Nishat

Exam

(4)

Example. XXXVb.

Theorem: In an equation with real coefficients imaginary roots occur in pairs.

Proof: suppose that $f(x)=0$ be an equation with real coefficients. it has an imaginary root. Let $a+ib$ be a root of the equation $f(x)=0$. we shall show that $a-ib$ is also a root of $f(x)$.

The factor of $f(x)$ corresponding to these two roots is

$$(x-a-ib)(x-a+ib)$$

$$=\{x-(a+ib)\}\{x-(a-ib)\}$$

$$=\{(x-a)+ib\}\{(x-a)-ib\}$$

$$=(x-a)^2+b^2$$

Therefore, $f(x)$ be devided by $(x-a)^2+b^2$. Let us denote the quotient by $Q(x)$ and the remainder by $Rx+R'$ if any.

$$\therefore f(x)=Q\{(x-a)^2+b^2\}+Rx+R' \quad \text{---} (a)$$

since $a+ib$ is a root of $f(x)$ then $f(a+ib)=0$

$$\therefore f(a+ib)=Q\{(a+ib-a)^2+b^2\}+R(a+ib)+R'$$

$$\Rightarrow 0=Q(-b^2+b^2)+Ra+iRb+R'$$

$$\Rightarrow 0=0+Ra+iRb+R'$$

$$\Rightarrow 0=Ra+R'+iRb$$

$$\therefore Ra+R'+iRb=0$$

Equating the real and imaginary parts on both sides,

$Ra+R'=0$ and $Rb=0$

since $b\neq 0$ then $R=0$

$$\therefore Ra+R'=0$$

$$\Rightarrow 0+R'=0$$

$$\therefore R'=0$$

Mareeful

B1CS

$\therefore f(x) = a(x)\{(x-a)^2+b^2\}$

As a result we say that $f(x)$ is exactly divisible by $\{(x-a)^2+b^2\}$.

Hence $x = a - ib$ is also a root of $f(x) = 0$.

Proved.

Descartes' Rule of signs :- The number of the real positive roots of the equation $f(x) = 0$ can not exceed the number of changes in the signs of the coefficients of the terms in $f(x)$ and the number of real negative roots can not exceed the number of changes in the signs of the coefficient of $f(-x)$.

Harmonical progression :- Three quantities a, b, c are said to be in Harmonical Progression when $\frac{a}{c} = \frac{a-b}{b-c}$.

** If the roots of $x^3 + 3px^2 + 3qx + r = 0$ are in harmonical progression, show that $2q^3 = r(3pq - r)$

solution :- Let a, b, c be the roots of the equation. since the roots in H.P then $\frac{a}{c} = \frac{a-b}{b-c} \Rightarrow a(b-c) = c(a-b)$

dividing by abc

$$\Rightarrow \frac{1}{c} - \frac{1}{b} = \frac{1}{b} - \frac{1}{a} \Rightarrow ab + bc = 2ac$$

$$\Rightarrow ab + bc + ca = 2ac + ac \Rightarrow ab + bc + ca = 3ac$$

$$\therefore ab + bc + ca = 3ac = \frac{3abc}{b} = \cancel{3}$$

$$\Rightarrow 3q = \frac{3r(-r)}{b} \Rightarrow b = -r/q$$

$\therefore ab+bc+ca = 3q$

$abc = -r$

since b is the root of the given equn, so $x = b$

putting the value of $x = b = -r/q$ in the equn $x^3 + 3px^2 + 3qx + r = 0$

$$\Rightarrow \frac{-r^3}{q^3} + \frac{3pr^2}{q^2} + 3q(-r/q) + r = 0$$

$$\Rightarrow -r^3 + 3pqr^2 - 3q^3 r + rq^3 = 0$$

$$\Rightarrow r^2 - \quad r + 3q^3 \quad = 0$$

$- r \quad r^2 = r(3pq - r)$

the equations:

