Network Theorems

Superposition Theorem

- The superposition theorem extends the use of Ohm’s Law to circuits with multiple sources.

- In order to apply the superposition theorem to a network, certain conditions must be met:

1. All the components must be **linear**, meaning that the current is proportional to the applied voltage.

Superposition Theorem

1. All the components must be **bilateral**, meaning that the current is the same amount for opposite polarities of the source voltage.

2. **Passive components** may be used. These are components such as resistors, capacitors, and inductors, that do not amplify or rectify.

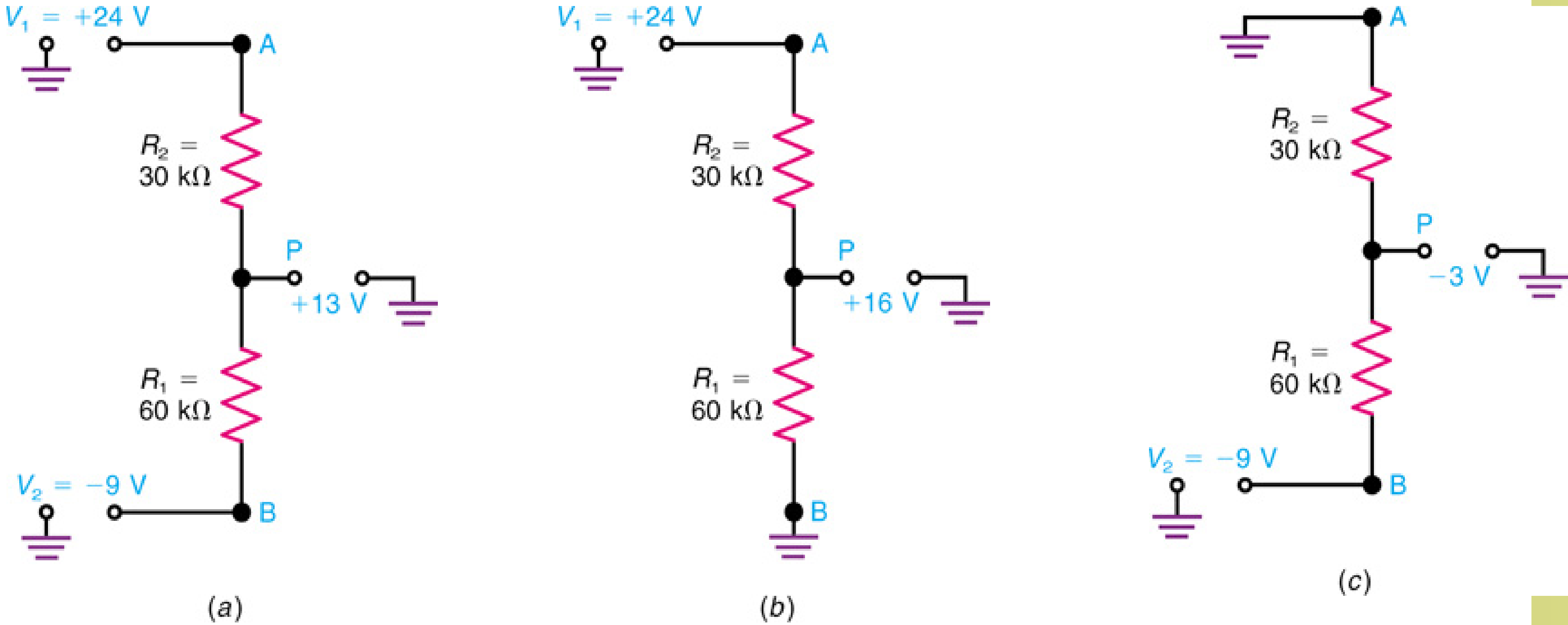
3. **Active components** may not be used. Active components include transistors, semiconductor diodes, and electron tubes. Such components are never bilateral.

Superposition Theorem

- In a linear, bilateral network that has more than one source, the current or voltage in any part of the network can be found by adding algebraically the effect of each source separately.

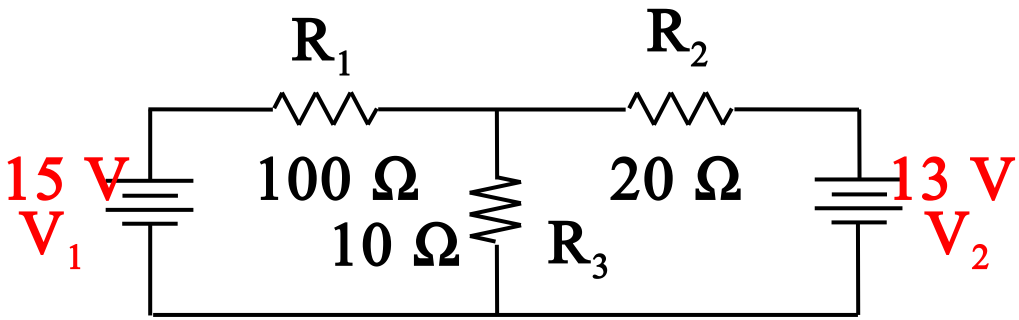
- This analysis is done by:
  - shorting each voltage source in turn.
  - opening each current source in turn.

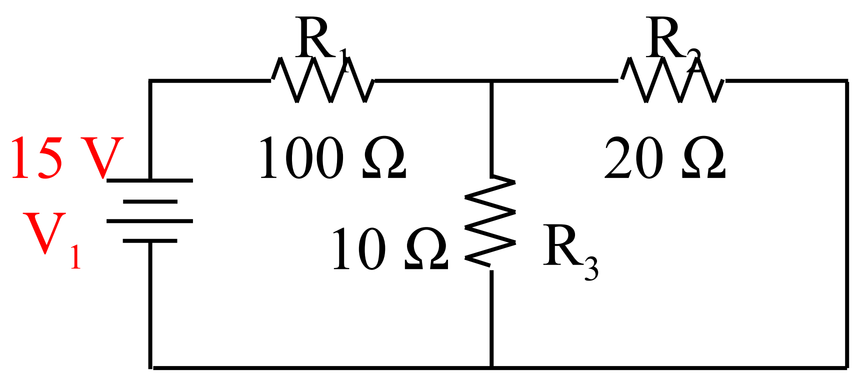
Superposition Theorem



Superposition theorem applied to a voltage divider with two sources V_1 and V_2. (*a*) Actual circuit with +13 V from point P to chassis ground. (*b*) V_1 alone producing +16 V at P. (*c*) V_2 alone producing −3 V at P.

