Capacitor : A capacitor is an eletric devise consisting of two conductors separted by an Insulating or dieletric medium and carrying equal and opposite charge. In a word it is a device for storing charge.

Capacitance : If a charge q is given to an isolated conductor its voltage is incresed by an amount V. For a given conductor the ratio Q/V is independent of Q and depends only on the size and shape of the conductor. The ratio Q/V is called capacitane of the conductor it is denoted by C.

$$C = Q/V$$

The unit of the capacitance is Farad. 1 Farad $= 10^6 \mu F$ and $1 \mu F = 10^{-6} F$

Capacitors in :

Fig (2) shows there capacitors of capacitanes C_1 C_2 and C_3 connected in Parallel let the terminal A and B be connected to a potential difference V the poteetial difference across each capacitor is the same. The charges on the three capacitors are respatively.

$$q_1 = C_1 V \qquad q_2 = C_2 V \qquad q_3 = C_3 V$$

The total charge on the system of capacitor is

$$q = q_1 + q_2 + q_3$$

$$q = C_1 V + C_2 V + C_3 V$$

$$q = V(C_1 + C_2 + C_3) \text{ ——— } (iii)$$

if C_p is the equivalent capacitor of the system

$$q = C_p V \text{ ——— (iv)}$$

(iii) and (iv) compair,

$$C_p V = V(C_1 + C_2 + C_3)$$

$$C_p = C_1 + C_2 + C_3$$

Capacitance of a parallel plate capacitor.

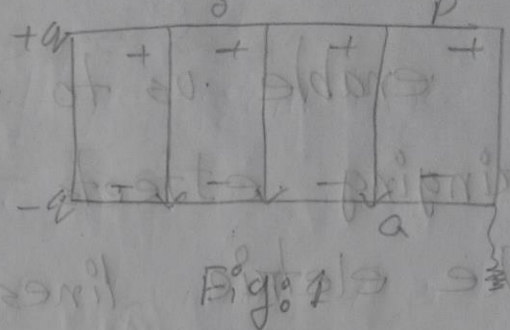


Fig: 1

Sol^n: The parallel plate capacitor consists of two parallel metal plates each of Area A and separated by a distance d (fig 1). The medium be tween the plutes is air. A charge +q is given to the plate p. it induces a charge -q on the upper surface of the earthed plate Q d is the kept smell compared with the plate dimension with the to enable us to ignove the fringing effects near the ends The eletric lines of force — starting from plate P and ending at the plate Q are parallel to each other and perpendicular to the plates. By the application of Gauss's low, Electric field at a point between the two plates

$$E = \frac{\sigma}{\epsilon_0}$$

Here σ = surface density of charge = q/A

Potential difference between the plates P and Q is

$$V = \int_d^0 -E \cdot dr$$

$$= -\int_d^0 \frac{\sigma}{\epsilon_0} \cdot dr$$

$$= \frac{\sigma}{\epsilon_0} \int_d^0 dr$$

$$= \frac{\sigma d}{\epsilon_0}$$

The capacitance of the parallel plate capacitor is.

$$C = \frac{q}{V}$$

$$C = \frac{q}{\frac{\sigma d}{\epsilon_0}}$$

$$C = \frac{\epsilon_0 q}{\sigma d}$$

$$C = \frac{\epsilon_0 q}{\frac{q}{A} d} \quad [q = \sigma A]$$

$$C = \frac{\epsilon_0 A}{d}$$

☐ capacitor of a parallel plate.

capacitor with a dielectric midium.

or, prove the relation for the capacitance of a parallel plate capacitor $C = \frac{\epsilon_0 A}{d - t + \frac{t}{k}}$ of plate area a separated by a distance d with dielectric slat of thiknsess 't' and constant 'k'

or, capacity of a parallel plate capacitor partially filled with dielectric.

