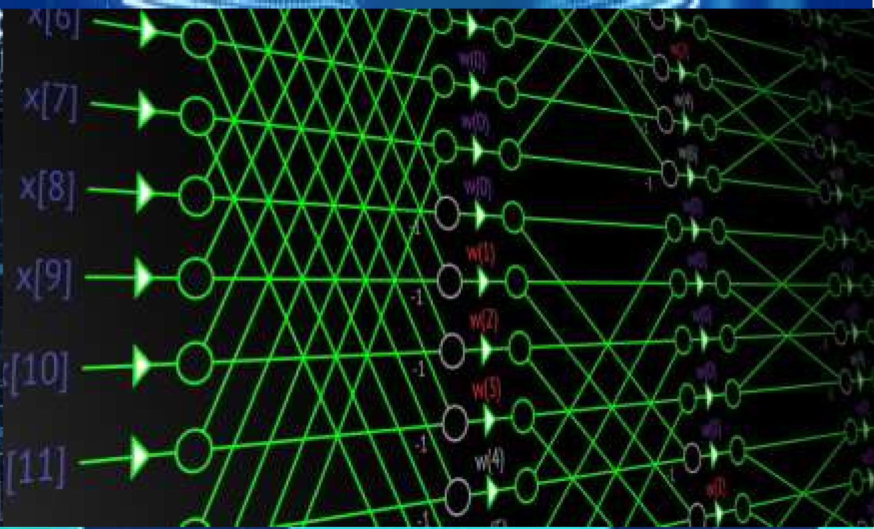
A Study Outline of

Digital

Signal

Processing



ABU SALEH MUSA MIAH(ABID)

A Guidebook of

Digital Signal Processing

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Digital Signal Processing

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Dedicated

To

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লেখকের কথা

Digital Signal Processing কোর্সটি বাংলাদেশের প্রায় সকল বিশ্ববিদ্যালয়ের সিলেবাসে স্থান লাভ করেছে,বাজারে টেক্সটবই থাকলেও গভীর ভাবে অনুধাবন ও পরীক্ষায় ভাল মার্কস উঠানোর মত সাজানো টেক্সটবই সহজলভ্য নয় । পরীক্ষার প্রস্তুতির স্বার্থে তাই, এই গাইডবইটি আপনাকে সহযোগিতা করবে বলে আশা করা যায় ।

এই বইয়ের পাঠক হিসেবে, আপনিই হচ্ছেন সবচেয়ে গুরুত্বপূর্ণ সমালোচক বা মন্তব্যকারী । আর আপনাদের মন্তব্য আমার কাছে মূল্যবান, কারন আপনিই বলতে পারবেন আপনার উপযোগী করে বইটি লেখা হলো কিনা অর্থাৎ বইটি কিভাবে প্রকাশিত হলে আরও ভাল হতো । সামগ্রিক ব্যাপারে আপনাদের যে কোন পরামর্শ আমাকে উৎসাহিত করবে ।

Engr. Abu Saleh Musa Miah (Abid)
Writer

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3.	**James Martin**	:	**Principles of Database Management**, *Prentice-hall Of India Pvt Ltd*
4.	**Ullman**	:	**Database Management systems**, *Prentice-Hall Publication.*
5.	**Abey**	:	**Oracle 8i a Beginners Guide**, *McGraw Hill.*

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Digital Signal Processing

CSE3131: Digital Signal Processing
75 Marks [70% Exam, 20% Quizzes/Class Tests, 10% Attendance]
3 Credits, 33 Contact hours, Exam. Time: 4 hours

Introduction: signals, systems and signal processing, classification of signals, the concept of frequency in continuous time and discrete time signals, analog to digital and digital to analog conversion, Sampling and quantization.
Discrete time signals and systems: Discrete time signals, discrete time systems, analysis of discrete time linear time invariant systems. Discrete time systems described by difference equations, implementation of discrete time systems, correlation and convolution of discrete time signals.
The z-transform: Introduction, definition of the z-transform, z-transform and ROC of infinite duration sequence, properties of z-transform inversion of the z-transform, the one-sided z-transform.
Frequency analysis of signals and systems: Frequency analysis of continuous time signals, Frequency analysis of discrete time signals, Properties of Fourier transform of discrete time signals, Frequency domain characteristics of linear time invariant system, linear time invariant systems as frequency selective filters, Inverse systems and deconvolution.
The Discrete Fourier Transform: The DFT, Properties of the DFT, Filtering method based on the DFT, Frequency analysis of signals using the DFT.
Fast Fourier Transform Algorithms: FFT algorithms, applications of FFT algorithm.
Digital Filters: Design of FIR and IIR filters.
Adaptive filters: Adaptive system, kalman filters, RLS adaptive filters, the steepest-descent method, the LMS filters.
Application of DSP: Speech processing, analysis and coding, Matlab application to DSP.

<u>Books Recommended:</u>

1. J. G. Prokis : Digital Signal Processing, Prentice-hall Of India
2. Defatta : Digital Signal Processing, Wiley India Pvt Ltd
3. R. G. Lyon : Understanding Digital Signal Processing, Orling Kindersley India

I

Important Question

1. Define signal ?
2. Define System?
3. Define complex exponential signal? Is it possible to represent a real cosine signal by a complex exponential signal? how?
4. Define signal processing?
5. Distinguish between deterministic and non-deterministic signals?
6. Define discrete signal? show that a discrete-time sinusoidal is periodic if its frequency is a rational number?
7. Explain the processing steps of A/D converter
8. Deduce the relationship between the frequency of analog signal and the frequency of discrete-time signal?
9. Briefly outline the advantages and dis advantages and dis advantages of digital signal processing as opposed to analog signal processing?
11.define quantization?
12.Defile bilinear transformations?

Exam Question:

1. What do you mean by DSP? Discuss the advantages and disadvantages of DSP. **5 Marks CSE-2004(OLD) CSE-2006 CSE-2010 CSE-2011**
2. Explain the following properties of the Linear Time-Invariant Digital Systems: (i) Time Invariance (ii) Unit Impulse System Response. **5 Marks CSE-2004 (OLD) CSE-2010**
3. Explain continuous-time and discrete time signals with examples. 5 Marks CSE-2004(OLD) CSE-2005
4. Define the following terms in case of discrete time signals with suitable examples: (i) Unit impulse sequence (ii) Unit-Step Sequence (iii) Exponential Sequence (iv) Unit sample Sequence. 9 Marks CSE-2005(OLD) CSE-2006 CSE-2011
5. Define the following operation in case of discrete time signals: (i) Shift-Operation and (ii) Transpose Operation. 6 Marks CSE-2004 (NEW)
6. What do you mean by Quantization and sampling? Describe sampling and quantization processes with examples. **8 Marks CSE-2005 CSE-2010**
7. Explain the following signal operations: (i) Signal Multiplication (ii) Signal Addition. **6 Marks CSE-2005**
8. What is a signal? Why speech signal cannot be described by a simple function ?4 Marks CSE-2007

9. Derive the equation of a continuous time signal with peak amplitude of 1 at a frequency f_n. Then express the sequence into discrete form and discuss with necessary figures. 4 Marks CSE-2007
10. What do you mean A/D and D/A conversion? Discuss the different steps of A/D conversion. 4 Marks CSE-2007
11. Considering two analog signals $x_i(t) = cos\, 2\pi\, F_i t\, , i = 1, 2;$ having frequencies $F_1 = 10\, Hz$ and $F_2 = 50\, Hz$. If the signals are sampled at $F_s = 40\, Hz$, show that $x_1(n) = x_2(n)$. 3 Marks CSE-2007
12. Differentiate between Digital and Analog signal processing. 3 Marks CSE-2008
13. Deduce the relationship between sampling frequency and signal frequency. **3 Marks CSE-2008**
14. Describe Unit step sequence and explain its applications with examples. 3 Marks CSE-2008
15. Discuss the process of signal denoising by down sampling. 6 Marks CSE-2008
16. Describe the sampling rate increasing and decreasing operations with examples. 4 Marks CSE-2008
17. What do you mean by signal and system? 2 Marks CSE-2009 CSE-2010
18. Consider the two sinusoids, $x_1(t) = 5\, cos(2\pi\, (100)t + \frac{\pi}{3})$ and $x_2(t) = 4\, cos(2\pi\, (100)t - \frac{\pi}{4})$. By applying phasor addition rule, show that the sum of the two signals will be, $x_3(t) = 5.536\, cos(2\pi\, (100)t +\, 0.2747)$. 5 Marks CSE-2009
19. A Discrete Time signal $x(n)$ is defined as:

$$x(n) = \begin{Bmatrix} 1 + \frac{N}{3}, -3 \leq N \leq -1 \\ 1, 0 \leq N \leq 3 \\ 0, otherwise \end{Bmatrix}$$

(i) Determine its value and sketch the signal $x(n)$. 2 Marks
(ii) Sketch the signal if the first fold $x(n)$ and then delay the resulting signal by 4 samples. 2 Marks
(iii) Sketch the signal if the first delay $x(n)$ by 4 samples and then fold the resulting signal. 2 Marks
(iv) Sketch the signal $x(-n + 4)$. 2 Marks
20. What do you mean by aliasing? **2 Marks CSE-2009**

Question: Give a brief introduction about the different part of digital signal processing system. Marks Question-2016 1(a)

Digital signal processing (**DSP**) is the used to perform a wide variety of signal processing operations. The signals processed in this manner are a sequence of numbers that represent samples of a continuous variable in a domain such as time, space, or frequency.

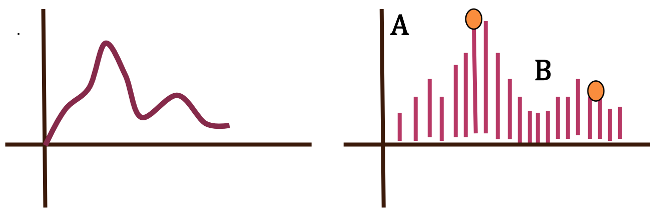
Different part of DSP:
Signal:

Signal is a means that conveys information. Signal is defined as any physical quantity that varies with time, space or any other variables, such as distance, position, temperature, pressure etc. Mathematically any signal can be described as a function of one or more independent variables

Signals can be analog or digital. Analog signals can have an infinite number of values in a range, digital signals can have only a limited number of values.

Signal sampling

Analog-to-digital converter (ADC). Sampling is usually carried out in two stages, discretization and quantization. Mathematically any signal can be described as a function of one or more independent variables. For example, the functions $x_1(t) = 5t$ and $x_2(t) = 20\,t^2$ describe two signals. One that varies linearly with the independent variable r (time) and a second that varies quadratically with t



The Nyquist–Shannon sampling theorem states that a signal can be exactly reconstructed from its samples if the sampling frequency is greater than twice the highest frequency component in the signal. In practice, the sampling frequency is often significantly higher than twice the Nyquist frequency.[4]

Theoretical DSP analyses and derivations are typically performed on discrete-time signal models with no amplitude inaccuracies (quantization error), "created" by the abstract process of sampling. Numerical methods require a quantized signal, such as those produced by an ADC. The processed result might be a frequency spectrum or a set of statistics. But often it is another quantized signal that is converted back to analog form by a digital-to-analog converter (DAC).

The basic components of signals are :-

Amplitude
Phase
Frequency

Domains

In DSP, engineers usually study digital signals in one of the following domains: time domain (one-dimensional signals), spatial domain (multidimensional signals), frequency domain, and wavelet domains. They choose the domain in which to process a signal by making an informed assumption (or by trying different possibilities) as to which domain best represents the essential characteristics of the signal and the processing to be applied to it.

Time and space domains

The most common processing approach in the time or space domain is enhancement of the input signal through a method called filtering.

Frequency domain

Signals are co**nverted from time or space domain to the frequency domain usually through use of the Fourier transform.** The Fourier transform converts the time or space information to a magnitude and phase component of each frequency. With some applications, how the phase varies with frequency can be a significant consideration. Where phase is unimportant, often the Fourier transform is converted to the power spectrum, which is the magnitude of each frequency component squared.

Z-plane analysis

The Z-transform provides a tool for analyzing stability issues of digital IIR filters. It is analogous to the Laplace transform, which is used to design and analyze analog IIR filters.

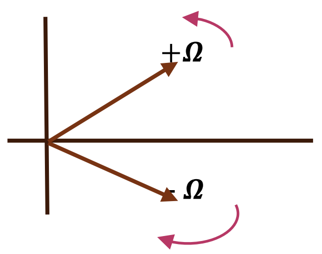
Wavelet

In numerical analysis and functional analysis, a discrete wavelet transform (DWT) is any wavelet transform for which the wavelets are discretely sampled. As with other wavelet transforms, a key advantage it has over Fourier transforms is temporal resolution: it captures both frequency *and* location information.The accuracy of the joint time-frequency resolution is limited by the uncertainty principle of time-frequency.

Question: Define complex exponential signal. Is it possible to represent a real cosine signal by a complex exponential signal? How ? 3 Marks CSE-2009

The complex-conjugate-exponential signal are some times called phasors , As time progresses the phesors rotate in opposite direction with angular frequency $(\pm\Omega)$ radiance per second.
A positive frequency corresponds to a counterclockwise uniform angular motion
A negative frequency simply corresponds to a clockwise angular motion.
complex exponential signal are represented as $e^{\pm j}\Omega$

which can be explained using Euler identity ,$e^{\pm j}(\Omega t+\theta)$ $=e^{\pm j}\emptyset = \cos\emptyset \pm$ jsign$\emptyset$

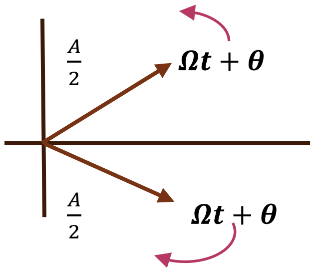


It is possible to represent a real cosine signal by a pair of complex-conjugate-exponential signals.

A sinusoidal signal
$x_a(t) = A\cos(\Omega_a t + \theta)$ can be expressed as

$$\frac{A}{2}\, e^{j(\Omega t+\theta)} + \frac{A}{2}\, e^{-j(\Omega t+\theta)}$$

So a real cosine signal can be obtained by adding to equal amplitude complex-conjugate-exponential signals.



As time progresses the phesors rotate in opposite direction with angular frequency $(\pm\Omega)$ radiance per second.

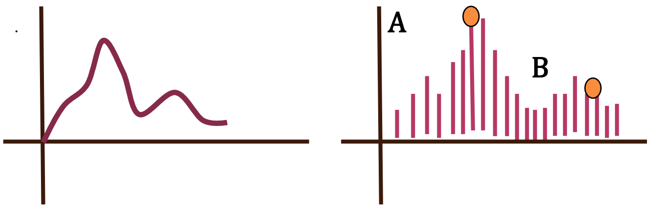
Question: 1(b)-2014 Describe basic elements of a digital signal processing system.
Question: Define Signal and signal processing. Distinguish between deterministic and non-deterministic signals. 3 Marks CSE-2010

Question: 1(a)-2014 What is signal and Signal processing?

Signal:

Signal is a means that conveys information. Signal is defined as any physical quantity that varies with time, space or any other variables, such as distance, position, temperature, pressure etc.

Mathematically any signal can be described as a function of one or more independent variables. For example, the functions $x_1(t) = 5t$ and $x_2(t) = 20\,t^2$ describe two signals. One that varies linearly with the independent variable r (time) and a second that varies quadratically with t



Examples of signals that we encounter frequently in our life are speech and physiological signals (ECG-provides information about condition of heart, EEG- provides information about condition of brain). Speech, ECG, EEG signals are examples of information bearing signals as a function of time.

Signals can be
analog or digital. Analog signals can have an infinite number of values in a range, digital signals can have only a limited number of values.

The basic components of signals are :-

- **Amplitude**
- **Phase**
- **Frequency**

Signal Processing:

Signal processing is any operation that changes the characteristics of a signal. These characteristics include the amplitude, shape, phase and frequency content of the signal.

Most of the signals encountered in science and engineering are analog in nature. That is, the signals are functions of a continuous variable such as time or space and usually take on values in a continuous range. Such signals may be processed directly by appropriate analog systems (such as filters or frequency analysers) or frequency multipliers for the purpose of changing their characteristics or extracting some desired information. In such a case we say that the signal has been processed. **The signal processing operation on an analog signal with the help of a analog signal processor where both the input signal and the output signal are in analog form, is called analog signal processing.** Similarly, **The signal processing operation on an analog signal with the**

help of A/D converter, digital signal processor and D/A converter, where both the input signal and the output signal are in analog form, is called digital signal processing.

Difference between deterministic and non-deterministic signals:

Following are some importance difference between deterministic and non-deterministic signals:

Deterministic Signal	Non-Deterministic Signal
Any signal that can be uniquely described by an explicit mathematical expression, a table of data, or a well-defined rule is called deterministic signal.	Any signals that either cannot be described to any reasonable degree of accuracy by explicit mathematical formulas, or such a description is too complicated to be of any practical use are called non-deterministic signal. The lack of such a relationship implies that such signals evolve in time in an unpredictable manner.
This term is used to emphasize the fact that all past, present and future values of the signal are known precisely, without any uncertainty.	This term is used to emphasize the fact that all past, present and future values of the signal are not known precisely, with unpredictability.
These signals are described in terms of explicit mathematical expression, a table of data, or a well-defined rule	These signals are described in terms of probabilities or with the help of their statistical properties.
Example:	**Example:** The output of a noise generator, the seismic signal and the speech signal are examples of non- deterministic signals.

21. Define discrete signal. Show that a discrete-time sinusoid is periodic if its frequency is a rational number. **3 Marks CSE-2010**

By definition , a discrete time signal x(n) is periodic with period N(N>0) if and only if,

X (n + N)=x(n) for all n ....(i)

The smallest value of N for which (i) is true is called fundamental period .

For a sinusoidal with frequency f_0 to be periodic, we should have ,

$$\text{Cos}[2\pi f_0 \ (\text{N+n}) + \boldsymbol{\theta}] = \text{Cos}[2\pi f_0 \,\text{n} + \boldsymbol{\theta}]$$

This relation is only possible if and only if there exist an integer K such that

$$2\pi f_0 \,\text{N} = 2\text{k}\,\pi$$

Or $f_0 = \frac{K}{N}$

Thus a discrete-time sinusoidal is periodic if its frequency can be expressed as the ratio of two integers.

22. Explain the processing steps of A/D converter. Deduce the relationship between the frequency of analog signal and the frequency of discrete time signal. **4 Marks CSE-2010**

23. Define bilinear transformation. What is the relation between digital frequency and analog frequency? Is this linear? **5 Marks CSE-2010**

24. Define each of the following terms as applied to discrete time signal processing systems: (i) Linearity (ii) Time-Invariance (iii) Causality and (iv) Stability. **4 Marks CSE-2011**

25. Explain general classification of discrete signals. 3 Marks CSE-2011

26. What are the different types of operations performed on discrete-time signals? 1 Marks CSE-2011

27. Let $x_1(n) = \{2,2,0,2,5,-3,2\}$ and $x_2(n) = \{5,5,5,5,5\}$. Write the mathematical expression for $x_1\,(n)$ and $x_2(n)$ by using unit sample sequence and unit step sequence respectively. 4 Marks CSE-2011

28. What are continuous and discrete signals? Give the classification of signals and explain them. 6 Marks CSE-2012

29. Define system. What are the types of system? 2 Marks CSE-2012

Question: Classify the following signals according to whether they are (1) one- or multidimensional: (2) single or multichannel, (3) continuous time or discrete time, and (4) Analog or digital (in amplitude). Give a brief explanation.
(a) Closing prices of utility stocks on the New York Stock Exchange.
(b) A color movie.
(c) Position of the steering wheel of a car in motion relative to car's reference frame.
(d) Position of the steering wheel of a car in motion relative to ground reference frame.
(e) Weight and height measurements of a child taken every month.

Solution:

(a) One dimensional, multichannel, discrete time, and digital.
(b) Multi dimensional, single channel, continuous-time, analog.
(c) One dimensional, single channel, continuous-time, analog.
(d) One dimensional, single channel, continuous-time, analog.
(e) One dimensional, multichannel, discrete-time, digital.

Question: 2(c)-2016 – Prove that the *The highest rate of oscillation in a discrete-time sinusoid is attained*

To illustrate this property, let us investigate the characteristics of the sinusoidal signal sequence

$$x(n) = \cos \omega_0 n$$

when the frequency varies from 0 to π. To simplify the argument, we take values of $\omega_0 = 0, \pi/8, \pi/4, \pi/2, \pi$ corresponding to $f = 0, \frac{1}{16}, \frac{1}{8}, \frac{1}{4}, \frac{1}{2}$, which result in periodic sequences having periods $N = \infty, 16, 8, 4, 2$. as depicted in Fig. 1.13. We note that the period of the sinusoid decreases as the frequency increases. In fact, we can see that the rate of oscillation increases as the frequency increases.

Question: 4(b)-2015 – Define Discrete time signal. Show that *A discrete-time sinusoid is periodic only if its frequency f is a rational number.*

Discrete time signals The discrete time signals are defined only at certain specific values of time .unlike analog signal , digital signal makes sudden change from value to value to reach one point to another.

When $w = \pi$ *(or $\omega = -\pi$) or, equivalently, $f = \frac{1}{2}$ (or $f = -\frac{1}{2}$)*

$$x(n+N) = x(n) \qquad \text{for all } n$$

Proof:

$$\cos[2\pi f_0(N+n)+\theta] = \cos(2\pi f_0 n+\theta)$$

This relation is true if and only if there exists an integer k such that

$$2\pi f_0 N = 2k\pi$$

or, equivalently,

$$f_0 = \frac{k}{N} \tag{1.3.11}$$

note that f

According to (1.3.11), a discrete-time sinusoidal signal is periodic only if its frequency f_0 can be expressed as the ratio of two integers (i.e., f_0 is rational).

Define system ?

Each signal is generated by an associated means such like voice sound is generated by forcing air through the vocal cords .this means or object which responds to the stimulus or force to generate signal is known as system .
A system can also be defined as a physical device that performs an operation on a signal.
For example,

Filters are used to reduce noise and interference corrupting a information-bearing signal.

Systems are classified according to their processing or action on signal.
Linear system :- the system operation is linear.
Non-linear :- the system operation is non-linear.

Digital SP system :- performs digital processing on a signal implemented by digital hardware(logic circuit) and a set of software.

Analog SP system :- performs processing by analog hardware.

We can summarize as any living source or physical devise (analog or digital) or software that is involved in generating or processing signal is called system

Define signal source ?

Any signal is generated by a system which responds to a stimulus or force. For voice signal this system is consists of vocal cords and vocal tract , sometimes called together vocal cavity and the stimulus is air.

The stimulus in combination with the signal system is calls signal source ,such as speech sources , image source.

Define the classification of signal

Signal is a means that conveys information. signal is defined as any physical quantity that varies with time, space or any other variables, such as distance ,position, temperature ,pressure etc.

Signal are classified in two types,

Signal

Deterministic

Non-Deterministic

The deterministic signals can be classified in more three types.

1.Continuous tine signal

2.discrete time signal

3.Digital signal

Question: 1(d)-2014 Distinguish deterministic and rangom signals

Distinguish between deterministic and non-deterministic signals?

Deterministic signals are those which can be described by mathematical description referred as signal model. any signal that can be uniquely described by an explicit mathematical expression , a table of data , or a well defined rules is called deterministic.

This term is used to emphasize the fact that all past , present , and future values of the signal are known precisely.
Non-Deterministic signals/ Random signals: are those that can not be described as mathematical model and are described in terms of probability and their statistical properties .these signals either cannot be described to any reasonable degree of accuracy by explicit mathematical formulas, or

such descriptions are too complicated to be of any practical use .this type of signal is known as non-deterministic or random signal. Example

The seismic signal

Speech signal

Question: 2(a)-2014 What do you mean by continuous time and discrete time signals?

Distinguish between continuous time and discrete time signal?

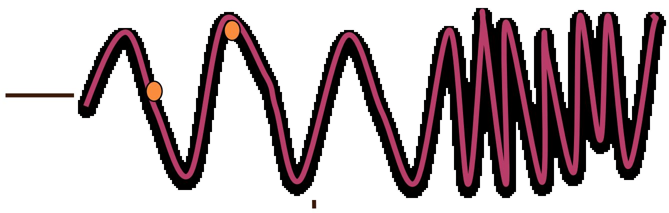
continuous time or analog signals An analog signal also known as continuous time signal has infinitely many levels of intensity over a period of time. As the wave moves from value A to value B, it passes through and includes an infinite number of values along its path.

$$x_a(t) = A\cos(\pi_a t + \theta)$$

Mostly the signals are represented as function of time (the change is measured against the time domain) . in analog signal both amplitude and time are continuous variable.

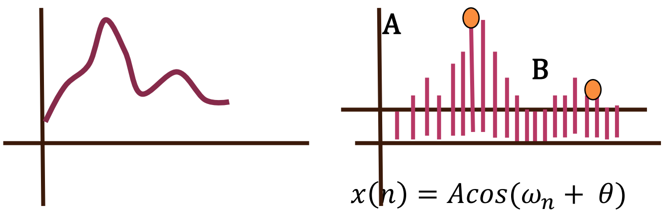
All most all the signal generated from various sources of nature are analog (continuous time) signals.

Example:-Electrocardiogram, Electroencephalogram , speech. **B**



Discrete time signals The discrete time signals are defined only at certain specific values of time .unlike analog signal , digital signal makes sudden change from value to value to reach one point to another.

Mostly the signals are represented as function of time and mathematically represented as a sequence of real or complex number (the change is measured against the time domain) . The discrete signal has values only at discrete instance of time(this time instance need not to be equidistance). the discrete time signal is obtained by time sampling a continuous time signal. here time is a continuous variable and amplitude is a discrete variable. also accumulating a variable over a period of time can lead us to a discrete time signal generation

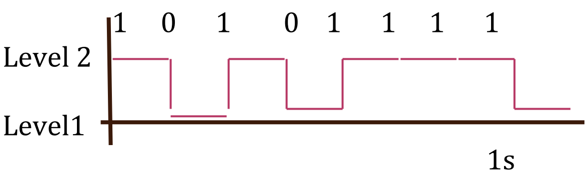


$$x(n) = A\cos(\omega_n + \theta)$$

Digital signal :

In digital signal the sampled value of analog magnitude is converted into a binary number. The digitization process consists of two steps sampling the signal and quantization. the quantization converts the sampled amplitude in binary code words

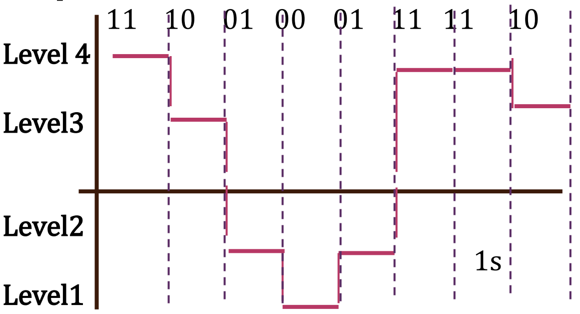
In general a positive voltage is encoded as a 1 and a 0 voltage as zero. here the signal can have two levels 0,1. And one bit can be sent per level.



8 bits sent in1 s,

Bit rate = 8 bps.one bit of data is sent per level.

A signal can have L levels,then each level can transmit $\log_2 L$ bits.
For example let us consider a digital signal with four levels then the signal will be able to send 2 bits per level.



16 bits sent in1 s,
Bit rate = 16 bps.Two bit of data is sent per level.

Define signal processing system?

Signal processing system is defined as a physical device that performs an operation on a signal . like noise reduction ,interference control , compression etc. in more practical sense signal processing system
Is an operator that maps an input sequence into an output sequence according the incorporated algorithms.
Example , filters , frequency analyzer ,frequency multipliers,

Define analog signal processing system?

The system operates on a signal directly on its analog form to change its characteristics or to extract some information is known as analog signal processing system.in such a case both input signal and output signal are in analog form.

the function of analog signal processor is to process direct on analog signal by functions such as attenuation ,amplification, filtering ,current to voltage conversion and voltage to current conversion.
Examples , , filters , frequency analyzer ,frequency multipliers.

Question1(a)-2015 Define digital signal processing? Advantage and disadvantage

Define digital signal processing system?

Digital Signal Processing is a field of numerical mathematics that is concerned with the processing of discrete signals
This area of mathematics deals with the principles that underlie all digital systems

The digital signal processing system includes a analog-to-digital converter ,a digital signal processor and as in most case signals are to be presented to user as analog signal it also includes a digital-to-analog converter .

Define basic components digital signal processing system?

Most of the signals generated from different sources are analog in nature ,but a digital signal processor which can perform a wide range of processing on signals need to get a digital signal as input. To perform processing digitally there is a need for the analog signal and the digital processor.

the first component of DSP system is an analog to digital converter.). the digital signal is obtained by time sampling a continuous time or analog signal. the sampled value of analog magnitude is converted into a binary number .

example DSP16xx,DSP32xx.

digital signal processor The digital signal processor can be a large programmable digital computer or a small microcontroller device programmed to perform the desired operations on the input signal. the programmable processing system provides a flexibility in frequent change in processing operation just changing the software .

digital to analog converter , in many applications like speech communication it is requested to present analog form signal ,then a digital to analog converter generates an equivalent analog signal .in signal analysis this component is not needed .

discuss the advantaged and disadvantage of DSP over ASP?

Advantages

- The DSP provides a flexibility in frequent change in processing operation simply changing the software .
- The DSP provides a much better control in accuracy requirements ,some terms related to this factor are word length, floating-point verses fixed-point arithmetic.
- In DSP the digital system can be cascaded without any loading problems.
- In this digital circuits can be reproduced easily in large quantities at comparatively lower cost.
- The digital circuits are less sensitive to tolerances of component values.
- These are easily transported because the digital signals can be processed off line.

- The signal is easily stored with out any deterioration or loss of signal fidelity.it makes the signal transportable and can be process off-line ..
- It is usually difficult to perform precise mathematical operations directly on signals in analog form, where DSP allows implementation of sophisticated signal processing algorithms.
- The output DSP dose not varies according to environmental factor like thermal effect.
- Digital implementation of signal processing system is farther cheaper then analog counterpart .

Digital out put x(n) analog signal x(t)

more over the basic advantage is easier , cheaper , reconfigurable and accuracy are the advantage of DSP.

Disadvantage

- For a well defined processing operation a hardwired implementation can be optimized, resulting in a cheaper signal processor and it will run faster than the multitasking digital device.

- Digital signal processing system has a overhead of ADC and DAC which in-turn slowdowns the entire processing.

- Signals having an extremely wide bandwidth requires a fast-sampling-rate ADC,and for some of these kinds od analog signals the digitally processing approach is beyond of the state of the art od digital hardware.

Some applications of DSP?

1. Speech processing
2. Signal transmission on telephone channel
3. Image processing and transmission
4. Seismology and geography
5. In oil exploration
6. In processing of signal from outer space
7. Determining the distance among planets or detecting the space routes .
8. Consumer applications.
9. Biomedical processing.
10. Security purposes.

Question: 1(c)-2014 Discuss about advantages of digital over analog signal processing.
Answer:
1. Precise signal level of the digital signal is not vital. This means that digital signals are fairly immune to the imperfections of real electronic systems which tend to spoil analog signals. As a result, digital CD's are much more robust than analog LP's.

2. Codes are often used in the transmission of information. These codes can be used either as a means of keeping the information secret or as a means of breaking the information into pieces that are manageable by the technology used to transmit the code, e.g. The letters and numbers to be sent by a Morse code are coded into dots and dashes.

3. Digital signals can convey information with greater noise immunity, because each information component (byte etc) is determined by the presence or absence of a data bit (0 or one). Analog signals vary continuously and their value is affected by all levels of noise.

4. Digital signals can be processed by digital circuit components, which are cheap and easily produced in many components on a single chip. Again, noise propagation through the demodulation system is minimized with digital techniques.

5. Digital signals do not get corrupted by noise etc. You are sending a series of numbers that represent the signal of interest (i.e. audio, video etc.)

6. Digital signals typically use less bandwidth. This is just another way to say you can cram more information (audio, video) into the same space.

7. Digital can be encrypted so that only the intended receiver can decode it (like pay per view video, secure telephone etc.)

8.Enables transmission of signals over a long distance.
9. Transmission is at a higher rate and with a wider broadband width. It is more secure.
11. It is also easier to translate human audio and video signals and other messages into machine language.
12. There is minimal electromagnetic interference in digital technology.
13. It enables multi-directional transmission simultaneously.

Question: 2(a)-2015 – Describe analog signal to digital signal (A/D) Conversion process in brief with figure Marks 5
Answer:

Explain the processing steps of A/D converter ?

THE analog to digital conversion is conducted in three steps

1. Sampling :-

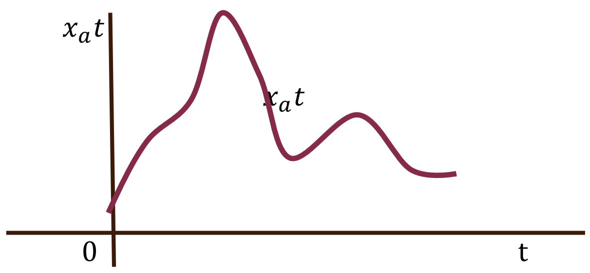
It is taking samples of continuous time signals at discrete time instance. here T is the sampling interval generally sampling rate is at least double of the frequency.

