Data Structure
Graph

Prepared By Manik Hosen

Graph Terminology

- Question: Define the following graph terms: (i) Adjacent Nodes (ii) Cycle (iii) Connected graph (iv) Weighted graph.
- Ans:

i. Adjacent Nodes: If e=(u,v) where u and v are endpoints of e then u and v are called Adjacent Nodes.
ii. Cycle: A cycle is a closed simple path with length 3 or more.
iii. Connected graph: A graph G is said to be connected if there is a path between any two of its nodes.
iv. Weighted graph: A graph G is said to be weighted if each edge e in G is assigned a nonnegative numerical value.

Sequential Representation

- Question: Discuss the sequential representation of graph with example.
- Ans: There are two standard ways of maintaining a graph G in the memory of computer. One way, called the sequential representation of G, is means of its adjacency matrix A.
- As example: Suppose G is a simple directed graph with 4 nodes then the sequential representation will be like: $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$

Adjacency Matrix

- Question: Consider the following adjacency matrix:
- $A=\begin{pmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}$. Now find out A^2, A^3, A^4, B_4 and from that make the path matrix and tell whether this is strongly connected or not.
- Ans: $A^2 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

$$A^3 = \begin{pmatrix} 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{pmatrix}$$

Adjacency Matrix

$$A^4 = \begin{pmatrix} 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix}$$

$$B_4 = \begin{pmatrix} 7 & 7 & 8 & 8 \\ 8 & 8 & 7 & 7 \\ 7 & 7 & 8 & 8 \\ 8 & 8 & 7 & 7 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Hence, This is strongly connected.

Directed Graph

- Question: What is Directed Graph? Explain.
- A directed graph G is that which has multiple edges and loops and each edge e is identified with an ordered pair (u,v) of nodes in G .
- If G is a directed graph with a directed edge e=(u,v) then,
  i. e begins at u and ends at v.
  ii. u is the origin and v is the destination.
  iii. u is a predecessor of v and v is a successor of u.
  iv. u is adjacent to v and v is adjacent to u.

Traversing a Graph

- Question: How many ways a graph G can be traversed?
- Ans: There are two standard ways a graph G can be traversed systematically.
- Question: What is the significance of STATUS field?
- Ans: STATUS field show the state of N and G during the execution of algorithms.

Depth-First Search

- Question: Consider the adjacency list of the graph G in the following table. Find the nodes that are reachable from node C using Depth-First Search.

Node	**Adjacency**	**Node**	**Adjacency**
A	G, E	E	C
B	C	F	A, B
C	F	G	B, C, E
D	C	H	D

Depth-First Search

- Ans: We want to find all the nodes reachable from the node C. The steps of Depth-First Search are given bellow:

a) Initially, push C onto the stack as follows: STACK: C
b) Pop and print the top element C and then push onto stack all the neighbors of C as follows: Print C STACK: F
c) Pop and print the top element F and then push onto stack all the neighbors of F as follows: Print F STACK: A, B
d) Pop and print the top element B and then push onto stack all the neighbors of B as follows: Print B STACK: A
e) Pop and print the top element A and then push onto stack all the neighbors of A as follows: Print A STACK: G, E
f) Pop and print the top element E and then push onto stack all the neighbors of E as follows: Print E STACK: G
g) Pop and print the top element G and then push onto stack all the neighbors of G as follows: Print G STACK:

Now stack is empty. So depth-first search is complete. The output is C, F, B, A, E, G
Hence, The nodes C, F, B, A, E, G are reachable from C.

Graph Terminology

- Define the following terms: (i)Degrees of a node (ii)Isolated node (iii)Path (iv)Multi graph.
- Ans:
  i. The degree of a node(u) is the number of edges containing u.
  ii. If u does not belong to any edge then u is called isolated node.
  iii. A path P of length n from a node u to a node v is defined as a sequence of n+1 nodes.
  iv. A graph containing multiple edges and loops is called multi graph.

Adjacency Matrix

- Question: Consider the following adjacency matrix:
- $A=\begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{pmatrix}$. Now find out A^2,A^3,A^4,B_4 and from that make the path matrix and tell whether this is strongly connected or not.
- Ans: $A^2 = \begin{pmatrix} 3 & 1 & 2 & 3 \\ 2 & 2 & 2 & 3 \\ 2 & 1 & 3 & 3 \\ 2 & 1 & 3 & 3 \end{pmatrix}$

$$A^3 = \begin{pmatrix} 6 & 4 & 8 & 9 \\ 7 & 4 & 7 & 9 \\ 7 & 3 & 8 & 9 \\ 7 & 3 & 8 & 9 \end{pmatrix}$$

Adjacency Matrix

$$A^4 = \begin{pmatrix} 19 & 10 & 23 & 27 \\ 20 & 11 & 23 & 27 \\ 20 & 10 & 24 & 27 \\ 20 & 10 & 24 & 27 \end{pmatrix}$$

$$B_4 = \begin{pmatrix} 28 & 16 & 34 & 40 \\ 30 & 18 & 32 & 40 \\ 30 & 14 & 36 & 40 \\ 30 & 14 & 36 & 40 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Hence, This is strongly connected.

