11 Feb, 18

Class: 01

$a_{11,12}$. a_{ij} , $i = 1, 2, 3, \dots, m$
$j = 1, 2, 3, \dots, n$

$A = [a_{ij}]$ is a matrix of order $m \times n$

Row size   Column size

3×4

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

22 element/entry

Real or imaginary number

Order 4 (Square matrix)

$\mathbb{N} = \{1, 2, 3, 4, \dots\}$

$\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3 \dots\}$

$\mathbb{Q}$ = The set of all rational no.s.

Rational $\rightarrow$ Ratio of two integers.

Ex: $\cdot 25 = \frac{1}{4} = \frac{2}{8} = \frac{25}{100}$.

$\sqrt{2}$ is not rational number

$\sqrt{2}$ " irrational number

$\mathbb{R}$ = The set of all real numbers

$\mathbb{C}$ = The set of all complex no.s

$(0,1)$

$= \{x : x \text{ is real and } 0<x<1\}$

$a \le x \le b$ means

$a \le x$ and $x \le b$

$(a<x$ or $a=x)$ and $(x<b$ or $x=b)$

$|x| \le 1 \Leftrightarrow -1 \le x \le 1$

*** $|x| > 1 \Leftrightarrow x>1$ or $x<-1$

$A = \{x : x>1 \text{ and } x<-1\}$

$= \phi$ (Null set)

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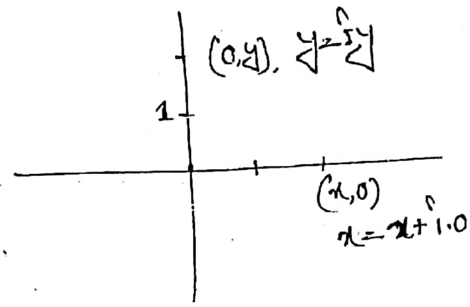
A no. of the form $x+iy$ where x and y are real numbers. $i=\sqrt{-1}$



$N \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$

A is a matrix over $\mathbb{R}$

B " " " over $\mathbb{C}$

$$B = \begin{bmatrix} 2 & 4 \\ 3 & i \end{bmatrix}$$

* Conditions for add two matrix
  (i) size same

Linear combination:

Defn: Let $A=[a_{ij}]$ and $B=[b_{ij}]$ be two matrices of the same order ~~m×m~~. $m \times n$

Let λ and μ be two scalares. Then we define $\lambda A + \mu B$ to be the matrix of order $m \times n$

Whose $(i\,j)$th element is $\lambda a_{ij} + \mu b_{ij}$ is called a linear combination of A and B.

$$\lambda A + \mu B = [c_{ij}]$$

$$c_{ij} = \lambda a_{ij} + \mu b_{ij}$$

$$A+B = A[ij] + B[ij].$$

* Matrix multiplication:

Condition:

have to same

(i) $\underbrace{m \times n}_{A}$ and $\underbrace{n \times p}_{B}$

$$A \times B = AB_{mp}$$

if, $A_{m \times n}$ and $B_{n \times m}$

$AB_{m \times m}$, $BA_{n \times n}$

Row matrix : $A_{1\times3} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

Column matrix : $A_{3\times1} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

Row vector

Column vector

$A_{1\times3} = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$

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Class: 02

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Defn: Two matrices A and B are said to be conformable for multiplications if the no. of columns of A is equal to the no. of rows of B. Thus the product matrix AB is defined when the no. of columns of A = the no. of rows of B.

Thus if A is a matrix of order $m \times n$ and B is a matrix of order $n \times p$ then AB is defined and its order will be $m \times p$.

The i-jth element of AB is the product of the ith row of A and the jth column of B.

Thus if $A = [a_{ij}]$ and $B = [b_{ij}]$ then the i-j th element of AB is $\sum_{k=1}^{n} a_{ik} b_{kj}$.

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Defn: Two matrices A and B are said to be equal if A and B have the same order and every element of A is equal to the corresponding element of B. In this case we write $A = B$.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 10 \end{bmatrix}$$

$A \neq B$

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Defn: Two matrices A and B are said to be equal if A and B have the same order and every element of A is equal to the corresponding element of B. In this case we write $A = B$.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

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$$A \neq B$$

Theorem: If A, B and C are matrices of order $m \times n$, $n \times p$ and $p \times q$ respectively then $(AB)C = A(BC)$. (Associative)

Proof: The orders of matrices AB and BC are $m \times p$ and $n \times q$ respectively. Hence, the orders of $(AB)C$ and $A(BC)$ are both equal to $m \times q$.