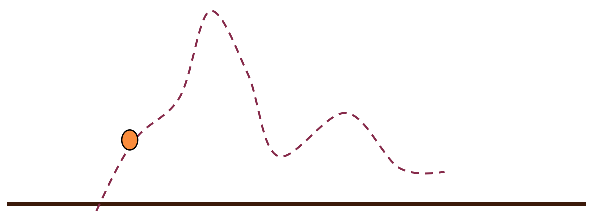
2. Quantization:-

the value of each signal sample is represented by a value selected from a finite set of possible values. The difference between unquantize sample x(n) and quantized sample $x_{q(n)}$ is called quantization error.

3. coding:-

in coding process each discrete value $x_{q(n)}$ is represented by a b-bit binary sequence.





Question: 2(b)-2014 State sampling theorem? What is anti aliasing?

Explain sampling analog signal ?

The periodic or uniform sampling is described as
$\boldsymbol{x(n) = x_a(nT)}$ $\quad -\infty < \boldsymbol{n} < \infty$, where x(n) is a discrete time signal by taking samples of analog signal $\boldsymbol{x_a(t)}$ ***every T*** second.

Sampling rate :- the time interval T between successive samples is called sampling period or sample interval. and the reciprocal of T , $\frac{1}{T} = F_s$ is called the sampling rate or sampling frequency.

Deduce the relationship between the frequency of analog signal and the frequency of discrete-time signal.

To establish the relationship let us consider an analog sinusoidal signal of form
$\boldsymbol{x_a(t) = A\cos(2\pi Ft + \theta)} \ldots\ldots (\boldsymbol{i})$ **when,** sampled periodically at a rate of , , $\frac{1}{T} = F_s$ samples per second
So (i) could be written as

$$\boldsymbol{x_a(nT) = A\cos(2\pi FnT + \theta)}\text{.....(ii)}$$

Comparing (ii) with x$\boldsymbol{(n) = A\cos(2\pi fn + \theta)}$
the general equation of discrete time sinusoidal signals
we find ,
f=FT

$$\boxed{f = \frac{F}{F_s} \text{.........(iii) [relation]}}$$

discrete time sinusoidal signals ,
$-\frac{1}{2} < f < \frac{1}{2}$ ....*(iv)*

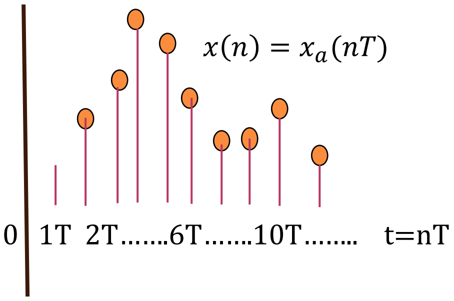
Now substituting from (iii) into (iv)

$$\boxed{-\frac{1}{2} < \frac{F}{F_s} < \frac{1}{2}}$$

$$\rightarrow -\frac{F_s}{2} < F < \frac{F_s}{2}$$

Thus we find that the frequency F of continuous time sinusoidal signals when sampled at a rate of F_s must fall in the above range.

We also can find that if the max frequency of discrete time sinusoidal is f then with a sampling rate F_s *the max frequency of discrete time sinusoidal is F is* $\frac{F_s}{2}$.



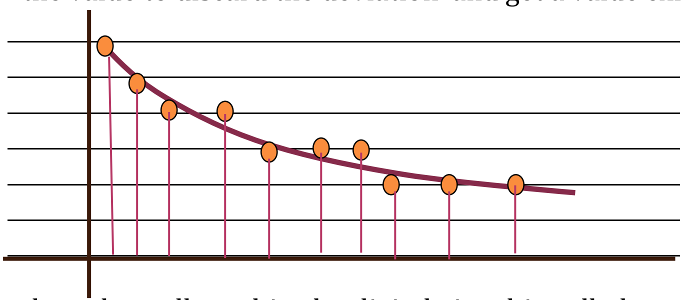
Explain quantization of continuous –amplitude signal ?

The process of converting a discrete time continuous amplitude signal into a digital signal by expressing each sample value as a finite number of digits is called quantization. The quantized value of actual sample value x(n) is $x_a(n) = Q[x(n)]$,

Here the quantization error is a sequence $e_a(n)$ *is the*
Difference between the quantized value and actual sample value.

$e_a(n) = x_a(n) - x(n)$ **, the instantaneous** quantization error can not exceed half of the quantization step.

As quantization converts the samples in values those are in a finite set. If any value that is originally is not in the set but is very near to any member of the set then a truncation or rounding is done on the value to discard the deviation and get a value existing in the set.



the values allowed in the digital signal is called quantization levels. And the distance Δ , between two successive quantization levels is called quantization step size or resolution . The rounding quantizer assaigns each sample of x(n) to the nearest quantization levels . and a truncation quantizer assaigns each sample of x(n) to the quantization level below to it.

quantization step:- . And the distance Δ , between two successive quantization levels is called quantization step size or resolution . it is never a random value rather it is determined from the maximum and minimum value in the set.

If x_{max} *and* x_{min} **represent the** maximum and minimum value of x(n) and L is the number of quantization levels then

$$\Delta = \frac{x_{max} - x_{min}}{L-1}$$

x_{max} *and* x_{min} **is called the dynamic range of the signal.**
L is also obtained from 2^{b+1} here the quantization has b bits of accuracy.

If the dynamic range of the signal is fixed ,increasing the number of quantization levels we can increase the accuracy of the quantization and decrease the step size.

SQNB

The quality of the output of A/D is usually measured by signal-to-quantization-noise-ratio.which provides the signal power p_x to the noise power p_q

SQNR=$\frac{p_x}{p_q}$

Expressed in decibles.SNQR increases approximately 6DB for each additional bit in wordlength.

Question: 1(a) – 2017

We know nyquest rate fs=F*2

Fs=2*3=6 hz

Question: A continuous time sinusoid $x_a(t)$ with fundamental period $T_p = \frac{1}{F_0}$ is sampled at a rate $F_s = \frac{1}{T}$ to produce a discrete-time sinusoid $x(n) = x_a(nT)$. (a) Show that $x(n)$ is periodic if $\frac{T}{T_p} = \frac{k}{N}$ (i.e. $\frac{T}{T_p}$ is a rational number) (b) If $x(n)$ is periodic, what is its fundamental period T_P in seconds? (c) Explain the statement: $x(n)$ is periodic if its fundamental period T_P, in seconds, is equal to an integer number of periods of $x_a(t)$. CSE-2011

Solution:

(a)

$$\begin{aligned} x(n) &= Acos(2\pi F_0 n/F_s + \theta) \\ &= Acos(2\pi(T/T_p)n + \theta) \end{aligned}$$

But $T/T_p = f \Rightarrow x(n)$ is periodic if f is rational.

(b) If x(n) is periodic, then f=k/N where N is the period. Then,

$$T_d = (\frac{k}{f}T) = k(\frac{T_p}{T})T = kT_p.$$

Thus, it takes k periods (kT_p) of the analog signal to make 1 period (T_d) of the discrete signal.

(c) $T_d = kT_p \Rightarrow NT = kT_p \Rightarrow f = k/N = T/T_p \Rightarrow$ f is rational $\Rightarrow$ x(n) is periodic.

Question: An analog signal $x_a(t) = sin(480\pi t) + 3\,sin(720\,\pi t)$ is sampled 600 times per second. (a) Determine the Nyquist sampling rate for $x_a(t)$. (b) Determine the folding frequency. (c) What are the frequencies, in radians, in the resulting discrete time signal $x(n)$? (d) If $x(n)$ is passed through an ideal D/A converter, what is the reconstructed signal $y_a(t)$? CSE-2011

Solution:

(a) $F_{max} = 360Hz$, $F_N = 2F_{max} = 720Hz$.
(b) $F_{fold} = \frac{F_s}{2} = 300Hz$.
(c)

$$\begin{aligned} x(n) &= x_a(nT) \\ &= x_a(n/F_s) \\ &= sin(480\pi n/600) + 3sin(720\pi n/600) \\ x(n) &= sin(4\pi n/5) - 3sin(4\pi n/5) \\ &= -2sin(4\pi n/5). \end{aligned}$$

Therefore, $w = 4\pi/5$.
(d) $y_a(t) = x(F_s t) = -2sin(480\pi t)$.

Question: A digital communication link carries binary-coded words representing samples of an input signal $x_a(t) = 3\cos 600\pi t + 2\cos 1800\pi t$. The link is operated at 10,000 bits/s and each input sample is quantized into 1024 different voltage levels. (a) What are the sampling frequency and the folding frequency? (b) What is the Nyquist rate for the signal $x_a(t)$? (c) What are the frequencies in the resulting discrete-time signal $x(n)$? (d) What is the resolution Δ ?

Solution:
(a)

$$\begin{aligned} \text{Number of bits/sample} &= log_2 1024 = 10. \\ F_s &= \frac{[10,000 \text{ bits/sec}]}{[10 \text{ bits/sample}]} \\ &= 1000 \text{ samples/sec}. \\ F_{fold} &= 500Hz. \end{aligned}$$

(b)

$$\begin{aligned} F_{max} &= \frac{1800\pi}{2\pi} \\ &= 900Hz; \\ F_N &= 2F_{max} = 1800Hz. \end{aligned}$$

(c)

$$\begin{aligned} f_1 &= \frac{600\pi}{2\pi}\left(\frac{1}{F_s}\right) \\ &= 0.3; \\ f_2 &= \frac{1800\pi}{2\pi}\left(\frac{1}{F_s}\right) \\ &= 0.9; \\ \text{But } f_2 &= 0.9 > 0.5 \Rightarrow f_2 = 0.1. \\ \text{Hence, } x(n) &= 3cos[(2\pi)(0.3)n] + 2cos[(2\pi)(0.1)n] \end{aligned}$$

(d) $\Delta = \frac{x_{max} - x_{min}}{m-1} = \frac{5-(-5)}{1023} = \frac{10}{1023}$.

Question: 1(b) – 2017

Consider the signal $x_a(t) = 3\,cos\,2000\pi t + 5sin600\,\pi t + 10cos\,12000\pi t$

(a) **Nyquest rate:**

$$we\ know\ cos(2 * \pi * 6000 * t)$$
$$F_{max} == 6000\ hz$$
$$Fs = 2 * F_{max}$$
$$Fs = 2 * 6000 = 12000\ Hz$$

(b) **If Sampling rate 5000 s/s what is the discrete time signal obtained after sampling.**

Given $F_s = 5000hz$

So discrete signal obtained: $f = \frac{F}{F_s}$

$$x(n) = 3\,cos\,2000\pi t\,/5000 + 5sin600\,\pi t/5000 + 10cos\,12000\pi t\,/5000$$

(c) **If $x(n)$ is passed through an ideal D/A converter, what is the reconstructed signal $y_a(t)$ or what is the analog signal $y_a(t)$ that we reconstruct from the samples if we use ideal interpolation**

$$y_a(t) = x(Fst) = 3\,cos\,2000\pi t + 5sin600\,\pi t$$

Question: 2(b) – 2015

Consider the signal $x_a(t) = 10sin400\,\pi t + 25cos\ 550\pi t - 15cos\ 450\pi t$

Determine the sampling rate to avoid aliasing and maximum magnitude of the signal.

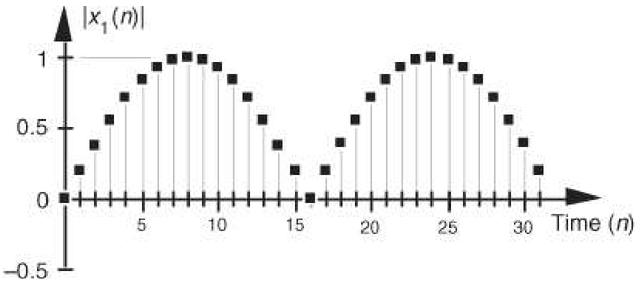
Fs=550

Maximum magnitude of signal:

The amplitude of a variable is the measure of how far, and in what direction, that variable differs from zero. Thus, signal amplitudes can be either positive or negative. The time-domain sequences in Figure 1–3 presented the sample value amplitudes of three different waveforms. Notice how some of the individual discrete amplitude values were positive and others were negative.

The magnitude of a variable, on the other hand, is the measure of how far, regardless of direction, its quantity differs from zero.

So magnitudes are always positive values. Figure 1–4 illustrates how the magnitude of the $x_1(n)$ time sequence in Figure 1–3(a) is equal to the amplitude, but with the sign always being positive for the magnitude. We use the modulus symbol (||) to represent the magnitude of $x_1(n)$. Occasionally, in the literature of digital signal processing, we'll find the term *magnitude* referred to as the *absolute value*.



Maximum magnitude of this signal 25

Question: 4(c) – 2015

Consider the signal $x_a(t) = 15sin250\,\pi t + 10cos\,300\pi t$

a) **Nyquist Rate: Fn=6000 Hz**
b) **Sampling 6000**

```
given sampling rate = nyquist frequency * 2
nyquist rate = given max frequency * 2
```

Question: 2(c) – 2014

Consider the signal $x_a(t) = 15sin250\,\pi t + 10cos\,300\pi t$

Sampling rate =300hz

Question: 3(a) – 2014

Consider the signal $x_a(t) = 3cos\,100\pi t$

a) **rate =100hz**
b) **Given Fs=200 Hz**

We know : $f = \frac{F}{F_s}$

Discrete signal from 200 hz sampling rate: $x_a(n) = 3cos\frac{100}{200}\pi t$

$$x_a(n) = 3cos\frac{1}{2}\pi t$$

Question: 4(c)-2015 – What is aliasing effect discuss it with an example? Why do you need antialiasing filter in signal processing.

Answer:

Question: 2(c) – 2015 if the number of quantization levels L=128, then how many bits per sample will be required.

L is also obtained from 2^{b+1} here the quantization has b bits of accuracy.

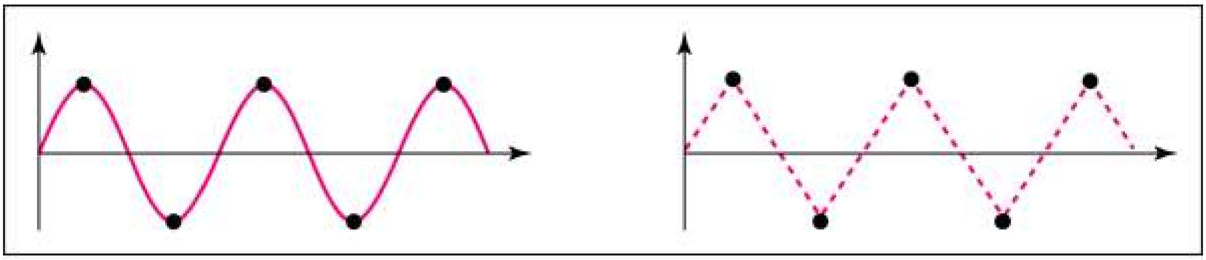
Here b=6 bits per sample will required.

Question: 2(a)-2016 – What is aliasing effect discuss it with an example? Why do you need antialiasing filter in signal processing.

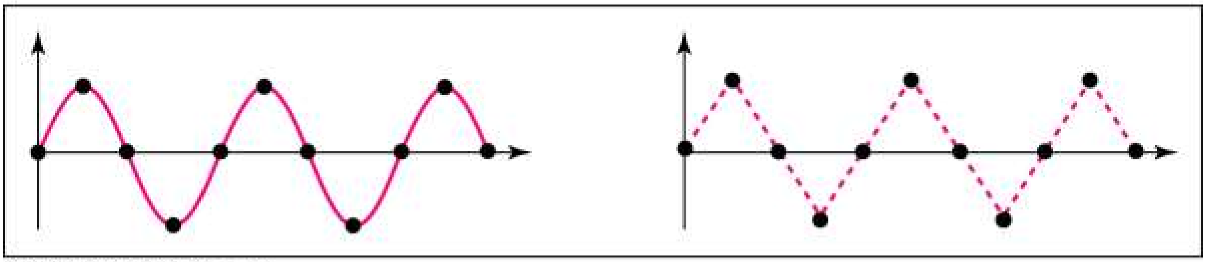
Answer:

- Aliasing refers to an effect that causes different signals to become indistinguishable (or *aliases* of one another) when sampled.
- Aliasing occurs when a system is measured at an insufficient sampling rate.

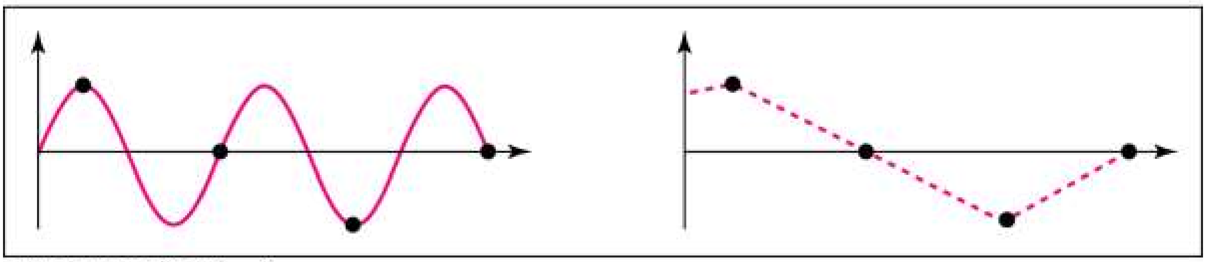
Example:



a. Nyquist rate sampling: $f_s = 2f$



b. Oversampling: $f_s = 4f$



c. Undersampling: $f_s = f$

Why do you need antialiasing filter in signal processing:

Recovery of a sampled sine wave for different sampling rates:

An anti-aliasing filter (AAF) is a filter used before a signal sampler to restrict the bandwidth of a signal to approximately or completely satisfy the Nyquist–Shannon sampling theorem over the band of interest.

Since the theorem states that unambiguous reconstruction of the signal from its samples is possible when the power of frequencies above the Nyquist frequency is zero, a real anti-aliasing filter trades off between bandwidth and aliasing.

A realizable anti-aliasing filter will typically either permit some aliasing to occur or else attenuate some in-band frequencies close to the Nyquist limit. For this reason, many practical systems sample higher than would be theoretically required by a perfect AAF in order to ensure that all frequencies of interest can be reconstructed, a practice called oversampling.

Question: 2(b)-2016 – Define up-sampling and down sampling with example? Why do you need to change sampling rage of signal.

Up Sampling:

Upsampling increases resolution, improves anti-aliasing filter performance and reduces noise. Some image or sound processing operations need high-resolution data to reduce errors.

For example, some guitar effects upsample the original data, apply the “effects” (actually filters) then downsample back the data. Picture zoom-in uses interpolation to increase the image size using the same data from the original image.

DJ tables change music tempo and pitch by upsampling tracks and playing back at different rates.

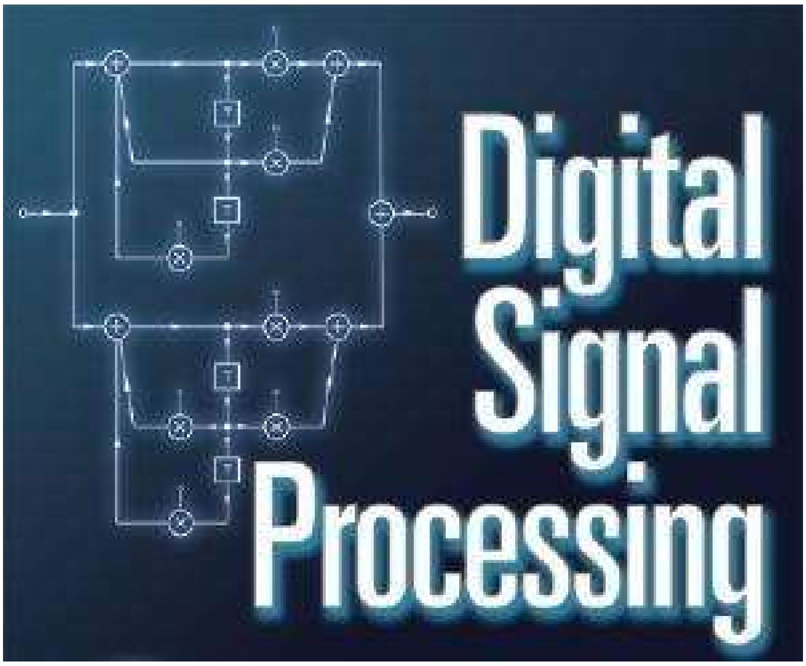
Down Sampling:

Downsampling helps reducing data size, compression or image reduction. For example, thumbnail extraction involves a decimation of the original picture.

Downsampling may violate the Nyquist rule, as the new sample rate may be less than twice the signal’s bandwidth, producing aliasing, so the data should go through a low-pass filter first.

Why need to change sampling range of signal:

Lots of reasons. Sometimes to match incoming and outgoing sample rates. Sometimes to save mips by always using the minimal rate. Sometimes changing sampling rate up or down sampling is used to get a nicer plot or better time resolution on zero crossings.



Important Question

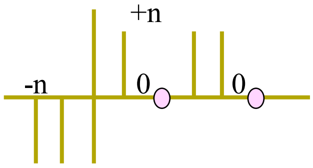
1. State and explain sampling theorem? What is Nyquest rate and aliasing ?
2. Consider the analog signal $x_a(t) = 3cos2000\pi t + 5sign\ 600\pi t + 10cos\ 1200\pi t$ now what is the nyquest rate for the signal? also if the sample this signal using a sampling rate Fs=5KHz , what is discrete-time signal obtained after sampling?
3. Define the following terms linearity, time-invariance , causality , stability?
4. An analog signal . $x_a(t) = sin480\pi t + cos720\pi t$ is sampled 600times per second what is the nyquest rate for the signal and folding frequency?
5. Define unit sample sequence and unite step sequence?
6. Explain general classification of discrete signals?

7. **What are the different types of operations performed on discrete time signals?**
8. **Let x1(n)= {2,2,0,0,5,-3,2} and x2(n)={5,5,5,5,5} write the . mathematical expression for x1(n) and x2(n) using unit sample sequence and unit step sequence?**

Discuss four representation types of discrete time signal

Functional representation :

$$X(n) = \begin{cases} 1 & n > 0 \\ 0 & n = 0 \\ -1 & n < 0 \end{cases}$$



Tabular representation :

n	……….-2 -1 0 1 2 3 4 5 ……
$x(n)$	………-1 -1 0 1 1 1 1 1……

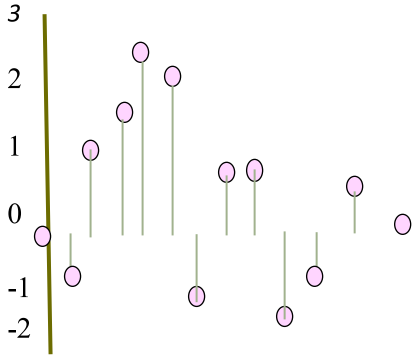
Sequence representation :

An infinite duration signal with a time origin (n=0) indicated by the symbol ↑ is represented as

$$x(n) = \{\ldots\ldots\ldots..0,0,1,4,1,1,1,4\ldots\ldots\}$$

A sequence x (n) which is 0 for n<0 can be represented as $(n) = \{0,1,4,1,1,1,4 \ldots.. \}$

Graphical representation :

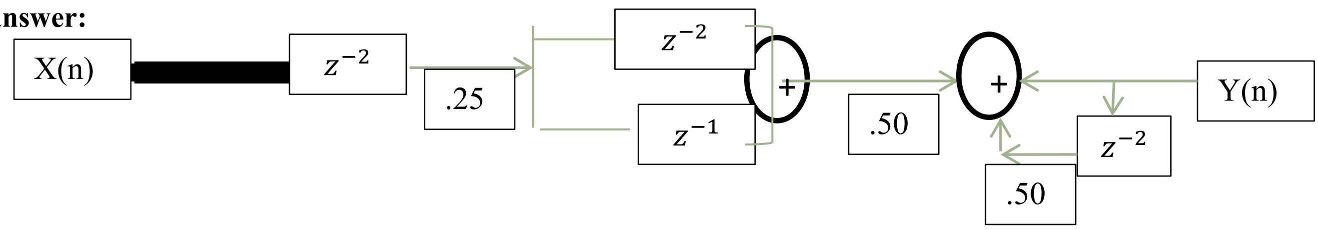


The time origin for the sequence x (n) is understood to be the left most point in the sequence where x(n) is 0 for n<0.

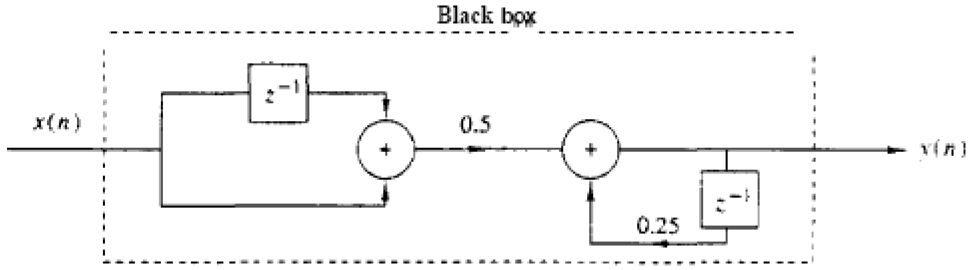
For finite representation $(n) = \{0,1,4,1,1,1,4\ \}$ which is a 7-point sequence.

Question:1(c) 2015 draw the block diagram of the following discrete time system
y[n]=0.5x[n-3]+ .25x[n-2]+ .5 x[n-4]+0.5y[n-2]

answer:



Question:4(c) 2014 sketch the block diagram of the following discrete time system
y[n]=1/4*y(n-1)+1/2* x(n)+ ½*x(n-1)



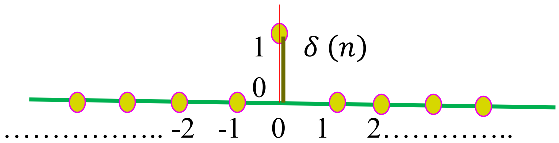
Question: 1(b) 2015: Discuss unit step and exponential sequence with figure

Unite sample sequence?

The unite sample sequence is denoted by $\delta\ (n)$ and is defined as

$$\delta\ (n) \equiv \begin{cases} 1\ , & for\ n = 0 \\ 0\ , & for\ n \neq 0 \end{cases}$$

The unit sample sequence is a signal that is zero every where except at n=0, where its value is unity (1). This is sometimes called a **unit impulse.**

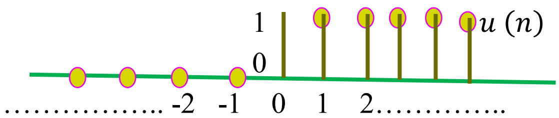


Unit step signal?

The unit step signal is denoted by $u(n)$ and is defined as,

$$u(n) \equiv \begin{cases} 1\ , & for\ n \geq 0 \\ 0\ , & for\ n < 0 \end{cases}$$

The signal is zero for each n< 0 and elsewhere it has a unity value.

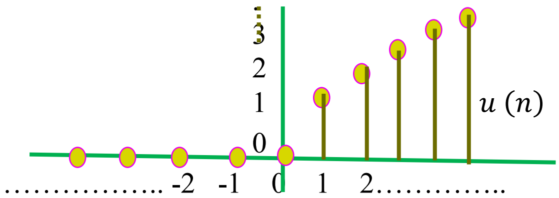


Unit ramp signal?

The unit ramp signal is denoted by $u_r(n)$ and is defined as,

$$u(n) \equiv \begin{cases} n\ , & for\ n \geq 0 \\ 0\ , & for\ n < 0 \end{cases}$$

The signal is zero for each n< 0 and elsewhere it has value same as n.



Exponential signals ?

The exponential signal is the sequence of the form

X(n)=a^n $\quad for\ all\ n$

If the parameter $\boldsymbol{a}$ is a real,then x(n) is a real signal.when the If the parameter $\boldsymbol{a}$ is a complex , it can be expressed as $a \equiv re^{j\theta}$ then we can express x(n) as, x(n)=$r^n\ e^{j\theta}\ = r^n(cos\theta n + j\ sin\theta\ n)$

For the graphical representation of complex signal real and imaginary parts are separately plotted.

Question: 3(b) 2014 The following two finite duration sequence represent two discrete signal

$\boldsymbol{x_1(n) = \{1, 5, -2, 3, -4, 6\}}$

$\boldsymbol{x_2(n) = \{1, 5, -2, 3, -4, 6\}}$

These two signal are not same. Index of first one is 0:5 and index of second one is -4:1

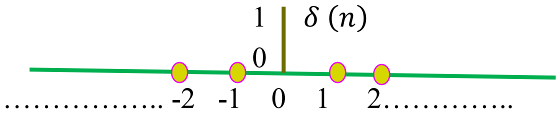
Question: 3(b) 2014 sketch $\boldsymbol{\delta(n), u(n), u_r(n) for -2 \leq n \leq 2 \; where \; \delta(n), u(n), u_r(n)}$ denote their usual meaning.

Here n=-2,-1, 0, 1, 2

$\boldsymbol{\delta(n)}$ is unit impulse:

$$\delta\,(n) \equiv \begin{cases} 1\;, & for\; n = 0 \\ 0\;, & for\; n = -2, -1, 1, 2 \end{cases}$$

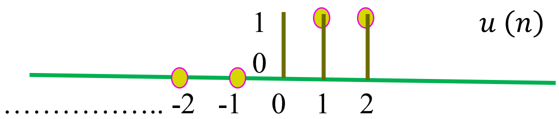
$\delta\,(n) = [0, 0, 1, 0, 0]$



$\boldsymbol{u(n)}$ is unite step:

$$u(n) \equiv \begin{cases} 1\;, & for\; n \geq 0 (0,1,2) \\ 0\;, & for\; n < 0\; (-2, -1) \end{cases}$$

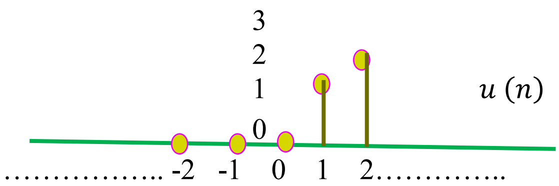
$u(n) = [0,0,1,1,1]$



$\boldsymbol{u_r(n)}$ is unit ramp:

$u_r(n) = [0,0,0,1,2]$

.



Questio: 3(d) determine the response of the following systemto the input signal

$$x(n) = \begin{cases} |n| & -3 \leq n \leq 3 \\ 0 & Otherwise \end{cases}$$

x(n)=[….0,0,3,2,1,0,1,2,3,0,0……]

(i) y(n)= x(n)=[….0,0,3,2,1,0,1,2,3,0,0……]

(ii) Y(n)=x(n-1)= [….0,0,3,2,1,0,1,2,3,0,0……]

(iii)

(iii) y(n)=x(n+1)= [….0,0,3,2,1,0,1,2,3,0,0……]

(iv) Y(n)=1/3[x(n)+x(n-1)+x(n+1)]

1/3*[0,3,5,3,4,3,4,5,1,2,3]

Question: 4(a) 2014 define energy and power of a signal

Discuss energy signal and power signal?

ENERGY SIGNAL :

The energy of a signal x(n) is denoted as E or E_x *and is* defined as ,

$$E \equiv \sum_{n=-\infty}^{\infty} |x(n)|^2$$

The energy of a signal can be finite or infinite.the signal x(n) is called energy signal if its energy is finite.

Many signals having infinite energy have a finite average power,the average power of a discrete signal is determined by.

$$P \equiv \lim_{n\to\infty} \frac{1}{(2N+1)} \sum_{n=-N}^{N} |x(n)|^2$$

POWER SIGNAL :

the average power of a discrete signal is determined by.

$$P \equiv \lim_{n\to\infty} \frac{1}{(2N+1)} \sum_{n=-N}^{N} |x(n)|^2$$

We define the energy of a signal in the finite interval

$$N \le n \le N \text{ as } E_N \equiv \sum_{n=-N}^{N} |x(n)|^2$$

Then the signal energy comes ,

$$E \equiv \lim_{N\to\infty} E_N$$

The average power of signal x(n) is

$$P \equiv \lim_{n\to\infty} \frac{1}{(2N+1)} E_N$$

If E is finite ,P = 0and if E is infinite the average power of the signal can be finite or infinite.if the signal average power P is finite and nonzero then the signal is called a power signal.

Determine the power and energy of unit step sequence

The average power of unit step sequence is

$$P \equiv \lim_{N\to\infty} \frac{1}{(2N+1)} \sum_{n=0}^{N} u^2(n)$$

$$= \lim_{N\to\infty} \frac{N+1}{(2N+1)}$$

$$= \lim_{N\to\infty} \frac{1+{}^{1}/_{N}}{2+{}^{1}/_{N}}$$

$$= {}^{1}/_{2}$$

The unit step sequence is a power signal having an infinite energy.

Discuss periodic signal and aperiodic signal?

Periodic signal :-

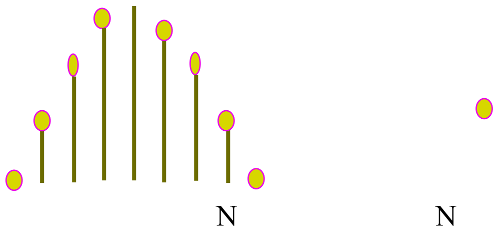
A signal x(n) is called periodic if it repeats it state after a certain time period N (N>0), means if it satisfies ,

X (n + N) = x(n) ……..(i) for all n

The smallest value of N for which (i) is true fundamental period.

