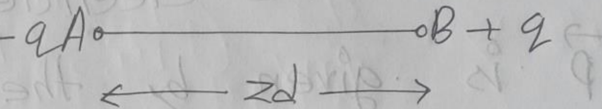
Electric dipole : Consider two charge $-q$ at Point A and $+q$ at point B. The distance between then being $2d$. fig(1) such a charge configaration is called an electri dipole.



Example : Hydrogen atom have one proton that positively charged and one electron that negatively charged so it is eletric dipole.

Electric dipole moment: If two equal and opposite point chargess are place at small distance apart such that one cannot cancels the otherfield Quitelyld is called an electric dipole moment. The electric dipole moment $\vec{p}$ is given by the product of any one of the charges and the distance between them.

$$\therefore p = q \times 2d$$

Direction of $\vec{p}$ is from $-q$ to $+q$ the units of $\vec{p}$ are Cm.

Find an electric field $\vec{E}$ at a point p due to a dipole at a distance r from the mid point of the dipole.

Sol^n:

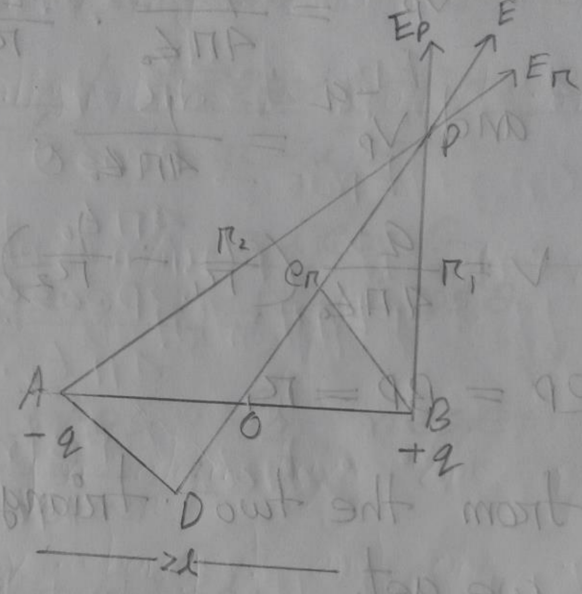


Fig: 2

⇒ let us consider two charge of dipole +q and -q at the distance AB = 2l from each other. The electric potential v at a point p at a distance r from the mid point of AB is

$$V = V_p^{+q} + V_p^{-q} \text{ ——— (i)}$$

$$\therefore V_p^{+q} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r_1}$$

$$\text{and } V_p^{-q} = \frac{1}{4\pi\epsilon_0} \cdot \frac{(-q)}{r_2}$$

$$\therefore V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) \text{ ——— (ii)}$$

let $CP = BP = r_1$

Now from the two triangle ΔAOD and ΔBOC we get,

$$\cos\theta = \frac{OD}{OA} \quad \therefore OD = l\cos\theta$$

$$\text{and } \cos\theta = \frac{OC}{OB} \quad \therefore OC = l\cos\theta$$

$$\therefore r_1 = r - l\cos\theta$$

$$r_2 = r + l\cos\theta$$

Now using the value of r_1 and r_2 is eqn (ii) we have,

$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{1}{r - l\cos\theta} - \frac{1}{r + l\cos\theta}\right)$$

$$V = \frac{q}{4\pi\epsilon_0}\left(\frac{r + l\cos\theta - r + l\cos\theta}{r^2 - l^2\cos^2\theta}\right)$$

$$V = \frac{q}{4\pi\epsilon_0} \cdot \frac{2l\cos\theta}{r^2 - l^2\cos^2\theta} \quad \text{--- (iii)}$$

$\because$ $l \ll r$ so $l^2\cos^2\theta$ is nesiesible.

