UIT2504 Artificial Intelligence

Basic Search Strategies

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State-Space Approach

- Given an initial state, and a set of actions, "search tree" can be visualized
- The number of possible actions determine the branching factor of the tree
- State space is the set of all states that are reachable from the initial state using a sequence of actions
- Which states will be leaf nodes in the search tree? — all goal states will be leaf nodes — there can also be "dead-end" states where no action is possible
- Where is the solution in the search tree? — normally a path from initial state to a goal state
- In some cases, goal state itself is a solution!

Problem Formulation

- A problem may be formulated in the state-space approach by defining the following four components:
  - Set of states, with the initial state
  - Set of possible actions, which may be abstracted by a successor function S. For any state x, $S(x)$ is the set of states reachable from x using an action
  - Goal test — a function that returns true if a given state is a goal state
  - Path cost — sum of all the costs of actions from initial state to a goal state

Illustration: 8-Puzzle

- *State*: Arrangement of eight tiles on a 3×3 grid — *Initial state* is some random arrangement of eight tiles on nine squares
- *Actions*: blank moves left, right, up, or down — branching factor of 4
- *Goal test*: state matches the given target configuration
- *Path cost*: unit cost for each action

Illustration: 8-Puzzle

3	2	7
5	8	
1	4	6

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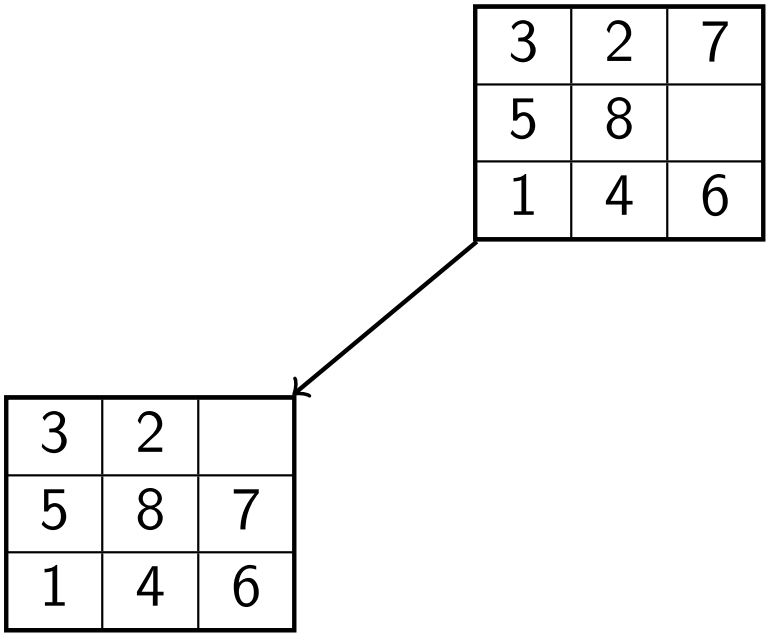

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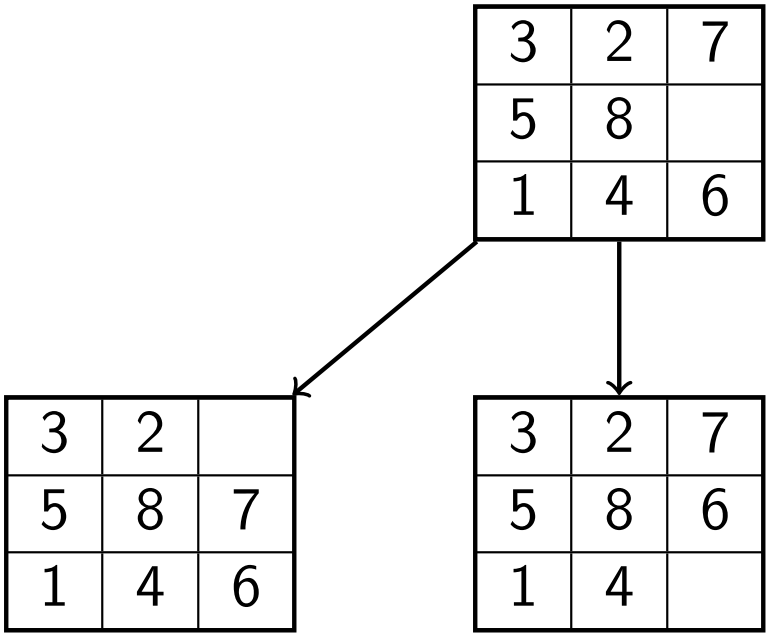