Show that periodic signals are power signals

The energy of a periodic signal over a signal period, (over the interval $0 \le n \le N-1$) on finite values over is finite is x(n) takes finite values over the period.however energy of the period $-\infty \le n \le \infty$ is infinite.the average power of the periodic signal is finite and is equal to the average power over a signal period.



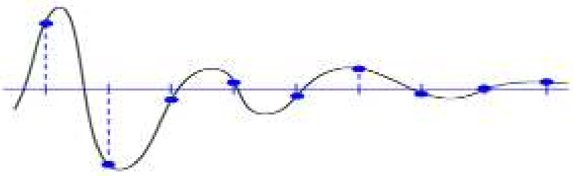
thus x(n) a periodic signal with N fundamental period have the average power P,

$$P = \lim_{N\to\infty} \frac{1}{\boldsymbol{N}} \sum_{n=0}^{N-1} |x(n)|^2$$

As periodic signal always have a finite power so it is a power signal. On the other hand energy of the period $-\infty \leq n \leq \infty$ is infinite.

aPeriodic or non-periodic signal :-

A signal x(n) is called aperiodic or non-periodic if there is no time span N , for which X (n + N) = x(n) is true for all n. or simply the signal do no repeats its state after any N seconds.



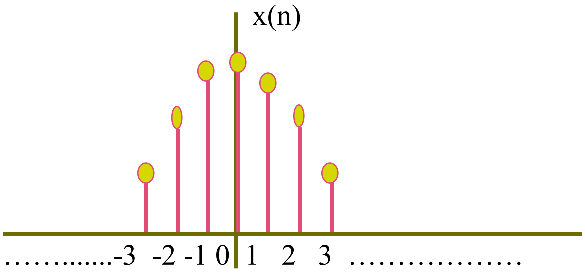
Discuss symmetric(even) signal and asymmetric(odd) signal?

symmetric(even) signal :-

a real valued signal is said to be symmetric(even) signal if x(-n)= x(n) ……(i) for all n

the even component of a signal is given by ,

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$



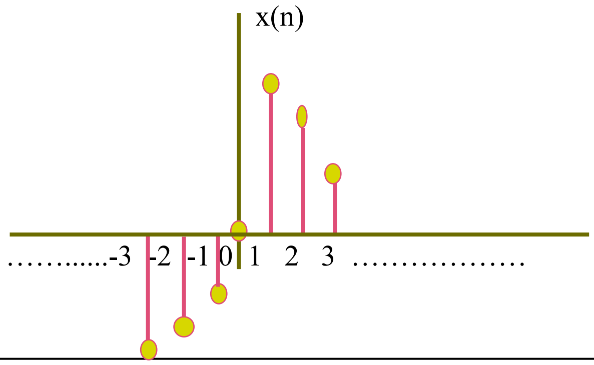
asymmetric(0dd) signal :-

a real valued signal is said to be asymmetric (odd) signal if x(-n)= -x(n) ……(i) for all n

the even component of a signal is given by ,

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

For odd signal x(0) = 0.



Any arbitrary signal can be represented as the sum of two signal component ,one of which is even and other odd.

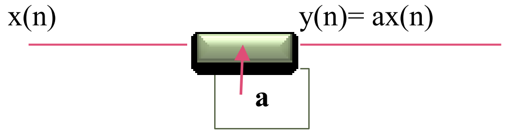
x(n) = $x_e(n) + x_o(n)$.

Discuss basic operation on discrete time signal?

The following operations can be carried out on an input signal to obtain the desired output signal.

Multiplication by a constant

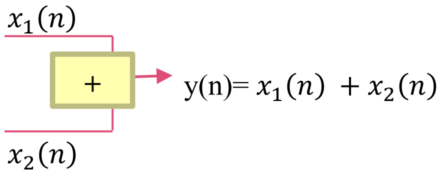
We can multiply a given sequence x(n) by a constant **a** ,resulting in a new sequence y(n) and **a** may ne positive or negative constant.

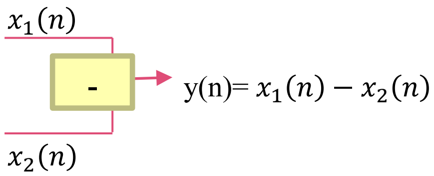


if x(n) = {3,2,4,5} and a=2 then y(n) ={6,4,8,10}

Addition and subtraction

Two sequence $x_1(n)$ *and* $x_2(n)$ can be added or subtracted producing a new sequence y(n).



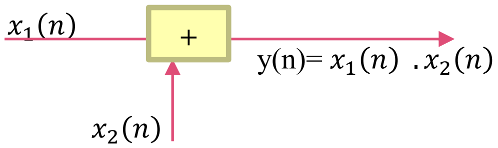


Example if $x_1(n)$ ={6',4,8,10} and $x_2(n)$ ={3',2,4,5}

Then y(n) = = $x_1(n) + x_2(n)$ = {9(6+3),6,12,15}

Multiplication operation ?

Multiplication operation of two or more input sequences are done by ,multiplying the corresponding n^{th} samples to form the n^{th} sample in the output sequence. The output sequence y(n)= $x_1(n)\ x_2(n)$….



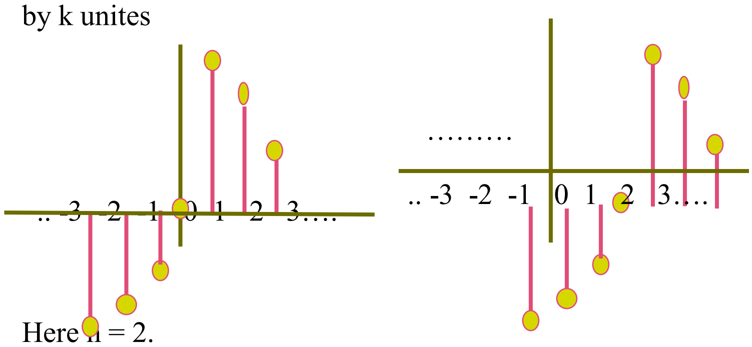
Example if $x_1(n)$ ={6',4,8,10} and $x_2(n)$ ={3',2,4,5}

Then y(n)= $x_1(n) \cdot x_2(n)$= {18(6 * 3),8,32,50}

Division can be done in the same way.

Shifting operation ?

A signal x(n) may be shifted in time by replacing the independent variable **n by (n-k)** ,where k is an integer.
If K is a positive number , the time shift results in a delay of the signal by k units of time, or shifted right by k unites



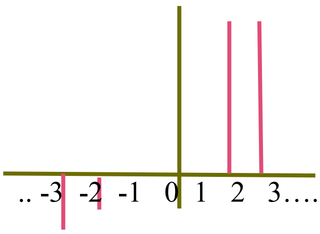
Here n = 2.

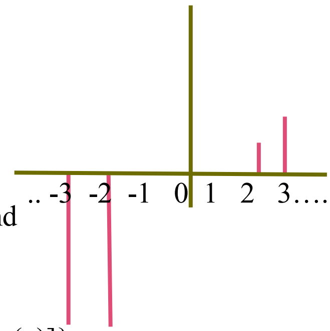
If K is a negative integer , the time shift results in an advance of the signal by | k | units of time. or the signal is shifted left for k units.

Time reversal or folding operation ?

The operation time reversal or folding of the signal with respect the time origin n=0 is to replace the independent variable n by –n.

Folding is defined as y(n)= x(-n) | y(-1)=x(1)
|y(1)=x(-1)
|y(0)=x(0)





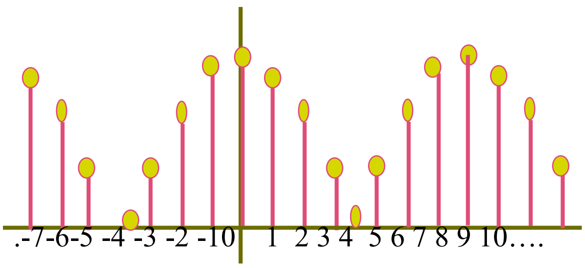
The operation of folding and time delaying are not commutative means
TD{FD[x(n)]} $\neq$ FD{TD[x(n)]}

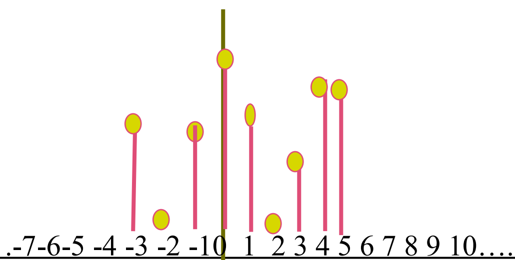
Time scaling or down sampling operation ?

It is done by replacing the independent variable n by μn where μ is an integer.

The out put y(n)= x(μ)

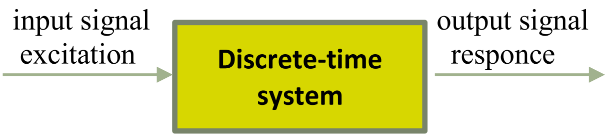
Represent y(n)=x(2n) ; the out put will have value like y(0)=x(0) , y(1)=x(2) , y(2)=x(4),…….and y(-2)=x(-4)……….. y(n) will hold values for only those n for which x(n) signal has a value at 2n .





Discuss discrete time system?

A discrete time system is a device or algorithm that operates on a discrete-time signal,called the input or excitation,according to some well-defined rule and prescribed procedure ,to produce another discrete-time signal called output or response.



the system transforms x(n) into y(n),

$$y(n) \equiv ד x(n) where\ ד\ is\ the\ transformation\ or$$

$processing\ done\ on\ x(b)$.

Discuss the input output discretion of DTS?

The input=output discretion of DTS consists of mathematical expression or a rule , which explicitly defines the relations between the input and output signal ,the internal structure has no deal with this.

It is like ד

$$x(n) \rightarrow y(n \rightarrow)$$

Examples,

1. $y(n) = x(n) \ldots \ldots identity\ system$
2. $y(n) = x(n-1) \ldots \ldots unit\ delay\ system$
3. $y(n) = x(n+1) \ldots \ldots init\ advance\ system$
4. $y(n) = \frac{1}{3}[x(n+1) + x(n) + x(n-1) \ldots \ldots$

   $average\ filter\ system$
5. $y(n) = midean\ [x(n+1), x(n), x(n-1) \ldots$

   $midean\ system$
6. $y(n) = \sum_{k=-\infty}^{n} x(k) = x(n) + x(n-1) + \qquad x(n-2) \ldots$

Find the response of the following system to the input,

$$x = \begin{cases} |n|, & -3 \le n \le 3 \\ 0\ , & otherwise \end{cases}$$

The input sequence is

x(n) = {….0,3,2,1,0,1,2,3,0….}

1. y(n) = {….0,3,2,1,0,1,2,3,0….}
2. y(n) = {….0,3,2,1,0,1,2,3,0….}
3. y(n) = {….0,3,2,1,0,1,2,3,0….}
4. Y(0)= $1 + 0 + {}^{1}/_{3} = {}^{2}/_{3}$

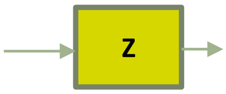
   Y(1)= $0 + 1 + {}^{2}/_{3} = {}^{3}/_{3} = 1$
5. y(n) = {….0,2,2,1,1,1,2,2,0,0….} , each

sample is median of three inputs n-1,n,n+2, for the first one it is(0+3+2)/3 = 2(grounded).

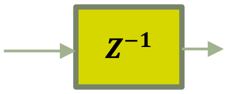
6. y(n) = {….0,3,5,6,6,7,9,12,0….}
$\uparrow$

the accumulator computes the current sum for all the past samples.

Block diagram for unit advance ,



Block diagram for unit delay



Page 59 example 2.2.3 prokish

Discuss the classification od DTS?

Static - dynamic system

Static system:-
A discrete time system is called static or memoryless if its output at any instance n depends at most on the input samples at the same time, but not on past or future samples or the input.

Y(n)= ax(n) or y(n)= nx(n) +bx^3 n are examples of static systems. There is no need to store past inputs or outputs inorder to compute the current output.

Dynamic system:-
The system is said to be dynamic , if the output of a system at time n is completely determined by the input samples in the interval n-N to n (N>=0) , the system is said to have memory of duration N. if N is 0 then the system is static .

If 0<N<∞ the system has finite memory. Or if N = ∞ the system is said to be infinite .

Y(n)=x(n)+3x(n-1)

$y(n) = \sum_{k=-\infty}^{n} x(k) = x(n) + x(n-1) +$ $x(n-2)$ ... Accumulator
The above systems are dynamic system 1 is of finite and 2 is of infinite memory.

Time-invariant and time –variant

Time-invariant :-
A system is called time-invariant if its input-output characteristics do not changes with time. for example there is a system τ excited by x(n) produces response y(n),now if we made a k unit delay in input then the system will generate y(n-k) for x(n-k) . means we will get the same result as response x(n) only delayed by the same k units in times that the in put was delayed.

The time-invariant or shift invariant system is as follows , τ

x(n) $\longrightarrow$ **y(n)**

implies τ

x(n-k) $\longrightarrow$ **y(n-k)**

so if for every input signal x(n) and every delay k
y(n,k) = $\tau[x(n-k)]$, and **y(n,k) = y(n-k)** is true then the system is time invariant or if
y(n,k) $\neq y(n-k)$ then the system is called

Time-variant .
Like ,
y(n)= nx**(n)…………….(i)**
now k unit delay in input
y(n,k)=nx(n-k) …………(iii)
delaying k unit in output i

y(n-k)=(n-k) x(n-k)……..(iii)

as the RHS of ii and iii are different so the system is time variant .
page 62 prokish

Question:6(b) -2013 Determine whether the following systems are time invariant or time variant

a. $\boldsymbol{y(n) = nx(n)}$
b. $\boldsymbol{y(n) = x(n) + x(n+1)}$

Answer: a. Time Invariant

Answer:b. Time Variant

Linear non linear system

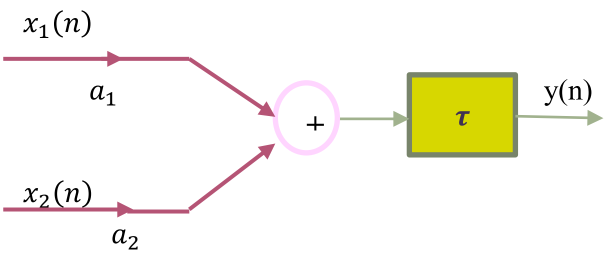
A system is said to be linear if it satisfies the of superposition principals .the principal requires that the response of a system to a weighted sum of signals be equal to the corresponding weighted sum of responses of the system to each of the individual input signals.

A system is linear if and only if,

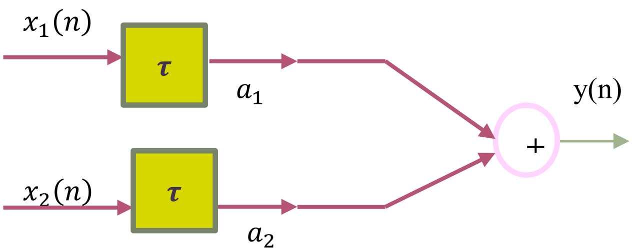
$$\tau\,[a_1x_1(n) + a_2x_2(n)\,] = a_1\tau[\,x_1(n)] + a_2\tau[\,x_2(n)]$$

For any arbitrary input sequence $x_1(n)$ and $x_2(n)$,also arbitrary constant a_1 *and* a_2.

The relation demonstrates the additive and multiplicative properties of a linear system.



The system operates on the sum of signals



The system operates on individual signal and the adds the weighted responses.

If both the results are same the system is called linear otherwise the system in non linear.

Question:2(a) -2017 consider a continuous time system which has input of signal x(t) and output of y(t)=x(t)u(t)

a. **Is this system time invariant justify your answer**

This is time invariant, because we know A system is said to be Time Invariant if its input output characteristics do not change with time, and here u(t) is a unit impulse, multiply with 1 is not change.

b. **Is this system Linear justify your answer**

This system is linear because we know that A linear system follows the laws of superposition. This law is necessary and suffic

- Law of additivity
- Law of homogeneity

*This system is a linear system because it maintained second condition.*ient condition to prove the linearity of the system. Apart from this, the system is a combination of two types of laws –

Question:2(b) -2017 Determine if the system describe by the following input-output eqution are linear or non-linear

a. $\boldsymbol{y(n) = Ax(n) + B}$ **Linear**

b. $\boldsymbol{y(n) = e^{x(n)}}$ **non-linear**

Answer: $\boldsymbol{y(n) = Ax(n) + B}$

(d) Assuming that the system is excited by $x_1(n)$ and $x_2(n)$ separately, we obtain the corresponding outputs

$$\begin{aligned} y_1(n) &= Ax_1(n) + B \\ y_2(n) &= Ax_2(n) + B \end{aligned} \tag{2.2.40}$$

A linear combination of $x_1(n)$ and $x_2(n)$ produces the output

$$\begin{aligned} y_3(n) &= T[a_1x_1(n) + a_2x_2(n)] \\ &= A[a_1x_1(n) + a_2x_2(n)] + B \\ &= Aa_1x_1(n) + a_2Ax_2(n) + B \end{aligned} \tag{2.2.41}$$

On the other hand, if the system were linear, its output to the linear combination of $x_1(n)$ and $x_2(n)$ would he a linear combination of $y_1(n)$ and $y_2(n)$, that is,

$$a_1y_1(n) + a_2y_2(n) = a_1Ax_1(n) + a_1B + a_2Ax_2(n) + a_2B \tag{2.2.42}$$

Clearly, (2.2.41) and (2.2.42) are different and hence the system fails to satisfy the linearity test.

The reason that this system fails to satisfy the linearity test is not that the system is nonlinear (in fact, the system is described by a linear equation) hut the presence of the constant B. Consequently, the output depends on both the input excitation and on the parameter $B \neq 0$. Hence, for $B \neq 0$, the system is not relaxed. If we set $B = 0$, the system is now relaxed and the linearity test is satisfied.

Answer: $(n) = e^{x(n)}$ **:**

Note that the system described by the input–output equation

$$y(n) = e^{x(n)} \tag{2.2.43}$$

is relaxed. If $x(n) = 0$, we find that $y(n) = 1$. This is an indication that the system is nonlinear. This, in fact, is the conclusion reached when the linearity test. is applied.

Question:1(c) -2016 Determine if the system describe by the following input-output eqution are linear or non-linear

a. $y(n) = x(n) + \frac{1}{x(n-1)}$

b. $y(n) = nx(n)$

Answer: $y(n) = x(n) + \frac{1}{x(n-1)}$

This is linear system, because these identical

Answer: $y(n) = nx(n)$

(a) For two input sequences $x_1(n)$ and $x_2(n)$, the corresponding outputs are

$$\begin{aligned} y_1(n) &= nx_1(n) \\ y_2(n) &= nx_2(n) \end{aligned} \tag{2.2.31}$$

A linear combination of the two input sequences results in the output

$$\begin{aligned} y_3(n) &= T[a_1x_1(n) + a_2x_2(n)] = n[a_1x_1(n) + a_2x_2(n)] \\ &= a_1nx_1(n) + a_2nx_2(n) \end{aligned} \tag{2.2.32}$$

On the other hand, a linear combination of the two outputs in (2.2.31) results in the output

$$a_1y_1(n) + a_2y_2(n) = a_1nx_1(n) + a_2nx_2(n) \tag{2.2.33}$$

Since the right-hand sides of (2.2.32) and (2.2.33) are identical, the system is linear.

Question:4(d) -2014 Determine if the system describe by the following input-output eqution are linear or non-linear

a. $y(n) = nx(n)$ **Linear**
b. $y(n) = x(n^2)$
c. $y(n) = x^2(n)$

$Answer: y(n) = x(n^2)$

The responses of the system to two separate input signals are

$$y_1(n) = x_1^2(n)$$
$$y_2(n) = x_2^2(n) \qquad (2.2.37)$$

The response of the system to a linear combination of these two input signals is

$$\begin{aligned} y_3(n) &= T[a_1x_1(n) + a_2x_2(n)] \\ &= [a_1x_1(n) + a_2x_2(n)]^2 \qquad (2.2.38) \\ &= a_1^2x_1^2(n) + 2a_1a_2x_1(n)x_2(n) + a_2^2x_2^2(n) \end{aligned}$$

On the other hand, if the system is linear, it would produce a linear combination of the two outputs in (2.2.37), namely,

$$a_1y_1(n) + a_2y_2(n) = a_1x_1^2(n) + a_2x_2^2(n) \qquad (2.2.39)$$

Since the actual output of the system, as given by (2.2.38), is not equal to (2.2.39), the system is nonlinear.

$Answer: y(n) = x^2(n)$

The output of the system is the square of the input. (Electronic devices thathave such an input-output characreristic and are called square-law dev~ces.)
From our previous discussion it is clear that such a system is memoryless. We now illustrate that this system is nonlinear.

As in part (a), we find the response of the system to two separate input signals $x_1(n)$ and $x_2(n)$. The result is

$$y_1(n) = x_1(n^2)$$
$$y_2(n) = x_2(n^2) \qquad (2.2.34)$$

The output of the system to a linear combination of $x_1(n)$ and $x_2(n)$ is

$$y_3(n) = T[a_1x_1(n) + a_2x_2(n)] = a_1x_1(n^2) + a_2x_2(n^2) \qquad (2.2.35)$$

Finally, a linear combination of the two outputs in (2.2.36) yields

$$a_1y_1(n) + a_2y_2(n) = a_1x_1(n^2) + a_2x_2(n^2) \qquad (2.2.36)$$

By comparing (2.2.35) with (2.2.36), we conclude that the system is linear.

Causal non causal system

Causal system :-

A system is said to be causal if the output of the system at any time n depends only on past and present inputs like [x(n),x(n-1),x(n-2)……….] but not on the future inputs like [x(n+1),x(n+2)……….]
In mathematical term the output of causal system satisfies the following equation.

Y(n)=F[x(n),x(n-1) ,x(n+2)……….]
Where F[…] is an arbitrary function.
Example ,
1.y(n)=x(n)-x(n-1)

$$2.\ \ y(n) = \sum_{k=-\infty}^{n} x(k)$$

nonCausal system :-

A system is said to be noncausal if the output of the system at any time n depends not only on past and present inputs like [x(n),x(n-1),x(n-2)………] but also on the future inputs like [x(n+1),x(n+2)………] .

y(n)=x(2n)

y(n)=median(x(n-1),x(n),x(n+1))

the system y(n) = x(-n) is also a non-causal system ,as when n= -1 the output will depend on x(n).

Question: 3(a) -2016 Describe stable and unstable system with example.

Stable unstable

An arbitrary relaxed system is said to be stable(bounded input- bounded output BIBO) , if and only if every bounded input produces a bounded output.

Which means that there exists some finite numbers say M_x *and* M_y *such that* $|x(n)| \leq M_x \quad < \infty$

And $|y(n)| \leq M_y \quad < \infty$

And a system generating an unbounded(infinite) output for a bounded input is termed as unstable system.

Discuss the correlation of discrete time signals?

A mathematical operation that closely resembles convolution is correlation. two signal sequences are involved in correlation ,the operation is to measure the degree to which the two signals are similar .generally for object detection in space it is used a lot.

If x(n) is the transmitted signal then if the any object reflects the signal,the received signal y(n) consists of a delayed version of the transmitting signal and some additive noise,

Y(n) $=\alpha\, x(n-D) + \omega(n) \ldots\ldots\ldots..(i)$

Here α *is the attenuation factor representation the round trip signal loss,* D *is the round trip delay,* $\omega(n)$ *is the noise.*

Correlation provides us a means to extract required information,

Suppose that we have two signal sequences x(n) and y(n) .each of which have finite energy.the correlation of x(n) and y(n) is a sequence $r_{xy}(l)$, which is defined as,

$$r_{xy}(l) \;=\; \sum_{k=-\infty}^{\infty} x(n)y(n-l)\ ]$$

$$[l = \cdots - 2, -1,0,1,2 \ldots \ldots.]$$

Or equivalently ,

$$r_{xy}(l) \;=\; \sum_{k=-\infty}^{\infty} x(n+l)y(n)\ ]$$

Again ,

$$r_{yx}(l) \;=\; \sum_{k=-\infty}^{\infty} y(n)x(n-l)\ ]$$

Or equivalently ,

$$r_{yx}(l) \;=\; \sum_{k=-\infty}^{\infty} y(n+l)x(n)\ ]$$

Now ,

$$r_{yx}(-l) \;=\; \sum_{k=-\infty}^{\infty} y(n)x(n+l)$$

$$= \sum_{k=-\infty}^{\infty} x(n+l)y(n)$$

$= r_{xy}(l)$

$$\therefore\;\; r_{yx}(-l) = r_{xy}(l)$$

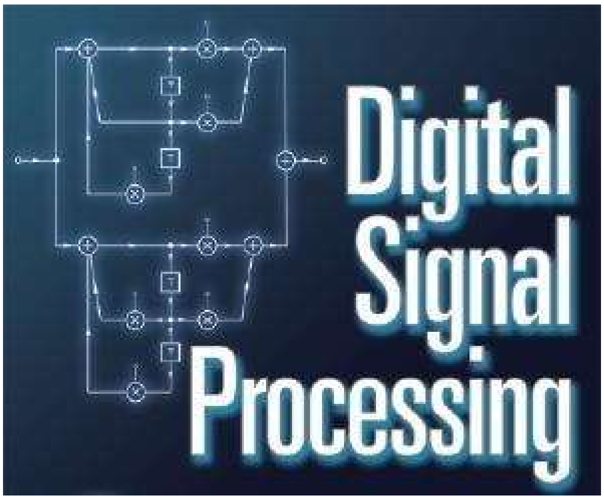
Again consider ,

$$x(l) * \; y(-l) \;=\; \sum_{k=-\infty}^{\infty} x(k)y(-[l-k])$$

$$= \sum_{k=-\infty}^{\infty} x(k)y(k-l)$$

$= r_{xy}(l)$

$\therefore\; \boldsymbol{r_{xy}(l) \;\; = x(l) * \; y(-l)}$



Important Question

1. Question:3(b) -2016 Define impulse response of system. How impulse are used to know the behavior any arbitory system. Discuss with suitable example 3
2. Question:3(b) -2016 Explain convolution sum of signal.-2.75
3. Question:5(a) 2014 discuss the process of computing the convolution between the input signal and impulse response of the system
4. Question: 5(b) 2014 what is the importance of convolution in signal processing
5. Question: 3(c)2016 Consider the input x[n]={1 0 2 3} h[n]={1 2 1 3} find out the convolution sum.

Define LTI system?

The digital signal processing is concerned exclusively with a particular class of system known as linear time invariant system. LTI systems are characterized in time domain simply by their response to a unit sample sequence . to set as input to any LTI system any arbitrary signal is need to be decomposed and represented as in weighted sum of unit sample sequences.

Question:3(b) -2016 Define impulse response of system. How impulse are used to know the behavior any arbitory system. Discuss with suitable example 3

Impulse response: Impulse response is the response of a system to a unit impulse at its input.

Any signal can be considered(approximated) as a stream of impulses (These impulses will be immeasurably close to each other in continuous signals, and will be 1 sample apart for discrete signals). These are not unit impulses. (they have magnitude not necessarily 1). So the response of the system to this impulse should be a scaled form of 'unit impulse response'.

So the output of the system for a certain signal can be considered as the superposition of impulse response due to all the impulses in the input signal. This process of scaling the unit impulse response according to the i/p signal values and adding them all is convolution.

How impulse are used to know the behavior any arbitory system.

Question:3(b) -2016 Explain convolution sum of signal.-2.75

Define convolution sum?

For a linear time invariant system ,input sequence is x(n) , impulse response is h(n) then the convolution sum is determined as

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

And is denoted as y(n)=x(n)*h(n). where * denotes the convolution operation.

Question: 5(b) 2014 what is the importance of convolution in signal processing

Importence of convolution :

Convolution is a mathematical way of combining two signals to form a third signal. It is the single most important technique in Digital Signal Processing. Using the strategy of impulse decomposition, systems are described by a signal called the impulse response.

Convolution is important because it relates the three signals of interest: the input signal, the output signal, and the impulse response. Convolution provides the mathematical framework for DSP;

Define properties of convolution sum?

The convolution operation has the following properties.

Commutative :-

The convolution operation is Commutative,

x(n)*h(n)= h(n)*x(n)

proof:

from the definition of convolution , .

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

let (n- k)=p , now as $k \to \infty$ $so\ p \ \to -\infty\ and\ as$ $k \to -\infty\ so\ p\ \to \infty\ and$.

$$x(n) * h(n) = \sum_{p=-\infty}^{\infty} x(n-p)\, h(p)$$

$$= \sum_{p=-\infty}^{\infty} h(p)\, x(n-p)$$

= h(n)*x(n)

Hence it is cumulative.

Distributive with respect to addition :-

The convolution operation is distributive with respect to addition ,
x(n)$[h_1(n) + h_2(n)] = x(n) * h_1(n) + x(n) * h_2(n)$

proof:
from the definition of convolution

x(n)$[h_1(n) + h_2(n)]$

$$= \sum_{k=-\infty}^{\infty} x(n)\,[h_1(n-k) + h_2(n-k)]$$

$$= \sum_{k=-\infty}^{\infty} x(n)\,[h_1(n-k)] + \sum_{k=-\infty}^{\infty} x(n)\,[h_2(n-k)]$$

$$= x(n) * h_1(n) + x(n) * h_2(n)$$

Thus , The convolution operation is distributive with respect to addition.

Associative :-

The convolution operation is associative,

$$x(n) * [y(n) * h(n)] = [x(n) * y(n)] * h(n)$$

Proof :

$$x(n) * [y(n) * h(n)] = \sum_{k=-\infty}^{\infty} \{x(k)[y(n-k)] * h(n-k)\}$$

Since convolution is commutative
y(n-k)*h(n-k)= h(n-k)*y(n-k)

hence ,

$$x(n) * [y(n) * h(n)] = \sum_{k=-\infty}^{\infty} \{x(k)[h(n-k)] * y(n-k)\}$$

$$= \sum_{k=-\infty}^{\infty} x(k) \sum_{p=-\infty}^{\infty} h(p)\,y[(n-k)-p]$$

$$= \sum_{p=-\infty}^{\infty} h(p) \sum_{k=-\infty}^{\infty} x(k)\,y[(n-k)-p]$$

$$= \sum_{k=-\infty}^{\infty} h(p)[x(n-p) * y(n-p)]$$

=h(n)*[x(n)*y(n)]
=[x(n)*y(n)]*h(n)
Thus convolution is associative.

Question:5(a) 2014 discuss the process of computing the convolution between the input signal and impulse response of the system

Define process of computing convolution sum?

The processes of computing convolution between x(k) and h(k) involves the following steps
1.folding:- fold h(k) about k=0 to get h(-k)
2.shifting :- shift h(-k) by n0 right if n0 is positive or left otherwise to obtain .h(n0-k).
Multiplication :-multiply x(k) by h(n0-k) to obtain the product sequence $u_{n_o}(k) \equiv x(k)h(n0-1)$
Summation :- sum all the values of product sequence $u_{n_o}(k)$ to obtain the value of the response at time n=n0.

Define three different process of computing convolution sum?

Mathematical approach

Let length of sequence x(n) is **N1**,lower limit of x(n) is **n1**,upper limit of x(n) is **n2**.
length of sequence h(n) is **N2**,lower limit of x(n) is **p1**, upper limit of x(n) is **p2.**

(a)Let length of sequence y(n) is :-
length of sequence x(n) is
\+ length of sequence h(n) is +1

n=N1+N2-2

(b) lower limit of y(n) is :-
lower limit of x(n) is + lower limit of h(n) is

y(n)=n1+p1

(c) upper limit of y(n) is :-
upper limit of x(n) is + upper limit of h(n) is

y(n)=n2+p2

now for the mathematical approach ,

$$y(n) = \sum_{k=n1 \text{ lower limit of } x(n)}^{n2 \text{ upper limit of } x(n)} x(k)\, h(n-k)$$

And this n will vary from n1+p1 to n2+p2
n1+p1 $\leq n \leq$ n2+p2

a try to understand upper limit and lower limit

let y(n)= {1,3,4,1,2,3,4,5}
-4 -3 -2 -1 0 1 2 3
here the low limit is -4 and the upper limit is 3

another example {3, -4}

here the lower limit is -1 and upper limit is 0.

the input sequence x(n)={3^,1,2,-1} sequence is applied to discrete time processor with unit sample response h(n)={3^,2,1}

Length of y(n)=4+3-1 =6

Lower limit of y(n)= 0+0=0 ,
Upper limit of y(n)=3+2 =5

$$y(n) = \sum_{k=n1(0)}^{n2(3)} x(k)\,h(n-k)$$

n=0,

$$y(0) = \sum_{k=n1(0)}^{n2(3)} x(k)\,h(-k)$$

=x(0)h(0) + x(1)h(-1) + x(2)h(-2) + x(3)h(-3)
=3.3+1.0+2.0+(-1).0
=9
n=1,

$$y(0) = \sum_{k=n1(0)}^{n2(3)} x(k)\,h(1-k)$$

=x(0)h(1) + x(1)h(0) + x(2)h(-1) + x(3)h(-2)
=3.2+1.3+2.0+(-1).0
=9

n=2,

$$y(0) = \sum_{k=n1(0)}^{n2(3)} x(k)\,h(2-k)$$

=x(0)h(2) + x(1)h(1) + x(2)h(0) + x(3)h(-1)
=3.1+1.2+2.3+(-1).0
=11

n=3,

$$y(0) = \sum_{k=n1(0)}^{n2(3)} x(k)\,h(3-k)$$

=x(0)h(3) + x(1)h(2) + x(2)h(1) + x(3)h(0)
=3.0+1.1+2.2+(-1).3
=2

n=4,

$$y(0) = \sum_{k=n1(0)}^{n2(3)} x(k)\,h(2-k)$$

=x(0)h(4) + x(1)h(3) + x(2)h(2) + x(3)h(1)
=3.0+1.0+2.1+(-1).2
=0

n=5,

$$y(0) = \sum_{k=n1(0)}^{n2(3)} x(k)\,h(2-k)$$

=x(0)h(5) + x(1)h(4) + x(2)h(3) + x(3)h(3)
=3.0+1.0+2.0+(-1).1
= -1

Thus , y(n) = {9^,9,11,2,0,-1}

Question: 3(c)2016 Consider the input x[n]={1 0 2 3} h[n]={1 2 1 3} find out the convolution sum.

