

## B-section

Q) state and explain Faradays law of induction. Deduce its differential form -  
or Deduce Faradays law of electromagnetism  
induction from -

### Faradays law of induction :

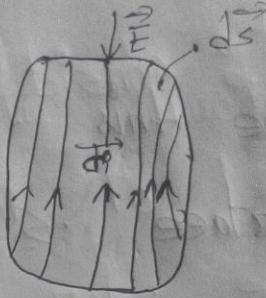
If the magnetic flux  $\phi_B$  through an area bounded by a closed conducting loop changes with time a current an emf are produced in the loop, thus process is called induction the induced emf is  $\Sigma = \frac{d\phi_B}{dt}$

which is Faradays law of induction if  
the loop is replaced by a closely  
packed coil of  $N$  turn the induced  
emf is  $\Sigma = -N \frac{d\phi}{dt}$

let  $C$  is a closed circuit and  $S$   
any surface with  $C$  as its boundary  
let  $\vec{B}$  the magnetic flux density  
in the neighbourhood the circuit,  
the magnetic flux through a  
small area

$$d\vec{s} \text{ is } \vec{B} \cdot \vec{d}$$

$$\therefore \phi_B = \int_S \vec{B} \cdot \vec{d}s$$



The line integral of the electric field round the circuit field round the circuit gives the induced emf.

$$\Sigma = \oint \vec{E} \cdot d\vec{s} \quad \textcircled{II}$$

According to Faradays law induced emf

$$\Sigma = - \frac{d\phi_B}{dt}$$

$$\Sigma = - \frac{d}{dt} \int \vec{B} \cdot d\vec{s} \quad \textcircled{III}$$

From equation \textcircled{II} and \textcircled{III}

$$\oint \vec{E} \cdot d\vec{s} = - \frac{d}{dt} \int \vec{B} \cdot d\vec{s} \quad \textcircled{IV}$$

$$\text{or } \oint \vec{E} \cdot d\vec{s} = - \int \cdot \frac{d\vec{B}}{dt} \cdot d\vec{s} \quad \textcircled{V}$$

From Stokes theorem,

$$\int \vec{E} \cdot d\vec{s} = \int_{\text{curl}} \vec{E} \cdot d\vec{s} \quad \textcircled{VI}$$

$$\int_{\text{curl}} \vec{E} \cdot d\vec{s} = - \int \frac{d\vec{B}}{dt} \cdot d\vec{s}$$

Since this must be true for all  
sources it follows that,

$$\text{curl } \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$
$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (\text{vii})$$

equation (vii) is the differentiation of  
Faraday's law.

Q) State and explain Amperes law

Statement: The line integral of  $\oint \vec{B} \cdot d\vec{l}$

of closed curve is equal to  $\mu_0$  times the net current through the area bounded by the curve. That is

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 i$$

Hence,

$\vec{B} \Rightarrow$  magnetic induction

$dI \Rightarrow$  current electron vector.

$\mu_0 \Rightarrow$  Permittivity constant

$i \Rightarrow$  current

Proof: Consider a path consisting a circle of radius "r" centered on the wire. For any point on the circle the magnitude of the magnetic field  $B$  is same  $\vec{B}$  and  $d\vec{l}$  which is always tangential to the integration of the path,

$$\oint \vec{B} \cdot d\vec{l} = \oint B dl$$

$$\text{or } \vec{B} \cdot d\vec{l} = B dl$$

$$\text{or } \oint \vec{B} \cdot d\vec{l} = B \times 2\pi r \quad \text{--- (1)}$$

