Renyi's Entropy

Use the probability in other ways

Shannon's entropy

- Take P and use it as log(1/p)

Generalize

- Take P and use it as f(p)

Renyi's entropy

- Take P and use it as p□, with □□1

Various information measures

- Weighted entropy
- Havrda - Charvat entropy
- Tsallis entropy

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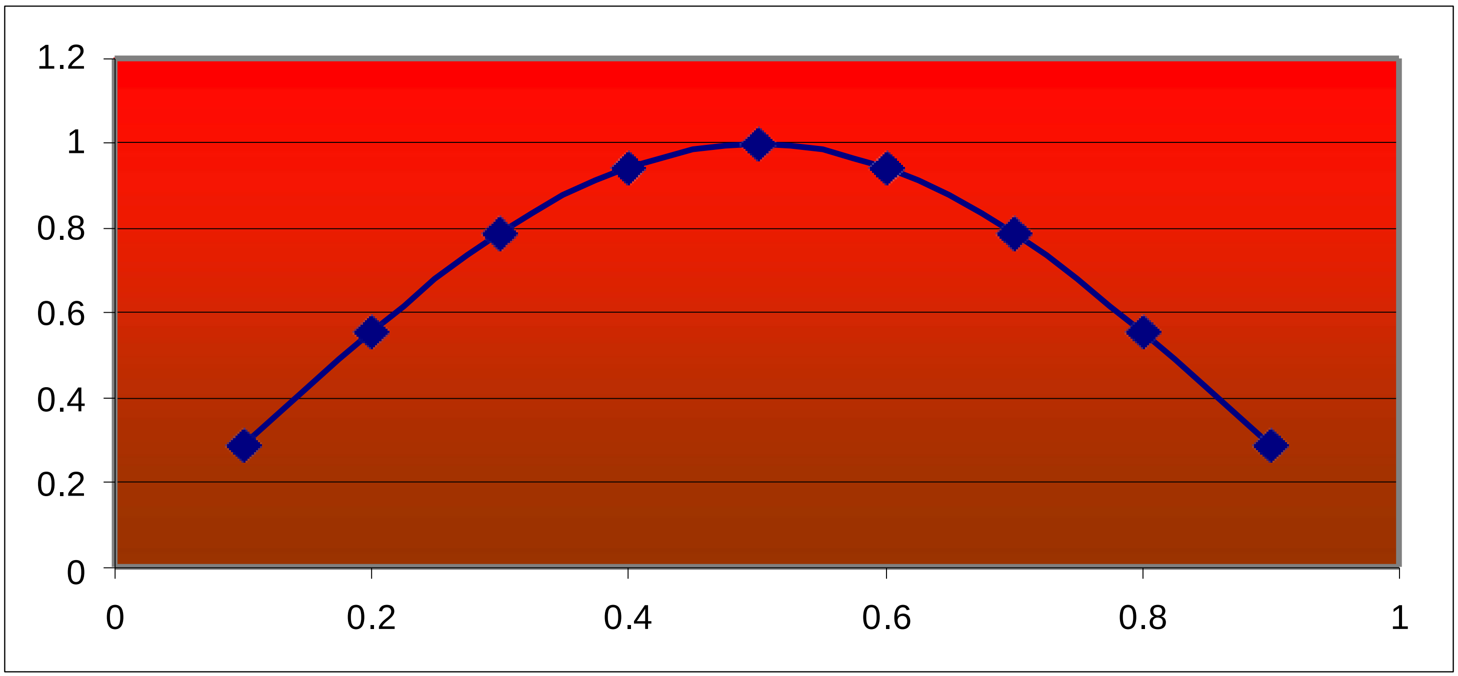
Renyi entropy

- Free parameter: □or q
- Denoted as $H_{R\square}$or H_{Rq}

$$H_\alpha(X) = \frac{1}{1-\alpha}\log_2\left(\sum_{i=1}^{n} p_i^\alpha\right)$$

H_{R2} versus p (i.e. □=2)

- H_{R2}=-log □$_i$ p_i^2
- Binary outputs with p and (1-p)
- $H_{R2}(\mathbf{X}) = -\log\{p^2 + (1-p)^2\}$