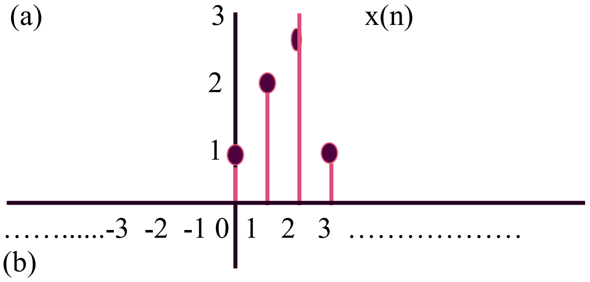
graphical approach :-

let us consider h(n)={1,2,1, -1}

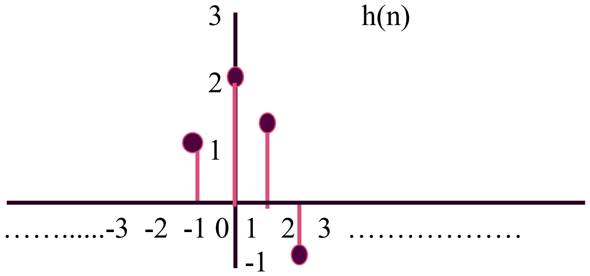
and x(n)={1,2,3,1}

now the graphical interpretation oh x(n) and h(n) is given bellow ,

(a)

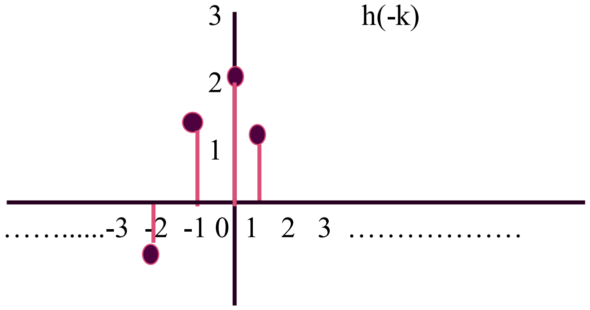


(b)



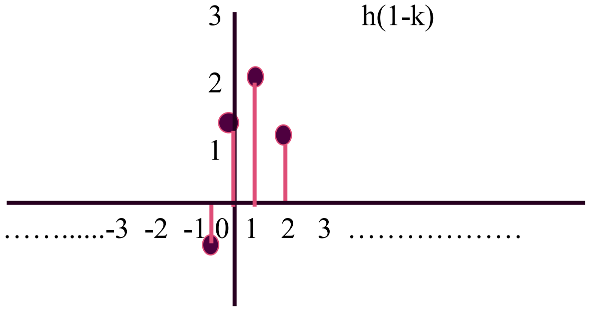
First step :- fold h(k) to get h(-k)

(c)

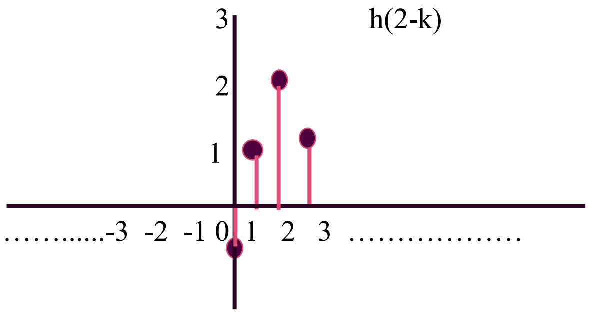


Step 2:- shift to right h(k) by 1 to get h(1-k)

(e)

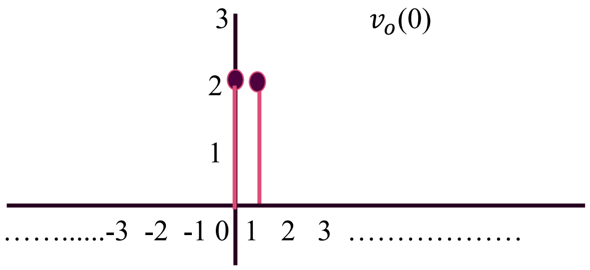


Step 3:- shift to right h(1-k) by 1 to get h(2-k)
(g)



Similarly we can find y(3)=3 ,y(4)= -2 , y(5) =- 1,as the upper bound is 2+3 = 5 so we will stop here. and as the lower bound is 0+(-1) = -1 we have to find y (-1). For that we have to shift h(-k) to left by 1 position to get h(-1-k)

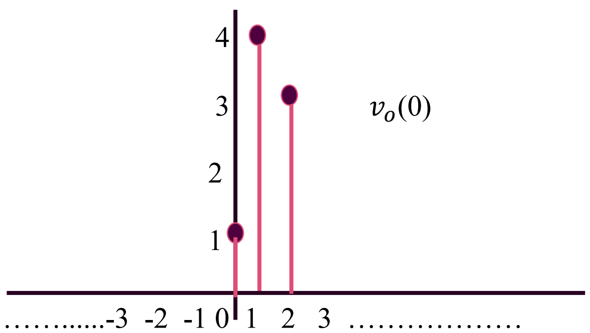
(d) now for y(0) we find the product sequence
x(k)h(-k) means (a).(c)



only 0th and 1st positions in both sequence holds non zero values so the product will have value in only those placed .

now y(0)= sum of all terms in $v_o(0)$ =2+2 = 4

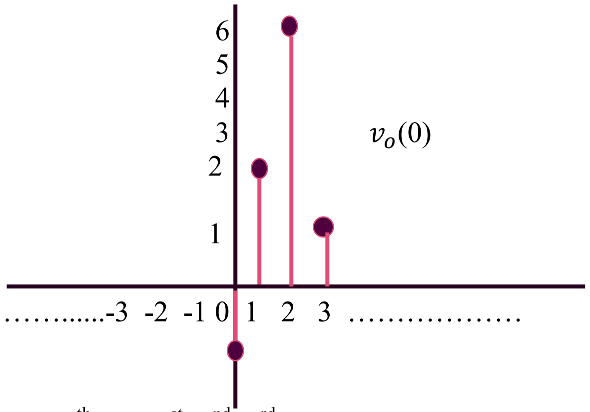
(f) now for y(1) we find the product sequence
x(k)h(1-k) means (a).(e)



only 0th and 1st ,2nd positions in both sequence holds non zero values so the product will have value in only those placed .

now y(1)= sum of all terms in $v_1(1)$ =1+4+3 = 8

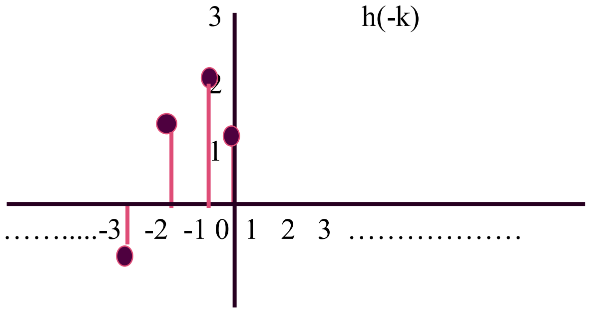
(h) now for y(2) we find the product sequence x(k)h(2-k) means (a).(g)



only 0th and 1st ,2nd ,3rd positions in both sequences holds non zero values so the product will have value in only those placed .

now y(2)= sum of all terms in $v_1(2)$ = -1+2+6+1 = 8

Step 6:- shift to right h(1-k) by 1 to get h(2-k)
(i)



As the lower limit of y(n) is -1 we should stop here.

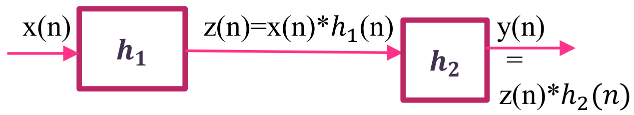
First we have to fold the impulse response sequence h(k) to get h-(k).
Then we will right shift it continuously to get h(1-k),h(2-k)………for values left to the origin we need to shift the h(-k) to left to get h(-1-k), h(-2-k) ……….
Then we will find products v(0)= x(k)h(-k), v(1)= x(k)h(1-k), v(2)= x(k)h(2-k),……… again for v(-1)= x(k)h(-1-k), v(-2)= x(k)h(-2-k), …..
finally adding all terms in v(1) gives y(1) and as follows to all.

Discuss the connection of LTI System ?

There are two types of connections in LTI system

Cascade system: -

In this types of connection output of (n-1)th system is connected to the input of the nth system. Let z(n) is the output of the first system and y(n) is the output of the second system and h(n) is the response of the entire system



here z(n) = x(n)*h_1(n)

y(n)= z(n)*$h_2(n)$

= [x(n)*h_1(n)]*$h_2(n)$

= x(n)*[h_1(n) *$h_2(n)$]

Hence the response of the entire system is given by ,

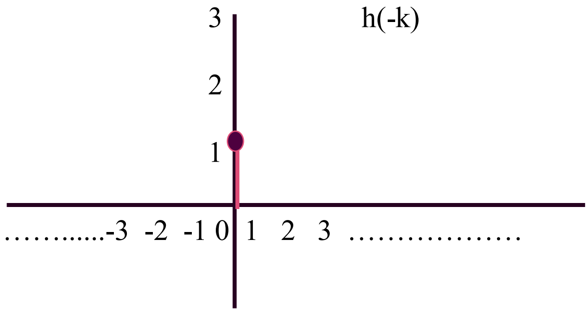
[h_1(n) *$h_2(n)$]

Parallel system:-

In this types of connections each system produces individual output .and finally the are added and the output of entire system is generated.

Step 6:- shift to right h(1-k) by 1 to get h(2-k)

(i)



only 0th positions in both sequences holds non zero values so the product will have value in only 0th placed .

now y(-1)= sum of all terms in v_1(-1) = 1

If $y_1\ (n) is\ the\ output\ of\ system\ 1\ (h_1) is\ the\ responce, and\ y_2\ (n)\ is\ the\ output\ of\ system\ 2\ (h_2)\ is\ the\ responce\ , also\ y(n) is\ the\ output\ of\ entire\ system\ and\ h(n) is\ the\ responce$

Then we get y(n) as below,

y(n)= $y_1\ (n) + y_2(n)$

$$= \sum_{k=-\infty}^{\infty} h_1(k)\, x(n-k) + \sum_{k=-\infty}^{\infty} h_2(k)\, x(n-k)$$

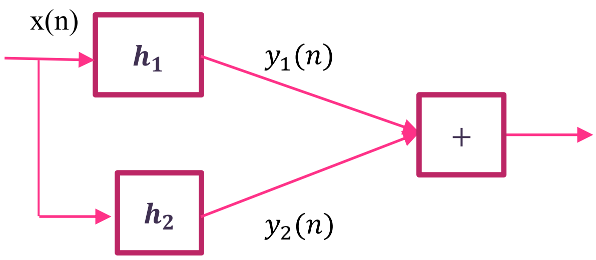
$$\sum_{k=-\infty}^{\infty} [h_1(k) + h_2(k)]\, x(n-k)$$

Hence the output y(n) is given by

$$[h_1(n) + h_2(n)] * x(n)$$

And the response of the system as

$$[h_1(n) + h_2(n)]$$



Types of LTI System ?

Casual anti-causal

The output of a causal system depends at most on current or past inputs like x(n),x(n-1),x(n-2)……..
But never on future inputs.
The output of a LTI system is given by convolution of x(n) and h(n) [unit sample response],
y(n)=x(n)*h(n)

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{-1} x(k)\, h(n-k) + \sum_{k=0}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{-1} h(k)\, x(n-k) + \sum_{k=0}^{\infty} h(k)\, x(n-k)$$

$$= h(-1)x(n+1) + h(-2)x(n+2) + h(-3)x(n+3) \ldots.$$
$$+ h(1)x(n-1) + h(2)x(n-2) + h(3)x(n-3) \ldots \ldots$$

Here n+2 , n+2 …are the future inputs so for the causality properties h(-1) = h(-2) =……=0
Means h(n) =0 ,when n< 0

Hence for LTI system causality condition is given by

$$h(n) = 0 \quad , when\, n < 0$$

output of a causal discrete LTI system

$$y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k)$$

output of an anti-causal discrete LTI system

$$y(n) = \sum_{k=-\infty}^{-1} h(k)\, x(n-k)$$

Stable unstable

For a bounded set input is a system generates a bounded output then it is called **stable** system and **unstable** otherwise

Output of LTI system is given by,
y(n)=x(n)*h(n)

$$= \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

$$= \sum_{k=-\infty}^{\infty} h(k)\, x(n-k)$$

If the input sequence is bounded then the necessity condition for stability is

$$\sum_{k=-\infty}^{\infty} |h(k)| < \infty$$

Question:3(a)-2017 given the impulse response of an LTI system is $h(n) = \begin{cases} a^n, n \geq 0 \\ b^n, n < 0 \end{cases}$ determine the range of values of a and b for which the system is stable.
Anser:

$$\sum_{n=-\infty}^{\infty} |h(n)| = \sum_{n=0,n\,even}^{\infty} |a|^n$$
$$= \sum_{n=0}^{\infty} |a|^{2n}$$
$$= \frac{1}{1-|a|^2}$$

Stable if $|a| < 1$

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Static dynamic

If the output of any system only depends on the current inputs and no storage is needed then the system is called static and dynamic otherwise.
$y(n) = \frac{1}{5}\, x(n)$ is a static system.

FIR and IIR

Depending on the impulse response sequence the system is divided into two classes ,if the impulse response sequence is of finite duration then the system is called FIR,

If h(n) is non-zero for n=0 to n- N-1, then the equation for FIR system is given by

$$y(n) = \sum_{k=0}^{n-1} h(k)\, x(n-k)$$

if the impulse response sequence is of infinite duration then the system is called IIR,

If h(n) is non-zero for n>0 , then the equation for IIR system is given by

$$y(n) = \sum_{k=0}^{\infty} h(k)\, x(n-k)$$

Recursive non recursive

There are some system which not only requires the past and current inputs to generate desired output, but also it needs the already available past outputs .

Consider a system tries to calculate cumulative average of x(n) in the interval $0 \leq k \leq n$
Defined as ,

$$y(n) = \frac{1}{n+1} \sum_{k=0}^{n} x(k)$$

To compute this the system needs to store all the input samples, and with the increase of n the storage requirement increases in time.

This could be simplified as storing some past outputs like y(n-1)

$$y(n) = \frac{1}{n+1}\sum_{k=0}^{n} x(k)$$

$$(n+1)y(n) = \sum_{k=0}^{n} x(k)$$

$$= \sum_{k=0}^{n-1} x(k) + x(n)$$

$= \ ny(n-1) \ + x(n)$

$$\therefore y(n) = \frac{n}{n+1} y(n-1) + \frac{1}{n+1} x(n)$$

In general , a system who's output y(n) at time n depends on any number of past output values y(n-1), y(n-2),…………is called a recursive system

The general difference equation of LTI system?

The zero state response or relaxed response of a recursive system is given by ,

$$y_{zs}(n) = \sum_{k=0}^{n} a^k x(n-k)$$

The general difference equation for LTI system is expressed as,

$$\sum_{k=0}^{N} a^k x(n-k) = \sum_{k=0}^{M} b^k x(n-k)$$

a^k , b^k $\rightarrow$ *coessicients*
y(n) $\rightarrow$ *output*
$x(n) \rightarrow$ *input*
N→order of the system

Now N=1,M=0,$a_0 \ = 1, b_0 \ = 1, -a_1 = a$
And y(n)=ay(n-1) +x(n)
$a_0\, y(n) + a_1\, y(n-1)$= $b_{0x} x(n)$
$\therefore\ a_0\, y(n) = \ -a_1\, y(n-1) + b_{0x} x(n)$

$$= -\sum_{k=0}^{N} a^k x(n-k) + \sum_{k=0}^{M} b^k x(n-k)$$

Now $a_0 \ = 1$

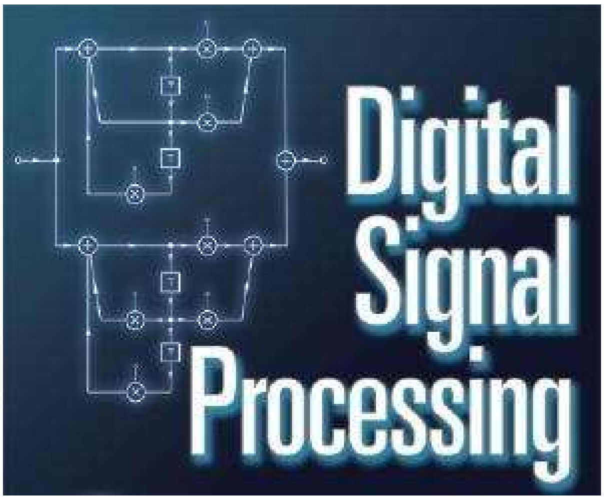
$$y(n) = -\sum_{k=0}^{N} a^k x(n-k) + \sum_{k=0}^{M} b^k x(n-k)$$

Consider,
y(n)= - $a_1 y(n-1) + \ b_0$ x(n)
y(0)= -$a_1 y(-1) + \ b_0$ x(0)
y(1)= -$a_1 y(0) + \ b_0$ x(1)
= -$a_1 \{-a_1 y(-1) + \ b_0$ x(0) } + b_0 x(1)

= $(-a_1)^2$ y(-1) + $(-a_1)$ b_0 x(0) +b_0 x(1)

Y(2) = $(-a_1)^3$ y(-1)+ $(-a_1)^2$ b_0 x(0) +$(-a_1)$ b_0 x(1) + b_0 x(2)

$$y(n) = (-a_1)^{n+1}\, y(-1) + \sum_{k=0}^{n} a^k x(n-k)$$



Important Question

(1) Define z-transform? what is ROC ? why it is important in Z transform?

(2) Briefly describe properties of z transform.

(3) Significance of ROC in Z-plane ? determine the z-transform of $x(n) = \left(\frac{1}{2}\right)^{n} u(n)$

(4)prove that the z-transform of x(n-k) is$Z^{-k}X(Z)$

Find the z transform of x(n)=[2,4,5,7,0,1]

Question 6(a) 2015 Define Z transform? Explain Scaling property of Z transform

what define z-transform?

The basic idea now known as the Z-transform was known as Laplace earlier . To produce a digital signal from an original analog signal ,The standard method is to sample the signal periodically and digitize it with an A to D converter using a standard number of bits 8, 16 etc. Digital signal processing is primarily concerned with the processing of these sampled signals . z-transform was first introduced to handle complex variables in the previous transforming equations , though X(z) is a complex function having real and imaginary parts z-transforms provides a straight forward way of computing the output of an LTI system.

The z-transform of a discrete signal x(n) is defined as,

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \quad \text{....(i)}$$

It transforms the time domain signal x(n) into its into its complex plane representation X(Z).

It is also denoted as X(Z)=Z{x(n)} ....(ii)

The relationship between x(n) and X(Z) is denoted as

x(n)<—>X(Z) ....(iii)

since Z-transform is an infinite power series ,it exists only for those values of Z for which this series converges .the region of convergence of X(Z) is the set of values of z for which X(Z) attains a finite values.

The z-transform is simply an alternative representation of a signal .where we see that the coefficient of Z^{-n},is the value of the signal at time n. And the exponents of z contains the time information we need to identify the sample of the signal.

Example 3.1.1

Determine the ;-transforms of the following *finire-duration* signals.

(a) $x_1(n) = (1, 2, 5, 7, 0, 1)$

(b) $x_2(n) = \{1, 2, \underset{\uparrow}{5}, 7, 0, 1\}$

(c) $x_3(n) = \{0, 0, 1, 2, 5, 7, 0, 1\}$

(d) $x_4(n) = \{2, 4, \underset{\uparrow}{5}, 7, 0, 1\}$

(e) $x_5(n) = \delta(n)$

(f) $x_6(n) = \delta(n-k), k > 0$

(g) $x_7(n) = \delta(n+k), k > 0$

Solution From definition (3.1.1), we have

(a) $X_1(z) = 1 + 2z^{-1} + 5z^{-2} + 7z^{-3} + z^{-5}$, ROC: entire z-plane except $z = 0$

(b) $X_2(z) = z^2 + 2z + 5 + 7z^{-1} + z^{-3}$, ROC: entire :-plane except $z = 0$ and $z = \infty$

(c) $X_3(z) = z^{-2} + 2z^{-3} + 5z^{-4} + 7z^{-5} + z^{-7}$, ROC: entire z-plane except $z = 0$

(d) $X_4(z) = 2z^2 + 4z + 5 + 7z^{-1} + z^{-3}$, ROC: entire z-plane except $z = 0$ and $z = \infty$

(e) $X_5(z) = 1$[i.e., $\delta(n) \xleftrightarrow{z} 1$], ROC: entire z-plane

(f) $X_6(z) = z^{-k}$[i.e., $\delta(n-k) \xleftrightarrow{z} z^{-k}$], $k > 0$, ROC: entire z-plane except $z = 0$

(g) $X_7(z) = z^k$[i.e., $\delta(n+k) \xleftrightarrow{z} z^k$], $k > 0$, ROC: entire z-plane except $z = \infty$

Question: 5(c) 2016 Find the z transform of x(n)=[2,4,$\underset{\uparrow}{5}$,7,0,1]

We know z transform $X(z) = \sum_{n=-\infty\infty}^{\infty} x(n)z^{-n}$

$$X(z) = \sum_{n=-2}^{3} x(n)z^{-n}$$

$$X(z) = 2z^2 + 4z + 5 + 7z^{-1} + 0z^{-2} + z^{-3}$$

Question: 5(c) 2016 Define Z transform? Find the z transform of x1(n)=[1,2,4,5,7,0,1] x2(n)=[1,2,4,5,7,0,1] x3(n)=[0, 0,1, 2,4,5,7,0,1]

We know z transform $X(z) = \sum_{n=-\infty\infty}^{\infty} x(n)z^{-n}$

$$X(z) = \sum_{n=-2}^{3} x(n)z^{-n}$$

$$X(z) = 2z^2 + 4z + 5 + 7z^{-1} + 0z^{-2} + z^{-3}$$

Example: Determine the z transform of the following signal

$$\boldsymbol{x(n) = \left(\frac{1}{2}\right)^n u(n)}$$

Solution:

Solution The signal $x(n)$ consists of an infinite number of nonzero values

$$x(n) = \{1, (\tfrac{1}{2}), (\tfrac{1}{2})^2, (\tfrac{1}{2})^3, \ldots, (\tfrac{1}{2})^n, \ldots\}$$

The z-transform of $x(n)$ is the infinite power series

$$X(z) = 1 + \tfrac{1}{2}z^{-1} + (\tfrac{1}{2})^2 z^{-2} + (\tfrac{1}{2})^n z^{-n} + \cdots$$

$$= \sum_{n=0}^{\infty} (\tfrac{1}{2})^n z^{-n} = \sum_{n=0}^{\infty} (\tfrac{1}{2}z^{-1})^n$$

This is an infinite geometric series. We recall that

$$1 + A + A^2 + A^3 + \cdots = \frac{1}{1-A} \qquad \text{if } |A| < 1$$

Consequently, for $|\tfrac{1}{2}z^{-1}| < 1$, or equivalently, for $|z| > \tfrac{1}{2}$, $X(z)$ converges to

$$X(z) = \frac{1}{1 - \tfrac{1}{2}z^{-1}} \qquad \text{ROC: } |z| > \tfrac{1}{2}$$

Question: 4(a) 2017 Determine the z transform of the following signal

$$x(n) = \left(\frac{1}{3}\right)^{n-1} u(n-1)$$

Solution:

We know x(n) consist of infinite number non-zero values

$$x(n) = \{1, \left(\frac{1}{3}\right), \left(\frac{1}{3}\right)^2, \left(\frac{1}{3}\right)^3, \left(\frac{1}{3}\right)^4, \left(\frac{1}{3}\right)^5, \left(\frac{1}{3}\right)^6, ,,,,,,,,,,,,,, \left(\frac{1}{3}\right)^n\}$$

The z transform of x(n)

$$x(z) = \{1, \left(\frac{1}{3}\right)z^{-1}, \left(\frac{1}{3}\right)^2 z^{-2}, \left(\frac{1}{3}\right)^3 z^{-3}, ,,,,,,,,,,,,,, \left(\frac{1}{3}\right)^n z^{-n}\}$$

$$x(z) = \sum_{n=0}^{\infty} \left(\frac{1}{3} z^{-1}\right)^n$$

This is a infinite geometric series we recall

$,1 + A + A^2 + A^3 + \cdots \ldots \ldots \ldots .. = \frac{1}{1-A}$ if $|A| < 1$

For consequently for $\left|\frac{1}{3} z^{-1}\right| < 1$ *or equivalently, for* $|z| > 1/2$ X(Z) converges to

$X(z) = \frac{1}{1-\frac{1}{3}z^{-1}}$ ROC: $|z| > 1/3$

Question: 6(b) 2015 Determine the z transform of the following signal

$$x(n) = [3(2^n) - [3(2^n)u(n)$$

Solution:

Solution If we define the signals

$$x_1(n) = 2^n u(n)$$

and

$$x_2(n) = 3^n u(n)$$

then $x(n)$ can be written as

$$x(n) = 3x_1(n) - 4x_2(n)$$

According to (3.2.1). its :-transform is

$$X(z) = 3X_1(z) - 4X_2(z)$$

From (3.1.7) we recall that

$$\alpha^n u(n) \stackrel{z}{\longleftrightarrow} \frac{1}{1-\alpha z^{-1}} \qquad \text{ROC: } |z| > |\alpha| \tag{3.2.2}$$

By setting $\alpha = 2$ and a = 3 in **(3.2.2)**. we obtain

$$x_1(n) = 2^n u(n) \longleftrightarrow X_1(z) = \frac{1}{1-2z^{-1}} \qquad \text{ROC: } |z| > 2$$

$$x_2(n) = 3^n u(n) \stackrel{z}{\longleftrightarrow} X_2(z) = \frac{1}{1-3z^{-1}} \qquad \text{ROC: } |z| > 3$$

The intersection of the ROC of $X_1(z)$ and $X_2(z)$ is $|z| > 3$. Thus the overall transform X(z) is

$$X(z) = \frac{3}{1-2z^{-1}} - \frac{4}{1-3z^{-1}} \qquad \text{ROC: } |z| > 3$$

Example 3.1.4

Determine the z-transform of the signal

$$x(n) = -\alpha^n u(-n-1) = \begin{cases} 0, & n \geq 0 \\ -\alpha^n, & n \leq -1 \end{cases}$$

Solution From the definition (3.1.1) we have

$$X(z) = \sum_{n=-\infty}^{-1} (-\alpha^n) z^{-n} = -\sum_{l=1}^{\infty} (\alpha^{-1} z)^l$$

where $l = -n$. Using the formula

$$A + A^2 + A^3 + \cdots = A(1 + A + A^2 + \cdots) = \frac{A}{1 - A}$$

when $|A| < 1$ gives

$$X(z) = -\frac{\alpha^{-1} z}{1 - \alpha^{-1} z} = \frac{1}{1 - \alpha z^{-1}}$$

provided that $|\alpha^{-1} z| < 1$ or, equivalently, $|z| < |\alpha|$. Thus

$$x(n) = -\alpha^n u(-n-1) \overset{z}{\longleftrightarrow} X(z) = -\frac{1}{1 - \alpha z^{-1}} \qquad \text{ROC}: |z| < |\alpha| \qquad (3.1.9)$$

The ROC is now the interior of a circle having radius $|\alpha|$. This is shown in Fig. 3.3.

Example 3.1.3

Determine the z-transform of the signal

$$x(n) = \alpha^n u(n) = \begin{cases} \alpha^n, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

Solution From the definition (3.1.1) we have

$$X(z) = \sum_{n=0}^{\infty} \alpha^n z^{-n} = \sum_{n=0}^{\infty} (\alpha z^{-1})^n$$

If $|\alpha z^{-1}| < 1$ or equivalently, $|z| > |\alpha|$, this power series converges to $1/(1 - \alpha z^{-1})$.

Thus we have the z-transform pair

$$x(n) = \alpha^n u(n) \overset{z}{\longleftrightarrow} X(z) = \frac{1}{1 - \alpha z^{-1}} \qquad \text{ROC}: |z| > |\alpha| \qquad (3.1.7)$$

The ROC is the exterior of a circle having radius $|\alpha|$. Figure 3.2 shows a graph of the signal $x(n)$ and its corresponding ROC*Note that, in general, a need not be real.

If we set $a = 1$ in (3.1.7), we obtain the z-transform of the unit step signal

$$x(n) = u(n) \overset{z}{\longleftrightarrow} X(z) = \frac{}{1 - z^{-1}} \qquad \text{ROC}: |z| > 1 \qquad (3.1.8)$$

Explain Scaling property of Z transform

Properties of z-transform?

Linearity

the linearity property can be easily be generalized for an arbitrary number of signals.it implies that Z-transform of a linear combination of signals is the same as linear combination of their Z-transforms.

So $\boldsymbol{Z[ax_1(n)+bx_2(n)] = aX_1(Z)+\ bX_2(Z)}$..........(i) will be true.

By definition : $Z[ax_1(n)+bx_2(n)] = \sum_{n=-\infty}^{\infty} Z[ax_1(n)+bx_2(n)]Z^{-n}$

$=\sum_{n=-\infty}^{\infty}[ax_1(n)]Z^{-n}+\sum_{n=-\infty}^{\infty}[b\,x_2(n)]Z^{-n}$

$= \boldsymbol{aX_1(Z)+\ bX_2(Z)}$

It is obvious that the ROC of the linear combination of x[n] and y[n] should be the intersection of the their individual ROCs $R_x \cap R_y$, in which both x(z) and y(z) exist.

Time shift

If x(n) $\overset{z}{\longleftrightarrow}$ X(z)

Then

x(n-k) $\overset{z}{\longleftrightarrow}$ Z^{-k}X(z)

so Z[x(n-k)]= $Z^{-k}X(z)$ will be true

from definition ,

Z[x(n-k)]= $\sum_{n=-\infty}^{\infty} x(n-k)\ Z^{-n}$

Let n-k=t ,

$= \ \sum_{t=-\infty}^{\infty} x(t)\ Z^{-(t+k)}$

$= \ Z^{-k}\sum_{t=-\infty}^{\infty} x(t)\ Z^{-t}$

$=Z^{-k}X(z)$

The ROC of Z^{-k} is the same that of X(z) except for

Z=0 if k › 0

Z=∞ if k ‹0

Scaling Properties:

If

$x(n) \overset{z}{\longleftrightarrow} X(z) \qquad \text{ROC}: r_1 < |z| < r_2$

then

$$a^n x(n) \overset{z}{\longleftrightarrow} X(a^{-1}z) \qquad ROC: |a|r_1 < |z| < |a|r_2 \qquad (3.2.9)$$

for any constant a, real or complex.

Proof: From the definition (3.1.1)

$$Z\{a^n x(n)\} = \sum_{n=-\infty}^{\infty} a^n x(n) z^{-n} = \sum_{n=-\infty}^{\infty} x(n)(a^{-1}z)^{-n}$$

$$= X(a^{-1}z)$$

Since the ROC of $X(z)$ is $r_1 < |z| < r_2$, the ROC of $X(a^{-1}z)$ is

$$r_1 < |a^{-1}z| < r_2$$

or

$$|a|r_1 < |z| < |a|r_2$$

To better understand the meaning and implications of the scaling property, we express a and z in polar form as $a = r_0 e^{j\omega_0}$, $z = re^{j\omega}$, and we introduce a new complex variable $w = a^{-1}z$. Thus $Z\{x(n)\} = X(z)$ and $Z\{a^n x(n)\} = X(w)$. It can easily be seen that

$$w = a^{-1}z = \left(\frac{1}{r_0} r\right) e^{j(\omega-\omega_0)}$$

This change of variables results in either shrinking (if $r_0 > 1$) or expanding (if $r_0 < 1$) the z-plane in combination with a rotation (if $\omega_0 \# 2k\pi$) of the z-plane (see Fig. 3.6). This explains why we have a change in the ROC of the new transform where $|a| < 1$. The case $|a| = 1$, that is, $a = e^{j\omega_0}$ is of special interest because it corresponds only to rotation of the z-plane.

Correlation

$Z[R_{xy}(k)] = X_1(Z)X_2(Z^{-1})$ will be true.