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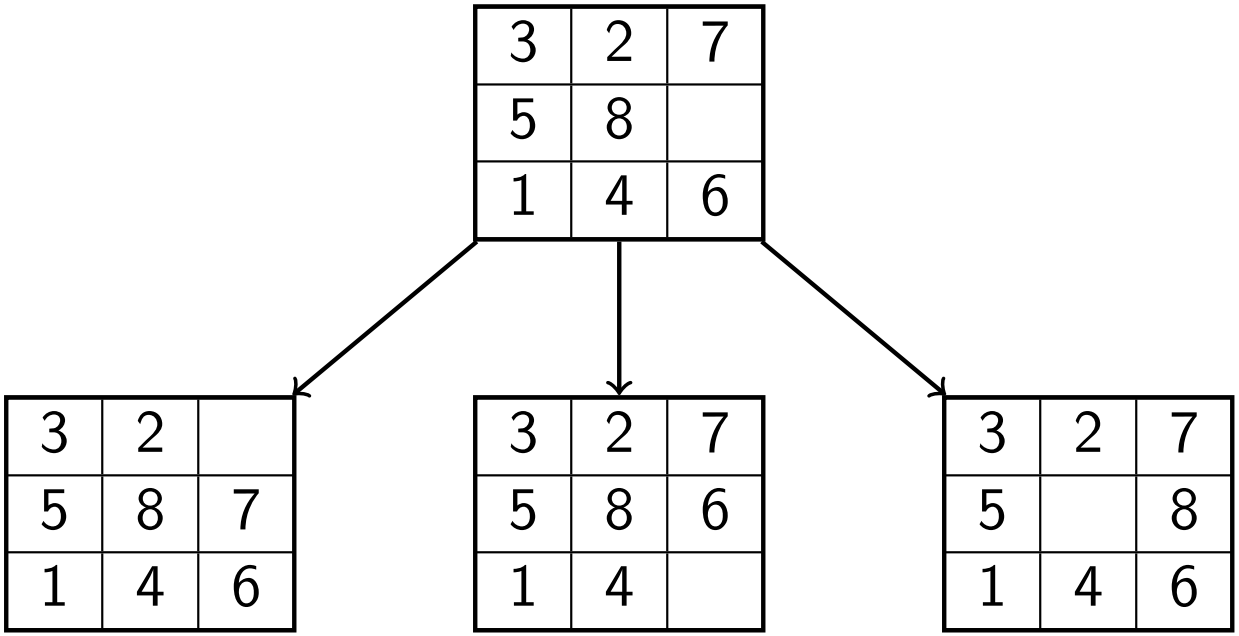