Hence the orders of $(AB)C$ and $A(BC)$ are equal.

~~Suppose, D = AB and E = BC~~

$\cancel{AB = [d_{ij}] \text{ and } BC = [e_{ij}]}$

Suppose, $A = [a_{ij}]$, $B = [b_{ij}]$, $C = [c_{ij}]$, $AB = [d_{ij}]$ and $BC = [e_{ij}]$

Then, $d_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}$ and $e_{ij} = \sum_{l=1}^{p} b_{il} c_{lj}$

Hence, the i, j th element of $(AB)C$

$$= \sum_{l=1}^{p} d_{il} \cdot c_{lj}$$

$$= \sum_{l=1}^{p} \left(\sum_{k=1}^{n} a_{ik} b_{kl} \right)$$

$$= \sum_{k=1}^{n} \sum_{l=1}^{p} \left(a_{ik} b_{kl} c_{lj} \right)$$

and the i-j th element of

$$A(BC) = \sum_{k=1}^{n} a_{ik} e_{kj} = \sum_{k=1}^{n} a_{ik} \sum$$

$$= \sum_{k=}^{n} \sum_{l=1}^{p} \left(a_{ik} b_{kl} c_{lj} \right)$$

and the i-j th element of

$$A(BC) = \sum_{k=1}^{n} a_{ik} e_{kj}$$

$$= \sum_{k=1}^{n} a_{ik} \sum_{l=1}^{p} \left(a_{ik} b_{kl} c_{lj} \right)$$

Def: If A and B are square matrices of order n and if $AB = BA$ then we say that A and B commute.

Theorem: Let A be a matrix of order $m \times n$ and B and C matrices each of order $m \times p$ then $A(B+C) = AB + AC$.

Problem: If A and B are n-squared matrices then prove that A and B commute, if and only if $A-\lambda I$ and $B-\lambda I$ commute for every scalar λ.

$A(B+C) = AB+$

$E = A+B$

$(A+B)(C+D) = E(C$

$= EC + ED$

* $(-A)(-B) = AB$

$xy = 0$

$\Rightarrow x=0$ or $y=0$

$AB = 0$

Zero matrix, null matrix $\longrightarrow$ Every element is zero.

Proof: Suppose A and B commute. Then

$$AB = BA \quad \text{..........} \quad (i)$$

Now, $(A-\lambda I)(B-\lambda I) = AB - A(\lambda I) - (\lambda I)B + (\lambda I)(\lambda I)$

$= AB - \lambda(AI) - \lambda(IB) + \lambda^2(I^2)$

$= AB - \lambda A - \lambda B + \lambda^2 I$

$= BA - \lambda A - \lambda B + \lambda^2 I$ [varify (i)]

Defn: A square matrix $D = [d_{ij}]$ is said to be a diagonal matrix if $d_{ij} = 0$ for $i \neq j$. We sometimes denote this matrix by the notation

$$\text{Diag}[d_{11}, d_{22}, d_{33}, \ldots, d_n$$

Ex: $\text{Diag}[1,0,3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

Def: A diagonal matrix A is said to be a scalar matrix if all it's diagonal elements are equal.

Hence, for a scalar matrix A, ~~then~~ there exists a scalar λ such that $A = \lambda I$

Problem: If a diagonal matrix is an idempotent matrix then it's diagonal elements are 0 or 1.

$$\begin{bmatrix} a_1 & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix} \begin{bmatrix} a_2 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_2 \end{bmatrix} = \begin{bmatrix} a_1a_2 & 0 & 0 \\ 0 & b_1b_2 & 0 \\ 0 & 0 & c_1c_2 \end{bmatrix}$$

Proof: Suppose, $A = \text{Diag}[a_1, a_2, \cdots, a_n]$ is an idempotent matrix. Then $A^2 = A$

$$\Rightarrow \text{Diag}[a_1, a_2, \ldots, a_n] \cdot \text{Diag}[a_1, a_2, \cdots, a_n]$$
$$= \text{Diag}[a_1, a_2, \ldots a_n]$$

$$\Rightarrow \text{Diag}[a_1^2, a_2^2, \ldots, a_3^2] = \text{Diag}[a_1, a_2, \cdots, a_n]$$

Hence, $a_i^2 = a_i$ for $i = 1, 2, \cdots, n$

Thus $a_i^2 - a_i = 0$

$$\Rightarrow a_i(a_i - 1) = 0$$

$$\Rightarrow a_i = 0 \text{ or } a_i = 1.$$

Problem: If A is an idempotent matrix then prove that $I-A$ is also an idempotent matrix.

$$(I-A)^2 = (I-A)(I-A) = I - A - A + A^2$$

$$= I - A - A + A$$

$$= (I-A)$$

So, we have $(I-A)^2 = I-A$

Hence, $(I-A)$ is an idempotent matrix.

