* State and Prove the maximum power Transfer Theorem.

Ans: Maximum Power Transfer Theorem: This theorem may be stated as follow:—

"A load will receive maximum power from the network when its resistance is exactly equal to the thevenin resistance of the network applied to the load."

Prove the Maximum power transfer theorem:

Consider a fig ⓐ; for prove the maximum power transfer theorem.

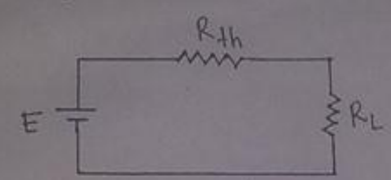


Fig-ⓐ

The load resistance will receive maximum power from the network when $R_L = R_{th}$

$\therefore$ The circuit current $I = \frac{E}{R_L + R_{th}} = \frac{E}{R_L + R_L}$ $[\because R_{th} = R_L]$

$$= \frac{E}{2R_L}$$

Now the maximum power of $P_{L\,max} = I^2 R_L$

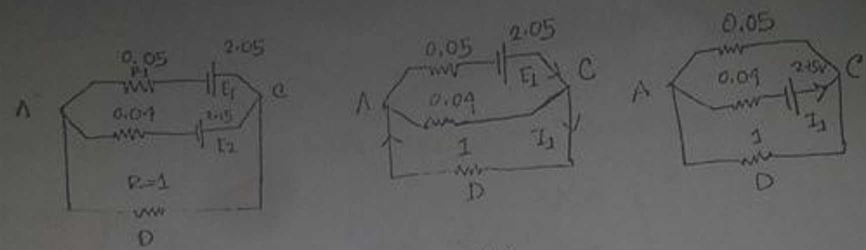
$$= \left(\frac{E}{2R_L}\right)^2 R_L$$

$$= \frac{E^2}{4R_L^2} \times R_L$$

$$= \frac{E^2}{4R_L}$$

$\therefore P_{L\,max} = \frac{E^2}{4R_L}$ or $\frac{E^2}{4R_{th}}$

3.



(b)

(a)

In Fig–b, E_2 has been removed. Resistance of 1Ω and 0.04Ω are in parallel across points A and C.

$\therefore R_{AC} = 1 \parallel 0.04 = \frac{1 \times 0.04}{1.04} = 0.035\Omega$ This resistance is in series with 0.05Ω. Hence total resistance offered to battery $R_{eq} = 0.05 + 0.038 = 0.088\Omega$.

$\therefore I = \frac{2.05}{0.088} = 23.3A$

Current through 1Ω resistance, $I_1 = 23.3 \times \frac{0.04}{1.04}$

$= \cancel{0.0896}$

$= 0.896$ A from C to A.

Again, in fig-c when E_1 is removed resistance would be, $R = 1 \parallel 0.05 = 1 \times \frac{0.05}{1.05} = 0.048\Omega$

Total resistance $= 0.04 + 0.048 = 0.088\Omega$

Current, $I = \frac{2.15}{0.088} = 24.4A$.

Again, $I_2 = 24.4 \times \frac{0.05}{1.05} = 1.16$ A

$\therefore$ Total current through $1\text{-}\Omega$ resistance,

$= I_1 + I_2 = 0.896 + 1.16 = 2.056A$ (Ans)

Resistance: Resistance is the property of substance due to which it oppose the flow of electricity through it.

The unit of resistance is ohm (Ω).

Conductance: Conductance is reciprocal of resistance.

Mathematicaly $G = \frac{1}{R}$

The unit of conductance is mho (siemens).

Capacitance: The property of a capaciton to store electricity may be called its capacitance.

Dept. of Computer Science and Engineering

Class Test 1 APEE1131 (Electrical Circuits and Electronics)-2016

Full Marks: 20 Time: 1 (One) Hour

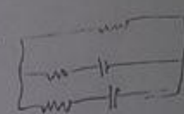
N.B. Answer all Questions. (4x5=20 Marks)

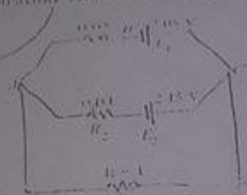
1. State and Prove Maximum Power Transform Theorem.

2. Use mesh analysis to find the current I_o in the circuit.

$4(I_1 + I_0 - I_2)$

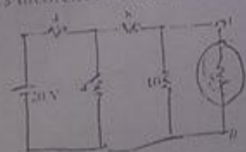
3. By using Superposition Theorem, find the current in resistance R.





2.05278

4. Apply Norton's theorem to calculate current flowing through 5 – Ω resistor.



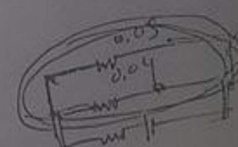


Fig-a

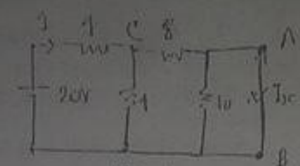


Fig-b

i) Remove 5Ω resistor and put a short across terminal A and B, as shown in fig-b

ii) As seen 10Ω resistance becomes short-circuited

iii) Let us now find I_{sc}. The battery sees a parallel combination of 4Ω and 8Ω in series with a 4Ω resistance

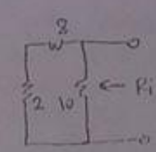
Total resistance $= 4 + 4 \| 8 = \frac{20}{3}\Omega$

Hence, $I = \frac{20}{20/3} = 3A$

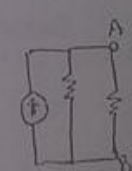
This current divides at the point C in Fig-b.

Current going along path CAB gives I_{sc}. This value $= 3 \times \frac{4}{12} = 1A$

(fig-c)



(d)



(e)

In fig-c,

Battery has been removed leaving behind its internal resistance which in this case is zero.

Resistance of the network looking into the terminals A, B:

$R_i = 10 \| 10 = 5\Omega$

iv) Hence, fig (e) gives the norton equivalent circuit.

v) Now join the 5Ω resistance back across terminal A and B.

$\therefore I_{AB} = 1 \times \frac{5}{10} = 0.5A$

*State and prove the Kirchhoff's laws?

Ans: Kirchhoff's Laws: There are two types of Kirchhoff's law. They are—

i) Kirchhoff's current law or KCL and

ii) " voltage " " KVL or Mesh law.

Kirchhoff's current law: This law may be state as follows:—

"In any electrical network the algebraic sum of the current meeting at a point or junction is zero."

Prove the Kirchhoff's current law: consider a figure ⓐ, for prove the Kirchhoff's current law.

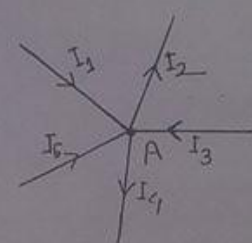


figure ⓐ

In figure ⓐ, The currents I_1, I_3 and I_5 enter the junction and I_2 and I_4 are leave.

