Find ~~the~~ $\frac{dy}{dx}$

① $x^x + y^x = 1$

② Diff. $x \sin^{-1} x$ with respect to $\sin^{-1} x$.

③ Diff $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$ w.r. t $\tan^{-1} x$.

④ $(\sin x)^{\cos x} + (\cos x)^{\sin x} =$

⑤ $\sin y = x \sin(a+y)$

Soln :

$$x = \frac{\sin y}{\sin(a+y)}$$

Diff. w.r to y

$$\frac{dx}{dy} = \frac{\sin(a+y)\cos y - \sin y \cos(}{\sin^2(a+y)}$$

$$\frac{dz}{dy} = \frac{\sin(a+y-y)}{\sin^2(a+y)}$$

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin(a)}$$

(4) Soln:

$$(\sin x)^{\cos x} + (\cos x)^{\sin x} = y$$

Let, $y_1 + y_2 = y$

$$\frac{dy_1}{dx} + \frac{dy_2}{dx} = \frac{dy}{dx} \quad \text{---} \quad (i)$$

where,

$$y_1 = (\sin x)^{\cos x}$$

Taking log on both sides.

$$\log y_1 = \log (\sin x)^{\cos x}$$

$$= \cos x \log \sin x$$

Diff. w.r. to x

~~$\log y_1 =$~~

$$\log \frac{1}{y_1} = \cos x \frac{\cos x}{\sin x} + -\sin x$$

$$= \cos x \cot x - \sin x \log(\sin x)$$

$$\therefore \frac{dy_1}{dx} = y_1 \left(\cos x \cot x - \sin x \log(\sin x) \right)$$

$$\frac{dy_1}{dx} = (\sin x)^{\cos x} \left(\cos x \cot x - \sin x \log(\sin x) \right)$$

d.

~~(2) $\tan^{-1}\frac{\sqrt{1+x^2}-1}{x}$~~

(2) $x^{\sin^{-1}x}$

Soln:

Let, $y = x^{\sin^{-1}x}$

and $z = \sin^{-1}x$

Let, $\log y = \sin^{-1}x \log x$

or, Diff. w.r. to x

$$\text{or, } \frac{1}{y}\frac{dy}{dx} = \sin^{-1}x \cdot \frac{1}{x} + \frac{1}{\sqrt{1-x^2}}\log x$$

$$\text{or, } \frac{dy}{dx} = y\left(\frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right)$$

$$\therefore \frac{dy}{dx} = x^{\sin^{-1}x}\left(\frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right) \quad \text{---(1)}$$

again,

$$z = \sin^{-1} x$$

Diff. w.r.to x

$$\frac{dz}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dx}{dz} = \sqrt{1-x^2} \quad \text{---(II)}$$

$$\frac{dy}{dz} = \frac{dy}{dx} \cdot \frac{dx}{dz}$$

$$= x \sin^{-1} x \left\{ \frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1-x^2}} \right\} \sqrt{1-x^2}$$

(using (I) and (II))

$$= x \sin^{-1} x \left\{ \frac{\sqrt{1-x^2}}{x} \sin^{-1} x + \log x \right\}$$

Ans