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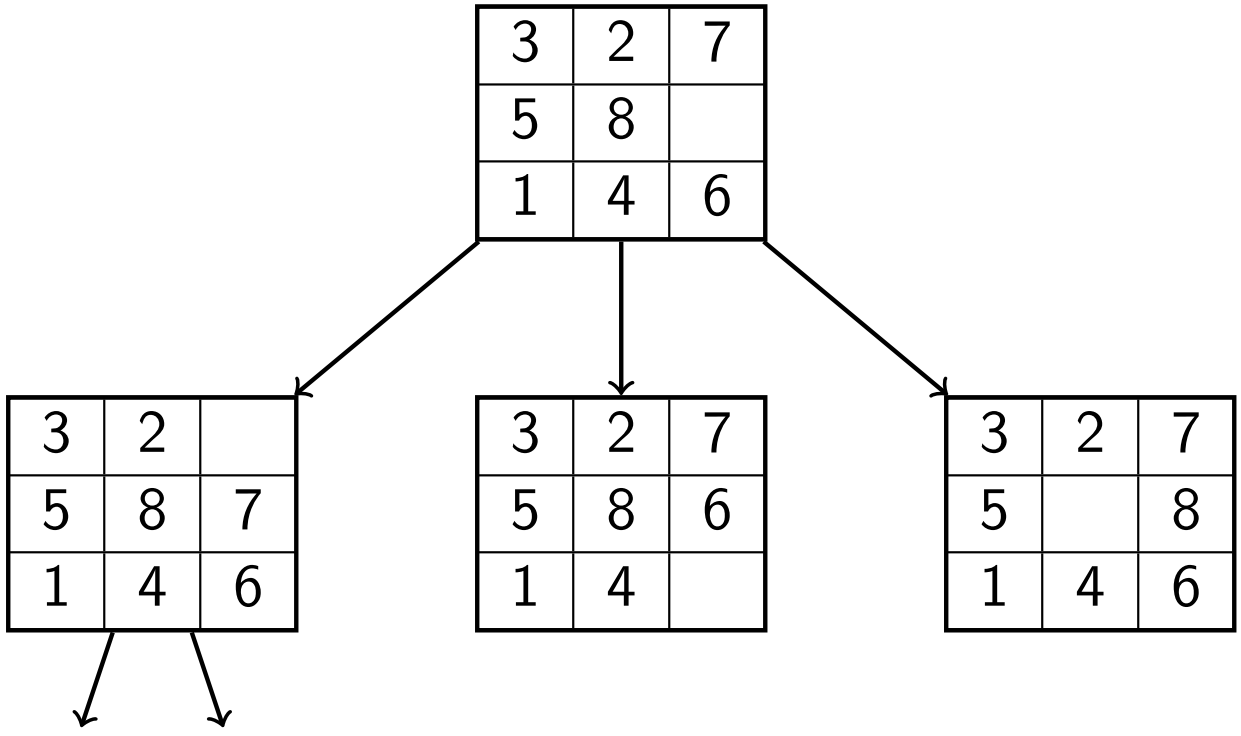

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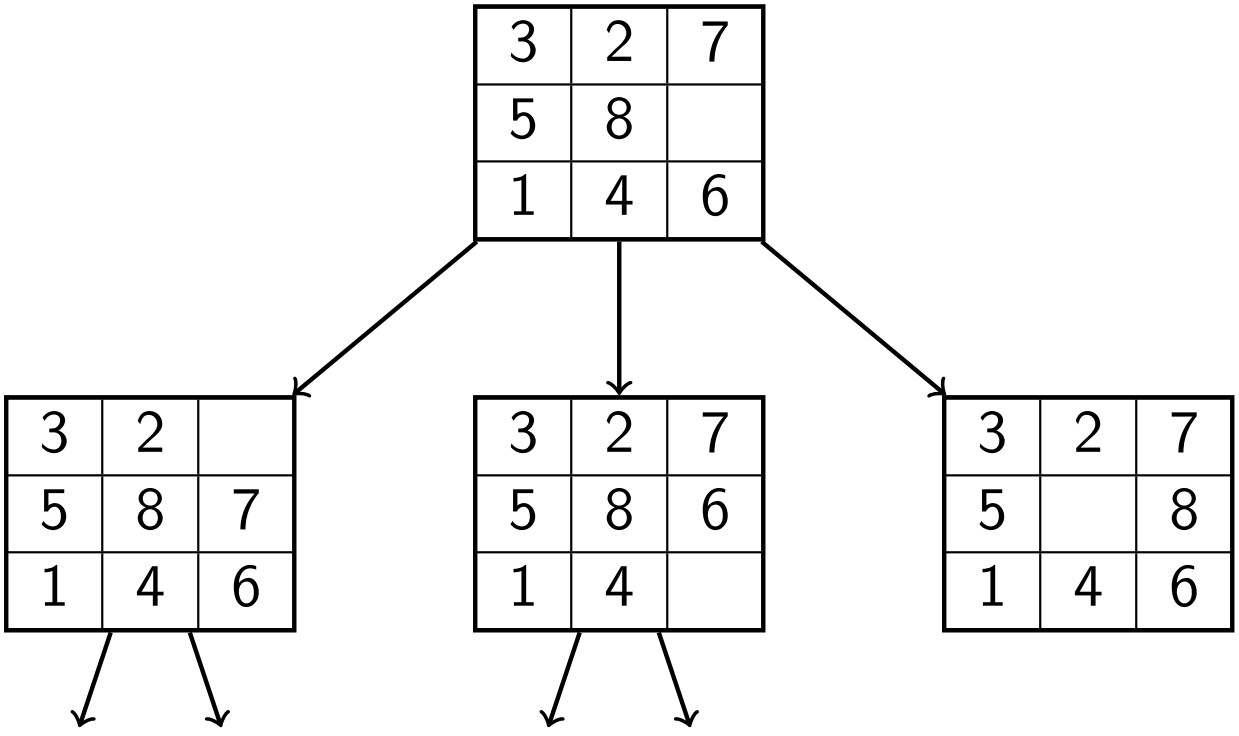

