Sujan Math. 01728050803

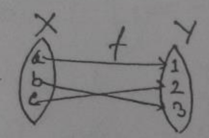
Define: Function, Domain, Range, one-one function onto-function, Equal function, Constant function, Identity function, Inverse function, Product function. Give an example of each.

Ans:-
07,08,07,04, 06 V.V.I.P

Function: let X and Y be two non-empty set. Then a function $f: X \rightarrow Y$ is a corespondence by which each element of X coresponds to a unique elements of Y.

$f: X \rightarrow Y$ read as f is a function X to Y or f is a mapping from X to Y. where X is called domain and Y is called co-domain. It is denoted by $Y = f(x)$. where x is independent and y is dependent variable.

Example:



Here $f: X \rightarrow Y$ is a function.

Domain: let X and Y be two sets and f be a function from X to Y. Then the set of X is called domain of the function. Domain is denoted by Df.

Range: let $f(x) = y$ be a function. Here y is the value of $f(x)$. So the set of all the value of y, of the function is called range of the function.

05,06 V.I.P

One-one function: let $f: A \rightarrow B$ is a function. The function is called one-one function if different elements in domain A have different image point in B. Then f is one-one

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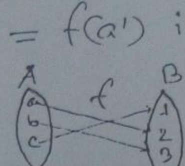
function if $f(a) = f(a')$ implise $a = a'$

Example:

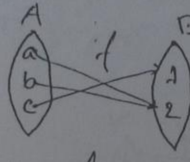


Here f is a one-one function from A to B.

09.08.07 xxx V.I.P

Onto-Function: Let $f: A \to B$ is a function. The function f is called onto function if every element in B is the image point of at least one element in A.

Example:



Here f is a onto function.

xx **Equal Function:** Let f and g be two function define as the same domain D. Then f and g said to be equal function if $f(a) = g(a)$ for every $a \in D$. That is $f = g$

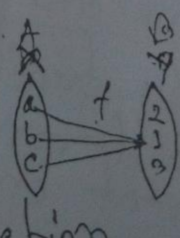
Example: Let $f(x) = y$ and $g(x) = y$ be two function, then they are equal function.

.08.07 x V.I.P

Constant function: Let $f: A \to B$ is a function. The function is called a constant function if the range of f consists of only one element.

Example: If $f(x) = y$. then

$f(1) = y$, $f(2) = y$ .... etc.



$f: A \to B$ is a constant function.

V.I.P **Identity function:** Let $f: A \to A$ be a function. The function is called identity function if every element in A in the image point of itself. It is denoted by I or I_A.

Example:

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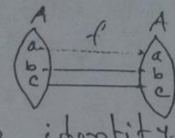
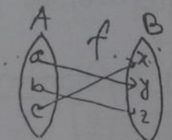
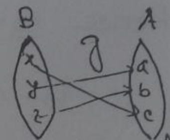


Here f is a identity function.

07.08.07. *** V.V.I.P **Inverse function:** Let $f: A \to B$ is a one-one and onto function. Then the function $g: B \to A$ is called inverse function of f. If $f \circ g = I_B$ and $g \circ f = I_A$ then $f^{-1} = g$

Example:

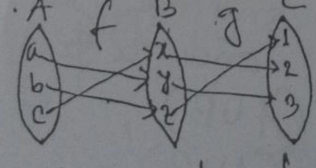




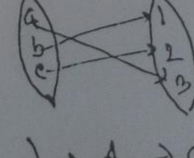
Here f and g are inverse function to each other.