$3x^4 - 10x^3 + 4x^2 - x - 6 = 0$, one root being $\frac{1+\sqrt{-3}}{2}$.

sol^n: The given equation is

$$3x^4 - 10x^3 + 4x^2 - x - 6 = 0 \longrightarrow (1)$$

one roots of ① is $\frac{1}{2}(1+\sqrt{-3})$; Hence the other is $\frac{1}{2}(1-\sqrt{-3})$

[since the imaginary roots occour in pairs]

we have,

the quadratic factor is

$$\left\{x - \tfrac{1}{2}(1+\sqrt{-3})\right\}\left\{x - \tfrac{1}{2}(1-\sqrt{-3})\right\}$$

$$= \left(x - \frac{1}{2} - \frac{\sqrt{-3}}{2}\right)\left(x - \frac{1}{2} + \frac{\sqrt{-3}}{2}\right)$$

$$= \left(x - \frac{1}{2}\right)^2 - \left(\frac{\sqrt{-3}}{2}\right)^2 = x^2 - x + \frac{1}{4} + \frac{3}{4} = x^2 - x + 1$$

Now, from ①

$$3x^2(x^2 - x + 1) - 7x(x^2 - x + 1) - 6(x^2 - x + 1) = 0$$

$$\Rightarrow (x^2 - x + 1)(3x^2 - 7x - 6) = 0$$

Hence the other two roots ~~is~~ are obtained from

$$3x^2 - 7x - 6 = 0$$

$$\Rightarrow (x-3)(3x+2) = 0 \Rightarrow x = 3, -2/3.$$

Hence the roots of ① are

$$3, -2/3, \tfrac{1}{2}(1+\sqrt{-3}), \tfrac{1}{2}(1-\sqrt{-3}) \quad \text{(Ans)}.$$

2. $6x^4 - 13x^3 - 35x^2 - x + 3 = 0$, one root being $2 - \sqrt{3}$.

sol^n: The given equation is

$$6x^4 - 13x^3 - 35x^2 - x + 3 = 0 \longrightarrow ①$$

one root of ① is $2 - \sqrt{3}$ Hence another one is $2 + \sqrt{3}$

we have, $x = 2+\sqrt{3}$ and $x = 2-\sqrt{3}$

$$\therefore \ (x-2-\sqrt{3})(x-2+\sqrt{3})$$
$$= (x-2)^2-(\sqrt{3})^2 = x^2-4x+4-3$$
$$= x^2-4x+1$$

Now, from ①

$$6x^2(x^2-4x+1)+11x(x^2-4x+1)+3(x^2-4x+1)=0$$
$$\Rightarrow (x^2-4x+1)(6x^2+11x+3)=0$$

Hence, the other roots are obtained from

$$6x^2+11x+3=0$$
$$\Rightarrow 6x^2+9x+2x+3=0 \Rightarrow (2x+3)(3x+1)=0$$
$$\therefore \ x = -3/2, -\frac{1}{3}$$

Thus the roots of ① are —

$$-3/2, -1/3, 2+\sqrt{3}, 2-\sqrt{3}$$ (Ans).

3. $x^4+4x^3+5x^2+2x-2=0$, one root being $-1+\sqrt{-1}$.

soln: Given that

$$x^4+4x^3+5x^2+2x-2=0 \longrightarrow ①$$

One root of ① is $-1+\sqrt{-1}$; Hence the another one is $-1-\sqrt{-1}$

we have, $(x+1-\sqrt{-1})(x+1+\sqrt{-1})$

$$\Rightarrow (x+1)^2-(\sqrt{-1})^2$$
$$\Rightarrow x^2+2x+1+1$$
$$\Rightarrow x^2+2x+2$$

$$x^4+4x^3+5x^2+2x-2 = x^2(x^2+2x+2)+2x(x^2+2x+2)-1(x^2+2x+2)$$

$$= (x^2+2x+2)(x^2+2x-1)$$

Hence, the other two roots are obtained from

$$x^2+2x-1=0$$

$$\Rightarrow x = \frac{-2 \pm \sqrt{4+4}}{2} = -1 \pm \sqrt{2}$$

Hence, the roots of ① are, $-1 \pm \sqrt{-1}$, $-1 \pm \sqrt{2}$ (Ans)

