Electrical Circuit and Electronics

Network Analysis

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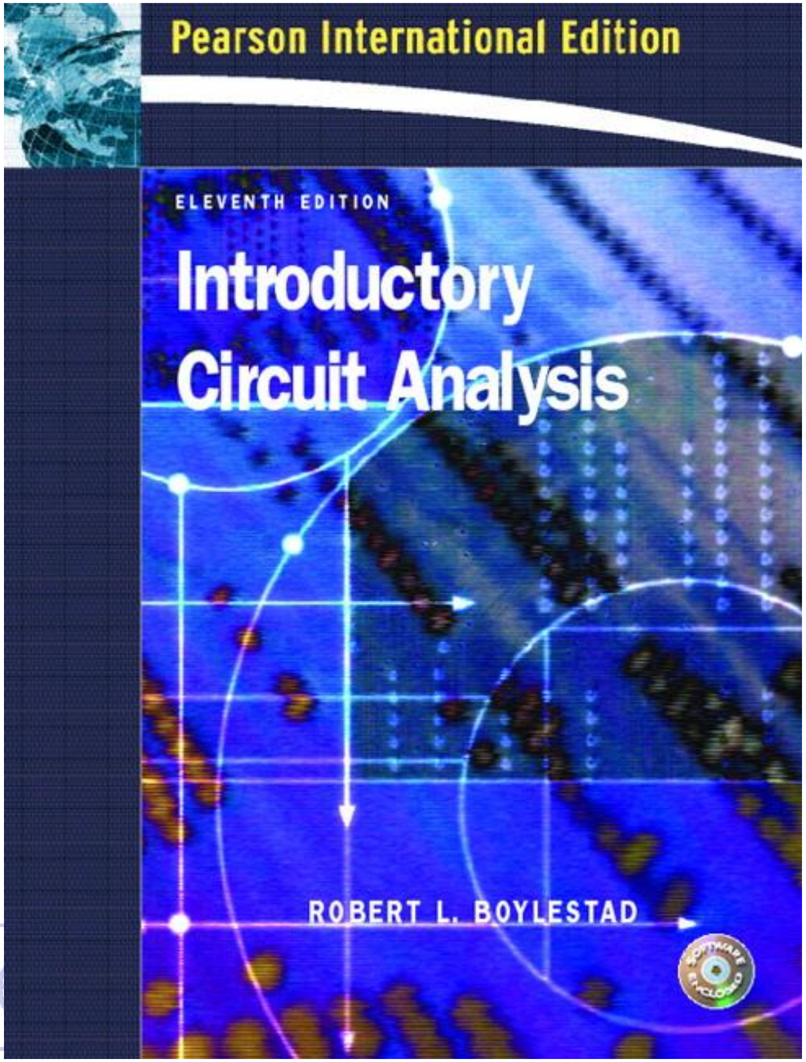
Reference Books Recommended

- **Introductory Circuit Analysis**

  - *Robert L. Boylestad*

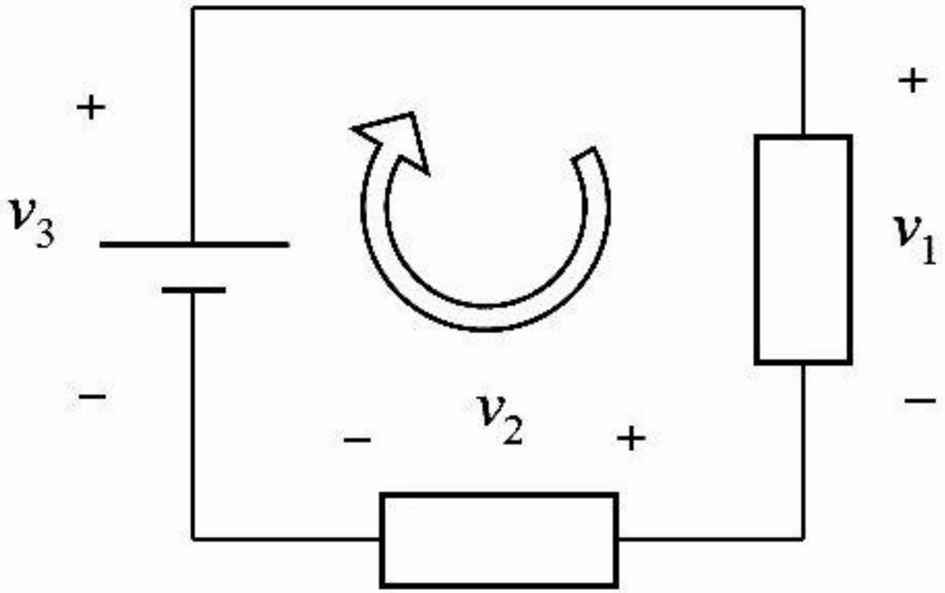


Kirchhoff’s Law

- Kirchhoff's Voltage Law (KVL)
- Kirchhoff's Current Law (KCL)

Kirchhoff's Voltage Law (KVL)

"The algebraic sum of all voltages around any closed path in a circuit is zero" (positive for a voltage rise, negative for a voltage drop.



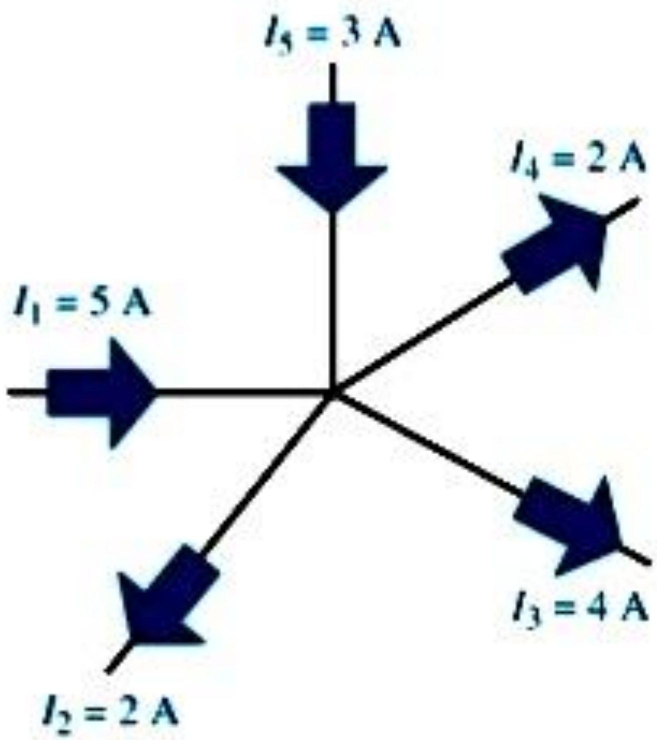
$$+v_1 + v_2 - v_3 = 0$$

$$-v_1 - v_2 + v_3 = 0$$

$$v_1 + v_2 = v_3$$

Kirchhoff’s Current Law (KCL)

KCL: The total current entering a node is equal to total current leaving the node.



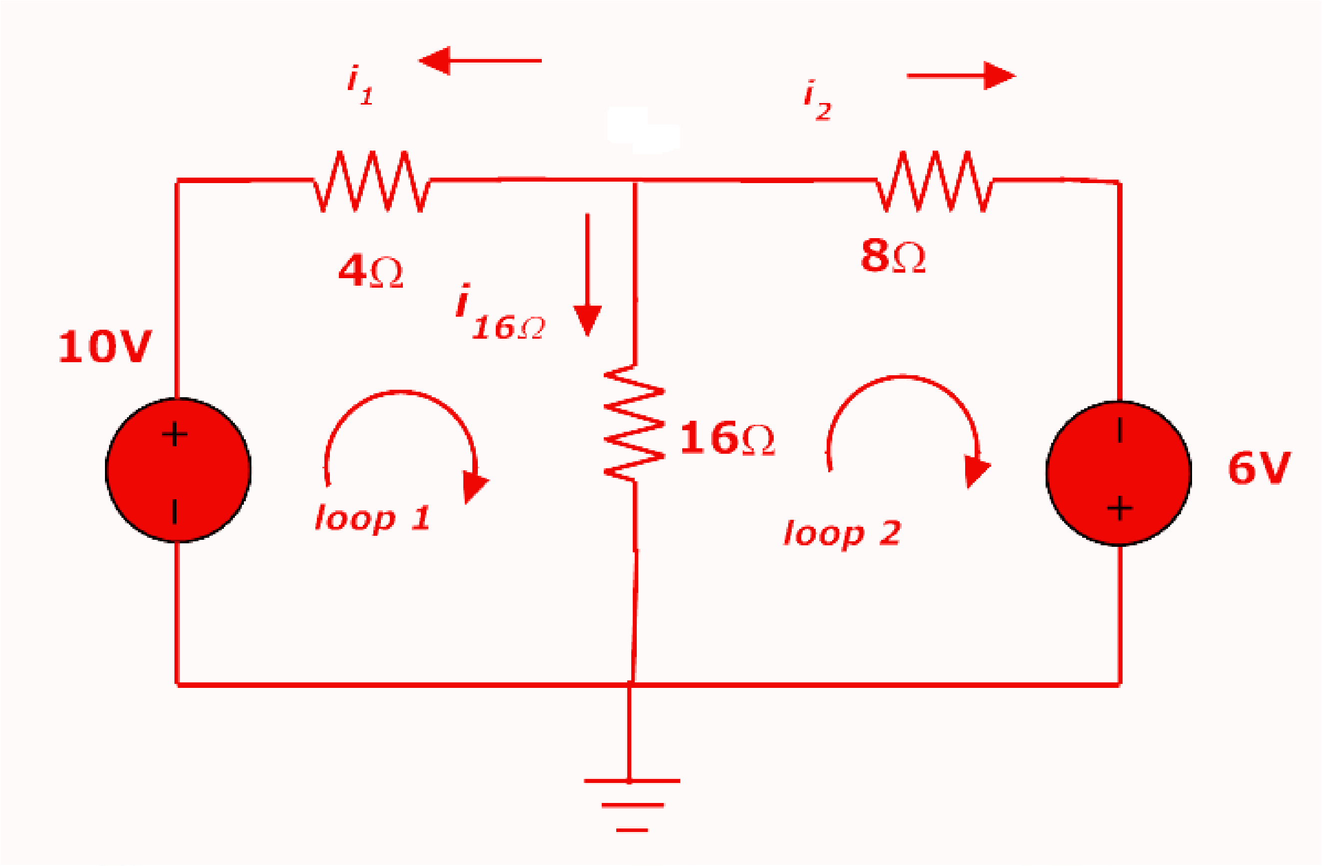
$$\sum I_{in} = \sum I_{out}$$

$$I_1 + I_5 = I_2 + I_3 + I_4$$

Ref: Boylestad: Page 203: Article 56.5

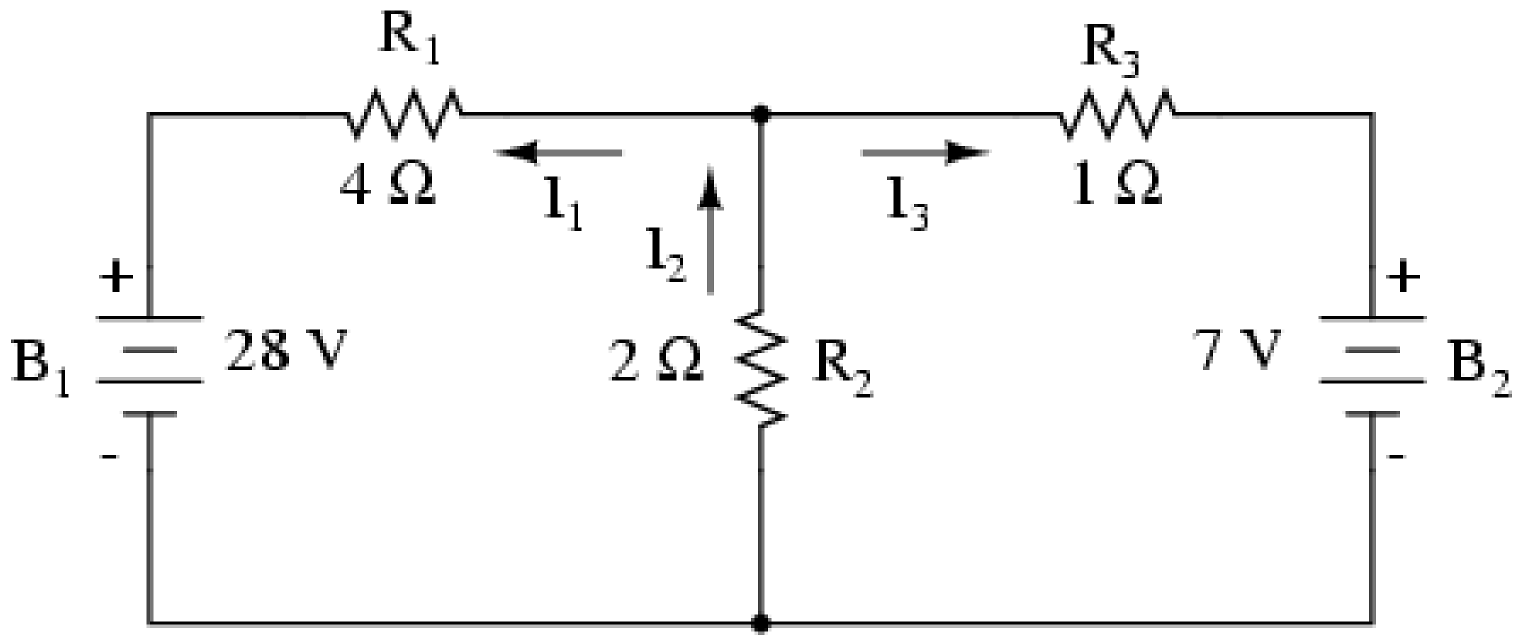
Practice Problem

Find $\boldsymbol{i_{16\Omega}}$ from the following circuit



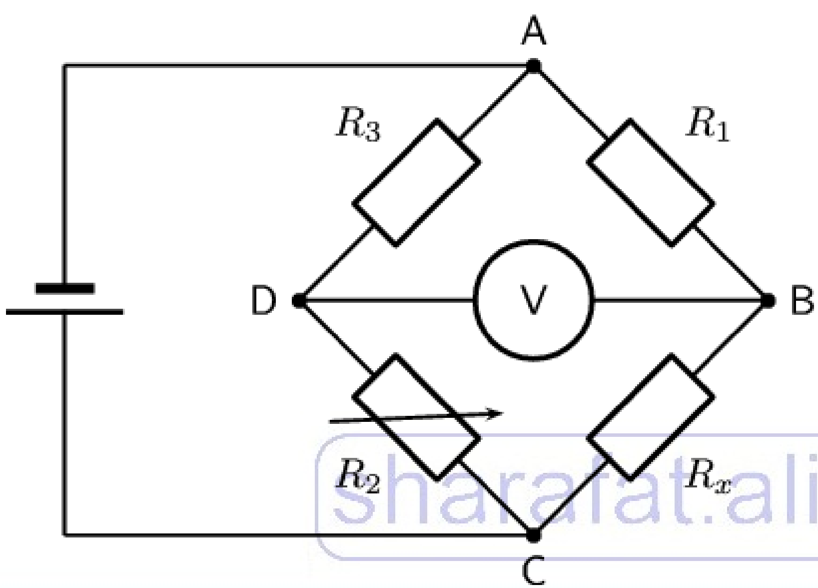
Practice Problem

Find I_1 , I_2 , I_3 from the following circuit



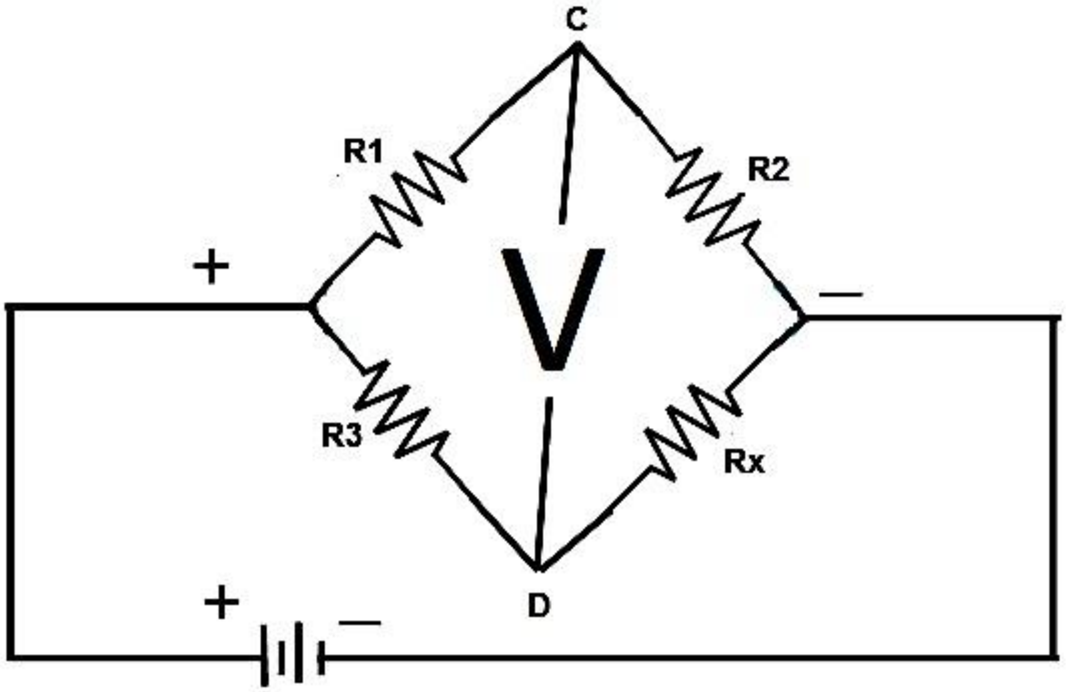
Wheatstone Bridge

A Wheatstone bridge is an electrical circuit used to measure an unknown electrical resistance. The Wheatstone Bridge can still be used to measure very low values of resistances down in the milli-Ohms range. A Wheatstone bridge circuit has two input terminals and two output terminals consisting of four resistors configured in a diamond-like arrangement.



If the measured voltage V is 0, then $R_2/R_3=R_x/R_1$

Wheatstone Bridge



For this bridge balanced condition voltage at points C and D must be equal. Hence, no current flows through the galvanometer. For getting the balanced condition one of the resistors must be variable.

Wheatstone Bridge

The voltage at point **D** $= V \times R_X / (R_3 + R_X)$

The voltage at point **C** $= V \times R_2 / (R_1 + R_2)$

The voltage (V) across galvanometer or between C and D is,

$$V_{CD} = V \times R_X / (R_3 + R_X) - V R_2 / (R_1 + R_2)$$

When the bridge is balanced $V_{CD} = 0$,

So, $V \times R_X / (R_3 + R_X) = V R_2 / (R_1 + R_2)$

$$R_X R_1 + R_X R_2 = R_2 R_3 + R_2 R_X$$

$$R_1 R_X = R_2 R_3$$

$$R_2/R_1 = R_X/R_3$$

Wheatstone Bridge Applications:

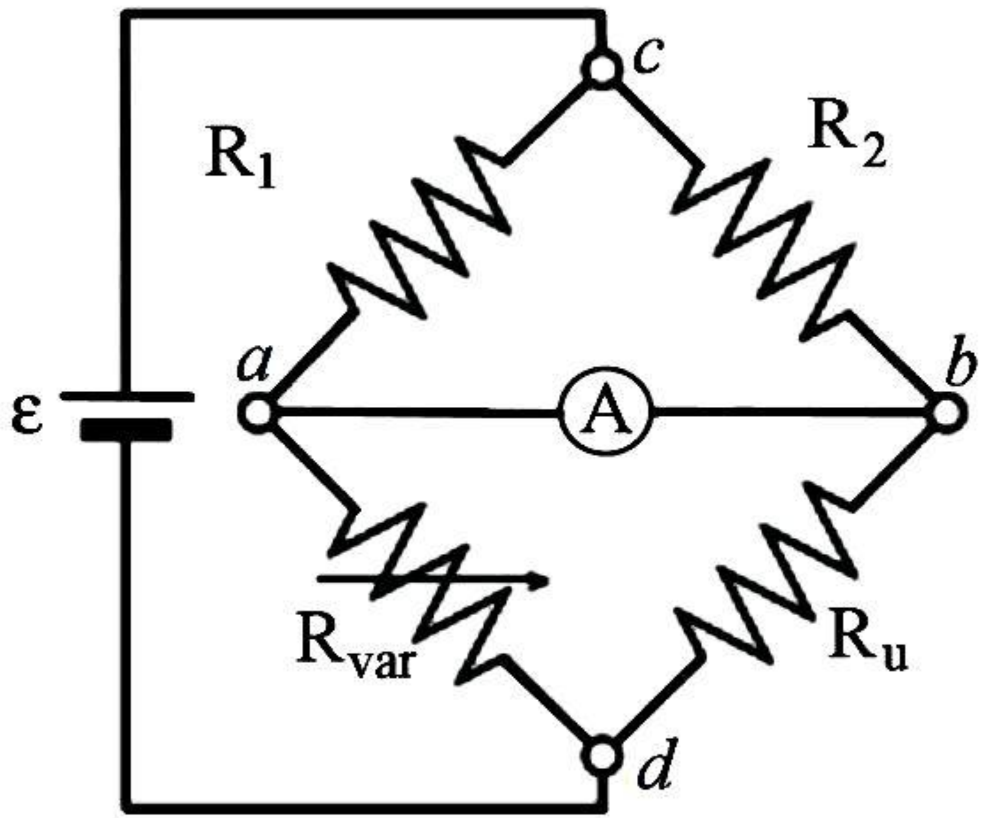
- The Wheatstone bridge is used for measuring the very low resistance values precisely.

- Wheatstone bridge along with operational amplifier is used to measure the physical parameters like temperature, strain, light, etc.

- We can also measure the quantities capacitance, inductance and impedance using the variations on the Wheatstone bridge.

Practice Problem for Wheatstone Bridge

A circuit consists of two resistors with resistances R_1 = 6.0 ohms and R_2 = 1.5 ohms , a variable resistor, the resistance R_{var} of which can be adjusted, a resistor of unknown value R_u , and 9.0 volt battery connected as shown in the figure. When R_{var} is adjusted to 12 ohms, there is zero current through the ammeter. What is the unknown resistance R_u?



