Karnagh-Map Method

A karnaugh map is a pictorial methode used to minimize boolean expression without using boolean algebra theorem and equation manipulations. It is also called veith diagram.

* k-map for two variable

where $x' = y' = 0$

$x = y = 1$

y \ x	0	1
0	$x'y'$	xy'
1	$x'y$	xy

m_0	m_1
m_2	m_3

① Example:

$F(x, y) = x'y + xy' + xy$

$\therefore F(x, y) = x + y$

y \ x	0	1
0		1
1	1	1

K map for three variables

① $F(x, y, z) = xy'z + xyz$

x \ yz	00	01	11	10
0	0	0	0	0
1	0	1	1	0

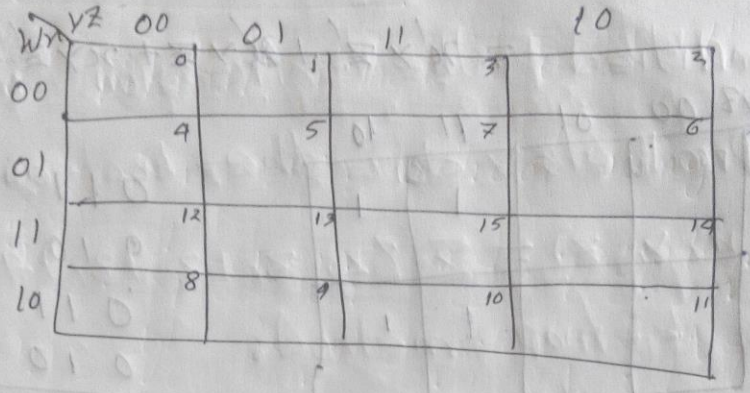
$F(x, y, z) = xz$

② $F(x, y, z) = xy'z + xyz + x'yz$

x \ yz	00	01	11	10
0	0	0	1	0
1	0	1	1	0

$F(x, y, z) = xz + yz$

* k-map for four variables

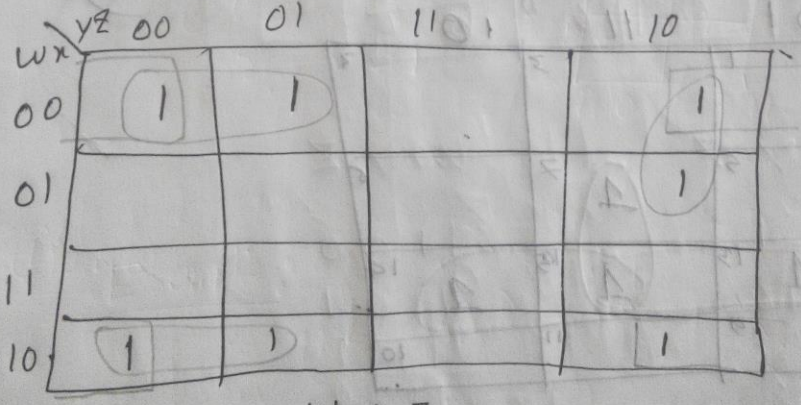


ex:10: $F(W,X,Y,Z) = W'X'Y' + X'YZ' + W XYZ + WX'Y'$

$= W'X'Y'(Z+Z') + (W+W')\cdot X'YZ' + W'XYZ' + WX'Y'(Z+Z')$

$= W'X'Y'Z + W'X'Y'Z' + WX'YZ' + W'X'YZ' + W'XYZ' + WX'YZ + WX'Y'Z'$

$=$



$F(WX,Y,Z) = W'YZ + X'Z + XY'$

WXYZ
0000
0001
1000
1001

$X'Y'$

WXYZ
0010
0110

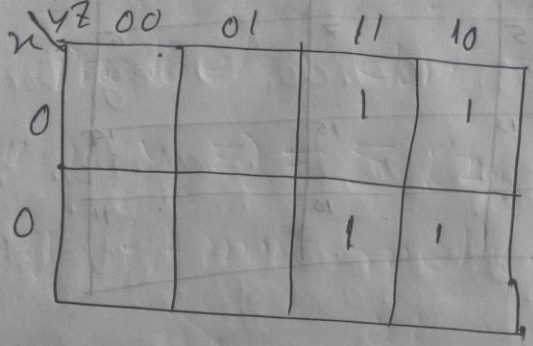
$W'YZ'$

WXYZ
0000
0010
1000
1010

$X'Z'$

* $F(x, y, z) = \Sigma(2, 3, 6, 7)$

