Question 1   midterm Exam S 11   ELEC 442-6601

a) $y_1[n] = x_1[2n]$ $y_2[n] = x_2[2n]$

$x_3[n] = a x_1[n] + b x_2[n]$ $y_3[n] = x_3[2n]$

4 For linearity we should show that $y_3[n] = a y_1[n] + b y_2[n]$

$y_3[n] = x_3[2n] = a x_1[2n] + b x_2[2n] = a y_1[n] + b y_2[n] = y_3[n]$

$\boxed{\text{The system is linear}}$

4

$y_1[n] = x_1[2n]$ and $y_2[n] = x_2[2n]$

we should prove that if $x_2[n] = x_1[n-N]$ then

$y_2[n] = y_1[n-N]$ for being time invariant.

$x_2[n] = x_1[n-N] \Rightarrow x_2[2n] = x_1[2n-N] \Rightarrow y_2[n] = x_1[2n-N]$

$y_1[n] = x_1[2n] \Rightarrow y_1[n-N] = x_1[2n-2N]$

Therefore $y_2[n] \neq y_1[n-N]$

and therefore system is $\boxed{\text{time Variant}}$

b) since the system is not LTI, we have to use general criteria for causality and memoryless.

$y[n] = x[2n] \Rightarrow y[1] = x[2] \Rightarrow$ ~~system~~ output depends on future values of input and system is $\boxed{\text{not causal.}}$

$y[n] = x[2n] \Rightarrow y[1] = x[2]$ and $y[-1] = x[-2] \Rightarrow$

$\Rightarrow$ output depends on future and past values of input and therefore the system $\boxed{\text{has memory}}$

question 2 MIDTERM EXAM 511 ELEC 442-0001

a) From table 2.3: $(0.8)^n u[n] \xleftrightarrow{FT} \frac{1}{1-0.8e^{-jw}}$

From table 2.2: $(0.8)^{n-2} u[n-2] \xleftrightarrow{FT} \frac{1 \times e^{-j2w}}{1-0.8e^{-jw}}$

Therefore: $h[n] = (0.8)^n u[n] + (0.8)^{n-2} u[n-2] \xleftrightarrow{FT} H(e^{jw}) = \frac{1+e^{-j2w}}{1+0.8e^{-jw}}$

$$H(e^{jw}) = \frac{Y(e^{jw})}{X(e^{jw})} = \frac{1+e^{-j2w}}{1+0.8e^{-jw}}$$

$$Y(e^{jw})(1+0.8e^{-jw}) = X(e^{jw})(1+e^{-j2w})$$

$$Y(e^{jw}) + 0.8e^{-jw}Y(e^{jw}) = X(e^{jw}) + e^{-j2w}X(e^{jw})$$

From table 2.2: $\boxed{y[n] + 0.8y[n-1] = x[n] + x[n-2]}$

b) since the system is LTI:

we should have $\sum_{n=-\infty}^{+\infty} |h[n]| < \infty$ to be stable.

$$\sum_{n=-\infty}^{+\infty} \left| (0.8)^n u[n] + (0.8)^{n-2} u[n-2] \right| = \sum_{n=0}^{\infty} |(0.8)^n| + \sum_{n=2}^{\infty} |(0.8)^{n-2}|$$

$$= \frac{1}{1-0.8} + \frac{1}{1-0.8} = 10 \Rightarrow \sum_{n=-\infty}^{+\infty} |h[n]| = 10 < \infty$$

$\boxed{\text{The system is stable.}}$

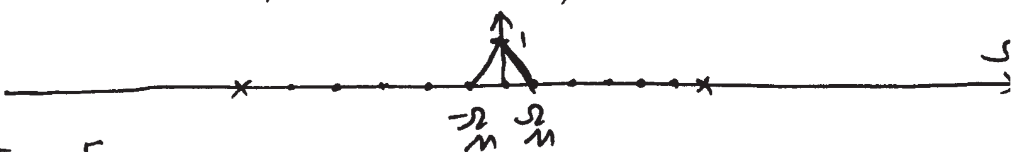
Question 3) (Q3) ELEC442-66

$\Omega_M = 8\pi \times 10^3 = \frac{1}{6}\Omega_s$

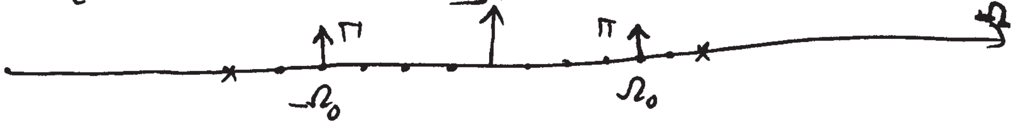
1) $\Omega_s = \frac{2\pi}{T} = 48\pi \times 10^3$