Solution for Practice Problem

When there is no current flowing through the ammeter, the potential difference

V (b) - V (a) = 0 . Therefore:

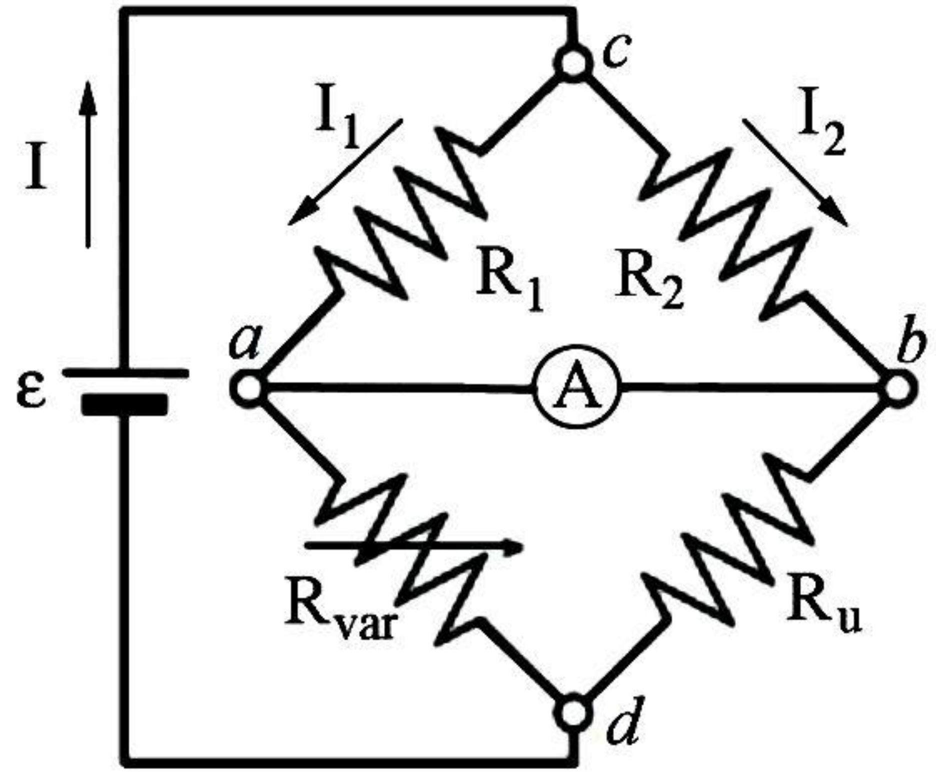
V (c) - V (a) = V (c) - V (b)

If we assign currents as shown in the figure then:

V (c) - V (a) = I_1R_1

and

V (c) - V (b) = I_2R_2



Solution for Practice Problem

Thus when the ammeter measures zero current

$I_1R_1 = I_2R_2$

$\therefore (I_1/I_2) = (R_2/R_1)$

In a similar fashion,

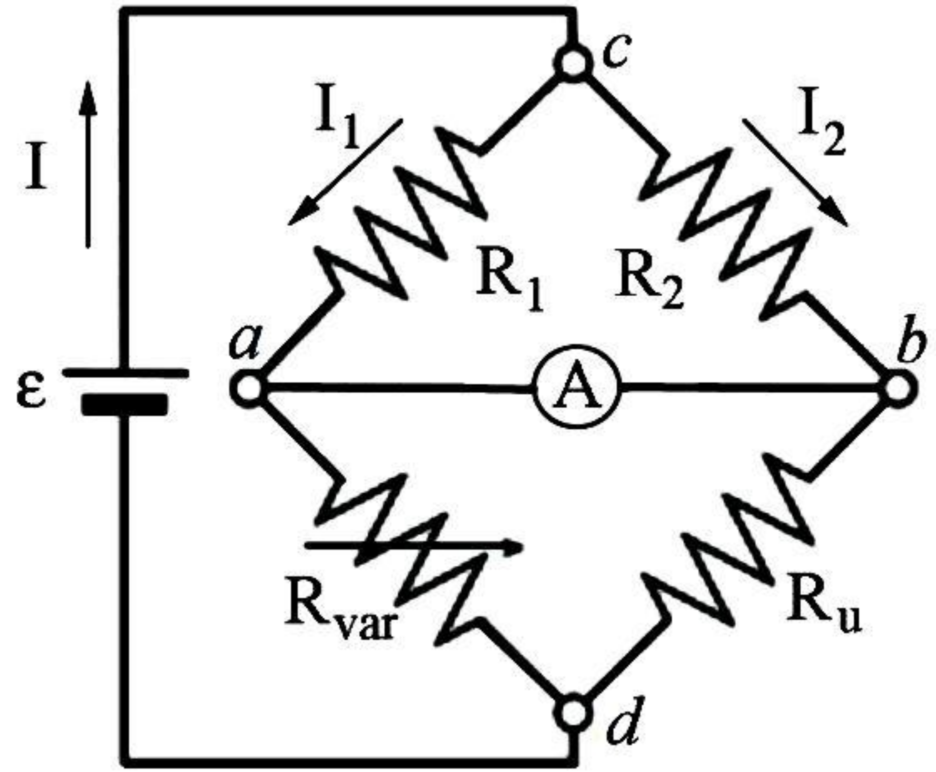
$V(a) - V(d) = V(b) - V(d)$

which implies that

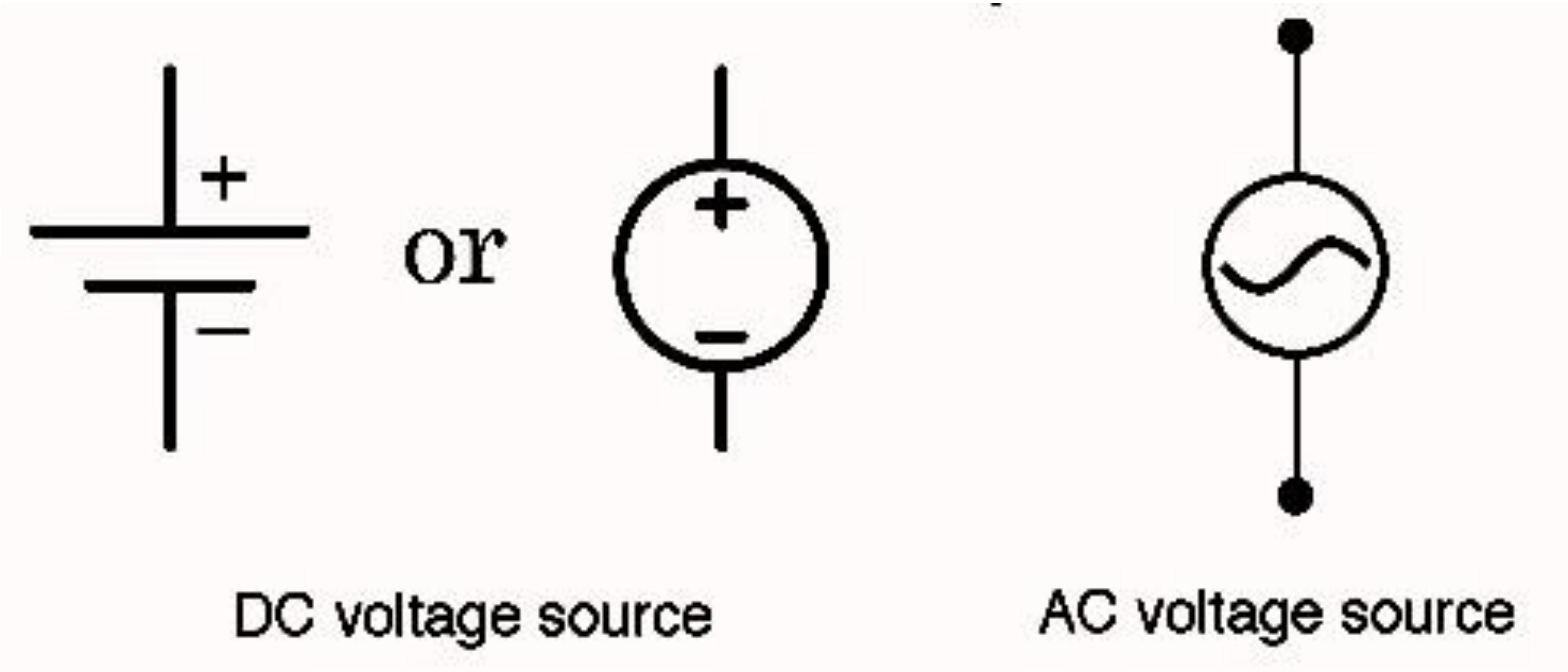
$I_1R_{var} = I_2R_u$

$R_u = (I_1/I_2)R_{var} = (R_2/R_1)\,R_{var}$

$\therefore R_u = (1.5/6)12\ \Omega = 3\ \Omega$



Voltage Source



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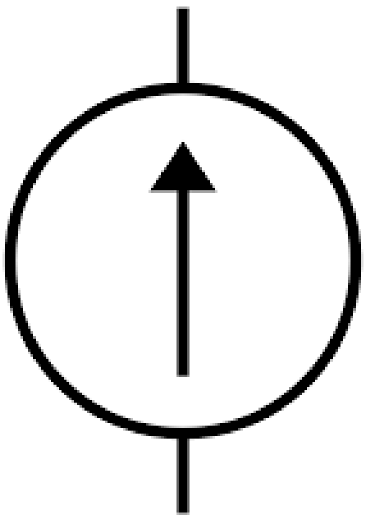
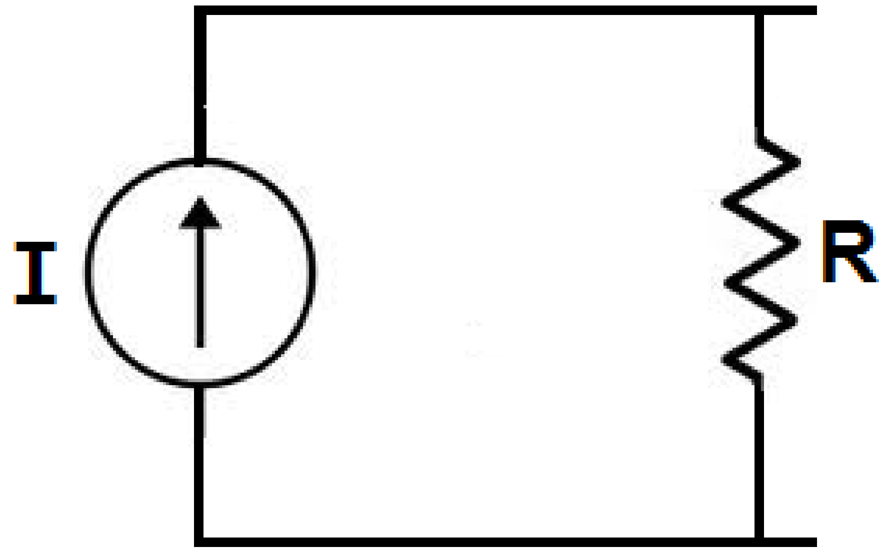
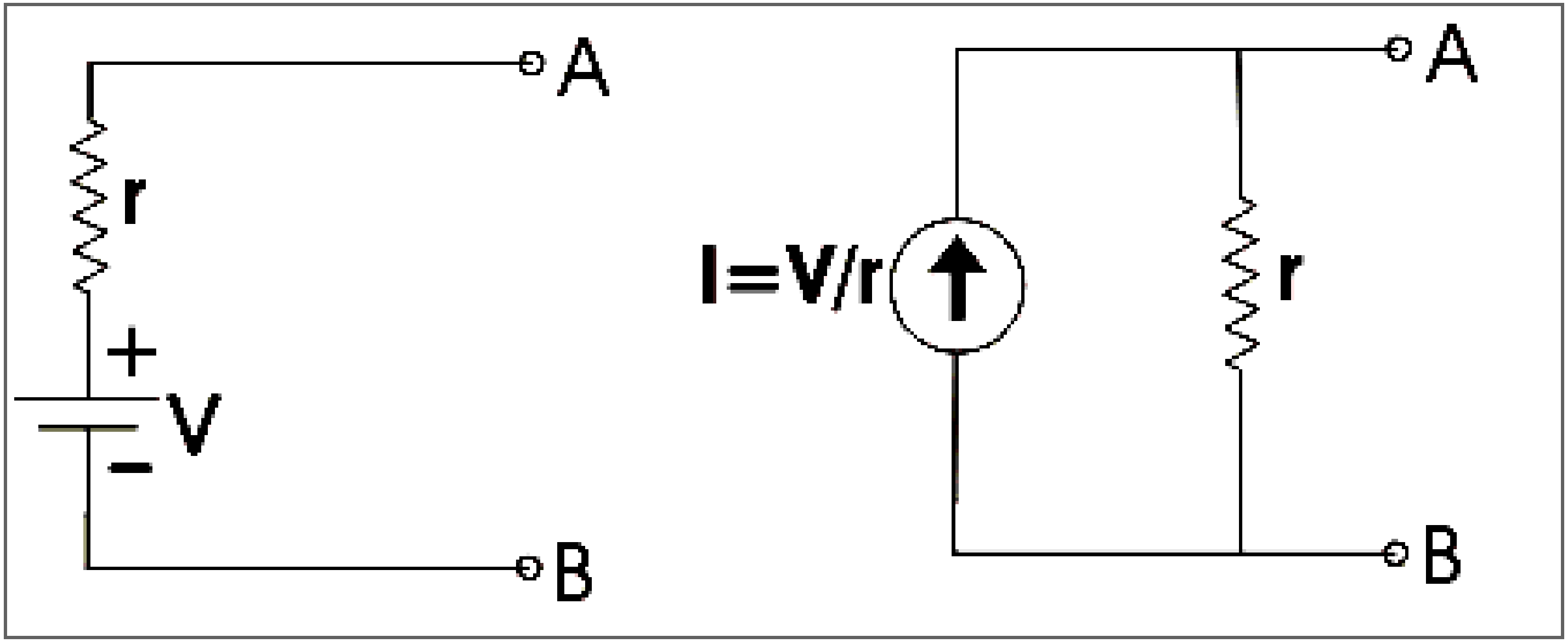
Current Source





Voltage Source → Current Source



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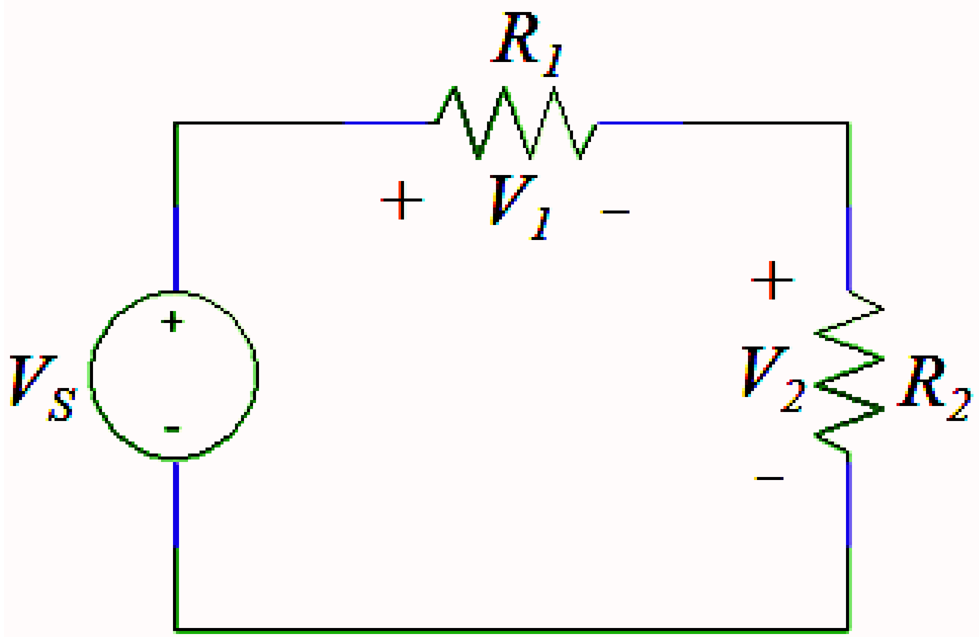
Voltage Divider Rule (VDR)

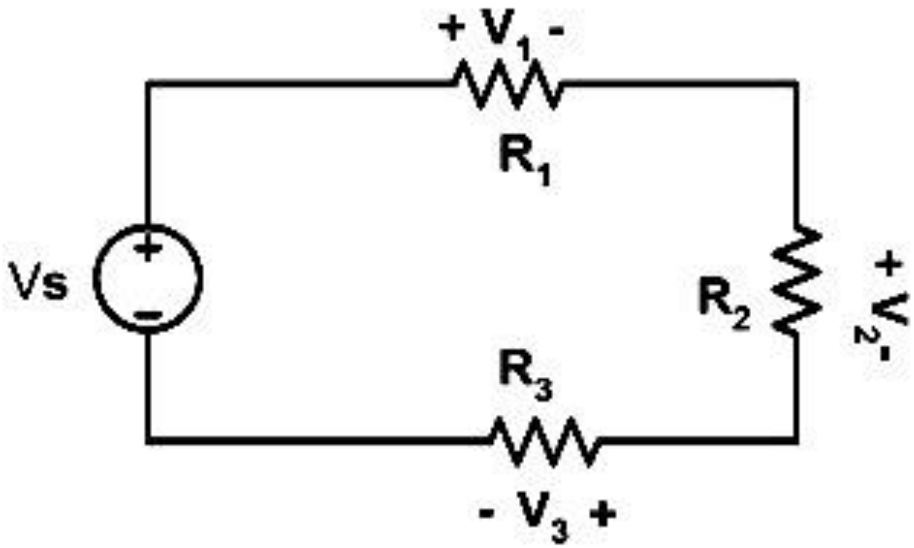
- To calculate the voltage drop across each resistor, use the following equations:



$$V_1 = \frac{R_1}{R_1 + R_2} V_S$$

$$V_2 = \frac{R_2}{R_1 + R_2} V_S$$

Voltage Divider Rule (VDR)



$$V_1 = \frac{R_1}{R_1+R_2+R_3} V_s$$

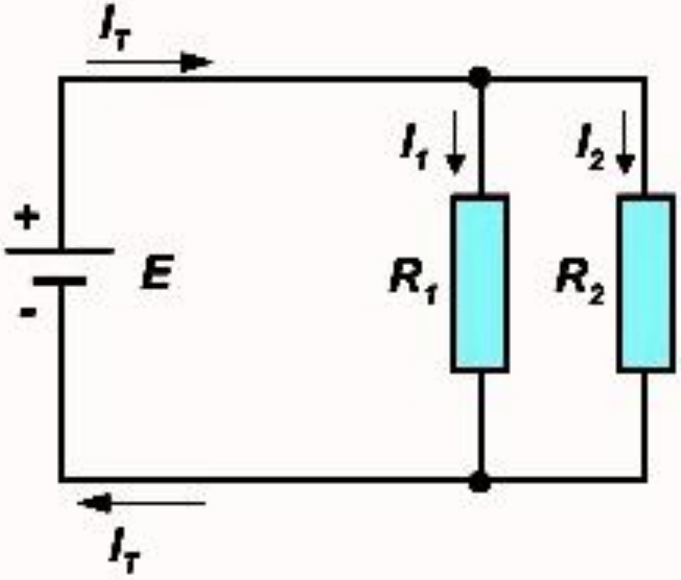
$$V_2 = \frac{R_2}{R_1+R_2+R_3} V_s$$

$$V_3 = \frac{R_3}{R_1+R_2+R_3} V_s$$

Current Divider Rule (CDR)

In parallel circuits the current I_T divides up through the various branch networks, I_1, I_2.

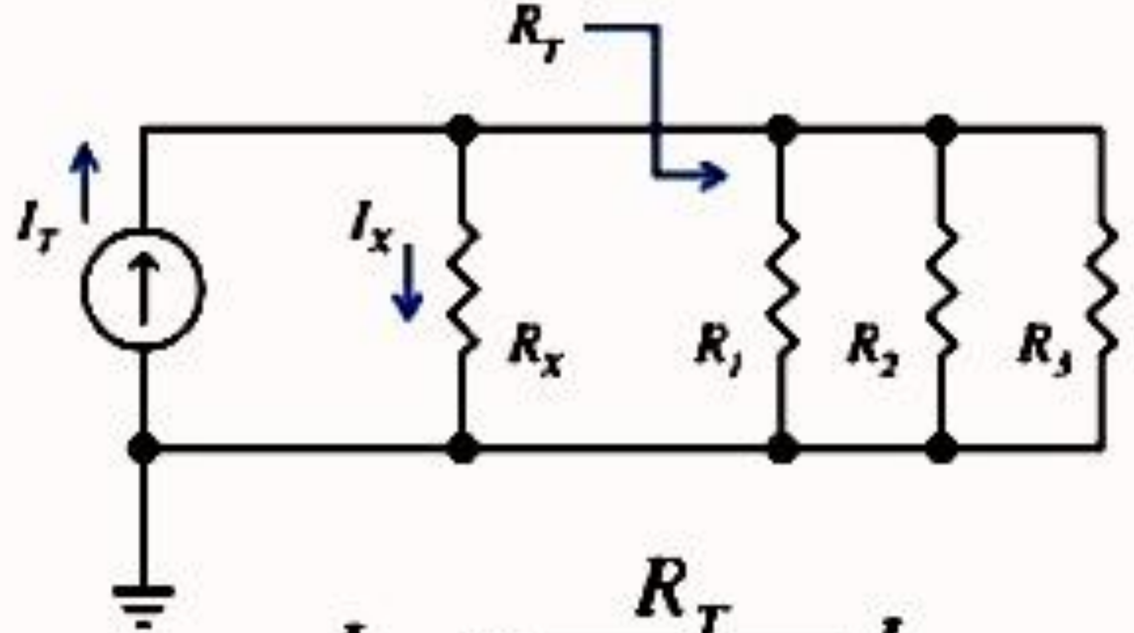
The ratio between any two branch currents is the inverse ratio of the branch resistances.



$$I_1 = I_T \frac{R_2}{R_1 + R_2}$$

$$I_2 = I_T \frac{R_1}{R_1 + R_2}$$

Current Divider Rule (CDR)



$$I_X = \frac{R_T}{R_X + R_T} I_T$$

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Network Theorems

- **Superposition Theorem** *(Ref: Boylestad: Page 345)*
- **Millman's Theorem** *(Ref: Boylestad: Page 376)*
- **Reciprocity Theorem** *(Ref: Boylestad: Page 381)*
- **Thévenin's Theorem** *(Ref: Boylestad: Page 353)*
- **Norton's Theorem** *(Ref: Boylestad: Page 363)*
- **Maximum Power Transfer Theorem** *(Ref: Boylestad: Page 367)*

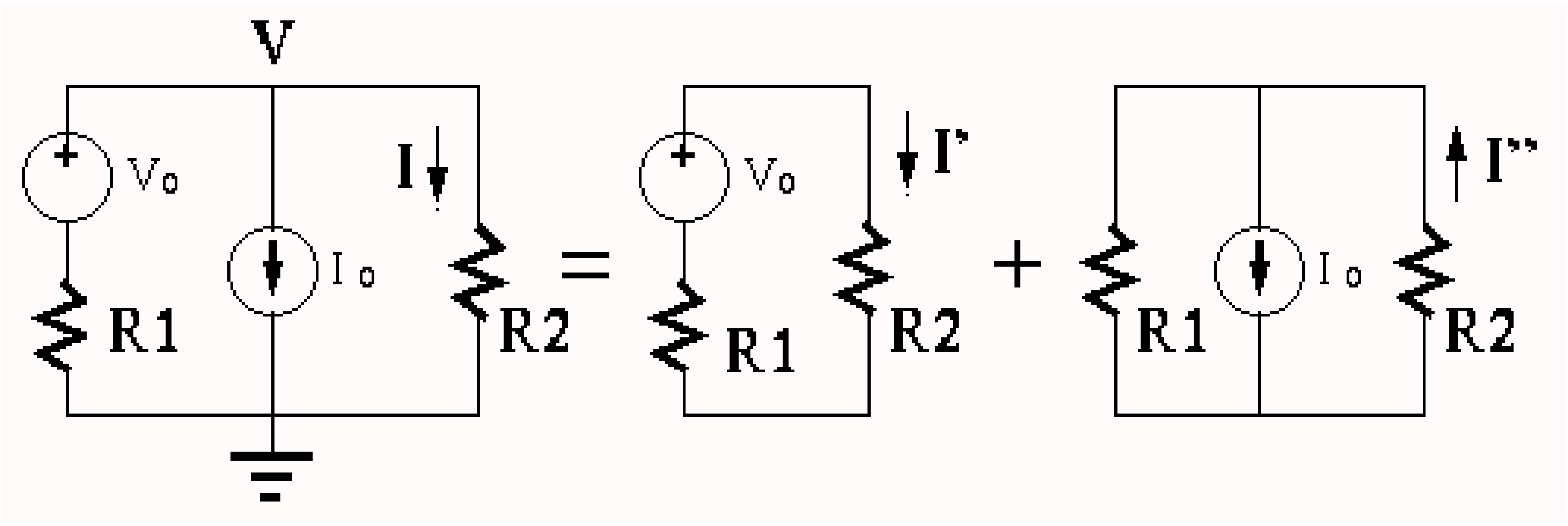
Superposition Theorem

The current through, or voltage across, any element of a network is equal to the algebraic sum of the currents or voltages produced independently by each source.

In general, the theorem can be used to do the following:

- Analyze networks that have two or more sources that are not in series or parallel.
- Reveal the effect of each source on a particular quantity of interest.
- For sources of different types, apply a separate analysis for each type, with the total result simply the algebraic sum of the results.

Superposition Theorem

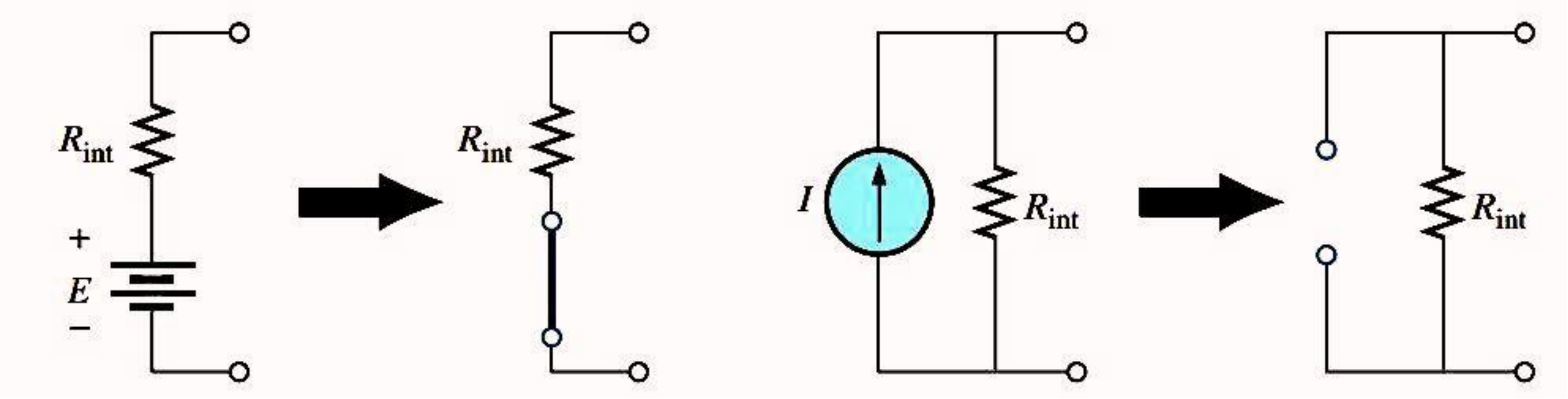


Superposition Theorem

- when removing a voltage source from a network schematic, replace it with a direct connection (short circuit) of zero ohms. Any internal resistance associated with the source must remain in the network.

