

# **Knowledge Representation & Predicate Logic**

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# Knowledge Representation

- Knowledge representation (KR) is an important issue in both cognitive science and artificial intelligence.
  - ❑ In cognitive science, it is concerned with the way people store and process information.
  - ❑ In artificial intelligence (AI), main focus is to store knowledge so that programs can process it and achieve human intelligence.

# Knowledge Representation

- A knowledge representation is most fundamentally a *substitute* for the thing itself, used to enable an entity to determine consequences by **reasoning** about the world.
- **Reasoning** is the use of symbolic representations of some statements in order to derive new ones.

# What is predicate logic (PL)?

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- A **predicate logic** is an expression of one or more variables defined on some specific domain. A **predicate** with variables can be made a proposition by either assigning a value to the variable or by quantifying the variable.

# Predicate logic (PL):

- A **predicate** is a statement that contains variables (**predicate** variables), and they may be true or false depending on the values of these variables.
- **Predicate logic** is the generic term for symbolic formal systems like first-order **logic**, second-order **logic**, many-sorted **logic**, or infinitary **logic**.

# Predicate Logic (PL):

- The Predicate Logic has three logical notions.
  - 1) Term,
  - 2) Predicate
  - 3) Quantifier

## (1). Term:

- a constant (single individual or concept i.e., 5, john etc.),
- a variable that stands for different individuals
- n-place function  $f(t_1, \dots, t_n)$ , where  $t_1, \dots, t_n$  are terms.
- A function is a mapping that maps **n terms** to a term.

# Predicate Logic (PL):

- **Predicate:**

- a relation that maps  $n$  terms to a truth value (T) or false value (F).

- **Quantifier:**

- Universal ( $\forall$ ) quantifier
- Existential ( $\exists$ ) quantifier

➤  $\forall$  and  $\exists$  are used for conjunction with variables.

# Examples

- “*x loves y*” is represented as **LOVE**(*x*, *y*) which maps it to true or false, when *x* and *y* get instantiated to actual values.
  - “*john’s father loves john*” is represented as **LOVE**(*father*(john), john).
- ❖ Here *father* is a **function** that maps *john* to his father.



# Examples

- *x is greater than y* is represented in predicate calculus as **GT(x, y)**.
- It is defined as follows:
$$\begin{aligned}\text{GT}(x, y) &= \text{T, if } x > y \\ &= \text{F, otherwise}\end{aligned}$$
- Symbols like **GT** and **LOVE** are called **predicates**.

❖ Predicates two terms and map to T or F depending upon the values of their terms.

## Examples – Cont..

- Translate the sentence "**Every man is mortal**" into Predicate formula.
- Representation of statement in predicate form
  - ❖ "**x is a man**" and "**MAN(x)**,
  - ❖ "**x is mortal**" by **MORTAL(x)**
- Every man is mortal :  
 **$(\forall x) (MAN(x) \rightarrow MORTAL(x))$**

Here,  $\forall x$  is read as "**for all x**" and  $\rightarrow$  is read as "**implies**".

# Syntax and semantics for Propositional Logic

- **Valid statements or sentences in PL(Predicate Logic) are determined according to the rules of propositional syntax.**
- This syntax governs the combination of basic building blocks such as propositions and logical connectives.
- **Propositions are elementary atomic sentences.**

# Cont.....

- **Propositional Logic may be either true or false but may take on no other value.**
- **Examples (Simple propositions):**
  - ❖ It is raining.
  - ❖ It is a shining day.
  - ❖ Snow is white.
  - ❖ Snow is black.
  - ❖ People live on the Earth.
  - ❖ People live on the Moon.

# Cont....

- **Examples (Compound propositions):**
  - ❖ It is raining and the wind is blowing.
  - ❖ If you study hard you will be rewarded.
  - ❖ The sum of 10 and 20 is not 40.
  - ❖ The sum of 20 and 10 is 40.
- T and F are special symbols having the values true and false.

# Conti...

- **Logical Connectives:**

Symbol	Meaning
$\sim$	for not or <b>negation</b>
$\&$	for <b>and</b> or <b>conjunction</b>
$\vee$	For <b>or</b> or <b>disjunction</b>
$\rightarrow$	For <b>if ... then</b> or <b>implication</b>
$\leftrightarrow$	For <b>if and only if</b> or <b>double implication</b>

# Syntax

- The syntax of PL is defined recursively as follows:
- T and F are formulas.
- IF P and Q are formulas, the following are also formulas:
- $(\sim P)$
- $(P \& Q)$
- $(P \vee Q)$
- $(P \rightarrow Q)$
- $(P \leftrightarrow Q)$

# Example

- Represent the following facts in predicate logic:
- *(i). All employees earning Tk. 3,00,000/= or more per year have to pay taxes.*
- $\forall x ((E(x) \ \& \ GE(i(x), 300000)) \rightarrow T(x))$
- *(ii). People only try to assassinate rulers they are not loyal to.*
- $\exists y: \forall x : \text{person}(x) \wedge \text{ruler}(y) \wedge \text{tryassassinate}(x, y) \rightarrow \neg \text{loyalto}(x, y)$
- *(iii). John likes all kinds of food.*
- *Like (john, all-kinds-of-food)*



# Assignment-1

Write PL for the following sentences:-

1. Every elephant is gray.
2. There is a white alligator.
3. All students are smart.
4. Every student in this class has visited India and Nepal.
5. Some roses are black.

# Semantics

- **The semantics or meaning of a sentence is just the value true or false: that is, it is an assignment of a truth value to the sentences.**
- An interpretation for a sentence or group of sentences is an assignment of the truth value to each propositional symbol.

# Conti...

- **Example:** Consider the statement  $= (P \ \& \ \sim Q)$
- Clearly, there are four distinct interpretations for this sentences.

Interpretation	P	Q
1	True	False
2	True	True
3	False	True
4	False	False

# Semantic Rules for statements

Consider **t** and **t'** denotes true statements, **f** and **f'** denotes false statements, and **a** is any statement.

Rules Number	True Statements	False Statements
1.	$T$	$F$
2.	$\neg f$	$\neg t$
3.	$t \& t'$	$f \& a$
4.	$t \vee a$	$a \& f$
5.	$a \vee t$	$f \vee f'$
6.	$a \rightarrow t$	$t \rightarrow f$
7.	$f \rightarrow a$	$t \leftrightarrow f$
8.	$t \leftrightarrow t'$	$f \leftrightarrow t$
9.	$f \leftrightarrow f'$	

## Example:

- Let  $I$  assign true to  $P$ , false to  $Q$  and false to  $R$  in statement  $((P \ \& \ \neg Q) \rightarrow R) \vee Q$ .
- What is the meaning of the statement?

### Answer:

- Rule 2 gives  $\neg Q$  as true.
- Rule 3 gives  $(P \ \& \ \neg Q)$  as true.
- Rule 6 gives  $(P \ \& \ \neg Q) \rightarrow R$  as false.
- Rule 5 gives the statement  $((P \ \& \ \neg Q) \rightarrow R) \vee Q$  value as false.

# Assignment-2

- Find the meaning of the statement  
 $(\neg P \vee Q) \wedge R \rightarrow S \vee (\neg R \wedge Q)$   
for each of the interpretations given below.
- (a).  $I_1$  : P is true, Q is true, R is false, S is true.
- (b).  $I_2$  : P is true, Q is false, R is true, S is true.

• **THE END**

• **THANKS**