Def: The ~~congu~~ conjugate of a matrix $A = [a_{ij}]$ denoted by $\bar{A}$ is the matrix whose ij-th element is the complex conjugate of the i-j-th element of A. Thus $\bar{A} = [\bar{a}_{ij}]$

$$A = \begin{bmatrix} 1+i & 3 \\ 4 & 6-i \end{bmatrix}$$

$$\bar{A} = \begin{bmatrix} 1-i & 3 \\ 4 & 6+i \end{bmatrix}$$

Def: The ~~congug~~ conjugate transpose of an $m \times n$ matrix $A = [a_{ij}]$ denoted by A^{θ} is the matrix $(\bar{A})'$. It is clear that $(\bar{A})' = (\bar{A'})$

Ex: Let $A = \begin{bmatrix} 1-i & 3+2i & 6 \\ 4+3i & -2i & 2-i \end{bmatrix}$

then $\bar{A} = \begin{bmatrix} 1+i & 3-2i & 6 \\ 4-3i & 2i & 2+i \end{bmatrix}$ and $(\bar{A})' = \begin{bmatrix} 1+i & 4-3i \\ 3-2i & 2i \\ 6 & 2+i \end{bmatrix} = A^{\theta}$

Defⁿ: A square matrix A is said to be Hermitian if $A^{\theta} = A$

if

$\overline{a_{ji}} = a_{ij}$

then Hermitian

$\overline{\overline{a_{ji}}} = \overline{a_{ij}}$

$a_{ji} = a_{ij}$

Theorem: The diagonal elements of a Hermitian matrix are all real.

Proof: Suppose the matrix $A = [a_{ij}]$ is hermitian,

Then, $\overline{a_{ij}} = a_{ji} \quad \forall i, j$

In particular, if $i = j$ then

$$a_{ii} = \overline{a_{ii}} \quad \forall i$$

Hence, all the diagonal elements are real.

Ex: Let,

$$A = \begin{bmatrix} 2 & 2+i & 3+4i \\ 2-i & -1 & 6i \\ 3-4i & -6i & 4 \end{bmatrix}$$

then A is a hermitian matrix.

Defn: A square matrix A is said to be skew hermitian if $A^{\theta} = -A$, i.e if $\overline{a_{ij}} = -a_{ji}$.

$\overline{a_{ii}} = -a_{ii}$

$\Rightarrow a_{ii} + \overline{a_{ii}} = 0$

$z + \bar{z} = 2x$

Theorem: The diagonal elements of a skew-hermitian matrix are 0 or pure imaginary.

Ex: Let $A = \begin{bmatrix} 0 & 2+3i & 6i \\ -2+3i & 3i & 5 \\ 6i & -5 & 0 \end{bmatrix}$

then A is a skew hermitian matrix.

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Problem: For any square matrix A, prove that (i) $A + A'$ is symmetric

(ii) $A - A'$ is skew-symmetric

~~Ex: Let $A =$~~

Proof: We have

$$(A + A')' = A' + (A')'$$
$$= A' + A$$
$$= (A + A')$$

Hence, $A + A'$ is symmetric.

if $B' = -B$ then skew symmetric

Again, $(A - A')' = A' - (A')'$

$$= A' - A$$
$$= -(A - A')$$

Hence $A - A'$ is skew-symmetric.

Problem: For any symmetric matrix A and for any scalar λ, prove that λA is symmetric.

Proof: Let $A = [a_{ij}]$ be symmetric then the

ij-th element of $\lambda A = \lambda a_{ij}$

$= \lambda a_{ji}$

$=$ ji-th element of λA.

Hence, λA is symmetric.

Problem: For any skew symmetric

Proof:

$= -($ ji-th element of A

Hence λA is skew symmetric.

★★★

Problem: Every square matrix can be expressed uniquely as a sum of a symmetric matrix and skew-symmetric matrix.