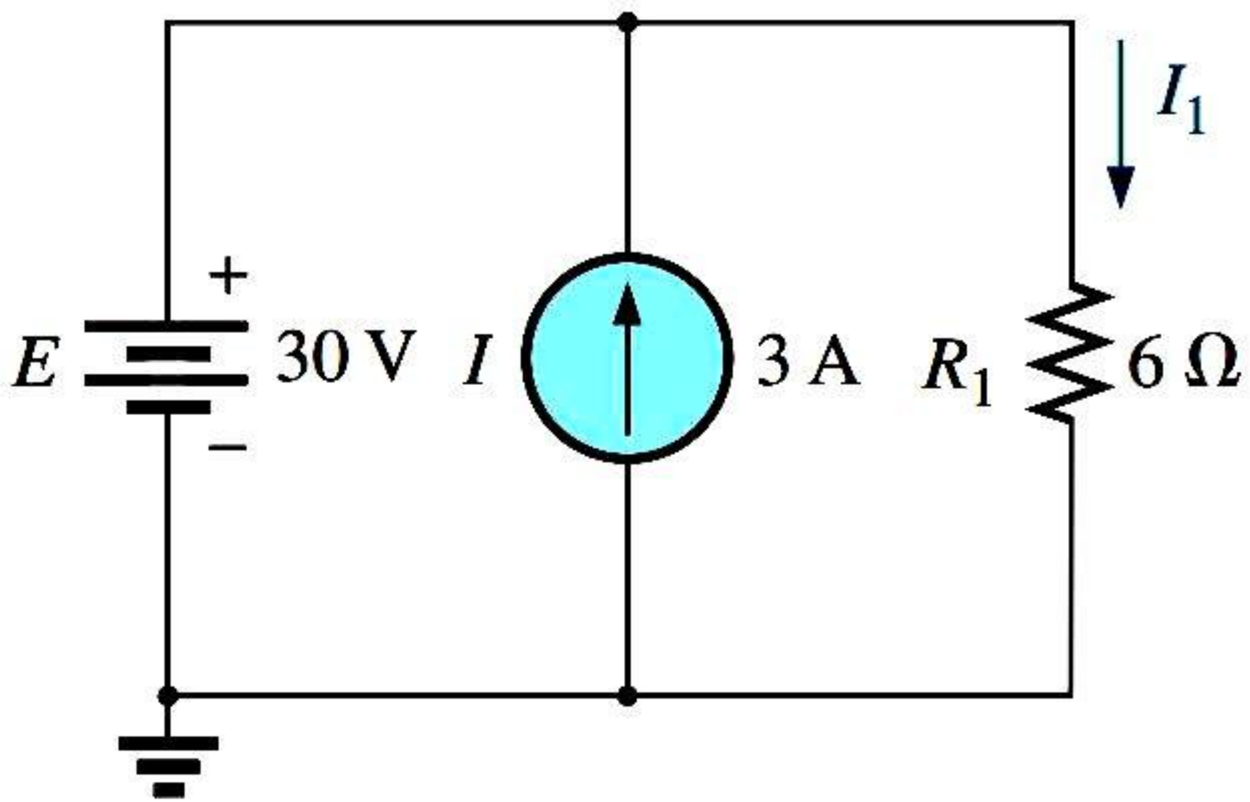
- when removing a current source from a network schematic, replace it by an open circuit of infinite ohms. Any internal resistance associated with the source must remain in the network.

Superposition Theorem

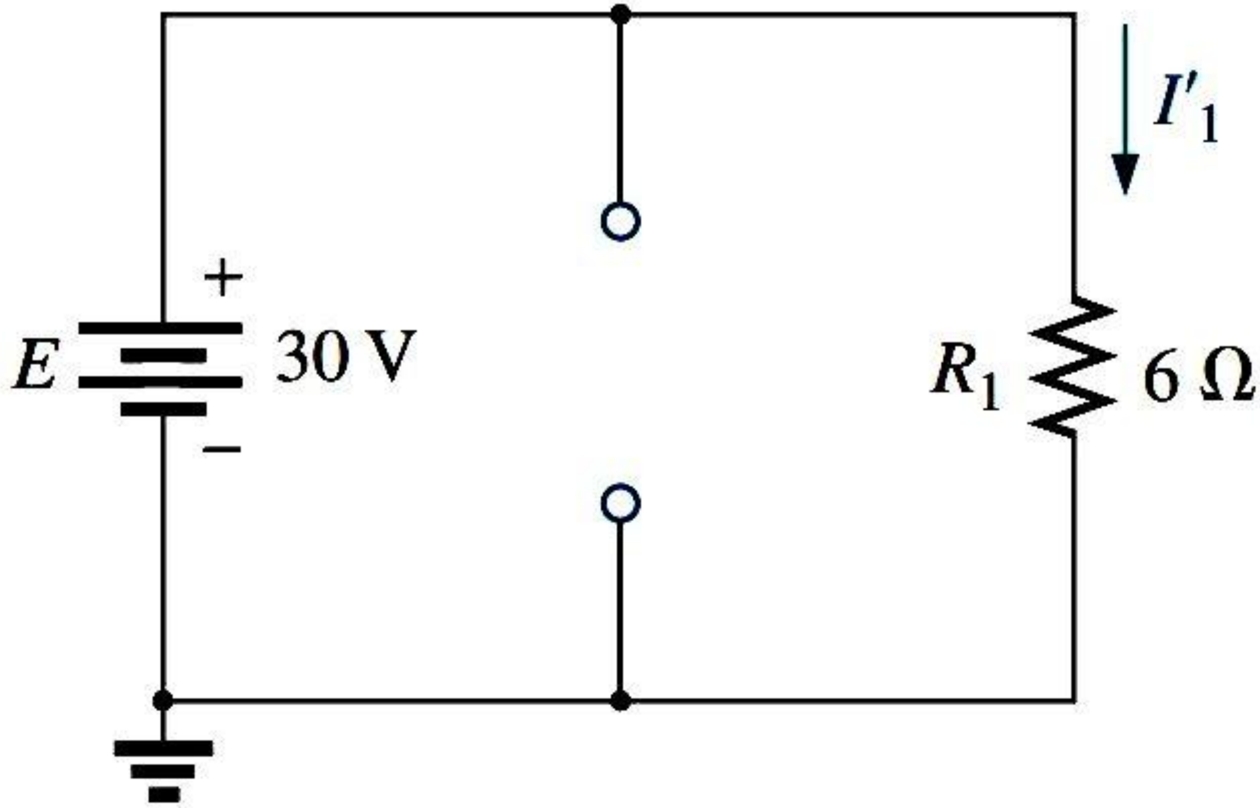


- **Practice Problem for Superposition Theorem:**

Using the superposition theorem, determine current I_1 for the network in the figure.

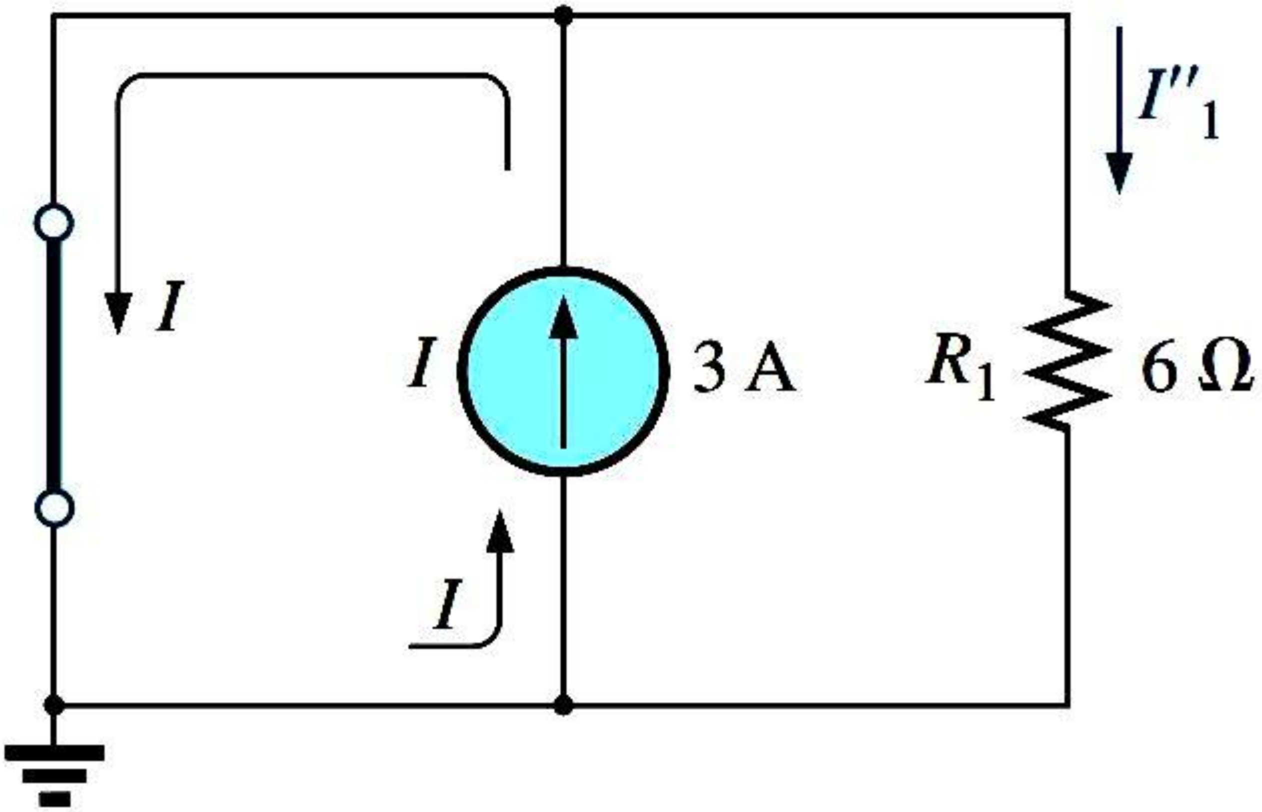


Superposition Theorem



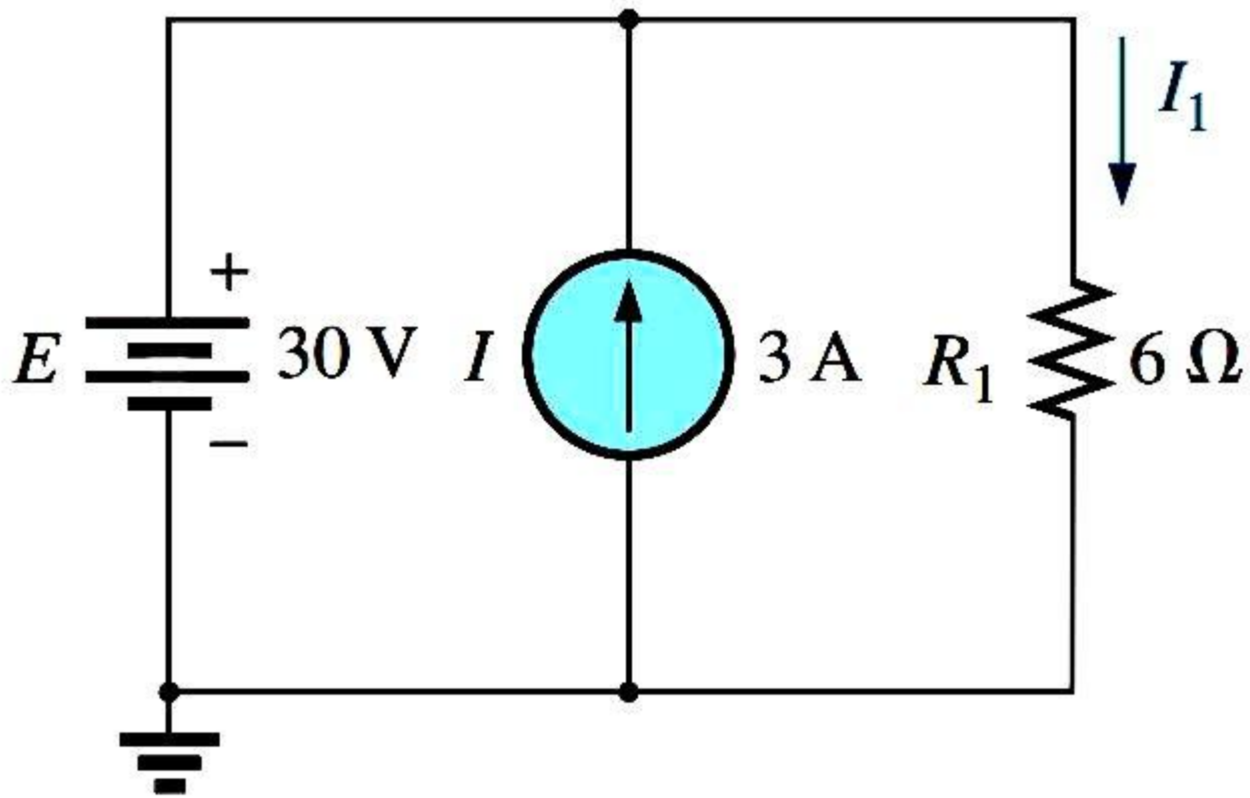
$$I'_1 = V_1 / R_1 = E / R_1 = 30V / 6\Omega = 5A$$

Superposition Theorem



$I''_1 = 0$ A

Superposition Theorem

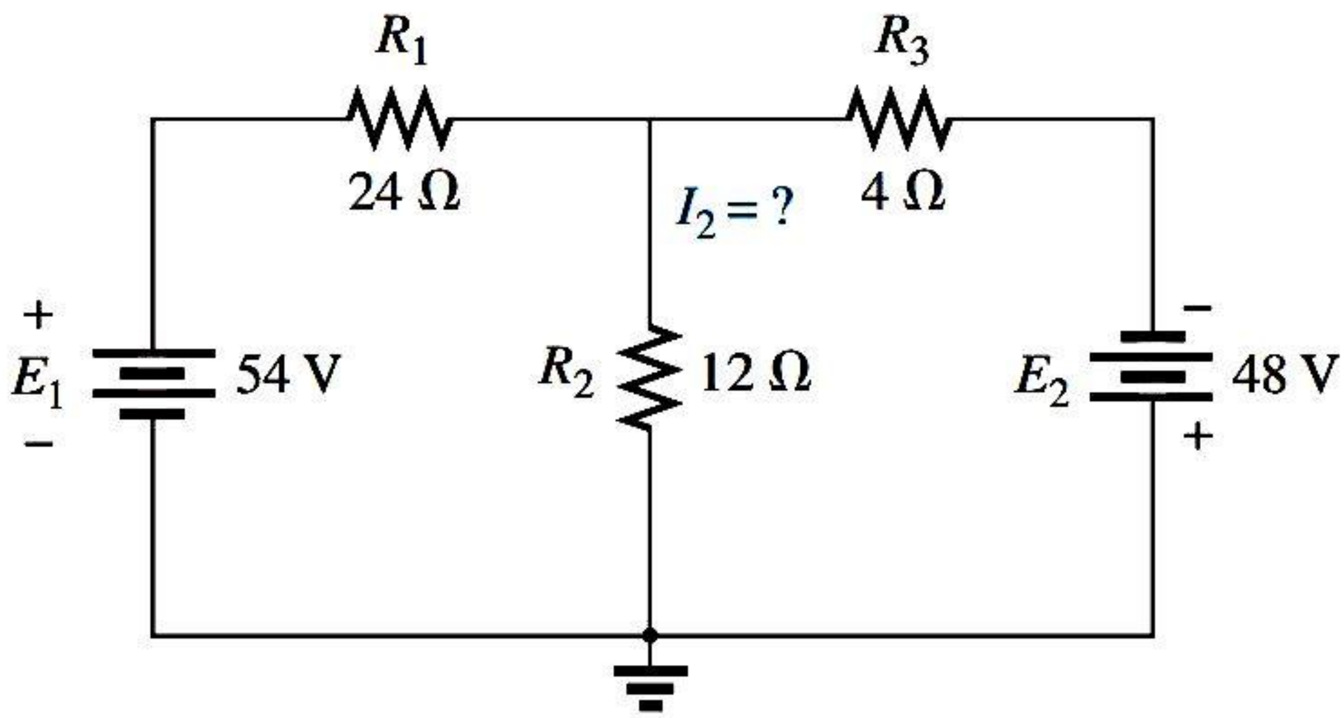


$I_1 = I'_1 + I''_1 = 5\text{ A} + 0\text{ A} = 5\text{ A}$

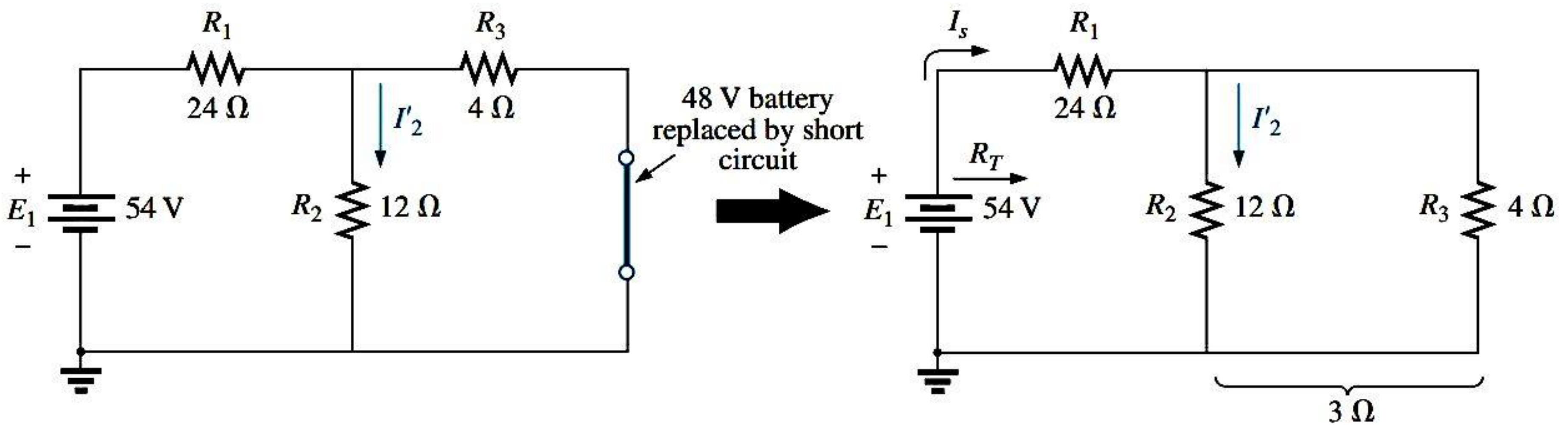
Superposition Theorem

- **Practice Problem for Superposition Theorem:**

Using the superposition theorem, determine the current through the 12 Ω resistor in the figure.



Superposition Theorem



The total resistance seen by the source

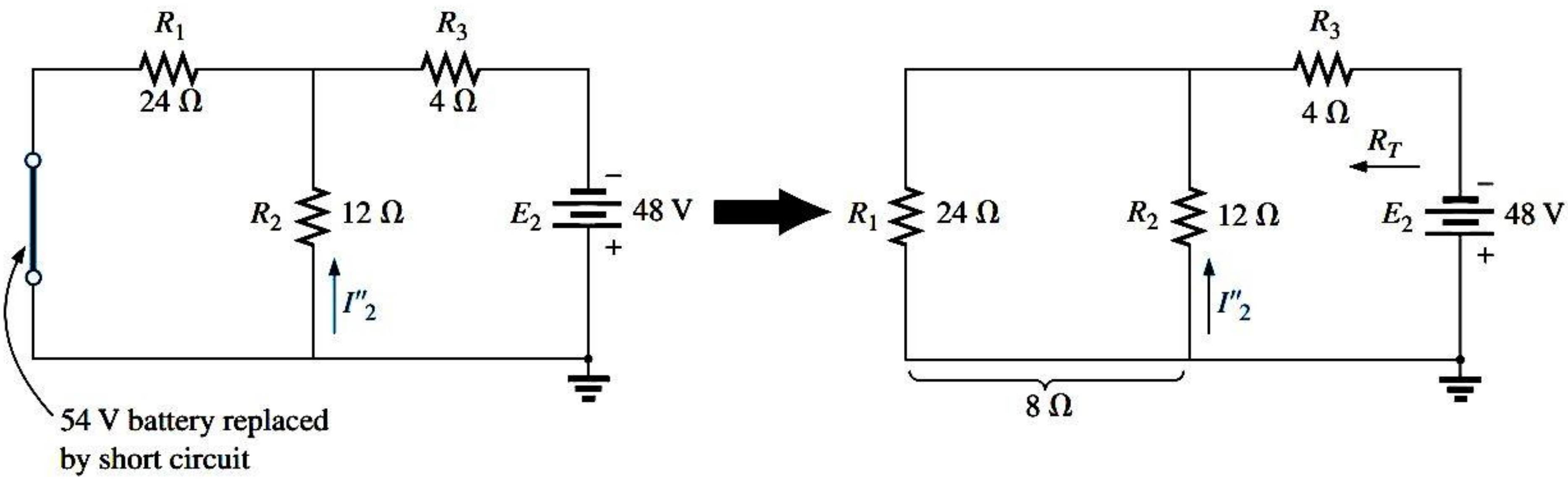
$$R_T = R_1 + R_2 \,||\, R_3 = 24\ \Omega + 12\ \Omega \,||\, 4\ \Omega = 27\ \Omega$$

$$I_s = E_1 / R_T = 54V / 27\Omega = 2\ A$$

Using the current divider rule

$$I'_2 = (R_3 \,.\, I_s) / (R_3 + R_2) = (4\Omega \,.\, 2A) / (4\Omega + 12\Omega) = 0.5\ A$$

Superposition Theorem



The total resistance seen by the source

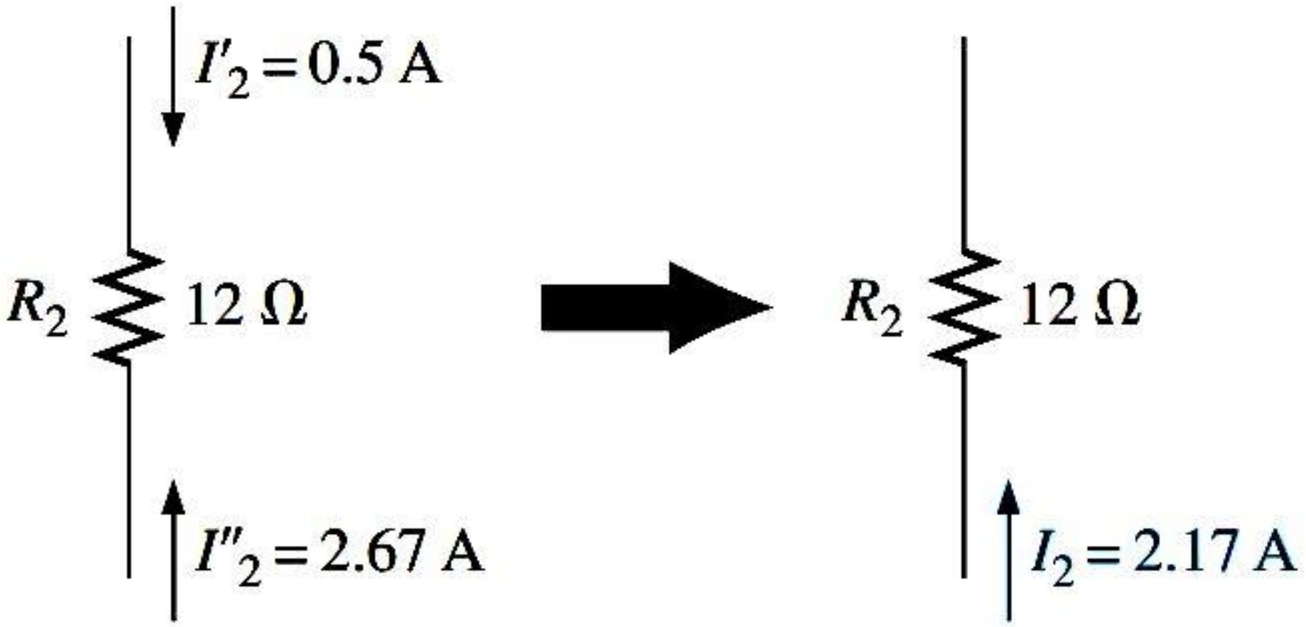
$$R_T = R_3 + R_2 \,||\, R_1 = 4\ \Omega + 12\ \Omega \,||\, 24\ \Omega = 12\ \Omega$$

$$I_s = E_2 / R_T = 48V / 12\Omega = 4\ A$$

Using the current divider rule

$$I''_2 = (R_1 \,.\, I_s) / (R_1 + R_2) = (24\Omega \,.\, 4A) / (24\Omega + 12\Omega) = 2.67A$$