$\Omega_0 = 32\pi \times 10^3 = \frac{2}{3}\Omega_s$ $\Omega_c = \frac{\pi}{T} = 24\pi \times 10^3 = \frac{\Omega_s}{2}$

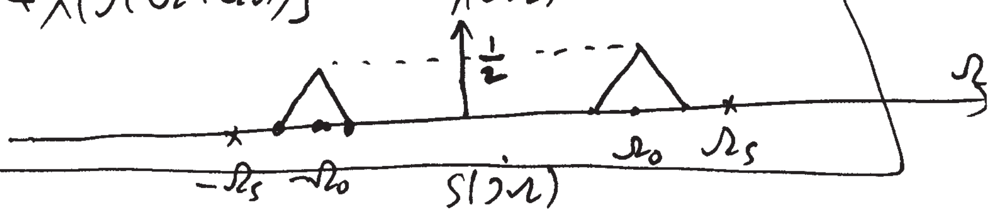
$x(t) \overset{FT}{\longleftrightarrow} X(j\Omega)$



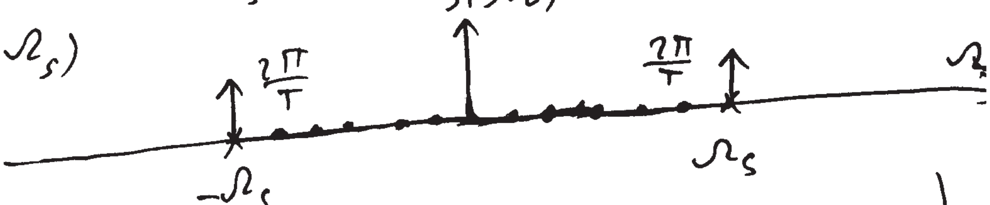
$\cos(2\pi f_0 t) = \cos(\Omega_0 t) \overset{FT}{\longleftrightarrow} \pi[\delta(\Omega-\Omega_0)+\delta(\Omega+\Omega_0)]$



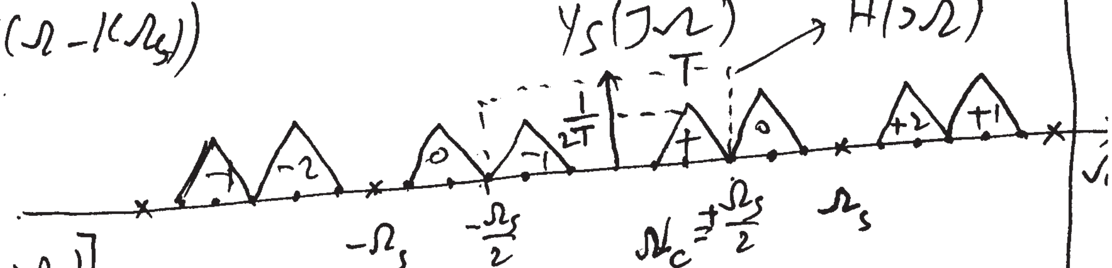
$$\boxed{\begin{aligned} Y(j\Omega) &= \frac{1}{2\pi} X(j\Omega) * \{\pi[\delta(\Omega-\Omega_0)+\delta(\Omega+\Omega_0)]\} \\ &= \frac{1}{2}[X(j(\Omega-\Omega_0)) + X(j(\Omega+\Omega_0))] \end{aligned}}$$