Special ** **Product function:** Let $f: A \to B$ and $g: B \to C$ be two function. If they consists a function $h: A \to C$. such that $g(f(a)) = h(a)$ for all element $a \in X$. Thus the function is called product function of f and g. It is denoted by $g \circ f = h$

Example:



Then



Then the product function $h: A \to C$

——x——

Tutorial Prove that (i) $B - A' = B \cap A$ (ii) $(A \cup B)$

Sol^n: Proof: (i) let $x \in B - A'$

$\Rightarrow x \in B$ and $x \notin A'$

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in (B \cap A)$

Hence $B - A' \subseteq B \cap A$

Conversely, let $x \in B \cap A$

$\Rightarrow x \in B$ and $x \in A$

$\Rightarrow x \in B$ and $x \notin A'$

$\Rightarrow x \in (B - A')$

Hence $B \cap A \subseteq B - A'$

$\therefore B - A' = B \cap A$ proved.

(ii) Proof: let $x \in (A \cup B)'$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \in (A' \cap B')$

Hence $(A \cup B)' \subseteq A' \cap B'$

Conversely, let $x \in A' \cap B'$

$\Rightarrow x \in A'$ and $x \in B'$

$\Rightarrow x \notin A$ and $x \notin B$

$\Rightarrow x \notin (A \cup B)$

$\Rightarrow x \in (A \cup B)'$

Hence $A' \cap B' \subseteq (A \cup B)'$

$\therefore (A \cup B)' = A' \cap B'$ proved.

When a function is said to be one-one and onto. একটি ফাংশন কখন, এক-এক ফাংশন এবং সর্বগ্রাহী ফাংশন হবে।

Ans:- <u>When one-one function</u>: Let the function $f: R \to R$ be defined by the formula

$$f(x) = x^2$$

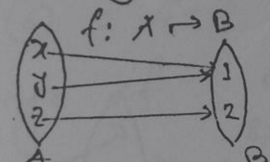
This function is not one-one function.

Since $f(-2) = 4$, $f(2) = 4$

But $f(x) = x^3$ is a one-one function.

<u>When onto-function</u>: Let f be a function of A into B.

By the following diagram.



$f(x) = 1, \; f(y) = 1, \; f(z) = 2$

Thus, $f(A) = \{1, 2\}$, $B = \{1, 2\}$

Since $f(A) \subseteq B$ and $f(A) = B$.

So the function ~~is~~ f is onto-function.

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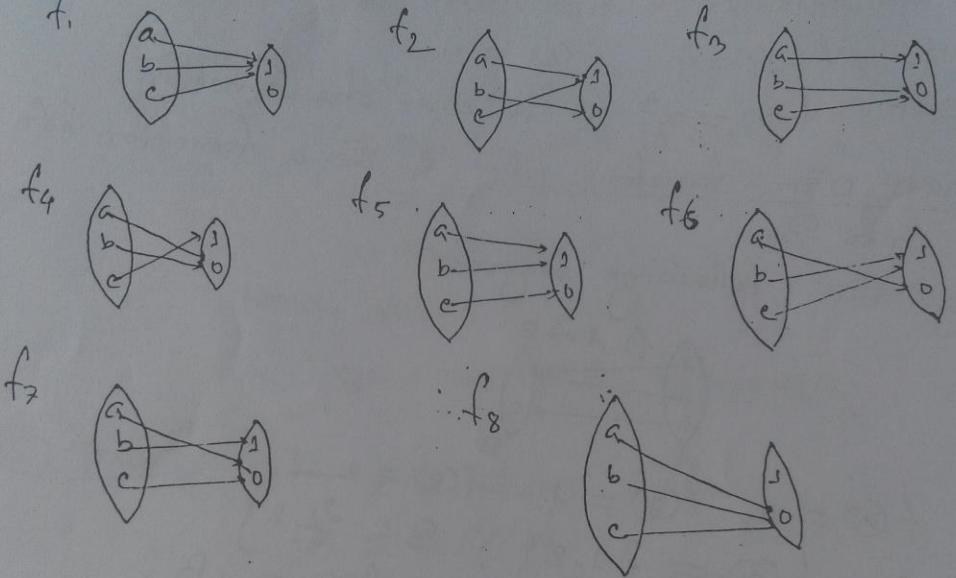
Tutorial 09.07 V.V.S.P

Let $A = \{a, b, c\}$ and $B = \{1, 0\}$. How many different function are there from A into B?