Proof: Suppose A is a square matrix.

put $B = \frac{1}{2}(A + A')$ and $C = \frac{1}{2}(A - A')$ then

$$B' = \frac{1}{2}(A + A')'$$
$$= \frac{1}{2}(A' + (A')')$$
$$= \frac{1}{2}(A' + A)$$
$$= \frac{1}{2}(A + A')$$
$$= B$$

and $C' = -C$

Hence, B is symmetric and c is skew-symmetric.

Now, $B + C = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$
$$= \frac{1}{2}(A + A' + A - A')$$
$$= A$$

Hence, A is a sum of a symmetric matrix B and a skew-symmetric matrix C.

Suppose, ~~there~~ $\exists$ a symmetric matrix D and a skew symmetric matrix E such that $A = D+E$

Then, $A' = (D+E)'$

$= D' + E'$

$= D - E$

$(A+A') = (D+E) + (D-E) = 2D$

and $A - A' = (D+E) - (D-E)$

$= 2E$

Hence, the sum B+C is unique.

Similarity :

Problem: Prove that every square matrix can be expressed uniquely as a sum of a hermitian matrix and a skew-hermitian matrix.

$$B = \frac{1}{2}(A + A^{\theta})$$

$$C = \frac{1}{2}(A - A^{\theta})$$

* Co-factor

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 3 & 5 \\ 0 & 1 & 3 \end{bmatrix}$$

$A = [a_{ij}]$

$A_{ij} = (-1)^{i+j} M_{ij}$

$A_{11} = 4$ $A_{12} = -(0-4)$

$A_{13} = 0$

$A_{21} = 5$, $A_{22} = 3$

$A_{23} = 0$

$A = [a_{ij}]$

$Cof(A) = [A_{ij}]$

$adj(A) = (cof(A))'$

$A\,adj(A) = |A| I$

$\downarrow$

Determinant of A into I

$\dfrac{A\,adj(A)}{|A|} = I$

$\hookrightarrow$ Identity

Class: 06

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$$A = \begin{bmatrix} a_{11} & a_{12} \cdots & a_{1n} \\ a_{21} & a_{22} \cdots & a_{2n} \\ \vdots & & \\ a_{n1} & a_{n2} \cdots & a_{nn} \end{bmatrix} \quad M_n = \begin{vmatrix} a_{22} & a_{23} & \cdots & a_{2n} \\ \vdots & & & \\ a_{n2} & a_{n3} & \cdots & a_{nn} \end{vmatrix}$$

$$\begin{array}{ll} & \hspace{5em} j\text{th} \\ i\text{th} & a_{i1} \; a_{i2} \cdots a_{ij} \cdots a_{in} \end{array}$$

M_{ij} is the determinant of the submatrix of A obtain by deleting the ith row and jth column of A.

Put, $A_{ij} = (-1)^{i+j} M_{ij}$. Then the matrix $[A_{ij}]$ is called coefficient matrix of A, and will be denoted by cof(A). The transpose of Cof(A) is called the adj of the matrix A and is denoted by adj(A).

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23}$$

$$= \sum_{K=1}^{3} a_{1K} A_{1K}$$

$$= \sum_{K=1}^{3} a_{3K} A_{3K}$$

$$= \sum_{i=1}^{3} a_{i1} A_{i1} \text{ (First column)}$$

$$= \sum_{i=}^{3} a_{i2} A_{i2} \text{ (Second Colum}$$

If $A = [a_{ij}]_{n \times n}$ and $Cof(A) = [A_{ij}]_{n \times n}$

then $|A| = \sum_{K=1}^{n} a_{1K} A_{1K}$ (First now)

$$= \sum_{K=1}^{n} a_{2K} A_{2K} \text{ (Second now)}$$

. . . . —

. . . . —

$$= \sum_{K=1}^{n} a_{nK} A_{nK}$$

$$= \sum_{i=1}^{n} a_{i1} A_{i1}$$

$$\vdots$$

$$= \sum_{i=1}^{n} a_{in} A_{in}$$

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$$

$$\Rightarrow \begin{vmatrix} a_{21} & a_{22} & a_{23} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} \cancel{a_{21}A_{11}} = 0$$

$$\sum_{k=1}^{n} a_{ik} A_{lk} = \begin{cases} |A| & \text{if } i = l \\ 0 & \text{if } i \neq l \end{cases}$$

$$= |A| \delta_{il}$$

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Similarly,

$$\sum_{i=1}^{n} a_{il} A_{ik} = \begin{cases} |A| & \text{if } l = k \\ 0 & \text{if } l \neq k. \end{cases}$$

