Probability Measure

Uncertainity

It is not certain that everything is uncertain

- B Pascal

- Uncertainty is a fundamental—and unavoidable—feature of daily life
- In order to deal with uncertainty intelligently, we need to be able to represent it and reason about it

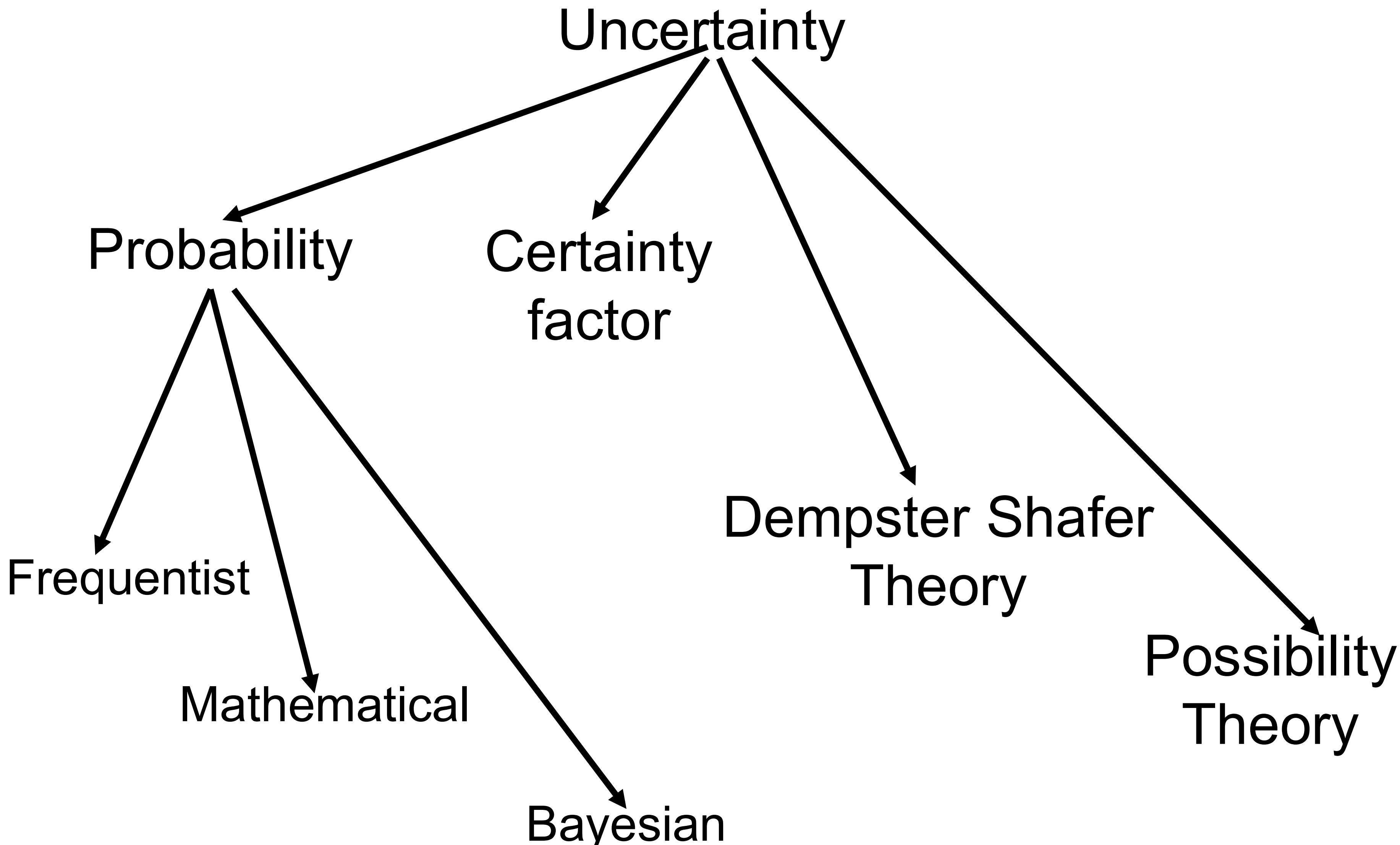
Expressing uncertainty in language

- Always
- Very often
- Usually
- Often
- Generally
- Frequently
- Rather often
- About as often as not
- Now and then

- Sometimes
- Occasionally
- Once in a while
- Not often
- Usually not
- Seldom
- Hardly ever
- Very seldom
- Rarely
- Almost never
- Never

Uncertain

- Going to Tambaram Railway station to catch train - about 30 KM away
- Starting 90 minutes before
- Shall we be in (on) time?
- Plan will get us to the airport in time, as long as
  - My car doesn't break down
  - Run out of fuel
  - I don't get into an accident
  - There are no accidents on the bridge
  - There's no earthquake, ...


Uncertainty
Probability
Certainty factor
Dempster Shafer Theory
Possibility Theory
Frequentist
Mathematical
Bayesian

Capturing uncertainity

- Probability
  - Dealing with chance
  - E.g. We know possible outcomes but which one is going to pop up each time

- Fuzzy
  - Dealing with truth
  - Truth is no more **Yes** or **No**
  - Full truth - Half truth – quarter truth

Look @ these statements

- I **support** the government on invalidating Rs. 1000/- and Rs. 500/-
- I **don't support** the government on invalidating Rs. 1000/- and Rs. 500/-
- I **am unaware** of government invalidating Rs. 1000/- and Rs. 500/-
- I **don't know** whether to support or oppose government on invalidating Rs. 1000/- and Rs. 500/-

Main limitation of Probability

- P = 1 (event happening is almost sure)
- P=0 (event NOT happening is almost sure)

- What about our ignorance about something?
- Impossible to represent ignorance in conventional probability theory

E.g. Probability

- A fair dice is thrown
- Possible events = 6
- P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = 1/6

- A fair dice is thrown 4 times
- Calculate P(atleast one six)

Solution

- 1 throw

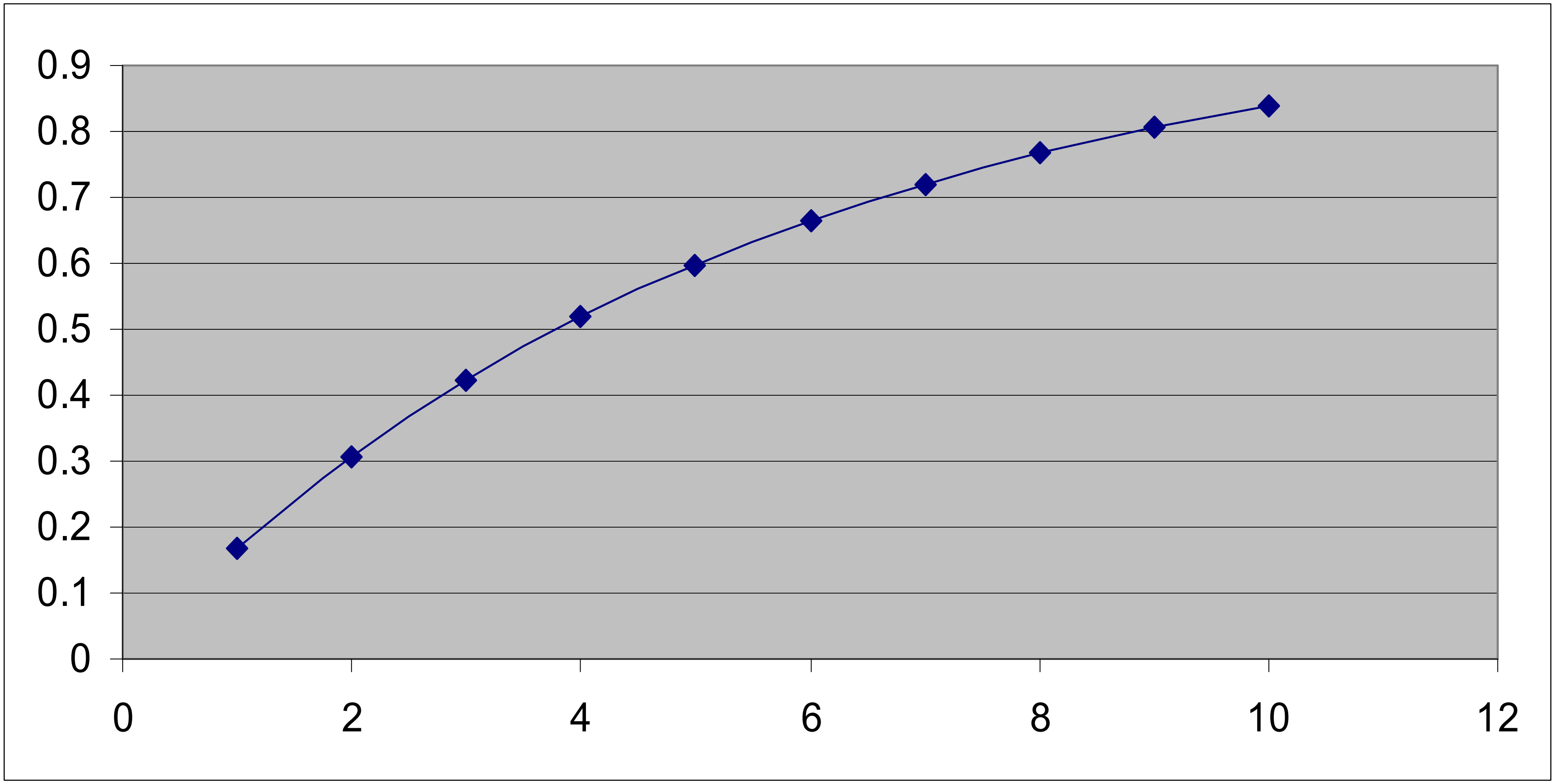
P(No Six) = 5/6

- 4 throws

P(No Six)=(5/6)* (5/6)* (5/6)* 5/6)=0.4823

P(atleast one six) = 1 – 0.4823 = 0.5177

Single dice throwing for n times and getting at least one six



Probability of getting 6 increases with no. of throws

E.g. Probability

- Two fair dice are thrown
- Calculate P (atleast double six)
- Possible events = 36
- Event set = {11, 12, 13, 14, 15, 16, 21, 22, 23, …,66}