4. $x^4+4x^3+6x^2+4x+5=0$, one roots being $\sqrt{-1}$

soln: Given equation

$$x^4+4x^3+6x^2+4x+5=0 \longrightarrow ①$$

one root of ① is $\sqrt{-1}$; Here the another one will be $-\sqrt{-1}$.

we have $(x-\sqrt{-1})(x+\sqrt{-1})$

$$= x^2+1$$

Now from ①

$$x^2(x^2+1)+4x(x^2+1)+5(x^2+1)$$

$$\Rightarrow (x^2+4x+5)(x^2+1)$$

Hence the other two roots are obtained from

$$x^2+4x+5=0$$

$$\Rightarrow x = \frac{-4 \pm \sqrt{16-20}}{2} = -2 \pm \sqrt{-1}$$

Thus the roots of ① are $-2 \pm \sqrt{-1}$, $\pm\sqrt{-1}$, (Ans)

5. solve the equation $x^5 - x^4 + 8x^2 - 9x - 15 = 0$ one root being and another $(1-2\sqrt{-1})$.

sol^n: the given equation

$$x^5 - x^4 + 8x^2 - 9x - 15 = 0 \longrightarrow (1)$$

two roots of (1) are $\sqrt{3}$ and $1-2\sqrt{-1}$; Hence two pairs of roots are $\sqrt{3}, -\sqrt{3}$ and $1-2\sqrt{-1}$, $1+2\sqrt{-1}$,

we have

$$\{(x-\sqrt{3})(x+\sqrt{3})\}\ \{(x-1+2\sqrt{-1})(x-1-2\sqrt{-1})\}$$

$$\Rightarrow (x^2-3)\ \{(x-1)^2-(2\sqrt{-1})^2\}$$

$$\Rightarrow (x^2-3)\ (x^2-2x+1+4)$$

$$\Rightarrow (x^2-3)\ (x^2-2x+5)$$

$$\Rightarrow x^4-2x^3+2x^2+6x-15.$$

Also,

$$x^5-x^4+8x^2-9x-15 = x(x^4-2x^3+2x^2+6x-15) + 1(x^4-2x^3+2x^2+6x-15)$$

$$= (x^4-2x^3+2x^2+6x-15)(x+1)$$

Hence the other roots is obtained from $x+1=0$

$\therefore x=-1$

Thus the roots of (1) are $-1, \pm\sqrt{3}, 1\pm2\sqrt{-1}$

(Ans)

From the equation whose roots are $\pm 4\sqrt{3}$, $5 \pm 2\sqrt{-1}$

sol^n: The given roots are $4\sqrt{3}, -4\sqrt{3}$, and $5-2\sqrt{-1}$, $5+2\sqrt{-1}$.

we have, from first pairs

$$(x-4\sqrt{3})(x+4\sqrt{3}) = x^2-48$$

Again from second pair

$$(x-5-2\sqrt{-1})(x-5+2\sqrt{-1})$$

$$\Rightarrow x^2-10x+25+4$$

$$\Rightarrow x^2-10x+29$$

Thus the required equation is,

$$(x^2-48)(x^2-10x+29)=0$$

$$\Rightarrow x^4-48x^2-10x^3+480x+29x^2-1392=0$$

$$\Rightarrow x^4-10x^3-19x^2+480x-1392=0 \quad \text{(Ans)}$$

11. Form the equation whose roots are $1\pm\sqrt{-2}$, $2\pm\sqrt{-3}$

sol^n: The given roots are

$1+\sqrt{-2}$, $1-\sqrt{-2}$, and $2+\sqrt{-3}$, $2-\sqrt{-3}$.