Now, we applying Kirchhoff's current law in node A, we get

$$I_1 - I_2 + I_3 - I_4 + I_5 = 0$$

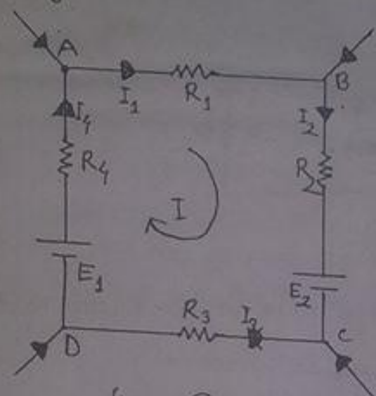
$$\Rightarrow I_1 + I_3 + I_5 = I_2 + I_4$$

P.T.O

Kirchhoff's voltage law: This law may be stated as follows:—

"The algebraic sum of the potential rises and drops around a closed path is zero."

Prove the Kirchhoffs voltage law: Consider a figure(b), for prove the Kirchhoffs voltage law.



figure(b)

In figure(b), we using voltage sign rules in the circuit ABCDA, we get

I_1R_1 is $-$ve (fall in potential)
I_2R_2 is $-$ve (" " ")
I_3R_3 is $+$ve (rise " ")
I_4R_4 is $-$ve (fall " ")
E_1 is $+$ve (rise " ")
E_2 is $-$ve (fall " ")

Now, we applying Kirchhoff's voltage law in circuit ABCDA, we get

$$-I_1R_1 - I_2R_2 + I_3R_3 - I_4R_4 + E_1 - E_2 = 0$$

$$\Rightarrow E_1 - E_2 = I_1R_1 + I_2R_2 - I_3R_3 + I_4R_4.$$

Explain the branch current method analysis.

Branch current method analysis: Consider a fig-(a) for explain the branch current method analysis.

In this circuit, there are two loop. In node A, we applying KCL, we get

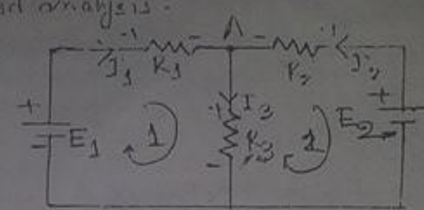


Fig-(a)

$$I_1+I_2-I_3=0 \quad \text{---(i)}$$

In loop 1, we applying KVL, we get

$$E_1-I_1R_1-I_3R_3=0 \quad \text{---(ii)}$$

Again,

In loop 2, we applying KVL, we get

$$E_2-I_2R_2-I_3R_3=0 \quad \text{---(iii)}$$

By calculation in equation (i) (ii) and (iii), we get

$$\left.\begin{aligned} I_1 &= ? \\ I_2 &= ? \\ I_3 &= ? \end{aligned}\right\} \text{Ans.}$$

* State and prove the superposition theorem?

Ans: <u>Superposition theorem</u>: This theorem may be stated as follows

"The current through or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source."

<u>Prove the superposition theorem</u>: Consider a fig ⓐ, for prove the superposition theorem.

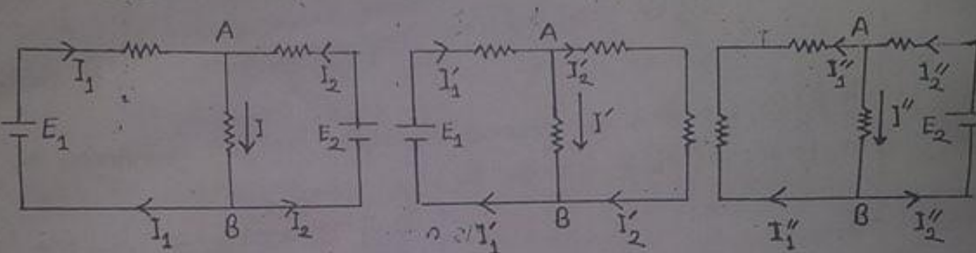


fig ⓐ fig ⓑ fig ⓒ

In fig ⓐ, I, I_1 and I_2 represents the values of currents and E_1 and E_2 represents the values of e.m.f source.

In fig ⓑ, represents condition obtained when E_1 battery source acts alone.

Similarly, In fig ⓒ, represents condition obtained when E_2 battery source acts alone.

By combining the current values of fig ⓑ and fig ⓒ actual values of fig ⓐ can be obtained.

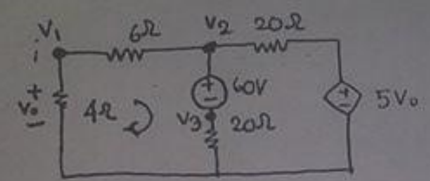
obviously $I_1 = I_1' - I_1''$ and $I_2 = I_2'' - I_2'$

$\therefore I = I' + I''$

ge – 115

3.8

Nodal Analysis



Applying KCL at node V_1

$$\frac{V_1}{4} + \frac{V_1 - V_2}{6} = 0$$

$$\Rightarrow 3V_1 + 2V_1 - 2V_2 = 0$$

$$\Rightarrow 5V_1 - 2V_2 = 0$$

Applying KCL at supermesh, we get,

$$\frac{V_1 - V_2}{6} - \frac{V_3}{20} - \frac{V_2 - 5V_0}{20} = 0$$

$$\Rightarrow \frac{V_1 - V_2}{6} - \frac{V_3}{20} - \frac{V_2 - 5V_1}{20} = 0$$

$$\Rightarrow 10V_1 - 10V_2 - 3V_3 - 3V_2 + 15V_1 = 0$$

$$\Rightarrow 25V_1 - 13V_2 - 3V_3 = 0$$

But at supernode, we can write, $V_2 - V_3 = 60$

$\therefore V_1 = 12$ | $V_0 = V_1$

$V_2 = 30$ | $V_0 = 12V$

$V_3 = -30$

* Alternative prove the Kirchhoffs voltage low ?

Ans: Kirchhoffs voltage low : This law may be stated as follows:

"The algebraic sum of the potential rises and drops arround a closed path is zero."

Prove the Kirchhoffs voltage low: Consider a fig ⑥, for prove the Kirchhoffs voltage law.

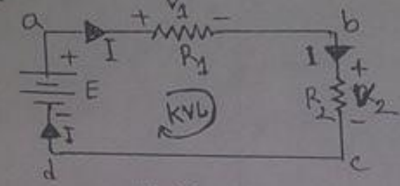


fig ⑥

In fig ⑥, we using voltage sign rules in the circuit abcda, we get

E is +ve (rises in potential)

V_1 is −ve (fall in potential)

V_2 is −ve (fall in potential)

Now, we applying Kirchhoffs voltage law in circuit abcda,

we get $E - V_1 - V_2 = 0$

$\Rightarrow E = V_1 + V_2$

The applied voltage of a serries circuit will equal the sum of the voltage drops of the circuit.

