Answer to the question no, 1

Domain: Let x and y be two sets and f be a func from x and y. Then the set of x is called domain of the function. Domain is denoted by Df.

Range: Let f(x) = y be a function. Here y is the value of f(x). so the set of all the value of y of the function is called Range of the function.

Function:

$f(x) = \frac{|x|}{x}$

Soln:

Given that,

$$f(x) = \frac{|x|}{x} \quad \text{---} (1)$$

when the set of x is zero. Then eqn (1) is undefine. so that the domain

$D_f = \mathbb{R} - \{0\}$

Now, $f(x) = \begin{cases} \frac{-x}{x} & \text{for } x < 0 \\ \frac{x}{x} & \text{for } x > 0 \end{cases}$

$= \begin{cases} -1 & \text{for } x < 0 \\ 1 & \text{for } x > 0 \end{cases}$

Hence Range $R_f = \{-1, 1\}$.

Answer to the question no, 2

☐ proved that Every differentiable f^n a continuous function.

or,

If $f'(a)$ is finite, then $f(x)$ is continuous proved it.

Soln:

Let $f(x)$ be a function

Given that, $f(x)$ is differentiable

Again,

Let $f(x)$ is differentiable $x = a$

$$\therefore f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \quad \text{——} (1)$$

Now we can write,

$$f(a+h)-f(a) = \frac{f(a+h)-f(a)}{h} \cdot h$$

$$\therefore \lim_{h\to 0} f(a+h)-f(a) = \lim_{h\to 0} \left\{ \frac{f(a+h)-f(a)}{h} \cdot h \right\}$$

$$\text{or, } \lim_{h\to 0} f(a+h) - \lim_{h\to 0} f(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} \cdot \lim_{h\to 0} h$$

$$\text{or, } \lim_{h\to 0} f(a+h) - f(a) = f'(a) \cdot \lim_{h\to 0} h \quad [\text{using } 1]$$

$$= f'(a) \cdot 0$$

$$= 0$$

$$\therefore \lim_{h\to 0} f(a+h) = f(a)$$

By the difination of continuity its show that $f(x)$ is continuous.

at $x = a$ (proved).

Answer to the question no, 3

* $\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$

soln:

Let $y = \left(\frac{\tan x}{x}\right)^{\frac{1}{x}}$

Taking oparator log on both sides

$\log y = \frac{1}{x} \log\left(\frac{\tan x}{x}\right)$

Taking $\lim_{x\to 0}$

$\lim_{x\to 0} \log y = \lim_{x\to 0} \frac{\log\left(\frac{\tan x}{x}\right)}{x}$

or, $\log\left(\lim_{x\to 0} y\right) = \lim_{x\to 0} \frac{\log\left(\frac{\tan x}{x}\right)}{x}$

Appling L. Hospital theorem

$\log \lim_{x\to 0} y = \lim_{x\to 0} \frac{\frac{x}{\tan x} \cdot \frac{x\sec^2 x - \tan x}{x^2}}{1}$

$= \lim_{x\to 0} \frac{x\sec^2 x - \tan x}{x\tan x}$

$$= \lim_{x\to 0} \frac{1 \cdot \sec^2 x + x(\tan^2 x) - \sec^2 x}{x\sec x + \tan x}$$

$$= \lim_{x\to 0} \frac{x\tan^2 x}{\sec^2 x + \tan^2 x}$$

$$= \lim_{x\to 0} \frac{\tan^2 x + x\sec^2 x}{x\sec^2 x + \tan^2 x + \sec^2 x}$$

$$= \lim_{x\to 0} \frac{\tan^2 x + x\sec^2 x}{2\sec^2 x + x\tan^2 x}$$

$$= \lim_{x\to 0} \frac{0}{2+0}$$

$$= \frac{0}{2}$$

$$= 0$$

$$\log \lim_{x\to 0} y = e^0$$

$$= 1$$

$$\lim_{x\to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x}} = 1$$

Answer to the question no, 4

solⁿ: Given that,

$$f(x) = \begin{cases} 3+2x & \text{for } -\frac{3}{2} \le x < 0 \\ 3-2x & \text{for } 0 \le x < \frac{3}{2} \\ -3-2x & \text{for } x \ge \frac{3}{2} \end{cases}$$