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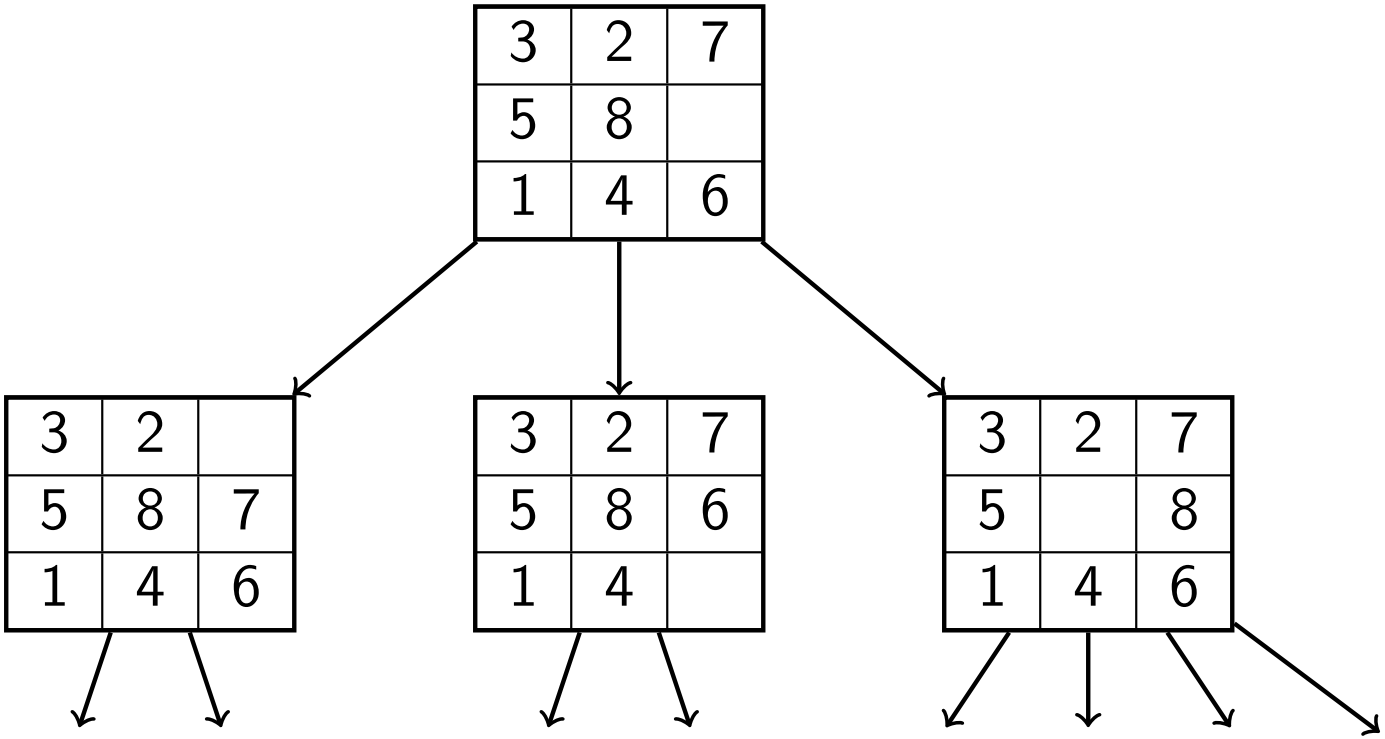