also dipole moment

$$P = q \times 2l$$

$$\therefore q = \frac{P}{2l}$$

$$\therefore V = \frac{P}{4\pi\epsilon_0}\left(\frac{2l\cos\theta}{2l\, r^2}\right)$$

$$V = \frac{1}{4\pi\epsilon_0} \cdot \frac{P\cos\theta}{r^2} \quad \text{--- (iv)}$$

we can express the vedial componet Er and transver componet E_θ easily from the above equation (iv)

$$E_r = -\frac{dv}{dr}$$

$$E_r = -\frac{p\cos\theta}{4\pi\epsilon_0}\frac{d}{dv}\left(\frac{1}{r^2}\right)$$

$$E_r = \frac{2p\cos\theta}{4\pi\epsilon_0 . r^3}$$

$$\therefore E_\theta = \frac{p\sin\theta}{4\pi\epsilon_0 . r^3}$$

Now total Electric field

$$E = \sqrt{E_r^2 + E_\theta^2}$$

$$= \sqrt{\left(\frac{2p\cos\theta}{4\pi\epsilon_0 . r^3}\right)^2 + \left(\frac{p\sin\theta}{4\pi\epsilon_0 . r^3}\right)^2}$$

$$= \frac{2p\cos\theta}{4\pi\epsilon_0 . r^3} + \frac{p\sin\theta}{4\pi\epsilon_0 . r^3}$$

17-08-21

$$= \frac{1}{4\pi\epsilon_0 r^3}\sqrt{4p^2\cos^2\theta + p^2\sin^2\theta}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + (\cos^2\theta + \sin^2\theta)}$$

$$= \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$$

θ is the genaral impression for electric field at any point at distance from the mid point of the dipole. At the point is in the axis of the dipole them $\theta = 0$. Electric field $E = \frac{2p}{4\pi\epsilon_0 r^3}$

At the point is perpendicular to the line of the mid point of the dipole them $\theta = 90^\circ$

$\therefore$ Electric field $E = \frac{p}{4\pi\epsilon_0 r^3}$

12-08-17

① Calculate the E potential and E field due to a dipole of dipole moment 4.5×10^{-10} c/m at a distance 1 meter from it

① on it's axis

Ⓘⓘ on it's perpendicular by sector

the E potential is given by.

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$

When point lies on the axis of the dipole

$$V = \frac{1}{4\pi\epsilon_0}\left(\frac{p\cos\theta}{r^2}\right)$$

$$= 9 \times 10^9 \left(\frac{4.5 \times 10^{-10}}{(1)^2}\right)$$

$$= 4.05 \text{ volts}.$$

Again field is given by

$$E = \frac{p}{4\pi\epsilon_0 r^3}\left(3\cos^2\theta + 1\right)^{\frac{1}{2}}$$

$$= \frac{2p}{4\pi\epsilon_0 r^3}$$

$$= \frac{2\times 9\times 10^{9}\times 4.5\times 10^{-10}}{(1)^3}$$

$$E = 8.1 \text{ volt/meter}$$

17-08-18

⊞ Electrie field due to an electrie dipole.

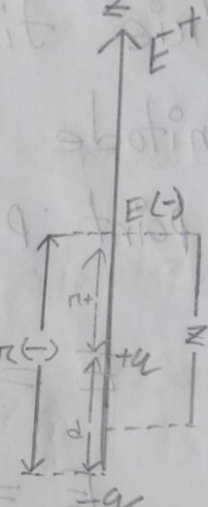


Fig: 1

Fig 2

Ans: shows two charges particles of magnitute q but opposite sign separated by a distance d. Find the eletrie field due to the dipole of (fig 1) at point P a distance z from the midpoint of the dipole and on the axis through the particle of the electrie field $\vec{E}$ at point $\vec{P}$ must lie along the dipole axis.

Applying superpositing principle for electric fields we find that the magnitude $\vec{E}$ of the electric field at point P is

$$E = E_+ - E_-$$

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r_+)^2} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{(r_-)^2}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{\left(z-\frac{d}{2}\right)^2} - \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{\left(z+\frac{d}{2}\right)^2}$$

$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{\left[z\left(1-\frac{d}{2z}\right)\right]^2} - \frac{1}{4\pi\varepsilon_0} \frac{q}{\left[z\left(1+\frac{d}{2z}\right)\right]^2}$$

$$E = \frac{q}{4\pi\varepsilon_0}\left[\left(1-\frac{d}{2z}\right)^{-2} - \left(1+\frac{d}{2z}\right)^{-3}\right]$$

Since we are interested to measurred the electric field of a dipole at distance that are large compared with the dimension of the dipole.