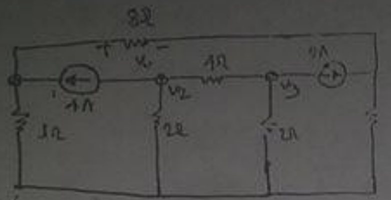
Kirchhoffs voltage law can also be written in the following form:

$$\Sigma V_{rises} = \Sigma V_{drops}$$

The sum of the voltage rises arround a closed path will always equal the sum of the voltage drops.

c-47 NODAL Alexander

3.24



Applying kcl at node v_1 we get

$$\frac{v_1}{1} + \frac{v_1 - v_4}{8} - 4 = 0$$

$$\Rightarrow 8v_1 + v_1 - v_4 - 32 = 0$$

$$\Rightarrow 9v_1 - v_4 = 32 \quad \text{—(i)}$$

Applying kcl at node v_2

$$\frac{v_2}{2} + \frac{v_2 - v_3}{4} + 4 = 0$$

$$\Rightarrow 2v_2 + v_2 - v_3 + 16 = 0$$

$$\Rightarrow 3v_2 - v_3 = -16 \quad \text{—(ii)}$$

Applying kcl at node v_3

$$\frac{v_3}{2} + 2 - \frac{v_2 - v_3}{4} = 0$$

$$\Rightarrow 2v_3 + 8 - v_2 + v_3 = 0$$

$$\Rightarrow v_2 - 3v_3 = 8 \quad \text{—(iii)}$$

Applying kcl at node we get

$$\frac{v_4}{1} + 2 - \frac{v_1 - v_4}{8} = 0$$

$$\Rightarrow 8v_4 + 16 - v_1 + v_4 = 0$$

$$\Rightarrow v_1 - 9v_4 = -16 \quad \text{—(iii)}$$

(i) × 9 − (iv)

$$81v_1 - 9v_4 = 288$$

$$\underset{(-)}{v_1} \underset{(+)}{- 9v_4} = \underset{(+)}{-16}$$

$$80v_1 = 304$$

$$v_1 = 3.8$$

$$\therefore v_1 - 9v_4 = -16$$

$$\Rightarrow 3.8 - 9v_4 = -16$$

$$\Rightarrow -9v_4 = -16 - 3.8$$

$$\Rightarrow v_4 = 2.2V$$

Again

(ii) − (iii) × 3

$$3v_2 - v_3 = -16$$

$$\underset{(-)}{3v_2} \underset{(+)}{- 9v_3} = \underset{(-)}{24}$$

$$8v_3 = -40$$

$$\Rightarrow v_3 = -5V$$

$$3v_2 - (-5) = -16$$

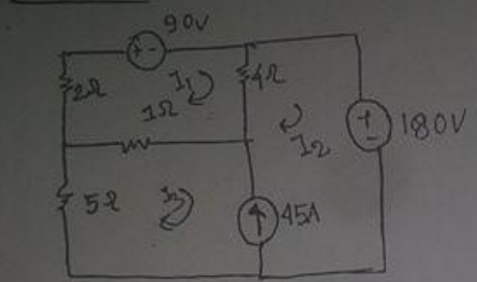
$$\Rightarrow 3v_2 + 5 = -16$$

$$\Rightarrow 3v_2 = -16 - 5 = -21$$

$$\therefore v_0 = v_1 - v_4 = 3.8 - 2.2 = 1.6V$$

$$\therefore v_0 = 1.6\,V \text{ (Ans)}$$

MESH :-



KVL at loop I_1

$90 + 4(I_1 - I_2) + 1(I_1 - I_3) + 2I_1 = 0$

$\Rightarrow 90 + 4I_1 - 4I_2 + I_1 - I_3 + 2I_1 = 0$

$\Rightarrow 7I_1 - 4I_2 - I_3 = -90$ —— (i)

Applying KVL at super node mesh we get

$180 + 5I_3 + 1(I_3 - I_1) + 1(I_2 - I_1) = 0$

$\Rightarrow -5I_1 + 4I_2 + 6I_3 = -180$

$\Rightarrow 5I_1 - 4I_2 - 6I_3 = 180$ —— (ii)

But supermesh, $I_2 - I_3 = 45$ —— (iii)

$\therefore I_1 = -20\,A$

$I_2 = -1A$

$I_3 = -46A$

$I_0 = I_3 - I_1$

$= -46 - (-20)$

$= -46 + 20$

$= -26$

$I_0 = -26\,A$ (Ans)

* State and prove the maximum power theorem?

Ans: Maximum power theorem: This theorem may be state as follows. "A load will receive maximum power from the network when its resistance is exactly equal to the thevenin resistance of the network applied to the load."

$$\text{That is } R_L = R_{th}$$

Prove the maximum power theorem: Consider a figure ⓐ for prove the maximum power theorem.

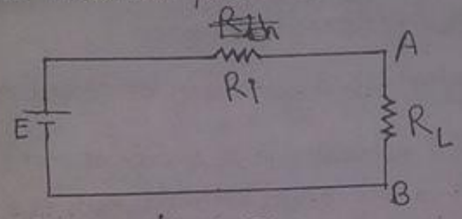


figure ⓐ

In figure ⓐ, load resistance R_L is connected across the terminal A and B of the network that consists of a emf E and internal resistance R_i. The load resistance R_L will receive maximum power from the network when $R_i = R_L$.

$\therefore$ The circuit current $I = \frac{E}{R_i + R_L}$ and

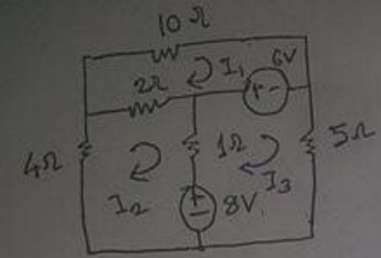
Power consumed by the load is $P_L = I^2 R_L$

$$= \left(\frac{E}{R_i + R_L}\right)^2 \times R_L$$

$$= \frac{E^2 R_L}{(R_i + R_L)^2}$$

P.T.O

MESH:-



Applying KVL at loop I_1 we get

$$2(I_1-I_2)+10I_1-6=0$$

$$\Rightarrow 2I_1-2I_2+10I_1-6=0$$

$$\Rightarrow 12I_1-2I_2=6 \quad \text{—— (i)}$$

Applying KVL at loop I_2

$$4I_2+2(I_2-I_1)+I_2-I_3+8=0$$

$$\Rightarrow 4I_2+2I_2-2I_1+I_2-I_3+8=0$$

$$\Rightarrow 2I_1-7I_2+I_3=8 \quad \text{—— (ii)}$$

Applying KVL at loop I_3, we get.

$$5I_3-8+1(I_3-I_2)+6=0$$

$$\Rightarrow 5I_3-8+I_3-I_2+6=0$$

$$\Rightarrow I_2-6I_3=-2 \quad \text{—— (iii)}$$

$I_1=0.329A$

$I_2=-1.025A$

$I_3=0.162A$

$\therefore I_0 = I_3-I_2$

$= 0.162+1.025$

$= 1.187$ A

Loop: Loop is a close path in a circuit in which no element or node is connected/encountered more than once.