Linked Representation

- Question: Discuss the linked representation of Graph with example.
- Ans: There are two standard ways of maintaining a graph G in the memory of computer. One way, called the linked representation of G, is means of its adjacency matrix A.
- For Example: Consider a graph G. In the table bellow shows each node in G followed by its adjacency list, which is its adjacency nodes.

Node	**Adjacency**
A	G, E
B	C
C	F

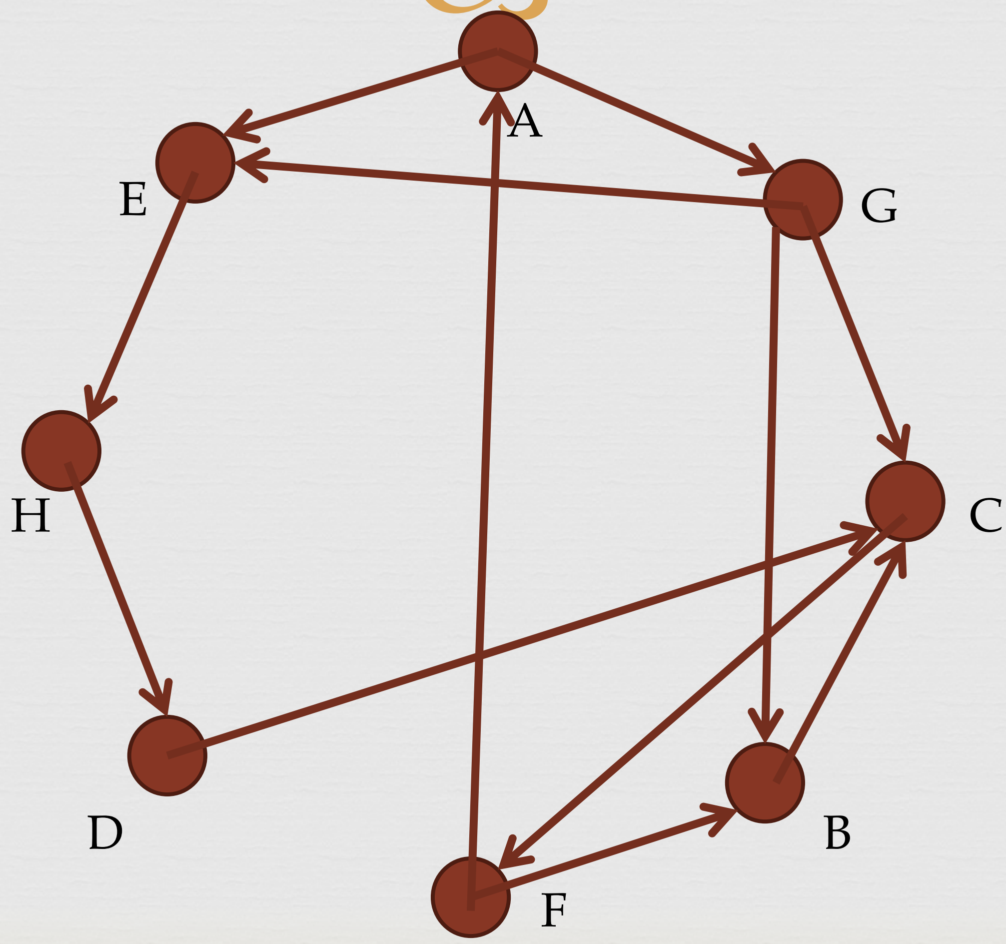
Breadth First Search

- Question: Consider the adjacency list of the graph G in the following table. Draw the graph and find out the path from A to H with minimum number of nodes along that using Breadth First Search nodes that are reachable from node C using Depth-First Search.

Node	**Adjacency**	**Node**	**Adjacency**
A	E, G	E	H
B	C	F	A, B
C	F	G	B, C, E
D	C	H	D

Breadth First Search

Graph:



Breadth First Search

- Ans: We want to find the minimum path P between A and H. The steps of Breadth First Search are given bellow:

a) Initially, add A to QUEUE and add NULL to ORIG as follows:

FRONT=1	QUEUE: A	REAR=1	ORIG:0

b) Remove the front element A from QUEUE by setting FRONT:=FRONT+1 and add to QUEUE the neighbors of A as follows:

FRONT=2	QUEUE: A, E, G	REAR=3	ORIG: 0, A, A

c) Remove the front element E from QUEUE by setting FRONT:=FRONT+1 and add to QUEUE the neighbors of A as follows:

FRONT=3	QUEUE: A, E, G, H	REAR=4	ORIG: 0, A, A, E

We stop as soon as H is added to QUEUE, since H is our final destination. Now we backtrack from H, using the array ORIG to find the path P,

Thus: $A \rightarrow E \rightarrow H$ is the required path.