Now,

$$A \operatorname{adj}(A) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \begin{bmatrix} A_{11} & A_{21} & \cdots & A_{n1} \\ A_{12} & A_{22} & \cdots & A_{n2} \\ \vdots & \vdots & & \vdots \\ A_{1n} & A_{2n} & \cdots & A_{nn} \end{bmatrix}$$

$$= \begin{bmatrix} \sum_k a_{1k} A_{1k} & \sum_k a_{1k} A_{2k} & \cdots & \sum_k a_{1k} A_{nk} \\ \sum a_{2k} A_{1k} & \sum a_{2k} A_{2k} & & \\ \vdots & & & \\ \sum_k a_{nk} A_{1k} & & & \end{bmatrix}$$

$$A \operatorname{adj}(A) = \begin{bmatrix} |A| & 0 & 0 & \cdots & 0 \\ 0 & |A| & 0 & \cdots & 0 \\ 0 & 0 & |A| & \cdots & 0 \end{bmatrix} = |A| I$$

$$|\lambda I| = \lambda^n$$

***

Problem: For any square matrix A, prove that: $A \cdot adj(A) = |A| I = adj(A) A$

$AB = I$

$BA = I$

Square matrix

Def^n: The inverse of a square matrix A is another square matrix (of the same order) B such that $AB = BA = I$

.Class

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$$A \cdot adj(A) = |A| I = adj(A) \cdot A$$

$$|AB| = |A||B|$$
$$= |B||A|$$
$$= |BA|$$

$$|\lambda I| = \lambda^n$$

$$\big| |A| I \big| = |A|^n$$

$$\big| A \ adj(A) \big| = \big| |A| I \big|$$

$$\Rightarrow |A| \, |adj(A)| = |A|^n$$

$$\Rightarrow |adj(A)| = |A|^{n-1}$$

Problem: For any n-squared matrix A, prove that

$$|adj(A)| = |A|^{n-1}$$

Properties of determinant:

- (i) $|AB| = |A|\,|B|$

(ii) $|\lambda A| = \lambda^n |A|$, n is the order of A

$$|\lambda A| = \left|\lambda \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}\right| = \begin{vmatrix} \lambda a_{11} & \lambda a_{12} & \lambda a_{13} \\ \lambda a_{21} & \lambda a_{22} & \lambda a_{23} \\ \lambda a_{31} & \lambda a_{32} & \lambda a_{33} \end{vmatrix}$$

$$= \lambda^3 \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ \vdots & \vdots & \vdots \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

(iii) $|I| = 1$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$A_{11} = a_{12}$

$A_{12} = -a_{21}$

$A_{21} = -a_{12}$

$a_{22} = a_{11}$

$$\mathrm{cof}(A) = \begin{bmatrix} a_{12} & -a_{21} \\ -a_{12} & a_{11} \end{bmatrix}$$

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Def: A square matrix A is said to be non-singular if $|A| \neq 0$.

Every non-singular matrix is invertible:

Proof: Let A be a non-singular matrix.

Then $|A| \neq 0$. Put $B = \frac{\operatorname{adj}(A)}{|A|}$. Then

$$AB = A \frac{\operatorname{adj}(A)}{|A|} = \frac{1}{|A|}\left(A \operatorname{adj}(A)\right)$$

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$$= \frac{1}{|A|} |A| I$$

$$= I$$

Similarly, $BA = I$. Hence B is the inverse of A. Thus A is invertible.

$$A^{-1} = \frac{\operatorname{adj}(A)}{|A|}$$

Theorem: For any n-squared matrix A and B $adj(AB) = adj(B) \cdot adj(A)$

Proof: We have $(AB) \cdot (adj(AB)) = |AB| I$

$$= |A||B| I \quad \text{---} (1)$$

Similarly, $adj(AB) \cdot (AB) = |A||B| I$. ——— (2)

Again, $(AB)(adj(B) \cdot adj(A)) = A(B\, adj(B))\, adj(A)$

$$= A(|B| I)\, adj(A)$$

$$= |B|\, A\, adj(A)$$

$$= |B||A| I$$

$$= |A||B| I \quad \text{---} (3)$$

Similarly,

$$(adj(B)\, adj(A))(AB) = |A||B| I \quad \text{---} (4)$$

From ~~(i), (ii), (iii) &~~ (1), (2), (3) & (4) we have

$$adj(AB) = adj(B) \cdot adj(A).$$

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Problem: For any n-squared non-singular matrices A and B, prove that

(i) AB is non-singular.