Proof ,by definition $Z[R_{xy}(k)] = Z[x(n) * y(-n)\quad]$

As , $x(-n) \longleftrightarrow X(Z^{-1})$

$x_1(n) * x_2(n) \quad \leftrightarrow \quad X(Z) = X_1(z)\ X_2(z)$

$Z[R_{xy}(k)] = R_{xy}(k)\ (Z) = X_1(z)\ X_2(z^{-1})$

Time advance

If x(n) is a causal signal then,

$$Z[x(n+1)] = ZX(Z) - Zx(0)$$

From definition,

$Z[x(n+1)] = \sum_{n=0}^{\infty} x(n+1)\, Z^{-n}$

Let ,n+1=p, $= \sum_{p=1}^{\infty} x(p)\, Z^{-p+1}$

$= Z\sum_{p=1}^{\infty} x(p) Z^{-p}$

$= Z\left[\sum_{p=0}^{\infty} x(p) Z^{-p} - x(0)\right]$

$= ZX(Z) - Zx(0)$

Multiplication by an exponential

$$Z[x(n)a^n] = X(a^{-1}Z)$$

From definition,

$Z[x(n)\, a^n] = \sum_{n=-\infty}^{\infty} x(n) a^n\, Z^{-n}$

$= \sum_{n=-\infty}^{\infty} x(n)(a^{-1}\, Z)^{-n}$

$= X(a^{-1}\, Z)$

Time reversal

If x(n) $\overset{z}{\longleftrightarrow}$ X(z) ROC: r_1‹|z|‹ r_2

Then

$$x(-n) \overset{z}{\longleftrightarrow} X(Z^{-1}) \text{ ROC: } \frac{1}{r_1} ‹ |z| ‹ \frac{1}{r_2}$$

from definition:

$$Z[x(-n)] = \sum_{n=-\infty}^{\infty} x(-n)\, Z^{-n}$$

Let n=-p, $= \sum_{p=-\infty}^{\infty} x(p)\, Z^{p}$

$$= \sum_{p=-\infty}^{\infty} x(p)\, (Z^{-1})^{-p}$$

$$= X(Z^{-1})$$

Convolution

$$x_1(n) \leftrightarrow X_1(z) \text{ and } x_2(n) \leftrightarrow X_2(z)$$

Then,

$$x(n) = x_1(n) * x_2(n) \quad \leftrightarrow \quad X(Z) = X_1(z)\; X_2(z)$$

the ROC of X(Z) is at least the intersection of $that\ for\ X_1(z)$ and $X_2(z)$

the convolution of $x_1(n)\ and\ x_2(n)\ \ is\ defined\ as$, $x(n) = x_1(n) * x_2(n) \; = \sum_{k=-\infty}^{\infty} x_1(k)x_2(n-k)$

the Z transform of x(n) is

$$Z[x(n)] = \sum_{n=-\infty}^{\infty} x(n)\, Z^{-n}$$

$$= \sum_{n=-\infty}^{\infty} [\sum_{k=-\infty}^{\infty} x_1(k)x_2(n-k)\, Z^{-n}$$

Upon interchanging the order of summation a we get

$$= \sum_{k=-\infty}^{\infty} x_1(k) [\sum_{n=-\infty}^{\infty} x_2(n-k)\, Z^{-n}]$$

Applying time shifting operation

$$= X_2(z) Z^{-k} \sum_{k=-\infty}^{\infty} x_1(k)$$

$$= X_2(z) \sum_{k=-\infty}^{\infty} x_1(k) Z^{-k}$$

$$= X_2(z) X_1(z)$$

Question: 2013 Explain poles and zeros in z-transform

Poles and Zeros

One of the most useful aspects of the z-transform analysis is the description of a system about the stability and steady state frequency response.

The zeros of a transfer function *H*(*z*) are:- the values of the variable *z* for which the transfer function (or equivalently its numerator) is zero. Therefore the zeros are the roots of numerator polynomial .

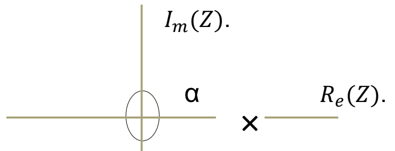
The poles of *H*(*z*):- are the values of the variable *z* for which *H*(*z*) is infinite, the denominator of *H*(*z*) is zero. Therefore the poles of *H*(*z*) are the roots of the denominator polynomial

For a stable system the poles should have a magnitude of less than one and lie inside the unit circle. The zeros represent the roots of the feed forward part of the transfer function of a system. for a system with real-valued coefficients ,and complex-valued poles or zeros always occur in complex conjugate pairs.

A useful graphical abstraction of the transfer function of a discrete-time system H(z) is the pole-zero plot in a complex polar plane. The location of the poles are usually shown by crosses (×) and the locations of the zeros by circles (O) .

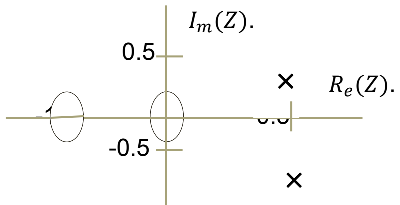
Example: $H(Z) = \frac{Z}{Z-\alpha}$ find the pole zero plot ,

System contains 1 zero at Z=0 and one ploe at Z= α



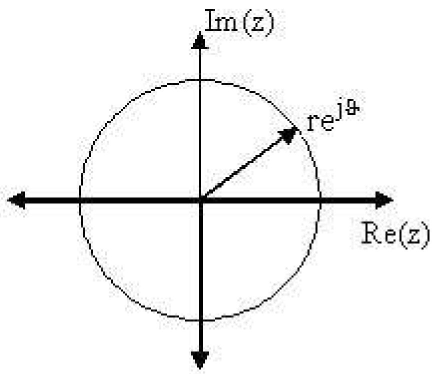
Example: $H(Z) = \frac{Z(Z+1)}{Z^2-Z+0.5}$ find the pole zero plot ,
System contains 2 zero at Z=0, Z=-1 and 2 poles at
Z=0.5+j0.5, Z=0.5-j0.5,

ystem contains 1 zero at Z=0 and one ploe at Z= α



The Z-Plane

Once the poles and zeros have been found for a given Z-Transform, they can be plotted onto the Z-Plane. The Z-plane is a complex plane with an imaginary and real axis referring to the complex-valued variable. The position on the complex plane is given by and the angle from the positive, real axis around the plane is denoted by . When mapping poles and zeros onto the plane, poles are denoted by an "x" and zeros by an "o". The below figure shows the Z-Plane,



Region of convergence

Region of convergence, known as the ROC,defines the region where the z-transform exists. The z-transform of a sequence is defined as

$$X(z) = \mathcal{Z}\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

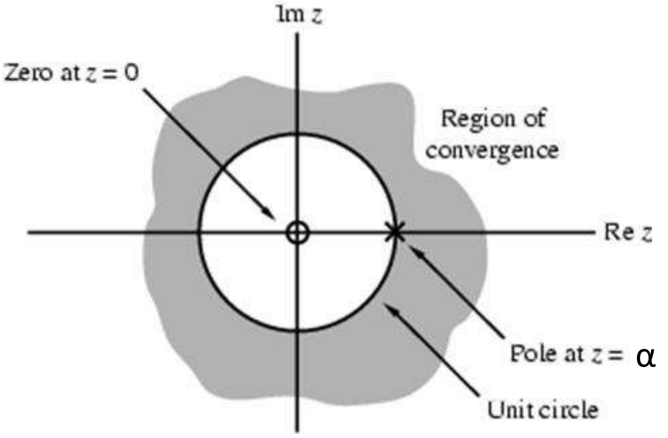
The ROC for a given x(n),is defined as the range of for which the z-transform converges(magnitude of X(Z) is finite). Since the z-transform is a power series, it converges when x(n)z^{-1} is absolutely summable . Stated differently,

$$\sum_{n=-\infty}^{\infty} x(n)Z^{-n} < \infty$$

must be satisfied for convergence. The ROC cannot contain any poles. By definition a pole is a where X(Z) is infinite. Since X(Z) must be finite for all Z for convergence, there cannot be a pole in the ROC.
Z-transform contains four types of ROC

(1) Right-sided signals

A right sided sequence, is one for which the values of x(n)=0,for n< n_o ,where n_o is negative or positive value .if $n_o \geq 0$,then the sequence is causal .for such a sequence Z-transform converges out side the circle.



,

Find ROC if x(n)=α^nu(n)

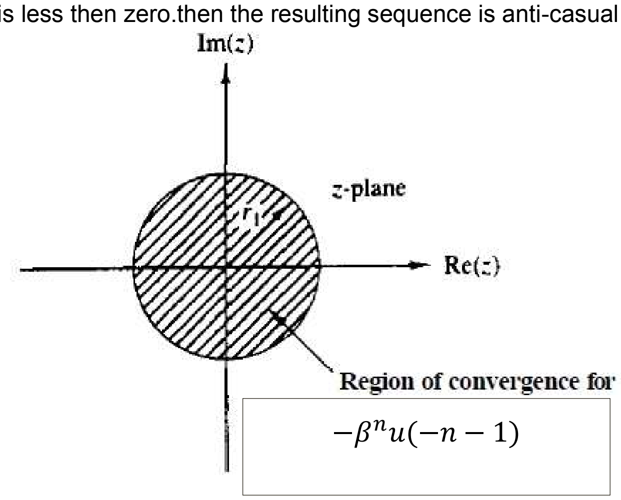
The z-transform is given by

$$X(Z) = \sum_{n=-\infty}^{\infty} \alpha^n u(n) Z^{-n} \quad = \sum_{n=0}^{\infty} (\alpha Z^{-1})^{\,n} \quad = \frac{1}{1-\alpha Z^{-1}} (i)$$

In this case it converses if |αZ^{-1}|<1 which implies |z|>| α |.the signal contains one pole at z= α,thus it converges
outside the circle with radius α.

(2) Left -sided signals

A left-sided sequence is one for which the values of x(n) =0 for n ≥ n_o ,,where n_o is positive or negative finite values.if n_o is less then zero.then the resulting sequence is anti-casual sequence,



For such a sequence Z-transform converges inside the circle.

X(Z) $= \sum_{n=-\infty}^{\infty} -\beta^n u(-n-1)\, Z^{-n}$

$= \sum_{n=-\infty}^{-1} -(\beta\, Z^{-1})^n$

$= 1 - \sum_{n=0}^{\infty} -(\beta^{-1} Z)^n$

$$X(Z) = 1 - \frac{1}{1-\beta^{-1}Z} = \frac{\beta^{-1}Z}{1-\beta^{-1}Z} = \frac{1}{1-\beta Z^{-1}}$$

In this case it converges if |$\beta^{-1}Z$ |<0,which implies |z| <| β |,hence it has one zero at Z=0 and one pole at Z= β and converges inside the circle of radius β .

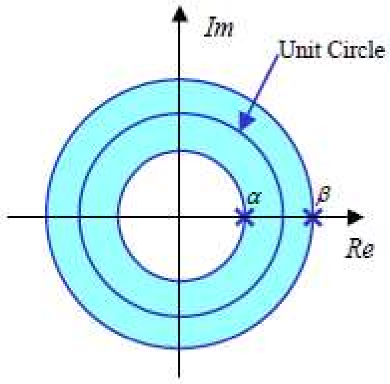
(3) two -sided signals

two sided signal is one in which signals extends from n= -∞ to $n = \infty$ such signal is given by

X(z)= $\sum_{n=-\infty}^{\infty} x(n)\, Z^{-n}$

$$=\sum_{n=-\infty}^{-1} x(n)\, Z^{-n} + \sum_{n=0}^{\infty} x(n)\, Z^{-n}$$

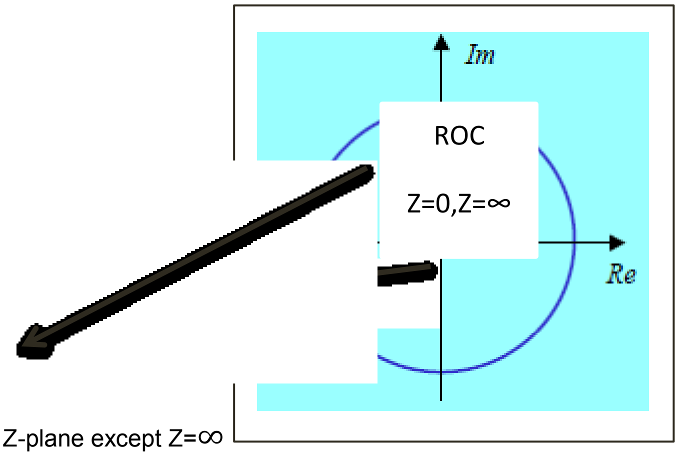
For the first power series (causal sequence) ROC is outside the circle of radius r1 , and For the 2nd power series (anti-causal sequence) ROC is inside the circle of radius r2



As shown by the shaded area of figure, the region of convergence corresponds to the area of α < z < β.

(4) Finite duration signals

If Z-transform converges everywhere in the z-plane except at Z=0 and at Z=∞ ,then the sequence is called finite duration sequence and n is positive , then ROC is the entire Z-plane except Z=0 and N is negative then



ROC is the entire Z-plane except Z=∞

Find the roc of x(n)= $\{\underset{\uparrow}{2,5,4}\}$,the z transform is given by

$$X(Z) = \sum_{n=-1}^{1} x(n) Z^{-1} = 2Z + 5 + 4Z^{-1}$$

For this case it converges everywhere except Z=0 and at Z=∞

Find the Z-transform for the given Sequence

X(n)= $\{\underset{\uparrow}{a},b,c,d,e.......\}$

$$a*(z^{-1})^0 + b*(z^{-1})^1 + c*(z^{-1})^2 + d*(z^{-1})^3$$

Again,

X(n)={a,b,c,d,e,f…….}
↑

$$a*(z^{-1})^{-2}+b*(z^{-1})^{-1}+c*(z^{-1})^{0}+d*(z^{-1})^{2}+e*(z^{-1})^{3}+f*(z^{-1})^{3}$$

Example: x(n)={2,5,0,6,0,9}
↑

$X(Z)=2(z^{-1})^{-1}+5(z^{-1})^{0}+0(z^{-1})^{1}+6(z^{-1})^{2}+0(z^{-1})^{3}+9(z^{-1})^{4}$

=2Z+5+6$z^{-2}+9z^{-4}$

System function of LTI system ?

Inverse z-transform

Question: 4(a) 2017,2016 what are the properties of ROC.
Answer:
Properties of ROC
Based on these observations, we can get the following properties for the ROC:

- If $x[n]$ is of finite duration, then the ROC is the entire z-plane (the z-transform summation converges, i.e., $X(z)$ exists, for any z) except possibly $z=0$ and/or $z=\infty$.
- The ROC of $X(z)$ consists of a ring centered about the origin in the z-plane. The inner boundary can extend inward to the origin in some cases, and the outer can extend to infinity in other cases.
- If $x[n]$ is right sided and the circle $|z|=r_0$ is in the ROC, then any finite z for which $|z|>r_0$ is also in the ROC.
- If $x[n]$ is left sided and the circle $|z|=r_0$ is in the ROC, then any z for which $0<|z|<r_0$ is also in the ROC.
- If $x[n]$ is two-sided, then the ROC is the intersection of the two one-sided ROCs corresponding to the two one-sided parts of $x[n]$. This intersection can be either a ring or an empty set.
- If $X(z)$ is rational, then its ROC does not contain any poles (by definition $X(z)\Big|_{z=z_p}=\infty$ dose not exist). The ROC is bounded by the poles or extends to infinity.
- If $X(z)$ is a rational z-transform of a right sided function $x[n]$, then the ROC is the region outside the out-most pole. If $x[n]=0$ for $n<0$ (causal), then the ROC includes $z=\infty$.
- If $X(z)$ is a rational z-transform of a left sided function $x[n]$, then the ROC is inside the innermost pole. If $x[n]=0$ for $n\geq 0$ (anti-causal), then the ROC includes $z=0$.
- Fourier transform $X(e^{j\omega})$ of discrete signal $x[n]$ exists if the ROC of the corresponding z-transform $X(z)$ contains the unit circle $|z|=1$ or $z=e^{j\omega}$.

Example 1:

$$X(z)=\sum_{n=-3}^{5} x[n]z^{-n}$$

When $z=0$, $z^{-n}=\infty$ for $n>0$, when $z=\infty$, $z^{-n}=\infty$ for $n<0$. Therefore neither $z=0$ nor $|z|=\infty$ are included in the ROC.

Example 2:

$$x[n]=a^{|n|}=a^n u[n]+a^{-n}u[-n-1]$$

The Z-transform is linear, and $X(z)$ is the sum of the transforms for the two terms:

$$\mathcal{Z}[a^n u[n]]=\frac{1}{1-az^{-1}},\qquad (|z|>|a|),\qquad \mathcal{Z}[a^{-n}u[-n-1]]=\frac{-1}{1-a^{-1}z^{-1}},\qquad (|z|<1/|a$$

If $|a|<1$, i.e., $x[n]$ decays when $|n|\rightarrow\infty$, the intersection of the two ROCs is $|a|<|z|<1/|a|$, and we have:

$$\mathcal{Z}[x[n]]=\frac{1}{1-az^{-1}}-\frac{1}{1-a^{-1}z^{-1}}=\frac{a^2-1}{a}\frac{z}{(z-a)(z-1/a)}$$

However, if $|a|>1$, i.e., $x[n]$ grows without a bound when $|n|\rightarrow\infty$, the intersection of the two ROCs is a empty set, the Z-transform does not exist.

Example 3: Given the following z-transform, find the corresponding signal:

$$X(z)=\frac{1}{(1-\frac{1}{3}z^{-1})(1-2z^{-1})}=-\frac{1/5}{1-\frac{1}{3}z^{-1}}+\frac{6/5}{1-2z^{-1}}$$

The two poles are $z_{p_1}=1/3$ and $z_{p_2}=2$, respectively. The $X(z)$ has three possible ROCs associated with three different time signals $x[n]$:

- The region outside the out-most pole $z_{p_2}=2$, with the corresponding right sided time function

$$x[n]=-\frac{1}{5}(\frac{1}{3})^n u[n]+\frac{6}{5}2^n u[n]$$

- The region inside the innermost pole $z_{p_1}=1/3$, with the corresponding left sided time function

$$x[n] = \frac{1}{5}(\frac{1}{3})^n u[-n-1] - \frac{6}{5}2^n u[-n-1]$$

- The ring between the two poles $1/3 < |z| < 2$, with the corresponding two sided time function

$$x[n] = -\frac{1}{5}(\frac{1}{3})^n u[n] - \frac{6}{5}2^n u[-n-1]$$

In particular, note that only the last ROC includes the circle $|z| = 1$ and the corresponding time function $x[n]$ has a discrete Fourier transform. Fourier transform of the other two functions do not exist.

Question: 4(b) 2017,2016 Define inverse Z transform. Find the inverse Z-transform of

$$X(z) = \frac{z+0.2}{(z+0.5)(z-1)}, |z| > 1$$

$$X(z) = \frac{z^2+z}{(z-1)(z-3)}, |z| > 3$$

Often, we have the :-transform $X(z)$ of a signal and we must determine the signal sequence. The procedure for transforming from the :-domain to the time domain is called the inverse *r-transform.* An inversion formula for obtaining $x(n)$ from X (z) can be derived by using the *Cauchy integral theorern.* which is an important theorem in the theory of complex variables. To begin, we have the :-transform defined ty (3.1.1) as

$$X(z) = \sum_{k=-\infty}^{\infty} x(k) z^{-k} \tag{3.1.12}$$

Suppose that we multiply both sides of (3.1.12) by z^{n-1} and integrate both sides over a closed contour within the ROC of $X(z)$ which rncloses the origin. Such a contour is illustrated in Fig. 3.5. Thus we have

$$\oint_C X(z) z^{n-1} dz = \oint_C \sum_{k=-\infty}^{\infty} x(k) z^{n-1-k} dz \tag{3.1.13}$$

where C denotes the closed contour in the ROC of $X(z)$, taken in a counterclock-wise direction. Since the series converges on this contour. we can interchange the order of integration and summation on the right-hand side of (3.1.13). Thus

(3.1.13) becomes

$$\oint_C X(z)z^{n-1}dz = \sum_{k=-\infty}^{\infty} x(k)\oint_C z^{n-1-k}dz \tag{3.1.14}$$

Now we can invoke the Cauchy integral theorem, which states that

$$\frac{1}{2\pi j}\oint_C z^{n-1-k}\,dz = \begin{cases} 1, & k = n \\ 0, & k \neq n \end{cases} \tag{3.1.15}$$

where C is any contour that encloses the origin. By applying **(3.1.15)**, the right-hand side of (3.1.14) reduces to $2\pi j x(n)$ and hence the desired inversion formula

$$x(n) = \frac{1}{2\pi j}\oint_C X(z)z^{n-1}\,dz \tag{3.1.16}$$

Although the contour integral in (3.1.16) provides the desired inversion formula for determining the sequence $x(n)$ from the z-transform, we shall not use (3.1.16) directly in our evaluation of inverse z-transforms. In our treatment we deal

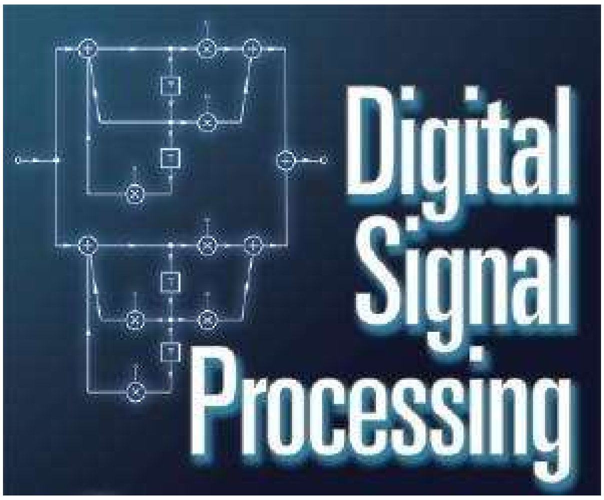
The basic idea of Z transform is “Given a Z-transform X(z) with its corresponding RoC, we can expand X(z) into a power series of the form

$$X(z) = \sum_{n=-\infty}^{\infty} c_n z^{-n}$$

which RoC”converges in the given

Find the inverse Z-transform of

$$X(z) = \frac{z + 0.2}{(z + 0.5)(z - 1)}, |z| > 1$$



Important Question

1. **What are the problems caused by data redundancy in a database? 4 Marks CSE-2013**
2. What is Normalization? State the goals of Normalization. Why is it necessary? **4 Marks CSE-2012 CSE-2011 CSE-2010 CSE-2005 CSE-2003 CSE-2003 (OLD)**
3. Explain First Normal Form (1NF) and Second Normal Form (2NF) with examples. **6 Marks CSE-2012 CSE-2010 CSE-2005 CSE-2003**
4. Define the following terms with examples:
   (a) Extraneous Attributes. **CSE- 2012 CSE-2011** (b) Lossless Join Decomposition. **CSE- 2012 CSE-2011 CSE-2002** (c) Functional Dependencies. **CSE-2011 CSE-2000** (d) Multivalued Dependencies. **CSE-2011** (e) Dependency Preservation. **CSE-2002**
5. Define Functional Dependency. What are Armstrong's axioms? Define Keys in terms of Functional Dependencies. **5 Marks CSE-2013 (Engg) CSE-2000**
6. What is Boyce-Codd Normal Form? Write the algorithm for decomposing a relation schema R into BCNF. **4 Marks CSE-2012 (Engg) CSE-2002**
7. What is Decomposition? Can you explain the formal definition of functional dependency? **1+2 CSE-2012 (Engg)**
8. How can you define the closure of a set of Functional dependencies? **3.75 Marks CSE-2012 (Engg)**
9. Define closure set. Compute four members of F^+ from the relation $R = \{A, B, C, G, H, I\}$ and the functional dependency set $F = \{A \to B, A \to C, CG \to H, CG \to I, B \to H\}$. **5 Marks CSE-2011**
10. Define Canonical Cover. Compute the canonical cover of the following of the following set $F = \{A \to BC, B \to C, A \to B, AB \to C\}$ of functional dependencies for relational schema $R = \{A, B, C\}$. **5 Marks CSE-2011**
11. Compute the closure of the following set F of functional dependencies for the following relational schema, $R = \{A, B, C, D, E\}$ and $F = \{A \to BC, CD \to E, B \to D, E \to A\}$. List candidate keys for R. **4 Marks CSE-2010 CSE-2009 CSE-2005 CSE-2002**
12. What criterion should be fulfilled for lossless join decomposition? **3 Marks CSE-2003 (OLD)**
13. Briefly describe 2NF and 3 NF with examples. **5 Marks CSE-2003 (OLD) CSE-2002**
14. What are the design goals of a good relational database design? Is it always possible to achieve these goals? Justify your answer. **5 Marks CSE-2003 (OLD) CSE-2000**
15. Discuss the problem of spurious tuples and how we may prevent it. **2 Marks CSE-2002**
16. Why Normalization is needed? What is a non-prime attribute? **2+1 CSE-2002**
17. What do you mean by the term "Decomposition" of a relation? What are the desirable properties of decomposition? **4 Marks CSE-2002 CSE-2000**
18. When can we say a relational schema is in 3NF? **2 Marks CSE-2000**

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Qustion: define magnitude and phase

- **Amplitude** is the peak value of a sinusoid in the time domain
- **Magnitude** is the absolute value of any value, as opposed to its phase.

With these meanings, you would not use *amplitude* for FFT bins, you would use *magnitude*, since you are describing a single value. The link would be that for a pure sinusoid, the signal amplitude would be the same as the magnitude of the appropriate FFT bin ('same as' depending on what scaling etc is used in the FFT implementation, but at the very least will be 'proportional to').
In saying all that, if you were to tell me about the amplitude of an FFT bin, I would know exactly what you were talking about.

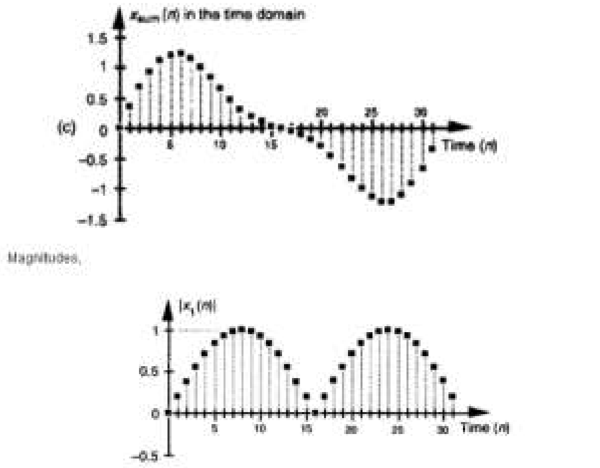


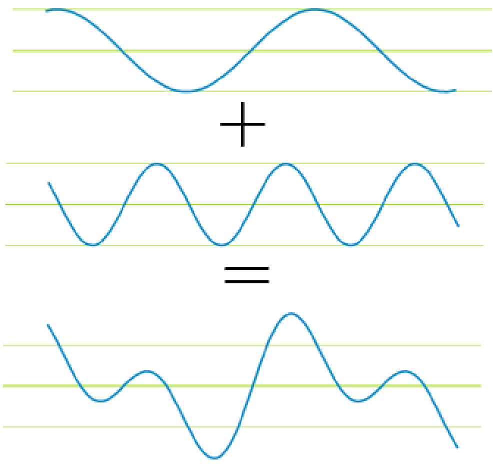
Figure 1-4 Magnitude samples, $|x_1(n)|$, of the time waveform in Figure 1-3(a)

Question: 4(a) 2016. What is fourier series ? Represent a fourier series with frequencies 1Khz, 2 Khz,…. Explain harmonic fourier series.

Fourier series: A **Fourier series** is a way of representing a periodic function as a (possibly infinite) sum of sine and cosine functions. It is analogous to a Taylor series, which represents functions as possibly infinite sums of monomial terms.

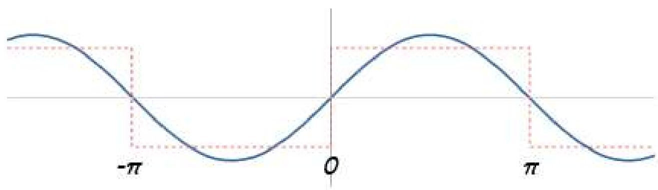
A Fourier series is an expansion of a periodic function $f(x)$ in terms of an infinite sum of sines and cosines. Fourier series make use of the orthogonality relationships of the sine and cosine functions.

Sine and cosine waves can make other functions!
Here two different sine waves add together to make a new wave:

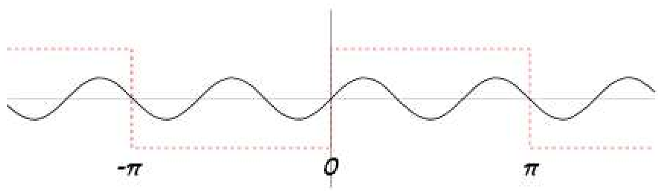


Try "sin(x)+sin(2x)" at the function grapher.

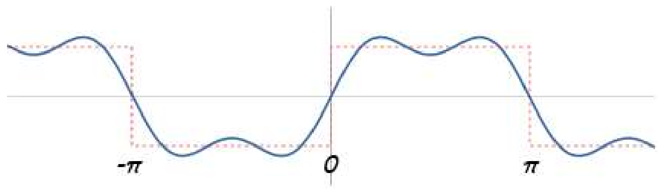
Start with **sin(x)**:



Then take **sin(3x)/3**:



And add it to make **sin(x)+sin(3x)/3**:



harmonic fourier series:

The computation and study of Fourier series is known as harmonic analysis and is extremely useful as a way to break up an *arbitrary* periodic function into a set of simple terms that can be plugged in, solved individually, and then recombined to obtain the solution to the original problem or an approximation to it to whatever accuracy is desired or practical. Examples of successive approximations to common functions using Fourier series are illustrated above.

In music, if a note has frequency f, integer multiples of that frequency, $2f, 3f, 4f$ and so on, are known as *harmonics*. As a result, the mathematical study of overlapping waves is called harmonic analysis. Harmonic analysis is a diverse field including such branches as Fourier series, isospectral manifolds (hearing the shape of a drum), and topological groups. Signal processing, medical imaging, and quantum mechanics are three of the fields that use harmonic analysis extensively.

Question: 4(b) 2016 What do you mean by bandwidth of a signal? Hwo can you extract a particular band of a signal

Bandwidth of a signal: Bandwidth is the range of frequencies -- the difference between the highest-frequency signal component and the lowest-frequency signal component -- an electronic signal uses on a given transmission medium. Like the frequency of a signal, bandwidth is measured in hertz (cycles per second). This is the original meaning of bandwidth, although it is now used primarily in discussions about cellular networks and the spectrum of frequencies that operators license from various governments for use in mobile services.

Extract a particular band of a signal:

which to me it means that given some periodic function:

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n cos(nx) + \sum_{n=1}^{\infty} b_n sin(nx)$$

we just need to extract the frequencies that are the largest and subtract them. For the sake of an example consider:

$f(x)=cos(2\pi x)+cos(4\pi x)+sin(3\pi x)+sin(7\pi x)$

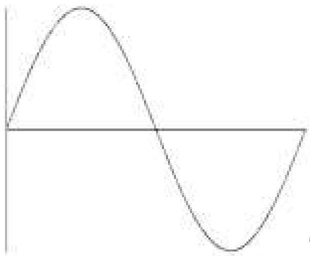
he frequency of a sin/cos is $f=n/2\pi$, thus, components of f with the largest and smallest n give the frequencies we want. So the band width B of f is:

$$B = f_{max} - f_{min}$$

$$B = \frac{n_{max}}{2\pi} - \frac{n_{min}}{2\pi} = \frac{7}{2\pi} - \frac{2}{2\pi}$$

Fourier series for continuous time signal

Any periodic continuous signal is given by X(t)=x(t+T) where T represents the period of signal , the duration of one complete cycle of x(t)



T

Any periodic signal x(t)can be represented as infinite series called Fourier series

Fourier series has following forms of representation

- Complex exponential Fourier series
- Sine-cosine form or trigonometric Fourier series

Complex exponential Fourier series representing a periodic signal x(t) is given by

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\Omega_o t}$$

Where $\Omega_o = 2\pi f_o = \frac{2\pi}{T}$ is called the fundamental angular frequency. f_o is the fundamental cycle frequency and c_k is the complex Fourier coefficient.

The complex set of c_k Is called the frequency spectrum of the signal

$$c_k = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\, e^{-jk\Omega_o t}\, dt$$

K is the number of complex signal in the composite signal spectrum.

We have

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\Omega_o t}$$

now

$$\int_{to}^{to+Tp} x(t)\, e^{-jl\Omega_o t}$$

© $\int_{to}^{to+Tp} x(t)\, e^{-j2\pi f_o l t}\, dt$

$$[\ \Omega_o = 2\pi f_o = \frac{2\pi}{T}\ ]$$

$$= \int_{to}^{to+Tp} e^{-j2\pi f_o l t}\ (\sum_{k=-\infty}^{\infty} c_k e^{j2\pi f_o k t})dt$$

$$= \sum_{k=-\infty}^{\infty} c_k\ \int_{to}^{to+Tp} e^{j2\pi(k-l)f_o t}\, dt$$

© $= \sum_{k=-\infty}^{\infty} c_k\ [\frac{e^{-j2\pi(k-l)f_o t}}{j2\pi(k-l)f_o t}]_{to}^{to+Tp}$

$$= \left| \begin{array}{l} 0, if\ k \neq l \\ c_l T_p\ , if\ k = l \end{array} \right.$$

© $\therefore\ c_l = \frac{1}{T_p}\int_{to}^{to+Tp} x(t)\, e^{-j2\pi f_o l t}\, dt$

© $\therefore\ c_k = \int_{Tp} x(t)\, e^{-j2\pi f_o k t}\, dt$

Find average power or power density of a continuous time periodic signal

Parsevals relation for continuous time periodic signal x(t) is given by

$$\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2\, dt = \sum_{k=-\infty}^{\infty} |c_k|^2$$

where, c_k is the complex Fourier coefficient of x(t) and T is the period of the signal.