Now magnitude of the electric field is proportional to the following current and inversely proportional to the distance  $r$ ,

$$B \propto \frac{i}{R}$$

where  $\frac{U_0}{2\pi}$  is  
the proportionality  
constant

$$B = \frac{U_0}{2\pi} \cdot \frac{i}{R} \quad (1)$$

Put the in equation ① then we get

$$\int \vec{B} \cdot d\vec{l} = \frac{U_0 i}{2\pi R} \times 2\pi R$$

$$\int \vec{B} \cdot d\vec{l} = U_0 i \quad (\text{proved})$$

■ Motional emf: Prove that  $\rho = \frac{B l v}{R}$

Sol<sup>n</sup>: Consider a rectangular loop at wire of with one end of which is in unit area field  $\vec{B}$  pointing at right angles to the plane of the loop. The experiment consist in Polling the loop to the right at speed  $v$  the flux  $\phi$  by the loop is

$$\phi_B = BA = B l_n$$

where  $l_n$  is the even of the loop which immersed in  $B$ . According to

Faraday law

$$\mathcal{E} = - \frac{d\phi B}{dt}$$

$$\text{or } \mathcal{E} = - \frac{d}{dt} (BL_v)$$

$$\text{or } \mathcal{E} = -BL \cdot \frac{dv}{dt}$$

$$\text{or } \mathcal{E} = BLv \quad \text{--- (1)}$$

Where  $v = -\frac{dv}{dt}$ , the induced emf is called motional emf. The emf  $\mathcal{E} = -BL$  sets up a current in the loop given by  $i = \frac{\mathcal{E}}{R} = \frac{BLv}{R}$

Where  $r$  is the resistance of the loop. the current in the loop will cause forces  $F_1$ ,  $F_2$  and  $F_3$  etc on the conductors.

Those  $\vec{F} = i\vec{l} \times \vec{B}$

Here,  $\vec{F}_2 = -\vec{F}_3$

$\vec{F}_2 + \vec{F}_3 = 0$  they connect each other

$F_1$  which is the force that opposes our effort,

to move the loop is given in

magnitude of  $F_1 = i l B \sin B$

or  $F_1 = i l B$

or  $F_1 = \frac{B l^2}{R} \times l_B$

or  $F_1 = \frac{B^2 l^2 r}{R}$

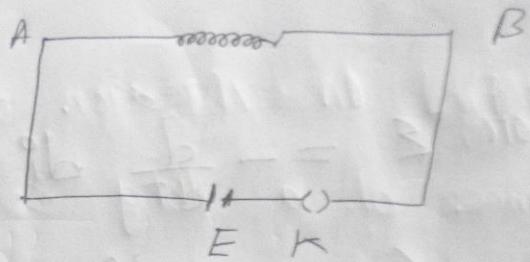
The agent that pulls the loop.

Must be work at the steady rate of

$$P = F_1 V = \frac{B^2 l^2 v^2}{R}$$

$$P = \frac{B^2 l^2 v^2}{R}$$

self inductance:



A inductance emf appears in a coil if the current in the same coil is changed this is called self induction and emf is produced its called self inductance.

$$\text{or } N\phi = Li$$

where  $N$  is the Number of terms

$$\text{or } Ni = L$$

where  $L$  is the proportional constant, it is called self inductance of the coil.  
From Faraday's law.

$$E = -\frac{d}{dt} (N\phi)$$

$$\text{or, } \mathcal{E} = - \frac{d}{dt} \frac{di}{dt}$$

$$\mathcal{E} = -L \frac{di}{dt}$$

$$\therefore L = - \frac{\mathcal{E}}{\frac{di}{dt}}$$

$$\frac{di}{dt} = 1 \text{ then } |\mathcal{E}| = L$$

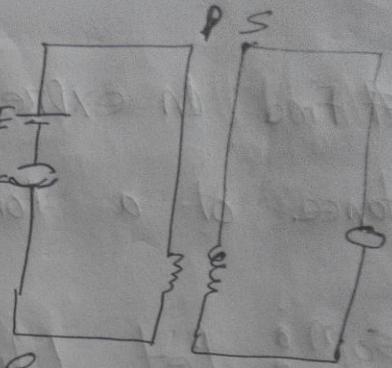
Unit of the self inductance is

Henry (H)