$z >> d$ therefore $\frac{d}{2z} << 1$

Using the binominal theorem,

$$\left[\left(1 + \frac{2d}{2z(1!)} + \cdots\right) - \left(1 - \frac{2d}{2z(1!)} + \cdots\right)\right]$$

Thus

$$E = \frac{q}{4\pi\epsilon_0 z^2}\left[\left(1 + \frac{d}{z} + \cdots\right) - \left(1 - \frac{d}{z}\right)\right.$$

Neglecting higher order term

Therefore

$$E = \frac{q}{4\pi\epsilon_0 z^2} \cdot \frac{2d}{z}$$

$$E = \frac{2qd}{4\pi\epsilon_0 z^3}$$

$$= \frac{qd}{2\pi\epsilon_0 z^3}$$

$$= \frac{p}{2\pi\epsilon_0 z^3}$$

The product qd is the magnitoded of a vector quantity is known as electri dipole moment $\vec{p}$.

4. A dipole is consisting of an electron and proton 4×10^{-10} m compute the electric fild at a distance of 2×10^{-8} m on a lie making an angle of $45°$ with the dipole axis from the center of the dipole.

(i) Dipole moment,

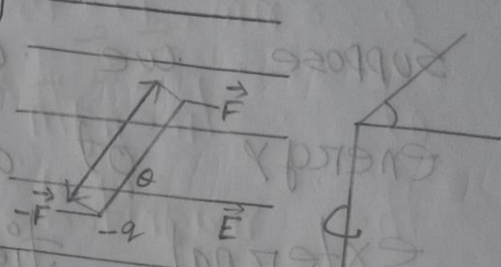
$$p = qd$$
$$= (1.6\times10^{-19})(4\times10^{-10})$$
$$= 6.4\times10^{-29}$$

(ii) Electric fild: $E = \frac{p}{4\pi\epsilon_0 r^3}\sqrt{3\cos^2\theta+1}$

$$= \frac{6.4\times10^{-29}}{9\times10^{-9}\times(2\times10^{-8})^3}\sqrt{3(45)^2+1}$$

□ A dipole is an exteronal electric field or prove that $\vec{\tau} = \vec{P} \times \vec{E}$

Sol^n: An eletric dipole of moment $\vec{P}$ makes an angle θ with a uniform external electric field $\vec{E}$ fig(1)



The two forceel F and −F acting on the two charges equal but opposite. so that the net force on this dipole is zero but there is a net torque about dipole given by an axis passing throug the centre o of the dipole given by.

$$\tau = Fd \sin\theta \text{ ——— (i)}$$

but $F = qE$

$$\tau = qdE \sin\theta$$

$$\tau = PE\sin\theta \text{ ——— (ii)} \quad [P = qd]$$

The torque can be written in vector form as $\vec{\tau} = \vec{p} \times \vec{E}$

Suppose we choose the potential energy of a dipole in an external field to be zero when $\theta = 90^\circ$, let W be the work done by an external agent to turn the dipole from its reference orientation to angle θ then.

$$W = \int_{90}^{\theta} \tau \, d\theta$$

$$\text{or } W = \int_{90}^{\theta} PE\sin\theta \cdot d\theta$$

$$\text{or } W = PE\left[-\cos\theta\right]_{90}^{\theta}$$

$$\text{or } W = -PE\cos\theta$$

This work done is stored as Potesitial energy in the dipole.

$$U(\theta) = -PE \cos\theta$$

$$U(\theta) = -\vec{P} \cdot \vec{E}$$

An electric dipole consists of Charge $+ze$ and $-ze$ separated by 0.78 nm. it is an electric field of strength $3.4 \times 10^{6} NC^{-1}$ calculate the magnitude of the torque on the the dipole when dipole moment is

(i) Parallel (ii) Perpendicular to (iii) Anti-Parallel to the electric field.