Two fair dice thrown 24 times

- Calculate the P of atleast one double 6
- Single throw

P (No double 6) = 35/36

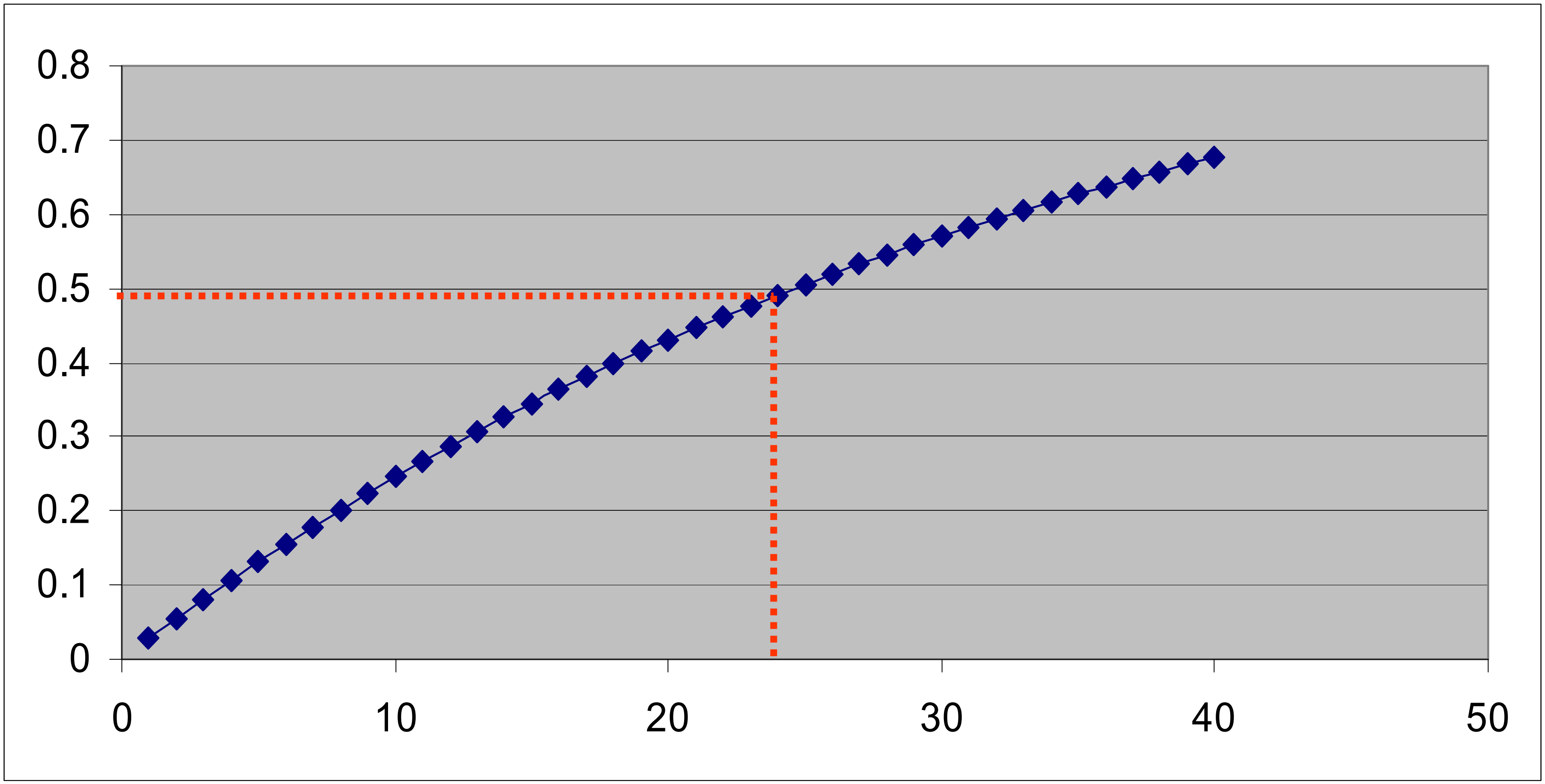
- 24 throws

P (No double 6) = $(35/36)^{24}$

P (atleast one double 6) = $1- (35/36)^{24}$

$= 0.4914$

Double dice thrown n times and getting atleast one double 6



Two fair dice thrown n times

- Calculate the P (both showing same numbers at least once) = 0.5
- Single throw

P (No same number) = 30/36

- 2 throws

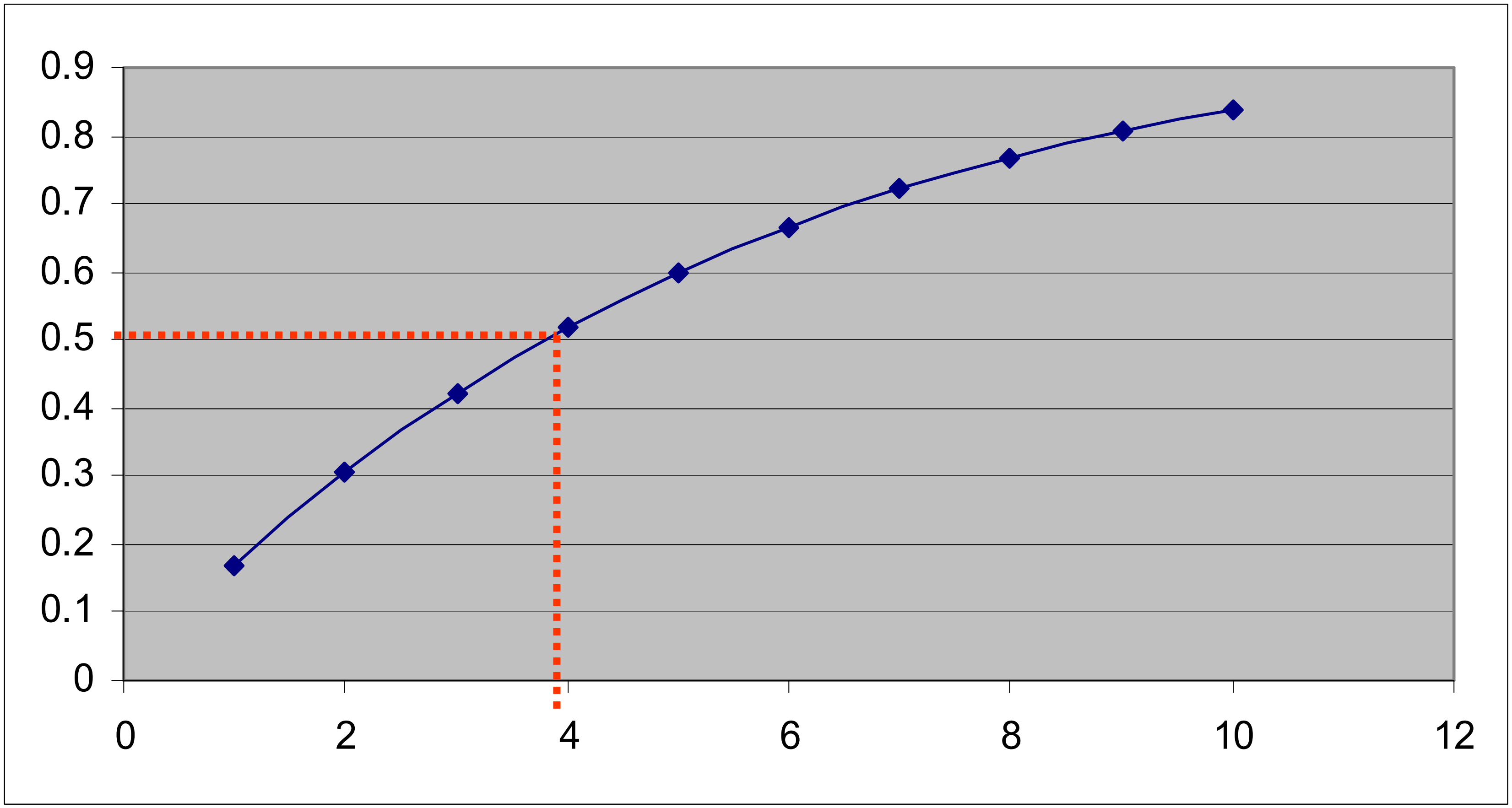
P (No same number) = $(30/36)^2$

- n throws