we have, from the first pair

$$(x-1-\sqrt{-2})(x-1+\sqrt{-2})$$

$$\Rightarrow (x-1)^2-(\sqrt{-2})^2$$

$$\Rightarrow x^2-2x+1+2$$

$$\Rightarrow x^2-2x+3$$

f

Again, from the second pair,

$(x-2-\sqrt{-3})\ (x-2+\sqrt{-3})$

$\Rightarrow (x-2)^2-(-\sqrt{3})^2$

$\Rightarrow x^2-4n+4+3 \Rightarrow x^2-4x+7$

Hence the required condition \equation is

$(x^2-2x+3)(x^2-4n+7)=0$

$\Rightarrow x^4-4x^3+7x^2-2x^3+8x^2-14x+3x^2-12x+21=0$

$\Rightarrow x^4-6x^3+18x^2-26x+21=0$

(Ans).

13. Find the nature of the roots of the equation,

$3x^4+12n^2+5n-4=0$

soln: Given that,

$3x^4+12n^2+5n-4=0 \longrightarrow (1)$

Let, $f(n)=3n^4+12n^2+5n-4=0$

Hence, there are one change of sign in $f(n)$

so, $f(n)$ has one positive root.

Now, $f(-n)=3x^4+12x^2-5n-4=0$

Here, there are one change of sign in $f(-n)$

so, $f(-n)$ has one negetive root.

Hence the equation has two real roots and the number of imaginary roots are 2. (Ans).

(4)

H.W CSE

Show that equation $2x^7 - x^4 + 4x^3 - 5 = 0$ has at least four imaginary roots.

Soln: The given equation is

$$2x^7 - x^4 + 4x^3 - 5 = 0 \longrightarrow (1)$$

Let $f(x) = 2x^7 - x^4 + 4x^3 - 5$

Here, there are three changes of sign in $f(x)$

so $f(x)$ has three positive roots.

Again, Let $f(-x) = -2x^7 - x^4 - 4x^3 - 5$

There are no change of sign in $f(-x)$, so,

$f(-x)$ has no negative roots.

Thus the given equation has three real roots.

Hence the number of imaginary roots is 4. (proved).

15. What may be inferred respecting the roots of the equation

$x^{10} - 4x^6 + x^4 - 2x - 3 = 0$

Soln: The given equation is

$$x^{10} - 4x^6 + x^4 - 2x - 3 = 0 \longrightarrow (1)$$

Let $f(x) = x^{10} - 4x^6 + x^4 - 2x - 3 = 0$

There are three changes sign in $f(x)$, so

$f(x)$ has three positive roots,

Again, $f(-x) = x^{10} - 4x^6 + x^4 + 2x - 3 = 0$

There are three changes of sign in $f(-x)$

so, $f(-x)$ has three negative roots.

Hence the number of real roots of (1) are 6

and so the number of imaginary roots are 4.

(Ans).

17. Find the condition that $x^3 - px^2 + qx - r = 0$ may have

(i) two roots equal but of opposite sign.

(ii) The roots in geometrical progression.

soln: Let the roots of (1) be $a, -a, b$

sum of the roots

$$a - a + b = p$$

$$\Rightarrow b = p$$

sum of the product of the roots taken two at a time.

$$-a^2 + ab - ab = q$$

$$\Rightarrow -a^2 = q$$

product of the roots

$$-a^2 b = r \Rightarrow qp = r$$

$$\therefore r = pq$$ (Ans) which is required condi:

15

Let $\frac{a}{r}, a, ar$ be the roots in geometrical progression of

sum of the roots,

$$\frac{a}{r}+a+ar=p$$

$$\Rightarrow a\left(1+r+\frac{1}{r}\right)=p \longrightarrow (2)$$

sum of the product of the roots taken two at a time.