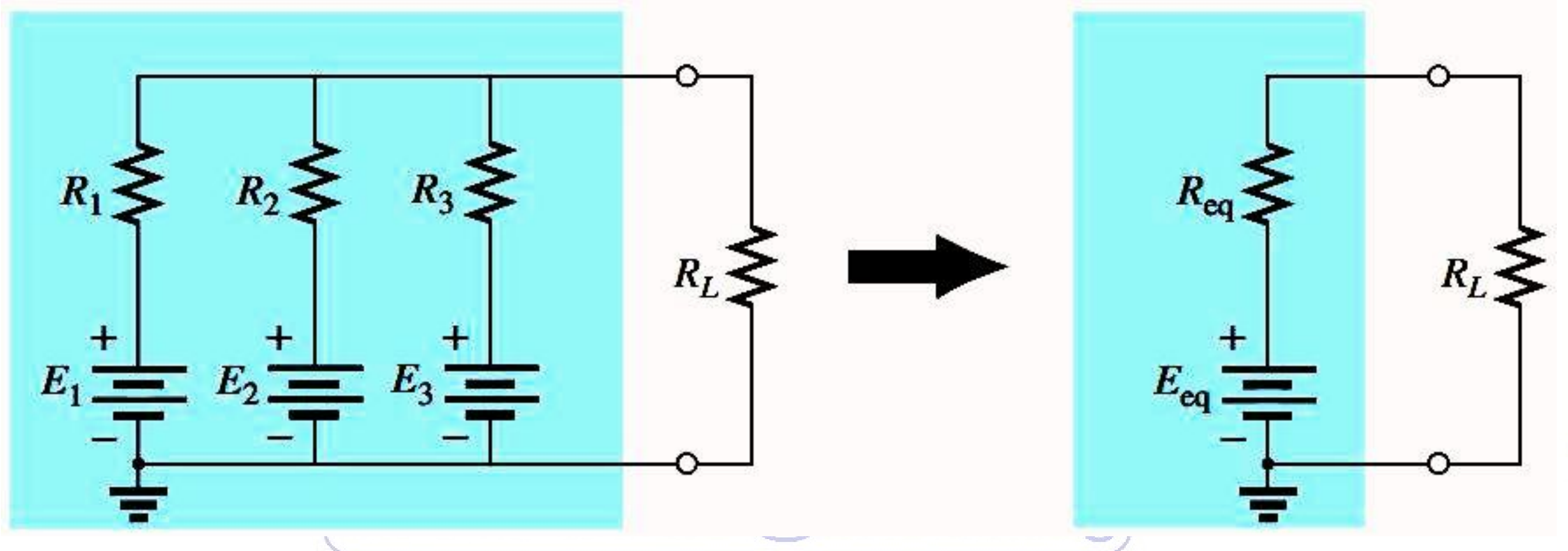
Superposition Theorem



I2= I''2 – I'2 = 2.67 A - 0.5 A = 2.17 A

Millman's Theorem

Any number of parallel voltage sources can be reduced to one voltage source.



Millman's Theorem

STEPS:

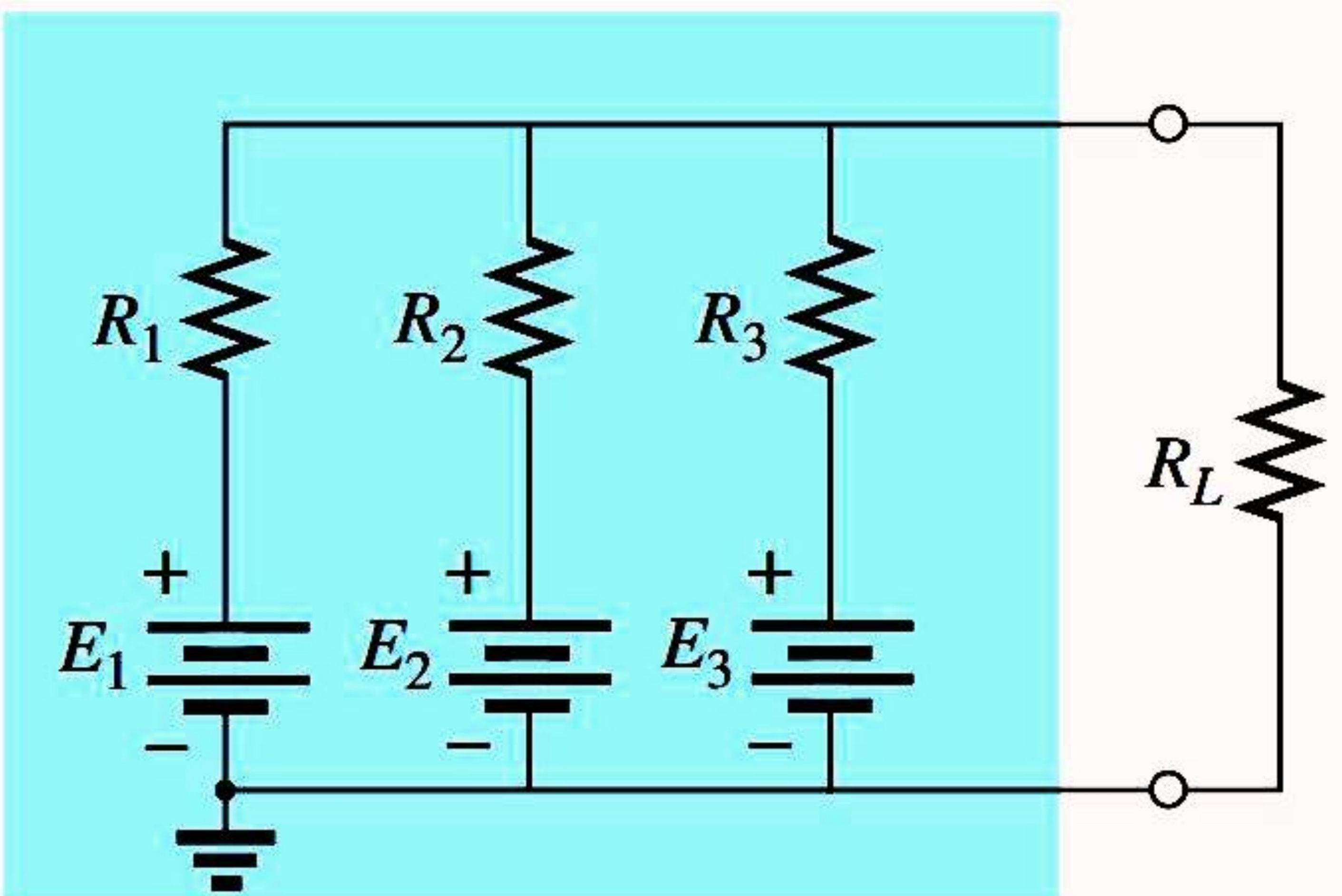
Step 1: Convert all voltage sources to current sources

Step 2: Combine parallel current sources

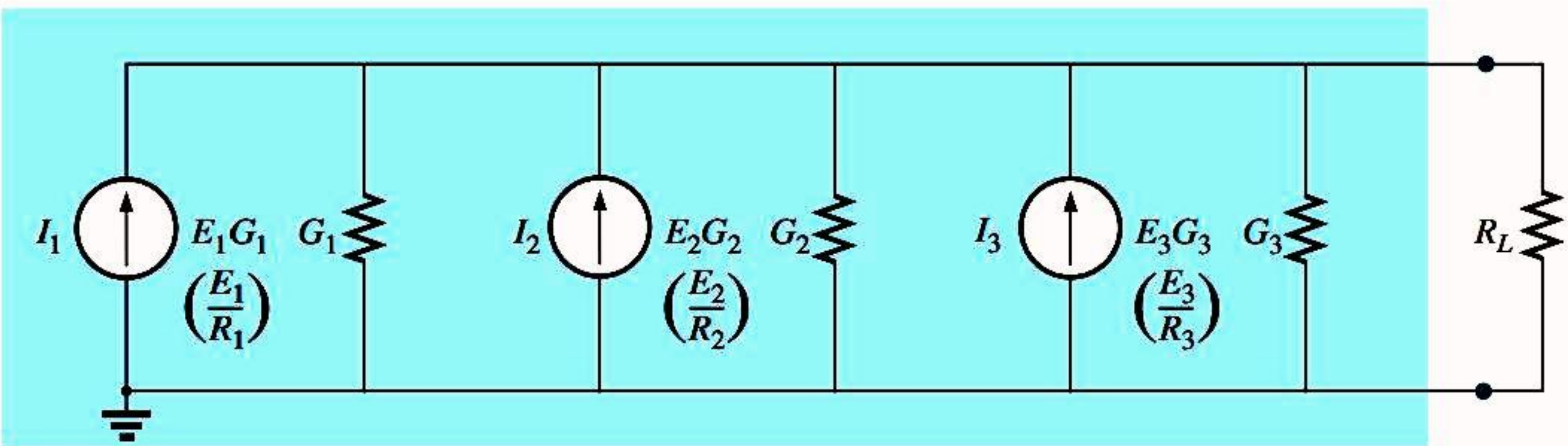
Step 3: Convert the resulting current source to a voltage source

Millman's Theorem

EXAMPLE:

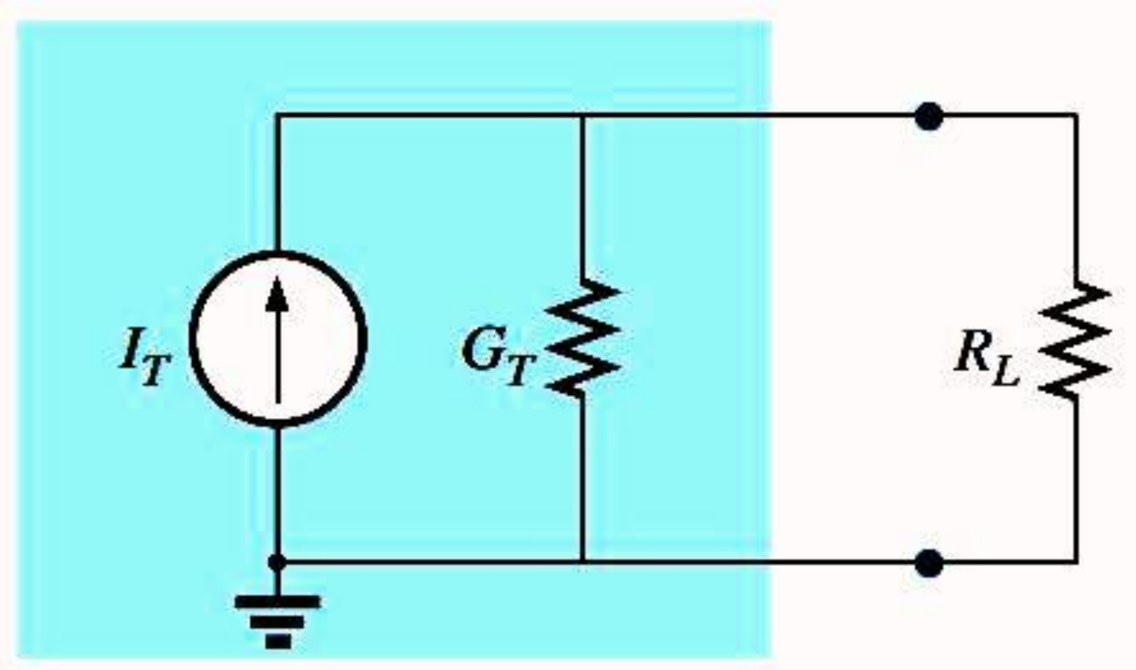


STEP 1: *Convert all voltage sources to current sources*



Millman's Theorem

STEP 2: *Combine parallel current sources*

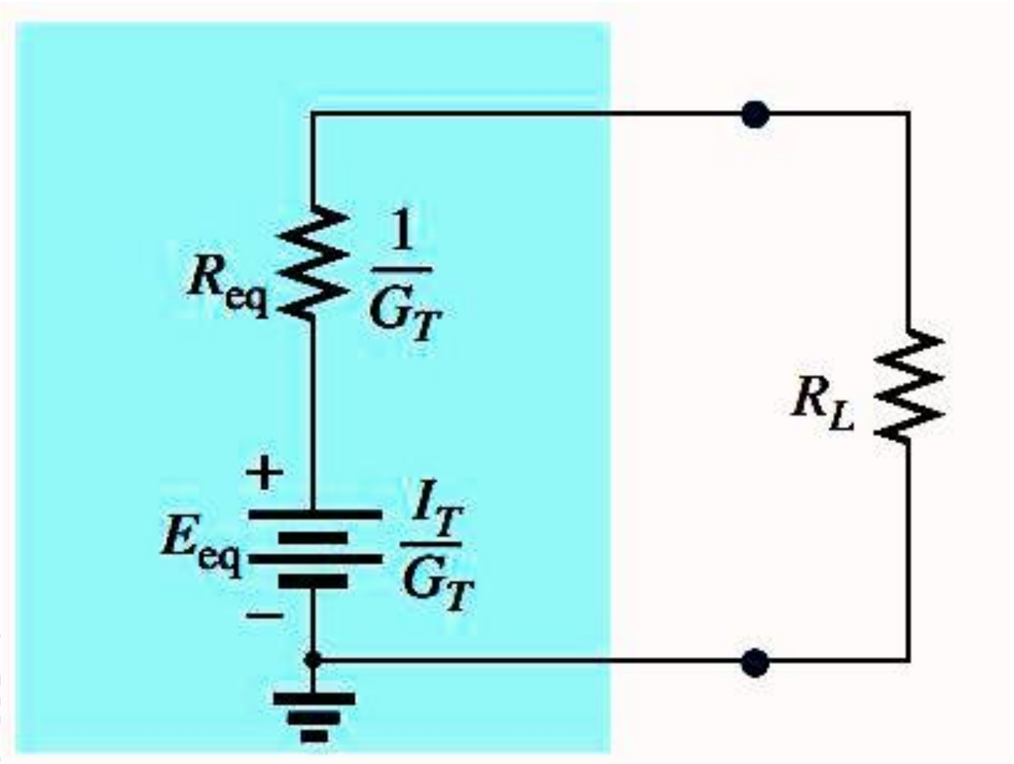


$$I_T = I_1 + I_2 + I_3$$

$$G_T = G_1 + G_2 + G_3$$

Millman's Theorem

STEP 3: *Convert the resulting current source to a voltage source*



Millman's Theorem

$$E_{eq} = \frac{I_T}{G_T} = \frac{\pm I_1 \pm I_2 \pm I_3 \pm \cdots \pm I_N}{G_1 + G_2 + G_3 + \cdots + G_N}$$

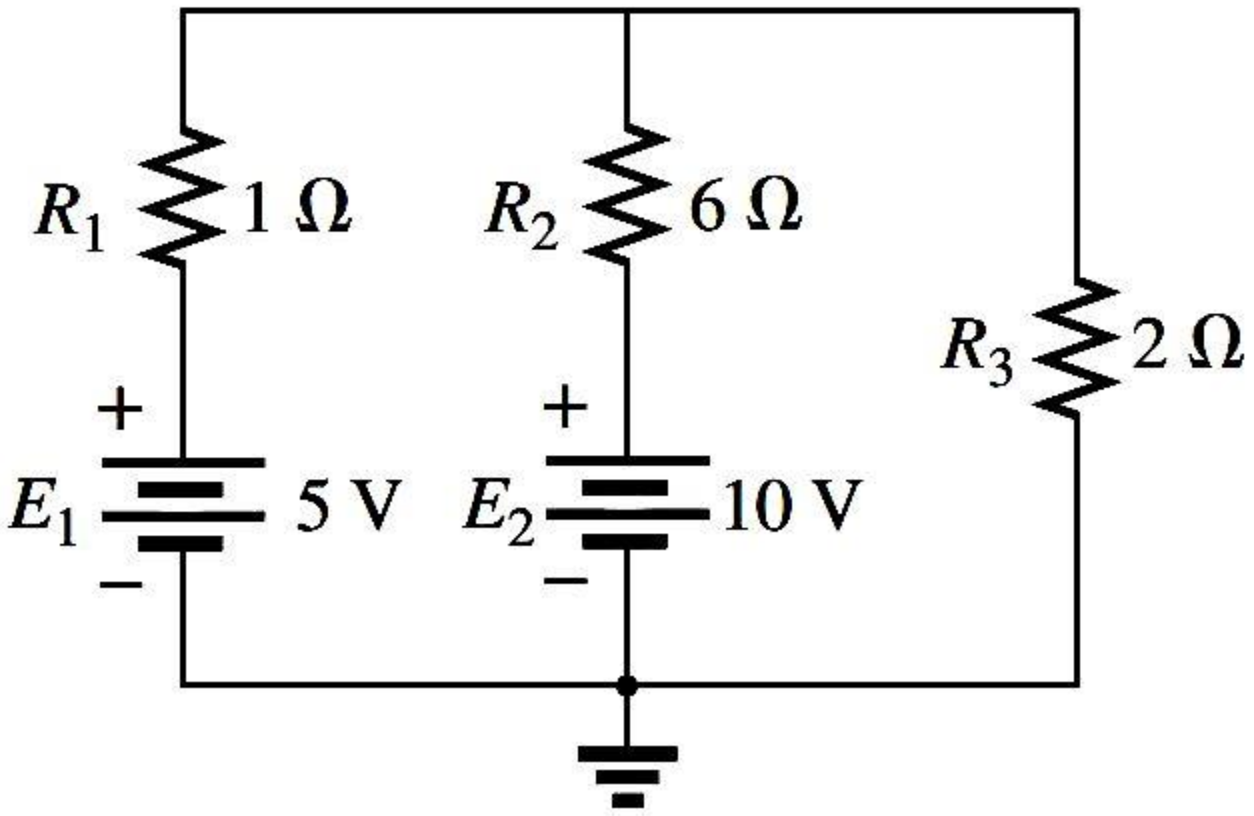
$$\boxed{E_{eq} = \frac{\pm E_1G_1 \pm E_2G_2 \pm E_3G_3 \pm \cdots \pm E_NG_N}{G_1 + G_2 + G_3 + \cdots + G_N}}$$

$$\boxed{R_{eq} = \frac{1}{G_T} = \frac{1}{G_1 + G_2 + G_3 + \cdots + G_N}}$$

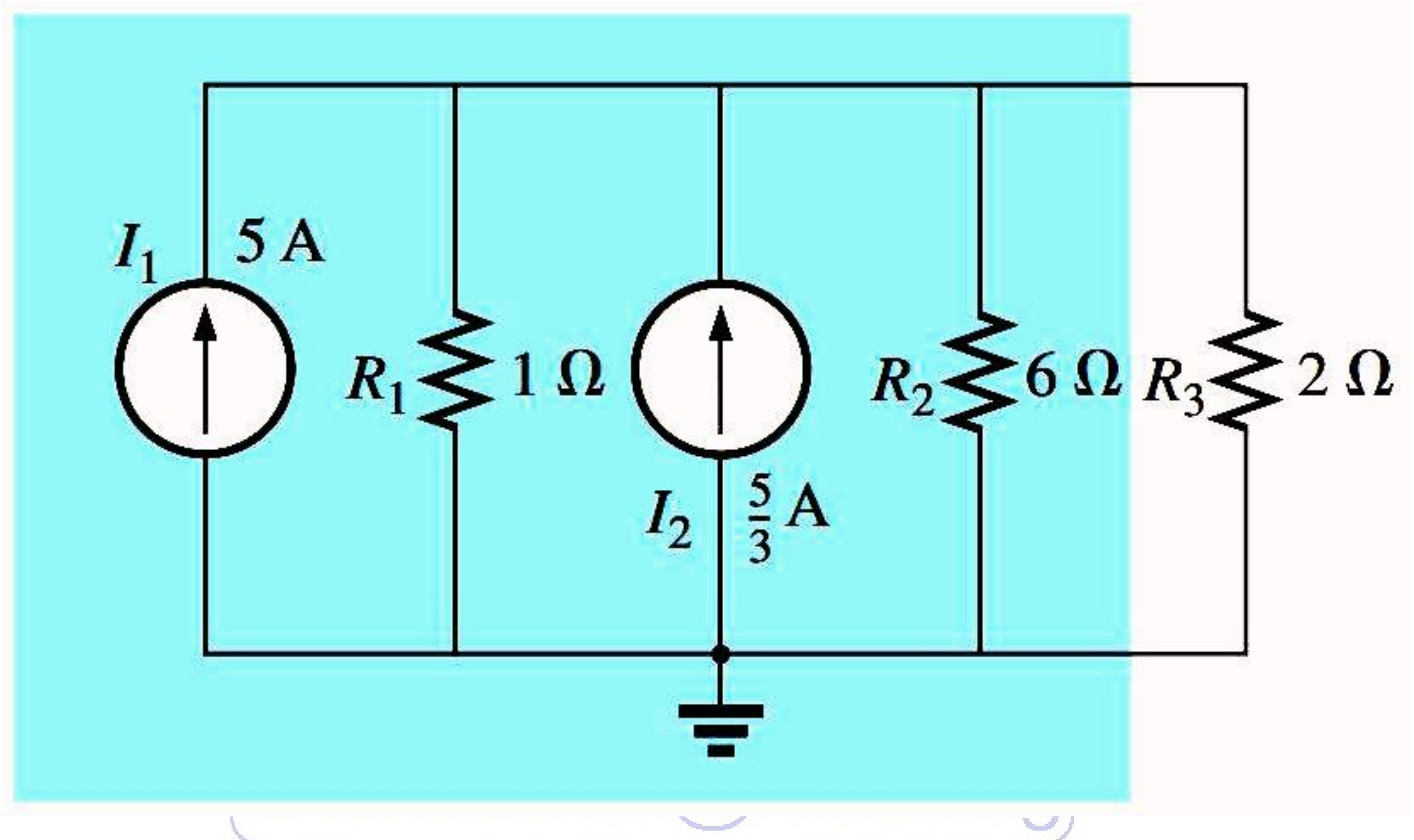
$$\boxed{E_{eq} = \frac{\pm \dfrac{E_1}{R_1} \pm \dfrac{E_2}{R_2} \pm \dfrac{E_3}{R_3} \pm \cdots \pm \dfrac{E_N}{R_N}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}}}$$

$$\boxed{R_{eq} = \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + \cdots + \dfrac{1}{R_N}}}$$