P (No same number) = $(30/36)^n$

P (atleast one same number in n throws) = $1- (30/36)^n$

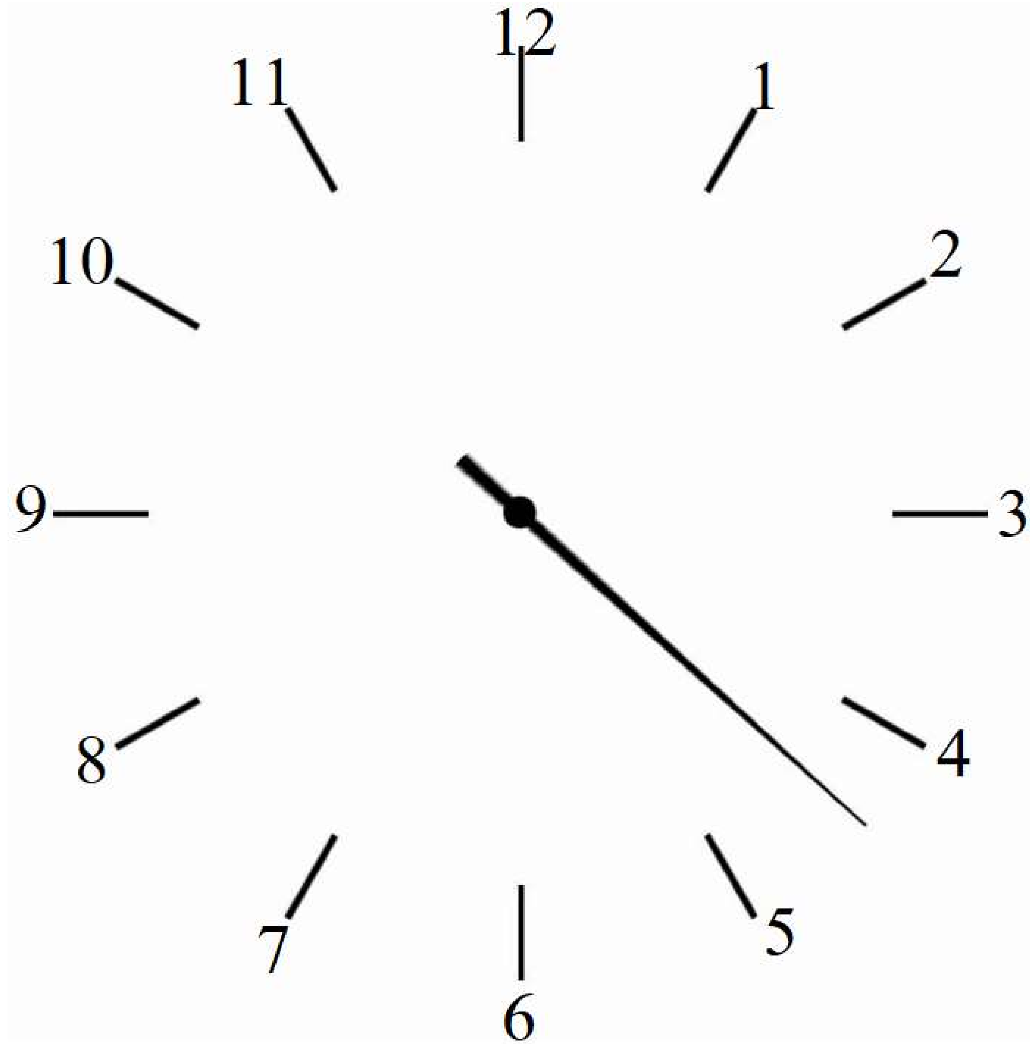
P=0.5 occurs after 4 throws



When you cannot count…

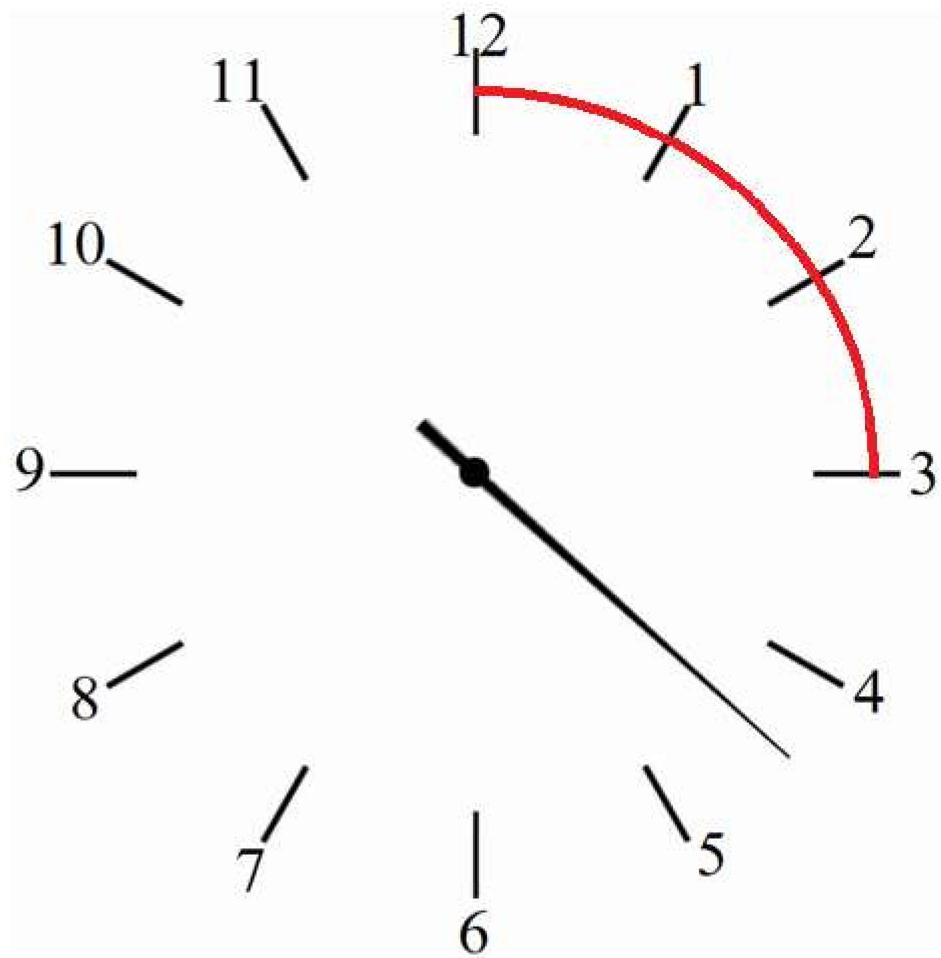
Randomly look @ the clock

- Find the probability of finding the pointer between 12 and 3



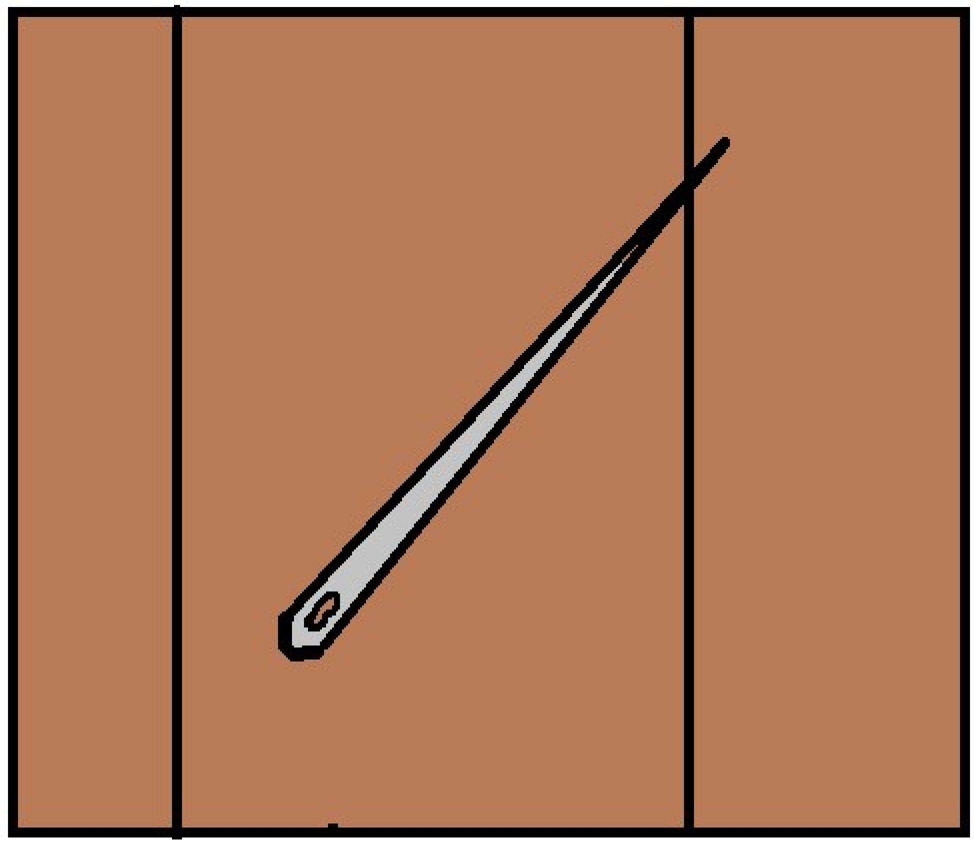
Randomly look @ the clock

- There are infinite positions between 12 and 3
- R – radius of the clock
- Total length = $2\pi R$
- Favorable length = $0.25*(2\pi R)$
- $P(12\text{-}3) = \dfrac{0.25*(2\pi R)}{2\pi R} = 0.25$