$$\frac{a}{r}\cdot a+\frac{a}{r}\cdot ar+a\cdot ar=q$$

$$\Rightarrow \frac{a^2}{r}+a^2+a^2r=q$$

$$\Rightarrow a^2\left(1+r+\frac{1}{r}\right)=q \longrightarrow (3)$$

product of the roots

$$\frac{a}{r}\cdot a\cdot ar=r$$

$$\therefore a^3=r.$$

$$(3)\div(2) \Rightarrow \frac{a^2\left(1+r+\frac{1}{r}\right)}{a\left(1+r+\frac{1}{r}\right)}=\frac{q}{p}$$

$$\Rightarrow a=q/p \Rightarrow \sqrt[3]{r}=\frac{q}{p}$$

$$\Rightarrow r=\frac{q^3}{p^3} \Rightarrow p^3r=q^3$$ ~~Ans~~. which is required condit

** 18. If the roots of the equation $x^4+px^3+qx^2+rx+s=0$ are in arithmetical progression, show that $p^3-4pq+8r=0$; and if they are in geometrical progression, show that $p^2s=r^2$.

Soln: The given equation

$$x^4+px^3+qx^2+rx+s=0 \longrightarrow (1)$$

Let $a-3d,\ a-d,\ a+d$ and $a+3d$ be the roots of (1)

sum of the roots

$$a-3d+a-d+a+d+a+3d=-p$$

$$\therefore\ a=-p/4 \longrightarrow (2)$$

sum of the product of the roots taken two at a time

$$(a-3d)(a-d)+(a-3d)(a+d)+(a-3d)(a+3d)+(a-d)(a+d)$$
$$+(a-d)(a+3d)+(a+d)(a+3d)=q$$

$$\Rightarrow a^2-3ad-ad-3d^2+a^2-3ad+ad-3d^2+a^2-3ad+3ad$$
$$-9d^2+a^2-ad+ad-d^2+a^2-ad+3ad-3d^2+a^2+ad$$
$$+3ad+3d^2=q$$

$$\Rightarrow 6a^2-10d^2=q$$

$$\Rightarrow 10d^2=\frac{6p^2}{16}-q$$
$$=\frac{3}{8}p^2-q \longrightarrow (3)$$

$6a^2-10d^2=$ for two

$4a^3-20ad^2=$ for three

sum of the product of the roots taken three at a time

$$(a-3d)(a-d)(a+d)+(a-3d)(a+d)(a+3d)+(a-3d)(a-d)(a+3d)$$
$$+(a-d)(a+d)(a+3d)=-r$$

$$\Rightarrow 4a^3-20ad^2=-r$$

$$\Rightarrow -4\cdot\frac{p^3}{64}-20\left(-p/4\right)\left(\frac{3p^2}{8}-q\right)=-r$$

$$\Rightarrow -\frac{p^3}{16}+\frac{3p^3}{16}+\frac{pq}{2}=-r$$

$$\Rightarrow 2p^3-8pq=-16r$$

$$\Rightarrow p^3-4pq+8r=0$$

(proved)

...d parts,

Let a/b^3, $\frac{a}{b}$, ab and ab^3 be the roots of (1)

sum of the roots,

$$\frac{a}{b^3}+\frac{a}{b}+ab+ab^3=-p$$

$$\Rightarrow a\left(\frac{1}{b^3}+\frac{1}{b}+b+b^3\right)=-p$$

$$\Rightarrow \frac{1}{b^3}+\frac{1}{b}+b+b^3=-p/a$$

sum of the product of the roots taken three at a time.

$$\frac{a}{b^3}\cdot\frac{a}{b}\cdot ab+\frac{a}{b^3}\cdot\frac{a}{b}\cdot ab^3+\frac{a}{b^3}ab\cdot ab^3+ab\cdot\frac{a}{b}\cdot ab^3=-r$$

$$\Rightarrow a^3\left(b+\frac{1}{b}+b^3+\frac{1}{b^3}\right)=-r \Rightarrow a^3 \times -p/a=-r$$

$$\therefore p=\frac{r}{a^2} \longrightarrow (2)$$

product of the roots $\frac{a}{b^3}, \frac{a}{b}, ab, ab^3 = s$

$$\Rightarrow a^4=s \Rightarrow a^2=\sqrt{s}$$

Now, from (2) we get, $p=\frac{r}{\sqrt{s}} \Rightarrow p\sqrt{s}=r$ (proved)

(V.I) 2a 15. If the roots of the equation $x^n-1=0$ are $1, \alpha, \beta, \gamma$ ...

