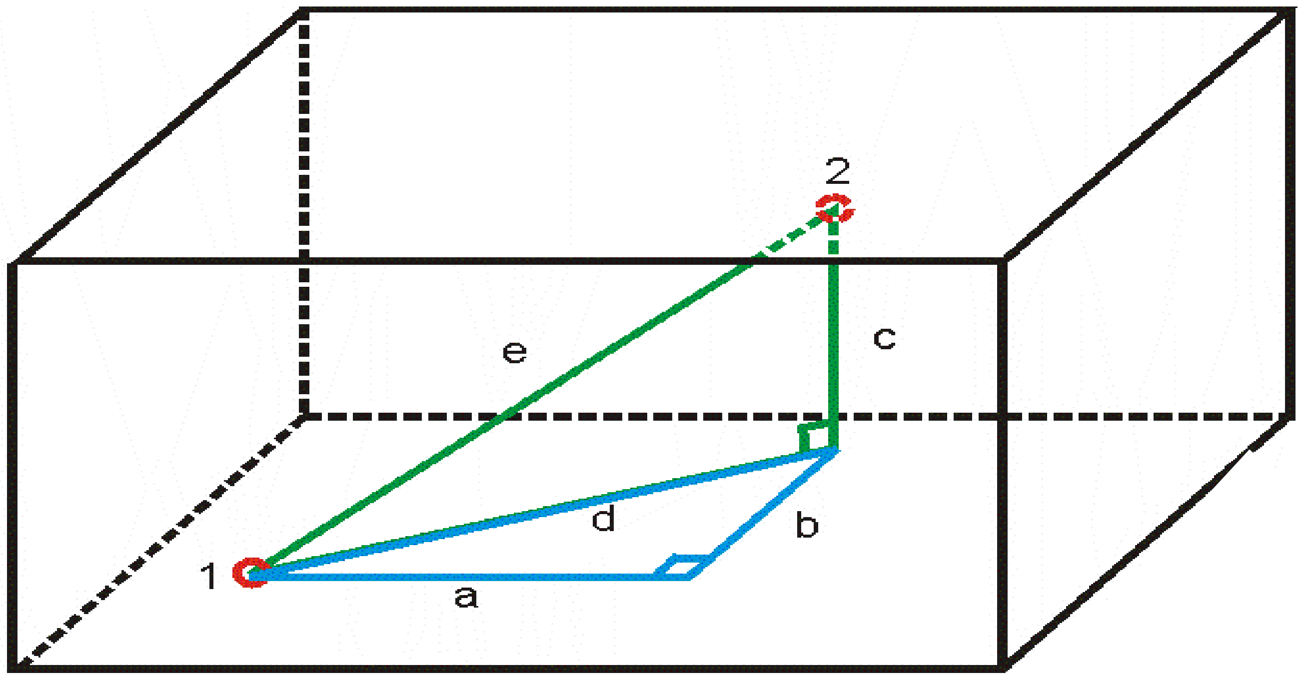
$$d = \sqrt{(q_1 - p_1)^2}$$

Euclidean distance in 2 D

$p=(p_1,p_2)$ & $q=(q_1,q_2)$



Euclidean distance in 3 D



$$p = (p_1, p_2, p_3), \;\&\; q = (q_1, q_2, q_3)$$

$$d = \sqrt{(q_1 - p_1)^2 + (q_2 - p_2)^2 + (q_3 - p_3)^2}$$

Distance in n D

$$d(p,q) = d(q,p)$$

$$= \sqrt{(q_{1-}p_1)^2 + (q_3 - p_3)^2 + \cdots + (q_n - p_n)^2}$$

$$= \sqrt{\sum_{i=1}^{n}(q_i - p_i\ )^2}$$

Digital Image

- A digital image ≡ pixels' matrix
- An area divided using grid
- Pixel – the smallest area

Distance between matrices

- Extend the idea of nD further

$$d(\mathbf{A},\mathbf{B}) = \sqrt{\sum_{i=1}^{n}\sum_{j=1}^{n}(a_{ij} - b_{ij})^2}$$