Ans:- we list all the function of A into B by diagram. In each function we assign either 1 or 0, but not both, to each element in A.



Tutorial V.V.S.P

Let the function $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 2x+1$, $g(x) = x^2 - 2$. Find formulas which define the product functions gof and fog.

Sol^n: we first compute $gof: \mathbb{R} \to \mathbb{R}$ function we use the definition of the product function as follows.

$$(gof)(x) \equiv g(f(x)) = g(2x+1) = (2x+1)^2 - 2$$

$$= 4x^2 + 4x + 1 - 2$$

$$= 4x^2 + 4x - 1 \quad \text{Ans:-}$$

Now we compute $f \circ g : \mathbb{R} \to \mathbb{R}$

$$
\begin{aligned}
(f \circ g)(x) \equiv f(g(x)) &= f(x^2 - 2) \\
&= 2(x^2-2)+1 \\
&= 2x^2 - 4 + 1 \\
&= 2x^2 - 3 \quad \text{Ans:-}
\end{aligned}
$$

Tutorial V.V.I.P

Let $A = \{1, 2, 3, 4\}$ and $f : A \to \mathbb{R}$ be defined by $f(x) = x+3$. Find the graph of f.

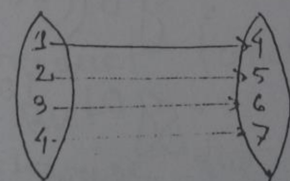
Soln: Given that, $A = \{1, 2, 3, 4\}$, $f(x) = x+3$

Now,

$$
\begin{aligned}
f(1) &= 1+3 = 4 \\
f(2) &= 2+3 = 5 \\
f(3) &= 3+3 = 6 \\
f(4) &= 4+3 = 7
\end{aligned}
$$

The range of $f = \{4, 5, 6, 7\}$

The graph of f



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গণিত বিভাগ
রাজশাহী বিশ্ববিদ্যালয়
Mob:- 01728050803

04.06.10

Let $f: A \to B$ and $g: B \to C$ have inverse func. $f^{-1}: B \to A$ and $g^{-1}: C \to B$. Then the composition $g \circ f: A \to C$ has an inverse function $f^{-1} \circ g^{-1}: C \to A$.

<u>Proof:</u> we must show that

$$(f^{-1} \circ g^{-1}) \circ (g \circ f) = 1 \text{ and } (g \circ f) \circ (f^{-1} \circ g^{-1}) = 1$$

Now $(f^{-1} \circ g^{-1}) \circ (g \circ f) = f^{-1} \circ (g^{-1} \circ (g \circ f))$

$$= f^{-1} \circ ((g^{-1} \circ g) \circ f)$$

$$= f^{-1} \circ (1 \circ f)$$

$$= f^{-1} \circ f = 1$$

we use the property that $g^{-1} \circ g$ is the identity function and that the product is 1.

Similarly,

$$(g \circ f) \circ (f^{-1} \circ g^{-1}) = g \circ (f \circ (f^{-1} \circ g^{-1}))$$

$$= g \circ ((f \circ f^{-1}) \circ g^{-1})$$

$$= g \circ (1 \circ g^{-1})$$

$$= g \circ g^{-1} = 1$$

<u>Hence proved.</u>

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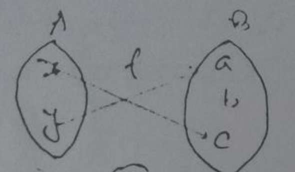
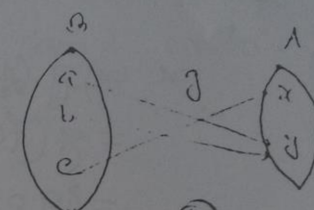
09. Let $f: A \to B$ and $g: B \to A$. Find the necessary condition for existing of f^{-1}.