(i) soll^n : when $\vec{P}$ is parallel to $\vec{E}$ then $\theta = 0^\circ$

$$\tau = PE\sin\theta$$

Here,

$d = 0.78\ nm$

$= 0.78 \times 10^{-9}\ m$

$q = 2e$

$= 2 \times 1.6 \times 10^{-19}\ C$

$P = qd$

$= 2 \times 1.6 \times 10^{-19} \times 0.78 \times 10^{-9}$

$= 2.49 \times 10^{-28}\ Cm$

$E = 3.4 \times 10^{6}\ NC^{-1}$

so that

$\tau = PE \sin\theta$

$= 2.49 \times 10^{-28} \times 3.4 \times 10^{6} \sin(0)$

$= 0$

(ii) when $\vec{P}$ is parpedicular to E

then $\theta = 90^\circ$

$$\tau = PE\sin\theta$$

$$= 2.49 \times 10^{-28} \times 3.4 \times 10^{6} \times \sin(90)^\circ$$

$$= 8.47 \times 10^{-28}\ Nm$$

(iii) when $\vec{P}$ is Anti-parallel to E then $\theta = 180^\circ$

$$\tau = PE\sin\theta$$

$$= 2.49 \times 10^{-28} \times 3.4 \times 10^{6} \times \sin(180)^\circ$$

$$= 0$$

A molecule of water vapour (H_2O) has an electric dipole moment of magnitude $P = 6.2 \times 10^{-30}$ cm. The dipole moment arises because the effective center of positive charge does not coin side with the effective center negative charge.

(i) How far apart are the effective centres of positive and negative charge in a molecule of H_2O.

(ii) What is the maximum torque on a molecule of H_2O in a eletric field of magnitude 1.5×10^{9} N/C

(iii) How much work ~~was~~ most an external agent do to rotate this molecule by 180° in the field staring from its folly oliged position for $\theta = 0$?

① Ans: There are 10 electron and corresponding 10 positive charge in this molecule. we can write for the magnitude of the dipole moment

$$p = q d$$

$$= (10 e) d$$

$$d = \frac{p}{10e}$$

$$= \frac{6.2 \times 10^{-30}}{10 \times 1.6 \times 10^{-19}}$$

$$= 3.9 \times 10^{-12} \, m$$

$$= 3.9 \, p.m$$

(ii) where $\theta = 90°$ the torqe is maximon

$$\tau = PE \sin\theta$$
$$= (6.2 \times 10^{-30}) \times (1.5 \times 10^{4}) \times \sin 90°$$
$$= 9.3 \times 10^{-26}$$

(iii) $U(\theta) = -PE \cos\theta$

$$W = U_{180} - U_{0}$$
$$= -PE \cos 180 + PE \cos 0°$$
$$= -PE(-1) + PE\, 0$$
$$= PE + PE$$
$$= 2PE$$
$$= 2 \times (6.2 \times 10^{-39}) \times (1.6 \times 10^{-19})$$
$$= 1.86 \times 10^{-25}$$

4 Superposition principle : It we have n point charges the act independtly in pairs. The force on any one of them. let us say q_1 is given by the vector som

$$\vec{F_1} = \vec{F}_{12} + \vec{F}_{13} + \vec{F}_{14} + \vec{F}_{15} - - - - + \vec{F}_n$$

where $\vec{F_1}$ Represents the force on charge q_1 by all the charges. F_{12} is the force acting on particle 1 owing to the presence of particle 2 and so on.

Gaussian surface :

Under necessary condition gauss's low is applying for uniform symmetrical and charge distributional surface this surface is called gaussian surface.

Characteristics of gaussians surface :

(i) The surface is hypothetical not real.

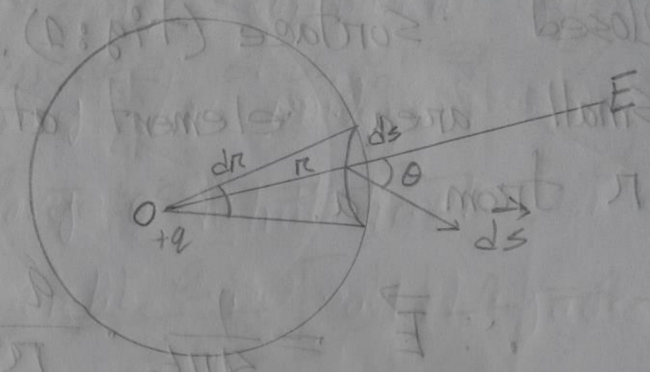
(ii) The surface may be circular, spherical or any other shape.