$= x'yz' + x'yz + xyz' + xyz$

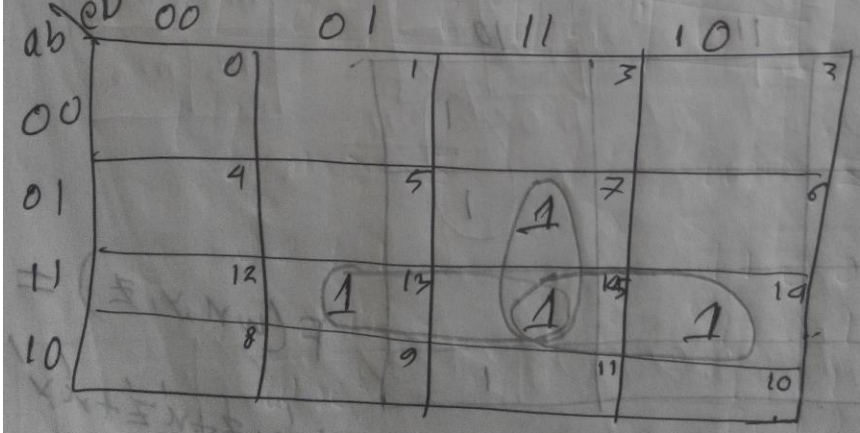


0 1 1
0 1 0
0 1 1
0 1 0

y

$F(x, y, z) = y$

* $F(a, b, e, d) = \Sigma(7, 13, 14, 15)$



0 1 1 1 1 1 1 1 1 1 1 0

bed abd abe

$F(a, b, e, d) = bed + abd + abe$

1) $F(A, B, C, D) = \Sigma(4, 6, 7, 15)$

AB \ CD	00	01	11	10
00				
01	1		1	1
11			1	
10				

0 1 1 1
1 1 1 1

B C D

0 1 1 0
0 1 0 0

A' B D'

0 1 1 1
0 1 1 0

A' B C

$F(A, B, C, D) = BCD + A'BD' + A'BC$

X X W
0 0 1 1
1 0 1 1
1 1 1 1
0 1 1 1

X W

X X W
1 1 0 0
0 1 0 0

X X W'

$XW + XXW' = 7$

Karnaugh – map method :

① Simplified the expression in sum of products for the boolean function $F(w, x, y, z) = \Sigma(2, 3, 12, 13, 14, 15)$ using k-map method.

wx \ yz	00	01	11	10
00	0 (0)	0 (1)	1 (3)	1 (2)
01	0 (4)	0 (5)	0 (7)	0 (6)
11	1 (12)	1 (13)	1 (15)	1 (14)
10	0 (8)	0 (9)	0 (11)	0 (10)

2 – 1 Group

4 – 1 Group

w	x	y	z
0	0	1	1
0	0	1	0

$w'x'y$

w	x	y	z
1	1	0	0
1	1	0	1
1	1	1	1
1	1	1	0

wx

$F = w'x'y + wx$

3. Simplified the expression in sum of products for the boolean function $F(x, y, z) = x'yz + x'yz' + xy'z' + xy'z$

Ans:

x \ yz	00	01	11	10
0	0	0	1	1
1	1	1	0	0

1	1	
x	y	z
0	1	1
0	1	0

$x'y$

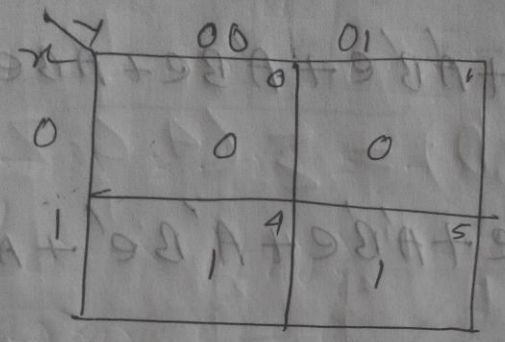
1	1	
x	y	z
1	0	0
1	0	1

xy'

$F = x'y + xy'$

Simplified the expression in sum of product for the $F(x,y) = x'y + xy'$

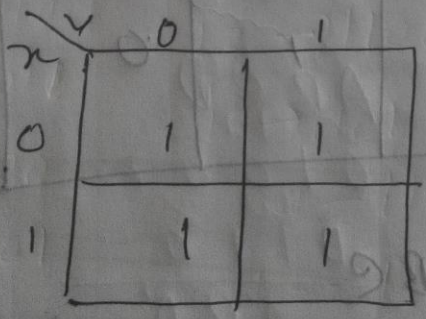
Ans: $F(x,y) = x'y' + xy'$



11
x y
100
101
---
x y'

$F = xy'$

Simplified $F(x,y) = x'y' + xy' + xy + x'y$



$F = 0$

⊞ $F(A, B, C) = A'C + A'B + AB'C + BC$

$= A'C(B+B') + A'B(C+C') + AB'C + BC(A+A')$

$= A'BC + A'B'C + A'BC + A'BC' + AB'C + ABC + A'BC$

$= A'BC + A'B'C + A'BC' + AB'C + ABC$

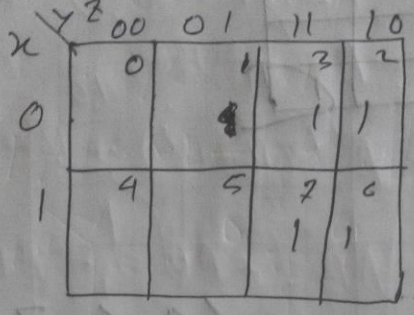
A \ BC	00	01	11	10
0	0	1	1	1
1	0	1	1	0

11
ABC
011
010
A'B

ABC
001
011
101
111
C

$F = A'B + C$

例1 $F(x, y, z) = \sum(2, 3, 6, 7)$



x y z

0 1 1

0 1 0

1 1 1

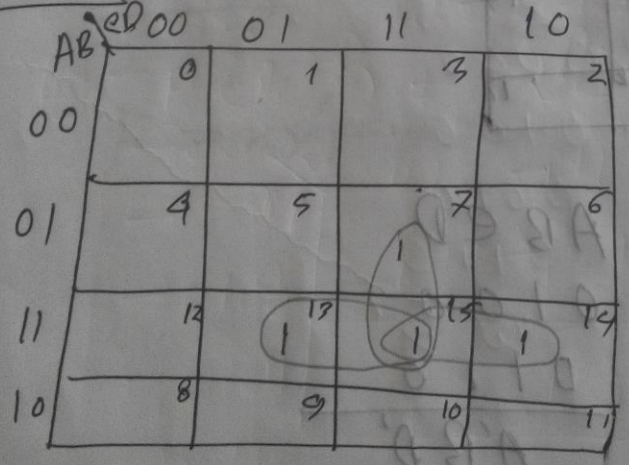
1 1 0

---

y

例2 $F(A, B, C, D) = \sum(7, 13, 14, 15)$

Ans:



A B C D

0 1 1 1

1 1 1 1

BCD

1 1 0 1

1 1 1 1

ABD

$F(A, B, C, D) = BCD + ABD + ABC$

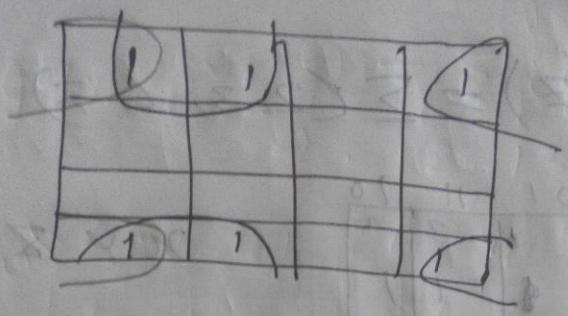
A B C D

1 1 1 1

1 1 1 0

ABC

㈣



㈤ $F(A, B, C, D) = \sum(4, 6, 7, 15)$

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

A B C D
0 1 1 1
1 1 1 1
B C D

A B C D
0 1 0 0
0 1 1 0
A' B D'

$F = BCD + A'BD'$

① $F(x, y, z) = \sum(0, 2, 4, 5, 6)$

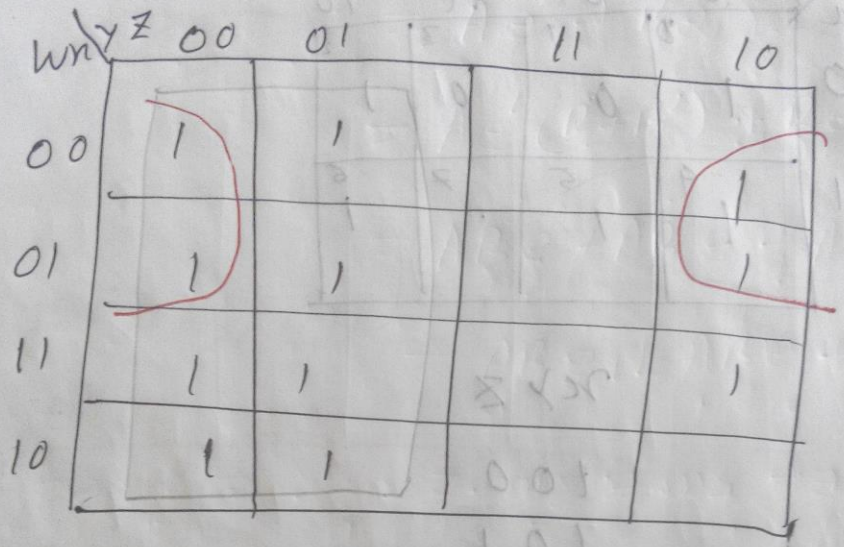
x \ yz	00	01	11	10
0	1 (0)	0 (1)	0 (3)	1 (2)
1	1 (4)	1 (5)	(7)	1 (6)

xyz
000
100
010
110
z'

xyz
100
101
xy'

$F = xy' + z'$

$F(w, x, y, z) = \sum(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$



$wxyz$

0 0 0 0
0 0 0 1
0 1 0 0
0 1 0 1
1 1 0 0
1 1 0 1
1 0 0 0
1 0 0 1

y'