$$(QH) \frac{b}{jb} = 3$$

## Mutual inductance :

Mutual inductance may be defined as the amount of emf induced in one coil per unit rate of change of current in another coil adjust to the first coil we can write,



$$\phi_2 \propto i_1$$

$$\text{or } N\phi_2 \propto i_1$$

$$\text{or } N\phi_2 = m i_1$$

where  $m$  is proportionality constant its called mutual inductance,

From Faraday's law,

$$\mathcal{E}_2 = -N \frac{d\phi_2}{dt}$$

$$\mathcal{E}_2 = - \frac{d}{dt} (N\phi_2)$$

$$\mathcal{E}_2 = - \frac{d}{dt} Ni_1$$

$$\mathcal{E}_2 = -m \frac{di_1}{dt}$$

$$M = - \frac{\mathcal{E}_2}{\frac{di_1}{dt}}$$

# Find an expression for the self-inductance of a long solenoid.

Soln: Let  $l$  be the length of a long solenoid with an area and total number of turns are  $N$ . When a current  $i$  flows through it the magnetic field inside it is given by

$$B = \frac{\mu_0 N i}{L}$$

Total flux through the solenoid

$$\phi_B = \frac{\mu_0 N i A}{L}$$

$$\Rightarrow \phi_B = \frac{\mu_0 N^2 A}{L}$$

$$\frac{1}{\mu_0} = 4\pi \times 10^{-7}$$

$$\frac{1}{A} = 10^{-4}$$

$$\frac{1}{L} = 10^{-2}$$

$$\frac{1}{i} = 10^{-3}$$

when the current  $i$  varies then  
 $\phi_B$  changes giving rise to the  
induced emf

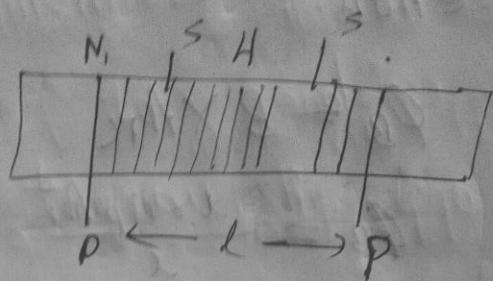
$$\begin{aligned} \epsilon &= -\frac{d}{dt} \left( \mu_0 \frac{N^2 A i}{L} \right) \\ \Rightarrow \epsilon &= -\frac{\mu_0 N^2 A}{L} \cdot \frac{di}{dt} \\ \Rightarrow \epsilon &= -L \cdot \frac{di}{dt} \quad [\text{from definition of } L] \end{aligned}$$

Therefore

$$\begin{aligned} L &= \frac{\mu_0 N^2 A}{L} \\ \Rightarrow L &= \mu_0 n^2 l A \quad [N = nl] \end{aligned}$$

where  $n$  are the number of turns  
per unit length. If the solenoid is  
wound a coil of constant permeability  
then  $L = \frac{\mu N^2 A}{L} \quad [\text{when } \mu = \mu_0]$

Q) Find the mutual inductance of two co-axial coil.



Sol<sup>n</sup>: Let us consider a long solenoid of area of cross-section  $A_p$  and length  $l$  which is called the Primary and  $n_p$  are the number of turn per unit length of coil. Another coil of  $N_s$  turns is wound inside the primary coil. The magnetic induction due to the current in primary is obtained by ampere law.

$$B = \frac{\mu n_p i}{l}$$

And the magnetizing flux =  $B \times \text{area of cross-section}$ .