Mesh: Mesh is one kind of loop that contains no other loop.

Electricity: Flow of electrons/charges through in conductor is called electricity:

Electronics: Flow of electrons/charges through in Vacum tude, gas, semi-conductor is called electronics.

Conductor: Conductor are those substance which easily flow of electronic current through them.

Semi-conductor: Semi-conductor are those substance whose electrical conductivity lies in between conductors and insulators.

~~Semi~~ Insulator: Insulator are those substance which donot flow of electronic current through them.

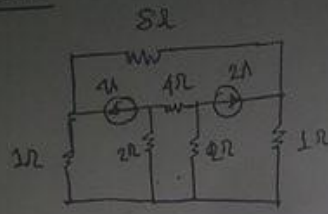
Diode: A diode is a ~~symbol~~ two terminal semi-conductor devices which ~~has the characteristics of a switch~~ that can conduct current in only one direction.

p-n junction diode: when a p-typ semi-conductor is joined to a n-type semi-conductor, the contact surface is called p-n junction diode.

Rectification: The process of converting ac voltage to dc voltage is called rectification.

Rectifier: A rectifier is an electronic devices which provides/conven ac voltage into a pulsating dc voltage.

MESH:-



Applying KVL at supermesh,

$I_1 + 8I_4 + I_3 + 2(I_3 - I_2) + 4(I_4 - I_2)$
$+ 2(I_1 - I_2) = 0$

$\Rightarrow I_1 + 8I_4 + I_3 + 2I_3 - 2I_2 + 4I_4 - 4I_2$
$+ 2I_1 - 2I_2 = 0$

$\Rightarrow 3I_1 - 8I_2 + 3I_3 + 12I_4 = 0$ —(i)

Applying KVL at loop I_2 we get,

$2(I_2 - I_1) + 4(I_2 - I_4) + 2(I_2 - I_3) = 0$

$\Rightarrow 2I_2 - 2I_1 + 4I_2 - 4I_4 + 2I_2 - 2I_3 = 0$

$\Rightarrow 2I_1 - 8I_2 + 2I_3 + 4I_4 = 0$

But in super mesh we can write

$I_4 - I_1 = 4$

$\therefore I_4 = 4 + I_1$ —(iii)

Again

$I_3 = I_4 + 2$ —(iv)

Again, ~~I_3~~ (i) − (ii)

$3I_1 - \cancel{8I_2} + 3I_3 + 12I_4 = 0$

$2I_1 - \cancel{8I_2} + 2I_3 + 4I_4 = 0$

$(-) \qquad (-) \qquad (-)$

$I_1 + I_3 + 8I_4 = 0$

$\Rightarrow I_1 + I_4 + 2 + 8(4 + I_1) = 0 \quad [\because I_3 = (I_4 + 2)]$

$\Rightarrow I_1 + 4 + I_1 + 2 + 32 + 8I_1 = 0 \quad [\because I_4 = 4 + I_1]$

$\Rightarrow 10I_1 = -38$

$\Rightarrow I_1 = 3.8A$

$\therefore I_4 = 4 + I_1$

$\Rightarrow I_4 = 4 + (-3.8)$

$= 4 - 3.8$

$I_4 = 0.2$

$I_1 + I_3 + 8I_4 = 0$

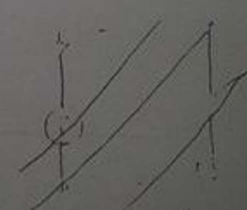
$-3.8 + I_3 + 8(0.2) = 0$

$I_3 = -1.6 + 3.8 = 2.2A$

$V_0 = 8I_4$

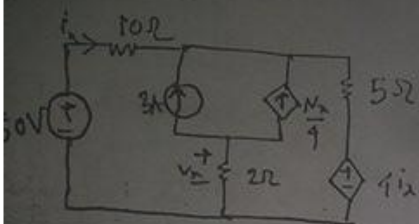
$= 8 \times 0.2$

$= 1.6A$



-129

5.69 NODAL



$V_x = V_2$, $i_x = \frac{50-V_1}{10}$

Applying kcl at V_1 we get

$\frac{50-V_1}{10} + 3 + \frac{V_x}{4} - \frac{V_1 - 4i_x}{5} = 0$

$\Rightarrow 100 - 2V_1 + 60 + 5V_x - 4V_1 + 16 i_x = 0$

$\Rightarrow 100 - 2V_1 + 60 + 5V_2 - 4V_1 + 16 \times \frac{50-V_1}{10} = 0$

$\Rightarrow 500 - 10V_1 + 300 + 25V_2 - 20V_1 + 400 - 8V_1 = 0$

$\Rightarrow 38V_1 - 25V_2 = 1200$ — (1)

Applying kcl at node V_2 we get

$3 + \frac{V_x}{4} + \frac{V_2}{2} = 0$

$\Rightarrow 12 + V_2 + 2V_2 = 0$

$\Rightarrow 3V_2 = -12V$

$\Rightarrow V_2 = -4V$

Putting the value of V_2 to the equation; we get,

$38V_1 - 25 \times (-4) = 1200$

$\Rightarrow 38V_1 + 100 = 1200$

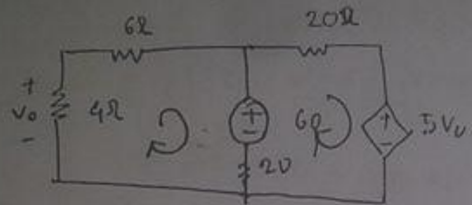
$\Rightarrow 38V_1 = 1100$

$\Rightarrow V_1 = 28.947V$

$\therefore i_x = \frac{50-V_1}{10} = \frac{50-28.947}{10} = 2.105A$ (Ans)

$V_x = V_2 = -4V$ (Ans)

Mesh :-



Applying KVL at Loop-1,

$4I_1+6I_1+60+20(I_1-I_2)=0$

$\Rightarrow 30I_1 - 20I_2 = -60$ —(i)