(iii) This surface must be closed.

(iv) This surface must be uniform and symmetric.

(v) charge is uniformly distributed through the surface.

□ state and prove gauss's low of Electro statistie.



Statement: The total flux of the eletrie field $\vec{E}$ over any closed surface is equal to $1/E_0$ times. The total net charge enclosed by the surface.

$$\phi = \oint \vec{E} \cdot \vec{ds} = q/E_0$$

$$\phi = \frac{q}{\epsilon_0}$$

here $\epsilon_0 \rightarrow$ Permitivity constant and $q \rightarrow$ total net charge enclosed by the surface.

Proof : Consider a single point charge $+q$ located at a point O inside a closed surface (fig: 1). Let ds be a small area element at a distance r from q.

$$\therefore \vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \cdot \hat{r}$$

The flux through the area ds is given by

$$d\phi = \vec{E} \cdot \vec{ds}$$

$$d\phi = E\,ds \cos\theta$$

$$d\phi = \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}\right) ds\cos\theta$$

$$d\phi = \frac{q}{4\pi\epsilon_0} \left(\frac{ds\cos\theta}{r^2}\right)$$

$$\left[\frac{ds\cos\theta}{r^2} = d\Omega = \text{solid angle}\right.$$

or, $d\phi = \frac{q}{4\pi\epsilon_0} d\Omega$

The total flux through the entire closed surface s is given by

$$\phi = \oint d\phi = \frac{q}{4\pi\epsilon_0} \oint d\Omega$$

$$\phi = \frac{q}{4\pi\epsilon_0} \left[\oint d\Omega = 4\Omega \right]$$

$$\phi = \frac{q}{4\pi\epsilon_0} \times 4\pi$$

$$\phi = \frac{q}{\epsilon_0}$$

$$\therefore \phi = \frac{q}{\epsilon_0} \text{ (Proved)}$$

□ Deduce Coulombs low from gauss's low:

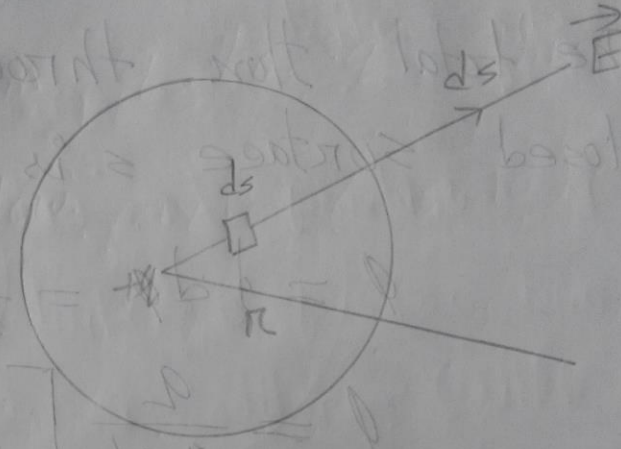


fig: 1

Soll^n: Let us apply Gauss's low to an isolated point charges $+q$ as shown in fig (1) constant a gaussian surface taking r as radius $\vec{E}$ is radiul on surface.

$$\vec{E}\cdot\vec{ds} = Eds\cos 0^\circ = Eds$$

From gauss's law

$$\oint \vec{E} \cdot \vec{ds} = \oint E ds = \frac{q}{\epsilon_0}$$

E is constant at all points on the surface

Thus,

$$E \oint ds = \frac{q}{\epsilon_0}$$

$$\text{or } E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$\therefore \vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

let us put a second point charge q' on this surface the force $\vec{F}$ that acts on this charge q' is

$$\vec{F} = q' E$$

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{q q'}{r^2} \hat{r}$$

This direction on this force is along the direction of $\vec{E}$.

i.e along the radius vector drawn from q to q'. This is Coolomb's low.