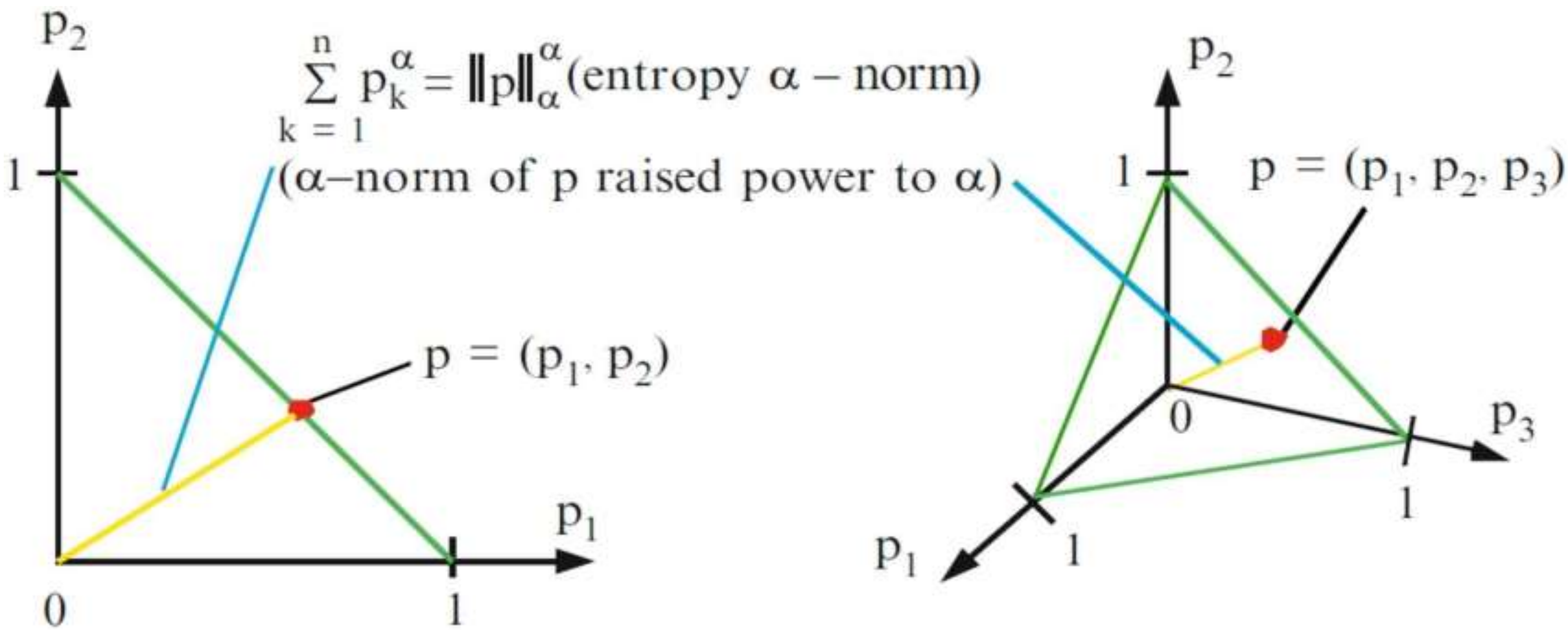


Information potential

$$H_\alpha(X) = \frac{1}{1-\alpha} \log \left(\sum_{k=1}^{N} p_k^\alpha \right) = -\log \boxed{\left(\sum_{k=1}^{N} p_k^\alpha \right)}^{\frac{1}{\alpha-1}}$$

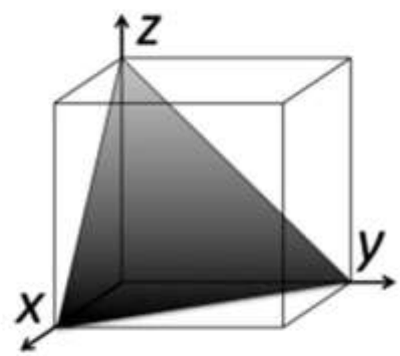
$$H_\alpha(X) = \frac{1}{1-\alpha} \log \left(V_\alpha(X) \right) = -\log \left(\sqrt[\alpha-1]{V_\alpha(X)} \right)$$

Renyi Entropy – Geometric view



$P_2 = 1 - P_1 = -P_1 + 1$ □m.x + c

Recall p values sum to 1



Renyi entropy in expectation form

$$H_{\alpha} = \frac{1}{1-\alpha}\log\left(\sum_{n} p^{\alpha}\left(I_n\right)\right) \quad with \ \alpha \neq 1 \ and \ \alpha > 0$$

$$H_{\alpha} = \frac{1}{1-\alpha} . \log\left[E\left(p^{\alpha-1}\right)\right]$$

Renyi divergence

$$H_\alpha(X) = \frac{1}{1-\alpha}\log_2\left(\sum_{i=1}^{n} p_i^\alpha\right)$$

Renyi entropy

H = $\frac{1}{\alpha \quad 1} log\left(E\left(p^{(\alpha-1)}\right)\right)$

Renyi divergence

$$D(P||Q) = \frac{1}{\alpha - 1} log\left(E\left(\frac{p}{q}\right)^{(\alpha-1)}\right)$$

Weighing factor is p

$$D(P||Q) = \frac{1}{\alpha - 1} log \sum \frac{p^{(\alpha)}}{q^{(\alpha-1)}}$$

With □=1, Renyi divergence=KLD