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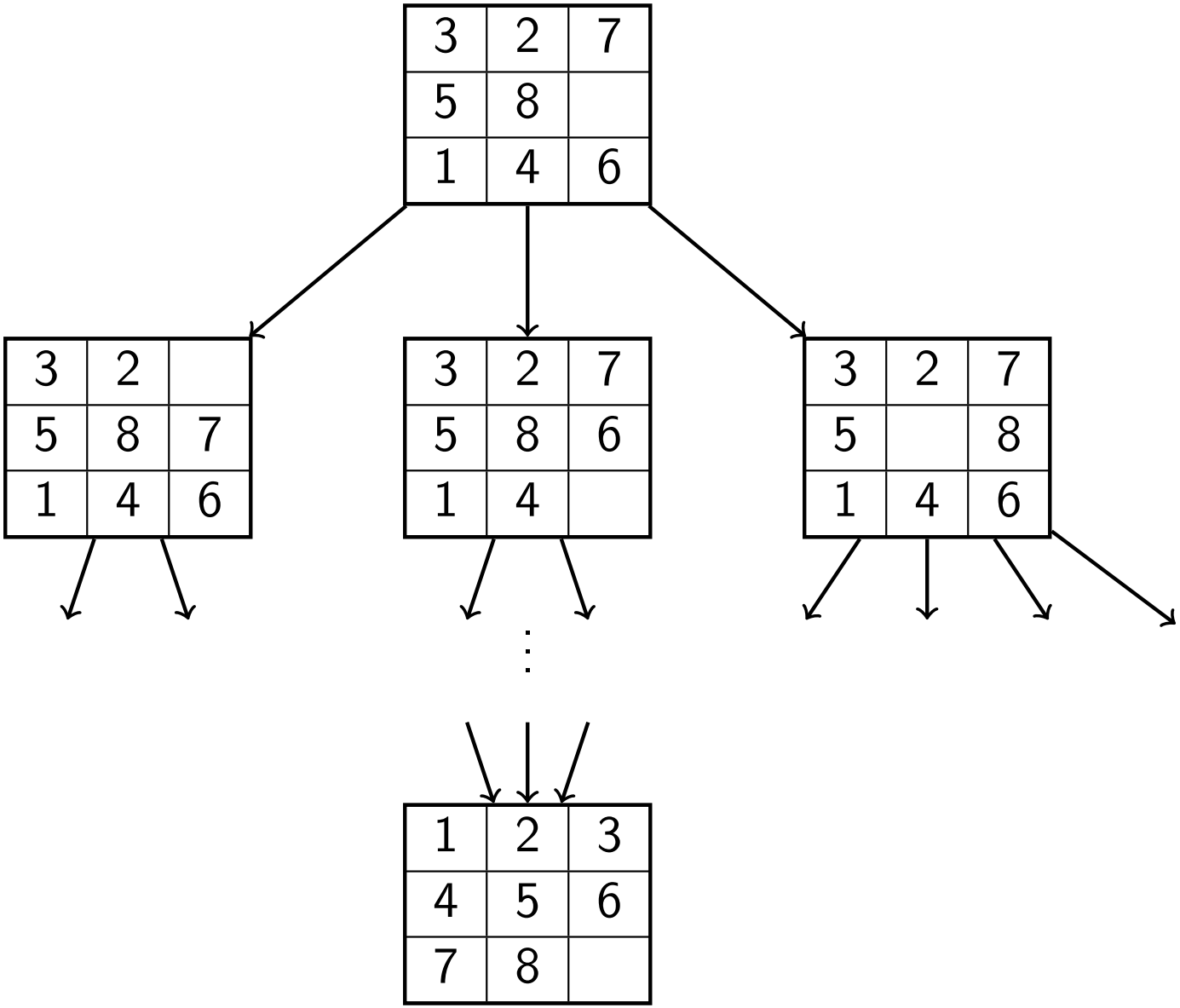

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  - Else, expand x and add the successor states $S(x)$ to the working set

Search Strategies

- Uninformed:
  - Breadth-First
  - Depth-First
  - Depth-Limited
  - Bi-Directional Search
- Informed (Heuristics):
  - Best-first Greedy
  - A^*
  - Local Search Strategies
- Constraint Satisfaction

Performance Measures

- Completeness
- Time Complexity
- Space Complexity
- Optimality

Questions?