Example application

- Digital image – Matrix
- Two images are given ≡ Two matrices are given
- If it is the same image distance between them is zero

e.g. – Distance between Test image & training image

test image

56	32	10	18
90	23	128	133
24	26	178	200
2	0	255	220

-

training image

10	20	24	17
8	10	89	100
12	16	178	170
4	32	233	112

=

pixel-wise absolute value differences

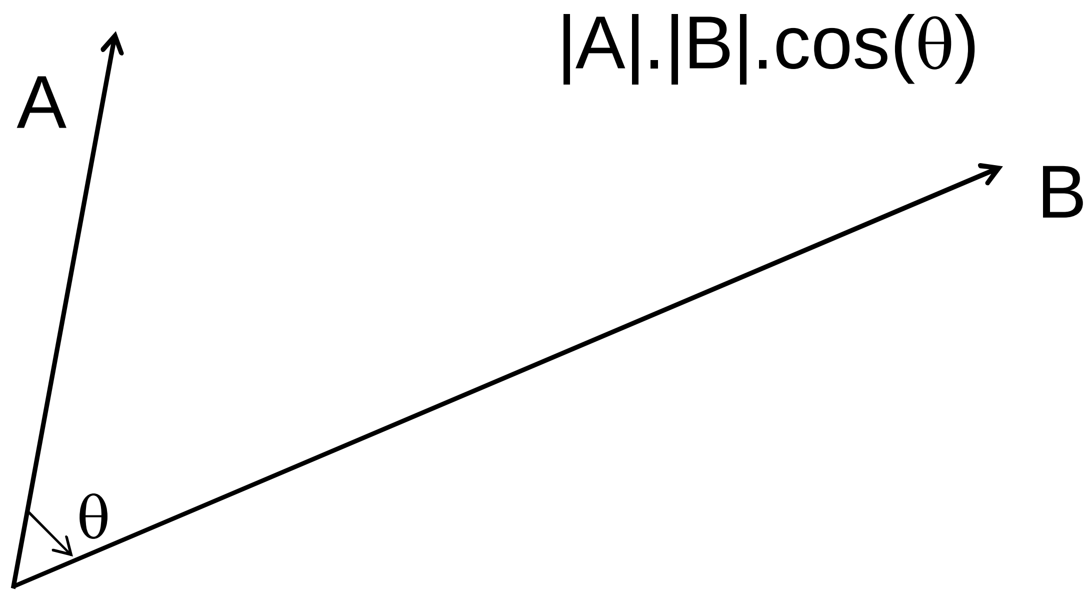
46	12	14	1
82	13	39	33
12	10	0	30
2	32	22	108

2116	144	196	1
6724	169	1521	1089
144	100	0	900
4	1024	484	11664

Square differences,
Sum up, & Take root
$\sqrt{26280} = 162.1$

Cosine ~~distance~~ similarity

- Dot product
- Inner product



Cosine similarity - two points (vectors)

Derived from cosine similarity

Cosine similarity = Inner product = dot product (for normalized vectors)

$$p = (p_1, p_2, p_3)\ \&\ q = (q_1, q_2, q_3)$$

$$similarity = (p_1.q_1) + (p_2.q_2) + (p_3.q_3)$$

$$similarity = p.q^T = \begin{bmatrix} p_1 & p_2 & p_3 \end{bmatrix} . \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

$$dis\tan ce = 1 - similarity$$

Vector to Matrix (inner product)

$$\begin{bmatrix} a_1 & a_2 \\ a_3 & a_4 \end{bmatrix} \cdot \begin{bmatrix} b_1 & b_2 \\ b_3 & b_4 \end{bmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3 + a_4 b_4$$

$$\mathbf{A} = \begin{pmatrix} 2 & 0 & 6 \\ 1 & -1 & 2 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 8 & -3 & 2 \\ 4 & 1 & -5 \end{pmatrix}$$

$$\begin{aligned} \langle \mathbf{A}, \mathbf{B} \rangle_{\mathrm{F}} &= 2 \cdot 8 + 0 \cdot (-3) + 6 \cdot 2 + 1 \cdot 4 + (-1) \cdot 1 + 2 \cdot (-5) \\ &= 16 + 12 + 4 - 1 - 10 \\ &= 21 \end{aligned}$$

Frobenius inner product

- Takes two matrices and returns a number
- Consider two matrices, A and B of size (mxn)
- **Extend the concept of dot product of vectors**

$$
\begin{aligned}
\langle \mathbf{A}, \mathbf{B} \rangle_{\mathrm{F}} = & \overline{A}_{11} B_{11} + \overline{A}_{12} B_{12} + \cdots + \overline{A}_{1m} B_{1m} \\
& + \overline{A}_{21} B_{21} + \overline{A}_{22} B_{22} + \cdots + \overline{A}_{2m} B_{2m} \\
& \vdots \\
& + \overline{A}_{n1} B_{n1} + \overline{A}_{n2} B_{n2} + \cdots + \overline{A}_{nm} B_{nm}
\end{aligned}
$$

Distance = 1 - similarity

Example application

- Two images are given ≡ Two matrices are given
- Cosine distance measures the distance between two images – works better than Euclidean

e.g.

test image

56	32	10	18
90	23	128	133
24	26	178	200
2	0	255	220

training image

10	20	24	17
8	10	89	100
12	16	178	170
4	32	233	112

560	640	240	306
720	230	11392	13300
288	416	31684	34000
8	0	59415	24640

Sum up: 177839