Booleam algebra and logic circuits

Booleam algebra: Booleam algebra is an algebra which deals with the binary number system. It is very useful in designing logic circuits which are used by the processors of computer system.

Compare Booleam algebra and general alge

(i) In booleam algebra are used only two binary digit 0 and 1.

(ii) It is used for simplification and design switching circuit.

(iii) It has two conditions true or false

(iv) **Example:** $1+1=1$

General algebra :

(i) In general algebra are used only ten digits 0 to 9.

(ii) It is used for solving mathematica Problem.

(iii) In general algebra, true or fals condition does not apply.

(iv) Example : $1+1=2$

Fundamental concept of B.A :

(i) Use of binary digits : 0 and 1
 F $\quad$ T

(ii) Logical addition : OR Gate (+)

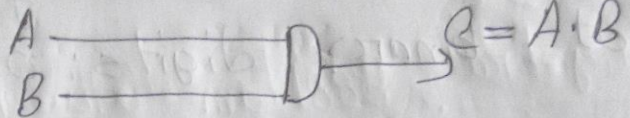
(iii) Logical multiplication : AND Gate (·)

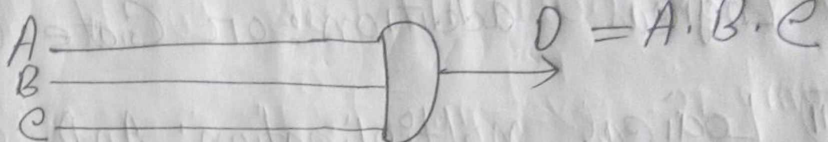
(iv) Complement : NOT gate (¯)

(v) Operator precedence :

$$A + B \cdot C$$
$$= (A+B) \cdot C \mid A + (B \cdot C)$$
$$(1 + 0) \cdot 0 = 0$$
$$1 + (0 \cdot 0) = 1$$

AND (·) :



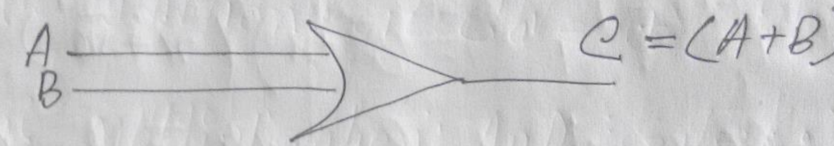


Truth table :

A	B	C = a·b
0	0	0
0	1	0
1	0	0
1	1	1

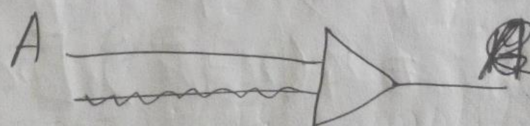
A	B	C	D = A·b·C
0	0	0	0
1	1	1	1

OR (+) :



A	B	C = A+B
0	0	0
0	1	1
1	0	1
1	1	1

NOT (¯) :



A	A
0	1
1	0

Postulates of B.A :

① $x+0=x$

$x \cdot 1=x$

⑪ $x+y=y+x$

$x \cdot y=y \cdot x$

⑪ $x+(y+z)=(x+y)+z$

$x \cdot (yz)=(x \cdot y) \cdot z$

④ $x+\bar{x}=1$

$x \cdot \bar{x}=0$

* $x(y+z)=(x \cdot y)+(x \cdot z)$

$x+(y \cdot z)=(x+y) \cdot (x+z)$

⑤ $x+x=x$

$x \cdot x=x$

⑥ $x+x \cdot y=x$

$x \cdot (x+y)=x$

⑦ $x \cdot (\bar{x}+y)=x \cdot y$

$x+(\bar{x} \cdot y)=x+y$

⑧ De-Morganis theorem

$\overline{x+y}=\bar{x} \cdot \bar{y}$

$\overline{x \cdot y}=\bar{x}+\bar{y}$

① Prove that; $x+x=x$

$$L.S = x+x$$
$$= (x+x).1$$
$$= (x+x)(x+\bar{x})$$
$$= x+(x.\bar{x}) \rightarrow (x+x).(x+\bar{x})$$
$$= x+0$$
$$= x = R.S$$

⑪ Prove that $x.x=x$

$$L.H.S = x.x$$
$$= x.x+0$$
$$= x.x+x.\bar{x}$$
$$= x(x+\bar{x}) \quad [x(x+\bar{x}) = x.x+x.\bar{x}$$
$$= x.1$$
$$= x$$

③ Prove that, $x + 1 = 1$

$$
\begin{aligned}
L.H.S &= x+1 \\
&= (x+1)\cdot 1 \\
&= (x+1)\cdot(x+\bar{x}) \\
&= x+(1\cdot \bar{x}) \quad [x+(1\cdot\bar{x}) = (x+1)(x+\bar{x}) \\
&= x+\bar{x} \\
&= 1
\end{aligned}
$$

