Conditional probability based classification

e.g.

Email spam filter

- We receive valid and spam mails
- $P(spam) = 2\%$
- Spam detector detects the spam mails
- But it has false positive problem
- $P(detected\ as\ spam | spam) = 98\%$
- $P(detected\ as\ spam | Not\text{-}spam) = 0.2\%$

$$P(spam \mid detected\ as\ spam)$$

$$= \frac{P(detected\ as\ spam \mid spam).P(spam)}{P(detected\ as\ spam)}$$

- Both good as well as spam mails can be detected as spam

$$P(detected\ as\ spam)$$
$$= P(detected\ as\ spam \cap spam)$$
$$+ P(detected\ as\ spam \cap NotSpam)$$

$P(detected\ as\ spam \cap spam)$

$= P(detected\ as\ spam|spam)P(spam)$

$= 0.99 * 0.02 = 0.0198$

$P(detected\ as\ spam \cap NotSpam)$

$= P(detected\ as\ spam|NotSpam)P(NotSpam)$

$= 0.002 * 0.98 = 0.00196$

$P(detected\ as\ spam) = 0.0198 + 0.00196$

$= 0.02176$

$$P(spam \,|detected\ as\ spam)$$

$$= \frac{P(detected\ as\ spam|spam).P(spam)}{P(detected\ as\ spam)}$$

$$= \frac{(0.98 * 0.02)}{0.02176} = 0.9 = 90\%$$

$$P(NotSpam \,|detected \; as \; spam)$$

$$= \frac{P(detected \; as \; spam|NotSpam).P(NotSpam)}{P(detected \; as \; spam)}$$

- Both good as well as spam mails can be detected as spam

$$P(detected \; as \; spam)$$
$$= P(detected \; as \; spam \cap spam)$$
$$+ P(detected \; as \; spam \cap NotSpam)$$

$P(detected\ as\ spam \cap spam)$
$= P(detected\ as\ spam|spam)P(spam)$
$= 0.99 * 0.02 = 0.0198$
$P(detected\ as\ spam \cap NotSpam)$
$= P(detected\ as\ spam|NotSpam)P(NotSpam)$
$= 0.002 * 0.98 = 0.00196$
$P(detected\ as\ spam) = 0.0198 + 0.00196$
$= 0.02176$

$$P(\text{detected as spam}|\text{NotSpam})$$

$$= \frac{P(\text{detected as spam} \cap \text{NotSpam})}{P(\text{NotSpam})} = \frac{0.002}{0.98}$$

$$= 0.002$$

$$P(\text{NotSpam }|\text{detected as spam})$$

$$= \frac{P(\text{detected as spam}|\text{NotSpam}).P(\text{NotSpam})}{P(\text{detected as spam})}$$

$$= \frac{0.002 * 0.98}{0.02176} = 0.09 = 9\%$$

$$P(NotSpam \mid detected\ as\ spam) = 9\%$$
$$P(Spam \mid detected\ as\ spam) = 90\%$$