Superposition Theorem

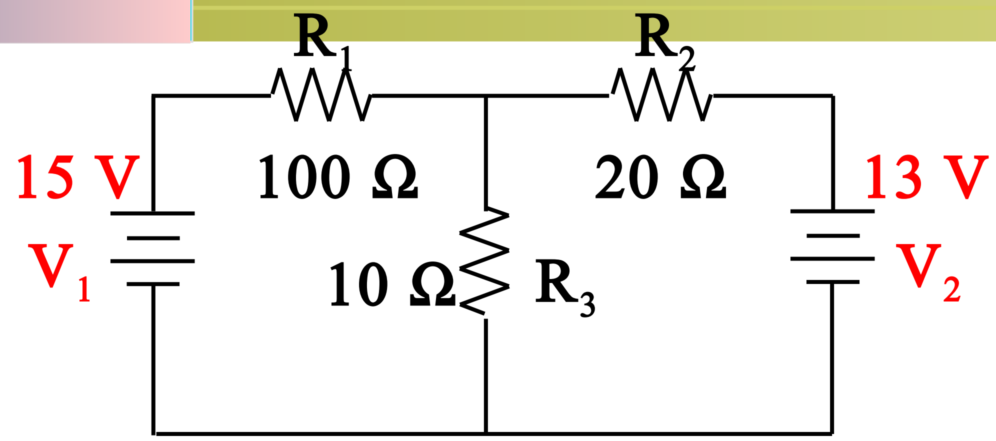


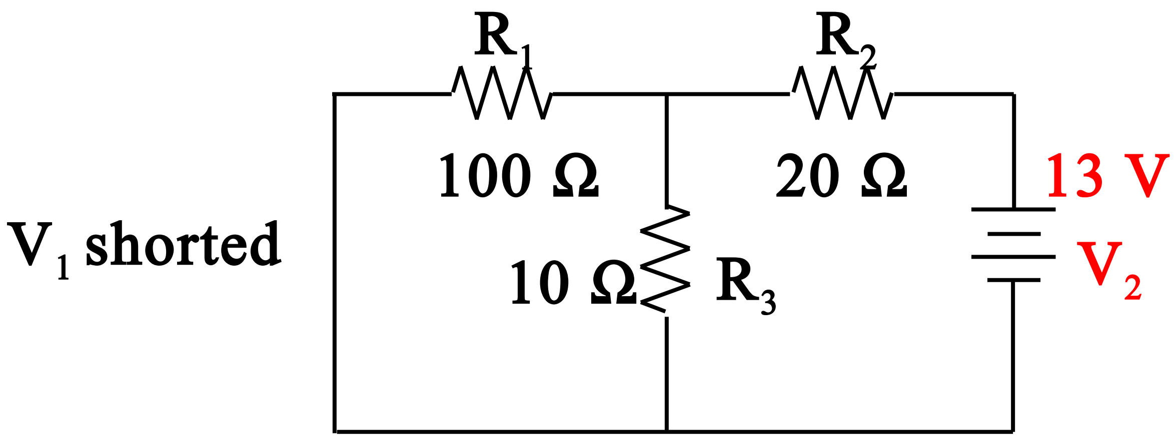


V_2 shorted

R_{EQ} = 106.7 Ω, I_T = 0.141 A and I_{R_3} = 0.094 A

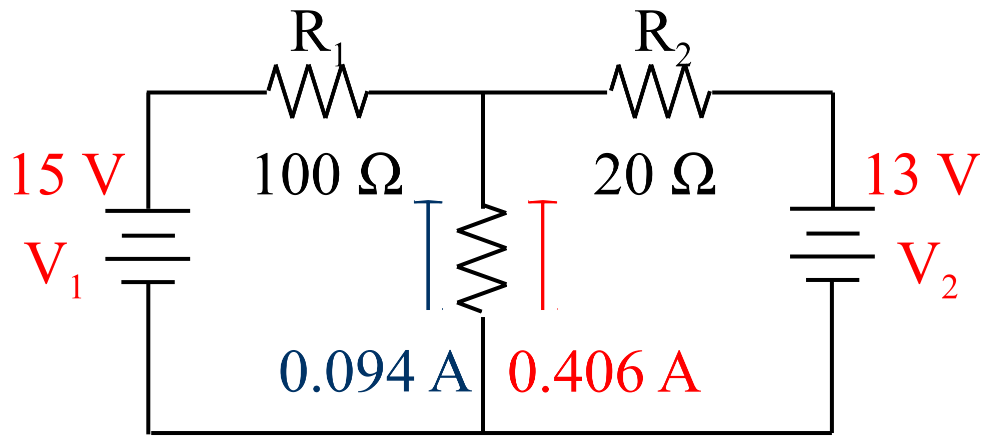
Superposition Theorem (Applied)





$R_{EQ} = 29.09\ \Omega$, $I_T = 0.447$ A and $I_{R_3} = 0.406$ A

Superposition Theorem (Applied)



With V_2 shorted

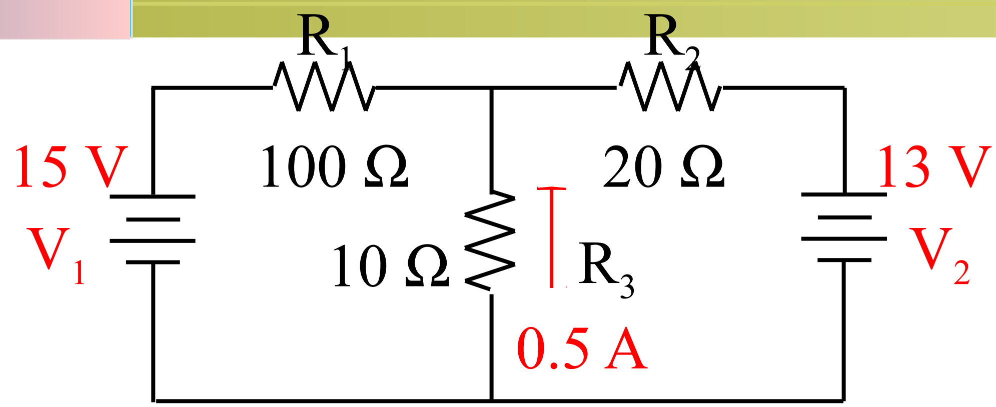
$R_{EQ} = 106.7\ \Omega$, $I_T = 0.141$ A and $I_{R_3} = 0.094$ A

With V_1 shorted

$R_{EQ} = 29.09\ \Omega$, $I_T = 0.447$ A and $I_{R_3} = 0.406$ A

Adding the currents gives $I_{R_3} = 0.5$ A

Superposition Method (Check)



With 0.5 A flowing in R_3, the voltage across R_3 must be 5 V (Ohm's Law). The voltage across R_1 must therefore be 10 volts (KVL) and the voltage across R_2 must be 8 volts (KVL). Solving for the currents in R_1 and R_2 will verify that the solution agrees with KCL.

$$I_{R_1} = 0.1 \text{ A and } I_{R_2} = 0.4 \text{ A}$$

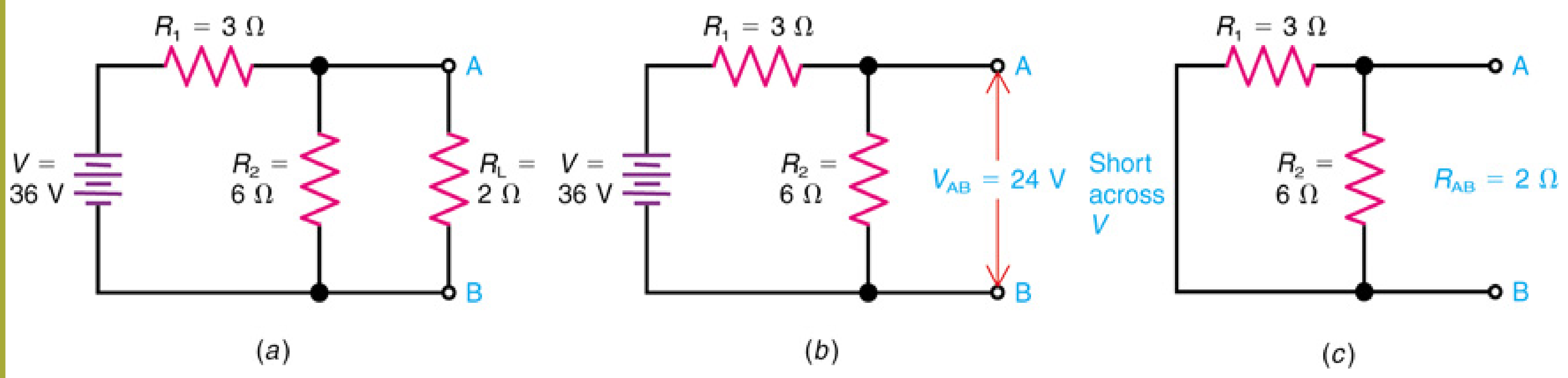
$$I_{R_3} = 0.1 \text{ A} + 0.4 \text{ A} = 0.5 \text{ A}$$

Thevenin's Theorem

- Thevenin's theorem simplifies the process of solving for the unknown values of voltage and current in a network by reducing the network to an equivalent series circuit connected to any pair of network terminals.

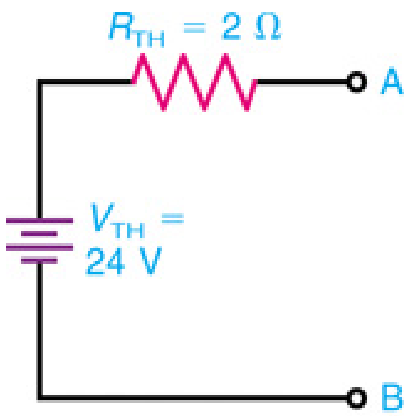
- Any network with two open terminals can be replaced by a **single voltage source (V_{TH})** and a **series resistance (R_{TH})** connected to the open terminals. A component can be removed to produce the open terminals.

Thevenin's Theorem

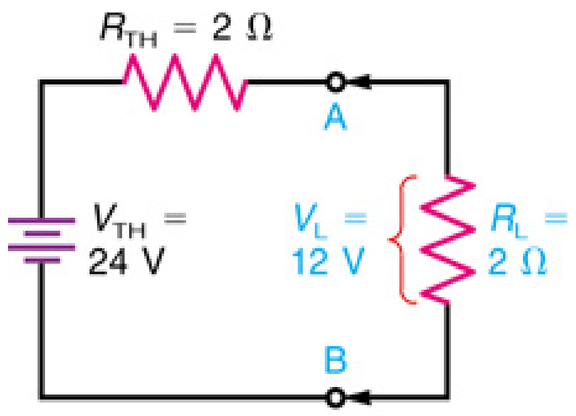


Application of Thevenin's theorem. (*a*) Actual circuit with terminals A and B across R_L. (*b*) Disconnect R_L to find that V_{AB} is 24V. (*c*) Short-circuit *V* to find that R_{AB} is 2Ω.