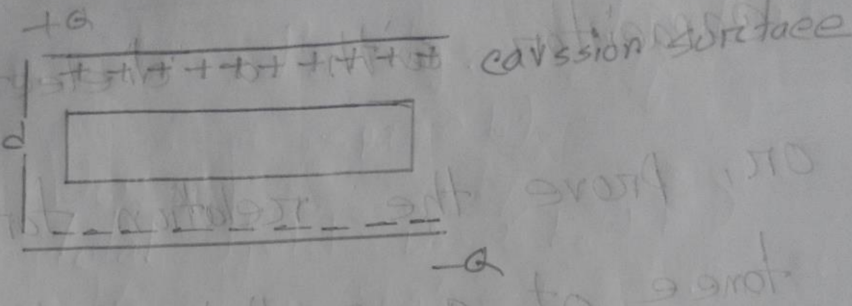


Fig: 1

⇒ Fig (1) shown a capacitor partilly filled with a dielectric material of thickness t. If Q is the charge on each plate of area A them value of electric fie For due to charge Q in the free space between plates and dielectr material is given by gauss's law,

$$\oint \vec{E} \cdot \vec{ds} = \varepsilon_0 E_0 A = Q$$

$$E_0 = \frac{Q}{\varepsilon_0 A} \quad \text{——— (i)}$$

and electric field E inside die electric is given by,

$$E = \frac{E_0}{k} = \frac{Q}{k \epsilon_0 A} \quad \text{--- (ii)}$$

E_0 is the value of the electric field the free space without dieletric which is (d-t) and E is the value of the electric field inside the dieleetrie of thickenss $\frac{1}{t}$ the potential differ ence between the two points is given

$$V = V_1 - V_2 = -\int_2^1 \bar{E} \cdot \bar{ds}$$

Hence the total potential difference between the plate's A and B

$$V_a = V_b = -\int \vec{E_0} \cdot du = \int \vec{E} \, dl$$

for free spaced-t   for thinkness t

Since the total field is $(\vec{E_0} + \vec{E})$

Therefore,

$$V_a - V_b = Et + E(d-t)$$

$E \cdot dl = -E\,dl$

similarly

$\vec{E_0} \cdot \vec{dl} = E_0 dl$

Substituting E_0 and E from eq^n and (ii)

$$V_a - V_b = \frac{Q}{A\epsilon_0}\left(d - t + \frac{t}{K}\right)$$

$\therefore$ capacitor C is given by

$$C = \frac{Q}{V_a - V_b} = \frac{\epsilon_0 A}{d - t(1 - 1/k)} = \frac{\epsilon_0 A}{\left(d - t + \frac{t}{k}\right)}$$

4 What do you mean by every density Derive on expression for the energy stoved by a charged capacitor.

$\underline{\underline{sol^n}}$: Energy density is defined as the quantity of the charge per unit area. S is denoted by σ

$\therefore \sigma = \frac{q}{A}$ where q - charge
A - area

Let q' be the charge and v the poten difference estabilished between the plates the capacitor at any instant during the proc of charging it an additional charge dq is given to the plates the work done by the battery is given by

$$dw = v'dq'$$

$$\text{or } dw = \left(\frac{q'}{c}\right)dq' \quad \Big| \quad v = \frac{q'}{c}$$

Total work done by the charge a capacitor to a charge q is,

$$w = \int dw = \int_0^q \frac{q'}{c}\, dq'$$

$$w = 1/c \int_0^q q'\, dq' \Rightarrow w = \frac{1}{c}\int_0^q q'dq'$$

$$\Rightarrow w = \frac{1}{2} \cdot \frac{q^2}{c}$$

This work done is stored as electrostatic potential energy in the capacitor.

$$u = \frac{1}{2} \cdot \frac{q^2}{c} \quad [q = cv$$

$$U = \frac{1}{2} cv^2$$

This energy can be recovered if the capacitor is allowed to discharge

Consider a parallel plates capacitor of A and plate separaetion d than energy of the capacitor $U = \frac{1}{2} AV^2$ $= \frac{1}{2}\left(\frac{\varepsilon_0 A}{d}\right) v^2$ volom of the between the plates Ad.