4 An electric dipole consist of opposite charge of magnitude of $2.0\times10^{-6}C$ seperated by 2.0 cm. The dipole is place in an extranal field $2.0\times10^{5}NC^{-1}$.

calculate

(i) Dipole moment

(ii) The maximum power of dipole

(iii) The work done two truk dipole through $180°$ starting from to position $\theta = 0°$

(i) **Soln:** Dipole moment $P = qd$

$= 2.0\times10^{-6}\times0.02$

$= 4\times10^{-8}$ cm

$q = 2.0\times10^{-6}$

$d = 2.0$ cm

$= 0.02$ m

(ii) Then $\theta = 90^\circ$ The maximum Power

$$\vec{\tau} = -PE\sin\theta$$

$$= -(4\times10^{-8})(2.0\times10^{5})(\sin 90^\circ)$$

(iii) $$\omega = U_{180} - U_{0^\circ}$$

$$= (-PE\cos180^\circ) - (-PE\cos 0)$$

$$= \{(-(4\times10^{-8})(2.0\times10^{5})\cos(180^\circ)\} - \{-(4\times10^{-8})(2.0\times10^{5})\cos(0^\circ)$$

☐ Charge is distributed uniformly over the surface of a sphere use gass's low to find the eletric field at the points

① outside the sphere

⑪ on the surface othe sphere.

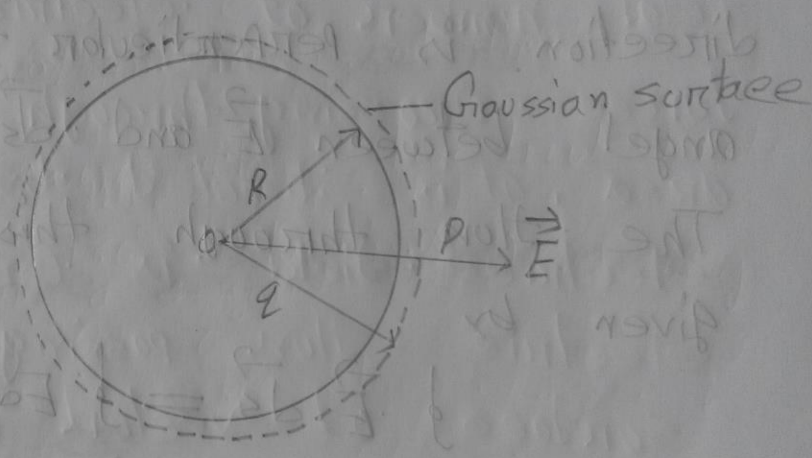


Fig:1

①

⇒ when the point P lies outside the sphere

Let P is a point at a distane r from the Centre O Fig(1) we have to find the eletric field $\vec{E}$ at P.

Draw a ~~Concentric~~ ~~sh~~ sphere (shown dotted) of various op with centre O. This is the Gaussian surface. At all points of this sphere the magnitude of the Electric field is the same and it direction is perpendicular to the surface angel between $\vec{E}$ and $\vec{ds}$ is zero.

The flux through this surface is given by

$$\oint \vec{E}.\vec{ds} = \oint E ds$$

$$= E \oint ds$$

$$= E(4\pi r^2) \quad (1)$$

By Gauss's law,

$$\oint \vec{E}.\vec{ds} = \frac{q}{\epsilon_0}$$

$$E(4\pi r^2) = \frac{q}{\epsilon_0}$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

In vector form,

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2} \hat{r}$$

Here the electric field at an exter[nal] point due to a uniformly charged sphere is the same as if the total charge [is] concentrated at this centre.

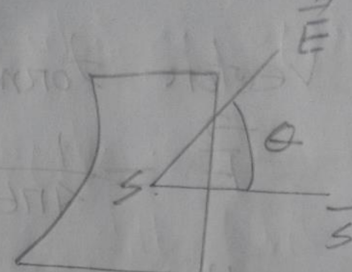
(ii)

$\Rightarrow$ when the point lies on the surface

Here $r = R$

$$\therefore E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{R^2}$$

Electric flux:



The electric flux is defined as the total number of electric lines of force crossing through the given area. It is denoted by ϕ_E. If the electric field is uniform the electric flux passing throoug a surface of vector area $\vec{s}$ is

$$\phi_E = \vec{E} \cdot s = Es \cos\theta$$

where $\vec{E}$ is the electric field s is the area of surface and θ is the angel between electric fild lins and normal to s.

The flax ot the electrie field
is seallen. It units Nm^2e^{-1}.