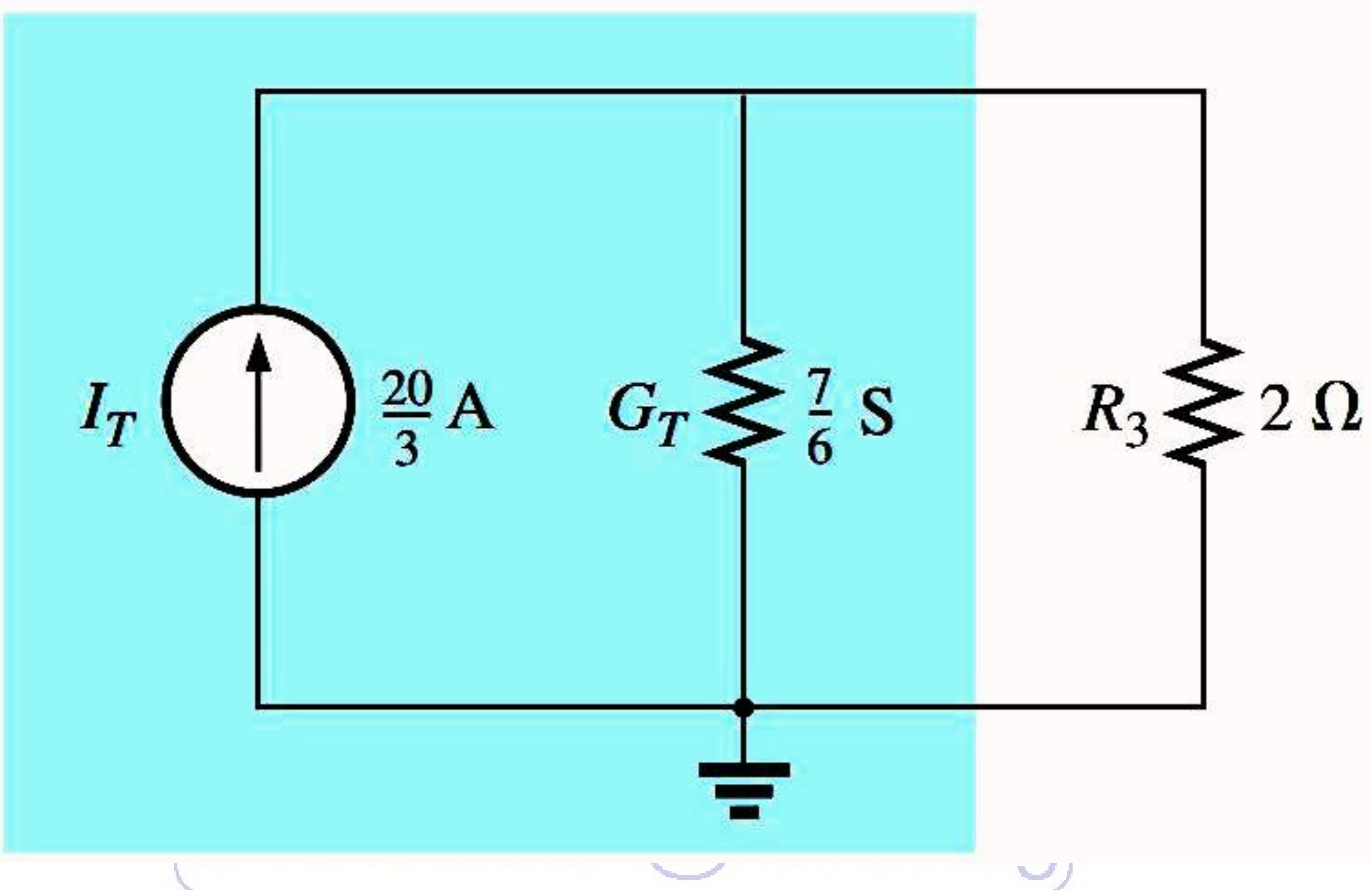
Millman's Theorem



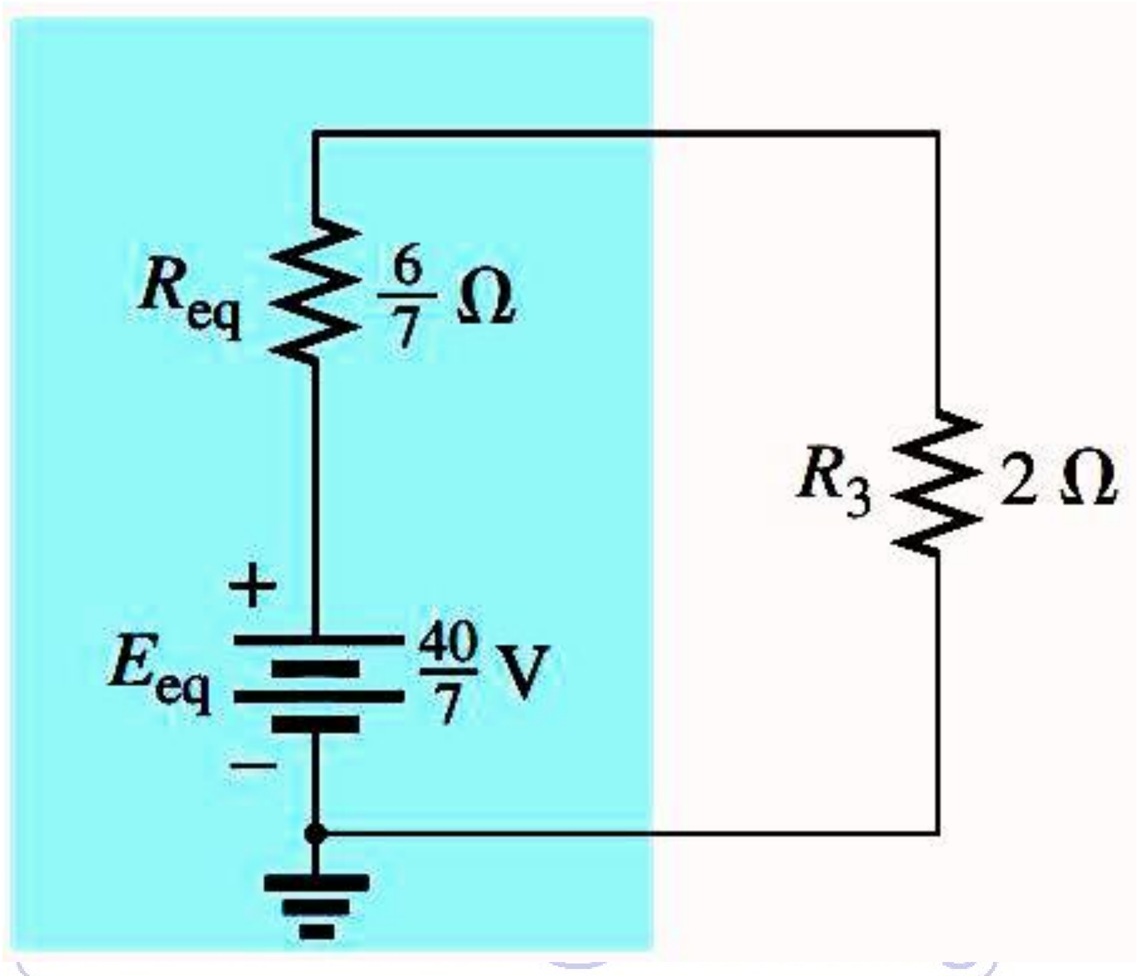
Millman’s Theorem



Millman's Theorem



Millman's Theorem



Millman’s Theorem

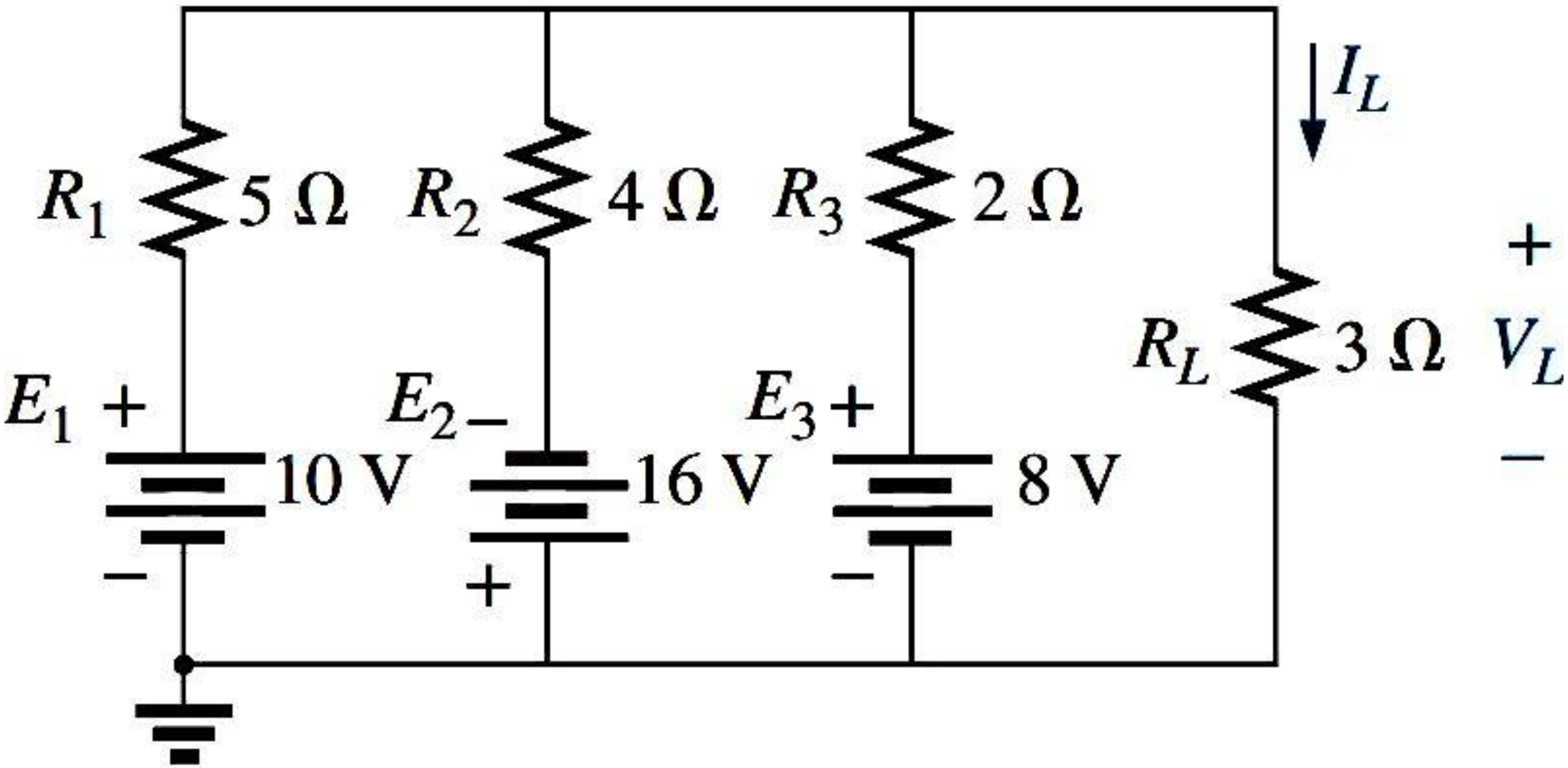
$$E_{\text{eq}} = \frac{+\dfrac{5\text{ V}}{1\,\Omega} + \dfrac{10\text{ V}}{6\,\Omega}}{\dfrac{1}{1\,\Omega} + \dfrac{1}{6\,\Omega}} = \frac{\dfrac{30\text{ V}}{6\,\Omega} + \dfrac{10\text{ V}}{6\,\Omega}}{\dfrac{6}{6\,\Omega} + \dfrac{1}{6\,\Omega}} = \mathbf{\frac{40}{7}\,V}$$

$$R_{\text{eq}} = \frac{1}{\dfrac{1}{1\,\Omega} + \dfrac{1}{6\,\Omega}} = \frac{1}{\dfrac{6}{6\,\Omega} + \dfrac{1}{6\,\Omega}} = \frac{1}{\dfrac{7}{6}\,\text{S}} = \mathbf{\frac{6}{7}\,\Omega}$$

Ref: Boylestad: Article 9.6: Page 376

Millman's Theorem

Using Millman's theorem, find the current through and voltage across the resistor R_L



Millman's Theorem

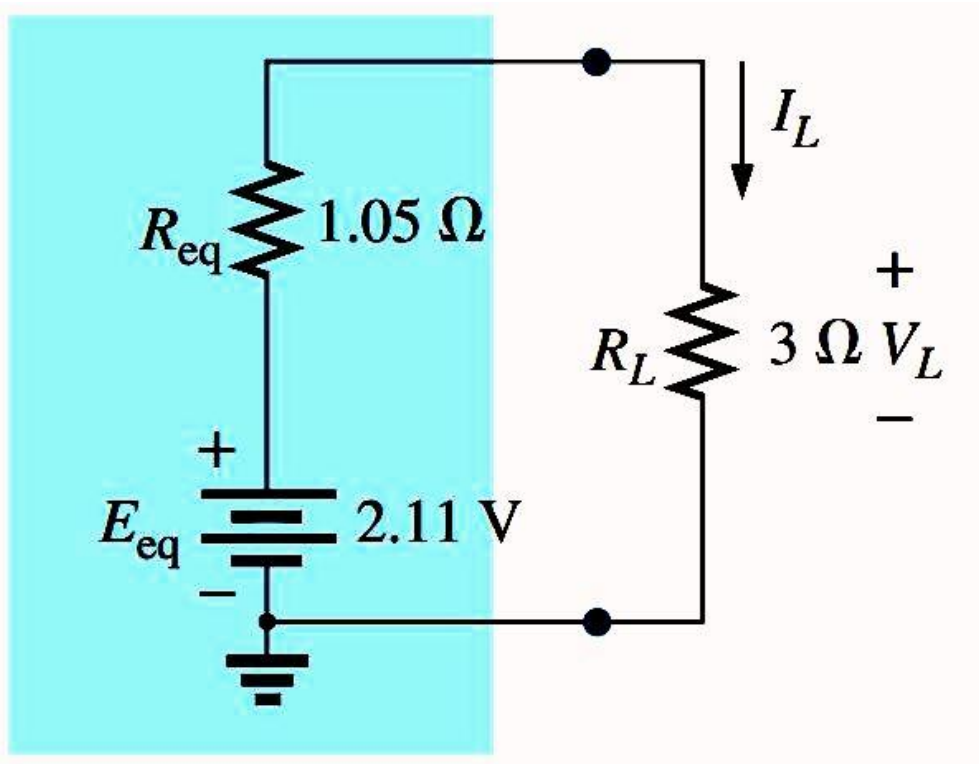
$$E_{eq} = \frac{+\dfrac{E_1}{R_1} - \dfrac{E_2}{R_2} + \dfrac{E_3}{R_3}}{\dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}}$$

$$E_{eq} = \frac{+\dfrac{10\text{ V}}{5\,\Omega} - \dfrac{16\text{ V}}{4\,\Omega} + \dfrac{8\text{ V}}{2\,\Omega}}{\dfrac{1}{5\,\Omega} + \dfrac{1}{4\,\Omega} + \dfrac{1}{2\,\Omega}} = \frac{2\text{ A} - 4\text{ A} + 4\text{ A}}{0.2\text{ S} + 0.25\text{ S} + 0.5\text{ S}}$$

$$= \frac{2\text{ A}}{0.95\text{ S}} = \mathbf{2.11\text{ V}}$$

Millman's Theorem

$$R_{eq} = \frac{1}{\dfrac{1}{5\,\Omega} + \dfrac{1}{4\,\Omega} + \dfrac{1}{2\,\Omega}} = \frac{1}{0.95\text{ S}} = \mathbf{1.05\,\Omega}$$

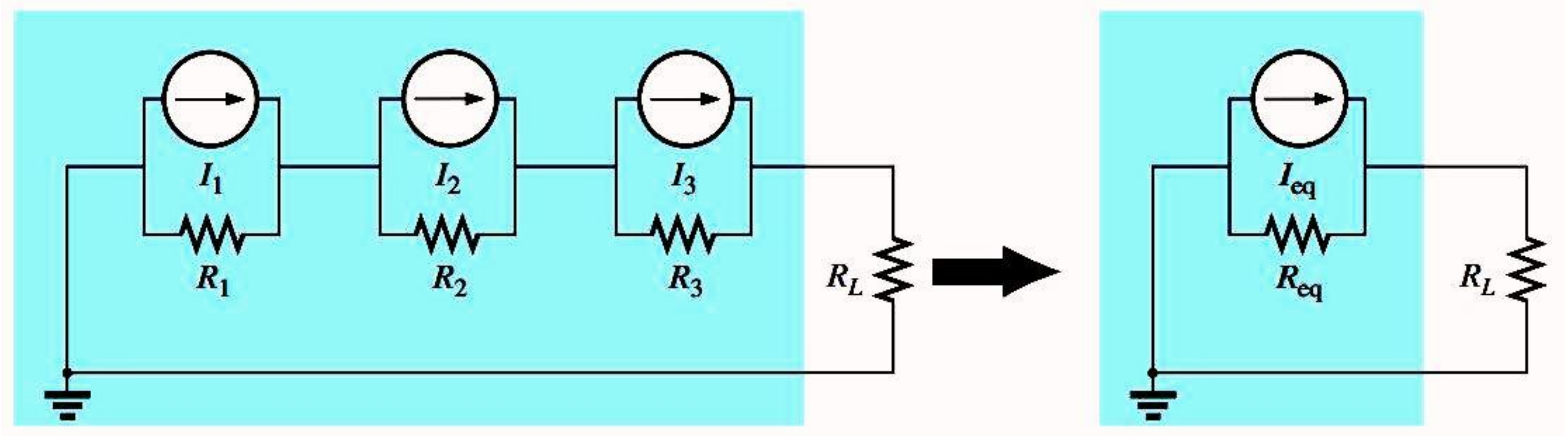


Millman's Theorem

$$I_L = \frac{2.11\ \text{V}}{1.05\ \Omega + 3\ \Omega} = \frac{2.11\ \text{V}}{4.05\ \Omega} = \mathbf{0.52\ A}$$

$$V_L = I_L R_L = (0.52\ \text{A})(3\ \Omega) = \mathbf{1.56\ V}$$

Duel Effect of Millman's Theorem



$$\boxed{I_{eq} = \frac{\pm I_1R_1 \pm I_2R_2 \pm I_3R_3}{R_1 + R_2 + R_3}}$$

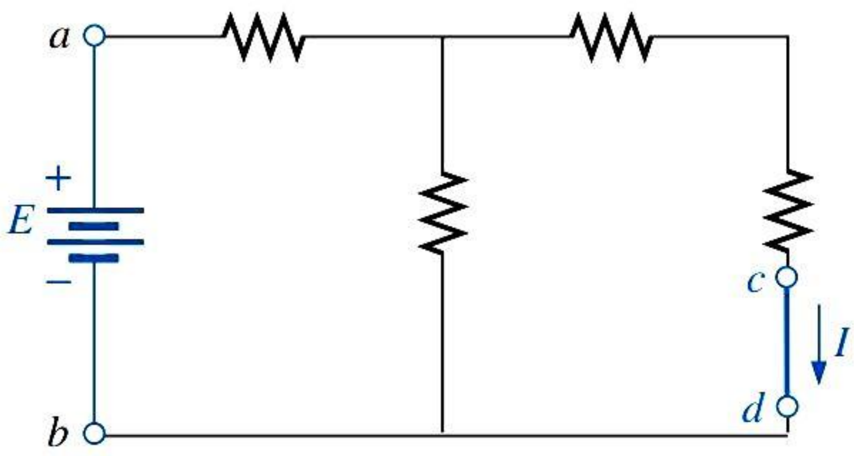
$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

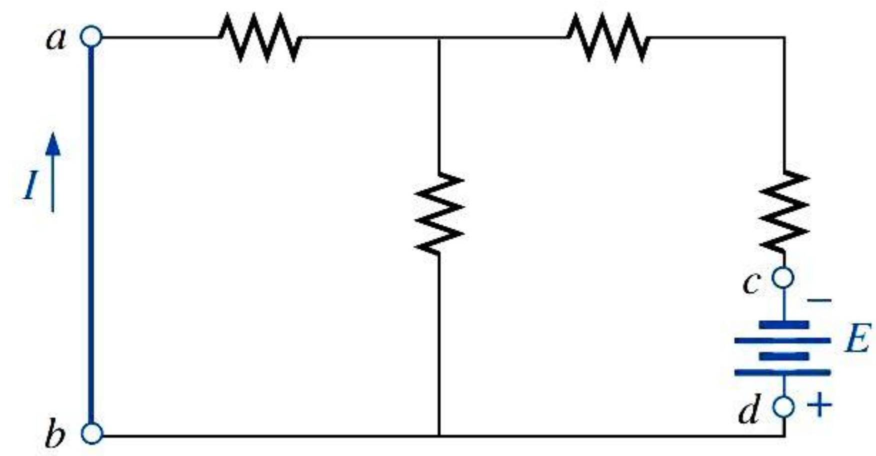
Reciprocity Theorem

The current I in any branch of a network, due to a single voltage source E anywhere else in the network, will equal the current through the branch in which the source was originally located if the source is placed in the branch in which the current I was originally measured.

(The reciprocity theorem is applicable only to single-source networks)

Reciprocity Theorem





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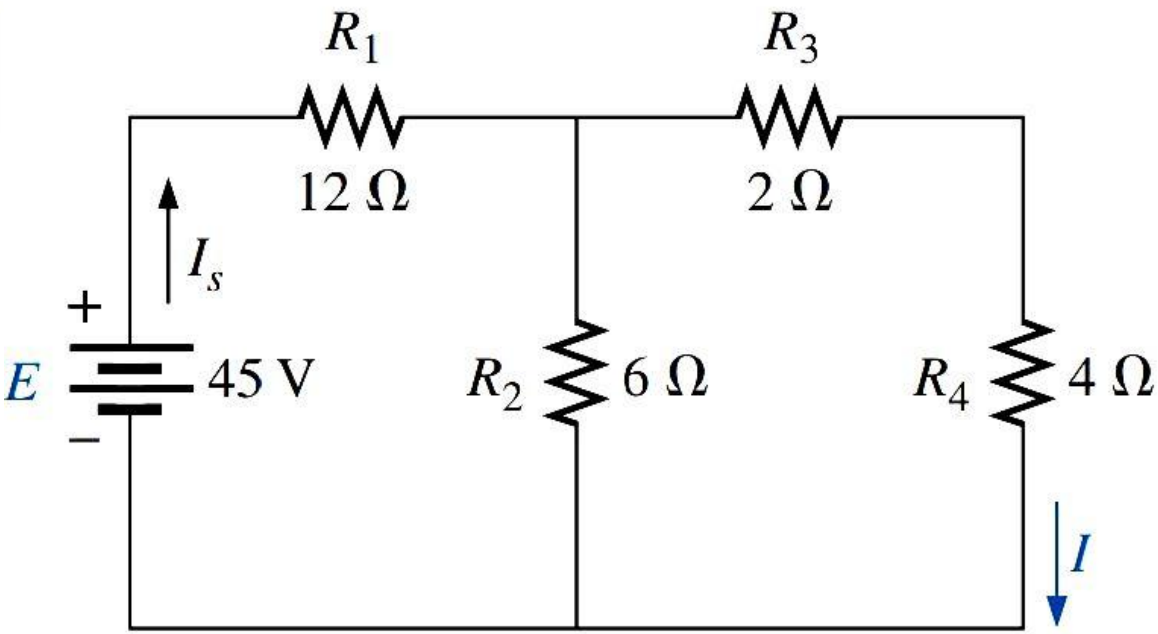
Reciprocity Theorem