Enegry density is the potential energy per unit volome,

$$U = \frac{V}{Ad}$$

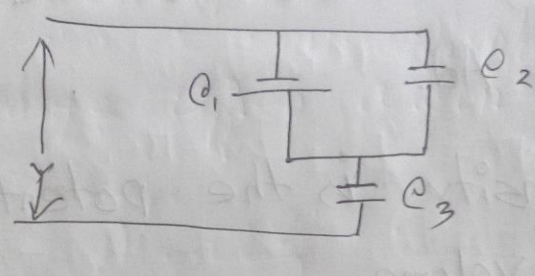
$$\text{or } U = \frac{1}{2}\left(\frac{\varepsilon_0 A}{d} v^2\right) \times \frac{1}{Ad}$$

$$\text{or } U = \frac{1}{2} \varepsilon_0 \left(\frac{V}{d}\right)^2$$

$$\text{or, } U = \frac{1}{2} \cdot \varepsilon_0 \left(\frac{v}{d}\right)^2$$

$$\therefore U = \frac{1}{2} \varepsilon_0 \left(\frac{V}{d}\right)^2$$

Find the equivalent capacitance of the combination as illustrated in the following figure assume $C_1 = 10\,\mu F$, $C_2 = 5\,\mu F$ $C_3 = 4\,\mu F$ and $V = 100$ volts.



Sol^n: Although C_1 and C_2 are parallel

$$C_P = C_1 + C_2$$
$$= 10 + 5$$
$$= 15$$

$$\frac{1}{C_S} = \frac{1}{C_P} + \frac{1}{C_3}$$
$$= \frac{1}{15} + \frac{1}{4}$$
$$= \frac{4+15}{60} = \frac{19}{60} = \frac{60}{19} = 3.1$$

(11) $C_s = \frac{1}{C_1} + \frac{1}{C_2}$

$= \frac{1}{10} + \frac{1}{5}$

$= 0.3$

$C_P = C_s + C_3$

$= 0.3 + 14$

$= 14.3$

Relative permitivity or Di-eleetrie constant : If C is the capacitance of the capacitor when a dieletrie material is present and C_0 is the capacitance when no dieletrie materials is present then the ratio C/C_0 is called the dieleetrie constant of the materials its denoted by k.

or $k = \frac{C}{C_0}$

k is the dimensionless factor by which the capacitance of a capacitor increases when a constant.

v.v.9

- Dielectric material is inserted in between the plates relative to its capacitance when no dielectric is present.

- Show that $\vec{D} = \epsilon_0\vec{E} + \vec{P}$ where symbols are use their usual meaning. or Find ~~the~~ out the relation between electric displacement and electric polarisation.

Solⁿ: The polarization is define as the induced surface charge per unit area it is denoted by $\vec{P}$ and $\vec{P} = \frac{q'}{A}$ ——— (1)

And eletrie displanet which depends only on the magnitude of charge and its distribution but it is independent of the nature of medium this vector is termed as eletris displanet its denoted by $\vec{D}$

$$\vec{D} = \frac{q}{A} \text{ ——— } (ii)$$

The dielectrie constant k for any medium is $k = \frac{E_0}{E}$

$$\text{or } E = \frac{E_0}{k} \left[E_0 = \frac{q}{\epsilon_0 A} \right]$$

$$\text{or } E = \frac{q}{k \epsilon_0 A} \text{ ——— } (iii)$$

$$\text{Again } E = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A} \text{ ——— } (iv)$$

From eq^n (3) and (4)

$$\frac{q}{k\epsilon_0 A} = \frac{q}{\epsilon_0 A} - \frac{q'}{\epsilon_0 A}$$

$$\text{or, } \frac{q}{\epsilon_0 A} = \frac{q}{k\epsilon_0 A} + \frac{q'}{\epsilon_0 A}$$

$$\text{or } \frac{q}{A} = \epsilon_0 \left(\frac{q}{k\epsilon A}\right) + \frac{q'}{A} \quad \text{———} (v)$$

From eq^n (i) (ii) (iii) and than we get

$$D = \epsilon_0 E + P \quad \text{————} (vi)$$

As E and P on the right hand side of equation (vi) are vector D is also a vector hence we can write

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

☐ Capacitor of a parallel plate capacitor with a dielectric midium.

or, Prove the relation for the capacitance of a parallel plate capacitor $C = \frac{\epsilon_0 A}{d - t + t/K}$ of plate area a separated by a distance d with dielectric slab of thiknsess 't' and constant 'k'.

or, capacity of a parallel plate capacitor partially filled with dielectric.