④ De morgan's theorem

ⓘ $\overline{x+y} = \bar{x}\cdot\bar{y}$

ⓘⓘ $\overline{x\cdot y} = \bar{x}+\bar{y}$

A	A'
0	1
1	0

$(\bar{+}) = \cdot$

$(\bar{\cdot}) = +$

(I) $\overline{x+y} = \bar{x} \cdot \bar{y}$

Input			Output			
x	y	$x+y$	$\overline{x+y}$	$\bar{x}$	$\bar{y}$	$\bar{x} \cdot \bar{y}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

(II) $\overline{x \cdot y} = \bar{x} + \bar{y}$

Input				Output		
x	y	$x \cdot y$	$\overline{x \cdot y}$	$\bar{x}$	$\bar{y}$	$\bar{x} + \bar{y}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

(i) $\overline{x+y} = \bar{x} \cdot \bar{y}$

When the OR som of two variables is invented, this is equal to inverting each variables individully and the AND ing these variables. That is $\overline{x+y} = \bar{x} \cdot \bar{y}$

(ii) $\overline{x \cdot y} = \bar{x} + \bar{y}$

When the AND som of two variables is invented, this is equal to inverting each variables individolly and the OR ing these variables.

The Principle of Duality:

		Colom 1	Colum 2	Colum 3	Colom 4
OR	Row1	$1+1=1$	$0+1=1$	$1+0=1$	$0+0=0$
AND	Row2	$0\cdot 0=0$	$1\cdot 0=0$	$0\cdot 1=0$	$1\cdot 1=1$

In Boolean algebra there is precise duality betn the operators AND ($\cdot$) and OR(+) and the digits 0 and 1.

Simplify the boolean functions to a minimum number of literals.

① $x+x'y=(x+x')\cdot(x+y)$

$=1\cdot(x+y) \longleftrightarrow 4$

$=(x+y)$

(ii) $x(x'+y) = (x \cdot x') + (x \cdot y)$

$= 0 + (x \cdot y) \longleftrightarrow 4$

$= xy$

(iii) $x'y'z + x'yz + xy'$

$= x'z(y'+y) + xy'$

$= x'z \cdot 1 + xy'$

$= x'z + xy'$

(4) $xy + x'z + yz$

$= xy + x'z + yz \cdot 1$

$= xy + x'z + yz \cdot (x + \bar{x}) \longleftrightarrow (4)$

$= xy + x'z + yzx + yzx'$

$= xy + x'z + xyz + x'yz$

$= xy(1+z) + x'z(1+y)$

$= xy \cdot 1 + x'z \cdot 1$

$= xy + x'z$

Q. Find the complement of the function $F = x'yz' + x'y'z$

Ans:

$$F' = (x'yz' + x'y'z)'$$

$$= (x'yz')' \cdot (x'y'z)'$$

$$= (x + y' + z) \cdot (x + y + z')$$

Q. Find the complement of the function

$$F = x(y'z' + yz)$$

$$F' = (x(y'z' + yz))'$$

$$= x' + (y'z' + yz)'$$

$$= x' + (y + z) \cdot (y' + z')$$

* $F = (xy'z) + (x'y'z') + xy$

$F' = ((xy'z) + (x'y'z') + xy)'$

$= (xy'z)' \cdot (x'y'z')' \cdot (xy)'$

$= (x' + y + z') \cdot (x + y + z) \cdot (x' + y')$

<u>Boolen function</u>: A Boolen function is an expression which formed with binary variables, the two binary operators OR and AND and the unary operator NOT Parentheses and equal sign.

For minimizing a boolean function we can use the method,

① <u>Canonical Form:</u>

① minterm (sum of product) $(x' \cdot y \cdot z') + (x' \cdot y'z')$

② Maxterm (Product of sum) $(x + y + z) \cdot (x + y' + z')$

minterm: A boolean quantity conisting of all terms ADNed together any combination of terms and complements is permissible provided all are included in the terms.

Example: Express the boolean function $F = A + B'.C$ in the sum of products (minterms) form.

$$F = A . B' . C$$

$$= A.(B.B') + B'.C(A + A')$$

$$= (AB + AB').(C + C') + B'CA + B'CA'$$

$$= ABC + ABC' + AB'C + AB'C' + AB'C$$

$$+ A'B'C$$

$= ABC + ABC' + AB'C + AB'C' + A'B'C$

$= 111 + 110 + 101 + 100 + 001$

$= m_7 + m_6 + m_5 + m_4 + m_1$

$= m_1 + m_4 + m_5 + m_6 + m_7$

$= \Sigma(1, 4, 5, 6, 7)$

Variables			Minterm		Maxiterm	
x	y	z	term	De	term	De
0	0	0	$x'y'z'$	m_0	$x+y+z$	M_0
0	0	1	$x'y'z$	m_1	$x+y+z'$	M_1
0	1	0	$x'yz'$	m_2	$x+y'+z$	M_2
0	1	1	$x'yz$	m_3	$x+y'+z'$	M_3
1	0	0	$xy'z'$	m_4	$x'+y+z$	M_4
1	0	1	$xy'z$	m_5	$x'+y+z'$	M_5
1	1	0	xyz'	m_6	$x+y+z'$	M_6
1	1	1	xyz	m_7	$x'+y'+z'$	M_7