Thevenin's Theorem



(*d*)



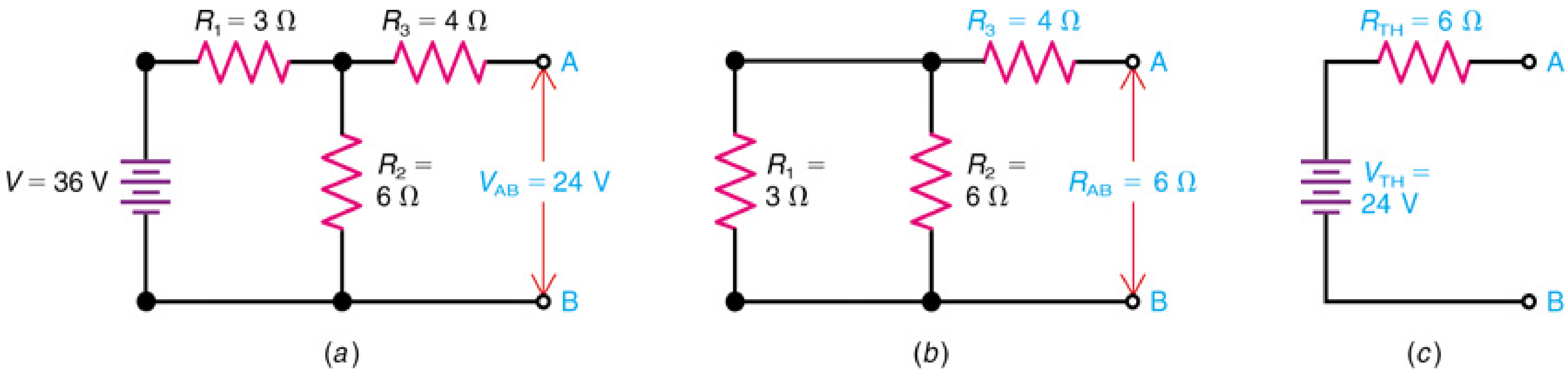
(*e*)

(*d*) Thevenin equivalent circuit. (*e*) Reconnect R_L at terminals A and B to find that V_L is 12V.

Thevenin’s Theorem

- Determining Thevenin Resistance and Voltage
  - R_{TH} is determined by shorting the voltage source and calculating the circuit’s total resistance as seen from open terminals A and B.
  - V_{TH} is determined by calculating the voltage between open terminals A and B.

Thevenin's Theorem



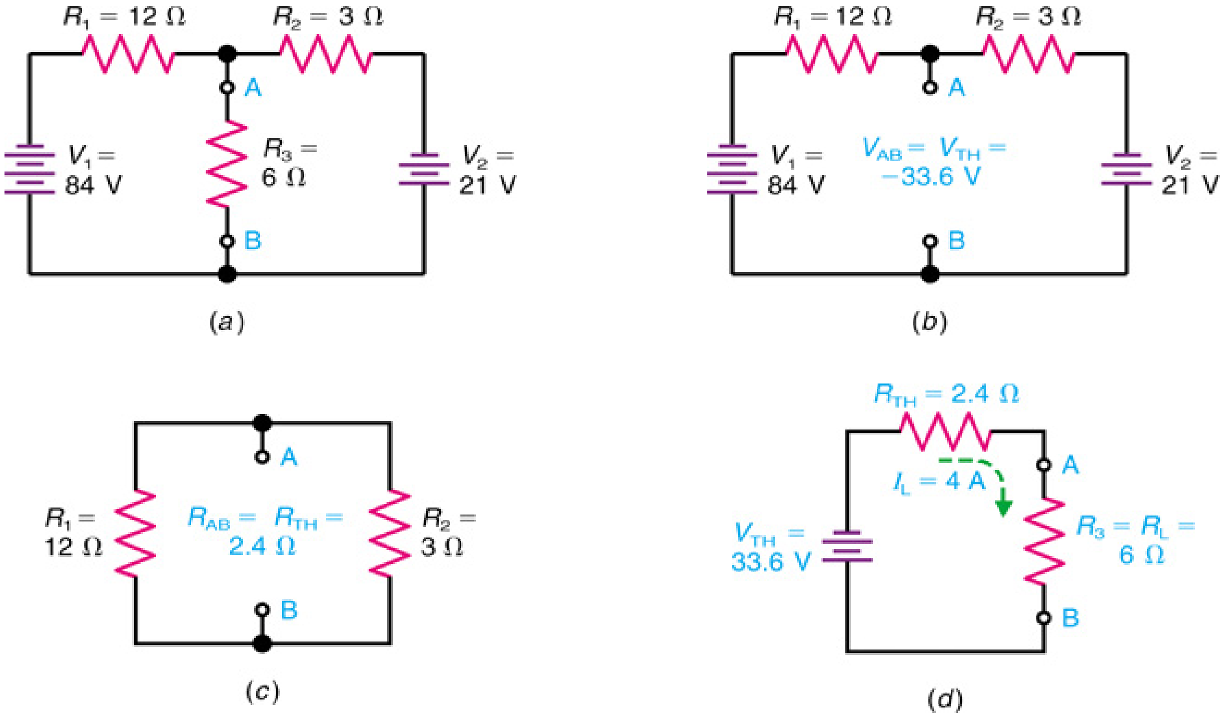
Note that $\mathbf{R_3}$ does not change the value of $\mathbf{V_{AB}}$ produced by the source V, but R_3 <u>does</u> increase the value of $\mathbf{R_{TH}}$.

(*a*) V_{AB} is 24V. (*b*) Now the R_{AB} is 2 + 4 = 6 Ω. (*c*) Thevenin equivalent circuit.

Thevenizing a Circuit with Two Voltage Sources

- The circuit in Figure can be solved by Kirchhoff's laws, but **Thevenin's theorem** can be used to find the current I_3 through the middle resistance R_3.
  - Mark the terminals A and B across R_3.
  - Disconnect R_3.
  - To calculate V_{TH}, find V_{AB} across the open terminals

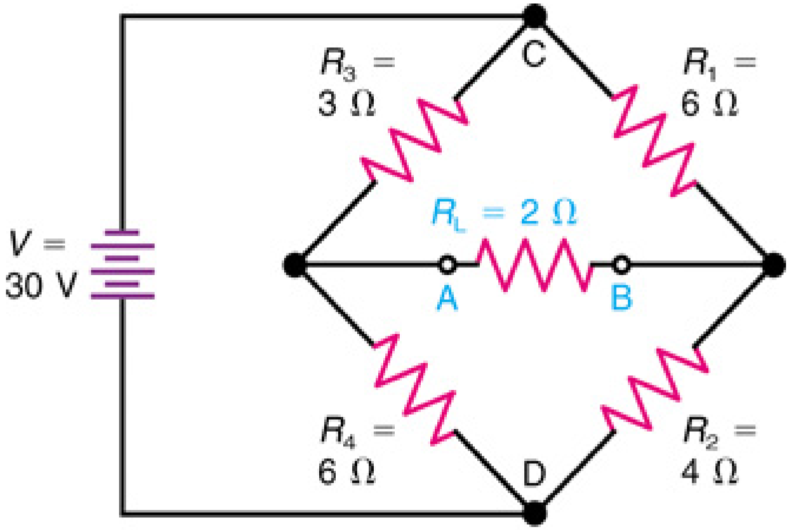
Thevenizing a Circuit with Two Voltage Sources



Thevenizing a circuit with two voltage sources V_1 and V_2. (*a*) Original circuit with terminals A and B across the middle resistor R_3. (*b*) Disconnect R_3 to find that V_{AB} is −33.6V. (*c*) Short-circuit V_1 and V_2 to find that R_{AB} is 2.4 Ω. (*d*) Thevenin equivalent with R_L reconnected to terminals A and B.

Thevenizing a Bridge Circuit

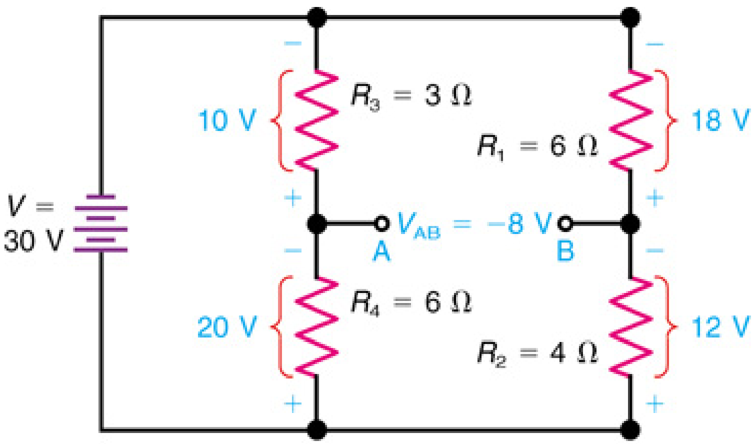
- A Wheatstone Bridge Can Be Thevenized.
  - Problem: Find the voltage drop across R_L.
  - The bridge is unbalanced and Thevenin's theorem is a good choice.
  - R_L will be removed in this procedure making **A** and **B** the Thevenin terminals.



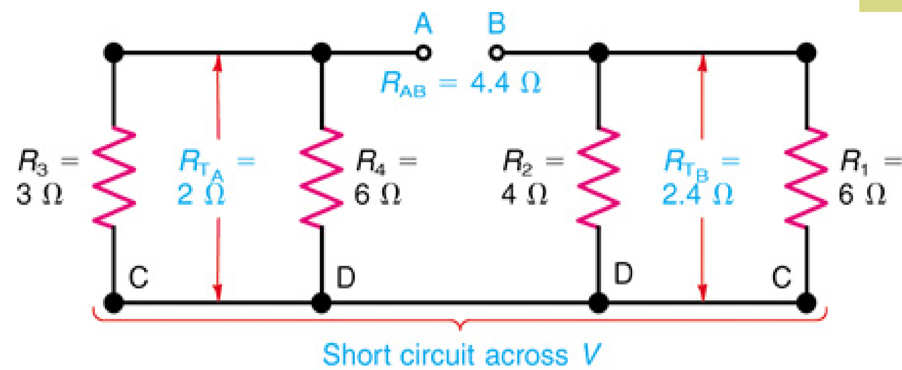
(*a*)

Thevenizing a bridge circuit. (*a*) Original circuit with terminals A and B across middle resistor R_L.

