

Bayes Classification

Conditional probability-based classifier

Classification

- Assigning a pattern \mathbf{x} to one of L classes, represented by labels C_1, \dots, C_L

Bayes rule-Maximum a posteriori (MAP) classifier

- Probability that \mathbf{x} belongs to class C_k is given by

$$P(C_k | \mathbf{x}) = \frac{P(C_k)p(\mathbf{x} | C_k)}{p(\mathbf{x})}$$

- $P(C_k | \mathbf{x})$ is the posterior probability of class C_k given that \mathbf{x} was observed,
- $P(C_k)$ is the prior probability of class C_k , $p(\mathbf{x} | C_k)$ is the likelihood function
- $p(\mathbf{x})$ is the PDF of \mathbf{x} which plays the role of a normalization factor

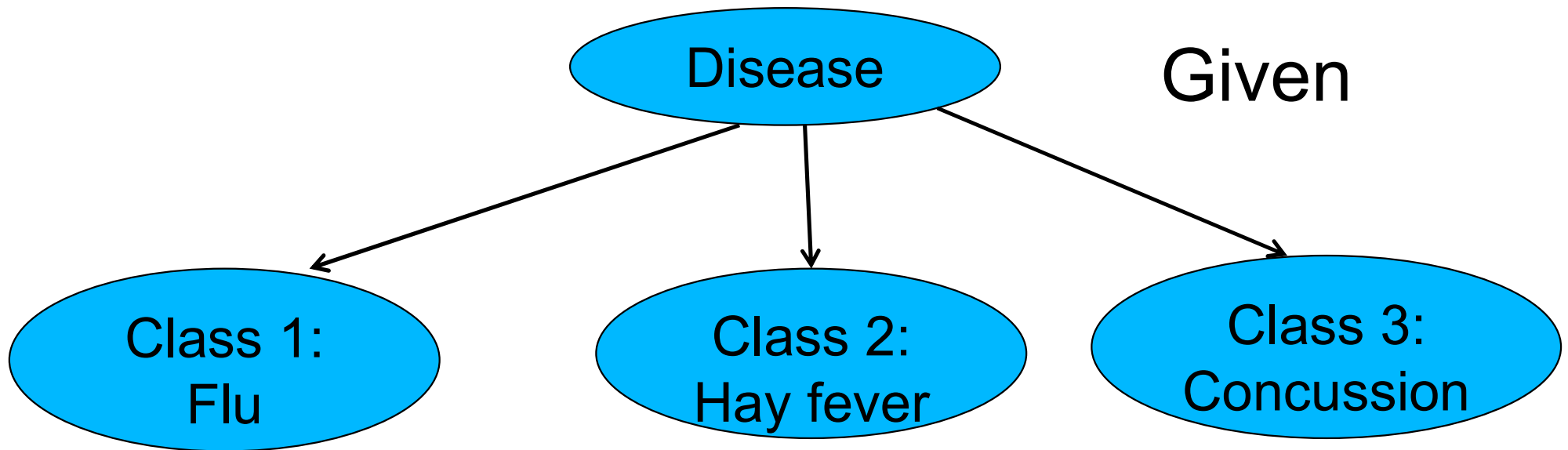
$$\mathbf{x} \in C_k \quad \text{if} \quad k = \arg \max_{l=1, \dots, L} P(C_l | \mathbf{x})$$

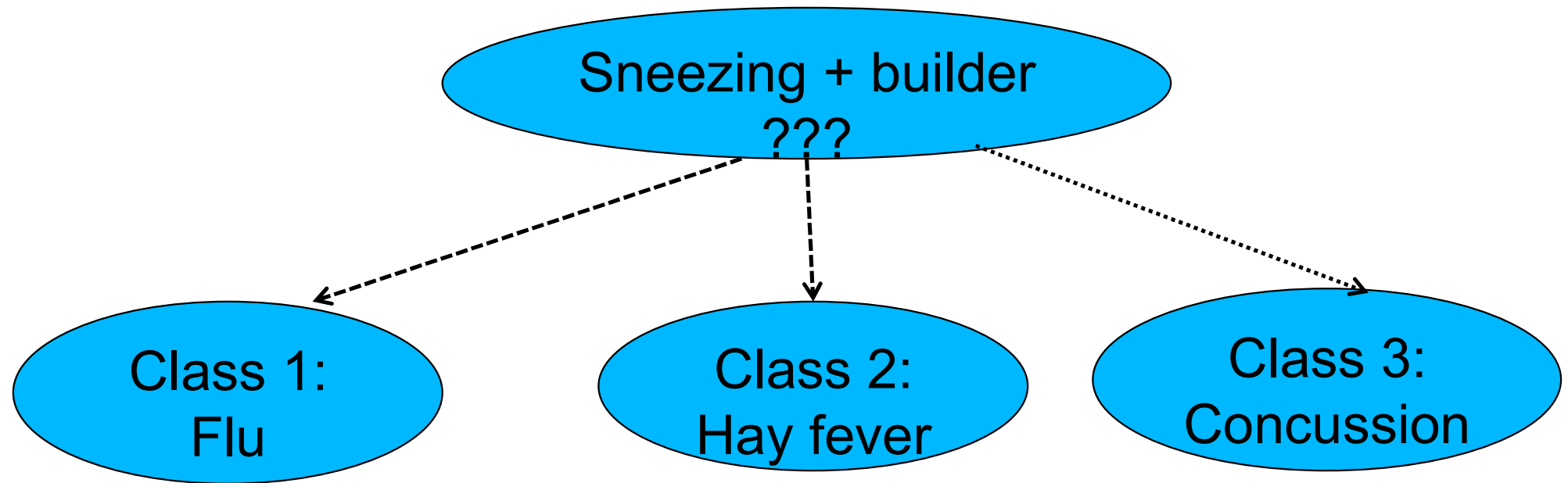
What is the disease?