$wxyz$

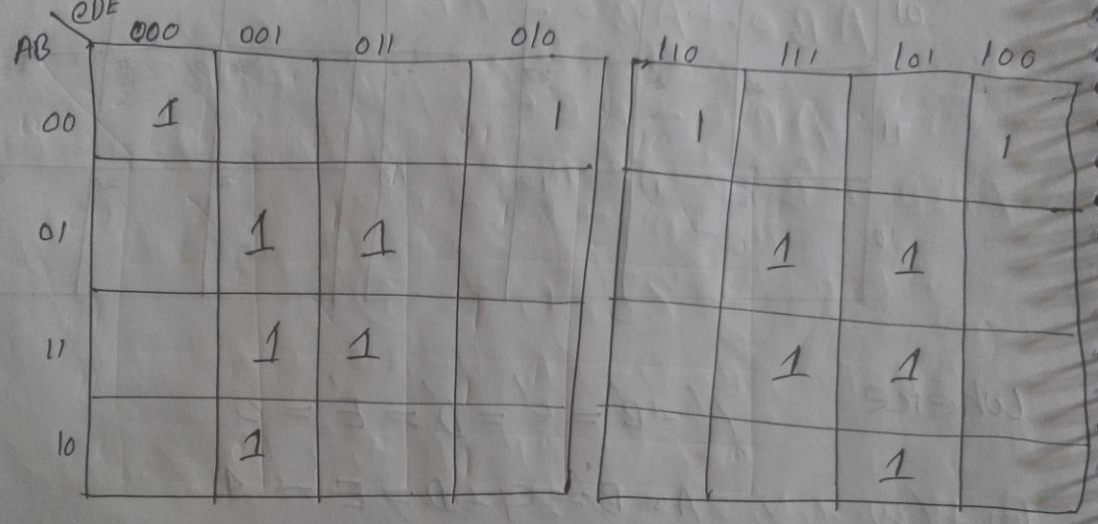
0 0 0 0
0 1 0 0
0 0 1 0
0 1 1 0

$w'z'$

$F = w'z' + y'$

Q. Simplify $B + f(ABCDE) = \Sigma(0, 2, 4, 6, 9, 11, 13, 15, 17, 21, 25, 27, 29, 31)$ using k-map.

Ans:



ABCDE
00000
00010
00110
00100
A'B'E'

ABCDE
01001
01011
11001
11011
01111
01101
11111
11101
BE

ABCDE
01001
11001
10001
01101
11101
10101
D'E

Given that,

The Boolean function $F(w, x, y, z) = \Sigma(1, 3, 7, 11, 15)$ and the Don't care condition $d(w, x, y, z) = \Sigma(0, 2, 5)$

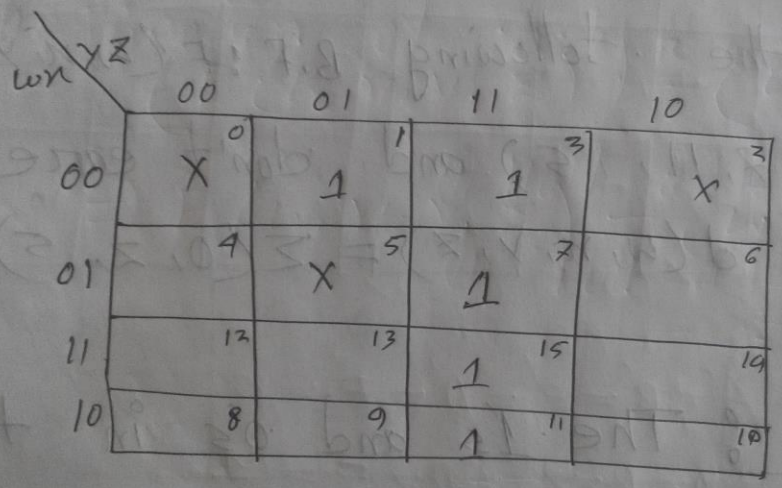


Fig: 1 Combaing 1s

w	x	y	z
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1

$w'z$

w	x	y	z
0	0	1	1
0	1	1	1
1	1	1	1
1	0	1	1

yz

$$\therefore F(w, x, y, z) = w'z + yz$$
$$= z(w' + y)$$

wx \ yz	00	01	11	10
00	X (0)	1 (1)	1 (3)	X (2)
01	0 (4)	X (5)	1 (7)	0 (6)
11	0 (12)	0 (13)	1 (15)	0 (14)
10	0 (8)	0 (9)	1 (11)	0 (10)

Fig: z: 0'

wxyz
1100
1101
1000
1001
wy'

wxyz
0000
0100
1100
1000
0010
0110
1110
1010
z