Applying KVL at loop I_2 we get,

$20I_2+5V_0+20(I_2-I_1)-60=0$

$\Rightarrow 20I_2+5(-4I_0)+20I_2-20I_1-60=0$

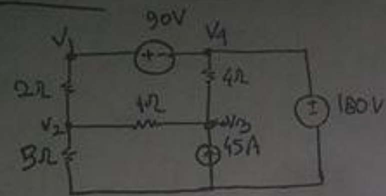
$\Rightarrow -40I_1+40I_2=60$

$\therefore I_1=3A \qquad I_2=-1.5A$

$\therefore V_0 = -4I_1 = -4(-3) = +12V$ (Ans)

.-120

3.44 → NODAL



Applying KCL at super Node:

$$\frac{V_1-V_2}{2} + \frac{V_4-V_3}{4} = 0$$

$$\Rightarrow \frac{V_1-V_2}{2} + \frac{180-V_3}{4} = 0$$

$$\Rightarrow 2V_1 - 2V_2 + 180 - V_3 = 0$$

$$\Rightarrow 2V_1 - 2V_2 - V_3 = -180 \quad \text{—— (i)}$$

Applying KCL at node V_2 we get,

$$\frac{V_1-V_2}{2} - \frac{V_2-V_3}{1} - \frac{V_2}{5} = 0$$

$$\Rightarrow 5V_1 - 5V_2 - 10V_2 + 10V_3 - 20V_2 = 0$$

$$\Rightarrow 5V_1 - 17V_2 + 10V_3 = 0$$

Applying KCL at V_3 we get,

$$\frac{V_4-V_3}{4} + \frac{V_2-V_3}{1} + 45 = 0$$

$$\Rightarrow \frac{180-V_3}{4} + \frac{V_2-V_3}{1} + 45 = 0$$

$$\Rightarrow 180 - V_3 + 4V_2 - 4V_3 + 180 = 0$$

$$\Rightarrow 4V_2 - 5V_3 = -360 \quad \text{—— (iii)}$$

But super node, we have to write

$$V_1 - V_4 = 90$$

$$\Rightarrow -180 + V_1 = 90$$

$$\Rightarrow V_1 = 90 + 180 = 270V \quad \text{—— (iv)}$$

Putting the value of V_1 to the equation.

$$2V_1 - 2V_2 - V_3 = -180$$

$$\Rightarrow 2 \times 270 - 2V_2 - V_3 = -180$$

$$\Rightarrow 2V_2 + V_3 = 180 + 540 = 720 \quad \text{— (v)}$$

$\therefore V_2 = 231.428V$

$\therefore V_3 = 257.142V$

$$\therefore i_o = \frac{V_2 - V_3}{1} = \frac{231.428 - 257.142}{1} = -25.71\,A \text{ (Ans)}$$

* Explain the voltage divider rule.

Ans: Voltage divider rule: Consider a fig (a) for explain the voltage divider rule.

In fig (a), there are three resistances in series connected across a voltage V.

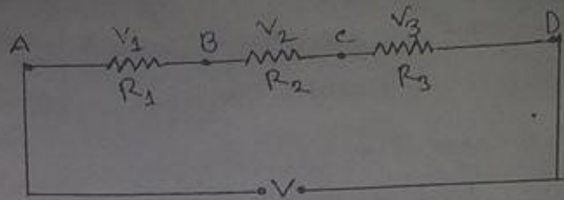


fig (a)

Total resistance $R_T = R_1 + R_2 + R_3$

According to voltage divider rule, various voltage drops are

$$V_1 = \frac{VR_1}{R_T} = \frac{VR_1}{R_1+R_2+R_3}$$

Similarly $V_2 = \frac{VR_2}{R_T} = \frac{VR_2}{R_1+R_2+R_3}$

and $V_3 = \frac{VR_3}{R_T} = \frac{VR_3}{R_1+R_2+R_3}$

* Explain the mesh analysis?

Ans: Mesh analysis: consider a fig (a) for explain the mesh analysis.

In fig (a), this circuit has three meshes which contains resistance and independent voltage sources.

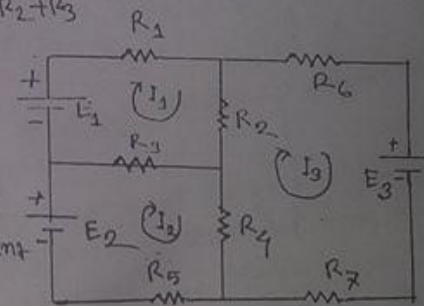


fig (a)

Applying KVL to mesh (i), we get,

$E_1 - I_1R_1 - (I_1 - I_3)R_2 - (I_1 - I_2)R_3 = 0$ ------ (i)

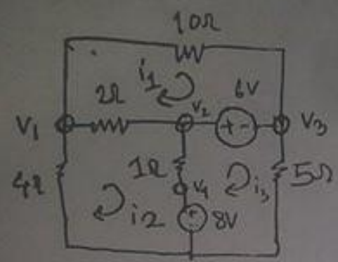
Similarly, from mesh (ii), we get,

$E_2 - (I_2 - I_1)R_3 - (I_2 - I_3)R_4 - I_2R_5 = 0$ ------ (ii)

and, from mesh (iii), we get,

$E_3 - I_3R_6 - (I_3 - I_1)R_2 - (I_3 - I_2)R_4 - I_3R_7 = 0$ ------ (iii) ge-120

3.41



Applying kcl at node V_1

$$\frac{V_1}{4} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{10} = 0$$

$$\Rightarrow 5V_1 + 10V_1 - 10V_2 + 2V_1 - 2V_3 = 0 \quad \text{— (i)}$$

Applying kcl at super node we get.

$$\frac{V_1 - V_3}{10} + \frac{V_1 - V_2}{2} - \frac{V_2 - V_4}{1} - \frac{V_5}{5} = 0$$

$$\Rightarrow \frac{V_1 - V_3}{10} + \frac{V_1 - V_2}{2} - \frac{V_2 - 8}{1} - \frac{V_3}{5} = 0$$

$$\Rightarrow V_1 - V_3 + 5V_1 - 5V_2 - 10V_2 + 80 - 2V_3 = 0$$

$$\Rightarrow 6V_1 - 15V_2 - 3V_3 = -80 \quad \text{— (ii)}$$

But super node we have to write

$$V_2 - V_3 = 6 \quad \text{— (iii)}$$

$V_1 = 4.102V$

$V_2 = 6.811V$

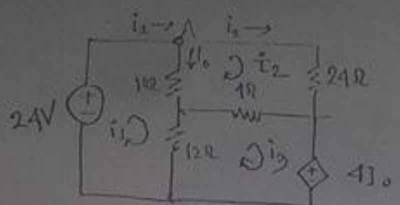
$V_3 = 0.811V$

$$\therefore I_o = \frac{8 - V_2}{1} = \frac{8 - 6.811}{1} = 1.2$$

$$\therefore I_o = 1.2A$$

Class Test

2. Use mesh analysis to find current I_o in the circuit.



We apply KVL to the three meshes in turn.