Flu or Hay-fever or concussion?

SYMPTOM	OCCUPATION	AILMENT
sneezing	nurse	Flu
sneezing	farmer	Hay-fever
headache	builder	concussion
headache	builder	flu
sneezing	teacher	flu
headache	teacher	concussion
sneezing	builder	???

It is a classification problem.
There are three classes.
Which class he/she belongs to?





What do we need to know...

$$P(\text{flu}|\text{sneezing}, \text{builder}) = ?$$

$$P(\text{hay} - \text{fever}|\text{sneezing}, \text{builder}) = ?$$

$$P(\text{concussion}|\text{sneezing}, \text{builder}) = ?$$

Compute the probability of flu

$$P(\text{flu}|\text{sneezing}, \text{builder}) = \frac{P(\text{sneezing}, \text{builder}|\text{flu}).P(\text{flu})}{P(\text{sneezing}, \text{builder})}$$

- $P(\text{flu}) = 3/6 = 0.5$
- $P(\text{sneezing}|\text{flu}) = 2/3 = 0.67$
- $P(\text{builder}|\text{flu}) = 1/3 = 0.33$
- $P(\text{sneezing}, \text{builder}|\text{flu}) = 0.67 \times 0.33 = 0.22$
- $P(\text{sneezing}) = 3/6 = 0.5$
- $P(\text{builder}) = 2/6 = 0.33$
- $P(\text{sneezing}, \text{builder}) = 0.5 \times 0.33 = 0.165$

$$= \frac{0.22 \times 0.5}{0.165} = 0.67$$

Compute the probability of hay-fever

$$P(\text{hay} - \text{fever} | \text{sneezing}, \text{builder}) \\ = \frac{P(\text{sneezing}, \text{builder} | \text{hay} - \text{fever}) \cdot P(\text{hay} - \text{fever})}{P(\text{sneezing}, \text{builder})}$$

- $P(\text{hay-fever}) = 1/6 = 0.167$
- $P(\text{sneezing} | \text{hay-fever}) = 1/1 = 1$
- $P(\text{builder} | \text{hay-fever}) = 0/1 = 0$
- $P(\text{sneezing}, \text{builder} | \text{hay-fever}) = 1 \times 0 = 0$
- $P(\text{sneezing}) = 3/6 = 0.5$
- $P(\text{builder}) = 2/6 = 0.33$
- $P(\text{sneezing}, \text{builder}) = 0.5 \times 0.33 = 0.165$

$$= \frac{0 \times 0.167}{0.165} = 0$$

Compute the probability of concussion

$P(\text{concussion}|\text{sneezing}, \text{builder})$

$$= \frac{P(\text{sneezing}, \text{builder}|\text{concussion}) \cdot P(\text{concussion})}{P(\text{sneezing}, \text{builder})}$$

- $P(\text{concussion}) = 2/6 = 0.33$
- $P(\text{sneezing}|\text{concussion}) = 0/2 = 0$
- $P(\text{builder}|\text{concussion}) = 1/2 = 0.5$
- $P(\text{sneezing}, \text{builder}|\text{flu}) = 0 \times 0.5 = 0$
- $P(\text{sneezing}) = 3/6 = 0.5$
- $P(\text{builder}) = 2/6 = 0.33$
- $P(\text{sneezing}, \text{builder}) = 0.5 \times 0.33 = 0.165$

$$= \frac{0 \times 0.33}{0.165} = 0$$

Which class it belongs to?

- Max (0.67(flu), 0(hay-fever), 0(concussion))
- 67% probability for flu – if he/she is a builder and sneezing