$$\phi_B = B \times A_R$$

$$\phi_B = \frac{\mu_0 n_p A_i}{l}$$

The flux is linked with each turns of the secondary suppose there is no leakage of flux therefore the total flux linked with  $N_s$  of the secondary is given by

$$\phi_B' = N_s \cdot \phi_B$$

$$\phi_B' = N_s \frac{\mu_0 n_p A_i}{l}$$

$$\Rightarrow \frac{\mu_0 n_p n_s A_i}{l}$$

Then the induced emf in secondary

coil

$$E_2 = - \frac{d\phi_B}{dt}$$

$$E_2 = - \frac{\mu_0 n_s n_p A_p}{l} \cdot \frac{di}{dt}$$

$$\dot{E}_2 = - M \cdot \frac{di}{dt}$$

if there force  $M = \frac{\mu_0 n_s n_p A_p}{l}$

if the primary and secondary are wound over a core of material having permeability and there,

$$M = \mu_0 n_s A_p \quad \text{Privent} \quad \textcircled{1}$$

$$M = \mu_0 n_p A_p \quad \text{Gated coil} \quad \textcircled{2}$$

$$M = \mu_0 n_s n_p A_p \quad \text{Crossed coil} \quad \textcircled{3}$$

12-11

Galvanometer: The instrument by which the existence of current in a conductor and its amount can be determined is called a galvanometer.

The working principle of galvanometers depends on the effect of magnetic field on the flow of current or effect of current of a magnet.

3 type of galvanometers:

① moving coil galvanometer.

② Blalastic galvanometer

③ Dead-beat galvanometer.

Moving coil galvanometer: A galvanometer

by which current can be measured from the deflection of the coil that can be rotated when placed in the magnetic field of a magnet is called moving coil galvanometer.

Example. D' ansonval galvanometer.

Balistic galvanometer: An instrument designed to measure quantity of charge through it in terms of momentary current it is called ballistic galvanometer.

Dead-beat galvanometer: An instrument designed to measure quantity of current through it in terms of deflection of coil of  $N$  turns due to extended or continuously coherent  $i$  is called dead beat galvanometer.

\* Difference between Dead-beat and Ballistic galvanometer.

Dead beat	Ballistic
① It measures steady current	① It measures charge
② The steady deflection measures the current	② The throw measures the charge
③ The coil is wound on a metal frame to increase electromechanical damping	③ The coil is wound on a non-magnetic frame to reduce electromagnetic damping
④ The coil is non-oscillatory due to large damping	④ The coil is oscillatory due to small damping
⑤ The coil rotates due to constant torque	⑤ The momentary passage of charge causes impulse on the coil. The torque is zero when the coil rotates.

\*\*\* A solenoid levying on aircore  
 and 10 cm long has 100 turns and  
 its area of cross-section is 5 sq cm.  
 Find the co-efficient of self  
 induction of the solenoid.

Sol<sup>n o</sup>

We know,

$$L = \frac{\mu_0 N^2 A}{l}$$

$$= \frac{4\pi \times 10^{-7} \times 10^4 \times 5 \times 10^{-9}}{0.1}$$

$$= 62.8 \times 10^{-6} \text{ m}$$

Hence,

$$l = 10 \text{ cm}$$

$$= 0.1 \text{ m}$$

$$N = 100$$

$$A = 5 \text{ sq. cm}$$

$$= 5 \times 10^{-4} \text{ m}^2$$

Q) A solenoid of length 30 cm  
 and area of cross-section 10sq-cm  
 has 1000 turns wound over a  
 core of constant permeability 600.  
 another coil of 500 turns is wound  
 over same coil at its middle  
 calculate the mutual inductance of  
 the coil.

Soln:

$$\begin{aligned}
 M &= \frac{\mu_0 N_1 N_2 A}{l} \\
 &= \frac{4\pi \times 10^{-7} \times 600 \times 1000 \times 500 \times 10 \times 10^{-4}}{0.3} \\
 &= 1.257
 \end{aligned}$$

Hence,

$$l = 30 \text{ cm}$$

$$= 30 \times 10^{-2} \text{ m}$$

$$A = 10 \text{ sq-cm}$$

$$N_1 = 1000$$

$$N_2 = 500$$

$$\mu_r = 600$$