UIT2504 Artificial Intelligence

Genetic Algorithms

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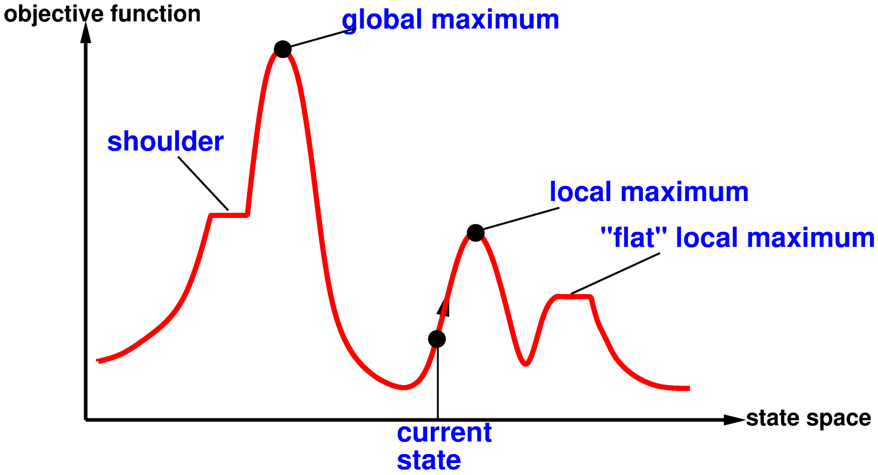
Iterative improvement algorithms

- In several problems, the path is irrelevant and a goal-state is a solution we are looking for
- With state space = set of "complete" configurations,
  - Find an optimal configuration (eg. TSP, maximal matching in a bipartite graph, "weights" that minimize error on the examples)
  - Find a configuration that satisfies some constraints (eg. Timetable generation, *n*-queens problem, stable matching)
- In such cases, we can use iterative improvement algorithms — "keep a single current state and try to improve it"

Outline of Hill Climbing Algorithm

```
def hill_climbing(problem):
    current = problem.initial()
    while True:
        neighbor = max( problem.children(current) )
        if problem.value(neighbor) <=
                                    problem.value(current):
            break
        current = neighbor
    return current
```

Hill Climbing Search



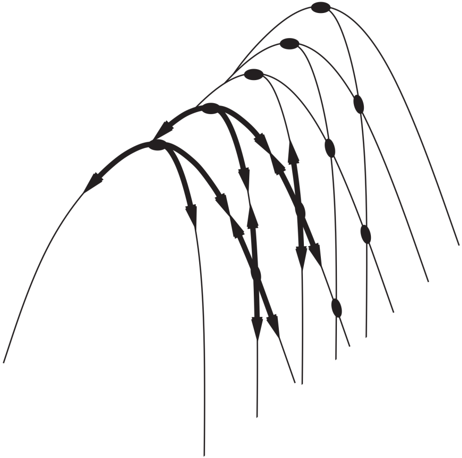
Issues in Greedy Local Search

- Local maxima / minima
- Ridges — sequence of local maxima that is very difficult for the greedy algorithm to navigate
- Plateaux — flat local maximum or a shoulder

Local maxima is a serious problem

Empirical analysis of 8-queens problem reveals that the greedy hill-climbing algorithm gets stuck 86% of the time

Ridges



Variations of Hill Climbing

- Random sideways moves — can escape from a shoulder, but gets trapped in a local maxima — number of sideways moves may be limited — number of instances solved for the 8-queens problem increases from 14% to 94%
- Stochastic Hill Climbing — chooses at random, among all uphill moves — probability of selection can depend on the steepness of the ascent — example of a Randomized algorithm
- First Choice Hill Climbing — randomly generate the successors, until one better than the current is generated
- Random Restart — enough restarts may make this algorithm complete — if each hill-climbing has a probability p of success, then $1/p$ restarts are expected — for 8-queens, $p \approx 0.14$, and so roughly 7 restarts are expected

Simulated Annealing

- One interesting variation of hill climbing is to adopt the concept of simulated annealing
- For example, conside a ball set to roll on a state landscape
- The ball simply follows the rules of gravity and moves towards nearby valley
- The ball needs to make some uphill moves to escape from local minima!
- Imagine applying just enough force for it to escape from all local minima but not from global minima
- How to find that “just enough force”?