Example - 2 : $\bar{A} \cdot (B + \bar{C})$

$$= A'B + A'C'$$

$$= A'B(C + C') + A'C'(B + B')$$

$$= A'BC + A'BC' + A'C'B + A'C'B'$$

$$= 011 + 010 + 001 + 000$$

$$= m_3 + m_2 + m_1 + m_0$$

$$= m_0 + m_1 + m_2 + m_3$$

$$= \Sigma(0, 1, 2, 3)$$

$$(A+B)\cdot C'$$

$$= AC' + BC'$$

$$= AC'(B+B') + BC'(A+A')$$

$$= ABC' + AC'B' + ABC' + A'BC'$$

$$= ABC' + AB'C' + ABC' + A'BC'$$

$$= 110 + 100 + 110 + 010$$

$$= m_6 + m_4 + m_6 + m_2$$

$$= m_2 + m_4 + m_6$$

$$= \sum(2, 4, 6)$$

Q. $F(A, B, C, D) = D(A' + B) + B'D$

$= A'D + BD + B'D$

$= A'D(B + B') + BD(A + A') + B'D(A + A')$

$= A'BD + A'B'D + ABD + A'BD + AB'D + A'B'D$

$= A'BD + A'B'D + ABD + AB'D$

$= 011 + 001 + 111 + 101$

$= m_3 + m_1 + m_7 + m_5$

$= m_1 + m_3 + m_5 + m_7$

$= \Sigma(1, 3, 5, 7)$

$F(A.B.C,D) = D(A'B) + B'D$

$= A'D + BD + B'D$

$= A'D(B+B') + (A+A')BD + (A+A')B'D$

$= A'BD + A'B'D + ABD + A'BD + AB'D + A'B'D$

$= A'BD(C+C') + A'B'D(C+C') + ABD(C+C') + A'BD(C+C') + (AB'D)(C+C') + A'B'D(C+C')$

$= A'BCD + A'BC'D + A'B'CD + A'B'C'D + ABCD + ABC'D + A'BCD + A'BC'D + AB'CD + AB'C'D + A'B'CD + A'B'C'D$

$= A'BCD + A'BC'D + A'B'CD + A'B'C'D + ABCD + ABC'D + AB'CD + AB'C'D$

$= 0111 + 0101 + 0011 + 0001 + 1111 + 1101 + 1011 + 1001$

$= m_7 + m_5 + m_3 + m_1 + m_{15} + m_{13} + m_{11} + m_9$

$= m_1 + m_3 + m_5 + m_7 + m_9 + m_{11} + m_{13} + m_{15}$

$= \Sigma(1, 3, 5, 7, 9, 11, 13, 15)$

$F(x, y, z) = 1$

$$= x + x'$$

$$= x\cdot(y + y') + x'(y + y')$$

$$= xy + xy' + x'y + x'y'$$

$$= xy\cdot(z + z') + xy'(z + z') + x'y(z + z') + x'y'(z + z')$$

$$= xyz + xyz' + xy'z + xy'z' + x'yz + x'yz' + x'y'z + x'y'z'$$

$$= 111 + 110 + 101 + 100 + 011 + 010 + 001 + 000$$

$$= m_7 + m_6 + m_5 + m_4 + m_3 + m_2 + m_1 + m_0$$

$$= m_0 + m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7$$

$$= \Sigma(0, 1, 2, 3, 4, 5, 6, 7)$$

③ $F(A,B,C) = (A'+B) \cdot (B'+C)$

$$= (A'+B)(C+C') + (B'+C)(A+A')$$

$$= A'C + A'C' + BC + BC' + B'A + B'A' + CA + CA'$$

$$= A'C(B+B') + A'C'(B+B') + BC(A+A') + BC'(A+A') + B'A(C+C') + B'A'(C+C') + CA(B+B') + CA'(B+B')$$

$$= A'BC + A'B'C + A'BC' + A'B'C' + ABC + A'BC' + ABC' + ABC' + AB'C + AB'C' + A'B'C + A'B'C' + ABC + AB'C + A'BC + A'B'C$$

$$= A'BC + A'B'C + A'BC' + A'B'C' + ABC + ABC' + AB'C + AB'C'$$

$$= 011 + 001 + 010 + 000 + 111 + 110 + 101 + 100$$

$$= m_3 + m_1 + m_2 + m_0 + m_7 + m_6 + m_5 + m_4$$

$$= m_0 + m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7$$