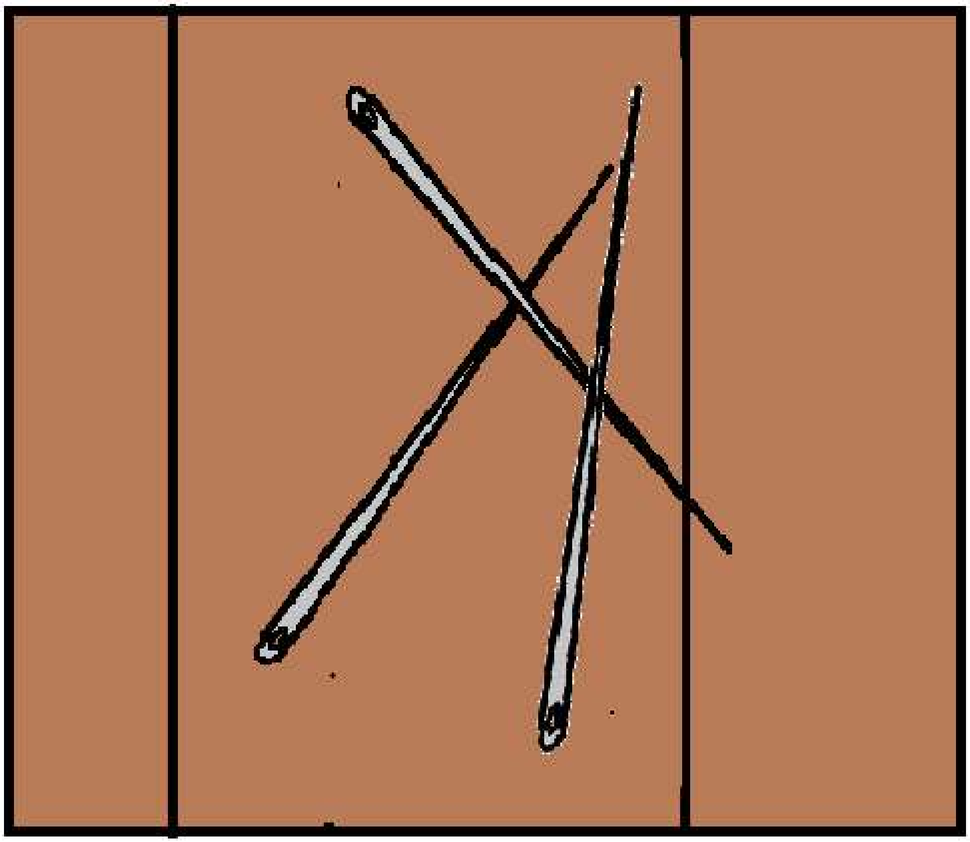


One more e.g.

- A needle drops randomly on a floor
- The floor has regular stripes
- What's the probability that the needle hits the stripe?



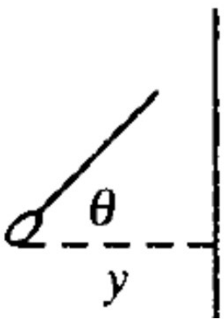
Understand the problem

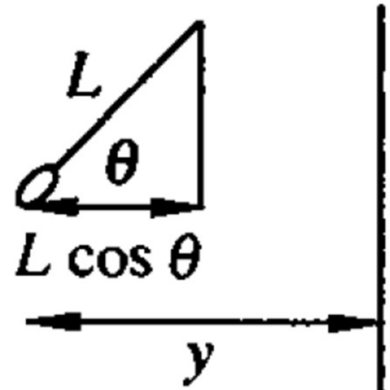


- Length of the needle
- Angle

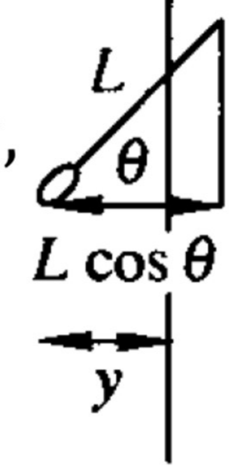
Deciding parameters

- Length of the needle (L)
- Angle (θ)

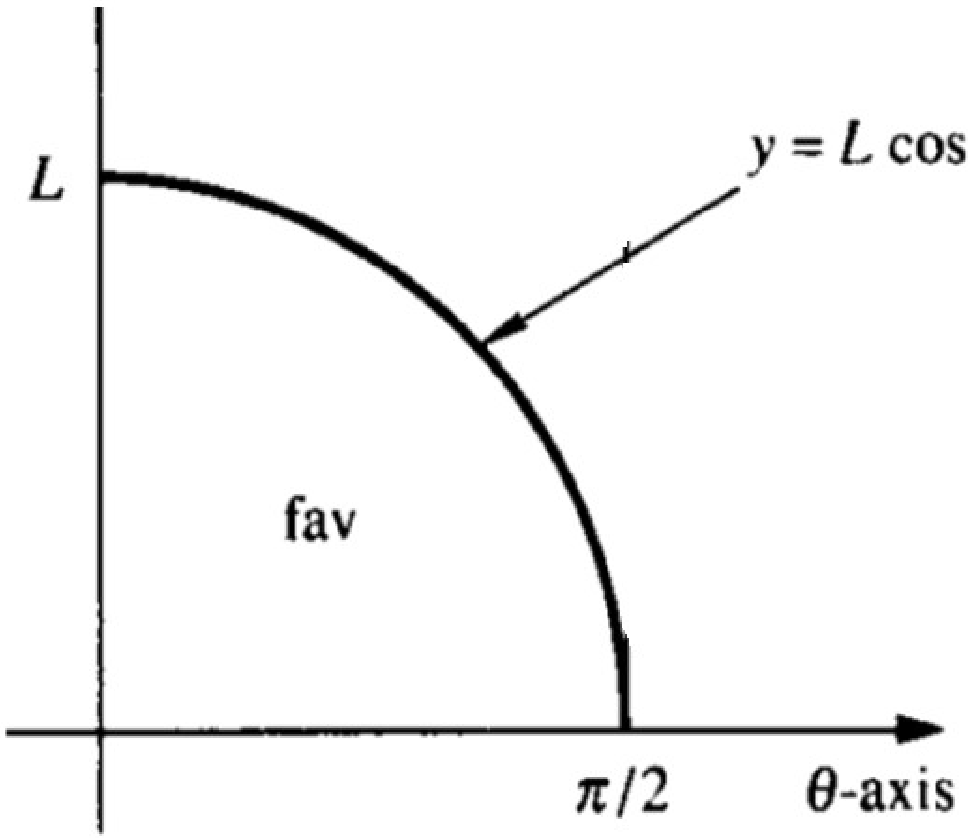




Needle hits the crack,
$$if \quad L.\cos(\theta) \geq y$$



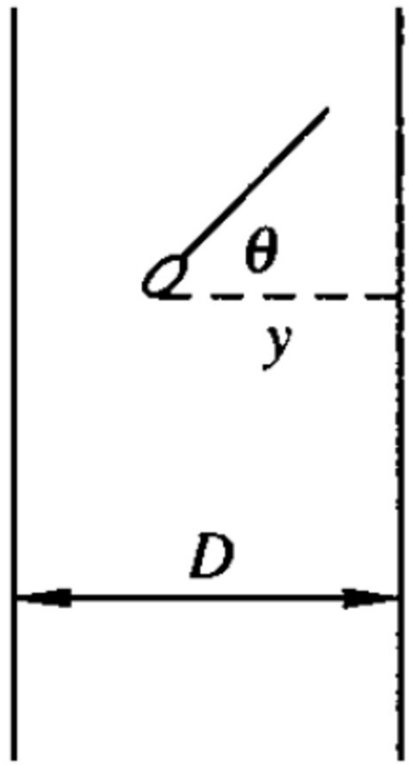
How to get the probability?