For mesh 1 →

$-24 + 10(i_1 - i_2) + 12(i_1 - i_3) = 0$

$\Rightarrow 11i_1 - 5i_2 - 6i_3 = 12$ —— (i)

For mesh -2,

$24i_2 + 4(i_2 - i_3) + 10(i_2 - i_1) = 0$

$\Rightarrow 5i_1 - 19i_2 - 2i_3 = 0$ —— (ii)

For mesh -3,

$4I_o + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$

But, at node A, $I_o = i_1 - i_2$

So that

$4(i_1 - i_2) + 12(i_3 - i_1) + 4(i_3 - i_2) = 0$

$\Rightarrow -i_1 - i_2 + 2i_3 = 0$ —— (iii)

∴ Calculating these equations, we get,

$i_1 = \frac{15}{8}$, $i_2 = \frac{3}{8}$, $i_3 = \frac{9}{8}$

Thus, $I_o = i_1 - i_2 = \frac{15}{8} - \left(\frac{3}{8}\right) = 1.5A$ (Ans)

* State and prove the norton's theorem ?

Ans: Norton's theorem: This theorem may be stated as follows :—

" Any two terminal bilateral dc network can be replaced by an equivalent circuit that consisting of a current source and a parallel resistance."

Prove the norton's theorem: Consider a figure ⓐ, for prove the norton's theorem.

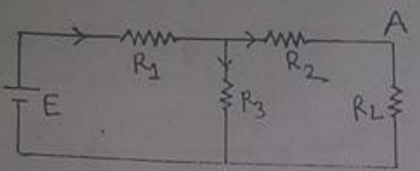


fig ⓐ

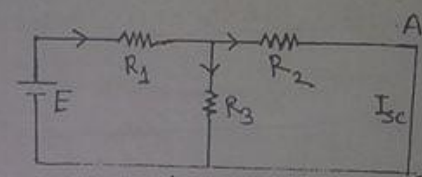


fig ⓑ

Firstly, we will remove the load resistance R_L from the terminal A and B by the short circuit. As a result, the circuit becomes as shown in fig ⓑ.

Now, we will calculate the short circuit current. It is also called norton's current.

The total resistance in this circuit $R_c = R_1 + (R_2 \| R_3) = R_1 + \frac{R_2 R_3}{R_2 + R_3}$

$$= \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2 + R_3}$$

The total current in this circuit $I_c = \frac{E}{R_c} = \frac{E \times (R_2 + R_3)}{R_1R_2 + R_2R_3 + R_3R_1}$

$\therefore$ Short circuit current $I_{sc} = \frac{I_c R_3}{R_2 + R_3} = \frac{\frac{E \times \cancel{(R_2+R_3)}}{R_1R_2 + R_2R_3 + R_3R_1} \times R_3}{\cancel{(R_2+R_3)}}$

$$= \frac{E R_3}{R_1 R_2 + R_2 R_3 + R_3 R_1}$$

To find the equivalent resistance, remove the voltage source by the short circuit. As a result, the circuit becomes as shown in fig ⓒ.

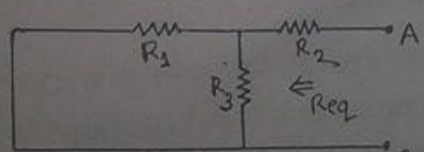


fig ⓒ

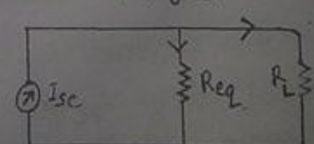


fig ⓓ

P.T.O

For P_L to be maximum $\frac{dP_L}{dR_L} = 0$

$$\therefore \frac{d}{dR_L}\left\{\frac{E^2 R_L}{(R_i+R_L)^2}\right\} = 0$$

$$\Rightarrow \frac{d}{dR_L}\left[E^2\left\{R_L(R_i+R_L)^{-2}\right\}\right] = 0$$

$$\Rightarrow E^2 \cancel{\frac{d}{dR_L}}\left\{R_L(-2)(R_i+R_L)^{-3} + (R_i+R_L)^{-2}\cdot 1\right\} = 0$$

$$\Rightarrow E^2\left\{\frac{1}{(R_i+R_L)^2} - \frac{2R_L}{(R_i+R_L)^3}\right\} = 0$$

$$\Rightarrow (R_i+R_L)^3 - 2R_L(R_i+R_L)^2 = 0$$

$$\Rightarrow (R_i+R_L)^3 = 2R_L(R_i+R_L)^2$$

$$\Rightarrow R_i+R_L = 2R_L$$

$$\Rightarrow R_i = 2R_L - R_L$$

$$\Rightarrow R_i = R_L$$

$$\therefore \text{Maximum power of } P_{L\,max} = \frac{E^2 R_L}{(R_i+R_L)^2}$$

$$= \frac{E^2 R_L}{(R_L+R_L)^2}$$

$$= \frac{E^2 R_L}{(2R_L)^2}$$

$$= \frac{E^2 R_L}{4R_L^2}$$

$$= \frac{E^2}{4R_L}$$

$$= \frac{E^2}{4R_i} \quad \left[\because R_L = R_i\right]$$

* State and prove the thevenins theorem?

Ans: Thevenin's theorem: This theorem may be stated as follows :—

"Any two terminal bilateral dc network can be replaced by an equivalent circuit that consisting of a voltage source and a series resistance."

Prove the thevenins theorem: Consider a fig(a) for prove the thevenin theorem.

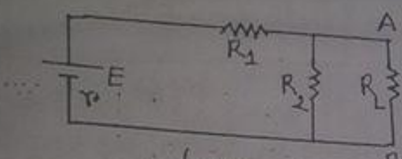


fig (a)

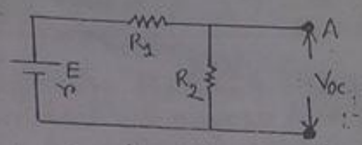


fig (b)

Firstly, we will remove the load resistance R_L from the terminal A and B by the open circuit. As a result, the circuit becomes as shown in fig(b)

Now, we will calculate the open circuit voltage V_{oc}. It is also called thevenins voltage. $\therefore V_{oc} = \frac{E \times R_2}{R_1 + R_2 + r}$ [By voltage divider rule]

To find the equivalent resistance, remove the voltage source by the internal resistance r. As a result, the circuit becomes as shown in fig(c).