Proof

$$\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2\, dt$$

=

$$= \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \quad * \quad x^*(t)\ \ dt$$

$$\left| \begin{array}{l} z = x + iy \\ z^* = -x - iy \end{array} \right. * conjugate$$

$$= \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\sum_{k=-\infty}^{\infty} c_k{}^{*} e^{-jk\Omega_o t} dt$$

$$= \sum_{k=-\infty}^{\infty} c_k{}^{*}\left[\frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)e^{-j2\pi k f_o t}\, dt\right]$$

$$= \sum_{k=-\infty}^{\infty} c_k{}^{*} * c_k$$

$$= \sum_{k=-\infty}^{\infty} |c_k|^2$$

Question: 4(c)-2016 Define fourier and inverse fourier fourier integral? Why do you need fourier transform of a signal? Write down the convolution property of fourier transform

Define fourier and inverse fourier fourier integral:

Fourier series for continuous time aperiodic signal

Let x(t) be continuous time aperiodic signal

Where x(t)=$\lim_{Tp\to\infty} x_p(t)$; $x_p(t)\ is\ a\ periodic\ signal$

now

$$x_p(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_o t}$$

$$c_k = \frac{1}{T_p}\int_{T_p} x_p(t)\, e^{-j2\pi k F_o t}\, dt$$

$$c_k = \frac{1}{T_p}\int_{-\frac{T_p}{2}/\infty}^{\frac{T_p}{2}/\infty} x_p(t)\, e^{-j2\pi k F_o t}\, dt$$

$$= \frac{1}{T_p} X(F)$$

$$\left| X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi k F_o t} dt \right.$$

$$c_k T_p = X(F)$$

$$c_k = \frac{1}{T_p} X\left(\frac{k}{T_p}\right) \ldots\ldots\ldots..(i)$$

$$x_p(t) = \sum_{k=-\infty}^{\infty} c_k e^{j2\pi k F_o t} \ldots\ldots\ldots\ldots(ii)$$

$$x_p(t) = \frac{1}{T_p}\sum_{k=-\infty}^{\infty} X\left(\frac{k}{T_p}\right) e^{j2\pi k F_o t}$$

$$= \sum_{k=-\infty}^{\infty} X(k\Delta F) e^{j2\pi k \Delta F t}\, \Delta F$$

$$let\ \Delta F = \frac{1}{T_p}, T_p \to \infty$$

$$\lim_{T_p \to \infty} x_p(t) = \lim_{\Delta F \to 0} \sum_{k=-\infty}^{\infty} X(k\Delta F) e^{j2\pi k\Delta F t}\, \Delta F$$

$$x(t) = \int_{-\infty}^{\infty} X(F)\, e^{j2\pi k\Delta F t} df \ldots\ldots\ldots..(i)$$

$$k\Delta F = F, if\ T_p \to \infty\ \ then\ \Delta F = 0$$

$$x(F) = \int_{-\infty}^{\infty} X(F)\, e^{-j2\pi F t} dt \ldots\ldots\ldots..(ii)$$

These two equations (i) and (ii) are called Fourier pair.

Why do you need fourier transform of a signal?

Fourier transforms are important in signal processing. The simplest, hand waving answer one can provide is that it is an *extremely* powerful mathematical tool that allows you to view your signals in a different domain, inside which several difficult problems become very simple to analyze.

Its ubiquity in nearly every field of engineering and physical sciences, all for different reasons, makes it all the more harder to narrow down a reason. I hope that looking at some of its properties which led to its widespread adoption along with some practical examples and a dash of history might help one to understand its importance.

Question: 2016 Write down the convolution property of fourier transform:

Multiplication and Convolution Properties

If $x(t) \stackrel{\text{F.T}}{\longleftrightarrow} X(\omega)$

& $y(t) \stackrel{\text{F.T}}{\longleftrightarrow} Y(\omega)$

Then multiplication property states that

$x(t).\, y(t) \stackrel{\text{F.T}}{\longleftrightarrow} X(\omega) * Y(\omega)$

and convolution property states that

$x(t) * y(t) \stackrel{\text{F.T}}{\longleftrightarrow} \frac{1}{2\pi} X(\omega).\, Y(\omega)$

Energy of Fourier series for continuous time aperiodic signal

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt --- time$$

$$= \int_{-\infty}^{\infty} |x(F)|^2 \, dF --- frequency$$

Fourier series for discrete time periodic signal

Frequency range of CT signal is $-\infty \; to \; \infty$

Frequency range of DT signal is -R to R or 0 to 2R.

Frequency component of DT is $\frac{2R}{N}$

For a periodic signal x(n+N)=x(n) N is the period of the signal

Fourier series for discrete time periodic signal is defined as

$$x(n) = \sum_{k=0}^{N-1} c_k e^{j2\pi k n/N}$$

$$= \sum_{n=0}^{N-1} c_k e^{j2\pi k n/N} \qquad = \Big|_0^N$$

$$k = 0, \pm N, \pm 2N..$$

Now

$$\sum_{n=0}^{N-1} c_k e^{j2\pi k n/N} = \sum_{n=0}^{N-1} \sum_{k=0}^{N-1} c_k e^{j2\pi(k-l)\, n/N}$$

$$= \sum_{k=0}^{N-1} c_k \quad \Big|_0^N$$

$$= N c_k$$

$$\therefore c_k = {}^{1}/_{N} \sum_{n=0}^{N-1} x(n) e^{-j2\pi k n/N}$$

Fourier transformation for discrete time aperiodic signal

Let us consider $x_{N_o}(n)$ is a periodic sequence with fundamental period N_o,and is formed by repeating a non-periodic signal x(n).

Discrete fourier series of $x_{N_o}(n)$ is

$$x_{N_o}(n) = \sum_{k=N_o} c_k e^{j2\pi k\, n/N_o}$$

And spectrum coefficient

$$c_k = {}^{1}\!/_{N_o} \sum_{n=N_o} x(n)e^{-j2\pi k^{n}/N_o}$$

But $x_{N_o}(n) = x(n)\ \ for - N_1 \le n \le N_1$
So we can write as

$$c_k = {}^{1}\!/_{N_o} \sum_{n=-N_1}^{N_1} x(n)e^{-j2\pi k^{n}/N_o}$$

Since x(n)=0 outside the interval $-N_1 \le n \le N_1$,

$$and\ \omega_o = \frac{2\pi}{N_o}$$

So it can be written as

$$c_k = {}^{1}\!/_{N_o} \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega_o kn}$$

Putting the value of c_k in the F-series

$$x_{N_o}(n) = \sum_{k=N_o} \left[{}^{1}\!/_{N_o} \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega_o kn} \right] e^{j2\pi k\, {}^{n}/N_o}$$

Since

$\omega_o = \frac{2\pi}{N_o}$ or $N_o = \frac{2\pi}{\omega_o}$

$$x_{N_o}(n) = \sum_{k=N_o} \left[{}^{\omega_o}\!/_{2\pi} \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega_o kn} \right] e^{j2\pi k\, {}^{n}/N_o}$$

Since
$\omega_o = \frac{2\pi}{N_o}$,and if we take limit as $N_o\ approaches\ to\ \infty$, $x_{N_o}(n)$ becomes non-periodic, and $\omega_o\ becomes\ small\ quantity$
The summation becomes an integral and the total integration will always have width of 2 π.
X(n)=$\int_{-\pi}^{\pi} \left[{}^{d\omega}\!/_{2\pi} \sum_{n=-\infty}^{\infty}\ x(n)e^{-j\omega n} \right] e^{j\omega n}$

Let us define , $X(e^{\omega}) = \sum_{n=-\infty}^{\infty}\ x(n)e^{-j\omega n}$…….(i)
Then,

X(n)=$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{\omega})e^{j\omega n}\ \ d\omega$………(ii)
These two equations (i) and (ii) are called Fourier pair.

Example 4.2.4

Determine the Fourier transform and the energy density spectrum of the sequence

$$x(n) = \begin{cases} A, & 0 \leq n \leq L-1 \\ 0, & \text{otherwise} \end{cases} \quad (4.2.48)$$

which is illustrated in Fig. 4.16.

Solution Before computing the Fourier transform, we observe that

$$\sum_{n=-\infty}^{\infty} |x(n)| = \sum_{n=0}^{L-1} |A| = L|A| < \infty$$

Hence $x(n)$ is absolutely summable and its Fourier transform exists. Furthermore, we note that $x(n)$ is a finite-energy signal with $E_x = |A|^2 L$.

The Fourier transform of this signal is

$$X(\omega) = \sum^{L-1} Ae^{-j\omega n}$$

$$= A \frac{}{1-e^{-j\omega}}$$

$$= Ae^{-j(\omega/2)(L-1)} \frac{\sin(\omega L/2)}{\sin(\omega/2)}$$

For $\omega = 0$ the transform in (4.2.49) yields $X(0) = AL$, which by setting $\omega = 0$ in the defining equation for $X(\omega)$, or by using (4.2.49) to resolve the indeterminate form when $\omega = 0$.

The magnitude and phase spectra of $x(n)$ are

$$|X(\omega)| = \begin{cases} |A|L, & \omega = 0 \\ |A|\left|\dfrac{\sin(\omega L/2)}{\sin(\omega/2)}\right|, & \text{otherwise} \end{cases}$$

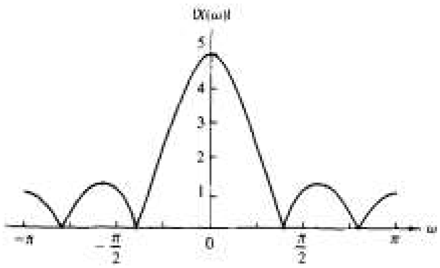
and

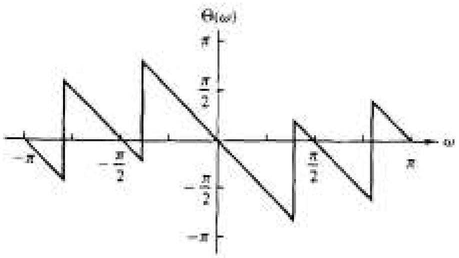
$$\measuredangle X(\omega) = \measuredangle A - \frac{\omega}{2}(L-1) + \measuredangle \frac{\sin(\omega L/2)}{\sin(\omega/2)}$$

where we should remember that the phase of a real quantity is ze positive and π if it is negative.

The spectra $|X(\omega)|$ and $\measuredangle X(\omega)$ are shown in Fig. 4.17 for L = 5. The energy density spectrum is simply the square of the (4.2.50).

There is an interesting relationship that exists between the of the constant amplitude pulse in Example 4.2.4 and the pe





Question: 5(a) 2017 consider the signal

$$x(n) = \begin{cases} A, & -M, \leq n \leq M \\ 0 & Elsewhere \end{cases}$$

i. **Determine the fourier transform of x(n) i.e X(w)**
ii. **Determine and plot the magnitude and phase of X(w)**

Solution:

Solution Clearly, $x(-n) = x(n)$. Thus $x(n)$ is a real and even signal. From (4.3.21) we obtain

$$X(\omega) = X_R(\omega) = A\left(1 + 2\sum_{n=1}^{M} \cos \omega n\right)$$

If we use the identity given in Problem 4.13, we obtain the simpler form

$$X(\omega) = A\frac{\sin(M + \frac{1}{2})\omega}{\sin(\omega/2)}$$

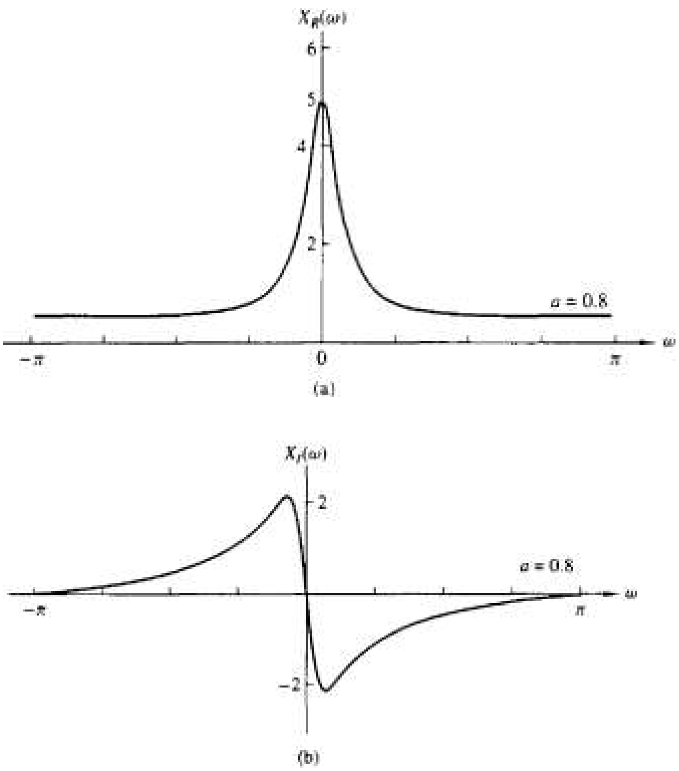
SInce ***X(w)*** **IS real.** the **magnitude and phase spectra are given by**

$$|X(\omega)| = \left|A\frac{\sin(M + \frac{1}{2})\omega}{\sin(\omega/2)}\right|$$

and

$$\measuredangle X(\omega) = \begin{cases} 0, & \text{if } X(\omega) > 0 \\ \pi, & \text{if } X(\omega) < 0 \end{cases}$$

Figure 4.32 shows the graphs for X(w).



Question: 5(b) 2017 Given $x_1(n) = x_2(n) = \{1, 1, 1\}$ and x(n)= $x_1(n) + x_2(n)$

a. **Determine X(w), the fourier transform of x(n)**
b. **By using X(w) determine x(n)**
c. **For n=0, find x(0)**

Fourier transform:

X(n)=[2,2,2]

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n}$$

$$X(\omega) = \sum_{n=0}^{2} \{2,2,2\}e^{-j\omega n}$$

$X(\omega)$ =2*$e^{-j\omega*0}$+2*$e^{-j\omega*1}$+2*$e^{-j\omega*2}$

=2+2*$e^{-j\omega}$+2*$e^{-j\omega*2}$

=2+2(cos(w)+jsin(w))+2(cos(2w)+jsin(2w))

b. Inverse Fourier transform

$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)e^{j\omega n}d\omega$

$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} \{2 + 2 * e^{-j\omega} + 2 * e^{-j\omega*2}\} e^{j\omega n}d\omega$$

a.). For n=0, find x(0)

$x(n) = \frac{1}{2\pi}\int_{-w_c}^{\omega_c} . e^{j\omega n}d\omega = \frac{sin\omega_c}{\pi n}$ $n \neq 0$

For n=0 we have

$x(0) = \frac{1}{2\pi}\int^{\omega_c} -\omega_c d\omega = \frac{\omega_c}{\pi}$

Hence

$$x(n) = \begin{cases} \frac{\omega_c}{\pi}, & n = 0 \\ \frac{\omega_c}{\pi}\frac{\sin \omega_c n}{\omega_c n}, & n \neq 0 \end{cases} \quad (4.2.36)$$

This transform pair is illustrated in Fig. 4.13.

Sometimes, the sequence $\{x(n)\}$ in (4.2.36) is expressed as

$$x(n) = \frac{\sin \omega_c n}{\pi n} \qquad -\infty < n < \infty \quad (4.2.37)$$

with the understanding that at $n = 0$, $x(n) = \omega_c/\pi$. We should emphasize, however, that $(\sin \omega_c n)/\pi n$ is not a continuous function, and hence L'Hospital's rule cannot be used to determine $x(0)$.

Now let us consider the determination of the Fourier transform of the sequence given by (4.2.37). The sequence $\{x(n)\}$ is not absolutely summable. Hence the infinite series

$$\sum_{n=-\infty}^{\infty} x(n)e^{-j\omega n} = \sum_{n=-\infty}^{\infty} \frac{\sin \omega_c n}{\pi n} e^{-j\omega n} \quad (4.2.38)$$

does not converge uniformly for all ω. However, the sequence $\{x(n)\}$ has a finite energy $E_x = \omega_c/\pi$ as will be shown in Section 4.3. Hence the sum in (4.2.38) is guaranteed to converge to the $X(\omega)$ given by (4.2.35) in the mean-square sense.

To elaborate on this point, let us consider the finite sum

$$X_N(\omega) = \sum_{n=-N}^{N} \frac{\sin \omega_c n}{\pi n} e^{-j\omega n} \quad (4.2.39)$$

Question: 6(a) 2017 :Determine the magnitude and phase of *H(w)* for the three-point moving average (MA) system

$$y(n) = \tfrac{1}{3}[x(n + 1) + x(n) + x(n - 1)]$$

and plot these two functions for $0 \leq \omega \leq \pi$.

Solution Since

$$h(n) = \{\tfrac{1}{3}, \underset{\uparrow}{\tfrac{1}{3}}, \tfrac{1}{3}\}$$

it **follows** that

$$H(\omega) = \tfrac{1}{3}(e^{j\omega} + 1 + e^{-j\omega}) = \tfrac{1}{3}(1 + 2\cos\omega)$$

Hence

$$|H(\omega)| = \tfrac{1}{3}|1 + 2\cos\omega| \qquad \textbf{(4.4.16)}$$

$$\Theta(\omega) = \begin{cases} 0, & 0 \le \omega \le 2\pi/3 \\ \pi, & 2\pi/3 \le \omega < \pi \end{cases}$$

Figure 4.37 illustrates the graphs of the magnitude and phase of $H(\omega)$. As indicated previously, $|H(\omega)|$ is an even function of frequency and $\Theta(\omega)$ is an odd function of

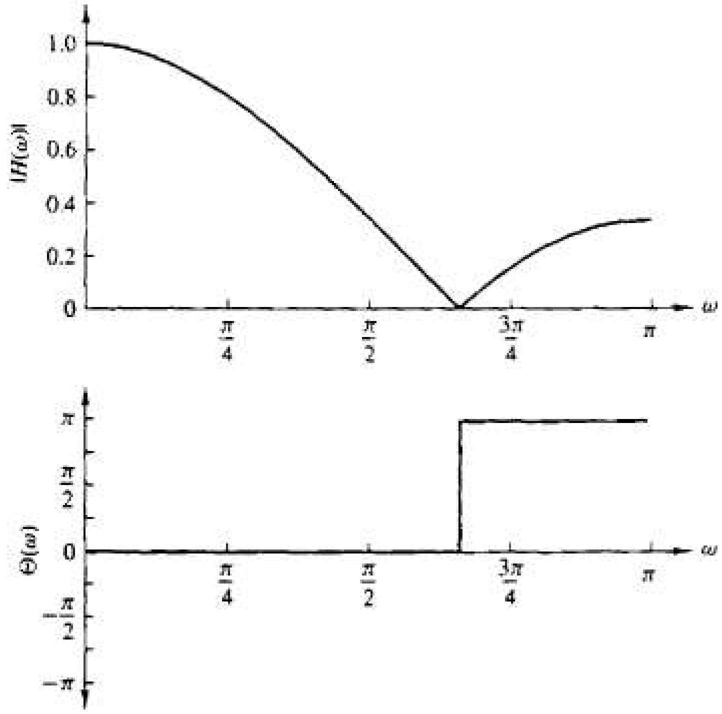
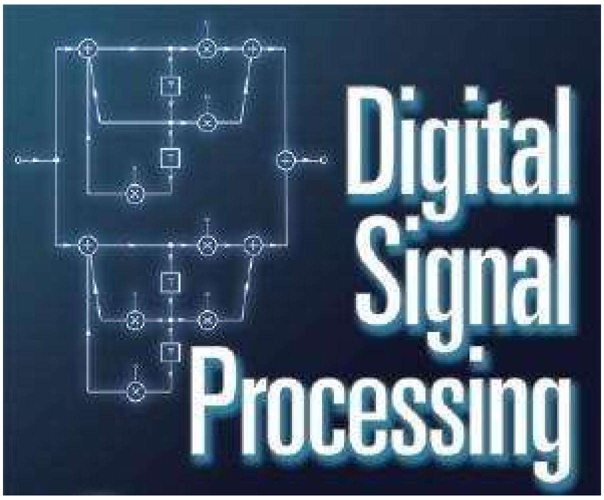


Figure 4.37 Magnitude and phase responses for the MA system in Example 4.4.2.



Important Question

1. **Question: 5(a) 2015 Describe DFT and IDFT. Why it is called DFT pair.**

Question 6(b) 2017: Given $x(n) = [2\ 4 - 1\ 6]n = 4(0, 1, 2, 3)$

   a. **Determine the X[k] for k=0,1,2,3**
   b. **Plot x(n) and $|x(k)|$**
      a. X(k)=$X\left(\frac{2\pi k}{N}\right) = \sum_{n=0}^{L-1} x(n)e^{-2\pi kn/N}$

$$\mathrm{X(k)} = \sum_{n=0}^{N-1} x(n)e^{-2\pi kn/N}$$

answer same as $x(\mathrm{m}) = \sum_{n=0}^{N-1} x(n)e^{-2\pi mn/N}$

here m=0,1,2,3 n=0,1,2,3 N=4

b. Plot x(n)

2. What are software RAID and Hardware RAID? How can you measure the performance of storage disk? **2+4 Marks CSE-2012**
3. Discuss shared mode lock, Exclusive Mode Lock and Compatible Lock. **6 Marks CSE-2012 CSE-2011 CSE-2007 CSE-2002**
4. What are main reasons of Starvation? How can you avoid starvation? **4 Marks CSE-2012 CSE-2007**
5. What are the different Techniques for Crash Recovery? Discuss the shadow paging technique of crash recovery. **7 Marks CSE-2012 CSE-2008 CSE-2005 CSE-2003 CSE-2000 CSE-2000 (OLD)**
6. Discuss Deadlock Detection Technique in database system. **3 Marks CSE-2012 CSE-2008 CSE-2003 CSE-2000 CSE-2000 (OLD)**
7. What do you mean by recovery system? Mention different types of log records. **4 Marks CSE-2012 CSE-2007**

Question: 5(a) 2015 Describe DFT and IDFT. Why it is called DFT pair.
DFT:

- The discrete Fourier transform (DFT) is one of the two most common, and powerful, procedures encountered in the field of digital signal processing. (Digital filtering is the other.)
- The DFT enables us to analyze, manipulate, and synthesize signals in ways not possible with continuous (analog) signal processing.
- The DFT's origin, of course, is the continuous Fourier transform X(f) defined as $X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt$
- where x(t) is some continuous time-domain signal.[]
- With the advent of the digital computer, the efforts of early digital processing pioneers led to the development of the DFT defined as the discrete frequency-domain sequence X(m), where

From Euler's relationship e–jø = cos(ø) –jsin(ø), Eq. (3-2) is equivalent to

$$X(m) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nm/N} .$$

DFT equation (rectangular form): →

$$X(m) = \sum_{n=0}^{N-1} x(n)[\cos(2\pi nm/N) - j\sin(2\pi nm/N)] .$$

We have separated the complex exponential of Eq. (3-2) into its real and imaginary components where

X(m)	=	the mth DFT output component, i.e., X(0), X(1), X(2), X(3), etc.,
m	=	the index of the DFT output in the frequency domain,
m	=	0, 1, 2, 3, . . ., N–1,
x(n)	=	the sequence of input samples, x(0), x(1), x(2), x(3), etc.,
n	=	the time-domain index of the input samples, n = 0, 1, 2, 3, . . ., N–1,
j	=	,
N	=	the number of samples of the input sequence and the number of frequency points in the DFT output.

Writing out all the terms for the first DFT output term corresponding to m = 0,

$$\begin{aligned}X(0) = x(0)\cos(2\pi\cdot 0\cdot 0/4) - jx(0)\sin(2\pi\cdot 0\cdot 0/4)\\ + x(1)\cos(2\pi\cdot 1\cdot 0/4) - jx(1)\sin(2\pi\cdot 1\cdot 0/4)\\ + x(2)\cos(2\pi\cdot 2\cdot 0/4) - jx(2)\sin(2\pi\cdot 2\cdot 0/4)\\ + x(3)\cos(2\pi\cdot 3\cdot 0/4) - jx(3)\sin(2\pi\cdot 3\cdot 0/4).\end{aligned}$$

For the second DFT output term corresponding to m = 1, Eq. (3-4a) becomes

$$\begin{aligned}X(1) = x(0)\cos(2\pi\cdot 0\cdot 1/4) - jx(0)\sin(2\pi\cdot 0\cdot 1/4)\\ + x(1)\cos(2\pi\cdot 1\cdot 1/4) - jx(1)\sin(2\pi\cdot 1\cdot 1/4)\\ + x(2)\cos(2\pi\cdot 2\cdot 1/4) - jx(2)\sin(2\pi\cdot 2\cdot 1/4)\\ + x(3)\cos(2\pi\cdot 3\cdot 1/4) - jx(3)\sin(2\pi\cdot 3\cdot 1/4).\end{aligned}$$

$$X(2) = x(0)\cos(2\pi\cdot 0\cdot 2/4) - jx(0)\sin(2\pi\cdot 0\cdot 2/4)$$
$$+ x(1)\cos(2\pi\cdot 1\cdot 2/4) - jx(1)\sin(2\pi\cdot 1\cdot 2/4)$$
$$+ x(2)\cos(2\pi v2\cdot 2/4) - jx(2)\sin(2\pi\cdot 2\cdot 2/4)$$
$$+ x(3)\cos(2\pi\cdot 3\cdot 2/4) - jx(3)\sin(2\pi\cdot 3\cdot 2/4).$$

$$X(3) = x(0)\cos(2\pi\cdot 0\cdot 3/4) - jx(0)\sin(2\pi\cdot 0\cdot 3/4)$$
$$+ x(1)\cos(2\pi\cdot 1\cdot 3/4) - jx(1)\sin(2\pi\cdot 1\cdot 3/4)$$
$$+ x(2)\cos(2\pi\cdot 2\cdot 3/4) - jx(2)\sin(2\pi\cdot 2\cdot 3/4)$$
$$+ x(3)\cos(2\pi\cdot 3\cdot 3/4) - jx(3)\sin(2\pi\cdot 3\cdot 3/4).$$

$X(m) = X_{real}(m) + jX_{imag}(m) = X_{mag}(m)$ at an angle of $X_ø(m)$,

the magnitude of X(m) is

$$X_{mag}(m) = |X(m)| = \sqrt{X_{real}(m)^2 + X_{imag}(m)^2}.$$

By definition, the phase angle of X(m), Xø(m), is

$$X_ø(m) = \tan^{-1}\left(\frac{X_{imag}(m)}{X_{real}(m)}\right).$$

The power of X(m), referred to as the power spectrum, is the magnitude squared where

$$X_{PS}(m) = X_{mag}(m)^2 = X_{real}(m)^2 + X_{imag}(m)^2.$$

DFT Example

Let's say we want to sample and perform an 8-point DFT on a continuous input signal containing components at 1 kHz and 2 kHz, expressed as

$$x_{in}(t) = \sin(2\pi\cdot 1000\cdot t) + 0.5\sin(2\pi\cdot 2000\cdot t + 3\pi/4).$$

- To make our example input signal xin(t) a little more interesting, we have the 2-kHz term shifted in phase by 135° (3p/4 radians) relative to the 1-kHz sinewave.
- With a sample rate of fs, we sample the input every 1/fs = ts seconds.
- Because N = 8, we need 8 input sample values on which to perform the DFT.
- So the 8-element sequence x(n) is equal to xin(t) sampled at the nts instants in time so that

$$x(n) = x_{in}(nt_s) = \sin(2\pi\cdot 1000\cdot nt_s) + 0.5\sin(2\pi\cdot 2000\cdot nt_s + 3\pi/4).$$

If we choose to sample xin(t) at a rate of fs = 8000 samples/s from Eq. (3-5), our DFT results will indicate what signal amplitude exists in x(n) at the analysis frequencies of mfs/N, or 0 kHz, 1 kHz, 2 kHz, . . ., 7 kHz. With fs = 8000 samples/s, our eight x(n) samples are

$$x(0) = 0.3535, \quad x(1) = 0.3535,$$
$$x(2) = 0.6464, \quad x(3) = 1.0607,$$
$$x(4) = 0.3535, \quad x(5) = -1.0607,$$
$$x(6) = -1.3535, \quad x(7) = -0.3535 .$$

Now we're ready to apply Eq. (3-3) to determine the DFT of our x(n) input. We'll start with m = 1 because the m = 0 case leads to a special result that we'll discuss shortly. So, for m = 1, or the 1-kHz (mfs/N = 1·8000/8) DFT frequency term, Eq. (3-3) for this example becomes

DFT equation (exponential form): →
$$X(m) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nm/N}.$$

DFT equation (rectangular form): →
$$X(m) = \sum_{n=0}^{N-1} x(n)[\cos(2\pi nm/N) - j\sin(2\pi nm/N)].$$

m=1 $\mathbf{X(1)=\sum_{n=0}^{7} x(n)\cos\left(2\pi n.\frac{1}{8}\right) - jx(n)\sin\left(2\pi n.\frac{1}{8}\right)}$

$$
\begin{aligned}
X(1) = \; & 0.3535 \cdot 1.0 && -j(0.3535 \cdot 0.0) && \leftarrow \text{this is the } n = 0 \text{ term} \\
& + 0.3535 \cdot 0.707 && -j(0.3535 \cdot 0.707) && \leftarrow \text{this is the } n = 1 \text{ term} \\
& + 0.6464 \cdot 0.0 && -j(0.6464 \cdot 1.0) && \leftarrow \text{this is the } n = 2 \text{ term} \\
& + 1.0607 \cdot -0.707 && -j(1.0607 \cdot 0.707) && \ldots \\
& + 0.3535 \cdot -1.0 && -j(0.3535 \cdot 0.0) && \ldots \\
& - 1.0607 \cdot -0.707 && -j(-1.0607 \cdot -0.707) && \ldots \\
& - 1.3535 \cdot 0.0 && -j(-1.3535 \cdot -1.0) && \ldots \\
& - 0.3535 \cdot 0.707 && -j(-0.3535 \cdot -0.707) && \leftarrow \text{this is the } n = 7 \text{ term}
\end{aligned}
$$

$$
\begin{aligned}
= \; & 0.3535 && + j0.0 \\
& + 0.250 && - j0.250 \\
& + 0.0 && - j0.6464 \\
& - 0.750 && - j0.750 \\
& - 0.3535 && - j0.0 \\
& + 0.750 && - j0.750 \\
& + 0.0 && - j1.3535 \\
& - 0.250 && - j0.250
\end{aligned}
$$

$$= 0.0 - j4.0 = 4 \angle -90°.$$

So we now see that the input $x(n)$ contains a signal component at a frequency of 1 kHz. Using Eqs. (3-7), (3-8), and (3-9) for our $X(1)$ result, $X_{\text{mag}}(1) = 4$, $X_{\text{PS}}(1) = 16$, and $X(1)$'s phase angle relative to a 1 kHz cosine is $X_{ø}(1) = -90°$.

m=2 $\mathbf{X(2)=\sum_{n=0}^{7} x(n)\cos\left(2\pi n.\frac{2}{8}\right) - jx(n)\sin\left(2\pi n.\frac{2}{8}\right)}$

$$
\begin{aligned}
X(2) = \; & 0.3535 \cdot 1.0 && -j(0.3535 \cdot 0.0) \\
& + 0.3535 \cdot 0.0 && -j(0.3535 \cdot 1.0) \\
& + 0.6464 \cdot -1.0 && -j(0.6464 \cdot 0.0) \\
& + 1.0607 \cdot 0.0 && -j(1.0607 \cdot -1.0) \\
& + 0.3535 \cdot 1.0 && -j(0.3535 \cdot 0.0) \\
& -1.0607 \cdot 0.0 && -j(-1.0607 \cdot 1.0) \\
& -1.3535 \cdot -1.0 && -j(-1.3535 \cdot 0.0) \\
& -0.3535 \cdot 0.0 && -j(-0.3535 \cdot -1.0)
\end{aligned}
$$

$$
\begin{aligned}
= \; & 0.3535 && + j0.0 \\
& + 0.0 && - j0.3535 \\
& - 0.6464 && - j0.0 \\
& - 0.0 && + j1.0607 \\
& + 0.3535 && - j0.0 \\
& + 0.0 && + j1.0607 \\
& + 1.3535 && - j0.0 \\
& - 0.0 && - j0.3535
\end{aligned}
$$

$$= 1.414 + j1.414 = 2 \angle 45°.$$

Here our input $x(n)$ contains a signal at a frequency of 2 kHz whose relative amplitude is 2, and whose phase angle relative to a 2 kHz cosine is 45°.