Thevenizing a Bridge Circuit



(b)



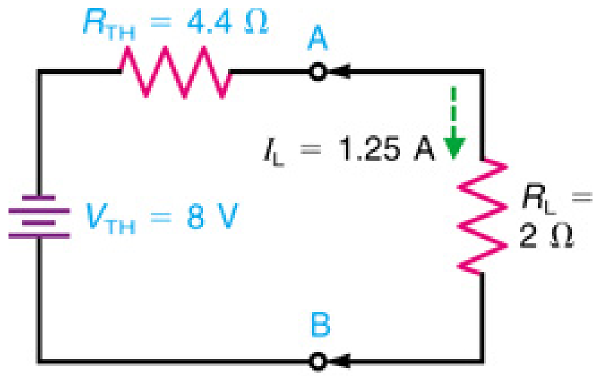
(c)

$$R_{AB} = R_{TA} + R_{TB} = 2 + 2.4 = 4.4\ \Omega$$

$$V_{AB} = -20 - (-12) = -8V$$

(*b*) Disconnect R_L to find V_{AB} of −8 V. (*c*) With source V short-circuited, R_{AB} is 2 + 2.4 = 4.4 Ω.

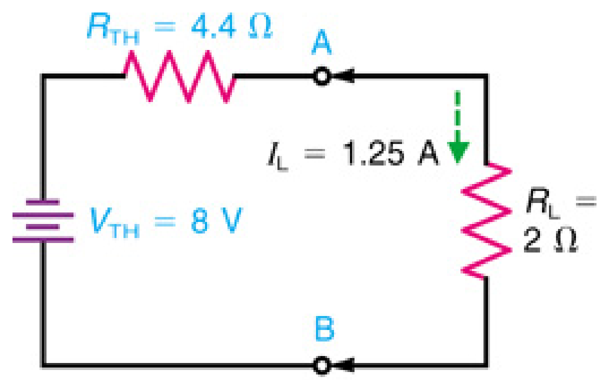
Thevenizing a Bridge Circuit



(*d*)

(*d*) Thevenin equivalent with R_L reconnected to terminals A and B.

Thevenizing a Bridge Circuit



(*d*)

(*d*) Thevenin equivalent with R_L reconnected to terminals A and B.

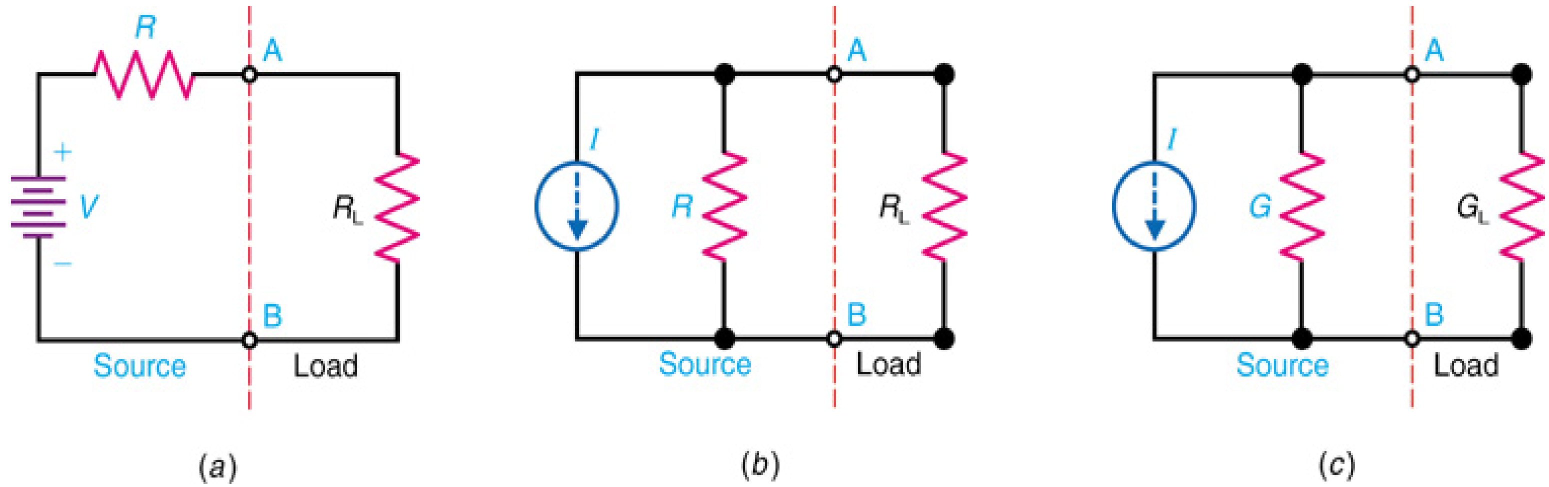
Norton’s Theorem

- Norton’s theorem is used to simplify a network in terms of currents instead of voltages.
- It reduces a network to a simple parallel circuit with a current source (comparable to a voltage source).
- Norton’s theorem states that any network with two terminals can be replaced by a single current source and parallel resistance connected across the terminals.
  - The two terminals are usually labeled something such as A and B.
  - The Norton current is usually labeled $\mathbf{I_N}$.
  - The Norton resistance is usually labeled $\mathbf{R_N}$.

Norton’s Theorem

- Example of a Current Source
  - The symbol for a current source is a circle enclosing an arrow that indicates the direction of current flow. The direction must be the same as the current produced by the polarity of the corresponding voltage source (which produces electron flow from the negative terminal).

Norton's Theorem



General forms for a voltage source or current source connected to a load R_L across terminals A and B. (*a*) Voltage source *V* with series *R*. (*b*) Current source *I* with parallel *R*. (*c*) Current source *I* with parallel conductance *G*.

Norton’s Theorem

- Example of a Current Source
  - In this example, the current *I* is provided constant with its rating regardless of what may be connected across output terminals A and B. As resistances are added, the current divides according to the rules for parallel branches (inversely to branch resistances but directly with conductances).
  - Note that unlike voltage sources, current sources are killed by making them open.

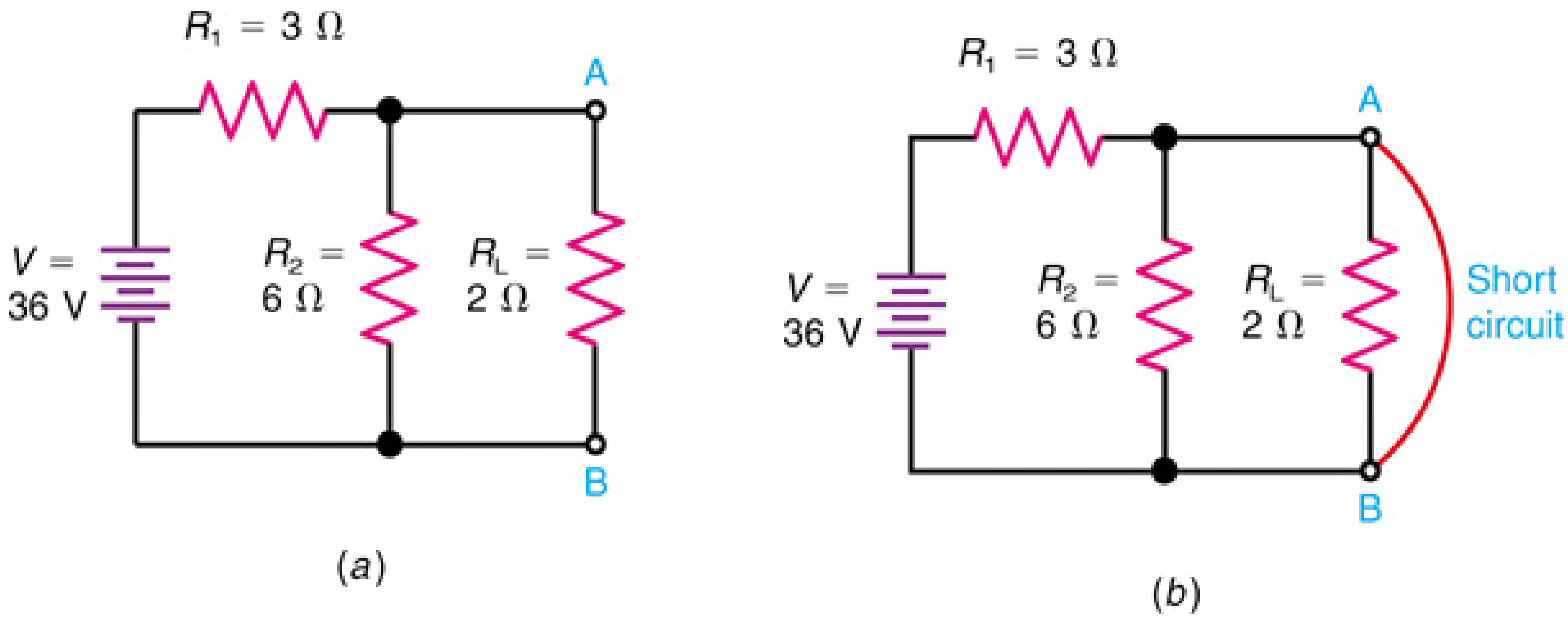
Norton's Theorem

- Determining Norton Current and Voltage
  - I_N is determined by calculating the current through a short placed across terminals A and B.