Show that $(1-\alpha)(1-\beta)(1-\gamma)\cdots=n$

soln: Since $1, \alpha, \beta, \gamma$ ... are the roots of the equation

$x^n-1=0$, we can write

$$x^n-1=(x-1)(x-\alpha)(x-\beta)(x-\gamma)\cdots$$

$$\Rightarrow \frac{x^n-1}{x-1}=(x-\alpha)(x-\beta)(x-\gamma)\cdots$$

$$\Rightarrow x^{n-1}+x^{n-2}+\cdots+x^2+x+1=(x-\alpha)(x-\beta)(x-\gamma)\cdots$$

putting $x=1$

$1+1+1+ \cdots +1$ (n terms) $= (1-\alpha)(1-\beta)(1-\gamma)\cdots$

$\therefore (1-\alpha)(1-\beta)(1-\gamma)\cdots = n$ (proved)

If a, b, c are the roots of the equation $x^3 - px^2 + qx - r = 0$. find the value of

20. $\sum a^2b^2$ 21. $(b+c)(c+a)(a+b)$

22. $\sum \left(b/c + \frac{c}{b}\right)$ 23. $\sum a^2 b$

Sol^n: The given equation $x^3 - px^2 + qx - r = 0 \longrightarrow (1)$

a, b, c are the roots of (1)

sum of the roots of (1)

$a+b+c = p \longrightarrow (2)$

sum of the product of the roots taken two at a time

$ab + bc + ca = q \longrightarrow (3)$

product of the roots

$abc = r \longrightarrow (4)$

20. $\sum a^2b^2 = a^2b^2 + b^2c^2 + c^2a^2$

$= (ab+bc+ca)^2 - 2abc(a+b+c)$

$= q^2 - 2rp$ (Ans)

$(b+c)(c+a)(a+b)$

$= (bc+ab+c^2+ac)(a+b)$

$= abc + a^2b + ac^2 + a^2c + b^2c + ab^2 + bc^2 + abc$

$= ab(a+b+c) + bc(a+b+c) + ca(a+b+c) - abc$

$= (a+b+c)(ab+bc+ca) - abc$

$= pq - r$ (Ans).

22. (V.V.I) $\sum \left(\frac{b}{c}+\frac{c}{b}\right) = \sum \frac{b^2+c^2}{bc} = \frac{b^2+c^2}{bc} + \frac{a^2+b^2}{ab} + \frac{a^2+c^2}{ac}$

$= \frac{a(b^2+c^2)+c(a^2+b^2)+b(a^2+c^2)}{abc}$

$= \frac{ab^2+ac^2+a^2c+b^2c+a^2b+bc^2}{abc}$

$= \frac{ab(a+b+c)+bc(a+b+c)+ca(a+b+c)-3abc}{abc}$

$= \frac{(a+b+c)(ab+bc+ca)-3abc}{abc}$

$= \frac{pq-3r}{r}$ (Ans).

23. $\sum a^2b = a^2b + a^2c + b^2c + b^2a + c^2a + c^2b$

$= (a+b+c)(ab+bc+ca) - 3abc$

$= pq - 3r$ (Ans).