m=3 $\mathbf{X(3)=\sum_{n=0}^{7} x(n)\cos\left(2\pi n.\frac{3}{8}\right) - jx(n)\sin\left(2\pi n.\frac{3}{8}\right)}$

$$
\begin{array}{ll}
X(3) = 0.3535 \cdot 1.0 & -j(0.3535 \cdot 0.0) \\
\quad + 0.3535 \cdot -0.707 & -j(0.3535 \cdot 0.707) \\
\quad + 0.6464 \cdot 0.0 & -j(0.6464 \cdot -1.0) \\
\quad + 1.0607 \cdot 0.707 & -j(1.0607 \cdot 0.707) \\
\quad + 0.3535 \cdot -1.0 & -j(0.3535 \cdot 0.0) \\
\quad - 1.0607 \cdot 0.707 & -j(-1.0607 \cdot -0.707) \\
\quad - 1.3535 \cdot 0.0 & -j(-1.3535 \cdot 1.0) \\
\quad - 0.3535 \cdot -0.707 & -j(-0.3535 \cdot -0.707)
\end{array}
$$

$$
\begin{array}{ll}
= 0.3535 & +j0.0 \\
\quad - 0.250 & -j0.250 \\
\quad + 0.0 & +j0.6464 \\
\quad + 0.750 & -j0.750 \\
\quad - 0.3535 & -j0.0 \\
\quad - 0.750 & -j0.750 \\
\quad + 0.0 & +j1.3535 \\
\quad + 0.250 & -j0.250
\end{array}
$$

$$= 0.0 - j0.0 = 0 \angle 0^\circ.$$

m=4 $\mathbf{X(4)=\sum_{n=0}^{7} x(n)\cos\left(2\pi n.\frac{4}{8}\right) - jx(n)\,sin\left(2\pi n.\frac{4}{8}\right)}$

$$
\begin{array}{ll}
X(4) = 0.3535 \cdot 1.0 & -j(0.3535 \cdot 0.0) \\
\quad + 0.3535 \cdot -1.0 & -j(0.3535 \cdot 0.0) \\
\quad + 0.6464 \cdot 1.0 & -j(0.6464 \cdot 0.0) \\
\quad + 1.0607 \cdot -1.0 & -j(1.0607 \cdot 0.0) \\
\quad + 0.3535 \cdot 1.0 & -j(0.3535 \cdot 0.0) \\
\quad - 1.0607 \cdot -1.0 & -j(-1.0607 \cdot 0.0) \\
\quad - 1.3535 \cdot 1.0 & -j(-1.3535 \cdot 0.0) \\
\quad - 0.3535 \cdot -1.0 & -j(-0.3535 \cdot 0.0)
\end{array}
$$

$$
\begin{array}{ll}
= 0.3535 & -j0.0 \\
\quad - 0.3535 & -j0.0 \\
\quad + 0.6464 & -j0.0 \\
\quad - 1.0607 & -j0.0 \\
\quad + 0.3535 & -j0.0 \\
\quad + 1.0607 & -j0.0 \\
\quad - 1.3535 & -j0.0 \\
\quad + 0.3535 & -j0.0
\end{array}
$$

$$= 0.0 - j0.0 = 0 \angle 0^\circ.$$

m=5 $\mathbf{X(5)=\sum_{n=0}^{7} x(n)\cos\left(2\pi n.\frac{4}{8}\right) - jx(n)\,sin\left(2\pi n.\frac{4}{8}\right)}$

$$
\begin{array}{ll}
X(5) = 0.3535 \cdot 1.0 & -j(0.3535 \cdot 0.0) \\
\quad + 0.3535 \cdot -0.707 & -j(0.3535 \cdot -0.707) \\
\quad + 0.6464 \cdot 0.0 & -j(0.6464 \cdot 1.0) \\
\quad + 1.0607 \cdot 0.707 & -j(1.0607 \cdot -0.707) \\
\quad + 0.3535 \cdot -1.0 & -j(0.3535 \cdot 0.0) \\
\quad - 1.0607 \cdot 0.707 & -j(-1.0607 \cdot 0.707) \\
\quad - 1.3535 \cdot 0.0 & -j(-1.3535 \cdot -1.0) \\
\quad - 0.3535 \cdot -0.707 & -j(-0.3535 \cdot 0.707)
\end{array}
$$

m=6

$$
\begin{array}{ll}
= 0.3535 & -j0.0 \\
\quad - 0.250 & +j0.250 \\
\quad + 0.0 & -j0.6464 \\
\quad + 0.750 & +j0.750 \\
\quad - 0.3535 & -j0.0 \\
\quad - 0.750 & +j0.750 \\
\quad + 0.0 & -j1.3535 \\
\quad + 0.250 & +j0.250
\end{array}
$$

$$= 0.0 - j.0 = 0 \angle 0^\circ.$$

$\mathbf{X(6)=\sum_{n=0}^{7} x(n)\cos\left(2\pi n.\frac{4}{8}\right) - jx(n)\,sin\left(2\pi n.\frac{4}{8}\right)}$

$$
\begin{aligned}
X(6) = {} & 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
& + 0.3535 \cdot 0.0 && - j(0.3535 \cdot -1.0) \\
& + 0.6464 \cdot -1.0 && - j(0.6464 \cdot 0.0) \\
& + 1.0607 \cdot 0.0 && - j(1.0607 \cdot 1.0) \\
& + 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
& - 1.0607 \cdot 0.0 && - j(-1.0607 \cdot -1.0) \\
& - 1.3535 \cdot -1.0 && - j(-1.3535 \cdot 0.0) \\
& - 0.3535 \cdot 0.0 && - j(-0.3535 \cdot 1.0) \\
= {} & 0.3535 && - j0.0 \\
& + 0.0 && + j0.3535 \\
& - 0.6464 && - j0.0 \\
& + 0.0 && - j1.0607 \\
& + 0.3535 && - j0.0 \\
& + 0.0 && - j1.0607 \\
& + 1.3535 && - j0.0 \\
& + 0.0 && + j0.3535 \\
= {} & 1.414 - j1.414 = 2 \angle -45^\circ.
\end{aligned}
$$

m=6 $\mathbf{X(7)}=\sum_{n=0}^{7} x(n)\cos\left(2\pi n.\frac{4}{8}\right) - jx(n)\sin\left(2\pi n.\frac{4}{8}\right)$

$$
\begin{aligned}
X(7) = {} & 0.3535 \cdot 1.0 && - j(0.3535 \cdot 0.0) \\
& + 0.3535 \cdot 0.707 && - j(0.3535 \cdot -0.707) \\
& + 0.6464 \cdot 0.0 && - j(0.6464 \cdot -1.0) \\
& + 1.0607 \cdot -0.707 && - j(1.0607 \cdot -0.707) \\
& + 0.3535 \cdot -1.0 && - j(0.3535 \cdot 0.0) \\
& - 1.0607 \cdot -0.707 && - j(-1.0607 \cdot 0.707) \\
& - 1.3535 \cdot 0.0 && - j(-1.3535 \cdot 1.0) \\
& - 0.3535 \cdot 0.707 && - j(-0.3535 \cdot 0.707) \\
= {} & 0.3535 && + j0.0 \\
& + 0.250 && + j0.250 \\
& + 0.0 && + j0.6464 \\
& - 0.750 && + j0.750 \\
& - 0.3535 && - j0.0 \\
& + 0.750 && + j0.750 \\
& + 0.0 && + j1.3535 \\
& - 0.250 && + j0.250 \\
= {} & 0.0 + j4.0 = 4 \angle 90^\circ.
\end{aligned}
$$

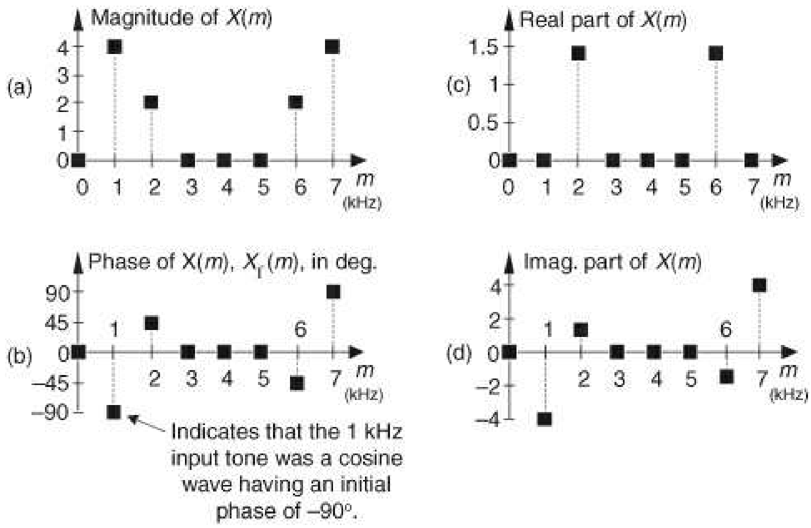


Figure 3-4 DFT results from Example 1: (a) magnitude of X(m); (b) phase of X(m); (c) real part of X(m); (d) imaginary part of X(m).

$$X(0) = \sum_{n=0}^{N-1} x(n)[\cos(0) - j\sin(0)].$$

$$X(0) = \sum_{n=0}^{N-1} x(n).$$

$$
\begin{aligned}
X(0) = & \ 0.3535 \cdot 1.0 & -j(0.3535 \cdot 0.0) \\
& + 0.3535 \cdot 1.0 & -j(0.3535 \cdot 0.0) \\
& + 0.6464 \cdot 1.0 & -j(0.6464 \cdot 0.0) \\
& + 1.0607 \cdot 1.0 & -j(1.0607 \cdot 0.0) \\
& + 0.3535 \cdot 1.0 & -j(0.3535 \cdot 0.0) \\
& - 1.0607 \cdot 1.0 & -j(-1.0607 \cdot 0.0) \\
& - 1.3535 \cdot 1.0 & -j(-1.3535 \cdot 0.0) \\
& - 0.3535 \cdot 1.0 & -j(-0.3535 \cdot 0.0)
\end{aligned}
$$

$$
\begin{aligned}
X(0) = & \ 0.3535 & -j0.0 \\
& + 0.3535 & -j0.0 \\
& + 0.6464 & -j0.0 \\
& + 1.0607 & -j0.0 \\
& + 0.3535 & -j0.0 \\
& - 1.0607 & -j0.0 \\
& - 1.3535 & -j0.0 \\
& - 0.3535 & -j0.0 \\
= & \ 0.0 - j0.0 = 0 \angle 0^\circ.
\end{aligned}
$$

Because cos(0) = 1, and sin(0) = 0

Question 6(b) 2017: Given $x(n) = [2\ 4 - 1\ 6] n = 4(0, 1, 2, 3)$

c. **Determine the X[k] for k=0,1,2,3**

d. **Plot x(n) and** $|x(k)|$

b. $X(k) = X\left(\frac{2\pi k}{N}\right) = \sum_{n=0}^{L-1} x(n) e^{-2\pi kn/N}$

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-2\pi kn/N}$$

answer same as $x(m) = \sum_{n=0}^{N-1} x(n) e^{-2\pi mn/N}$

here m=0,1,2,3 n=0,1,2,3 N=4

b. Plot x(n)

Question: 8(b) 2017 compute the DFT of a sequence x(n)={1,-1,1,-1}

Question 5(c)-2015 Find the DFT of the following signal

i. $x(n) = \delta(n)$

ii. $x(n) = a^n$

Consider the signal

$$x(n) = a^n u(n) \qquad 0 < a < 1$$

The spectrum of this signal is sampled at frequencies $\omega_k = 2\pi k/N$, $k = 0, 1, \ldots, N-1$. Determine the reconstructed spectra for $a = 0.8$ when $N = 5$ and $N = 50$.

Solution The Fourier transform of the sequence $x(n)$ is

$$X(\omega) = \sum_{n=0}^{\infty} a^n e^{-j\omega n} = \frac{1}{1 - ae^{-j\omega}}$$

Suppose that we sample $X(\omega)$ at N equidistant frequencies $\omega_k = 2\pi k/N$, $k = 0$, 1, . A' − 1. Thus we obtain the spectral samples

$$X(\omega_k) \equiv X\left(\frac{2\pi k}{N}\right) = \frac{1}{1 - ae^{-j2\pi k/N}} \qquad k = 0, 1, \ldots, N-1$$

i. $\boldsymbol{x(n) = \delta(n)}$

$\boldsymbol{X(\omega) = \sum_{n=0}^{\infty} e^{-j\omega n}}$

$\boldsymbol{= \frac{1}{1-e^{-j\omega}}=1}$

Question: 5(b) 2016 Explain the three basic properties of DFT.

Question: 2(b) 2013 discuss 2 properties of DFT.

DFT Properties:

1. **DFT Symmetry**

When the input sequence x(n) is real, as it will be for all of our examples, the complex DFT outputs for m = 1 to m = (N/2) – 1 are redundant with frequency output values for m > (N/2).
he mth DFT output will have the same magnitude as the (N−m)th DFT output.
The phase angle of the DFT's mth output is the negative of the phase angle of the (N−m)th DFT output. So the mth and (N−m)th outputs are related

$$X(m) = |X(m)| \text{ at } X_{\phi}(m) \text{ degrees}$$
$$= |X(N-m)| \text{ at } -X_{\phi}(N-m) \text{ degrees}$$

for 1 ≤ m ≤ (N/2)−1. We can state that when the DFT input sequence is real, X(m) is the complex conjugate of X(N−m), or

$$X(m) = X^*(N-m),^{\dagger}$$

$$X(N-m) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi n(N-m)/N} = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nN/N}e^{-j2\pi n(-m)/N}$$

$$= \sum_{n=0}^{N-1} x(n)e^{-j2\pi n}e^{j2\pi nm/N}.$$

$$X(N-m) = \sum_{n=0}^{N-1} x(n)e^{j2\pi nm/N}.$$

2. **DFT Linearity**

The DFT has a very important property known as linearity.
DFT of the sum of two signals is equal to the sum of the transforms of each signal;

if an input sequence x1(n) has a DFT X1(m) and another input sequence x2(n) has a DFT X2(m), then the DFT of the sum of these sequences

$x_{sum}(n) = x_1(n) + x_2(n)$ is $X_{sum}(m) = X(m) + X_2(m)$

$$X_{\text{sum}}(m) = \sum_{n=0}^{N-1} [x_1(n) + x_2(n)] e^{-j2\pi nm/N}$$

$$= \sum_{n=0}^{N-1} x_1(n) e^{-j2\pi nm/N} + \sum_{n=0}^{N-1} x_2(n) e^{-j2\pi nm/N} = X_1(m) + X_2(m).$$

3.Circular Convolution

Circular convolution. If

$$x_1(n) \xleftrightarrow[N]{\text{DFT}} X_1(k)$$

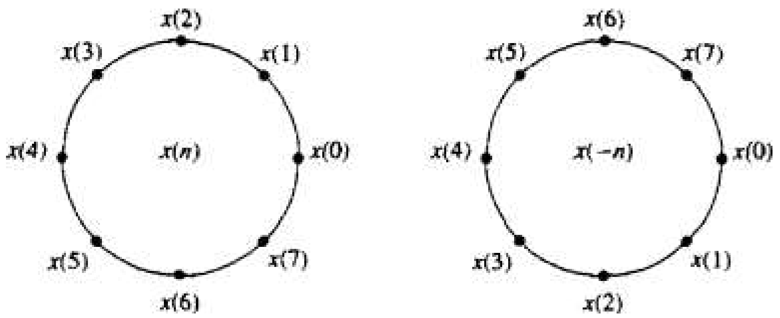
aod

$$x_2(n) \xleftrightarrow[N]{\text{DFT}} X_2(k)$$

then

$$x_1(n) \circledN x_2(n) \xleftrightarrow[N]{\text{DFT}} X_1(k)X_2(k) \qquad (5.2.41)$$

where $x_1(n) \circledN x_2(n)$ denotes the circular convolution of the sequence $x_1(n)$ and $x_2(n)$.



3. **Periodicity.**

If xin) and **X(k)** are an N-point DFT pair, then

x (n + N) = x (n) for all ***n*** (5.2.4)

X(k+N)=X(k) for all k (5.2.5)

These periodicities in **.x(n)** and X(k) follow immediately from formulas (5.2.1) and (5.2.2) for the DFT and IDFT. respectively.

We previously illustrated the periodicity property in the sequence ***x (n)*** for a given DFT. However, we had not previously viewed the DFT X(k) as a periodic sequence. In some applications it is advantageous to do this.

IDFT:

Inverse DFT

- ❖ DFT as transforming time-domain data into a frequency-domain representation.
- ❖ we can reverse this process and obtain the original time-domain signal by performing the IDFT on the *X*(*m*) frequency-domain values.
- ❖ The standard expressions for the IDFT are

$$x(n) = \frac{1}{N} \sum_{m=0}^{N-1} X(m) e^{j2\pi mn/N}$$

$$x(n) = \frac{1}{N}\sum_{m=0}^{N-1} X(m)[\cos(2\pi mn/N) + j\sin(2\pi mn/N)].$$

Why IDFT is called DFT pair:
The equation of DFT and IDFT is called DFT pair

Question: 8(c) 2017 compute the IDFT of a sequence x(n)={10,-2+2j,-2,-2,-2j}
Question:6(a) 2016 describe the relationship between the magnitude of a signal before and after DFT with an example
Question:6(b) 2016 what is leakage effect of DFT? Explain the reason behind the leakage problem with example

DFT Leakage

- DFTs are constrained to operate on a finite set of N input values,
- sampled at a sample rate of fs, to produce an N-point transform whose discrete outputs are associated with the individual analytical frequencies fanalysis(m) , with

$$f_{\text{analysis}}(m) = \frac{mf_s}{N}, \text{where } m = 0,1,2,\ldots,N-1.$$

- if the input has a signal component at some intermediate frequency between our analytical frequencies of mfs/N,
- say **1.5fs/N,** this input signal will show up to some degree in all of the N output analysis frequencies of our DFT! energy shows up in all of the DFT's output bins,.
- Engineers often refer to DFT samples as "bins." So when you see, or hear, the word bin it merely means a frequency-domain sample.) Let's understand the significance of this problem with another DFT example.

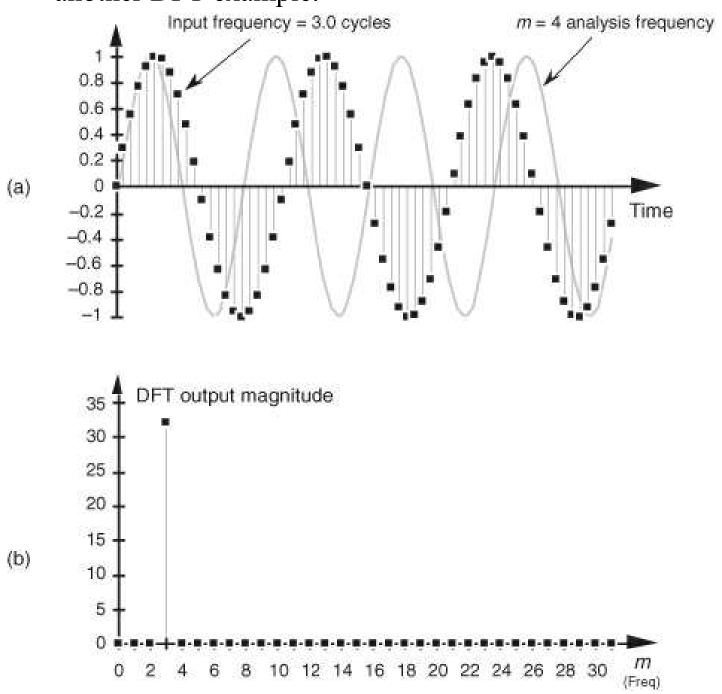
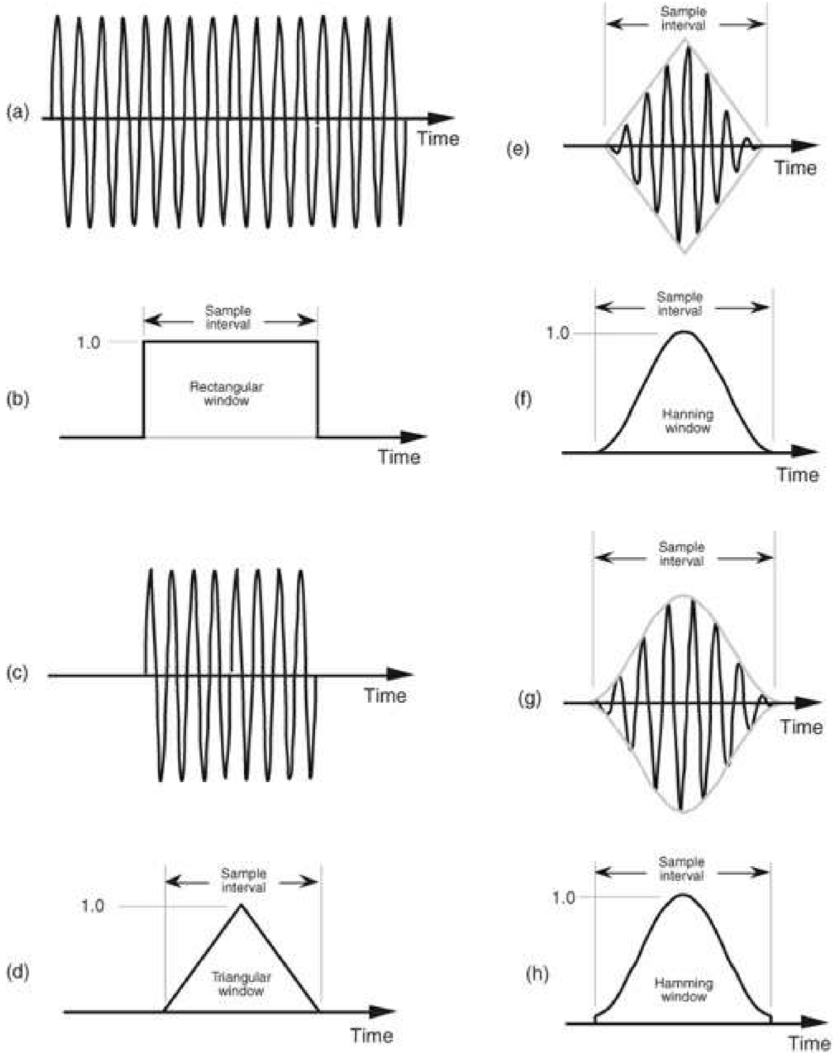


Figure 3-7 Sixty-four-point DFT: (a) input sequence of **thre**e cycles and the m = 4 analysis frequency sinusoid; (b) DFT output magnitude.

Question:6(c) 2016 Discuss the use of windows to solve the leakage effect. Show the responses of hanning and rectangular window I frequency domain(FD) and clearly describe the finding from spectrums

Question:7(c) 2013 discuss different types of window with equation and frequency response

Windows:



$$X_w(m) = \sum_{n=0}^{N-1} w(n) \cdot x(n) e^{-j2\pi nm/N}.$$

Rectangular window: (also called the uniform, or boxcar, window)

$$w(n) = 1, \text{ for } n = 0, 1, 2, \ldots, N-1.$$

Triangular window: (very similar to the Bartlett[3], and Parzen[4,5] windows)

$$w = \begin{cases} \dfrac{n}{N/2}, & \text{for } n = 0, 1, 2, \ldots, N/2 \\ 2 - \dfrac{n}{N/2}, & \text{for } n = N/2+1, N/2+2, \ldots, N-1. \end{cases}$$

Hanning window: (also called the raised cosine, Hann, or von Hann window)

$$w(n) = 0.5 - 0.5\cos\left(\frac{2\pi n}{N}\right),$$

$$\text{for } n = 0, 1, 2, \ldots, N-1.$$

Hamming window: $$w(n) = 0.54 - 0.46\cos\left(\frac{2\pi n}{N}\right),$$
for $n = 0, 1, 2, \ldots, N-1$.

Discrete Fourier Transform

- As the number of points in the DFT is increased to hundreds, or thousands, the amount of necessary number crunching becomes excessive.
- In1965 a paper was published by Cooley and Tukey describing a very efficient algorithm to implement the DFT[1].
- That algorithm is now known as the *fast Fourier transform* (FFT).†
- we'll show why the most popular FFT algorithm (called the *radix-2* FFT) is superior to the classical DFT algorithm,

Question: 6(c) 2017: Explain briefly when the overlap-add and overlap save methods used

Overlap-add method. In this method the size of the input data block is ***L*** points and the size of the Dms **and** IDFT is N = ***L*** + ***M*** - 1. To each data block append M - 1 zeros and compute the N-point DFI'. **Thus** the data blocks may be represented as

$$x_1(n) = \{x(0), x(1), \ldots, x(L-1), \underbrace{0, 0, \ldots, 0}_{M-1 \text{ zeros}}\} \qquad (5.3.13)$$

$$x_2(n) = \{x(L), x(L+1), \ldots, x(2L-1), \underbrace{0, 0, \ldots, 0}_{M-1 \text{ zeros}}\} \qquad (5.3.14)$$

$$x_3(n) = \{x(2L), \ldots, x(3L-1), \underbrace{0, 0, \ldots, 0}_{M-1 \text{ zeros}}\} \qquad (5.3.15)$$

and so on. The two N-point DFTs are multiplied together to form

$$Y_m(k) = H(k)X_m(k) \qquad k = 0, 1, \ldots, N-1 \qquad (5.3.16)$$

The IDFT yields data blocks of Iength N that are free of aliasing since the size of the DFTs and IDFT is N = L + A4 - 1 and the sequences are increased to N-points
by appending zeros to each block.

Overlap-save method. In this method the size of the input data blocks is ***N*** = L + ***M*** - 1 and the.size of the DFTs and IDFT are of length ***N.*** Each data block consists of the last ***M - 1*** data points of the previous data block followed by L new data points to form a data sequence of length N = ***L*** + M - 1. An N-point DFT is computed for each data block. The impulse response of the **FIR** filter is increased in length by appending L - 1 zeros and an N-point DFT of the sequence is computed once and stored. The multiplication of the two N-point DFTs ***{H(k))*** and ***{Xm(k)}*** for the **mtb** block of data yields

$$\hat{Y}_m(k) = H(k)X_m(k) \qquad k = 0, 1, \ldots, N-1 \qquad (5.3.7)$$

-

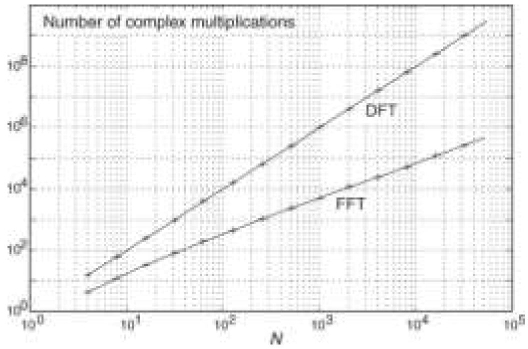
Relationship of the FFT to the DFT

- The radix-2 FFT algorithm is a very efficient process for performing DFTs under the constraint that the DFT size be an integral power of two.
- DFT Example 1 in Section 3.1 illustrated the number of redundant arithmetic operations necessary for a simple 8-point DFT.
- (For example, we ended up calculating the product of 1.0607 · 0.707 four separate times.) On the other hand, the radix-2 FFT eliminates these redundancies and greatly reduces the number of necessary arithmetic operations

For an 8 point DFT tells us that we'd have to perform N^2 or 64 complex multiplications.

The number of multiplications, for an N-point FFT is approximately $\frac{N}{2}.log_2 N$

When N = 512, for example, the DFT requires 114 times the number of complex multiplications than needed by the FFT. When N = 8192, the DFT must calculate 1260 complex multiplications for each complex multiplication in the FFT!



Question: 7(a) 2015 Explain Radix-2 FFT Algorithm

Derivation of the Radix-2 FFT Algorithm:

To see just exactly how the FFT evolved from the DFT, we return to the equation for an N-point DFT,

$$X(m) = \sum_{n=0}^{N-1} x(n)e^{-j2\pi nm/N}.$$

A straightforward derivation of the FFT proceeds with the separation of the input data sequence x(n) into two parts. When x(n) is segmented into its even and odd indexed elements, we can, then, break Eq. (4-11) into two parts a

Pulling the constant phase angle outside the second summation

$$X(m) = \sum_{n=0}^{(N/2)-1} x(2n)e^{-j2\pi(2n)m/N} + \sum_{n=0}^{(N/2)-1} x(2n+1)e^{-j2\pi(2n+1)m/N}.$$

$$X(m) = \sum_{n=0}^{(N/2)-1} x(2n)e^{-j2\pi(2n)m/N}$$

Well, here the equations become so long and drawn out that we'll use a popular notation to simplify things. We'll define to represent the complex phase-angle factor that is constant with N. So, Eq. (4-13) becomes

$$W_N = e^{-j2\pi/N}$$

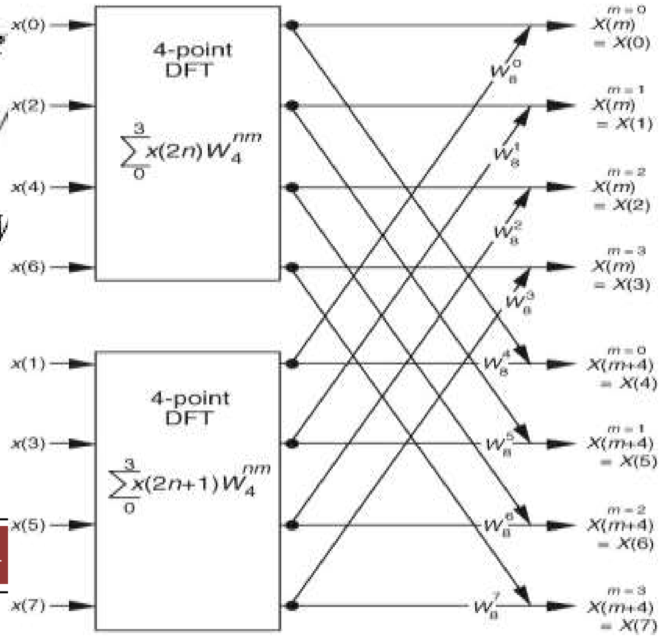
$$X(m) = \sum_{n=0}^{(N/2)-1} x(2n)W_N^{2nm} + W_N^m \sum_{n=0}^{(N/2)-1} x(2n+1)W_N^{2nm}.$$

Because

$$W_N^2 = e^{-j2\pi 2/(N)} = e$$

we can substitute $W_{N/2}$ for W

$$X(m) = \sum_{n=0}^{(N/2)-1} x(2n)W_{N/2}^{nm} + W$$



- ❖ Where m is in the range 0 to N/2−1. Index m has that reduced range because each of the two N/2-point
- ❖ DFTs on the right side of Eq. (4-15) are periodic in m with period N/2.

- So we now have two N/2 summations whose results can be combined to give us the first N/2 samples of an N-point DFT.
- We've reduced some of the necessary number crunching in Eq. (4-15) relative to
- Eq. (4-11) because the W terms in the two summations of Eq. (4-15) are identical.
- There's a further benefit in breaking the N-point DFT into two parts because the upper half of the DFT outputs is easy to calculate. Consider the X(m+N/2) output. If we plug m+N/2 in for m in Eq. (4-15), then

Figure 4-2 FFT implementation of an 8-point DFT using two 4-point DF

Because $-e^{-j2\pi m/N} = e^{-j2\pi(m+N/2)/N}$, the negative W twiddle factors before the second summation in Eq. (4-20') are implemented with positive W twiddle factors that follow the lower DFT in Figure 4-2.