  - R_N is determined by shorting the voltage source and calculating the circuit's total resistance as seen from open terminals A and B (same procedure as for R_{TH}).

Norton’s Theorem

- A Wheatstone Bridge Can Be Nortonized.

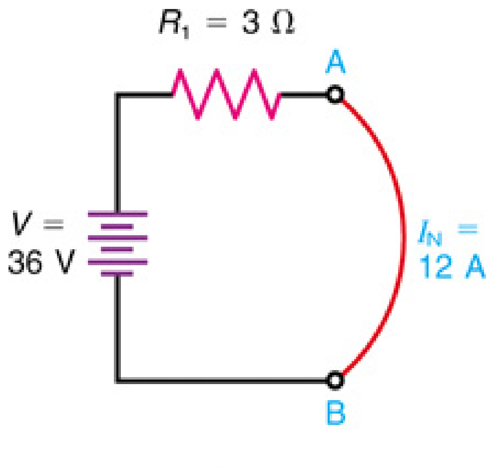


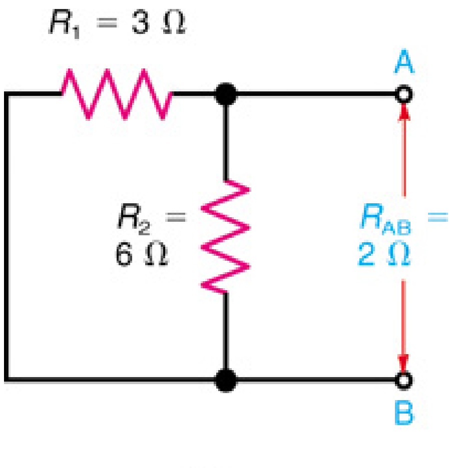
(*a*) Original circuit. (*b*) Short circuit across terminals A and B.

Norton’s Theorem

- The Norton Equivalent Circuit
  - Replace R_2 with a short and determine I_N.
  - Apply the current divider.
  - Apply KCL.
  - $R_N = R_{TH}$.
  - The current source provides 12 A total flow, regardless of what is connected across it. With no load, all of the current will flow in R_N. When shorted, all of the current will flow in the short.
    - Connect R_2.
    - Apply the current divider.
    - Use Ohm’s Law.

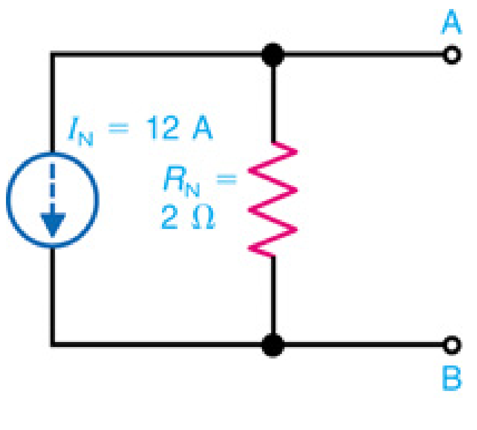
Norton's Theorem



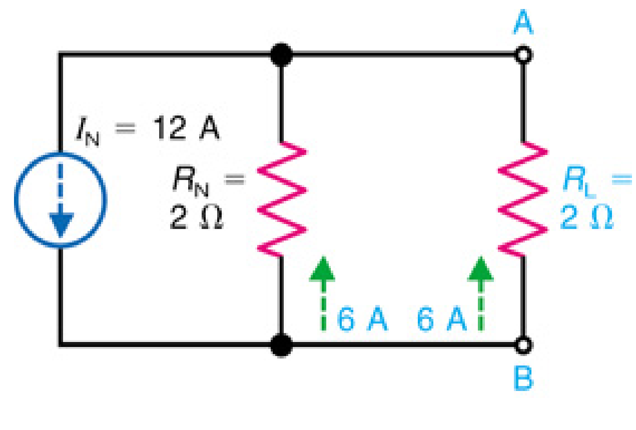


(*c*) The short-circuit current I_N is 36/3 = 12 A. (*d*) Open terminals A and B but short-circuit V to find R_{AB} is 2 Ω, the same as R_{TH}.

Norton's Theorem



(e)



(f)

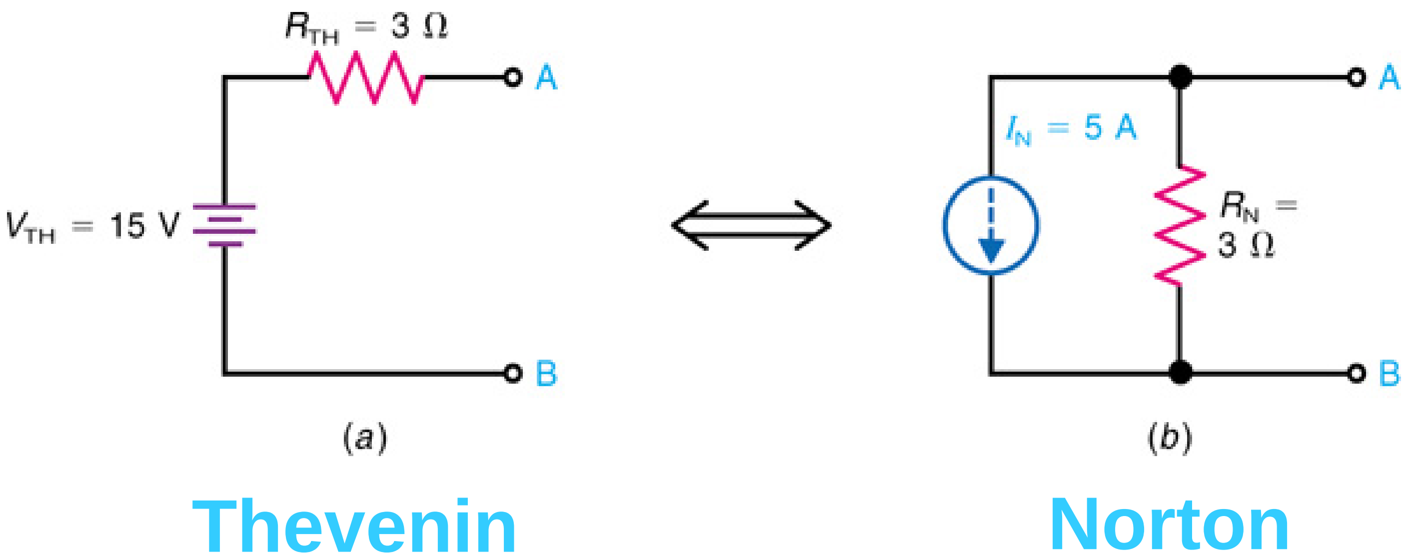
$I_L = I_N \times R_N/R_N + R_L = 12 \times 2/4 =$ 6 A

(*e*) Norton equivalent circuit. (*f*) R_L reconnected to terminals A and B to find that I_L is 6A.

Thevenin-Norton Conversions

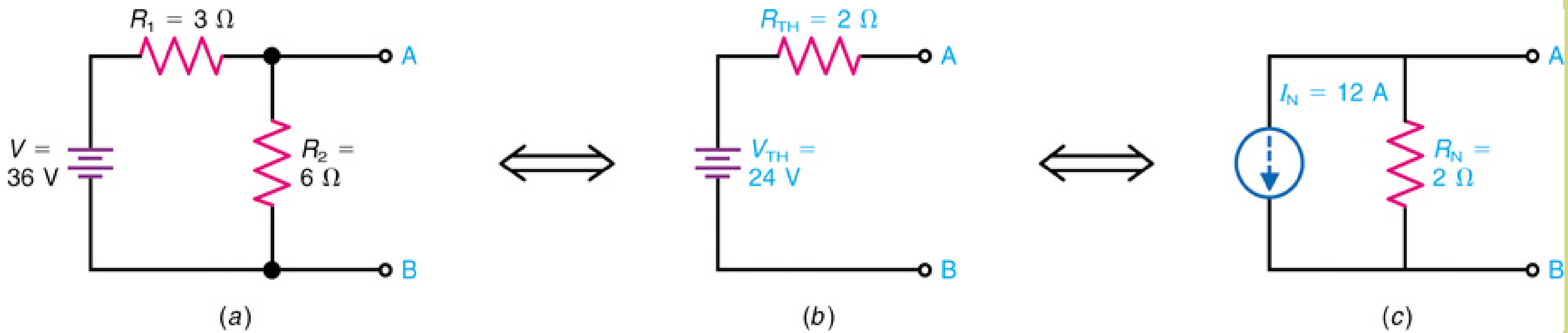
- **Thevenin's theorem** says that any network can be represented by a voltage source and series resistance.
- **Norton's theorem** says that the same network can be represented by a current source and shunt resistance.
- Therefore, it is possible to convert directly from a Thevenin form to a Norton form and vice versa.
- Thevenin-Norton conversions are often useful.