(ii) $(AB)^{-1} = B^{-1}A^{-1}$

Proof: (i) Since A and B are non-singular

$$|A| \neq 0, \quad |B| \neq 0$$

Now $|AB| = |A|\,|B| \neq 0$ Hence AB is non-singular.

(ii) Since
$$(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1}$$
$$= (AI)A^{-1}$$
$$= AA^{-1}$$
$$= I$$

and
$$(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B$$
$$= B^{-1}(IB)$$
$$= B^{-1}B = I.$$

$\cancel{|A| \neq 0, |B| \neq 0}$ Hence $(AB)^{-1} = B^{-1}A^{-1}$

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Ex: 33 Let, $A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}$ find a non-singular matrix P such that $P^{-1}AP$ is diagonal.

Soln:- The ch. equation of A is $|A - \lambda I| = 0$

or, $\lambda^2 - 3\lambda + 2 =$

or, $(\lambda - 2)(\lambda - 1) = 0$

$\therefore \lambda = 1, 2$

Let, $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ be an eigen vector of A corresponding to $\lambda = 1$,

then, $(A - I)X = 0$

$$\begin{bmatrix} 3 & -3 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

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$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_1 - x_2 = 0$$

put $x_2 = 1$ then we have $x_1 = 1$, Thus

$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is an eigenvector of A corresponding to $\lambda = 1$

Again, Let $X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ be an eigenvector of ~~…~~ of A corresponding to $\lambda = 2$.

Then, $(A - 2I)X = 0$

$$\Rightarrow \begin{bmatrix} 2 & -3 \\ 2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \xRightarrow{R_{21}(-1)} \begin{bmatrix} 2 & -3 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore 2x_1 - 3x_2 = 0$

put $x_2 = 2$, then we have $x_1 = 3$

Thus $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$ is an eigen vector of A corresponding to $\lambda = 2$

put $P = \begin{bmatrix}1 & 3\\ 1 & 2\end{bmatrix}$, then, $AP = \begin{bmatrix}4 & -3\\ 2 & -1\end{bmatrix}\begin{bmatrix}1 & 3\\ 1 & 2\end{bmatrix}$

$$= \begin{bmatrix}1 & 6\\ 1 & 4\end{bmatrix}$$

$\cancel{X_1 = \begin{bmatrix}1\\ 1\end{bmatrix},\ X_2 = \begin{bmatrix}3\\ 2\end{bmatrix}}$

$\cancel{P = \begin{bmatrix}x_1\end{bmatrix}}$

$\cancel{= \begin{bmatrix}1 & 3\\ 1 & 2\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}}$

$\cancel{= P\begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}}$

Now, $P^{-1} = \begin{bmatrix}-2 & +3\\ +1 & -1\end{bmatrix}$

$$\therefore P^{-1}AP = \begin{bmatrix}1 & 0\\ 0 & 2\end{bmatrix}$$

For a $A_{3\times3}$ $\lambda_1, \lambda_2, \lambda_3$ are eigenvalues of A

X_1, X_2, X_3 are eigenvectors of A corresponding to $\lambda_1, \lambda_2, \lambda_3$

$P = \begin{bmatrix}X_1 & X_2 & X_3\end{bmatrix}$

$$P^{-1}AP = \begin{bmatrix}\lambda_1 & 0 & 0\\ 0 & \lambda_2 & 0\\ 0 & 0 & \lambda_3\end{bmatrix}.$$

05 Mar, 18

Elementary matrices are non-singular:

E_{ij}, $E_i(\lambda)$, $E_{ij}(\lambda)$

Ans. $|E_{ij}| = -1$, $|E_i(\lambda)| = \lambda$ $|E_{ij}(\lambda)| = 1$

$$D = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & u \end{bmatrix}$$

$|D| = \lambda.\mu.u$

$$U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$|U| = 1.4.2$

Scanned by CamScanner

$E_{ij}^{-1} = ?$

$E_i(\lambda)^{-1} = ?$

$E_{ij}(\lambda)^{-1} = ?$

$E_{ij} E_{ij} = I$

$E_{ij}^{-1} = E_{ij}$

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$E_{23} \cdot E_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & & & \\ & & & \\ & & & \\ & & & \end{bmatrix}$$