UIT2504 Artificial Intelligence

Local Search Strategies

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Evaluating a state

- Sometimes, it may be possible to design an evaluation function $f(s)$ that evaluates the "badness" (to be minimized) or "goodness" (to be maximized) of a state s
- In such cases, the most desirable state may be chosen from the working set
- Working set is maintained as a priority queue based on the evaluation function f
- Obviously, the quality of search depends on the evaluation function f

Heuristics

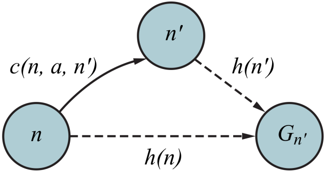
- Usually, such an evaluation function $f(s)$ is designed based on some heuristics $h(s)$ — estimation of cost of reaching a goal state from state s
- For example, can you think of a heuristics for the route finding problem in a map? — Straight line distance (SLD) from the current city to the destination city
- Heuristics should be an easy function to compute!
- $h(s^*)$ should be 0 for any goal state s^*

Admissible Heuristics

- A heuristic h is admissible if for every node n, $h(n) \leq h^*(n)$, where $h^*(n)$ is the **true** cost to reach the goal state from n.
- In other words, $f(n) = g(n) + h(n) \leq C^*$, where C^* is the optimal path cost
- An admissible heuristic **never overestimates** the cost to reach the goal, i.e., it is always **optimistic**
- The SLD heuristics and the two heuristics for sliding puzzle problem are examples of admissible heuristics

Consistent Heuristics

- A heuristics is consistent if for every node n, $h(n) \leq c(n, a, n') + h(n')$, where n' is a successor of n generated by some action a



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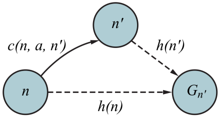

- When h is consistent, we can infer the following

$$
\begin{aligned}
f(n') &= g(n') + h(n') \\
&= g(n) + c(n, a, n') + h(n') \\
&\geq g(n) + h(n) \\
&\geq f(n)
\end{aligned}
$$

- That means, evaluation function f is monotonic — it is non-decreasing along any path
- Every consistent heuristics is also admissible

Questions?

Iterative improvement algorithms

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  - Find an optimal configuration (eg. TSP, maximal matching in a bipartite graph, "weights" that minimize error on the examples)

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- In such cases, we can use iterative improvement algorithms — "keep a single current state and try to improve it"

Example: TSP

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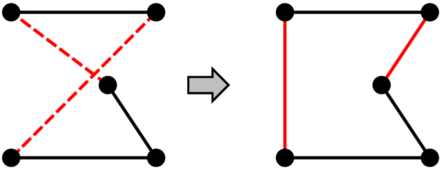
- Start with any complete state (any Hamiltonian circuit)
- Action: Pairwise exchanges — replace edges (u, v) and (u', v') with edges (u, v') and (u', v)



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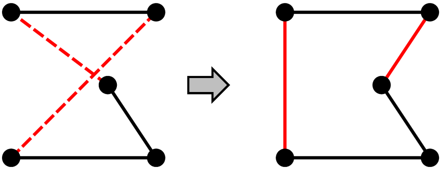

- Able to get within 1% of optimal solution very quickly, even with thousands of cities (good, as an approximation algorithm)

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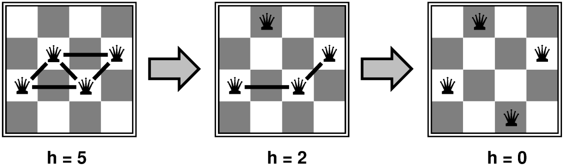
- Action: Move a queen, within its column, to reduce the number of conflicts



Example: *n*-queens problem

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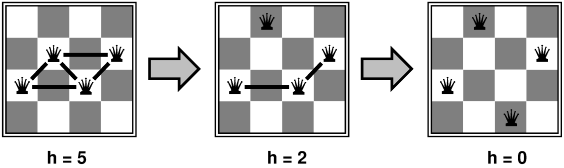
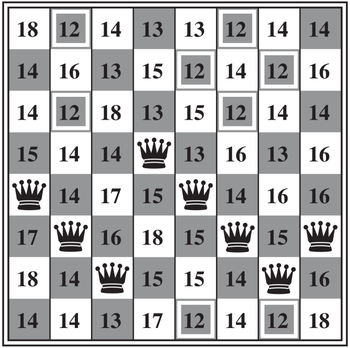


- Able to solve instantaneously even for very large *n*

Outline of Hill Climbing Algorithm

```
def hill_climbing(problem):
    current = problem.initial()
    while True:
        neighbor = max( problem.children(current) )
        if problem.value(neighbor) <=
                                     problem.value(current):
            break
        current = neighbor
    return current
```