1st we discuss $x=0$

Left hand limit $= \lim_{x \to 0^-} f(x)$

$= \lim_{x \to 0^-} (3+2x)$

$= (3+2\cdot 0)$

$= 3$

Right hand limit $= \lim_{x \to 0^+} f(x)$

$= \lim_{x \to 0^+} (3-2x)$

$= (3-2\cdot 0)$

$= 3$

Functional value $f(0) = f(x)$

$= 3-2x$

$= 3-2\cdot 0$

$= 3$

Therefore,

Left hand limit = Right hand limit = functional value

Hence $f(x)$ is continuous at $x=0$.

Again,

we shall discuss at $x=\frac{3}{2}$

$$L.H.L = \lim_{x \to \frac{3}{2}^-} f(x)$$

$$= \lim_{x \to \frac{3}{2}^-} (3-2x)$$

$$= \left(3-2\cdot\frac{3}{2}\right)$$

$$= (3-3)$$

$$= 0$$

$$R.H.L = \lim_{x \to \frac{3}{2}^+} f(x)$$

$$= \lim_{x \to \frac{3}{2}^+} (-3-2x)$$

$$= \left(-3-2\cdot\frac{3}{2}\right)$$

$$= (-3-3)$$

$$= -6$$

Functional value $f(\frac{3}{2}) = f(x)$

$$= (-3-2x)$$

$$= (-3-2\cdot\frac{3}{2})$$

$$= -6$$

since does not exists at $x = \frac{3}{2}$.

By the difinition of continuity $f(x)$ is discontinuous at $x = \frac{3}{2}$.

Answer to the question no, 5

soln: Given that,

$$f(x) = \begin{cases} 1 & \text{when } x < 0 \\ 1+\sin x & \text{when } 0 \le x \le \frac{\pi}{2} \\ 2+(x-\frac{\pi}{2})^2 & \text{"} \quad \frac{\pi}{2} \le x \end{cases}$$

when we discuss at $x = \frac{\pi}{2}$

$$L.H.L = \lim_{x\to\frac{\pi}{2}^-} f(x)$$

$$= \lim_{x\to\frac{\pi}{2}^-} (1+\sin x)$$

$= \left(1+\sin\frac{\pi}{2}\right)$

$= 2$

$R.H.L = \lim_{x\to\frac{\pi}{2}^+} f(x)$

$= \lim_{x\to\frac{\pi}{2}^+} \left(2+\left(x-\frac{\pi}{2}\right)^2\right)$

$= 2+\left(\frac{\pi}{2}-\frac{\pi}{2}\right)^2$

$= 2$

Functional value $f\left(\frac{\pi}{2}\right) = f(x)$

$= 2+\left(x-\frac{\pi}{2}\right)^2$

$= 2+\left(\frac{\pi}{2}-\frac{\pi}{2}\right)^2$

$= 2$

Therefore,

$L.HL = RHL = FV$

Hence, $f(x)$ is continuous at $x = \frac{\pi}{2}$

$Rf'\left(\frac{\pi}{2}\right) = \lim_{h\to 0} \frac{f\left(\frac{\pi}{2}+h\right) - f\left(\frac{\pi}{2}\right)}{h}$

$= \lim_{h\to 0} \frac{2+\left(\frac{\pi}{2}+h-\frac{\pi}{2}\right)^2 - (2+0)}{h}$