Sol^n: Let $f: A \to B$ and $g: B \to A$. Then g is the inverse function of f, i.e. $g = f^{-1}$, if the product function $\boxed{(g \circ f): A \to A}$ is the identity on $\underline{A}$ and $\boxed{(f \circ g): B \to B}$ is the identity function on $\underline{B}$. ([illegible])

Let $A = \{x, y\}$ and let $B = \{a, b, c\}$. Define a function $f: A \to B$ by the diagram (i)



(i)



(ii)

Now define a function $g: B \to A$ by the diagram (ii).

We compute $(g \circ f): A \to A$

$(g \circ f)(x) = g(f(x)) = g(c) = x$

$(g \circ f)(y) = g(f(y)) = g(a) = y$

Therefore the product function $(g \circ f)$ is identity function on A. But g is not the inverse function of f, because the product function $(f \circ g)$ is not the identity function on B. f not being an onto function.

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Theorem: If $f: A \to B$ and $g: B \to C$ be two onto function. then prove that the product function $(g \circ f): A \to C$ is onto.

Proof: let $x \in C$ any arbitrary. Since g is onto
Then there exist an element $y \in B \to g(y) = x$
Again, Since f is onto and $y \in B$.
Then there exist an element $z \in A \to f(z) = y$

Now.
$$x = g(y)$$
$$= g(f(z))$$
$$= (g \circ f)(z)$$
$$\therefore (g \circ f)(x) = x$$

Hence $g \circ f: A \to C$ is onto. proved.

Let $V = \{-2, -1, 0, 1, 2\}$ and let the function $g: V \to \mathbb{R}$ be defined by the formula $g(x) = x^2 + 1$
Find the rang of g and draw the diagram.

Soln: Given that $V = \{-2, -1, 0, 1, 2\}$

and $g(x) = x^2 + 1$

Now, $g(-2) = (-2)^2 + 1 = 4 + 1 = 5$

$g(-1) = (-1)^2 + 1 = 1 + 1 = 2$

$g(0) = 0 + 1 = 1$

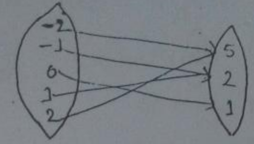
$g(1) = (1)^2 + 1 = 1 + 1 = 2$

P.T.O.

$g(2) = (2)^2 + 1 = 4 + 1 = 5$

The range of $g = \{5, 2, 1, 2, 5\}$ i.e $\{5, 2, 1\}$

Draw the graph.



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6. Let $f: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = 3x + 4$. prove that f is one-one and onto. Also find a formula that defines f^{-1}.

Sol^n: Solve for x in term of y.

$\therefore f(x) = 3x + 4$

$\Rightarrow y = 3x + 4$

$\Rightarrow y - 4 = 3x$

$\Rightarrow \dfrac{y-4}{3} = x$

$\therefore x = \dfrac{y-4}{3} \Rightarrow f^{-1}(y) = \dfrac{y-4}{3}$

Then the inverse function is $f^{-1}(x) = \dfrac{x-4}{3}$.

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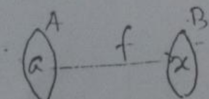
*** 08 Can a constant function be one-one?

Solution: If the domain of a function consists a single element, then the function is a constant and it is also one-one.



$f: A \longrightarrow B$

*** 07 Can a Constant function be onto?

solution: If the codomain of a function consists a single element then the function is a constant function and it is a onto.

*** $f: \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = x^3 - 1$ find f^{-1}

Soln: Let y be image of x under f.

That is $f(x) = y$

Since f is both one-one and onto the f has a inverse function.

That is $x = f^{-1}(y)$

$$f(x) = x^3 - 1$$

Now. $y = x^3 - 1$

$$\Rightarrow x^3 = y + 1$$

$$\Rightarrow x = (y+1)^{1/3}$$

$$\Rightarrow f^{-1}(y) = (y+1)^{1/3}$$

$$\therefore f^{-1}(x) = (x+1)^{1/3}$$ Ans:-

Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$ let the function $f: A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$. Then f is one-one and onto. Find a formula that defines f^{-1}.