Thevenin-Norton Conversions



Thevenin equivalent circuit in (*a*) corresponds to the Norton equivalent in (*b*).

Thevenin-Norton Conversions



Example of Thevenin-Norton conversions. (*a*) Original circuit (*b*) Thevenin equivalent. (*c*) Norton equivalent.

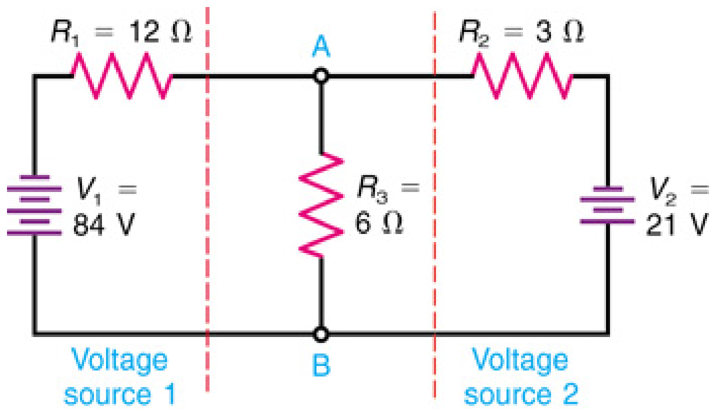
Conversion of Voltage and Current Sources

- Converting voltage and current sources can simplify circuits, especially those with multiple sources.
- Current sources are easier for parallel connections, where currents can be added or divided.
- Voltage sources are easier for series connections, where voltages can be added or divided.

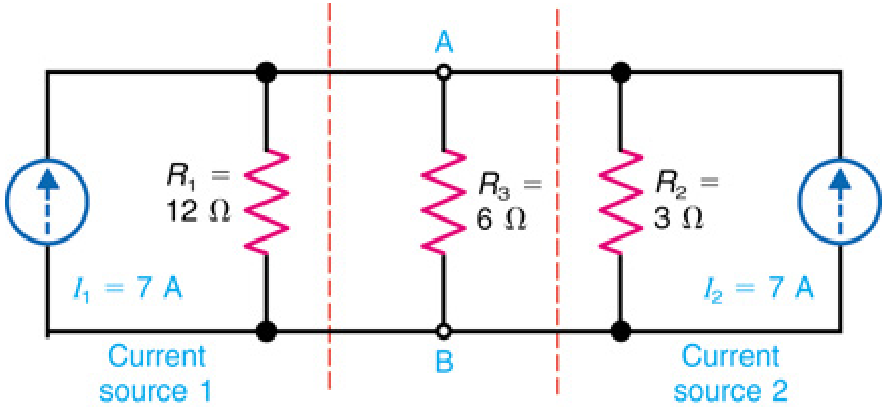
Conversion of Voltage and Current Sources

- **Norton** conversion is a specific example of the general principle that any voltage source with its series resistance can be converted to an equivalent current source with the same resistance in parallel.
- **Conversion** of voltage and current sources can often simplify circuits, especially those with two or more sources.
- **Current** sources are easier for parallel connections, where currents can be added or divided.
- **Voltage** sources are easier for series connections, where voltages can be added or divided.

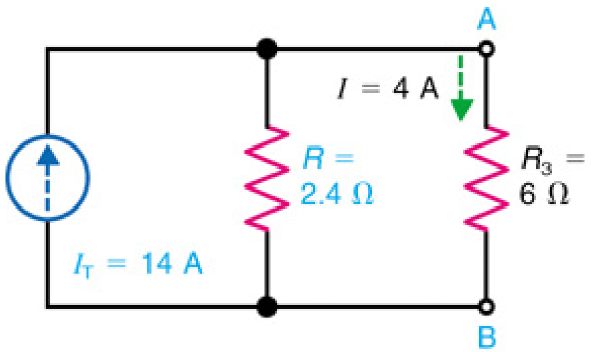
Conversion of Voltage and Current Sources



(a)



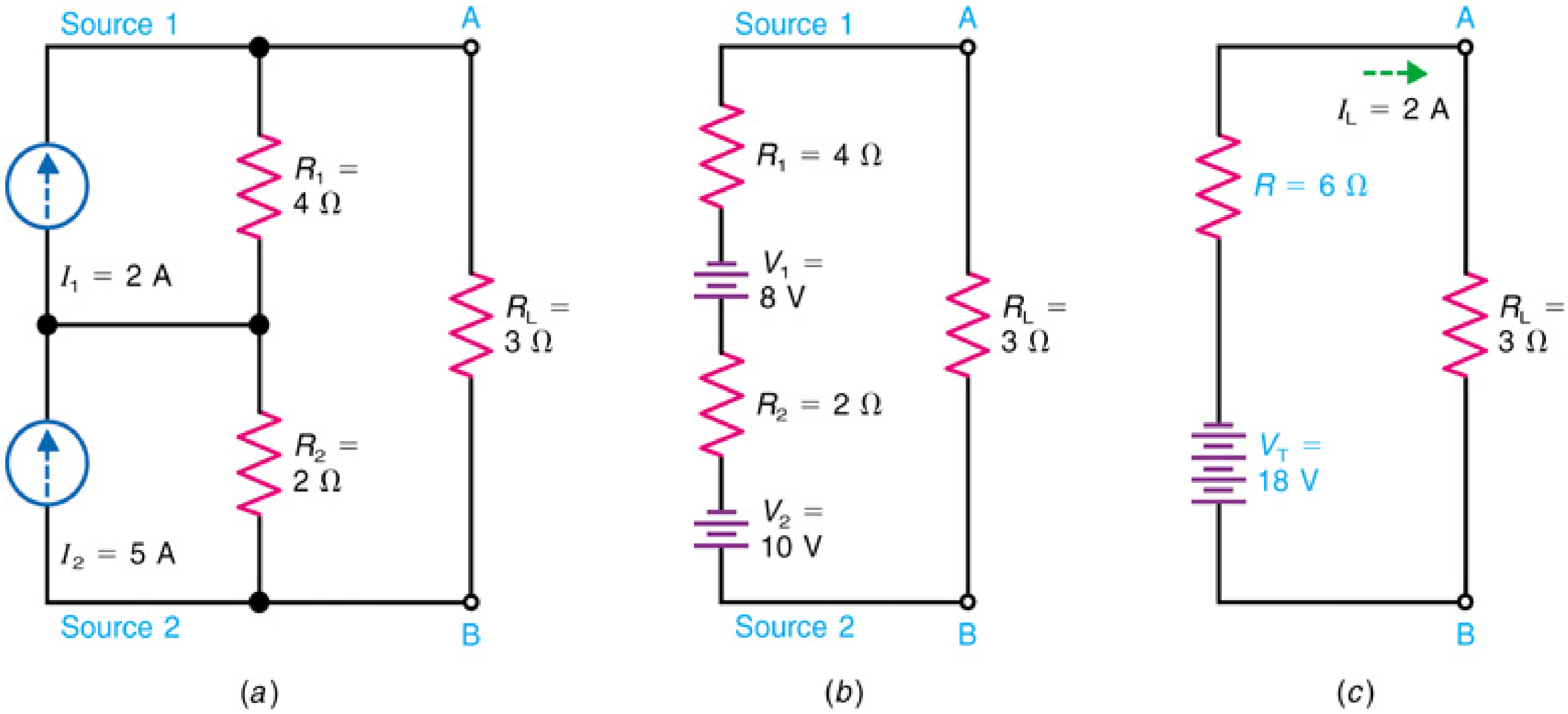
(b)



(c)

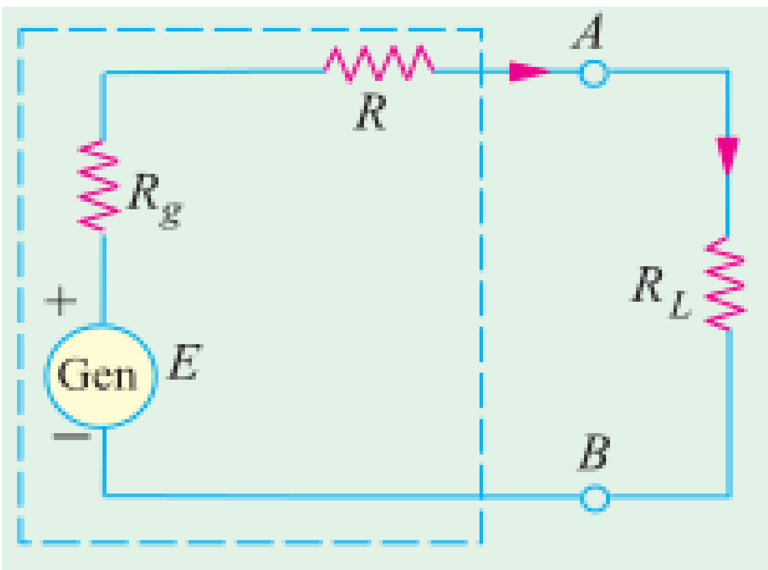
Converting two voltage sources in V_1 and V_2 in parallel branches to current sources I_1 and I_2 that can be combined. (*a*) Original circuit. (*b*) V_1 and V_2 converted to parallel current sources I_1 and I_2. (*c*) Equivalent circuit with one combined current source I_T.