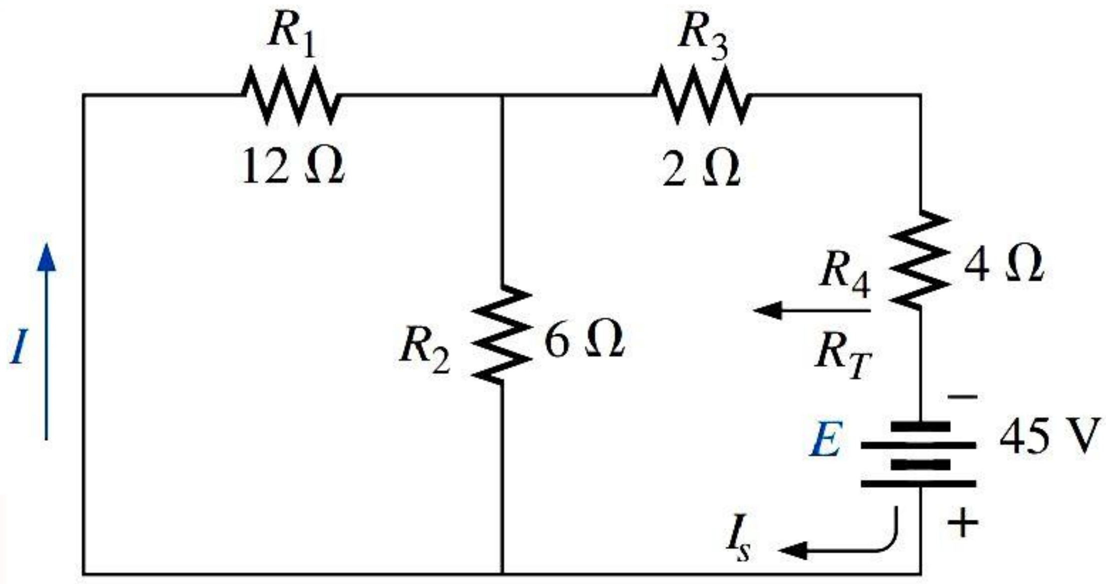


$$I = \frac{3\text{ A}}{2} = \mathbf{1.5\text{ A}}$$

$$R_T = R_1 + R_2 \parallel (R_3 + R_4) = 12\ \Omega + 6\ \Omega \parallel (2\ \Omega + 4\ \Omega)$$
$$= 12\ \Omega + 6\ \Omega \parallel 6\ \Omega = 12\ \Omega + 3\ \Omega = 15\ \Omega$$

$$I_s = \frac{E}{R_T} = \frac{45\text{ V}}{15\ \Omega} = 3\text{ A}$$

Reciprocity Theorem

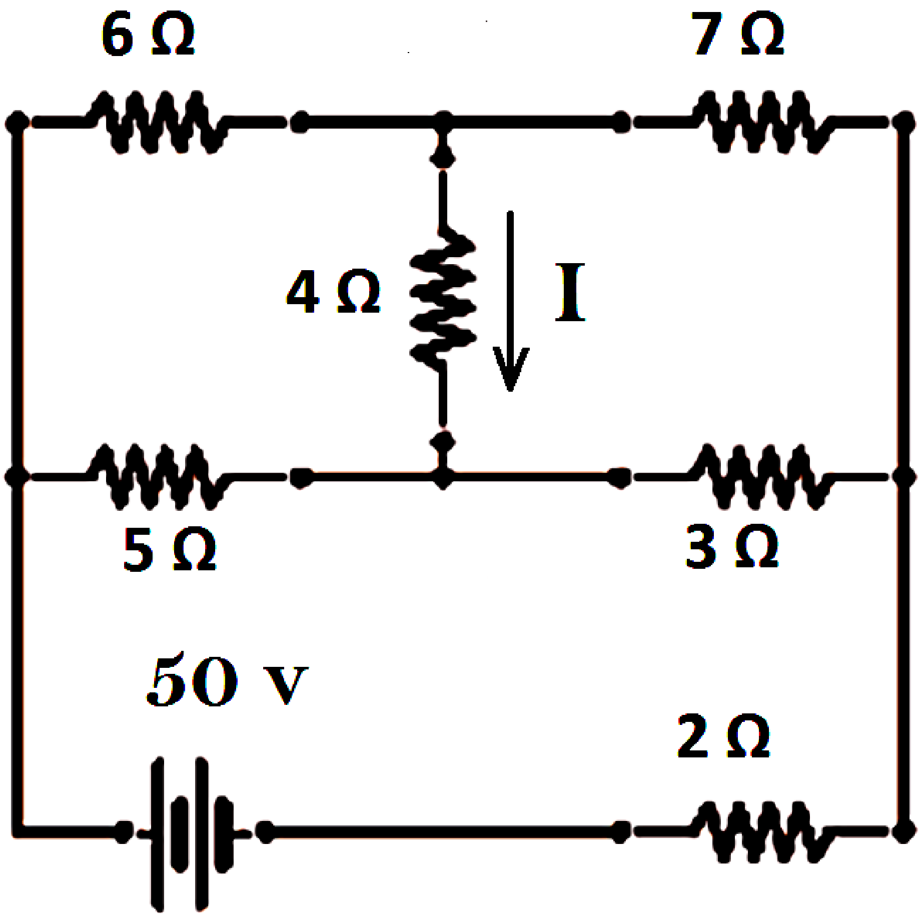


$$R_T = R_4 + R_3 + R_1 \parallel R_2$$
$$= 4\,\Omega + 2\,\Omega + 12\,\Omega \parallel 6\,\Omega = 10\,\Omega$$

$$I_s = \frac{E}{R_T} = \frac{45\text{ V}}{10\,\Omega} = 4.5\text{ A}$$

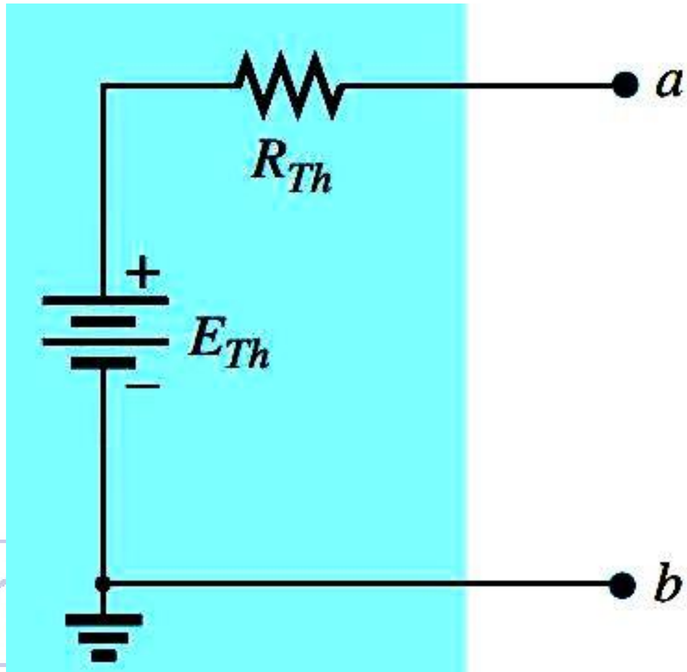
$$I = \frac{(6\,\Omega)(4.5\text{ A})}{12\,\Omega + 6\,\Omega} = \frac{4.5\text{ A}}{3} = \mathbf{1.5\text{ A}}$$

Practice for Reciprocity Theorem



Thévenin's Theorem

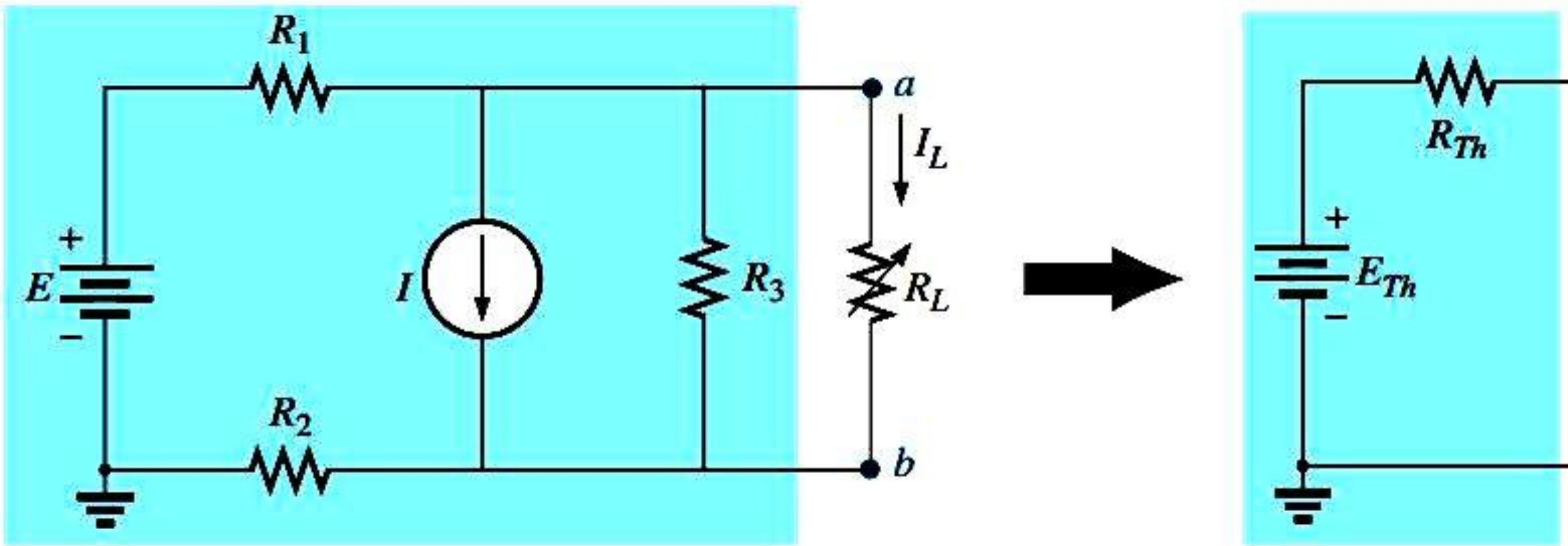
Any two-terminal dc network can be replaced by an equivalent circuit consisting solely of a voltage source and a series resistor as shown in the figure.



Applications of Thévenin’s Theorem:

- Analyze networks with sources that are not in series or parallel.
- Reduce the number of components required to establish the same characteristics at the output terminals.
- Investigate the effect of changing a particular component on the behavior of a network without having to analyze the entire network after each change.

Thévenin's Theorem

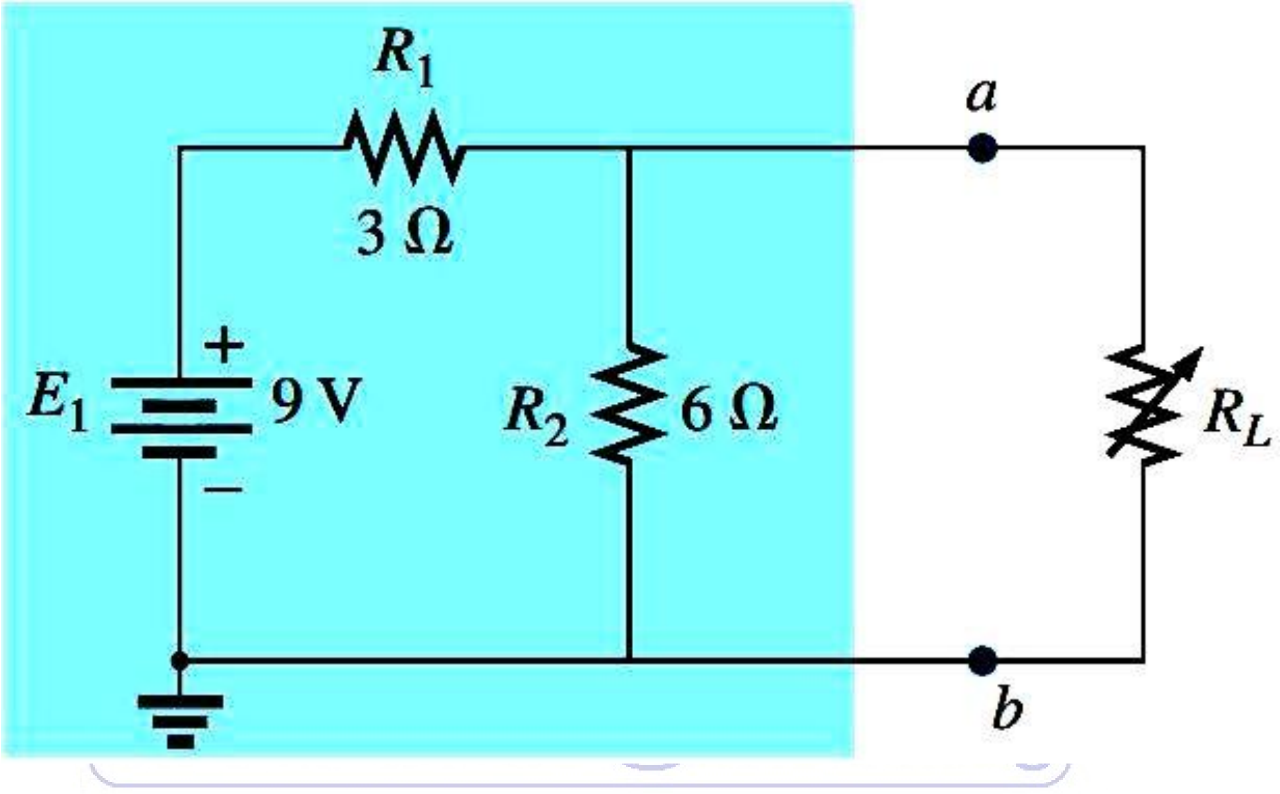


Thévenin's Theorem Procedure

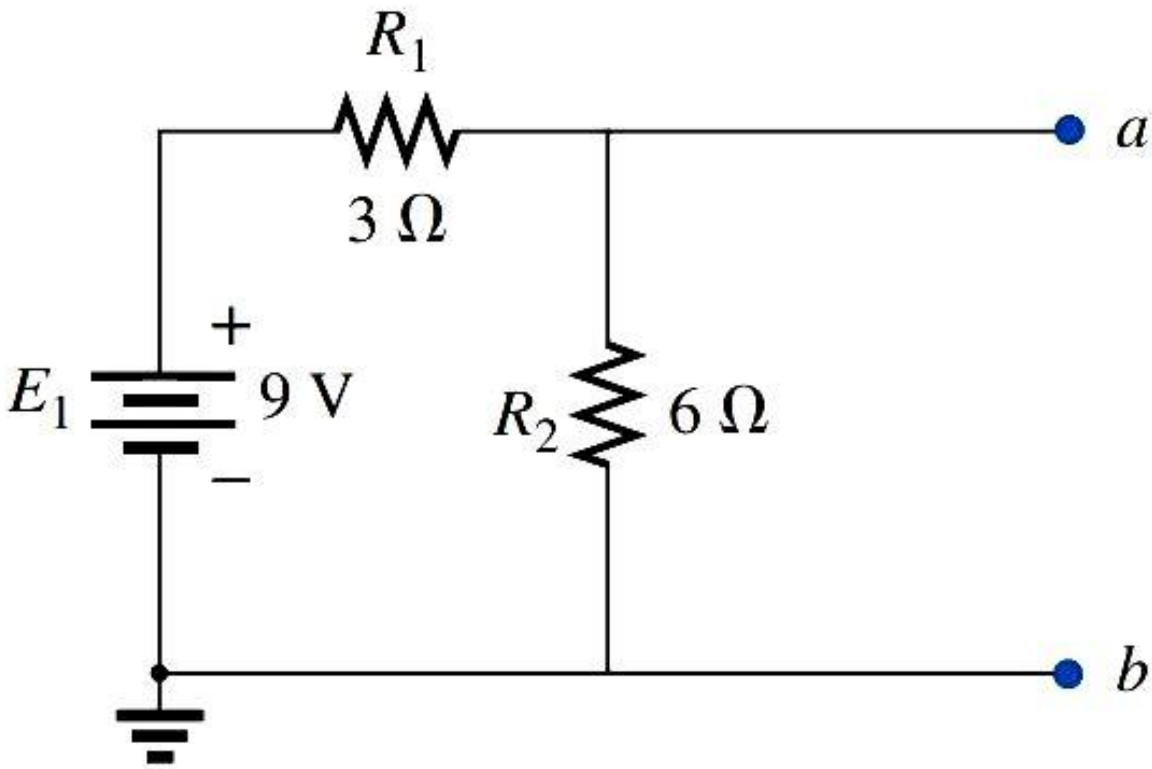
- Remove that portion of the network where the Thévenin equivalent circuit is found. In Figure, this requires that the load resistor R_L be temporarily removed from the network.
- Mark the terminals of the remaining two-terminal network.
- Calculate R_{Th} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuits) and then finding the resultant resistance between the two marked terminals.
- Calculate E_{Th} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals.
- Draw the Thévenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

Example of Thévenin's Theorem

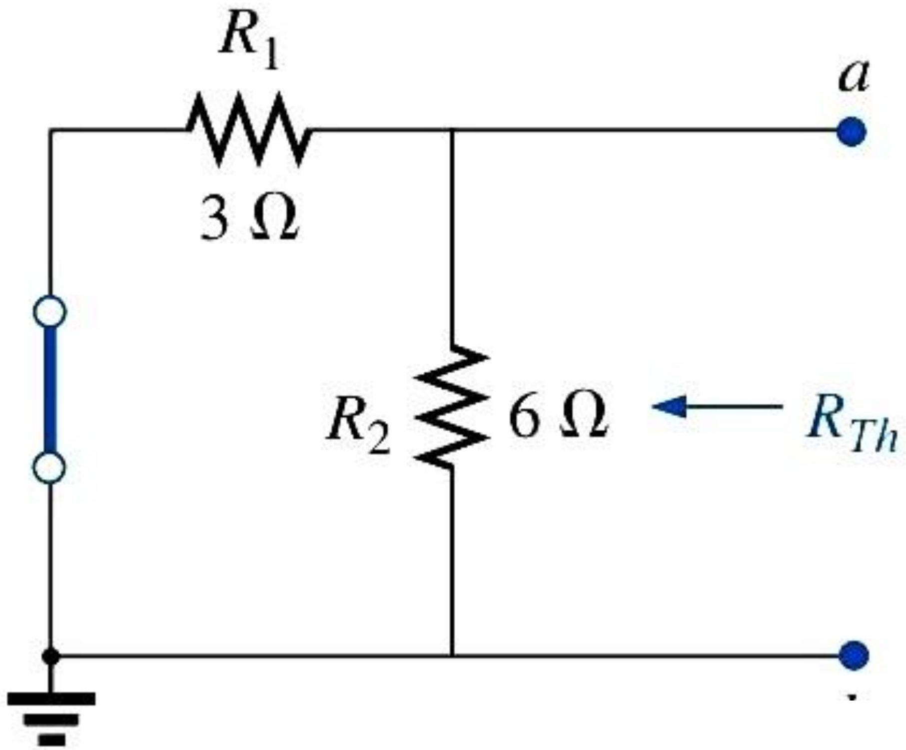
Find the current through R_L



Example of Thévenin's Theorem

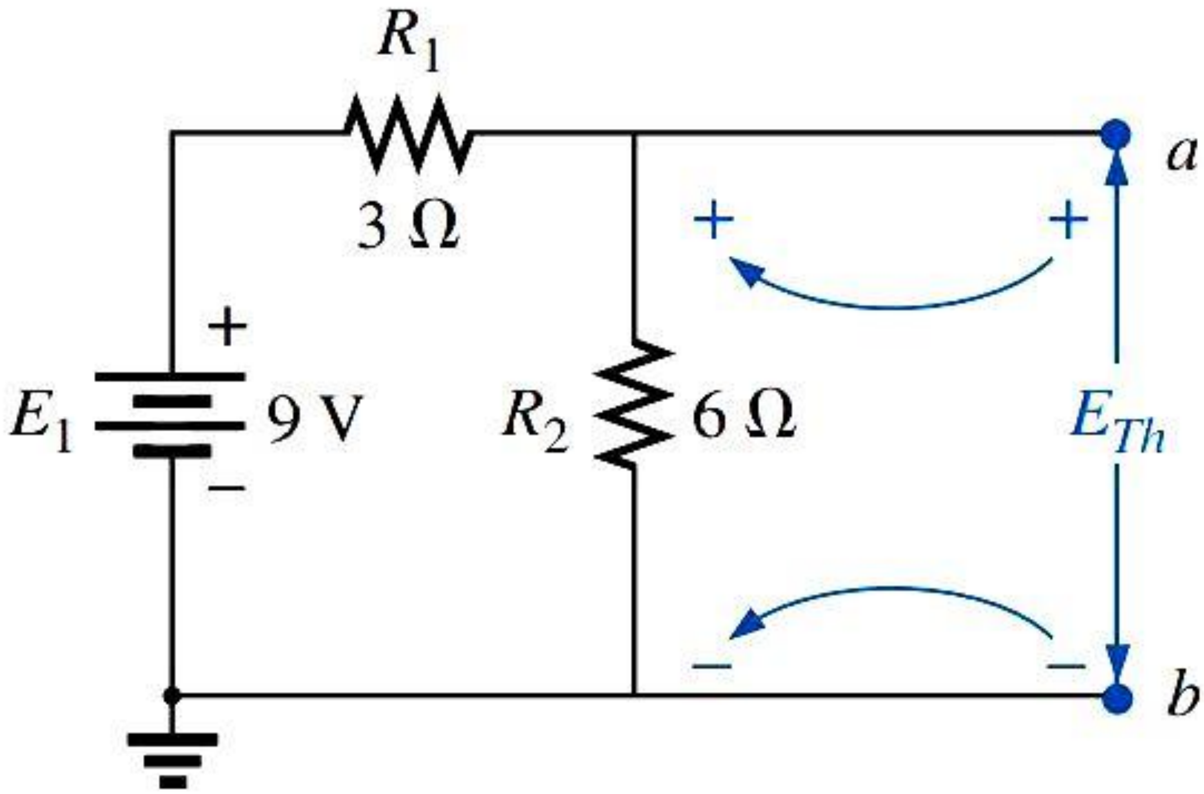


Example of Thévenin's Theorem



$$R_{Th} = R_1 \| R_2 = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \mathbf{2\ \Omega}$$

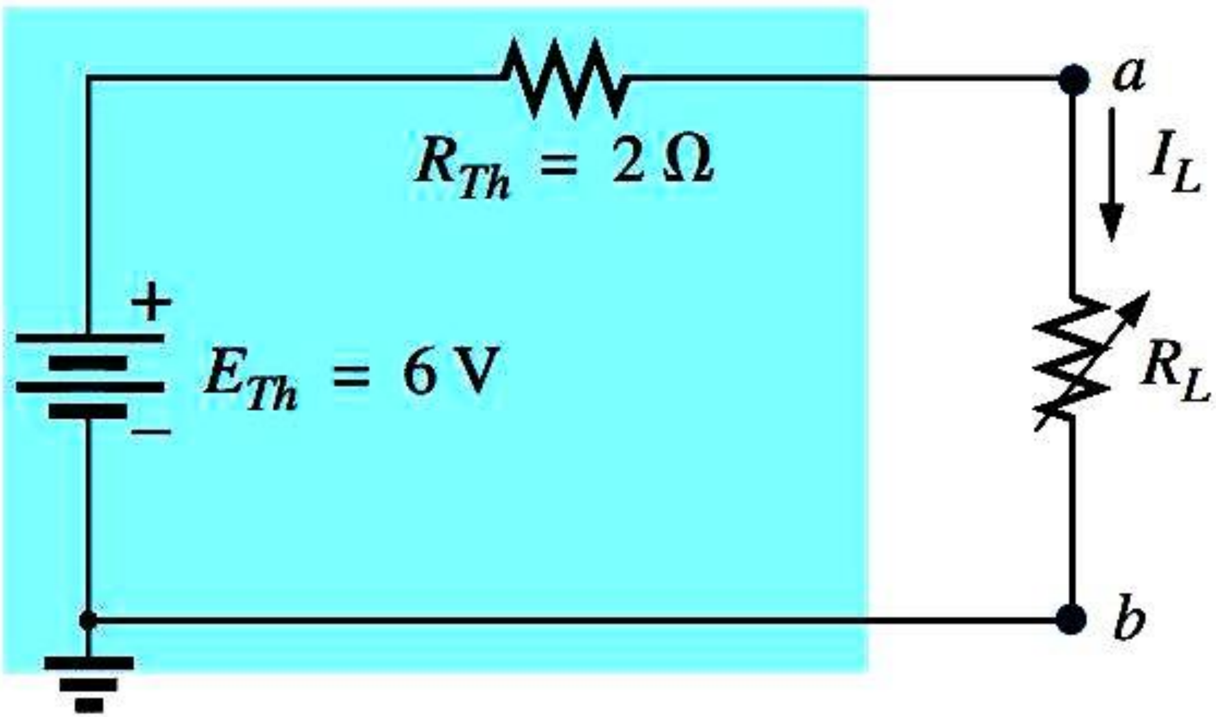
Example of Thévenin's Theorem



$$E_{Th} = \frac{R_2E_1}{R_2 + R_1} = \frac{(6\ \Omega)(9\ \text{V})}{6\ \Omega + 3\ \Omega} = \frac{54\ \text{V}}{9} = \mathbf{6\ V}$$