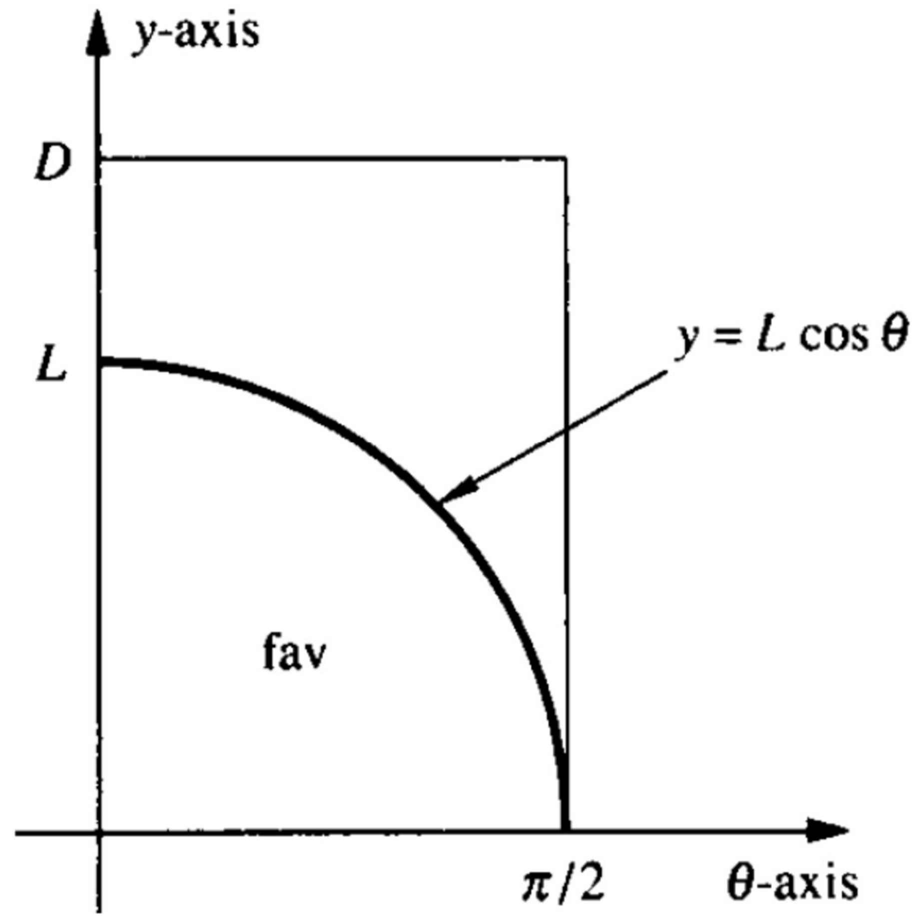


- We can measure L

Favorable area:

- θ varies from 0 to $\frac{\pi}{2}$
- L.cos(θ) varies from L to 0





- When θ changes, L.cos θ changes
- When θ changes, D remains constant
- Depict in graph

$$P(\text{needle hits a crack}) = \frac{\text{favorable area}}{\text{total area}}$$

$$= \left| \frac{\int_0^{\pi/2} L \cos\theta \, d\theta}{(\pi/2)D} \right|$$

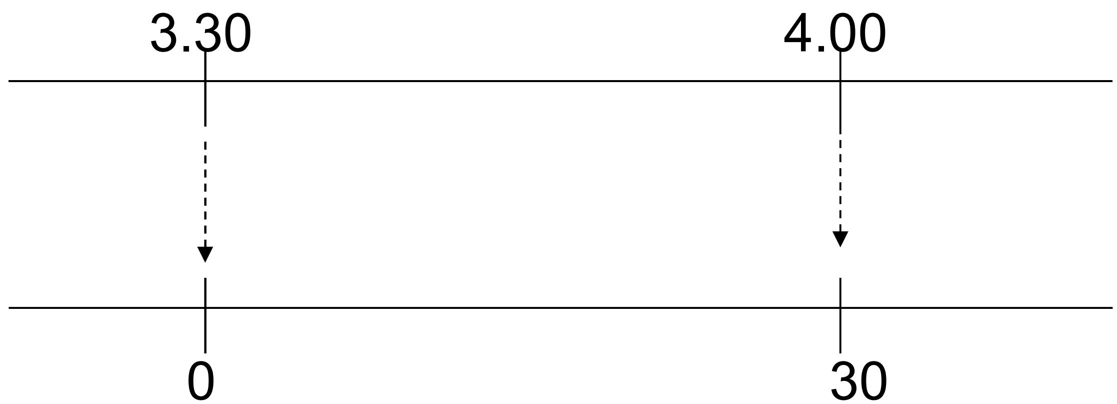
$$= \frac{L}{(\pi/2)D} = \frac{2L}{\pi D}$$

Do the lovers meet?

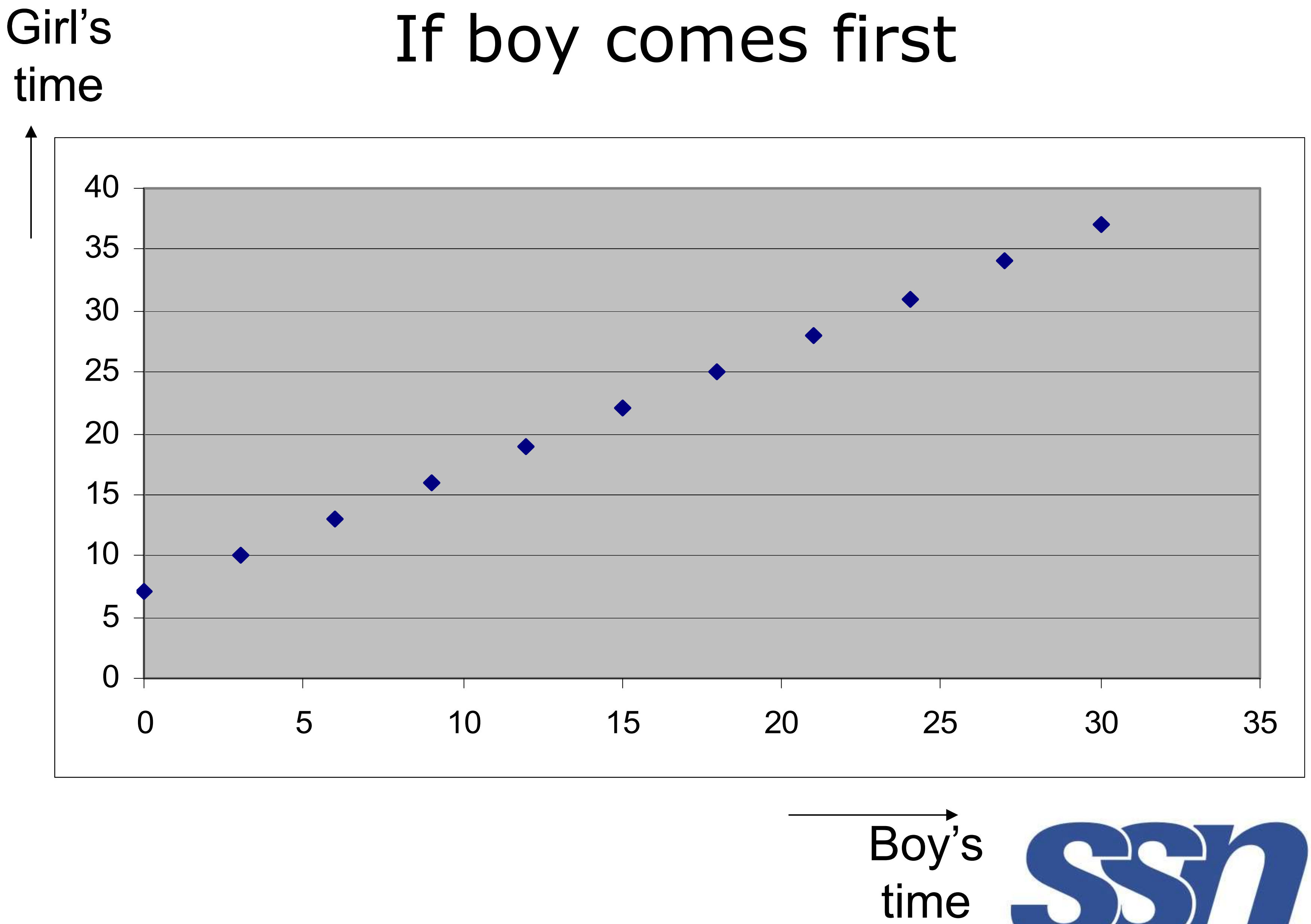
- A girl and boy to meet in coffee day between 3:30-and 4
- If girl arrives first, she'll wait five minutes for Boy, and then leave if he hasn't appeared by then
- If boy arrives first, however, he'll wait seven minutes for girl before leaving if she hasn't appeared by then
- Neither will wait past 4 o'clock
- What's the probability that girl and boy meet?

How to solve?