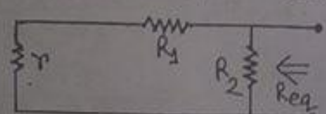


fig (c)

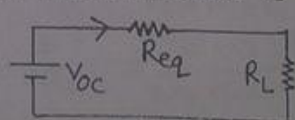


fig (d)

$\therefore$ The equivalent resistance in this circuit $R_{eq} = R_2 \| (R_1 + r) = \frac{R_2 \times (R_1 + r)}{(R_1 + R_2 + r)}$

The final thevenin's equivalent circuit as shown in fig(d). where open circuit voltage V_{oc} and the equivalent resistance R_{eq} are connected in series with the load resistance R_L.

$\therefore$ The current flow in load resistance $I = V/R = \frac{V_{oc}}{R_L + R_{eq}}$

1. In fig 9.85, a load resistance of R_L is connected across the terminals A and B of a network, which consist of a generator of e.m.f E and internal resistance

R_o and a series resistance R_o and series resistance R.

let, $R_i = R_g + R$
= internal resistance of the network as viewed from A and B.

According to this theorem R_L will absorb maximum power from the network $R_L = R_i$

Proof:- Circuit current $I = \frac{E}{R_L + R_i}$

Power consumed by the load is $P_L = I^2 R_L = \frac{E^2 R_L}{(R_L + R_i)^2}$

For P_L to be maximum $\frac{dP_L}{dR_L} = 0$

Differentiating eq (i) above, we have

$$\frac{dP_L}{dR_L} = E^2\left[\frac{1}{(R_L + R_i)^2} + R_L\left(\frac{-2}{(R_L + R_i)^3}\right)\right]$$

$$= E^2\left[\frac{1}{(R_L + R_i)^2} - \frac{2R_L}{(R_L + R_i)^3}\right]$$

$$0 = E^2\left[\frac{1}{(R_L + R_i)^2} - \frac{2R_L}{(R_L + R_i)^3}\right]$$

$2R_L = R_L + R_i$

$\Rightarrow R_L = R_i$

Max. Power $P = \frac{E^2 R_L}{4R_L^2} = \frac{E^2}{4R_L}$

$\therefore$ The equivalent resistance in this circuit $R_{eq} = R_2 + (R_1 \parallel R_3)$

$$= R_2 + \frac{R_1 R_3}{R_1 + R_3}$$

$$= \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1 + R_3}$$

The final norton's equivalent circuit as shown in fig (d). Where short circuit current source I_{sc} and the equivalent resistance R_{eq} are connected in parallel across the load resistance R_L.

$\therefore$ The current flow in load resistance $I_L = \frac{I_{sc} \times R_{eq}}{R_L + R_{eq}}$.

* State and prove the Reciprocity theorem?

Ans: Reciprocity theorem: This theorem may be stated as follows:—

"In any linear bilateral network, if a source of emf E in any branc produces a current I in any other branch, then the same emf E acting in the second branch would produce the same current I in the first branch."

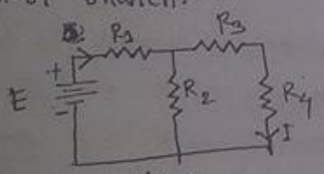


Fig-a

In Fig-a, the total resistance is

$$R_a = R_1 + R_2 \parallel (R_3 + R_4) = R_1 + \frac{R_2 R_3 + R_2 R_4}{R_2 + R_3 + R_4}$$

$$= \frac{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_3 + R_2 R_4}{R_2 + R_3 + R_4}$$

$$\therefore I = \frac{E}{R_a} = \frac{E(R_2 + R_3 + R_4)}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_3 + R_2 R_4} \quad \text{—(i)}$$

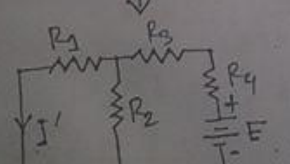


Fig-b

In fig-b, the total resistance is

$$R_b = (R_3 + R_4) \parallel R_2 + R_1$$

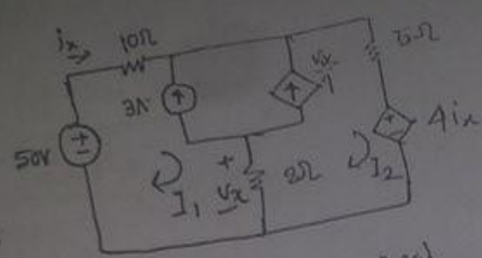
$$= \frac{R_2 R_3 + R_2 R_4}{R_2 + R_3 + R_4} + R_1 = \frac{R_2 R_3 + R_2 R_4 + R_1 R_2 + R_1 R_3 + R_1 R_4}{R_2 + R_3 + R_4}$$

$$\therefore I' = \frac{E}{R_b} = \frac{E(R_2 + R_3 + R_4)}{R_1 R_2 + R_1 R_3 + R_1 R_4 + R_2 R_3 + R_2 R_4} \quad \text{—(ii)}$$

From the equation (i) and (ii), we get $I = I'$.

Hence the theorem is proved.

MESH:-



Applying KVL at super Mesh

$-50+10I_1+5I_2+4ix=0$

$-50+10I_1+5I_2+4I_1=0$

$\Rightarrow 14I_1+5I_2=50$ —(1)

But super mesh

$I_2 = 3+\frac{V_x}{4}+I_1$

$\Rightarrow I_2 = 3+\frac{2(I_1-I_2)}{4}+I_1$

$\Rightarrow I_2 = 3+\frac{I_1-I_2}{2}+I_1$

$\Rightarrow 2I_2 = 6+I_1-I_2+2I_1$

$\Rightarrow 3I_1-3I_2=6$

$\therefore I_1 = 2.105A$

$\therefore I_2 = 4.105A$

$ix = 2(I_1-I_2)$

$= 2(2.105-4.05)$

$= 2(-2)$

$= -4$

$\therefore V_{ix} = -4V$ (Ans)

$i_x = I_1$

$\Rightarrow i_x = 2.105A$ (Ans)

*Explain the T-π or π-T conversions or Transformations.

Ans: <u>T-π or π-T conversions:</u> Consider a fig ⓐ. for explain the T-π or π-T conversions.