Ref: Boylestad: Example 9.6: Page 354

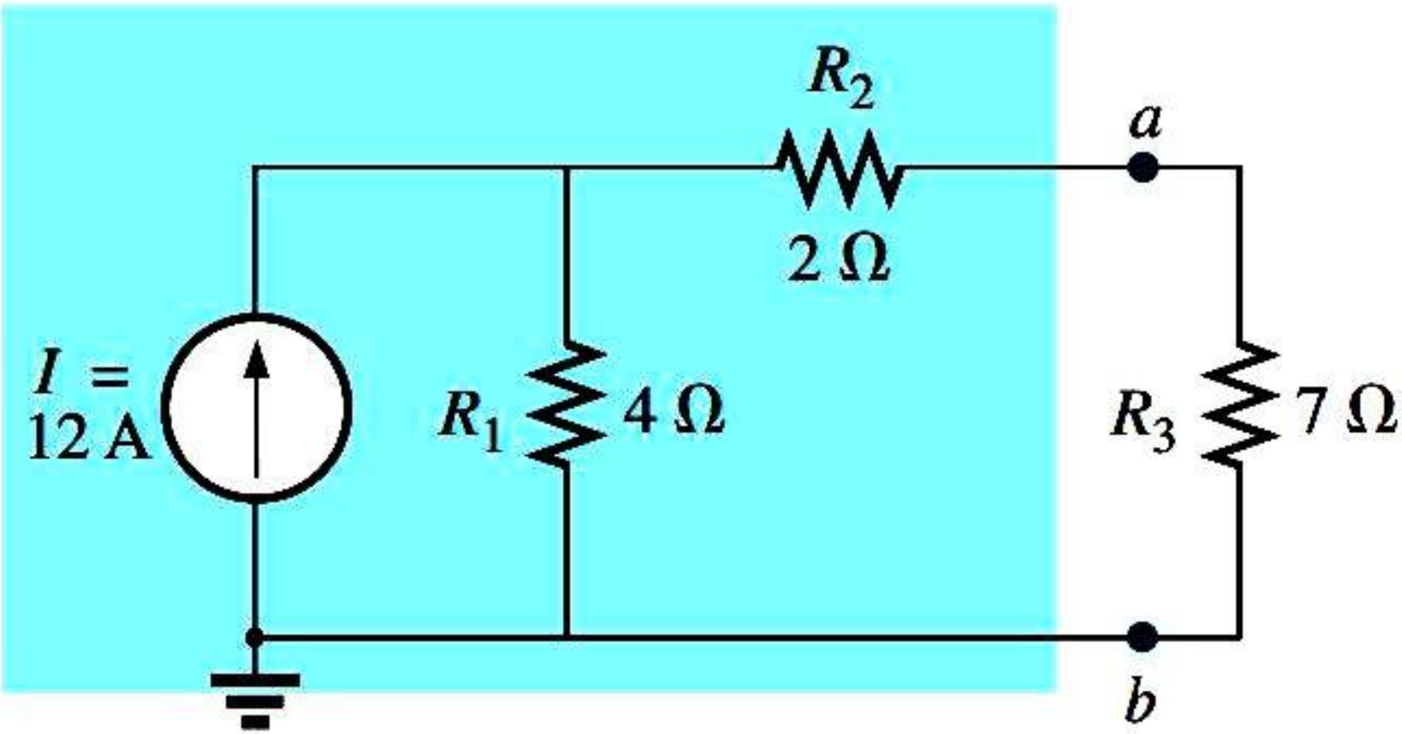
Example of Thévenin's Theorem



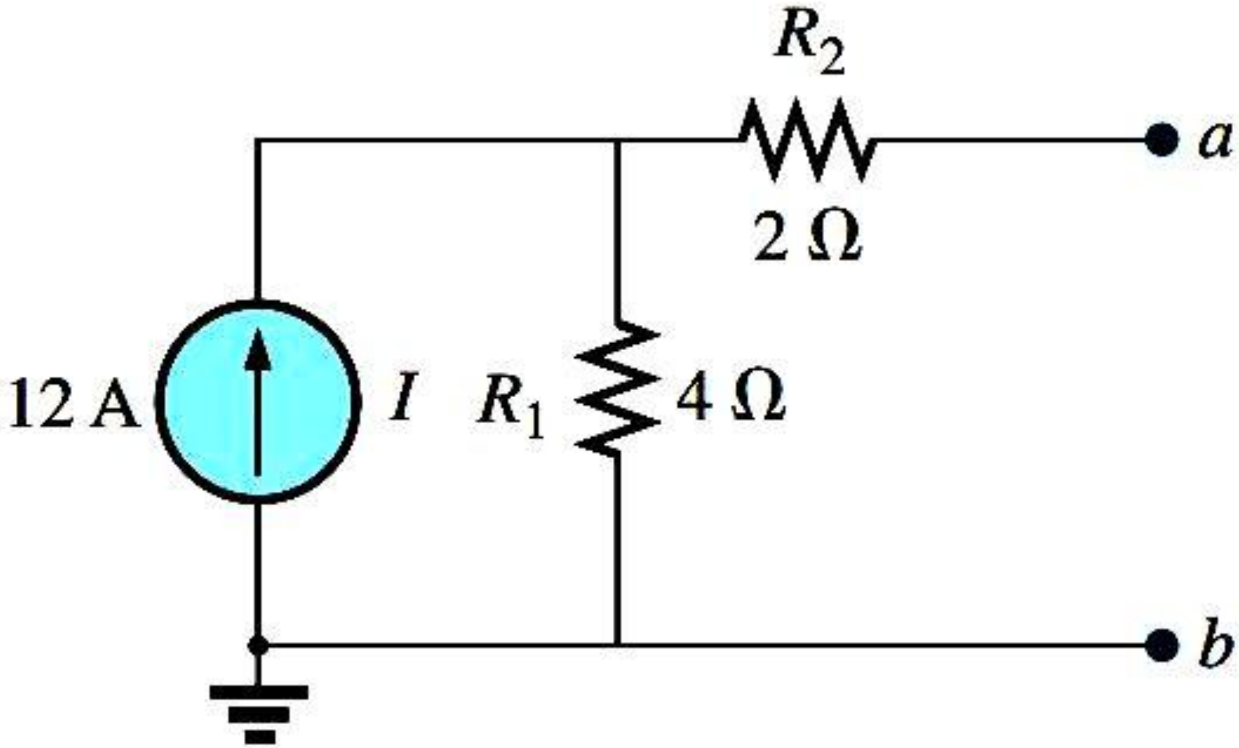
$$I_L = \frac{E_{Th}}{R_{Th} + R_L}$$

Example of Thévenin's Theorem

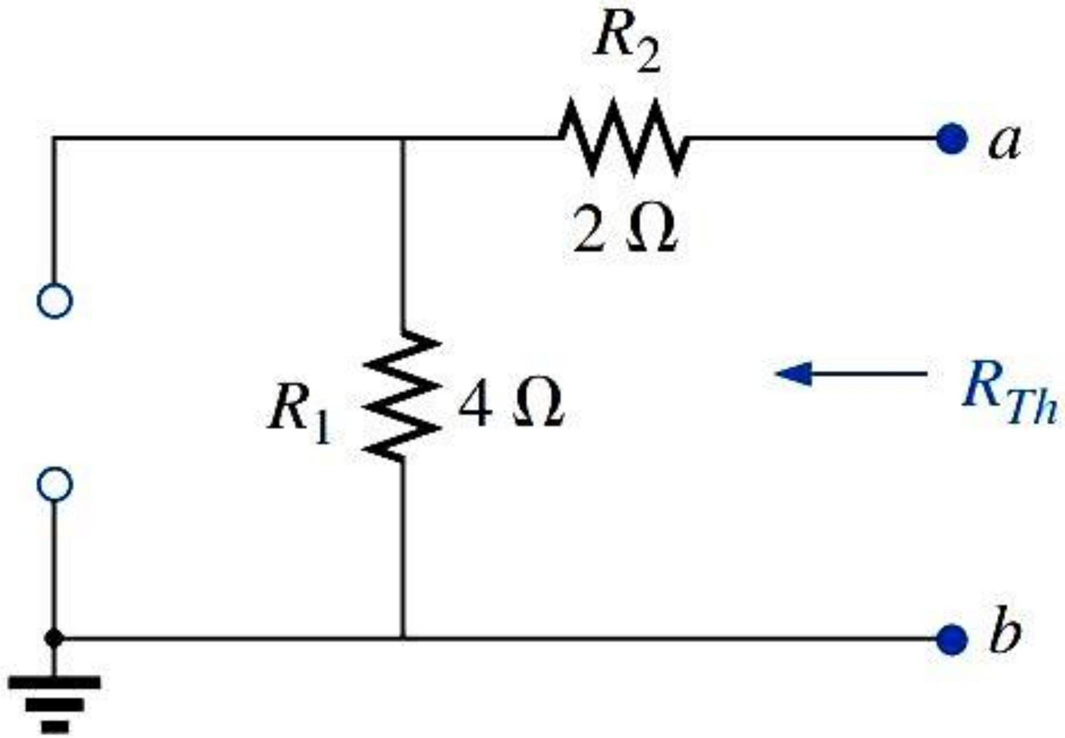
Find the Thévenin equivalent circuit for the network in the shaded area of the network in the figure.



Example of Thévenin's Theorem

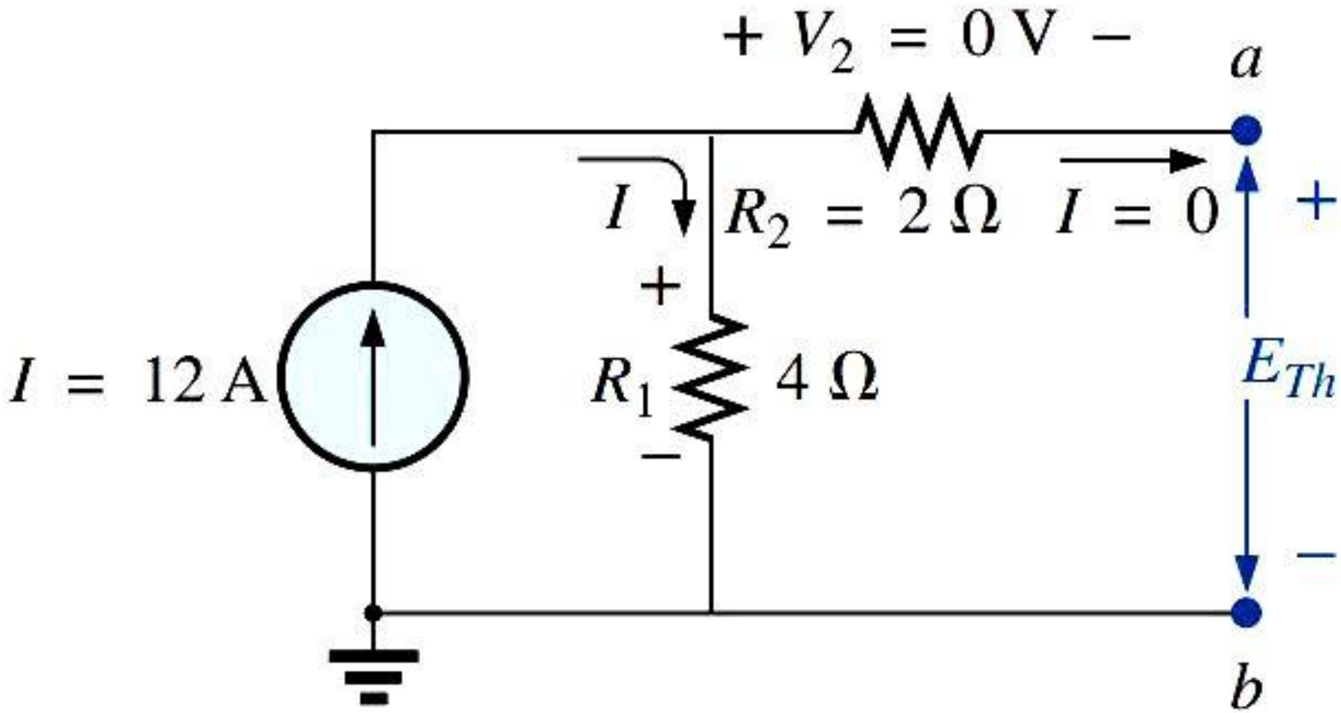


Example of Thévenin's Theorem



$$R_{Th} = R_1 + R_2 = 4\,\Omega + 2\,\Omega = \mathbf{6\,\Omega}$$

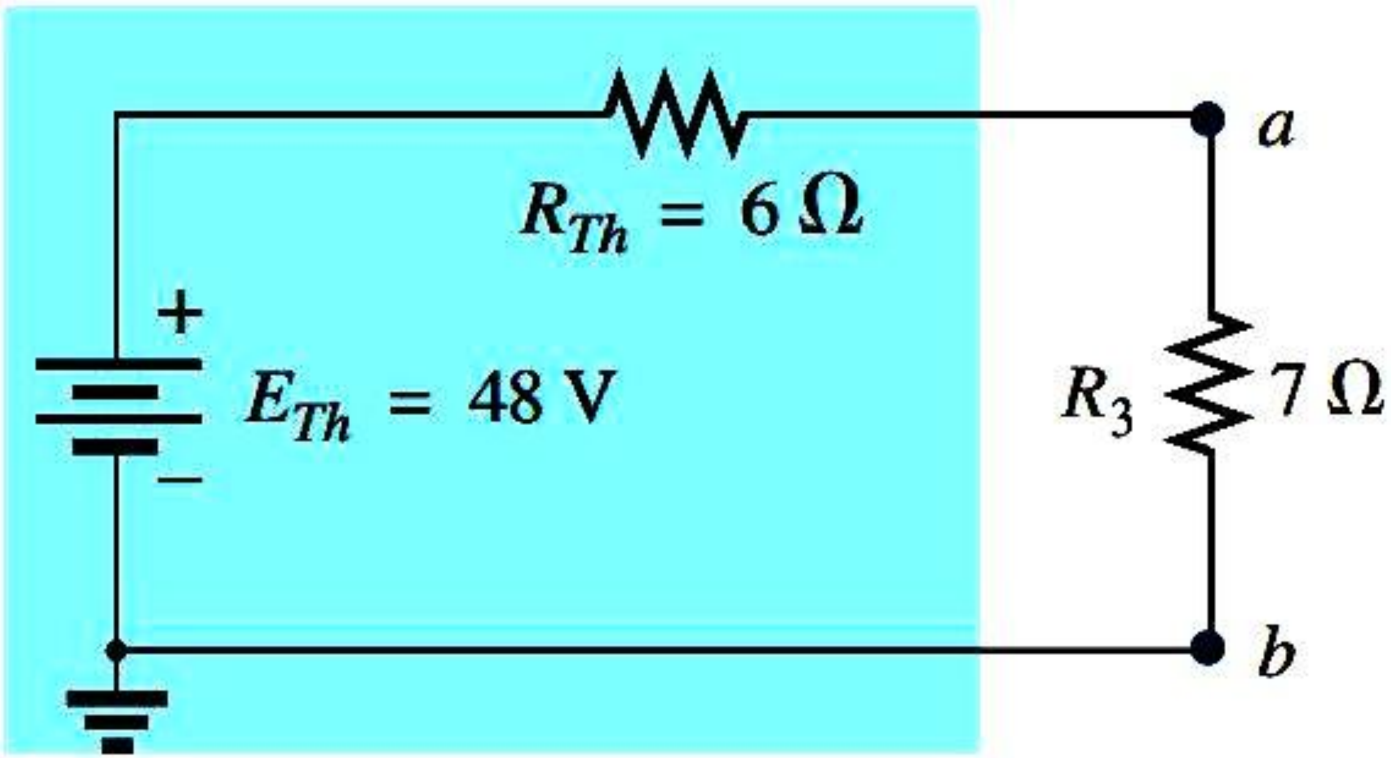
Example of Thévenin’s Theorem



$$V_2 = I_2R_2 = (0)R_2 = 0\text{ V}$$

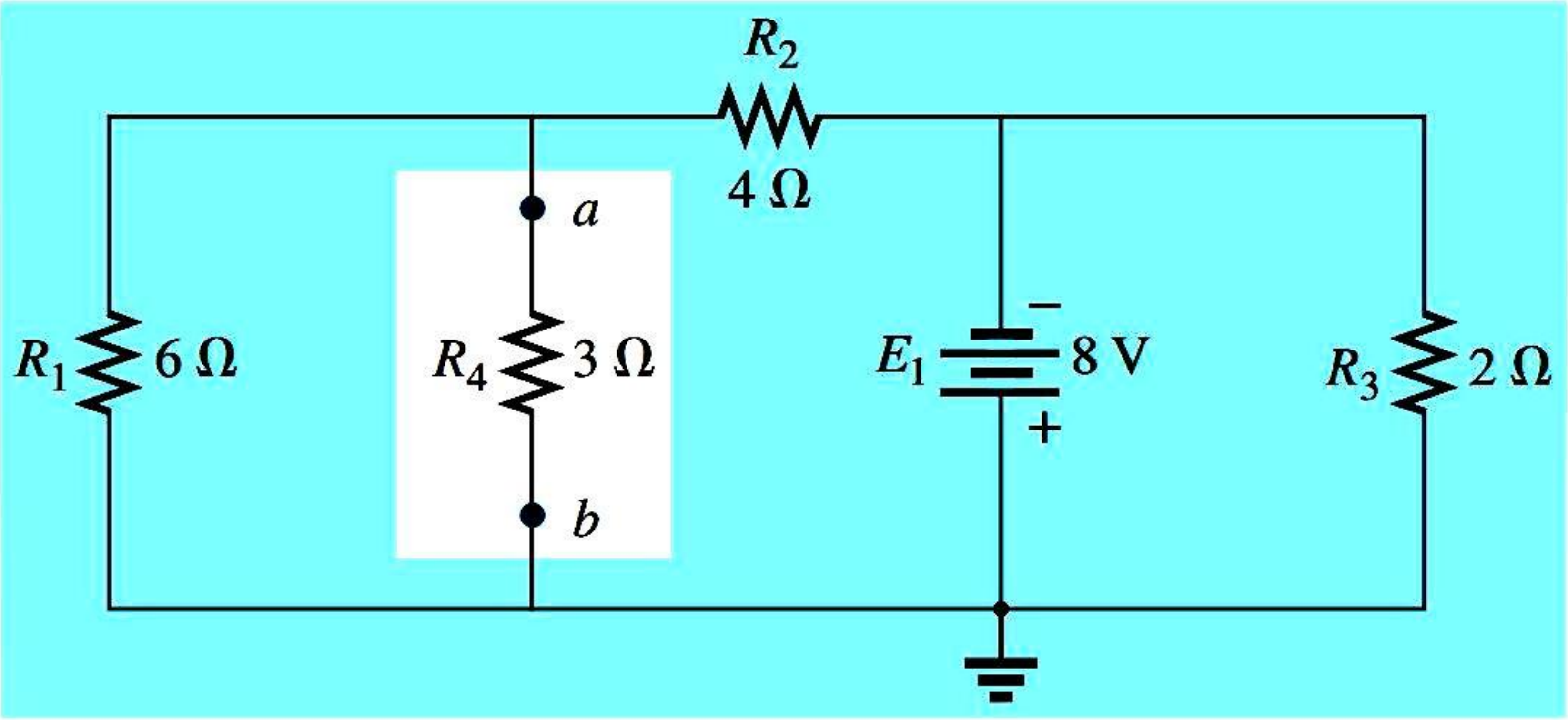
$$E_{Th} = V_1 = I_1R_1 = IR_1 = (12\text{ A})(4\ \Omega) = \mathbf{48\ V}$$

Example of Thévenin's Theorem



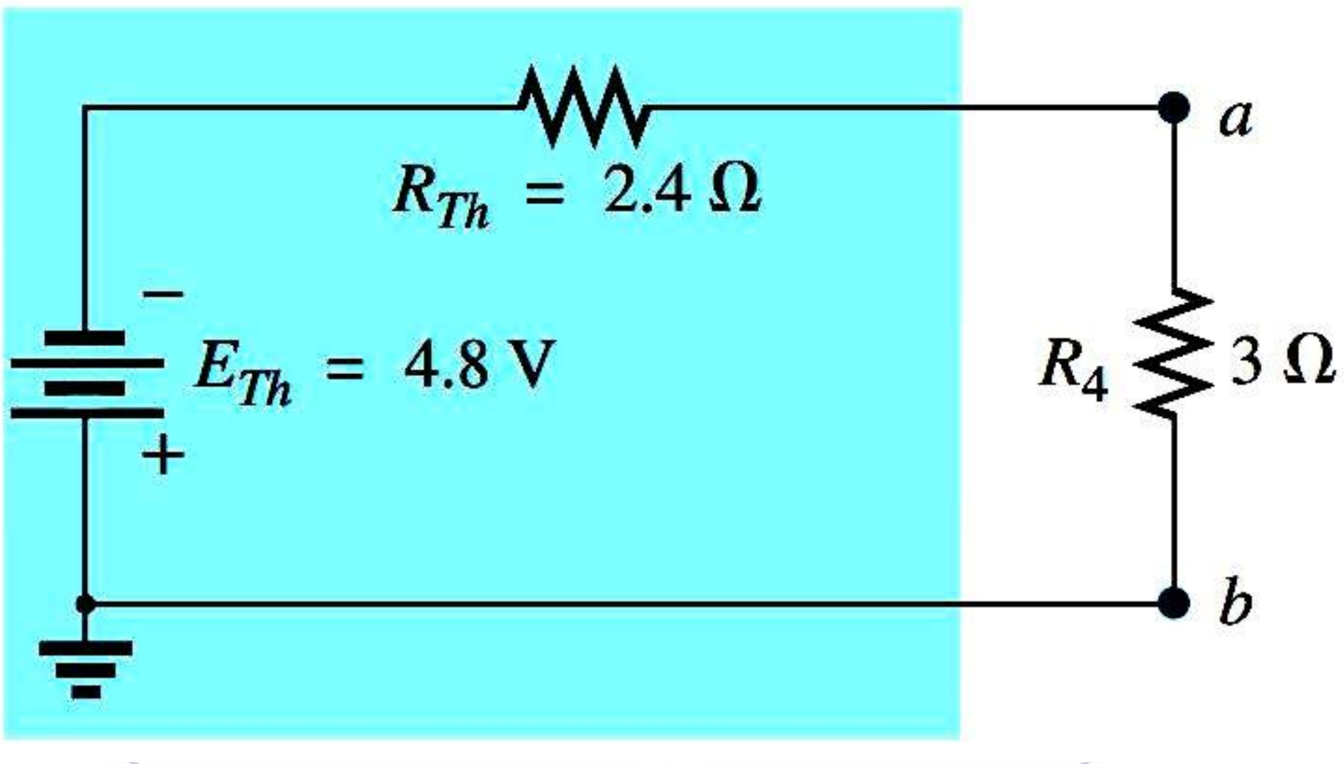
Example of Thévenin's Theorem

Find the Thévenin equivalent circuit for the network in the shaded area of the network in the figure.



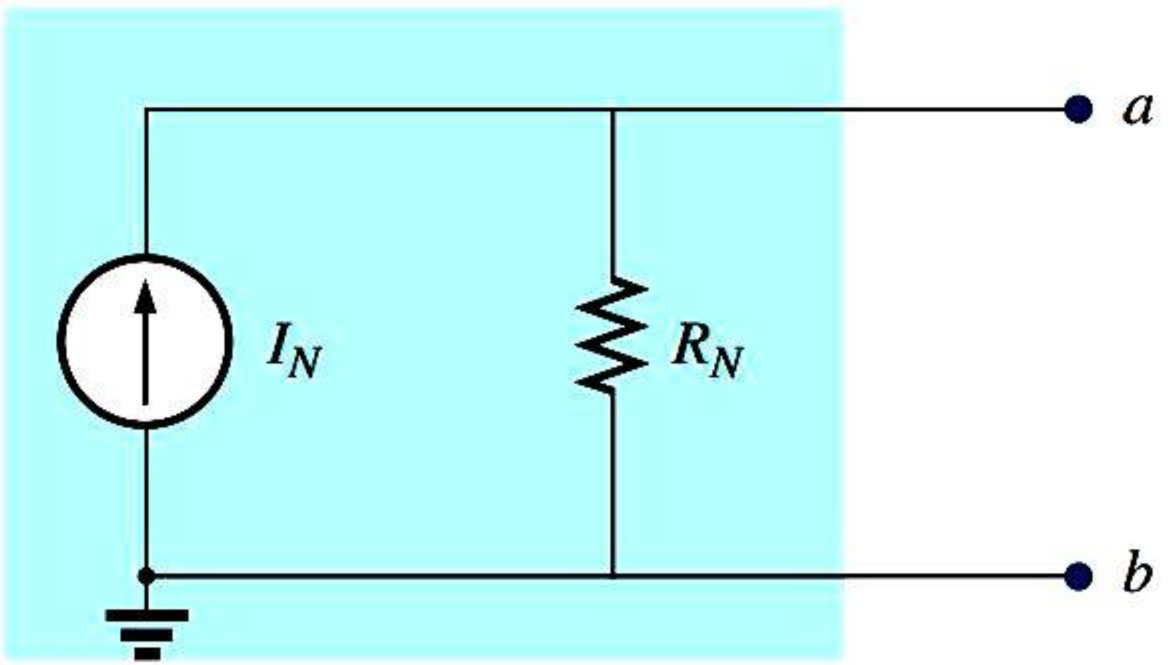
Example of Thévenin's Theorem

The Thévenin equivalent circuit for the network:



Norton's Theorem

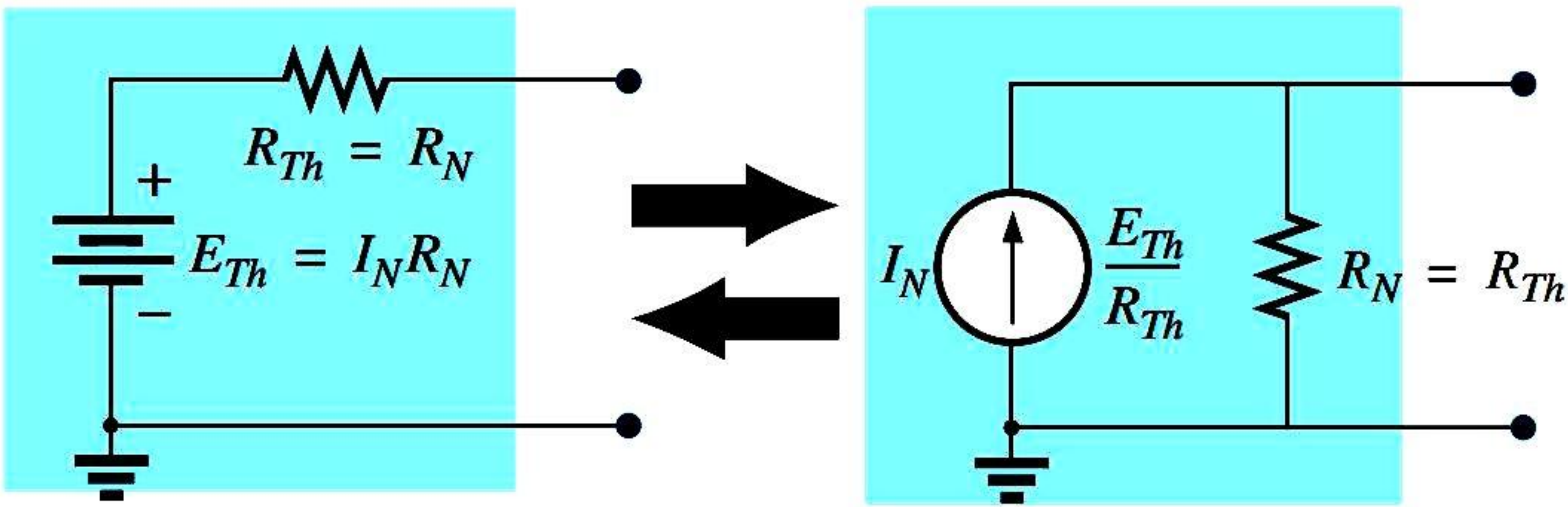
Any two-terminal linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor, as shown in the figure.