07-10

The area of each plate of an airfilled parallel plate capacitor is 1.1×10^{8} metre^{2}. What must be the separation between the plates if the capacitance is to be 1.0 farad.

Solⁿ:

$$C = \frac{\epsilon_0 A}{d}$$

$$d = \frac{\epsilon_0 A}{C}$$

$A = 1.1\times10^{8}$

$C = 1.0$

$$= \frac{8.85\times10^{-12}\times(1.1\times10^{8})}{1.0}$$

$$d = 9.73\times10^{-4}\,m$$

$$d = 9.73\times10^{-4}\,m\times10^{3}$$

$$d = 0.97\,mm$$

[4] Compute the energy stored in a 60 PF capacitor.

(i) when charged to a potential difference of 2KV

(ii) when the charge on each plate is 30nC.

Soln: (i) $E = \frac{1}{2} CV^2$

$= \frac{1}{2} \times 60 \times 10^{-12} \times (2000)^2$

$= 1.2 \times 10^{-4}$ J

(ii) $E = \frac{1}{2} CV^2$

$= \frac{1}{2} C \times \frac{Q^2}{C^2}$

$= \frac{1}{2} \frac{Q^2}{C}$

$= \frac{1}{2} \frac{30 \times 10^{-9} C}{60 \times 10^{-12}} = 7.5 \times 10^{-6}$ J

10-10

Electric Polarization :

Electric polarization is defined as the induced charge per unit area it is denoted by -

$\therefore$ Electric polarization $\vec{P} = \frac{q'}{A}$

Here,

q' = Induced charge.

A = Area of the surface.

$$\therefore \vec{P} = \frac{q'}{A}$$

$$\text{or } \vec{P} = \frac{q'd}{Ad}$$

$$\text{or } \vec{P} = \frac{\hat{P}}{V}$$

$\therefore$ Electric polarization = $\frac{\text{Induced dipole moment}}{\text{Volume}}$

Polarization is also defined by indoeed dipole moment per onit volome.

Electrie susceptibility: The polarization $\vec{P}$ in a homogeneeos isotropie dieletrie is dependent on the natore of the dieletrie it has the same direction as the resoltant electrie field also is dependent on the field. These result are sommarized by the equation —

$$\vec{P} = \chi_e \vec{E} \quad \text{———} \ (1)$$

Here χ_e a scallar quanty is called the electric susceptibility of the material

Permitivity: we know the electric displacement $\vec{D}$ for the dielectric medium as—

$$\vec{D} = \epsilon_0 E + \vec{P} \longrightarrow (1)$$

and for electric susceptibility

$$\vec{P} = \chi_e \vec{E} \longrightarrow (11)$$

we get equation (1) and (11)

$$D = \epsilon \vec{E} + \chi_e \vec{E}$$

$$D = \vec{E}(\epsilon_0 + \chi_e)$$

$$D = \vec{E}\,\epsilon$$

Where $\varepsilon = \varepsilon_0 + \chi_e$ and ε is called Permittivity of the material. For air or vacuum $D = 0$ and so $\chi_e = 0 \therefore \varepsilon = \varepsilon_0$

☐ Find the equivalent capacitance of the the combination illustrated in the following figure. Assume, $C_1 = 20\,\mu F$, $C_2 = 10\,\mu F$, $C_3 = 8\,\mu F$ and $V = 500$ volts.

Solⁿ:

$$C_P = C_1 + C_2$$
$$= 20 + 10$$
$$= 30\,\mu F$$

$$\frac{1}{C_S} = \frac{1}{C_P} + \frac{1}{C_3}$$
$$= \frac{1}{30} + \frac{1}{8}$$
$$= \frac{4 + 15}{120}$$
$$= \frac{19}{120}$$
$$= 6.31$$