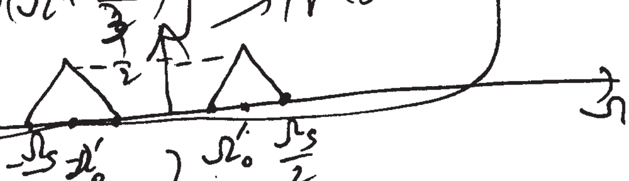
$s(t) \overset{FT}{\longleftrightarrow} S(j\Omega) = \frac{2\pi}{T}\sum_{k=-\infty}^{+\infty}\delta(\Omega - k\Omega_s)$



$$\begin{aligned} Y_s(j\Omega) &= \frac{1}{2\pi} X(j\Omega) * S(j\Omega) \\ &= \frac{1}{T}\sum_{K=-\infty}^{+\infty} Y(j(\Omega - K\Omega_s)) \\ &= \frac{1}{2T}\sum_{K=-\infty}^{+\infty}[X(j(\Omega-K\Omega_s-\Omega_0)) + X(j(\Omega-K\Omega_s+\Omega_0))] \end{aligned}$$



$Y_r(j\Omega) = Y_s(j\Omega) \cdot H(j\Omega) = \frac{1}{2}[X(j(\Omega - \frac{\Omega_s}{3})) + X(j(\Omega + \frac{\Omega_s}{3}))] \rightarrow Y_r(j\omega)$



b) $= \frac{1}{2\pi} X(j\Omega) * \{\pi[\delta(\Omega - \frac{\Omega_s}{3}) + \delta(\Omega + \frac{\Omega_s}{3})]\}$ $\Omega_s = 48\pi \times 10^3$

$\Rightarrow \boxed{y_r(t) = x(t) \cdot \cos(\frac{\Omega_s}{3}t)}$

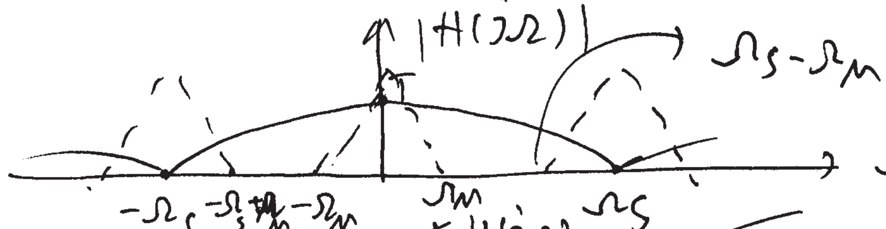
Question 4 $x_c(t) \to \otimes \to \boxed{H_r(j\Omega)} \to y_r(t)$ [Midterm Exam D11] ELEC442/6001

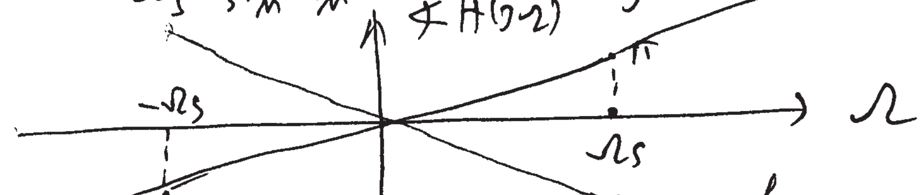
a) $y_p(t)=\sum_{n=-\infty}^{+\infty} y[n]\,h(t-nT)= h(t) * \left\{\sum_{n=-\infty}^{+\infty} y[n]\,\delta(t-nT)\right\}$

Therefore, $h(t)$ is acting as hold circuitry for the system which zero order hold. In the passband of the filter we should compensate for this zero order hold circuitry.

from table 4.2 & 4.1:

$$h(t) \xleftrightarrow{FT} \frac{2\sin\Omega T/2}{\Omega} e^{-j\Omega\frac{T}{2}} = H(j\Omega)$$