Conversion of Voltage and Current Sources



Maximum Power Transfer Theorem

A resistive load will abstract maximum power from a network when the load resistance is equal to the resistance of the network as viewed from the output terminals, with all energy sources removed leaving behind their internal resistances.



Proof. Circuit current $I = \dfrac{E}{R_L + R_i}$

Power consumed by the load is

$$P_L = I^2 R_L = \frac{E^2 R_L}{(R_L + R_i)^2}$$

For P_L to be maximum, $\dfrac{dP_L}{dR_L} = 0.$

Max. power is $P_{L\,max.} = \dfrac{E^2 R_L}{4 R_L^2} = \dfrac{E^2}{4 R_L} = \dfrac{E^2}{4 R_i}$

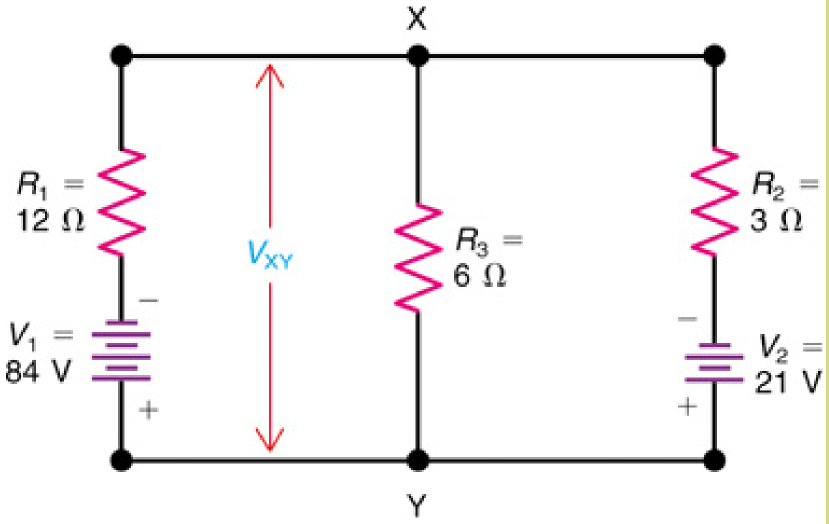
Millman’s Theorem

- Millman’s theorem provides a shortcut for finding the common voltage across any number of parallel branches with different voltage sources.
- The theorem states that the common voltage across parallel branches with different voltage sources can be determined by:

$$V_{XY} = \frac{\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} \ldots \text{etc}$$

- This formula converts the voltage sources to current sources and combines the results.

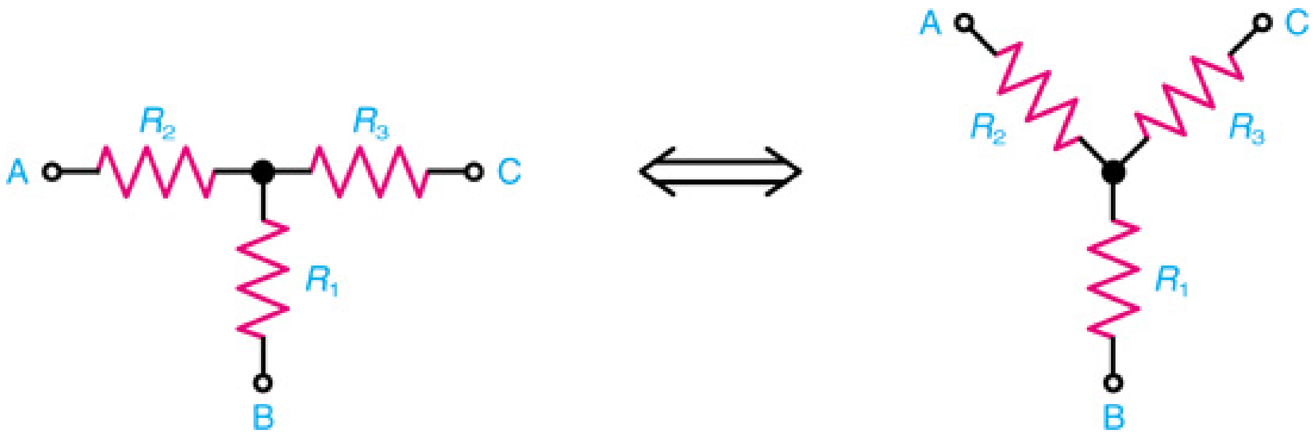
Millman’s Theorem



shown with parallel branches to calculate V_{XY} by Millman’s theorem.

T or Y and π or Δ Conversions

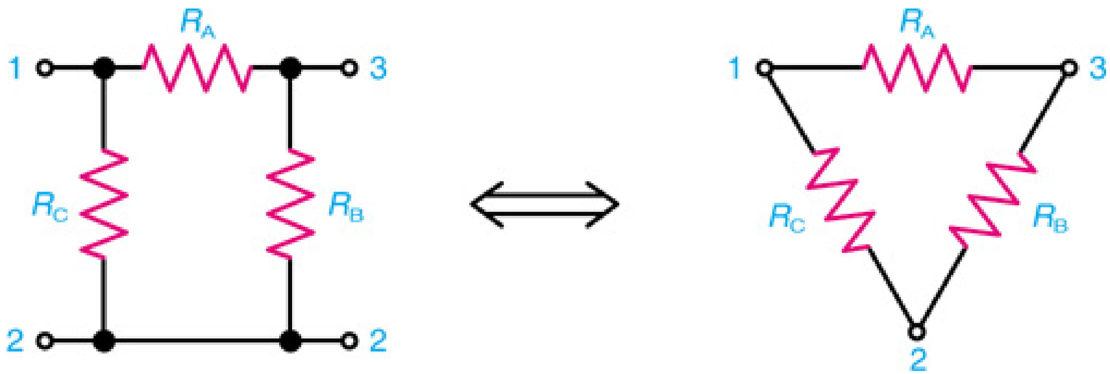
- Circuits are sometimes called different names according to their shapes.
- This circuit is the same circuit in both diagrams. The one on the left is a T (tee) network; the one on the right is a Y (wye) network.



The form of a T or Y network.

T or Y and π or Δ Conversions

- Both of the following networks are the same; the one on the left is called a pi (π), and the one on the right is called a delta (Δ), because the forms resemble those Greek characters.



The form of a π or Δ network.

T or Y and π or Δ Conversions

- The Y and Δ forms are different ways to connect three resistors in a passive network.
- When analyzing such networks, it is often useful to convert a Δ to a Y or vice-versa.

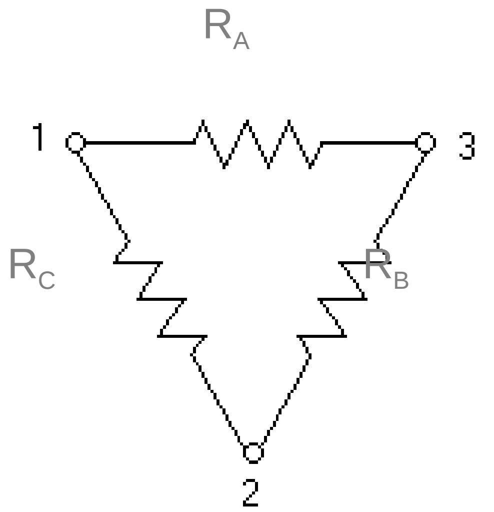
T or Y and π or Δ Conversions

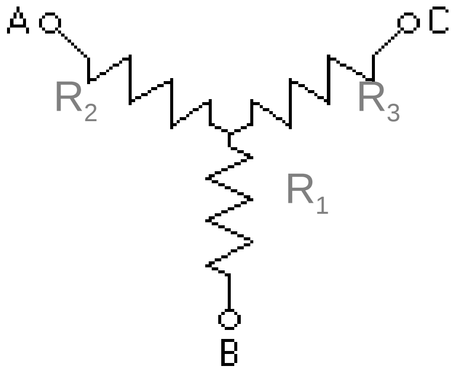
- Delta-to-Wye Conversion
  - A delta (Δ) circuit can be converted to a wye (Y) equivalent circuit by applying Kirchhoff's laws:
  - This approach also converts a T to a π network.