If we simplify Eqs. (4-20) and (4-20') to the form

$$X(m) = A(m) + W_N^m B(m)$$

$$A(m) = \sum_{n=0}^{(N/2)-1} x(2n)W_{N/2}^{nm} = \sum_{n=0}^{(N/4)-1} x(4n)W_{N/2}^{2nm} + \sum_{n=0}^{(N/4)-1} x(4n+2)W_{N/2}^{(2n+1)m}.$$

Because $W_{N/2}^{2nm} = W_{N/4}^{nm}$ we can express $A(m)$ in the form of two $N/4$-point DFTs, as

$$A(m) = \sum_{n=0}^{(N/4)-1} x(4n)W_{N/4}^{nm} + W_{N/2}^{m} \sum_{n=0}^{(N/4)-1} x(4n+2)W_{N/4}^{nm}.$$

Notice the similarity between Eqs. (4-23) and (4-20). This capability to subdivide an N/2-point DFT into two N/4-point DFTs gives the FFT its capacity to greatly reduce the number of necessary multiplications to implement DFTs. (We're going to demonstrate this shortly.) Following the same steps we used to obtained A(m), we can show that Eq.(4-21)'s B(m) is

$$B(m) = \sum_{n=0}^{(N/4)-1} x(4n+1)W_{N/4}^{nm} + W_{N/2}^{m} \sum_{n=0}^{(N/4)-1} x(4n+3)W_{N/4}^{nm}.$$

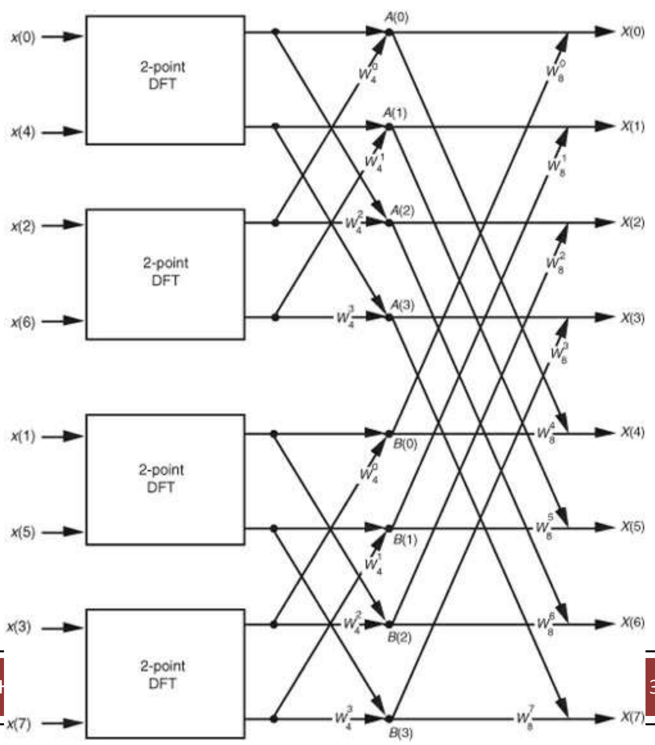


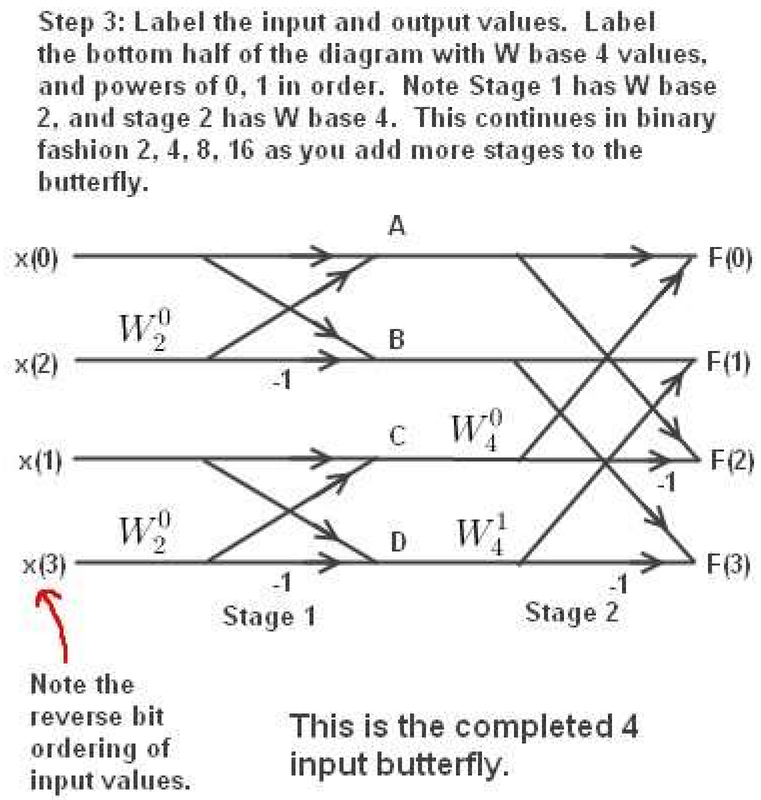
Figure 4-3 FFT implementation of an 8-point DFT as two 4-point DFTs and four 2-point DFTs

Question: 7(c) 2016 Draw the structure of FFT implementation for a 4-point DFT

Question: 7(d) 2015 construct a 4 input butterfly diagram of FFT

Answer:

Constructing A 4 Input Butterfly Diagram



The four output equations for the butterfly are derived below.

The equations derived from the 4 input butterfly

Stage 1:

$$A = x(0) + W_2^0 x(2)$$
$$B = x(0) - W_2^0 x(2)$$
$$C = x(1) + W_2^0 x(3)$$
$$D = x(1) - W_2^0 x(3)$$

Stage 2:

$$F(0) = A + W_4^0 C$$
$$F(1) = B + W_4^1 D$$
$$F(2) = A - W_4^0 C$$
$$F(3) = B - W_4^1 D$$

Substituting back in for A, B, C and D:

$$F(0) = x(0) + W_2^0 x(2) + W_4^0 (x(1) + W_2^0 x(3))$$
$$F(1) = x(0) - W_2^0 x(2) + W_4^1 (x(1) - W_2^0 x(3))$$
$$F(2) = x(0) + W_2^0 x(2) - W_4^0 (x(1) + W_2^0 x(3))$$
$$F(3) = x(0) - W_2^0 x(2) - W_4^1 (x(1) - W_2^0 x(3))$$

Question: 7(b) if N point =512, then how many computation are needed for DFT and FFT?
Also calculate the speed factor

DFT computation: N^2 =512*512=
FFT= $\frac{N}{2} log2(N)$ =256*8

Question: 7(a) 2015,2(a)2013Differentiate between DFT and FFT

FFT	DFT
FFT is abbreviated as Fast Fourier Transform.	DFT stands for Discrete Fourier Transform.
FFT is a much faster version of the DFT algorithm.	DFT is the discrete version of the Fourier Transform.
Various fast DFT computation techniques are collectively known as the FFT algorithm.	It is the algorithm that transforms the time domain signals to the frequency domain components.
It's an implementation of the DFT.	It establishes a relationship between the time domain and the frequency domain representation
Applications include integer and polynomial multiplication, filtering algorithms, computing isotopic distributions, calculating Fourier series coefficients, etc.	Applications of DFT include solving partial differential applications, detection of targets from radar echoes, correlation analysis, computing polynomial multiplication, spectral analysis, etc.

Question:7(a) 2016 what is zero stuffing technique. Explain its effect in signal analysis.

zeero padding allows one to use a longer FFT, which will produce a longer FFT result vector.
A longer FFT result has more frequency bins that are more closely spaced in frequency. But they will be essentially providing the same result as a high quality Sinc interpolation of a shorter non-zero-padded FFT of the original data.
This might result in a smoother looking spectrum when plotted without further interpolation.
Although this interpolation won't help with resolving or the resolution of and/or between adjacent or nearby frequencies, it might make it easier to visually resolve the peak of a single isolated frequency that does not have any significant adjacent signals or noise in the spectrum.

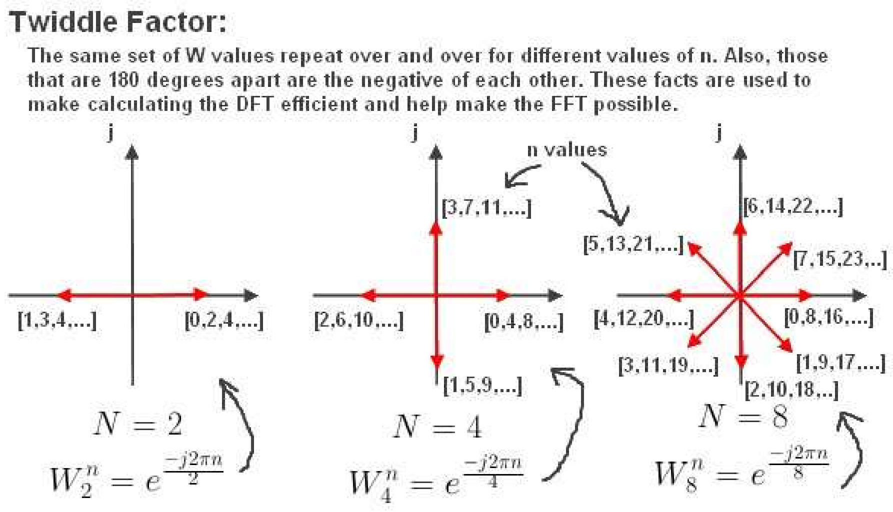
Zero-padding for cross-correlation, auto-correlation, or convolution filtering is used to not mix convolution results (due to circular convolution). Zero-padding provides a bunch zeros into which to mix the longer result. And it's far far easier to un-mix something that has only been mixed/summed with a vector of zeros.

Question:7(b) 2016 what is twiddle factor? Discuss the effect of twiddle factor in details in fft.

A **twiddle factor**, in fast Fourier transform (FFT) algorithms, is any of the trigonometric constant coefficients that are multiplied by the data in the course of the algorithm.

The twiddle factor, W, describes a "rotating vector", which rotates in increments according to the number of samples, N. Here are graphs where N = 2, 4 and 8 samples.

Effect of twiddle factor:



Question 7(c) 2014 Explain how we get benefit from redundancy and symmetry of the “Twiddle factor”

The Redundancy and Symmetry of the "Twiddle Factor"

As shown in the diagram above, the twiddle factor has redundancy in values as the vector rotates around. For example W for N=2, is the same for n = 0, 2, 4, 6, etc. And W for N=8 is the same for n = 3, 11, 19, 27, etc.
Also, the symmetry is the fact that values that are 180 degrees out of phase are the negative of each other. So for example, W for N =4 samples, where n = 0,4,8, etc, are the negative of n = 2,6,10, etc.
The Butterfly diagram takes advantage of this redundancy and symmetry, which part of what makes the FFT possible.

Question: 6(a) 2014 why do we need to convert a time domain signal into frequency domain
is
Answer:

Actually time domain gives us some issues when we design systems, solving mathematics problems like convolution (Filtering), reconstruct signals and reduce noise! However every domain has advantage and disadvantage; as example (in Control Theory) we can't know if system stable or not stable if we use Time Domain, so we convert the function of system to Frequency Domain or S-Domain to know that.
In other way, Telecom company can't make your voice travel to another country without making Sampling >> Quantization >> Encoding; all this steps in Time Domain! so Time Domain good to coded voice to make digital system understand it.
Because your call travel in air so of course you will get noise when phone receive it! to reduce this noise we convert the call signal from Time Domain to Frequency Domain and delete the noise frequencies! to make same thing in Time Domain you must know the value of this noise in very time interval and that not possible because it's a Gaussian noise

Question: 6(b) 2014 Distinguish between z-transform and DFT

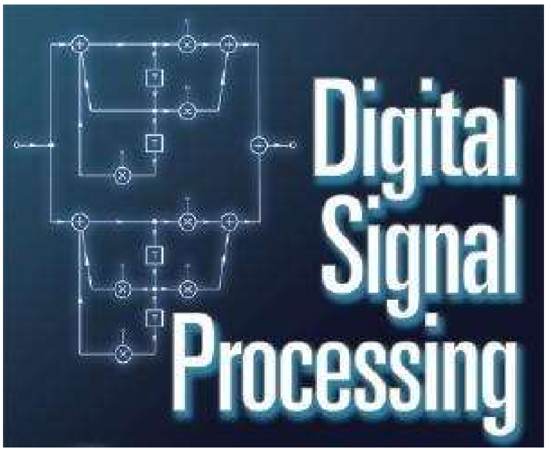
z-transform is digital equivalent of laplace transform and it is used for steady state analysis of signals/systems,while DFT is digital analog of fourier transform Actually, the Z transform is not really a proper transform, just a re-interpretation of the sequence of samples as coefficients of a formal Laurent series.
In some cases the formal Laurent series converges, if it does, it does so on an annular region in the complex plane. For useful signals (stable, summable, exponentially decaying) this annulus contains the unit circle, and the evaluation of the Laurent series on the unit circle corresponds to the Fourier series.
The interesting point of connecting a signal sequence to a periodic function on the unit circle is the inverse transformation, that many useful sequences are sequences of Fourier coefficients. And of course that convolution of signals corresponds to point-wise multiplication of the functions.

In discrete time signal, we use ZT or DTFT(discrete time fourier transform). The difference is that ZT is more general version. Z can be any complex. When ZT converges on unit circle(e^jw) then it becomes to fourier transform.

Question: 7(a) 2014 for 1024 samples, how many operation require for FFT?

$N \log_2 N$

$1024 \; log_2 \; 1024$



Important Question

1. Discuss the following operations in terms of relational algebra: (i) Project **CSE-2011 CSE-2009 CSE-2003 CSE-2002** (ii) Set Difference **CSE-2011** (iii) Cartesian Product **CSE-2011 CSE-2009 CSE-2003 CSE-2002** (iv) Outer-Join. **CSE-2011 CSE-2009** (V) Select Operation **CSE-2010 CSE-2008** (vi) Union Operation **CSE-2010 CSE-2008** (vii) Rename Operation **CSE-2010 CSE-2008** (viii) Natural Join Operation. **CSE-2010 CSE-2009 CSE-2008 CSE-2003 CSE-2002** (ix) Set intersection. **CSE-2003** (x) Set Difference. **CSE-2002**
2. Explain relational data model with example. **4 Marks CSE-2009**
3. Define schema diagram. Differentiate between Cartesian product and Natural Join with suitable example. **2+3 Marks CSE-2006**
4. Which operations are used to modify the database? Describe them with examples. **6 Marks CSE-2006**
5. What do you mean by relational algebra? What are the fundamental operations in relational algebra? Briefly explain. **7 Marks CSE-2005 CSE-2003 CSE-2003 (OLD)**
6. List two reasons why null values might be introduced into the database. **2 Marks CSE-2005**
7. What are left outer join, right outer join and full outer join? Explain with example. **5 Marks CSE-2004**
8. List two reasons why we may choose to define a view. **2 Marks CSE-2003 (OLD)**
9. Define Domain, Tuple Variable, Query Languages and foreign key. 4 Marks CSE-2007
10. What is a View? Why do we define a View? **1+2 Marks CSE-2007**
11. Why do we use a number of relations that are connected in some ways rather than just a relation that contains all the attributes? **2 Marks CSE-2007**
12. What are the reasons for the popularity of relational data model? **3 Marks CSE-2006**

- In earlier chapters we studied the theory of *discrete systems* in both the time and frequency domains.
- We will now use this theory for the *processing* of digital signals.
- To process signals, we have to *design and implement* systems called *filters*.
- The filter design issue is influenced by such factors as
  - The *type* of the filter: IIR or FIR
  - The *form* of its implementation: structures
  - Different filter *structures* dictate different design *strategies*.

Two Classes of Digital Filters:

a) Finite Impulse Response **(FIR),** non recursive, of the form

$$y(n) = h(0)x(n) + h(1)x(n-1) + \ldots + h(N)x(n-N)$$

With N being the order of the filter.

Advantages: always stable, the phase can be made exactly linear, we can approximate any filter we want;

Disadvantages: we need a lot of coefficients (*N large*) for good performance;

b) **Infinite Impulse Response (IIR)**, recursive, of the form

$$y(n) + a_1 y(n-1) + \ldots + a_N y(n-N) = b_0 x(n) + b_1 x(n-1) + \ldots + b_N x(n-N)$$

Advantages: very selective with a few coefficients;

Disadvantages: non necessarily stable, nonlinear phase.

FIR Digital Filter	IIR Digital Filter
FIR system has finite duration unit sample response. i.e h(n) = 0 for n<0 and n ≥ M Thus the unit sample response exists for the duration from 0 to M-1.	IIR system has infinite duration unit sample response. i. e h(n) = 0 for n<0 Thus the unit sample response exists for the duration from 0 to ∞.
FIR systems are non recursive. Thus output of FIR filter depends upon present and past inputs.	IIR systems are recursive. Thus they use feedback. Thus output of IIR filter depends upon present and past inputs as well as past outputs
Difference equation of the LSI system for FIR filters becomes $y(n)=\sum_{k=0}^{M} b_k\, x(n-k)$	Difference equation of the LSI system for IIR filters becomes $y(n)=-\sum_{k=1}^{N} a_k\, y(n-k)+\sum_{k=0}^{M} b_k\, x(n-k)$
FIR systems has limited or finite memory requirements.	IIR system requires infinite memory.

FIR Filter Structure

In general, an FIR system is described by the difference equation

$$y(n) = \sum_{k=0}^{M-1} b_k x(n-k)$$

or, equivalently, by the system function

$$H(z) = \sum_{k=0}^{M-1} b_k z^{-k}$$

FIR Systems are represented in four different ways

1. Direct Form Structures
2. Cascade Form Structure
3. Frequency-Sampling Structures
4. Lattice structures.

Classification of FIR filter:

Lowpass (LP), Highpass (HP), Bandpass (BP) and Bandreject "Notch" Filters

IIR Systems are represented in four different ways

1. Direct Form Structures Form I and Form II
2. Cascade Form Structure
3. Parallel Form Structure
4. Lattice and Lattice-Ladder structure

Question 7(a) 2017 Given $h(n) = \delta(n) + \delta(n-1)$associated phase and group delay as well

- **i. Show it is low pass filter**
- **ii. Explain why it is low pass filter**
- **iii. Determinate and plot th**

Question 8(a) 2017 Define analog filter and digital filter. Realized the second order digital filter

$$y(n) = 25cos(\omega_0)y(n-1) - 50y(n-2) + x(n) - 2cos(\omega_0)x(n-1)$$

Analog filter:

Analog filters, as mentioned earlier, do not need to convert the signal into a digital one which means they do not require any ADC or DAC converters. In such filters, signal stays in its genuine analog form throughout the processing. Resistor-Capacitor (RC) electronic networks perform the filtering.

This article explains the main differences between analog and digital filter base on factors such as representation, components, frequency response, stability, flexibility, adaptability, environmental changes, additive noise, cost, design, and applications.

Question 8(a) 2014 What is digital filter. Derive the expression for the frequency response of linear phase symmetric Fir filter.

Digital Filter:

In signal processing, a digital filter is a system that performs mathematical operations on a sampled, discrete-time signal to reduce or enhance certain aspects of that signal. This is in contrast to the other major type of electronic filter, the analog filter, which is an electronic circuit operating on continuous-time analog signals.

A digital filter will require an *Analogue to Digital Converter* (DAC). This converts the analogue voltage levels in the waveform into binary numbers.

Once those binary numbers are processed - or mathematically manipulated - they feed to another circuit called a DAC, which is a *Digital to Analogue Converter* and as its name suggests converts the binary values into voltage levels thereby reconstructing the signal back into an analogue form.

Linear phase fir filter:

"Linear Phase" refers to the condition where the phase response of the filter is a linear (straight-line) function of frequency (excluding phase wraps at +/- 180 degrees). This results in the *delay* through the filter being the same at all frequencies. Therefore, the filter does not cause "phase distortion" or "delay distortion". The lack of phase/delay distortion can be a critical advantage of FIR filters over IIR and analog filters in certain systems, for example, in digital data modems.

The symmetric-impulse-response constraint means that *linear-phase filters must be FIR filters*, because a causal recursive filter cannot have a symmetric impulse response.

We will show that every real symmetric impulse response corresponds to a *real* frequency response times a *linear phase term* $e^{-j\alpha\omega T}$, where $\alpha = (N-1)/2$ is the *slope* of the linear phase. Linear phase is often ideal because a filter phase of the form $\Theta(\omega) = -\alpha\omega T$ corresponds to phase delay

$$P(\omega) \triangleq -\frac{\Theta(\omega)}{\omega} = -\frac{-\alpha\omega T}{\omega} = \alpha T = \frac{(N-1)T}{2}$$

and group delay

$$D(\omega) \triangleq -\frac{\partial}{\partial\omega}\Theta(\omega) = -\frac{\partial}{\partial\omega}(-\alpha\omega T) = \alpha T = \frac{(N-1)T}{2}.$$

That is, both the phase and group delay of a linear-phase filter are equal to $(N-1)/2$ sample of plain delay *at every frequency*. Since a length N FIR filter implements $N-1$ samples of delay, the value $(N-1)/2$ is exactly half the total filter delay. Delaying all frequency components by the same amount *preserves the waveshape* as much as possible for a given amplitude response.

Difference between Analog and Digital Filter

Characteristics	Analog Filter	Digital Filter
Working signals	These filters work with analog or actual signals	These filters work with digital samples of the signal
Representation	These filters are represented by linear differential equations	These filters are represented by linear difference equations
Components	Implementing such filters requires resistors, inductors, and capacitors	Implementing such filters requires adders, subtractors, and delays
Frequency response	Approximation problem is computed in order to achieve the desired frequency response	Special coefficients are designed in order to meet the expected frequency response
Stable & Causal Response	Transfer function G(s) should be a rational function of laplace variable s, whose coefficients are real numbers.	Transfer function G(s) should be a rational function of z-transform z, whose coefficients are real numbers.
Stability & Causality in terms of Poles	Poles of transfer function should lie on left-half of s-plane	Poles of transfer function should lie inside the unit circle of z-plane
Environmental changes	Because of components tolerance, these filters are more sensitive to environmental changes	These types of filters are less sensitive towards environmental changes, noise, and disturbances.

Question 8(b) 2014 how can you design digital filter from analog filter?

Question:7(b) describe fir and IIR filtr briefly

Question 8(a) 2015 Define finite impulse response filter(FIR)? Explain FIR filter with equation and diagram.

FIR Filter:

Finite impulse response (FIR) filter is a filter whose impulse response (or response to any finite length input) is of finite duration, because it settles to zero in finite time.

This is in contrast to infinite impulse response (IIR) filters, which may have internal feedback and may continue to respond indefinitely (usually decaying).

Explain:

For a causal discrete-time FIR filter of order *N*, each value of the output sequence is a weighted sum of the most recent input values:

$$y[n] = b_0 x[n] + b_1 x[n-1] + \cdots + b_N x[n-N]$$
$$= \sum_{i=0}^{N} b_i \cdot x[n-i],$$

where:

- x[n]is the input signal,
- y[n] is the output signal,
- N is the filter order; an th-order filter has (N+1) terms on the right-hand side
- b_i is the value of the impulse response at the i'th instant for $0 \le i \le N$of an th-order FIR filter. If the
-
- filter is a direct form FIR filter then is also a coefficient of the filter.

This computation is also known as discrete convolution.

The in these terms are commonly referred to as *taps*, based on the structure of a tapped delay line that in many implementations or block diagrams provides the delayed inputs to the multiplication operations. One may speak of a *5th order/6-tap filter*, for instance.

The impulse response of the filter as defined is nonzero over a finite duration. Including zeros, the impulse response is the infinite sequence:

$$h[n] = \sum_{i=0}^{N} b_i \cdot \delta[n-i] = \begin{cases} b_n & 0 \le n \le N \\ 0 & \text{otherwise.} \end{cases}$$

If an FIR filter is non-causal, the range of nonzero values in its impulse response can start before $n = 0$, with the defining formula appropriately generalized

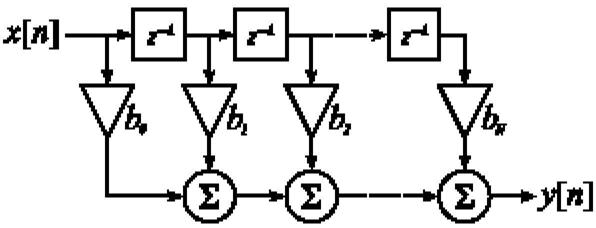
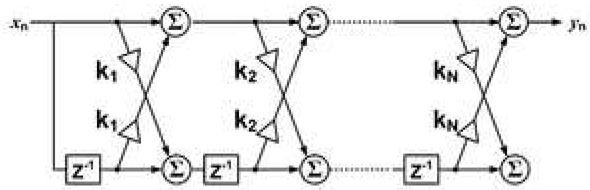


Figure: A direct form discrete-time FIR filter of order *N*. The top part is an *N*-stage delay line with *N* + 1 taps. Each unit delay is a z^{-1} operator in Z-transform notation.



A lattice-form discrete-time FIR filter of order *N*. Each unit delay is a z^{-1} operator in Z-transform notations.

IIR filter:

Infinite impulse response (**IIR**) is a property applying to many linear time-invariant systems. Common examples of linear time-invariant systems are most electronic and digital filters. Systems with this property are known as IIR systems or IIR filters, and are distinguished by having an impulse response which does not become exactly zero past a certain point, but continues indefinitely.

Question 8(b) 2015 Define Adaptive filter? Describe about kalman filter.

Question 7(c) 2013 Define Adaptive filter? Describe about kalman filter.

Question 8(b) 2016 Define Adaptive filter? Discuss LMS filter with necessary figure and equation.

Define Adaptive filter: An **adaptive filter** is a system with a linear filter that has a transfer function controlled by variable parameters and a means to adjust those parameters according to an optimization algorithm.

Because of the complexity of the optimization algorithms, almost all adaptive filters are digital filters. Adaptive filters are required for some applications because some parameters of the desired processing operation (for instance, the locations of reflective surfaces in a reverberant space) are not known in advance or are changing. The closed loop adaptive filter uses feedback in the form of an error signal to refine its transfer function.

Block diagram

The idea behind a closed loop adaptive filter is that a variable filter is adjusted until the error (the difference between the filter output and the desired signal) is minimized. The Least Mean Squares (LMS) filter and the Recursive Least Squares (RLS) filter are types of adaptive filter.

There are two input signals to the adaptive filter: d_K and x_K which are sometimes called the *primary input* and the *reference input* respectively.[3] The adaption algorithm attempts to filter the reference input into a replica of the desired input by minimizing the residual signal, ϵ_K . When the adaption is successful, the output of the filter y_K is effectively an estimate of the desired signal

d_K which includes the desired signal plus undesired interference and
x_K which includes the signals that are correlated to some of the undesired interference in

.

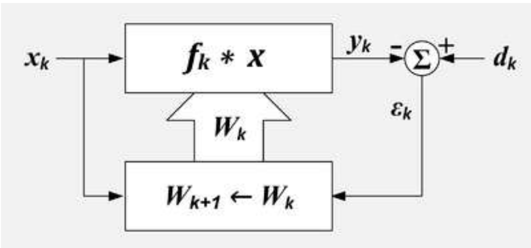
k represents the discrete sample number.

The filter is controlled by a set of L+1 coefficients or weights.

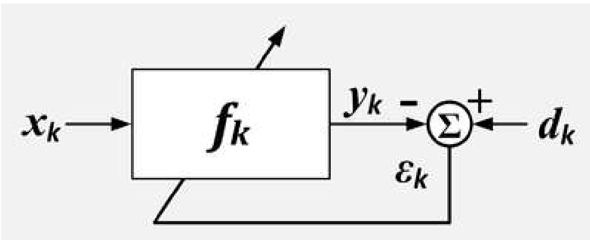
$\mathbf{W}_k = [w_{0k}, w_{1k}, \ldots, w_{Lk}]^T$ Represents the set or vector of weights, which control the filter at sample time k. where w_{lK} refers to the l th weight at k'th time.

ΔW_k represents the change in the weights that occurs as a result of adjustments computed at sample time k.

These changes will be applied after sample time k and before they are used at sample time k+1



Adaptive Filter. k = sample number, x = reference input, X = set of recent values of x, d = desired input, W = set of filter coefficients, ε = error output, f = filter impulse response, * = convolution, Σ = summation, upper box=linear filter, lower box=adaption algorithm



Adaptive Filter, compact representation. k = sample number, x = reference input, d = desired input, ε = error output, f = filter impulse response, Σ = summation, box=linear filter and adaption algorithm.

Discuss LMS filter:

1. Least Mean Square (LMS) Adaptive Filter Concepts

An adaptive filter is a computational device that iteratively models the relationship between the input and output signals of a filter. An adaptive filter self-adjusts the filter coefficients according to an adaptive algorithm. Figure 1 shows the diagram of a typical adaptive filter.

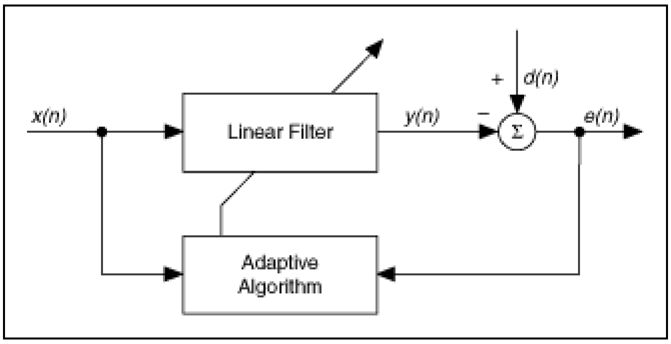


Figure 1. Typical Adaptive Filter

where *x(n)* is the input signal to a linear filter
y(n) is the corresponding output signal
d(n) is an additional input signal to the adaptive filter
e(n) is the error signal that denotes the difference between *d(n)* and *y(n)*.

The linear filter can be different filter types such as finite impulse response (FIR) or infinite impulse response (IIR). An adaptive algorithm adjusts the coefficients of the linear filter iteratively to minimize the power of *e*(*n*). The LMS algorithm is an adaptive algorithm among others which adjusts the coefficients of FIR filters iteratively. Other adaptive algorithms include the recursive least square (RLS) algorithms.

The LMS algorithm performs the following operations to update the coefficients of an adaptive FIR filter:

1. Calculates the output signal *y(n)* from the FIR filter.

$$y(n) = \vec{u}^{T}(n) \cdot \vec{w}(n)$$

where $\vec{u}(n)$ is the filter input vector and $\vec{u}(n) = [x(n)x(n-1)\ldots x(n-N+1)]^T$

$\vec{w}(n)$ is the filter coefficients vector and $\vec{w}(n) = [w_0(n)\ w_1(n)\ldots w_{N-1}(n)]^T$

2. Calculates the error signal *e(n)* by using the following equation: *e(n) = d(n)–y(n)*
3. Updates the filter coefficients by using the following equation:

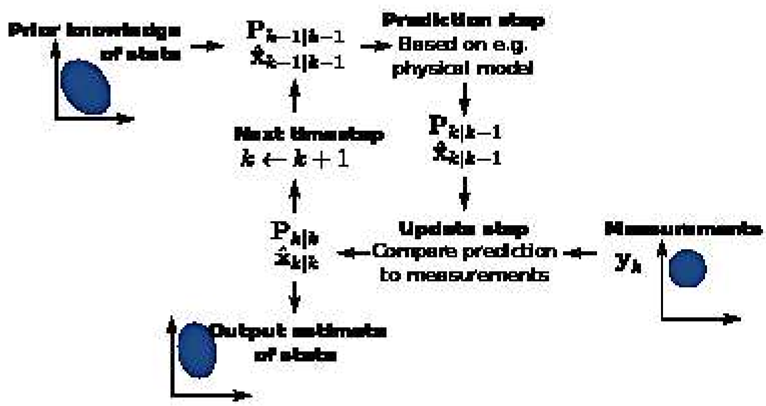
$$\vec{w}(n+1) = (1-\mu\alpha)\cdot\vec{w}(n) + \mu\cdot e(n)\cdot\vec{u}(n)$$

where μ is the step size of the adaptive filter
w(n) is the filter coefficients vector
u(n) is the filter input vector.

Kalman filter:

Kalman filtering, also known as **linear quadratic estimation** (**LQE**), is an algorithm that uses a series of measurements observed over time, containing statistical noise and other inaccuracies, and produces estimates of unknown variables that tend to be more accurate than those based on a single measurement alone, by estimating a joint probability distribution over the variables for each timeframe. The filter is named after Rudolf E. Kálmán, one of the primary developers of its theory.

The Kalman filter has numerous applications in technology. A common application is for guidance, navigation, and control of vehicles, particularly aircraft, spacecraft and dynamically positioned ships.[1] Furthermore, the Kalman filter is a widely applied concept in time series analysis used in fields such as signal processing and econometrics. Kalman filters also are one of the main topics in the field of robotic motion planning and control, and they are sometimes included in trajectory optimization.



The Kalman filter keeps track of the estimated state of the system and the variance or uncertainty of the estimate. The estimate is updated using a state transition model and measurements. $x_{k|k-1}$ denotes the estimate of the system's state at time step k before the k-th measurement y_k has been taken into account; $p_{k|k-1}$ is the corresponding uncertainty.

Question 8(c) 2015 What is window function? Write its application.

A **window function** is a mathematical function that is zero-valued outside of some chosen interval, normally symmetric around the middle of the interval, usually near a maximum in the middle, and usually tapering away from the middle.

Mathematically, when another function or waveform/data-sequence is "multiplied" by a window function, the product is also zero-valued outside the interval: all that is left is the part where they overlap, the "view through the window". Equivalently, and in actual practice, the segment of data within the window is first isolated, and then only that data is multiplied by the window function values. Thus, tapering, not segmentation, is the main purpose of window functions.

Applications

Spectral analysis

The Fourier transform of the function cos ωt is zero, except at frequency ±ω. However, many other functions and waveforms do not have convenient closed-form transforms. Alternatively, one might be interested in their spectral content only during a certain time period.

Windowing

Windowing of a simple waveform like cos ωt causes its Fourier transform to develop non-zero values (commonly called spectral leakage) at frequencies other than ω. The leakage tends to be worst (highest) near ω and least at frequencies farthest from ω.

Discrete-time signals

When the input waveform is time-sampled, instead of continuous, the analysis is usually done by applying a window function and then a discrete Fourier transform (DFT).

Question 8(c)2014 Writ the properties of FIR filter.

Answer:

Properties

An FIR filter has a number of useful properties which sometimes make it preferable to an infinite impulse response (IIR) filter. FIR filters:

- Require no feedback. This means that any rounding errors are not compounded by summed iterations. The same relative error occurs in each calculation. This also makes implementation simpler.
- Are inherently stable, since the output is a sum of a finite number of finite multiples of the input values, so can be no greater than $\sum b_i$ times the largest value appearing in the input.
- Can easily be designed to be linear phase by making the coefficient sequence symmetric. This property is sometimes desired for phase-sensitive applications, for example data communications, seismology, crossover filters, and mastering.

The main disadvantage of FIR filters is that considerably more computation power in a general purpose processor is required compared to an IIR filter with similar sharpness or selectivity, especially when low frequency (relative to the sample rate) cutoffs are needed. However, many digital signal processors provide specialized hardware features to make FIR filters approximately as efficient as IIR for many applications.

Question 8(c) 2016 Obtain the impulse response H(z) of a digital filter corresponding to an analog filter having impulse response of $h_a(t) = 0.5e^{-2t}$ and a sampling rate of 1 Khz by using the impulse invariance method

Question 8(c)2014 Writ the properties of FIR filter.

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