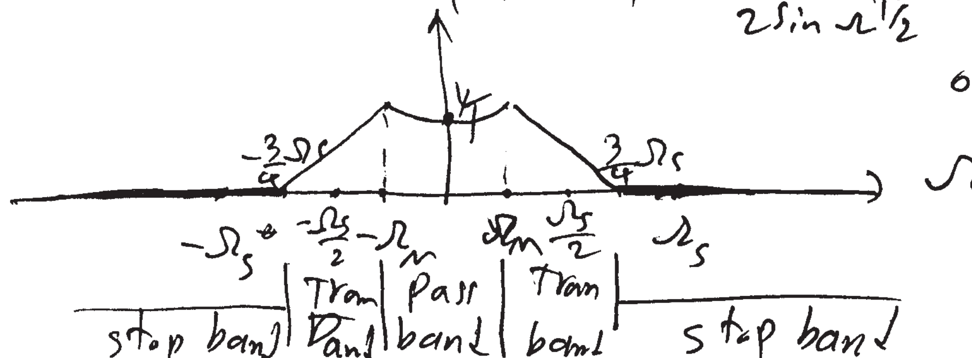




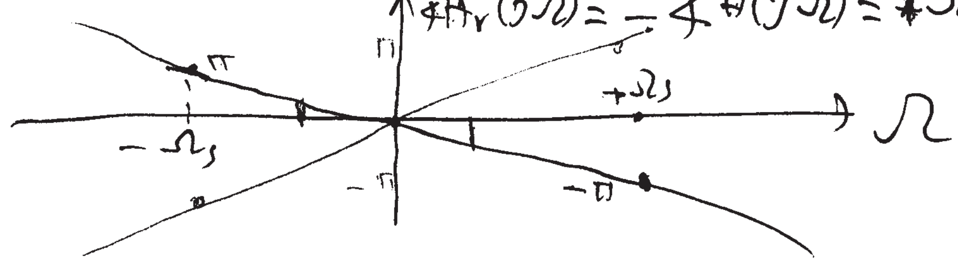
$H_r(j\Omega)$ should compensate in the region of $-\Omega_m$ to Ω_m.

Therefor the filter will be:

$$|H_r(j\Omega)| = \begin{cases} \dfrac{\Omega}{2\sin\Omega T/2} & \text{for } -\Omega_m < \Omega < \Omega_m \\ 0 & \text{otherwise} \end{cases}$$



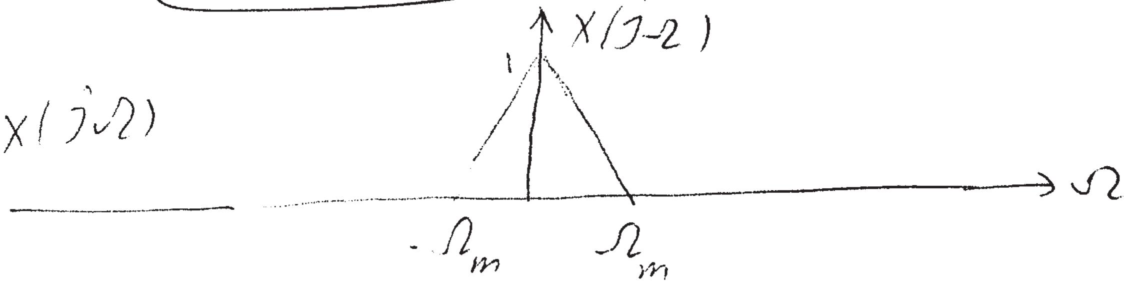
$\measuredangle H_r(j\Omega) = -\measuredangle H(j\Omega) = \Omega T/2$



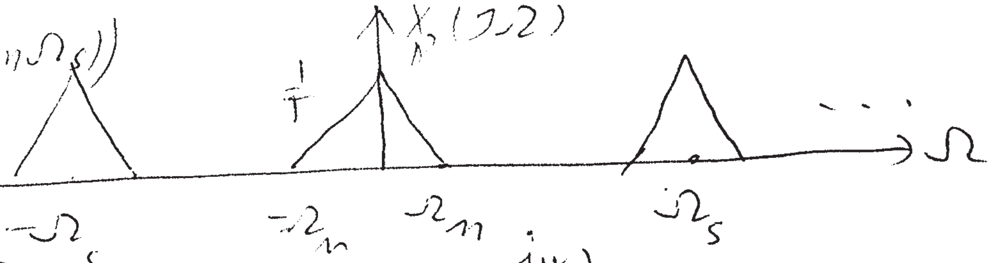
Question 4 | midterm Exam S11 | (ELEC 442/6601)

b)