$= \lim_{h\to 0} \frac{2+h^2-2}{h}$

$= \frac{0}{0}$

$= 0$

$$Lf'\left(\frac{\pi}{2}\right) = \lim_{h\to 0} \frac{f\left(\frac{\pi}{2}+h\right) - f\left(\frac{\pi}{2}\right)}{-h}$$

$$= \lim_{h\to 0} \frac{1+\sin\left(\frac{\pi}{2}+h\right) - \cancel{6}\,2}{-h}$$

$$= \lim_{h\to 0} \frac{1+\sin\frac{\pi}{2} - \sin h - 2}{-h}$$

$$= \lim_{h\to 0} \frac{1+1-\sin h - 2}{-h}$$

$$= \lim_{h\to 0} \frac{2-\sin h - 2}{-h}$$

$$= \frac{-\sin 0^\circ}{0}$$

$$= 0$$

$$= \lim_{h\to 0^-} \frac{1+\cos h - 2}{-h}$$

$$= \lim_{h\to 0^-} \frac{\cos h - 1}{-h}$$

$$= \lim_{h\to 0^-} \frac{-\sin h - 0}{-h}$$

$$= 0$$

Answer to the question no, 7

* $x\sin^{-1}x$

soln:

Let $y = x^{\sin^{-1}x}$

and $z = \sin^{-1}x$

Let, $\log y = \sin^{-1}x \log x$ ——— (ii)

Differentiate with respect to x

$$\text{or, } \frac{1}{y}\frac{dy}{dx} = \sin^{-1}x \cdot \frac{1}{x} + \frac{1}{\sqrt{1-x^2}}\log x$$

$$\text{or, } \frac{dy}{dx} = y\left(\frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right)$$

$$\therefore \frac{dy}{dx} = x^{\sin^{-1}x}\left(\frac{\sin^{-1}x}{x} + \frac{\log x}{\sqrt{1-x^2}}\right).$$

——— (1)

Again, $z = \sin^{-1} x$

Differentiate with respect to x

$$\frac{dz}{dx} = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{dx}{dz} = \sqrt{1-x^2} \quad \text{——— (11)}$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \cdot \frac{dx}{dz}$$

$$= x^{\sin^{-1} x} \left\{ \frac{\sin^{-1} x}{x} + \frac{\log x}{\sqrt{1-x^2}} \right\} \sqrt{1-x^2}$$

using 1 and 2

$$= x^{\sin^{-1} x} \left\{ \frac{\sqrt{1-x^2}}{x} \sin^{-1} x + \log x \right\}$$

Ans;

Answer to the question no,6

sol^n:

Let,

$$y = \tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$$

$$z = \tan^{-1} x$$

Let, $x = \tan\theta$

$$\therefore y = \tan^{-1} \frac{\sqrt{1+\tan^2\theta}+1}{\tan\theta}$$

$$= \tan^{-1} \frac{\sqrt{\sec^2\theta - 1}}{\tan\theta}$$

$$= \tan^{-1} \frac{\sec\theta - 1}{\tan\theta}$$

$$= \tan^{-1} \frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$$

$$= \tan^{-1} \frac{1-\cos\theta}{\cos\theta} \times \frac{\cos\theta}{\sin\theta}$$

$$= \tan^{-1} \frac{1-\cos\theta}{\sin\theta}$$

$$1-\cos\theta = 2\sin^2\frac{\theta}{2}$$

$$\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$$

$$= \tan^{-1} \frac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}$$

$$= \tan^{-1} \tan\frac{\theta}{2}$$

$$= \frac{\theta}{2}$$

$$y = \frac{1}{2}\tan^{-1} x$$

$$\frac{dy}{dz} = \frac{d}{d\tan^{-1}x}\left(\frac{1}{2}\tan^{-1}x\right)$$

$$= \frac{1}{2} \cdot \frac{d\tan^{-1}x}{d\tan^{-1}x}$$

$$= \frac{1}{2}$$

OR,

$$z = \tan^{-1} x$$

$$\frac{dy}{dz} = \frac{1}{1+x^2}$$

$$\therefore \frac{dy}{dz} = \frac{dy}{dx} \cdot \frac{dx}{dz}$$

$$= \frac{1}{2} \cdot \frac{1}{1+x^2} \left(1+x^2\right)$$

$$= \frac{1}{2}$$