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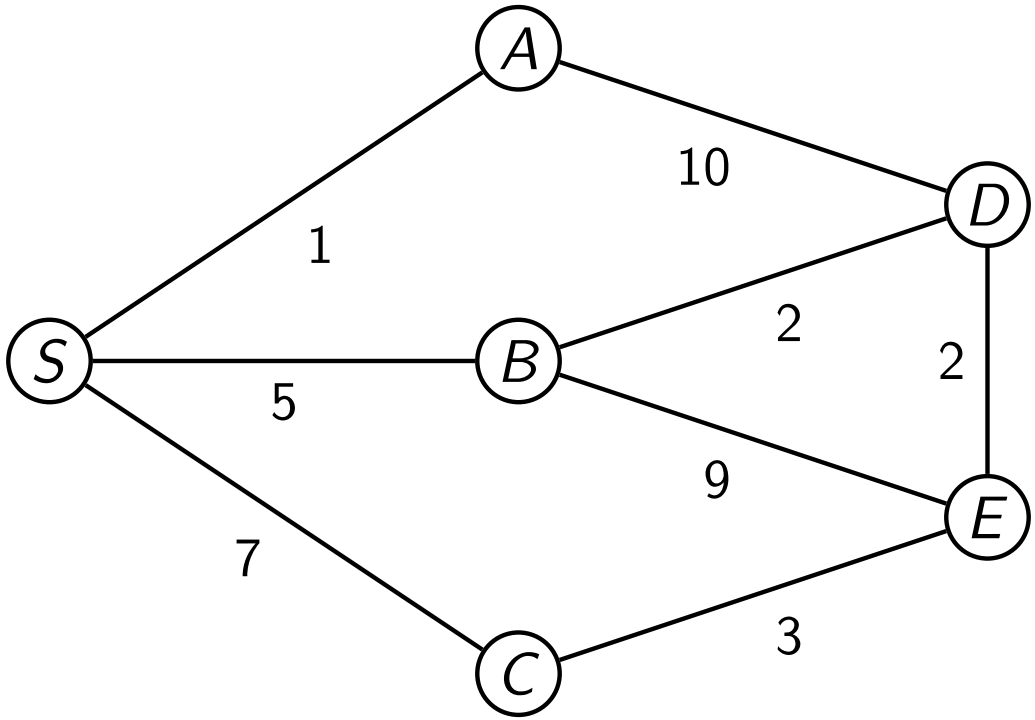
- All nodes at depth d are expanded before any node at depth $d + 1$

BFS: Illustration

- How does it work on our toy example?



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- From tree point of view, nodes are examined in the level order
- Each and every path is progressively examined!

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- Working set (fringe) is a partial order arranged in ascending order of cost (minheap)
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- Does this remind you of any graph alsorithm that you have learnt?