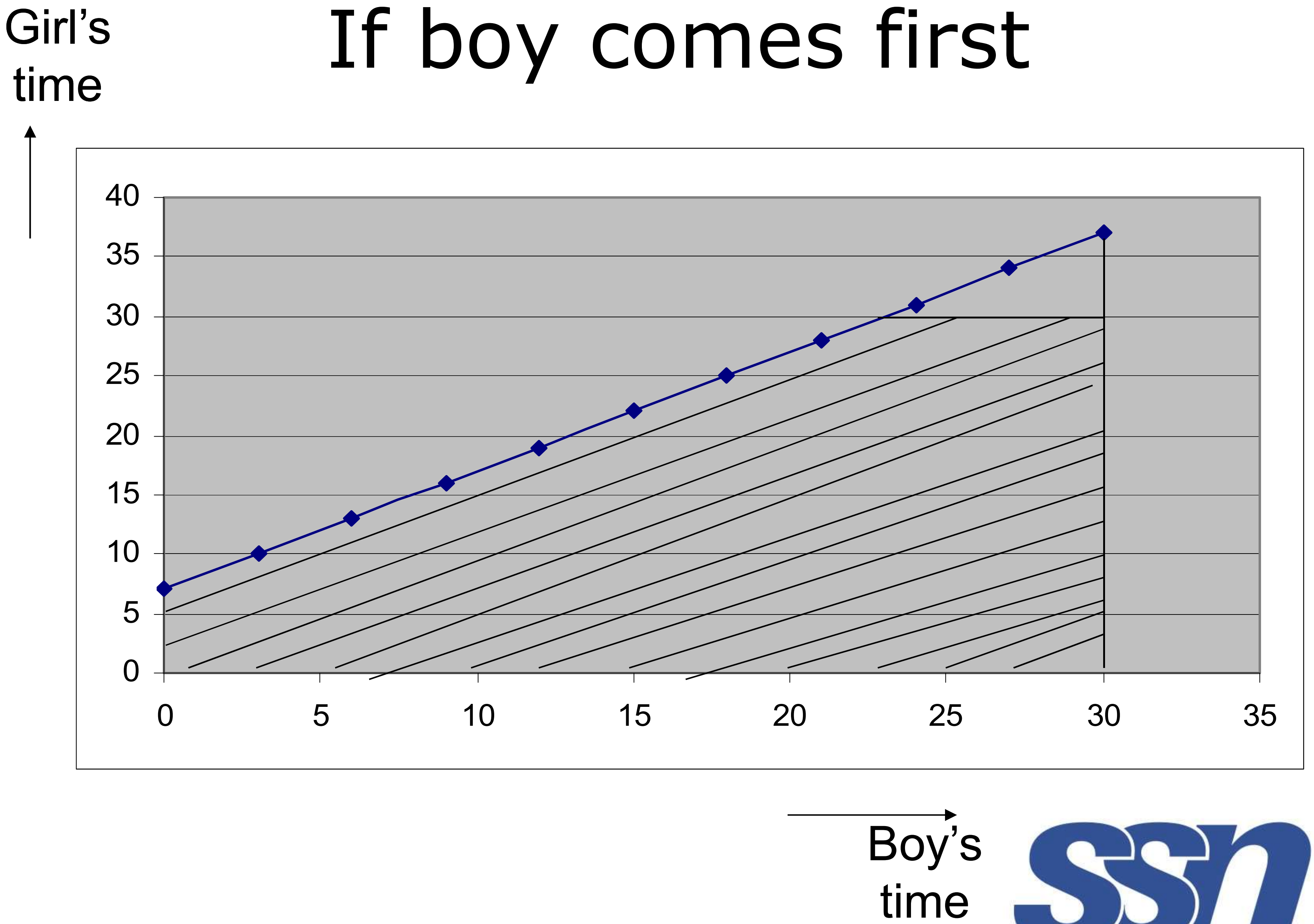
- Let's denote the arrival times of boy and girl by B and G
- They are random
- B and G are random variables
  - Range of B = Range of G = 0 to 30