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_C R_A}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$





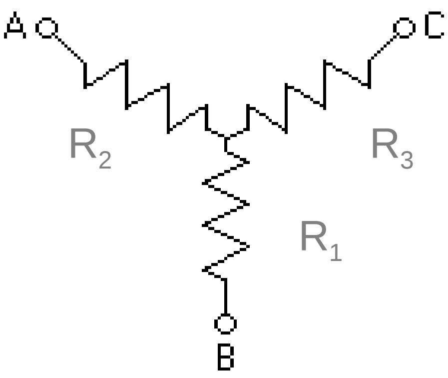
T or Y and π or Δ Conversions

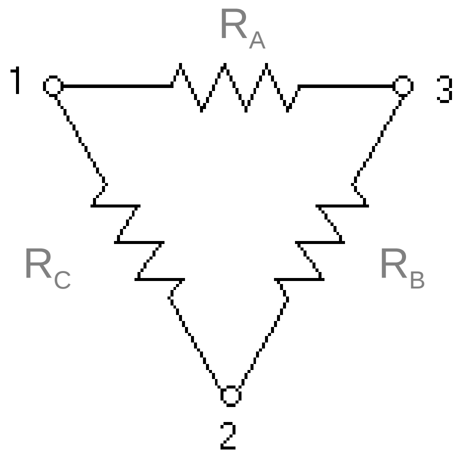
- Wye-to-delta Conversion
  - A wye (Y) circuit can be converted to a delta (Δ) equivalent circuit by applying Kirchhoff's laws:

$$R_A = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_1}$$

$$R_B = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_2}$$

$$R_C = \frac{R_1 R_2 + R_2 R_3 + R_1 R_3}{R_3}$$

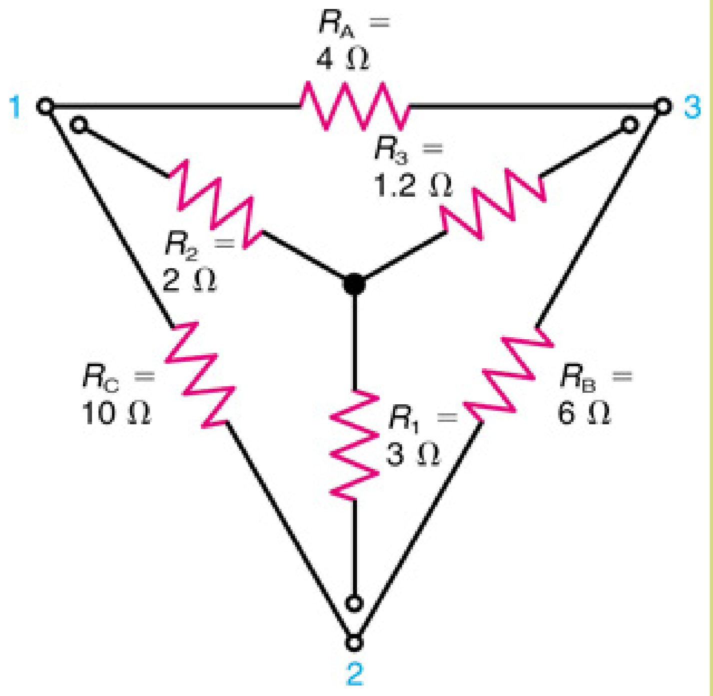




T or Y and π or Δ Conversions

Useful aid in using formulas:

- Place the Y inside the Δ.
- Note the Δ has three closed sides and the Y has three open arms.
- Note how resistors can be considered opposite each other in the two networks.
- Each resistor in an open arm has two adjacent resistors in the closed sides.



Conversion between Y and Δ networks.

T or Y and π or Δ Conversions

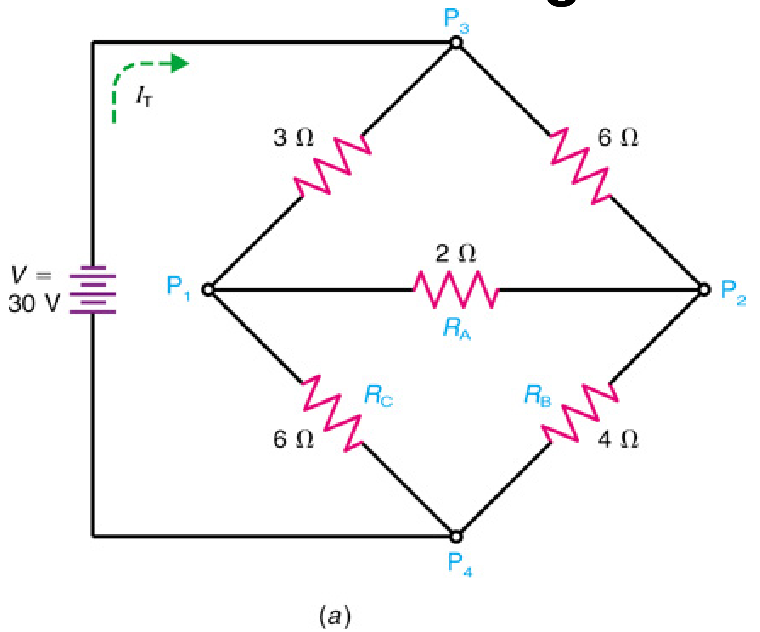
- In the formulas for the **Y-to-Δ** conversion, each side of the delta is found by first taking all possible cross products of the arms of the wye, using two arms at a time. (There are three such cross products.)
- The sum of the three cross products is then divided by the opposite arm to find the value of each side of the delta.
- Note that the numerator remains the same, the sum of the three cross products.
- Each side of the delta is calculated by dividing this sum by the opposite arm.

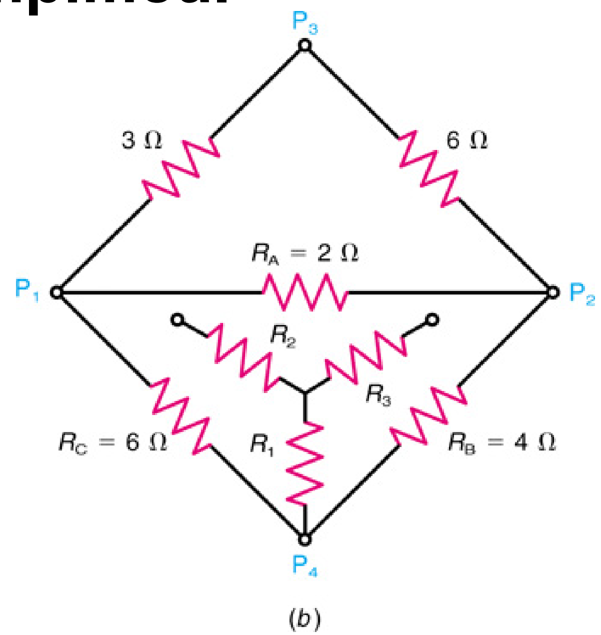
T or Y and π or Δ Conversions

- For the **Δ-to-Y** conversion, each arm of the wye is found by taking the product of the two adjacent sides in the delta and dividing by the sum of the three sides of the delta.
- The product of the two adjacent resistors excludes the opposite resistor.
- The denominator for the sum of the three sides remains the same in the three formulas.
- Each arm is calculated by dividing the sum into each cross product.

T or Y and π or Δ Conversions

A Wheatstone Bridge Can Be Simplified.

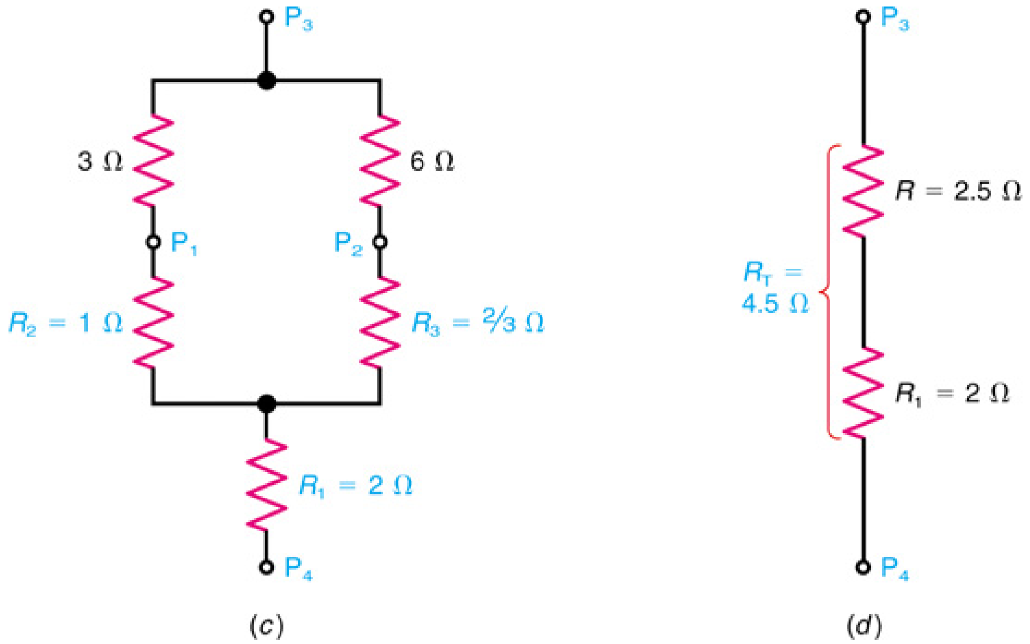




The total current I_T from the battery is desired. Therefore, total resistance R_T must be found.

Solving a bridge circuit by Δ-to-Y conversion. (*a*) Original circuit. (*b*) How the Y of $R_1R_2R_3$ corresponds to the Δ of $R_AR_BR_C$.

T or Y and π or Δ Conversions



$$\mathbf{R_T = R + R_1 = 2.5 + 2 = 4.5\ \Omega}$$

(*c*) The Y substituted for the Δ network. The result is a series-parallel circuit with the same R_T as the original bridge circuit. (*d*) R_T is 4.5Ω between points P_3 and P_4.