Simulated Annealing

- Similar to the metallurgical process of annealing
- Start with a high "temperature" — probability of selecting a bad move is high
- Slowly reduce the "temperature" — probability of selecting a bad move reduces slowly
- "Schedule" of reducing the temperature is very critical

function SIMULATED-ANNEALING($problem, schedule$) **returns** a solution state
  $current \leftarrow problem$.INITIAL
  for t = 1 **to** ∞ **do**
    $T \leftarrow schedule(t)$
    if T = 0 **then return** $current$
    $next \leftarrow$ a randomly selected successor of $current$
    $\Delta E \leftarrow$ VALUE($current$) – VALUE($next$)
    if $\Delta E > 0$ **then** $current \leftarrow next$
    else $current \leftarrow next$ only with probability $e^{-\Delta E/T}$

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- At each iteration, generate successors of all k states
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- Different from running k parallel searches!

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- Variation called stochastic beam search may be used — instead of k best from the frontier, randomly select k successors with probability directly related to their "fitness"

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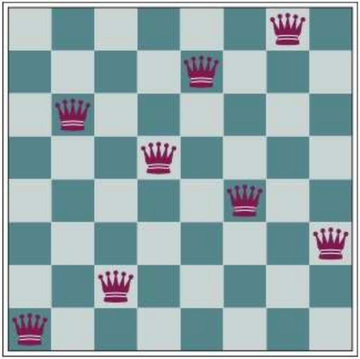
Genetic Algorithms — Representation



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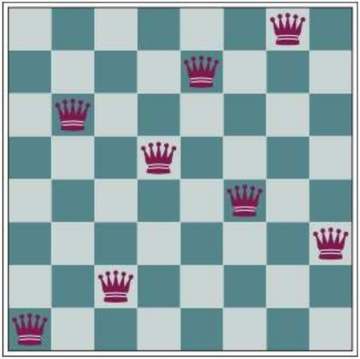

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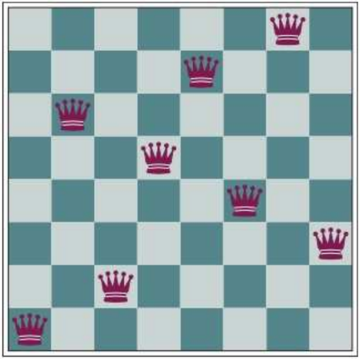

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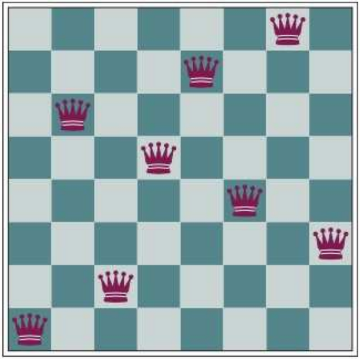

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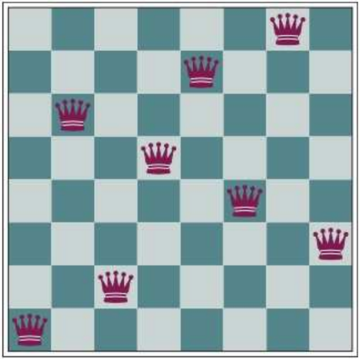
- Does the representation matter?

Genetic Algorithms — Fitness function



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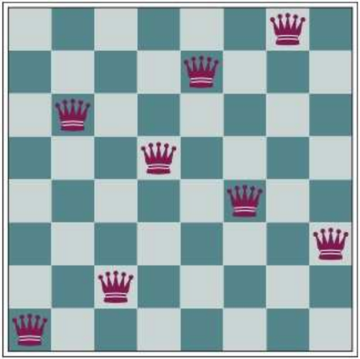

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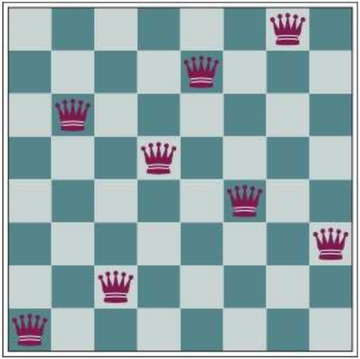

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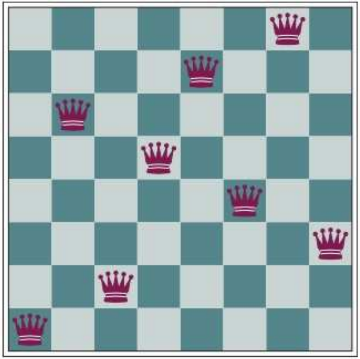

- Fitness of this individual may be evaluated using a heuristics — number of pairs of non-attacking queens (for maximization), or number of pairs of attacking queens (for minimization)