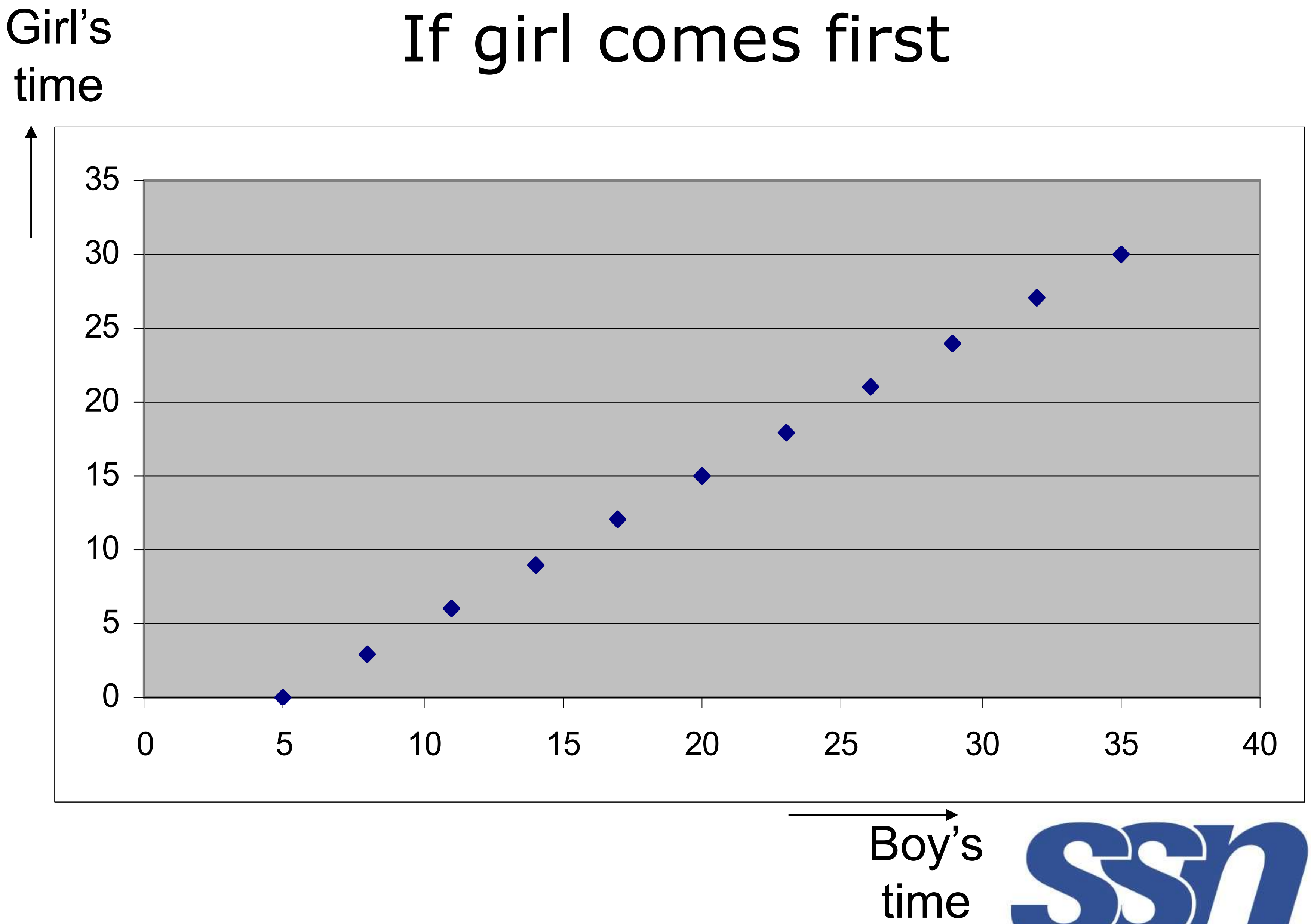
If boy comes first



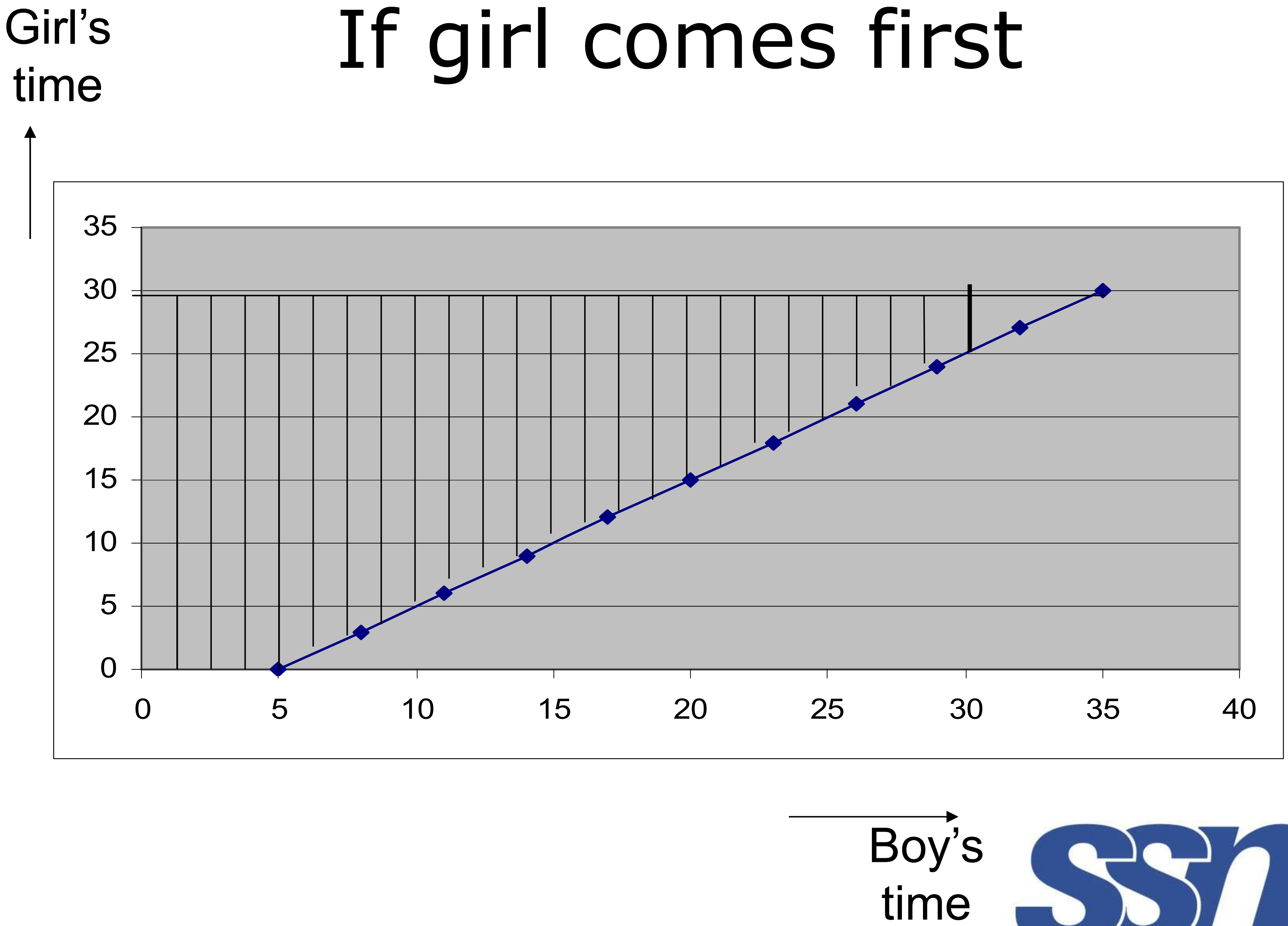
If boy comes first



If girl comes first



If girl comes first



if $G < B$,

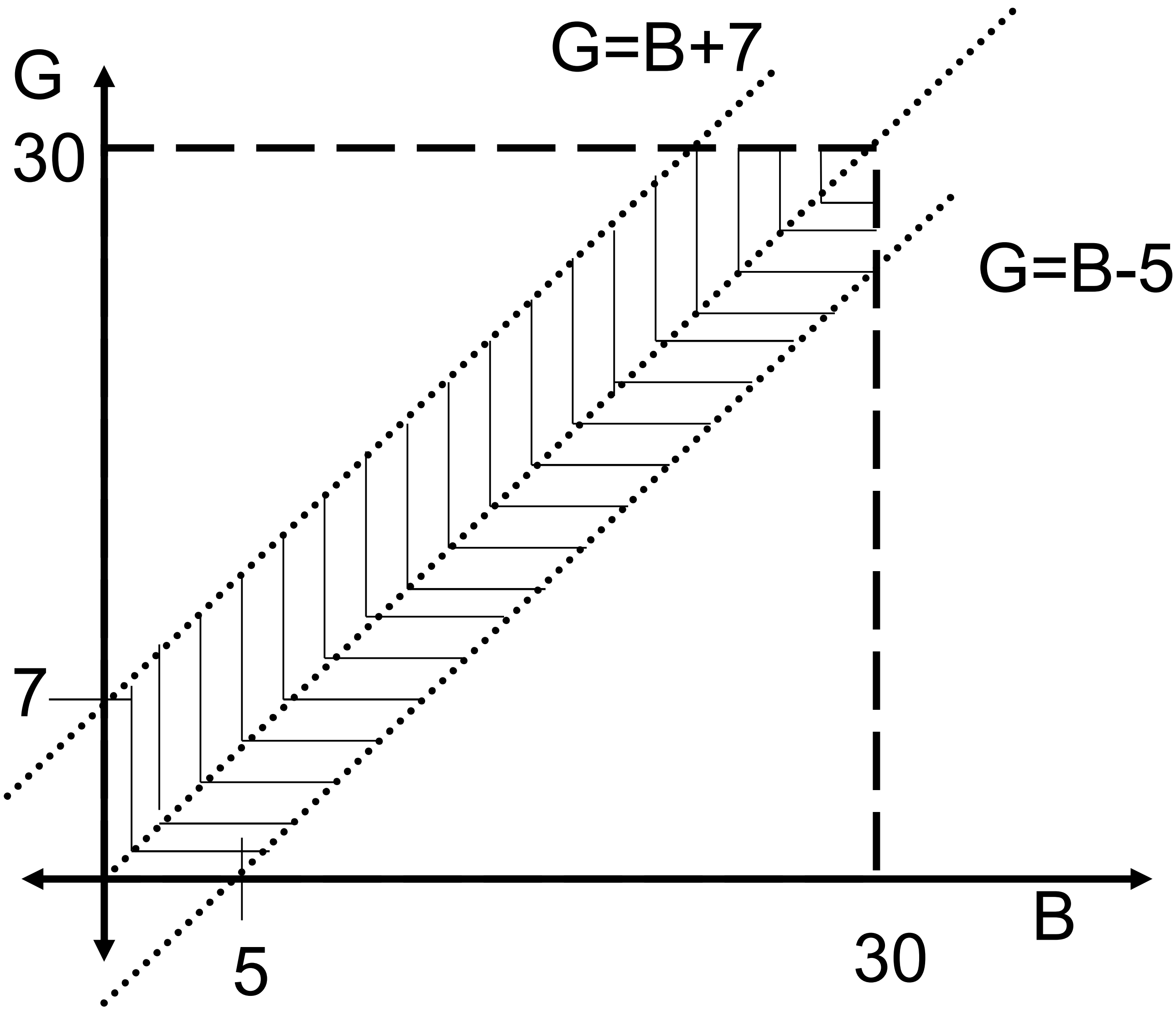
then girl and boy will meet *if* $B - G < 5$

&

if $G > B$,

then girl and boy will meet *if* $G - B < 7$

- There are infinite possibilities
- Represent them graphically
  - Two lines: $G=B-5$ & $G=B+7$


G=B+7
G
30
G=B-5
7
B
5
30

- Showing up time immaterial
- Will they wait? – That's important
- If they are going to wait up to 4 O' clock then definitely they'll meet irrespective of their arrival time
- Probability of meeting is the ratio between the (30x30) square area to the shaded area

<u>Shaded area in Upper triangle</u>

(½ x 30 x 30) – (½ x 23 x 23) = 185.5

<u>Shaded area in Lower triangle</u>

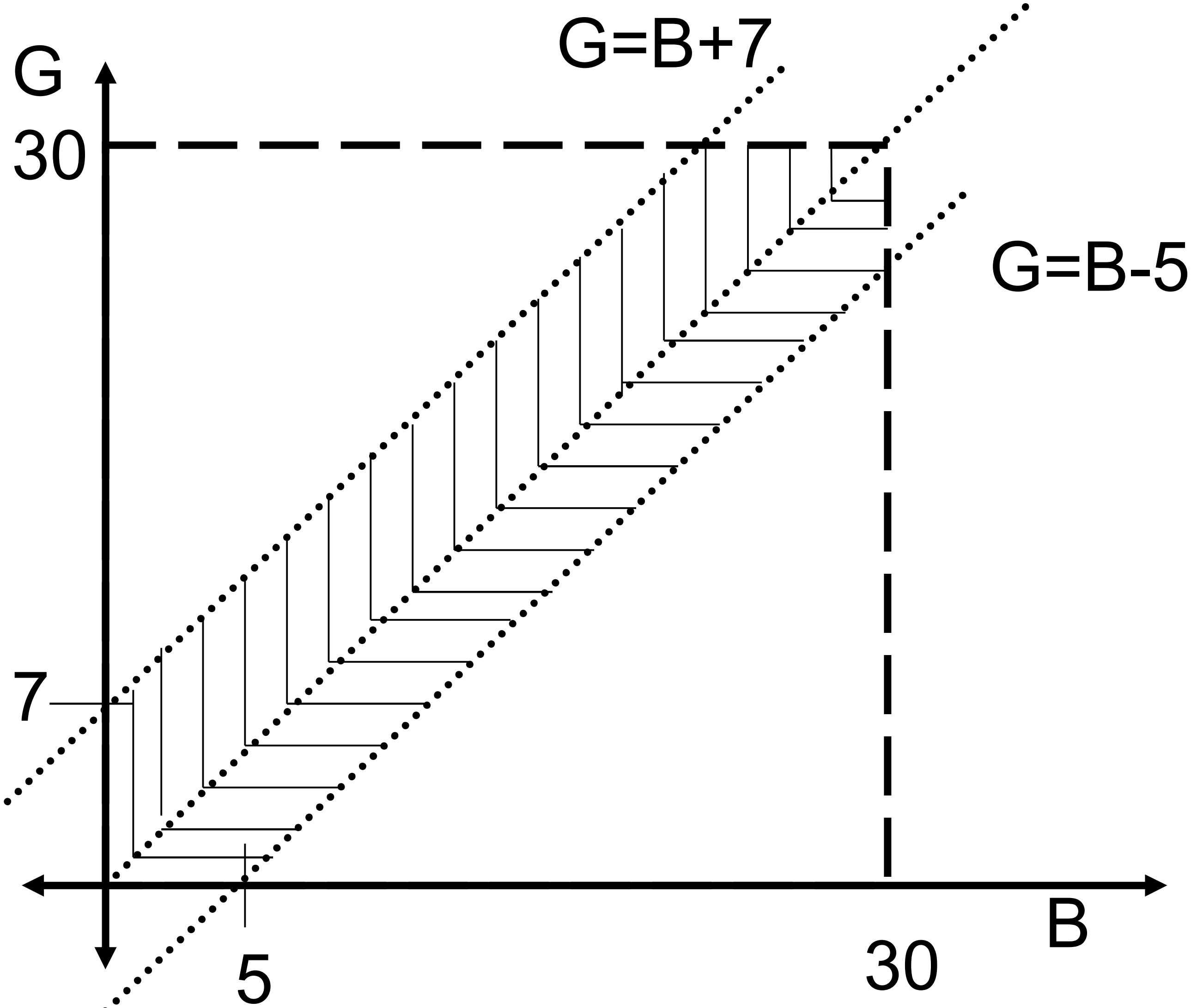
(½ x 30 x 30) – (½ x 25 x 25) =137.5

Shaded area = 323

Probability = 323/900 = 0.359 = 35% chance

Shall we make it technical?

