UIT2504 Artificial Intelligence

Heuristics Revisited

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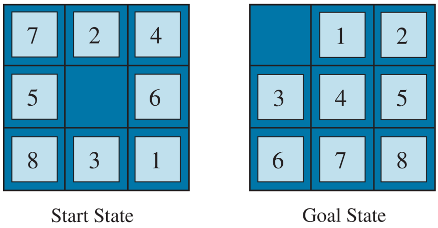
Heuristics — Revisited



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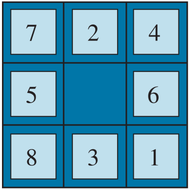

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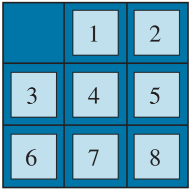

Goal State

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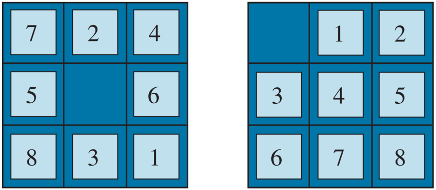

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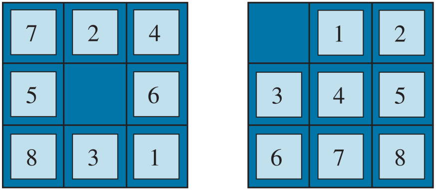

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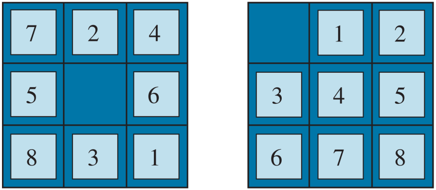

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- Use of good heuristics can drastically cut down the search space!

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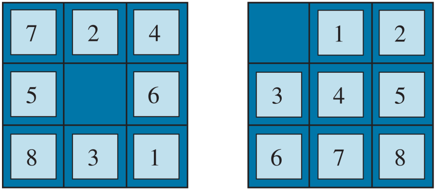

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- h_1: Number of misplaced tiles
- h_2: Sum of the (vertical + horizontal) distances of the tiles from their goal positions (total Manhattan distance)
- For the given start state, $h_1 = 8$, and $h_2 = 3 + 1 + 2 + 2 + 2 + 3 + 3 + 2 = 18$

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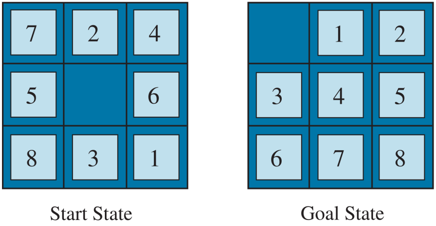

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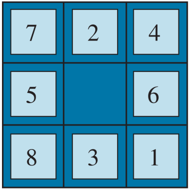

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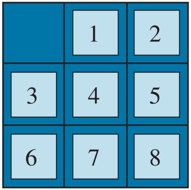

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- As discussed already, h_2 dominates h_1

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- A^* using h_2 does not expand more nodes than the one using h_1

Dominance matters!

	Search Cost (nodes generated)			Effective Branching Factor		
d	BFS	A*(h_1)	A*(h_2)	BFS	A*(h_1)	A*(h_2)
6	128	24	19	2.01	1.42	1.34
8	368	48	31	1.91	1.40	1.30
10	1033	116	48	1.85	1.43	1.27
12	2672	279	84	1.80	1.45	1.28
14	6783	678	174	1.77	1.47	1.31
16	17270	1683	364	1.74	1.48	1.32
18	41558	4102	751	1.72	1.49	1.34
20	91493	9905	1318	1.69	1.50	1.34
22	175921	22955	2548	1.66	1.50	1.34
24	290082	53039	5733	1.62	1.50	1.36
26	395355	110372	10080	1.58	1.50	1.35
28	463234	202565	22055	1.53	1.49	1.36

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- Exercise: Think of a relaxed problem where a tile can move from X to Y, if Y is blank. Solution to this relaxed problem gives yet another heuristics!
- When there are many heuristics with no clarity on which is better (dominant), we can think of a composite heuristics $h(n) = max\,\{h_1(n), h_2(n), \cdots, h_k(n)\}$

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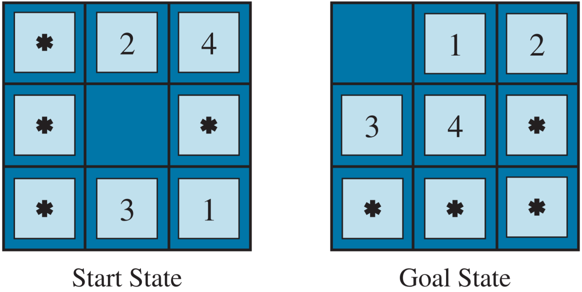

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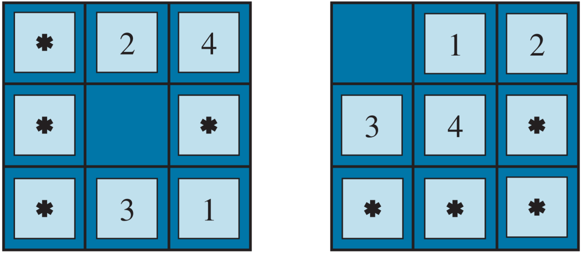

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- Solution to this subproblem turns out to be a better heuristics than the Manhattan distance for all the nodes that fit this pattern

Pattern databases

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- It is easy to construct such a database, as we can compute the actual cost of a pattern by working backwards from the goal states!
- Similarly, other patterns such as $5-6-7-8$ may be considered
- Heuristic values from different pattern databases may be easily composed (using max function) to form a stronger heuristics

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- In general, the answer is no, since the resulting heuristics may not be admissible
- However, if we count only the moves involving the tiles $1, 2, 3, 4$ in the $1-2-3-4$ and so on, then summation is possible

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- An efficient heuristic: $h_L(n) = \min_{L \in Landmarks} C^*(n, L) + C^*(L, goal)$
- This will be the best heuristics, if L is along the optimal path to goal. Otherwise, it may be an overestimate.

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- For DH, it is better if the landmarks are spread around the perimeter of the graph — arrange k pie-shaped wedges around the centroid and select the farthest vertex in each wedge

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- A neural network may be trained to map a state to a heuristic value

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- Several learning algorithms are available to learn these weights from examples

Questions?