Discrete Mathematics

Partial Orders

Partial Orderings I

Definition:

A relation R on a set S is called a partial order if it is reflexive,

antisymmetric and transitive. A set S together with a partial

ordering R is called a partially ordered set or poset for short

and is denoted

(S,R)

Partial Orderings II

#less than equal

#geater than equal

#subset

Hasse Diagrams

Hasse Diagrams:

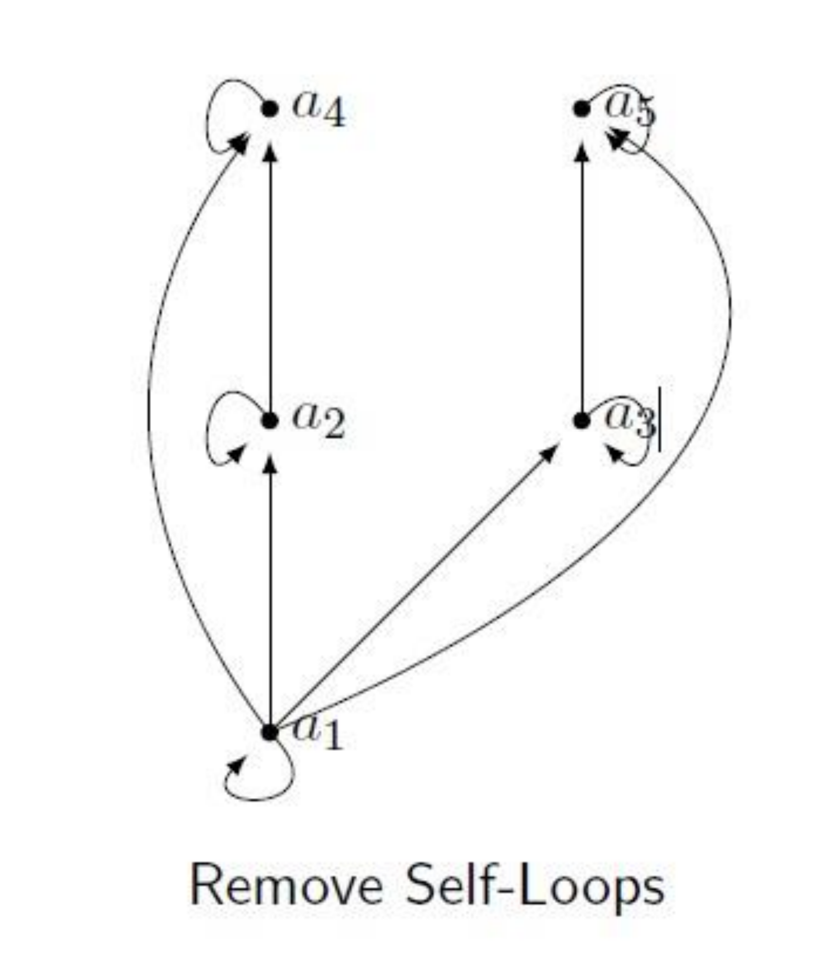
As with relations and functions, there is a convenient graphical representation for partial orders—Hasse Diagrams.

Consider the digraph representation of a partial order—since we know we are dealing with a partial order, we implicitly know that the relation must be reflexive and transitive. Thus we can simplify the graph as follows:

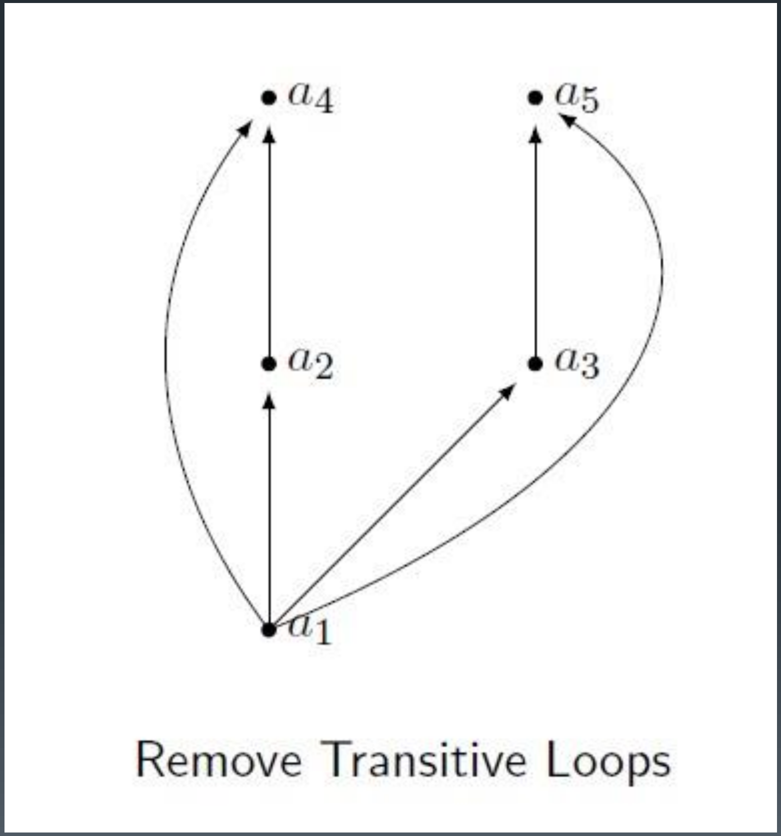
- Remove all self-loops.
- Remove all transitive edges.
- Make the graph direction-less—that is, we can assume that the orientations are upwards.

The resulting diagram is far simpler.