Graph Terminology

- Question: Define weighted graph and directed graph with example.
- Ans:
- Weighted graph: A graph G is said to be weighted if each edge e in G is assigned a nonnegative numerical value. Fig X is a weighted graph.
- Directed graph: A directed graph G is that which has multiple edges and loops and each edge e is identified with an ordered pair (u,v) of nodes in G . Fig Y is a directed graph.

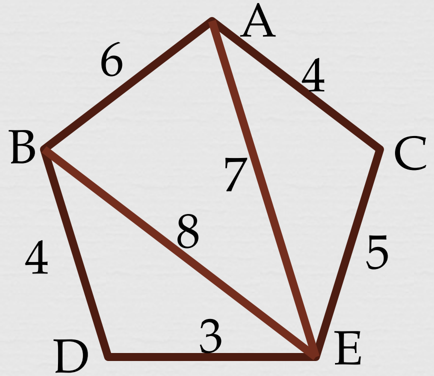


Fig: X

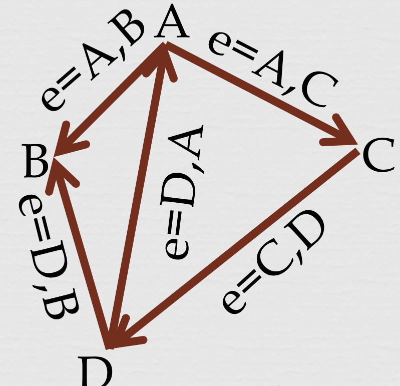


Fig: Y

Adjacency Matrix

- Question: Consider the following adjacency matrix:
- A=$\begin{pmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{pmatrix}$. Now find out A^2,A^3,A^4,B_4 and from that make the path matrix and tell whether this is strongly connected or not.
- Ans: $A^2 = \begin{pmatrix} 2 & 1 & 2 & 2 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$

$$A^3 = \begin{pmatrix} 4 & 3 & 4 & 4 \\ 0 & 1 & 0 & 0 \\ 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 \end{pmatrix}$$

Adjacency Matrix

$$A^4 = \begin{pmatrix} 8 & 7 & 8 & 8 \\ 0 & 1 & 0 & 0 \\ 4 & 4 & 4 & 4 \\ 4 & 4 & 4 & 4 \end{pmatrix}$$

$$B_4 = \begin{pmatrix} 15 & 11 & 15 & 15 \\ 0 & 4 & 0 & 0 \\ 7 & 7 & 8 & 8 \\ 8 & 8 & 7 & 7 \end{pmatrix}$$

$$P = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

Hence, This is not strongly connected.

Warshall's Algorithm

- Question: Use the Warshall's Algorithm to find the shortest path matrix of the weighted matrix given below:

$$W = \begin{pmatrix} 6 & 8 & 0 & 0 \\ 3 & 0 & 0 & 9 \\ 5 & 8 & 3 & 6 \\ 6 & 2 & 3 & 0 \end{pmatrix}$$

Ans: Applying the Warshall's Algorithm, we obtain the following matrices Q_0, Q_1, Q_2, Q_3, $Q_4 = Q$.

$$Q_0 = \begin{pmatrix} 6 & 8 & \infty & \infty \\ 3 & \infty & \infty & 9 \\ 5 & 8 & 3 & 6 \\ 6 & 2 & 3 & \infty \end{pmatrix} \qquad Q_1 = \begin{pmatrix} 6 & 8 & \infty & \infty \\ 3 & 11 & \infty & 9 \\ 5 & 8 & 3 & 6 \\ 6 & 2 & 3 & \infty \end{pmatrix}$$

$$Q_2 = \begin{pmatrix} 6 & 8 & \infty & 17 \\ 3 & 11 & \infty & 9 \\ 5 & 8 & 3 & 6 \\ 6 & 2 & 3 & 11 \end{pmatrix} \qquad Q_3 = \begin{pmatrix} 6 & 8 & \infty & 17 \\ 3 & 11 & \infty & 9 \\ 5 & 8 & 3 & 6 \\ 6 & 2 & 3 & 11 \end{pmatrix}$$

$$Q_4 = \begin{pmatrix} 6 & 8 & 20 & 17 \\ 3 & 11 & 11 & 9 \\ 5 & 8 & 3 & 6 \\ 6 & 2 & 3 & 11 \end{pmatrix};$$

Hence Q_4 is the shortest path matrix.

Graph Terminology

- Question: Define the following terms: Connected graph, Path, Weighted graph.
- Ans:
- Connected Graph: graph G is said to be connected if there is a path between any to of its nodes.
- Path: A path P of length n from a node u to a node v is defined as a sequence of n+1 nodes.
- Weighted graph: A graph G is said to be weighted if each edge e in G is assigned a nonnegative numerical value. Fig X is a weighted graph.

Overview

- Top Topics:
  - Depth First Search
  - Breadth First Search
  - Adjacency Matrix/Path Matrix
  - Basic Terminology
  - Warshall's Algorithm
  - Traversing
  - Directed Graph