Example: 8-queens problem



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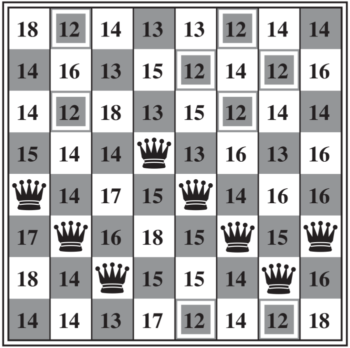

- One of the square with $h = 12$ is chosen and the corresponding queen is moved there

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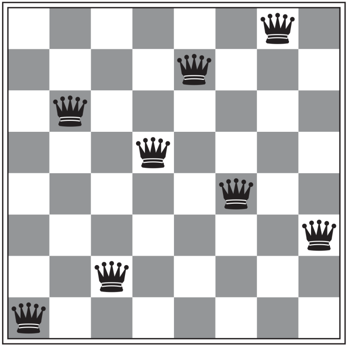

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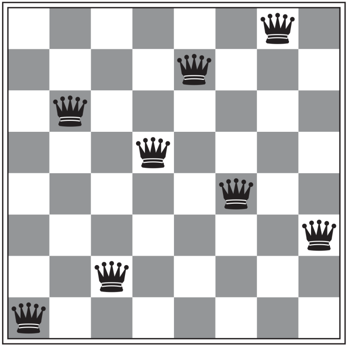

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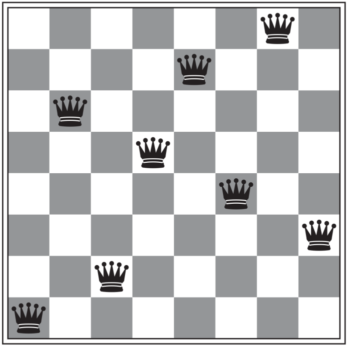
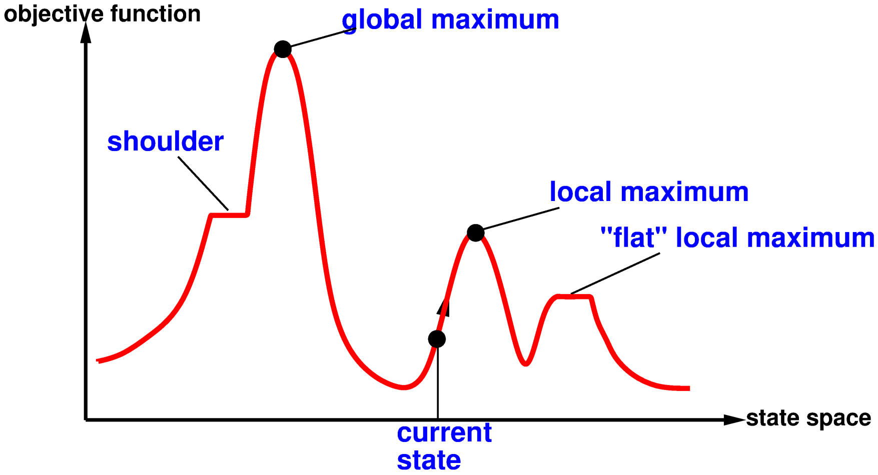


- $h(s) = 1$
- And, we have hit a local minimum!

Hill Climbing Search



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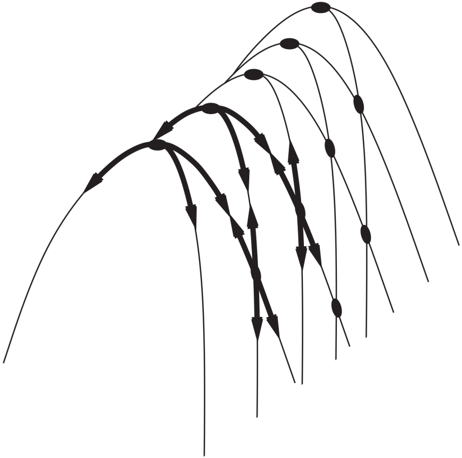
Local maxima is a serious problem

Empirical analysis of 8-queens problem reveals that the greedy hill-climbing algorithm gets stuck 86% of the time

Ridges



Variations of Hill Climbing

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- Imagine applying just enough force for it to escape from all local minima but not from global minima
- How to find that “just enough force”?

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- "Schedule" of reducing the temperature is very critical

Simulated Annealing

function SIMULATED-ANNEALING($problem$, $schedule$) **returns** a solution state
  $current \leftarrow problem$.INITIAL
  for t = 1 **to** ∞ **do**
    $T \leftarrow schedule(t)$
    if T = 0 **then return** $current$
    $next \leftarrow$ a randomly selected successor of $current$
    $\Delta E \leftarrow$ VALUE($current$) – VALUE($next$)
    if $\Delta E > 0$ **then** $current \leftarrow next$
    else $current \leftarrow next$ only with probability $e^{-\Delta E/T}$

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