If a, b, c, d are the roots of $x^4 + px^3 + qx^2 + rx + s = 0$ find the value of

24. $\sum a^2bc$ 25. $\sum a^4$

soln: The given equation

$$x^4 + px^3 + qx^2 + rx + s = 0 \longrightarrow (1)$$

sum of the roots $a+b+c+d = -p$

sum of the product of the roots taken two at a time

$$ab + ac + ad + bc + bd + cd = q$$

sum of the product of the roots taken three at a time

$$abc + abd + acd + bcd = -r$$

product of the roots $abcd = s$

24. $\sum a^2bc = a^2bc + a^2cd + a^2bd + b^2cd + b^2ac + b^2ad + c^2ad$
$+ c^2bd + c^2ab + d^2ab + d^2bc + d^2ca$

$= abc(a+b+c+d) + abd(a+b+c+d) + acd(a+b+c+d)$
$+ bcd(a+b+c+d) - 4abcd$

$= (a+b+c+d)(abc + abd + acd + bcd) - 4abcd$

$= (-p)(-r) - 4s$

$= pr - 4s$ (proved)

C-102-16574

MINTU

App. math.

25. Solve— Itself - same as 21. (EX-XXXV.a) - By synthetic division or Newton method.

Example: Show that the equation

$$\frac{A^2}{x-a}+\frac{B^2}{x-b}+\frac{C^2}{x-c}+\cdots+\frac{H^2}{x-h}=K$$ has no imaginary roots.

Solution:

If possible let $p+iq$ be the root of the equation $$\frac{A^2}{x-a}+\frac{B^2}{x-b}+\frac{C^2}{x-c}+\cdots+\frac{H^2}{x-h}=K \longrightarrow (1)$$

substitute the value of $x=p+iq$ in (1), we get

$$\frac{A^2}{p+iq-a}+\frac{B^2}{p+iq-b}+\frac{C^2}{p+iq-c}+\cdots+\frac{H^2}{p+iq-h}=K \longrightarrow (2)$$

we know that, the imaginary roots occurs in pairs. so $x=p-iq$ be the root of (1) also.

Again, substitute the value of $x=p-iq$ in (1), we get

$$\frac{A^2}{p-iq-a}+\frac{B^2}{p-iq-b}+\frac{C^2}{p-iq-c}+\cdots+\frac{H^2}{p-iq-h}=K \longrightarrow (3)$$

so (3) — (2) $\Rightarrow$

$$A^2\left(\frac{1}{p-a-iq}-\frac{1}{p-a+iq}\right)+B^2\left(\frac{1}{p-b-iq}-\frac{1}{p-b+iq}\right)+C^2\left(\frac{1}{p-c-iq}-\frac{1}{p-c+iq}\right)+\cdots+H^2\left(\frac{1}{p-h-iq}-\frac{1}{p-h+iq}\right)=0$$

$$\Rightarrow \frac{2iqA^2}{(p-a)^2+q^2}+\frac{2iqB^2}{(p-b)^2+q^2}+\frac{2iqC^2}{(p-c)^2+q^2}+\cdots+\frac{2iqH^2}{(p-h)^2+q^2}=0$$

$$\Rightarrow 2iq\left(\frac{A^2}{(p-a)^2+q^2}+\frac{B^2}{(p-b)^2+q^2}+\cdots+\frac{H^2}{(p-h)^2+q^2}\right)=0$$

$$\Rightarrow q\left(\frac{A^2}{(p-a)^2+q^2}+\frac{B^2}{(p-b)^2+q^2}+\cdots+\frac{H^2}{(p-h)^2+q^2}\right)=0$$

which is impossible unless $q=0$.

Newton Formula :-

$$S_1 + P_1 = 0$$

$$S_2 + P_1 S_1 + 2P_2 = 0$$

$$S_3 + P_1 S_2 + P_2 S_1 + 3P_3 = 0$$

$$S_4 + P_1 S_3 + P_2 S_2 + P_3 S_1 + 4P_4 = 0$$

$$S_5 + P_1 S_4 + P_2 S_3 + P_3 S_2 + P_4 S_1 + 5P_5 = 0$$

$$S_6 + P_1 S_5 + P_2 S_4 + P_3 S_3 + P_4 S_2 + P_5 S_1 + 6P_6 = 0$$

MINTU

App. Math