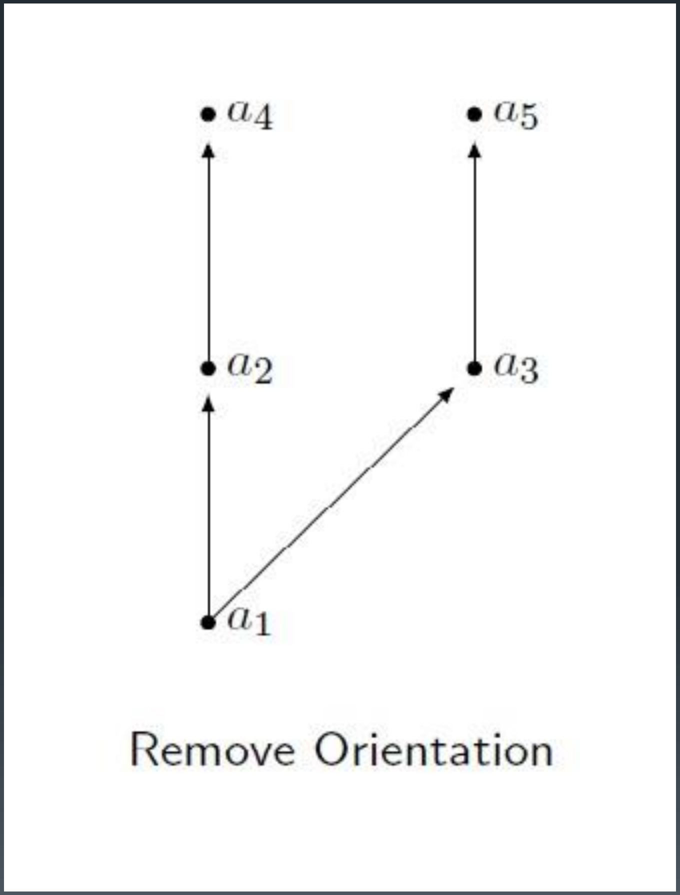
Hasse Diagram Example



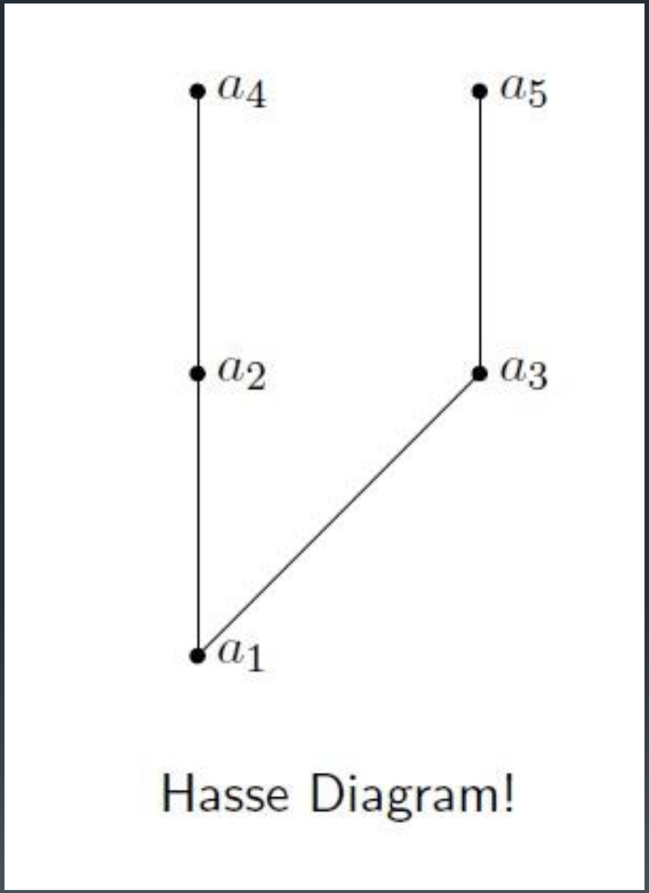
Hasse Diagram Example



Hasse Diagram Example



Hasse Diagram Example



Hasse Diagrams
Example

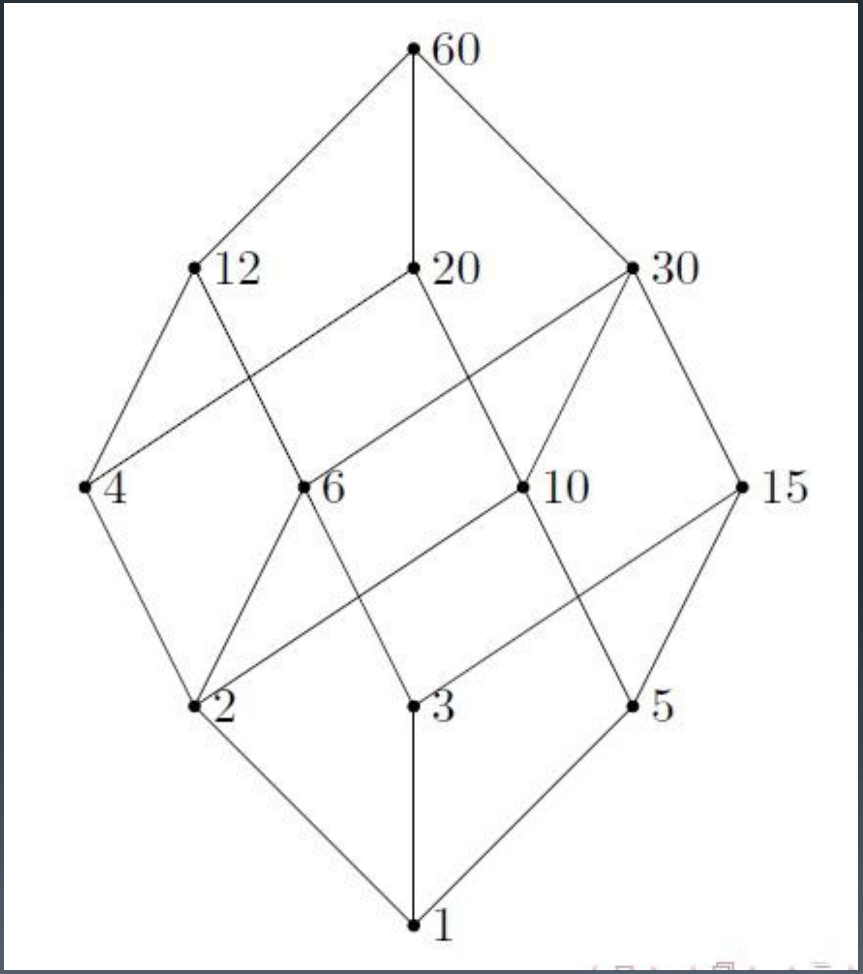
Of course, you need not always start with the complete relation in the partial order and then trim everything. Rather, you can build a Hasse directly from the partial order.

Example:

Draw a Hasse diagram for the partial ordering
$\{(a, b) \mid a \mid b\}$
on $\{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$
(these are the divisors of 60 which form the basis of the ancient Babylonian base-60 numeral system)

Hasse Diagrams

Example Answer:



Lattice :A lattice is a poset in which every pair of elements has a least upper bound (LUB) and a greatest lower bound (GLB).

- LUB(uper bound) = Least Common Multiple
- GLB (lower bound)= Greatest Common Denominator/Divisor