- Network protocol (TCP-IP)
- In a time slot I and you transmit some packets
  - 30 milliseconds
- Chosen time for transmission is random with in the slot
- How many packets? – Depending on this ending time varies
  - I have 7 packets & it takes 7 millisec
  - You have 5 packets & it takes 5 millisec
- What is the probability they won't collide?


G=B+7
G
30
G=B-5
7
5
30
B

- How long is my message? i.e. the number of packets are important
- I start the transmission some time after 3.30 and transmitting till 4 O' clock then definitely I and you'll collide
- 'Probability for not colliding' is the ratio between the (30x30) square area to the un-shaded area

<u>Un-shaded area in Upper triangle</u>

(½ x 23 x 23) = 264.5

<u>Un-shaded area in Lower triangle</u>

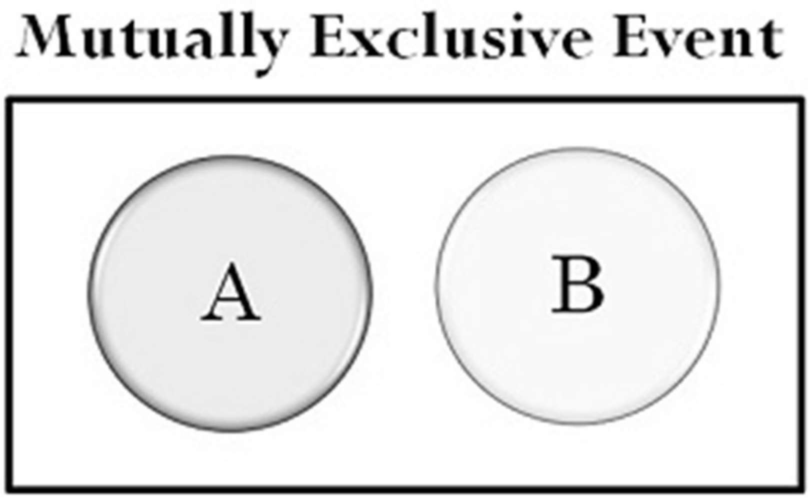
(½ x 25 x 25) =312.5

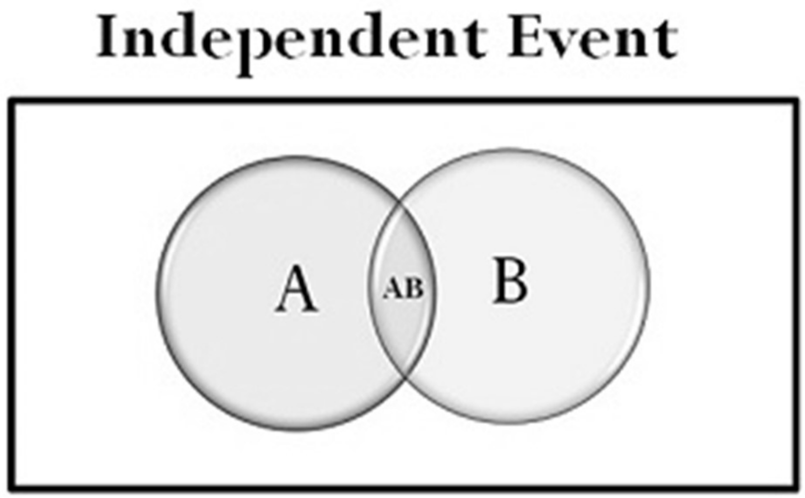
Un-shaded area = 577

Probability = 577/900 = 0.641 = 64% chance

Boy and girl should meet (shaded area)
My pa(o)cket and your pa(o)cket should not meet (unshaded area)

Mutually exclusive and independent events - difference

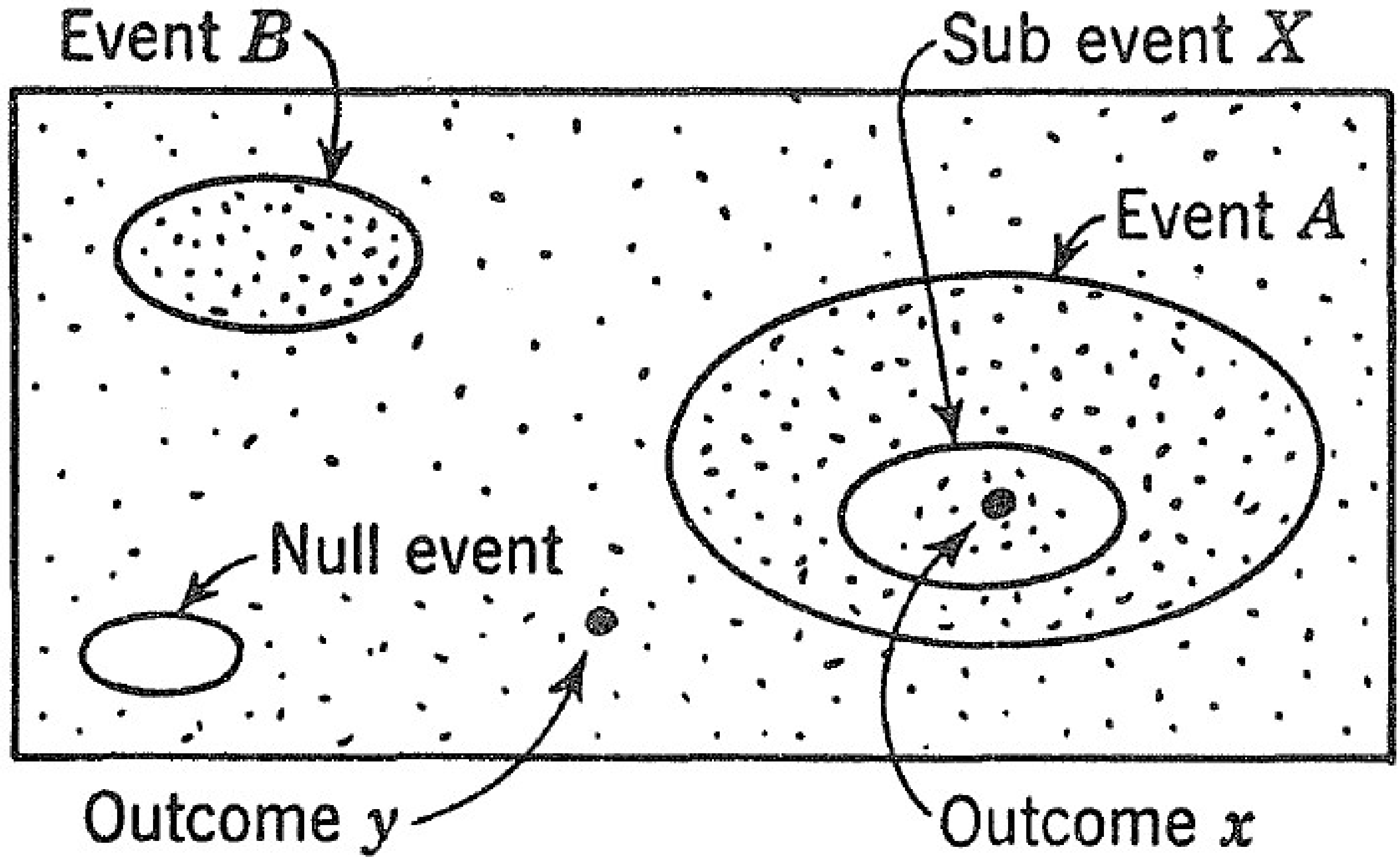




$P(A \cap B) = 0$

$P(A \cap B) = P(A).P(B)$

Probability Space



Probability measure = probability value (denoted as m or p)

$$\left.\begin{aligned} m\{A\} &\le m\{B\} \\ m\{A\} &= m\{B\} - m\{B - A\} \end{aligned}\right\} \qquad \text{if } A \subset B$$

$$m\{A'\} = m\{U - A\} = m\{U\} - m\{A\} = 1 - m\{A\}$$

$$m\{A \cup B\} = m\{(A - AB) \cup B\} = m\{A\} - m\{AB\} + m\{B\}$$

$$m\{A\} + m\{B\} \ge m\{AB\}$$

For three disjoint sets,

$$m\{A \cup B \cup C\} = m\{A\} + m\{B\} + m\{C\}$$