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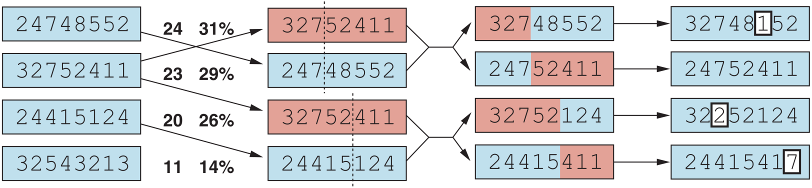
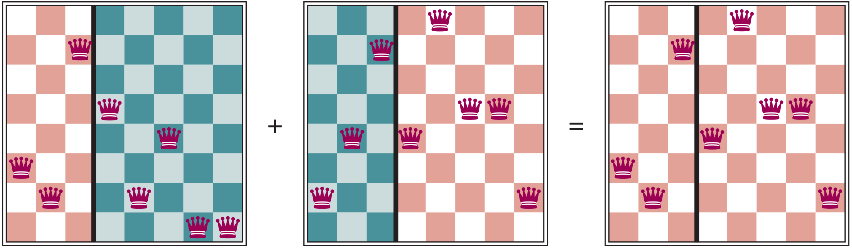
- Culling may be done to eliminate the “unfit” individuals and keep only, say k, best for the next generation of population

Genetic Algorithms — Operations



(a) Initial Population (b) Fitness Function (c) Selection (d) Crossover (e) Mutation

Genetic Algorithms — Crossover



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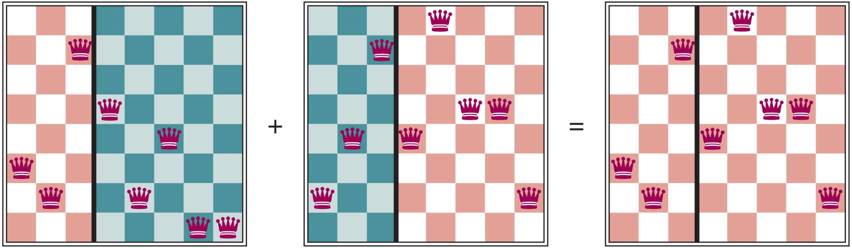

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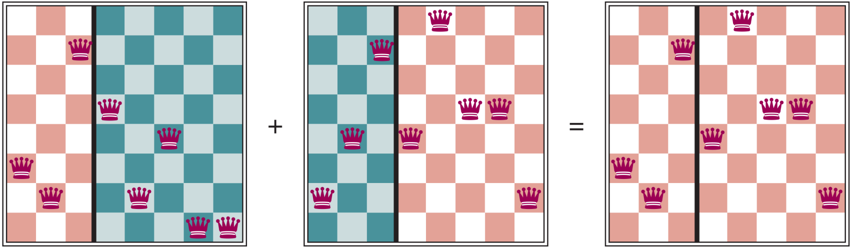

- Randomly generate crossover points
- Generate an offspring by taking one part of the string from one parent and the other part from the other parent (string)

function GENETIC-ALGORITHM(*population*, *fitness*) **returns** an individual
  repeat
    weights ← WEIGHTED-BY(*population*, *fitness*)
    population2 ← empty list
    for i = 1 **to** SIZE(*population*) **do**
      parent1, *parent2* ← WEIGHTED-RANDOM-CHOICES(*population*, *weights*, 2)
      child ← REPRODUCE(*parent1*, *parent2*)
      if (small random probability) **then** *child* ← MUTATE(*child*)
      add *child* to *population2*
    population ← *population2*
  until some individual is fit enough, or enough time has elapsed
  return the best individual in *population*, according to *fitness*

function REPRODUCE(*parent1*, *parent2*) **returns** an individual
  n ← LENGTH(*parent1*)
  c ← random number from 1 to n
  return APPEND(SUBSTRING(*parent1*, 1, c), SUBSTRING(*parent2*, $c+1$, n))

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- Genetic algorithms work better for problems where such schema make sense

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- The evaluation function $f(s)$ may be easily computed as:

$$f((x_1, y_1), (x_2, y_2), (x_3, y_3)) = \sum_{i=1}^{3} \sum_{c \in C_i} (x_i - x_c)^2 + (y_i - y_c)^2$$

where C_i is the set of cities whose closest airport is airport i

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- Such approaches are referred to as empirical gradient methods

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$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3} \right)$$

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- Current state vector x may be updated as

$$x \leftarrow x + \alpha \nabla f(x)$$

where α is a small constant called the step size (learning rate, in the

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- Elements H_{ij} are given by $\frac{\partial^2 f}{\partial x_i \partial x_j}$

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