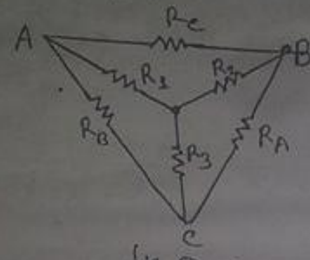
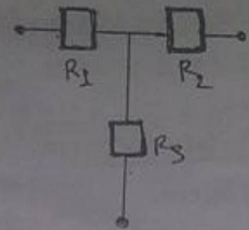
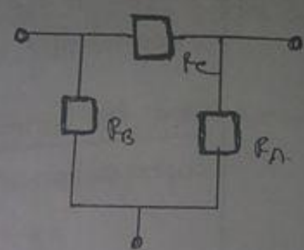


fig ⓐ



T configuration



π configuration

In fig ⓐ, we find the general equations for the resistances of the T in terms of those for the π :

$$R_1 = \frac{R_B R_c}{R_A + R_B + R_3} \quad \text{—— (i)}$$

$$R_2 = \frac{R_c R_A}{R_A + R_B + R_c} \quad \text{—— (ii)}$$

$$\text{and } R_3 = \frac{R_A R_B}{R_A + R_B + R_c} \quad \text{—— (iii)}$$

For the resistance of the π in terms of those for the T:

$$R_A = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \quad \text{—— (iv)}$$

$$R_B = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \quad \text{—— (v)}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \quad \text{—— (vi)}$$

If $R_A = R_B = R_c$, the equation (iii), would become the following

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_c} = \frac{R_A R_A}{R_A + R_A + R_A} = \frac{R_A^2}{3R_A} = \frac{R_A}{3}$$

$\therefore R_3 = R_A/3$

Similarly $R_1 = R_A/3$ and ~~$R_A/3$~~ $R_2 = \frac{R_A}{3}$

In general, therefore $R_T = \frac{R_\pi}{3}$ or $R_\pi = 3R_T$.

(d) 'Explain the nodal analysis with current source and voltage source.

Explain nodal analysis with current source:

consider a fig-ⓐ, for explain the nodal analysis with current source. In this circuit, there are three node and node c is the reference node.

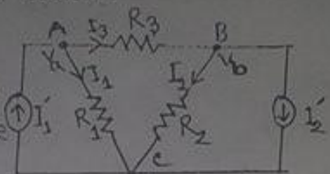


Fig-ⓐ

Now, we applying KCL in node A, we get

$$I_1' = I_1 + I_3 = \frac{V_a}{R_1} + \frac{V_a - V_b}{R_3} = \frac{V_a}{R_1} + \frac{V_a}{R_3} - \frac{V_b}{R_3} = V_a\left(\frac{1}{R_1} + \frac{1}{R_3}\right) - \frac{V_b}{R_3}$$

In node B, we get.

$$I_3 = I_2' + I_2 \Rightarrow I_2' = I_3 - I_2 = \frac{V_a - V_b}{R_3} - \frac{V_b}{R_2} = \frac{V_a}{R_3} - \frac{V_b}{R_3} - \frac{V_b}{R_2}$$

$$= \frac{V_a}{R_3} - V_b\left(\frac{1}{R_2} + \frac{1}{R_3}\right)$$

Explain nodal analysis with voltage source:

Consider a fig-ⓑ, for explain the nodal analysis with voltage source.
In this circuit, there are three node and node c is the reference node.

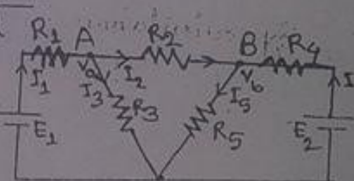


Fig-ⓑ

Now, we applying KCL in node A, we get,

$$I_1 = I_2 + I_3 \Rightarrow \frac{E_1 - V_a}{R_1} = \frac{V_a - V_b}{R_2} + \frac{V_a}{R_3} \Rightarrow \frac{E_1}{R_1} - \frac{V_a}{R_1} - \frac{V_a}{R_2} + \frac{V_b}{R_2} - \frac{V_a}{R_3} = 0$$

$$\Rightarrow \frac{E_1}{R_1} + \frac{V_b}{R_2} - V_a\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) = 0$$

In node B, we get.

$$I_5 = I_2 + I_4 \Rightarrow \frac{V_b}{R_5} = \frac{V_a - V_b}{R_2} + \frac{E_2 - V_b}{R_4} \Rightarrow \frac{V_a}{R_2} - \frac{V_b}{R_2} + \frac{E_2}{R_4} - \frac{V_b}{R_4} - \frac{V_b}{R_5} = 0$$

$$\Rightarrow \frac{V_a}{R_2}$$

$$\Rightarrow \frac{E_2}{R_4} + \frac{V_a}{R_2} - V_b\left(\frac{1}{R_2} + \frac{1}{R_4} + \frac{1}{R_5}\right) = 0.$$

* Explain the current divider rule.

Ans: Current divider rule: consider a fig ⓐ. for explain the current divider rule.

In fig ⓐ. There are two registors in parrallel connected across a voltage V.

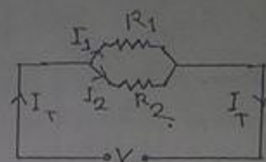


fig ⓐ

Total current $I_T = I_1 + I_2$

By ohm's law $V = I_1R_1 = I_2R_2 = \cdots\cdots = I_xR_x$

Using ohm's law $I_T = \frac{V}{R_T}$

$\Rightarrow I_T = \frac{I_xR_x}{R_T}$

Here $R_T = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$

$= \frac{R_1R_2}{R_1+R_2}$

When $x = 1$ then $I_T = \frac{I_1R_1}{R_T}$

$\Rightarrow I_1 = \frac{I_TR_T}{R_1} = \frac{I_T \times \frac{R_1R_2}{R_1+R_2}}{\cancel{R_1}}$

$\Rightarrow I_1 = \frac{I_TR_2}{R_1+R_2}$

Similarly $I_2 = \frac{I_TR_1}{R_1+R_2}$

Again, In fig ⓑ. There are three registors in parrallel connected accross a voltage V.

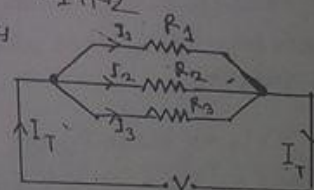


fig ⓑ

Total current $I_T = I_1 + I_2 + I_3$

Total resistance $R_T = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1}$

$= \frac{R_1R_2R_3}{R_1R_2 + R_2R_3 + R_3R_1}$

By ohm's law $V = I_1R_1 = I_2R_2 = I_3R_3 = \cdots\cdots = I_xR_x$

using ohm's law $I_T = \frac{V}{R_T} \Rightarrow I_T = \frac{I_xR_x}{R_T}$

when $x = 1$ then $I_T = \frac{I_1R_1}{R_T} \Rightarrow I_1 = \frac{I_TR_T}{R_1} = \frac{I_T \times \frac{\cancel{R_1}R_2R_3}{R_1R_2+R_2R_3+R_3R_1}}{\cancel{R_1}}$

$\therefore I_1 = \frac{I_TR_2R_3}{R_1R_2+R_2R_3+R_3R_1}$ Similarly $I_2 = \frac{I_TR_3R_1}{R_1R_2+R_2R_3+R_3R_1}$ and $I_3 = \frac{I_TR_1R_2}{R_1R_2+R_2R_3+R_3R_1}$