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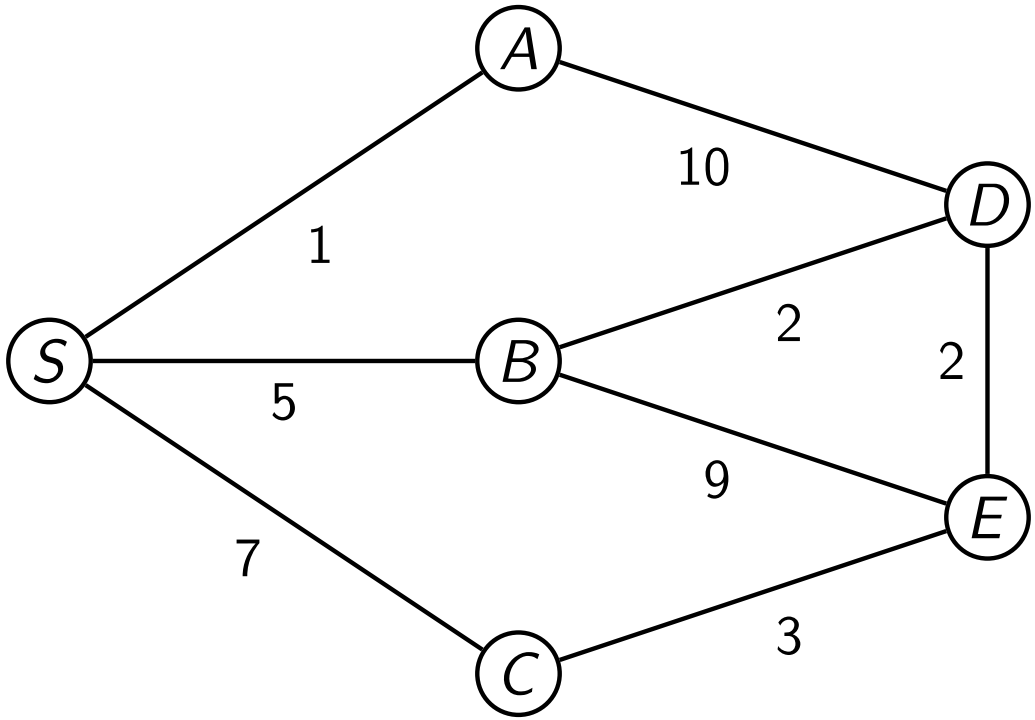

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- Selectively switches between the paths!

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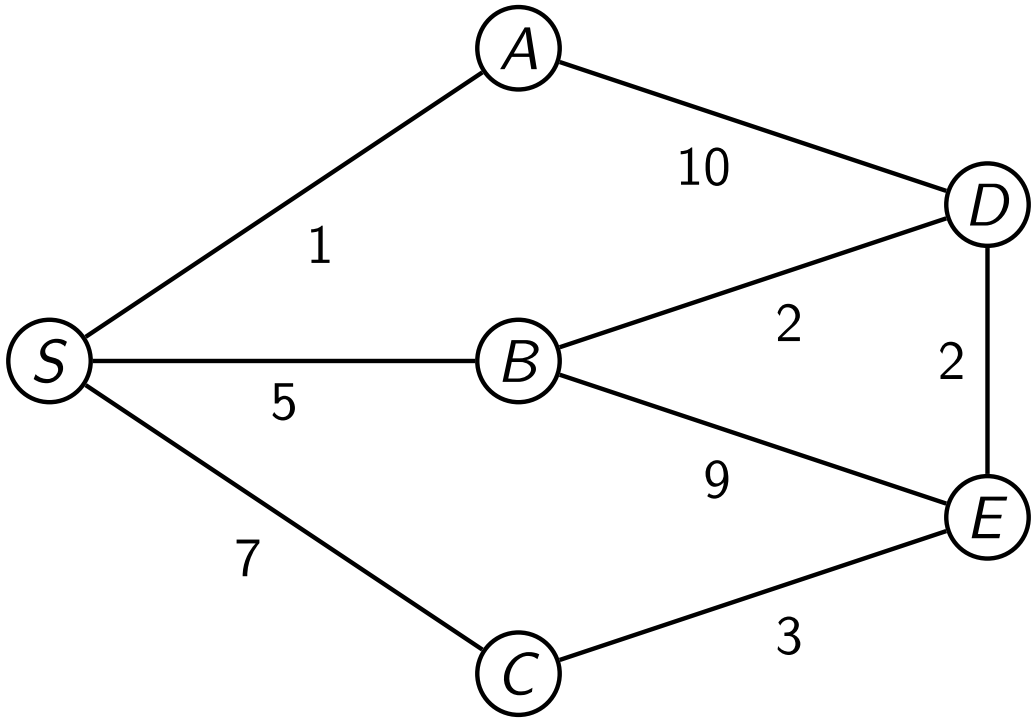

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- Can we design an algorithm that is complete (like BFS) and with linear space complexity (like DFS)?
- Can you suggest some modification to DFS to make it complete?