$x(t) \longleftrightarrow X(j\Omega)$

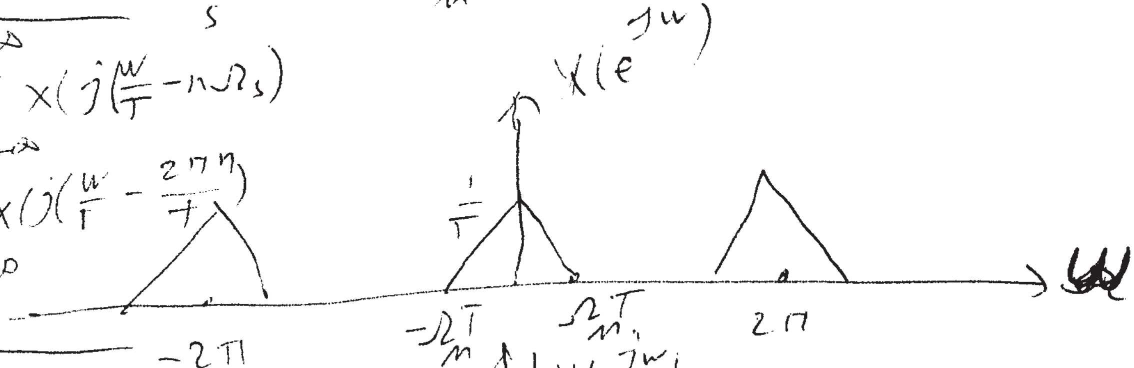


$X_p(j\Omega) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} X(j(\Omega - n\Omega_s))$



$X(e^{jw}) = \frac{1}{T}\sum_{n=-\infty}^{+\infty} X(j(\frac{w}{T} - n\Omega_s))$

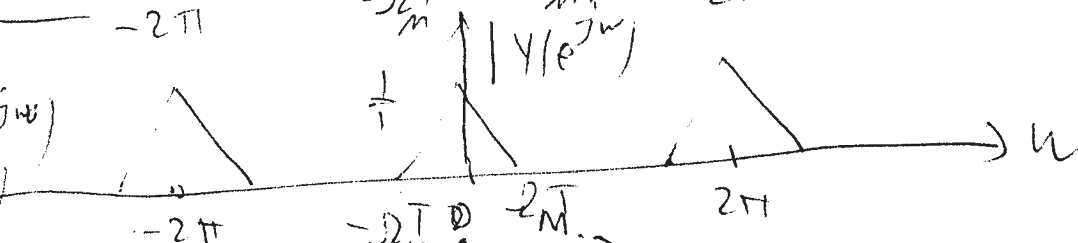
$= \frac{1}{T}\sum_{n=-\infty}^{+\infty} X(j(\frac{w}{T} - \frac{2\pi n}{T}))$



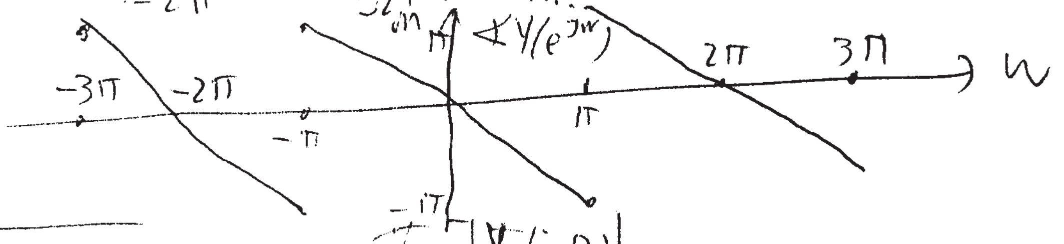
$y[n] = x[n-1]$

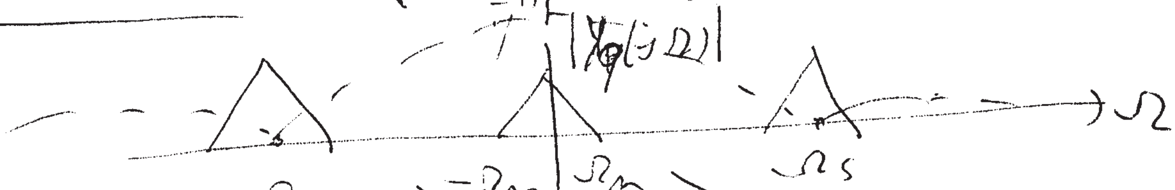
$Y(e^{jw}) = e^{-jw} X(e^{jw})$

$|Y(e^{jw})| = |X(e^{jw})|$



$\measuredangle Y(e^{jw}) = -w$





$\measuredangle Y_q(j\Omega)$