Solⁿ: Solve for x the term of y.

$\therefore f(x) = \frac{x-2}{x-3}$ where $f(x) = y \therefore x = f^{-1}(y)$

$\therefore y = \frac{x-2}{x-3}$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow xy - x = 3y - 2$

$\Rightarrow x(y-1) = 3y - 2$

$\Rightarrow x = \frac{3y-2}{y-1} = \frac{2-3y}{1-y}$

$\Rightarrow f^{-1}(y) = \frac{2-3y}{1-y}$

$\therefore f^{-1}(x) = \frac{2-3x}{1-x}$ Ans:-

If $f: A \to B$, $g: B \to C$ and $h: C \to D$ be three maps then $(h \circ g) \circ f = h \circ (g \circ f)$.

Proof: we shall ~~prove~~ that

$((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$

Now, $((h \circ g) \circ f)(x) = (h \circ g)(f(x))$

$= h(g(f(x)))$ —— (I)

Again, $(h \circ (g \circ f))(x) = h \circ (g \circ f)(x)$

$= h(g(f(x)))$ —— (II)

From (I) and (II) we get:

$((h \circ g) \circ f)(x) = (h \circ (g \circ f))(x)$

Hence $(h \circ g) \circ f = h \circ (g \circ f)$ proved.

*** Let the function $f: \mathbb{R} \to \mathbb{R}$ and $g: \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 3x + 1$, $g(x) = 2x - 3$. Find the product function formulas which define the (i) fog (ii) gof (iii) gog (iv) fof

Soln: Given that—

$f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$

(i) $fog(x) = f(g(x)) = f(2x-3)$
$= (2x-3)^2 + 3(2x-3) + 1$
$= 4x^2 - 12x + 9 - 6x - 9 + 1$
$= 4x^2 - 6x + 1$ Ans:-

(ii) $gof(x) = g(f(x)) = g(x^2+3x+1)$
$= 2(x^2+3x+1) - 3$
$= 2x^2 + 6x + 2 - 3$
$= 2x^2 + 6x - 1$ Ans:-

(iii) $gog(x) = g(g(x)) = g(2x-3)$
$= 2(2x-3) - 3$
$= 4x - 6 - 3$
$= 4x - 9$ Ans:-

(iv) $fof(x) = f(f(x)) = f(x^2+3x+1)$
$= (x^2+3x+1)^2 + 3(x^2+3x+1) + 1$
$= x^4 + 9x^2 + 1 + 6x^3 + 6x + 2x^2 + 3x^2 + 9x + 3 + 1$

$= x^4 + 6x^3 + 14x^2 + 15x + 5$ Ans:-

—x—

05 ... $f: A \to B$ is one-one and different $g: B \to C$ is one-one, then the product function $gof: A \to C$ is one-one.

Soln: Suppose $(gof)(a_1) = (gof)(a_2)$ for same $a_1, a_2 \in A$.

Then $(gof)(a_1) = (gof)(a_2)$

$\Rightarrow g(f(a_1)) = g(f(a_2))$

$\Rightarrow f(a_1) = f(a_2)$ [$\because$ g is one-one]

$\Rightarrow a_1 = a_2$ [$\because$ f is one-one]

$\therefore a_1 = a_2$

Therefore gof is also one-one.

*** Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Let the function $f: A \to B$ be defined by $f(x) = \frac{x-2}{x-3}$. Then f is one-one and onto. Find a formula that defines f^{-1}.

Soln: Let $y = f(x) = \frac{x-2}{x-3}$

$\Rightarrow y = \frac{x-2}{x-3}$

$\Rightarrow yx - 3y = x - 2$

$\Rightarrow x(y-1) = 3y - 2$

$\Rightarrow x = \frac{3y-2}{y-1} = \frac{2-3y}{1-y}$

$\therefore f^{-1}(y) = \frac{1-2y}{1-y}$

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rent img

$\therefore f^{-1}(x) = \frac{1-2x}{1-y}$

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