Norton's Theorem Procedure

- Remove that portion of the network across which the Norton equivalent circuit is found.
- Mark the terminals of the remaining two-terminal network.
- Calculate R_N by first setting all sources to zero (voltage sources are replaced with short circuits, and current sources with open circuits) and then finding the resultant resistance between the two marked terminals.
- Calculate I_N by first returning all sources to their original position and then finding the short-circuit current between the marked terminals.
- Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.

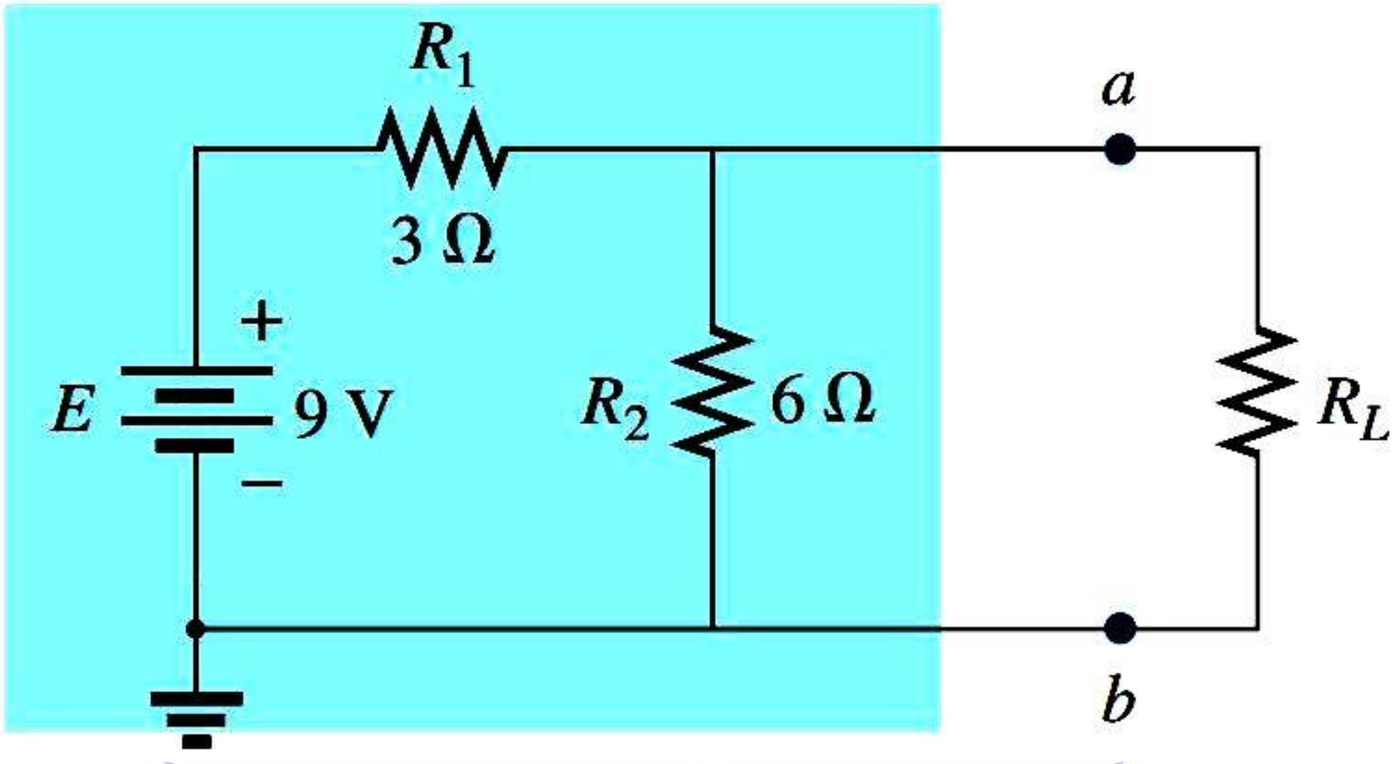
Norton's Theorem



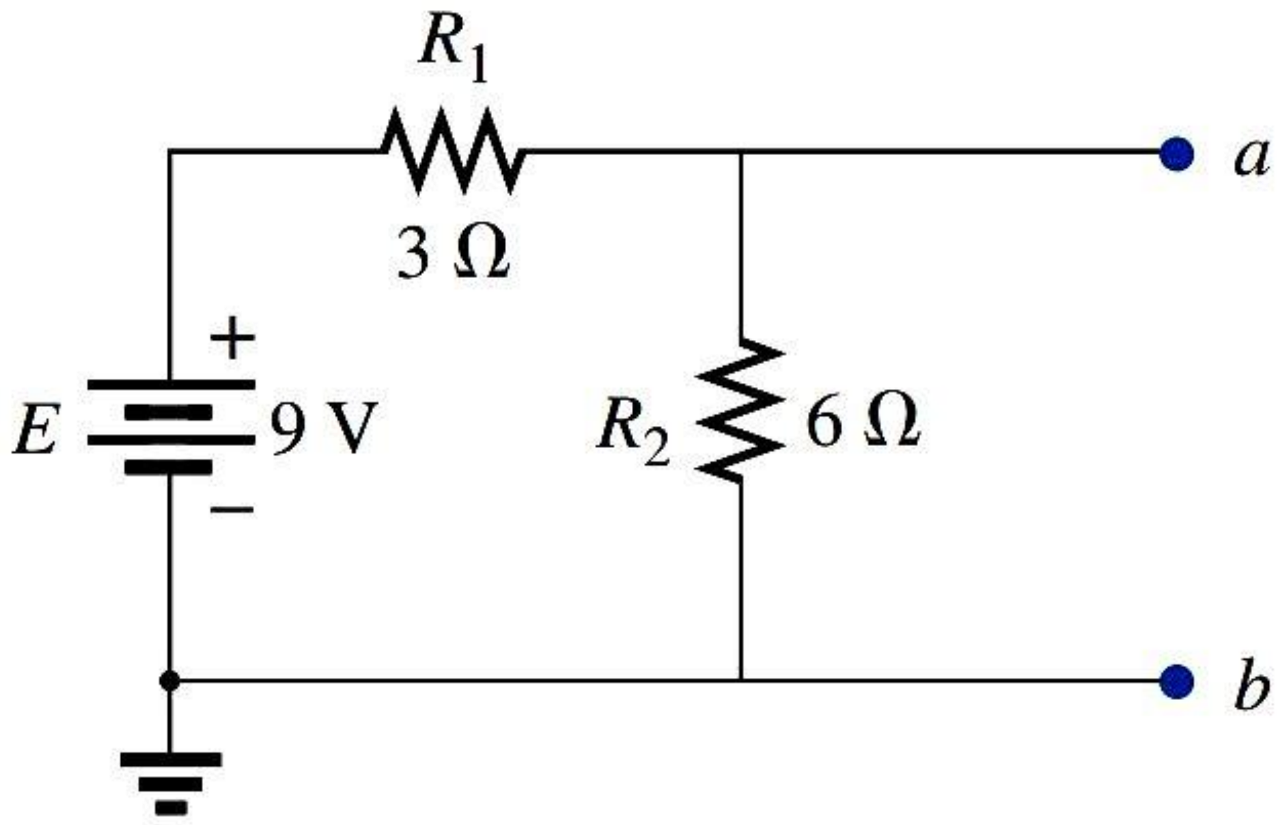
Converting between Thévenin and Norton equivalent circuits

Example of Norton's Theorem

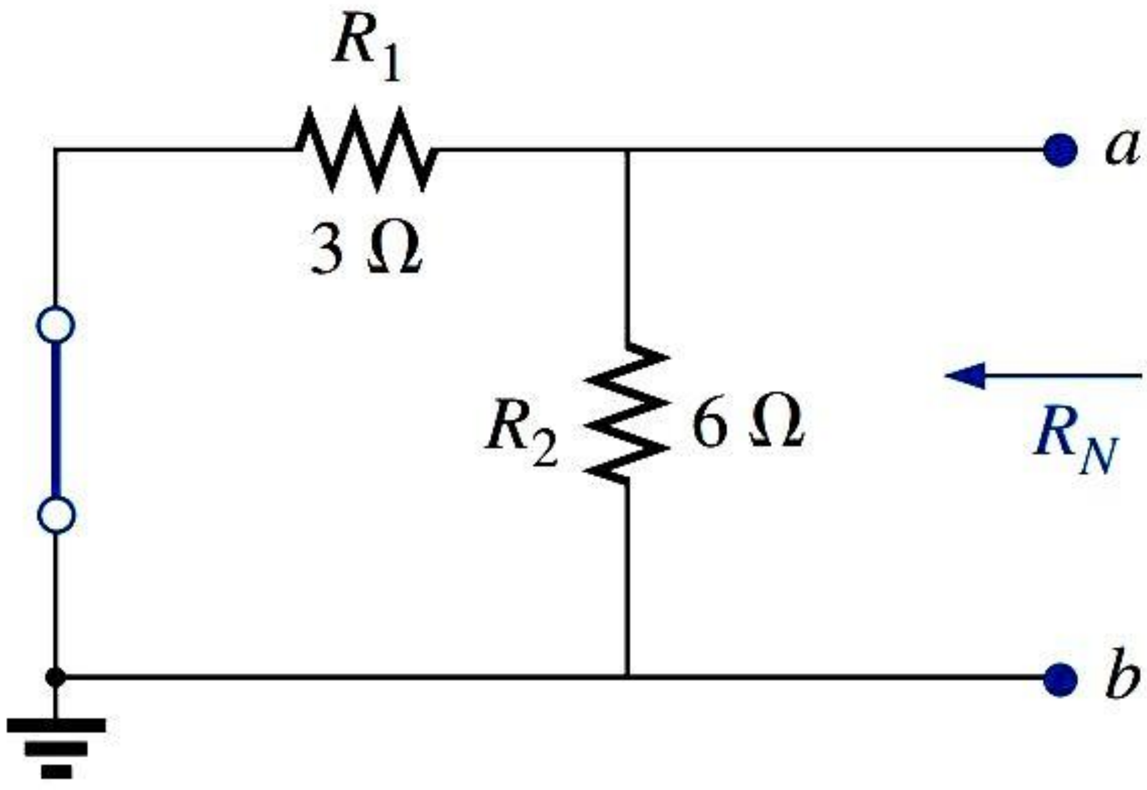
Find the Norton equivalent circuit for the network in the shaded area



Example of Norton's Theorem



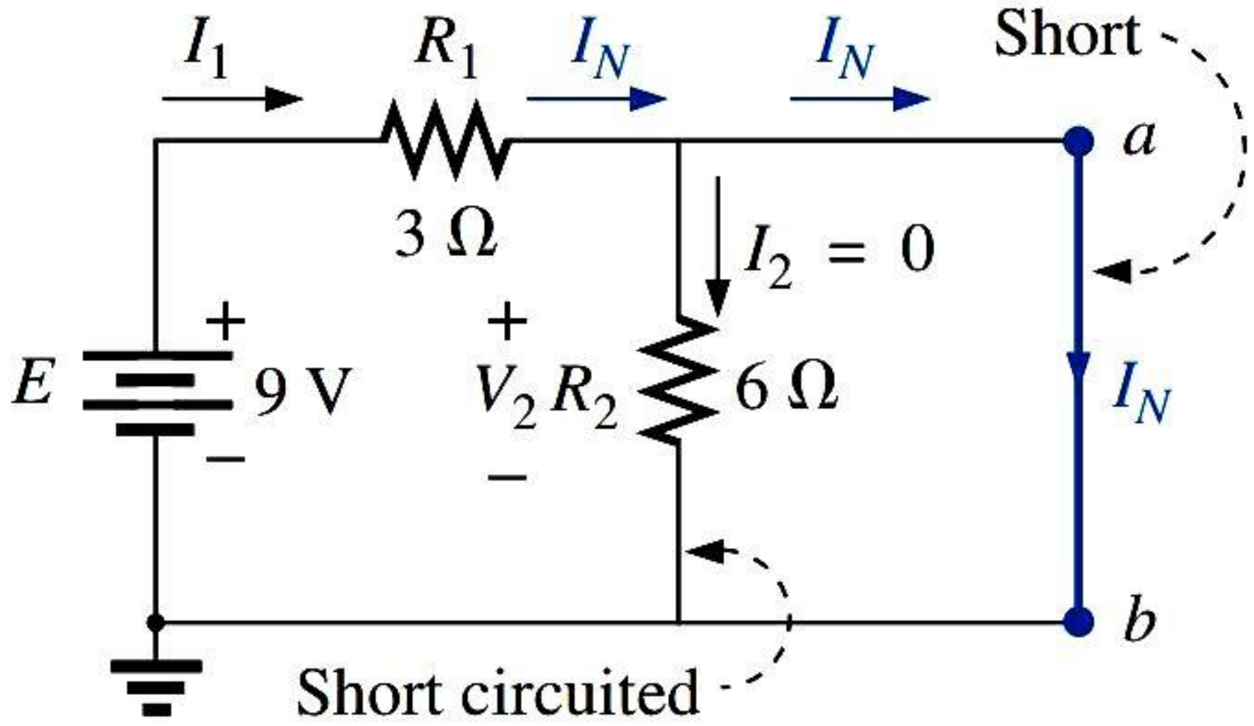
Example of Norton's Theorem



$$R_N = R_1 \parallel R_2 = 3\ \Omega \parallel 6\ \Omega = \frac{(3\ \Omega)(6\ \Omega)}{3\ \Omega + 6\ \Omega} = \frac{18\ \Omega}{9} = \mathbf{2\ \Omega}$$

Ref: Boylestad: Example 9.11: Page 364

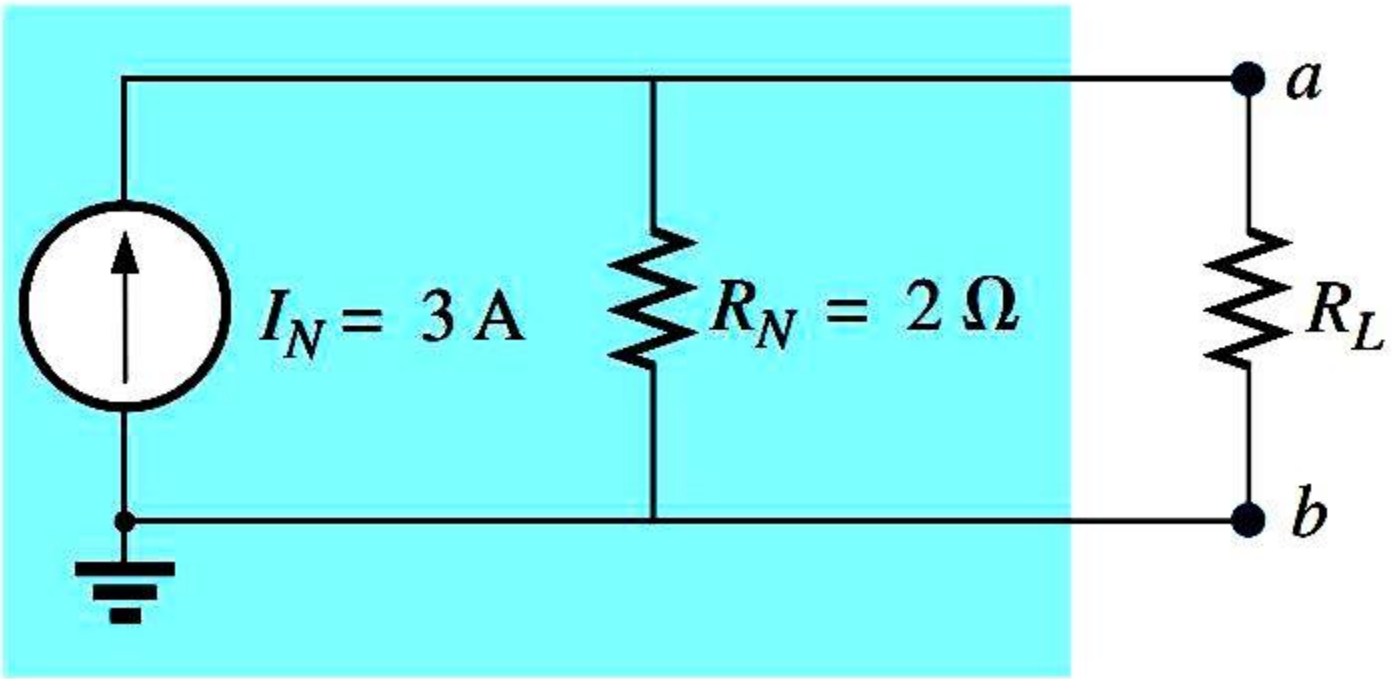
Example of Norton's Theorem



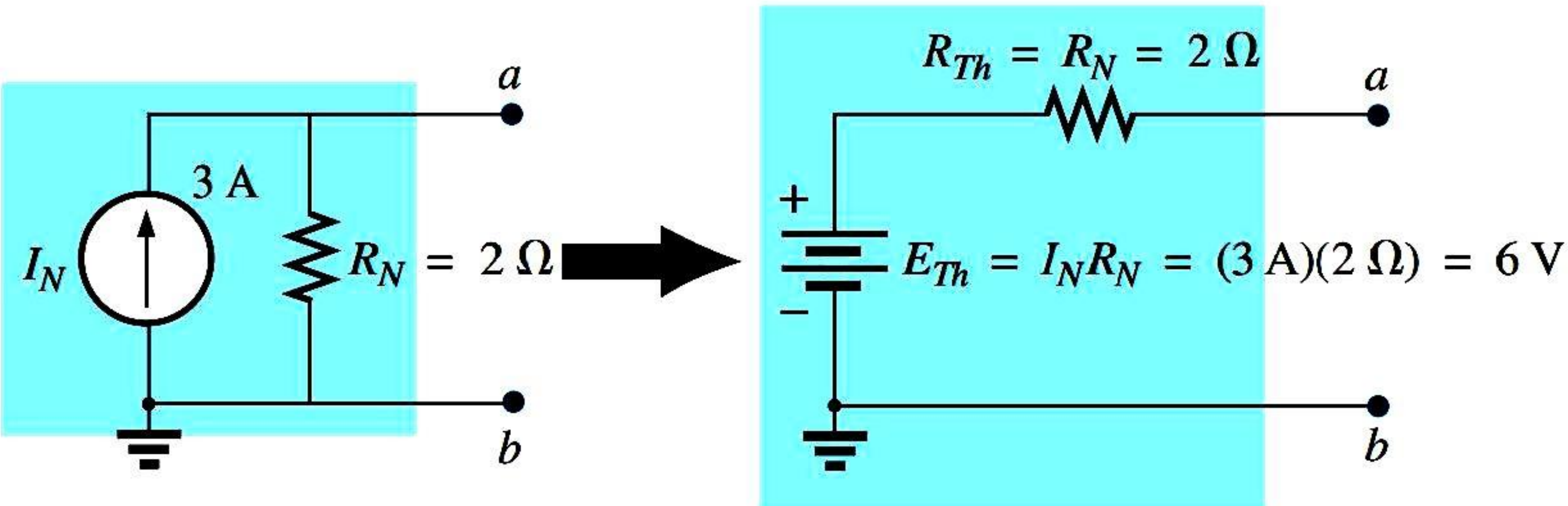
$$V_2 = I_2R_2 = (0)6\ \Omega = 0\ \text{V}$$

$$I_N = \frac{E}{R_1} = \frac{9\ \text{V}}{3\ \Omega} = \mathbf{3\ A}$$

Example of Norton's Theorem



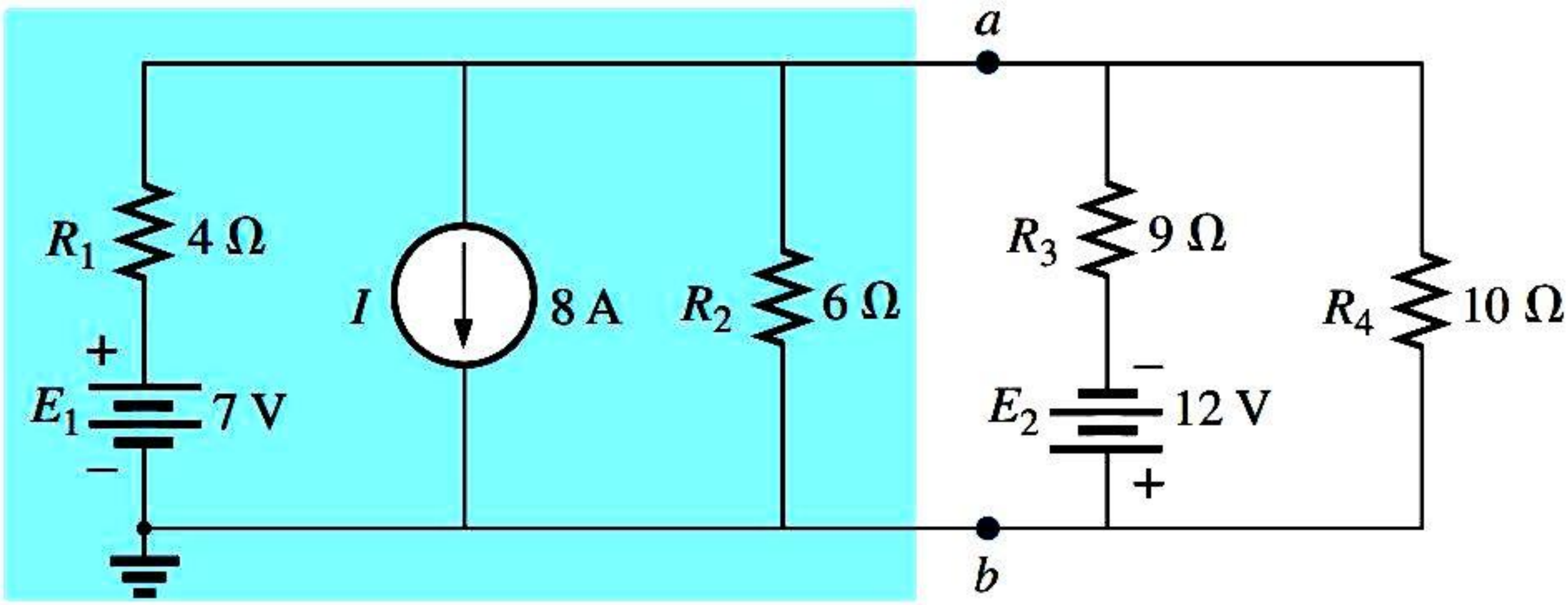
Example of Norton's Theorem



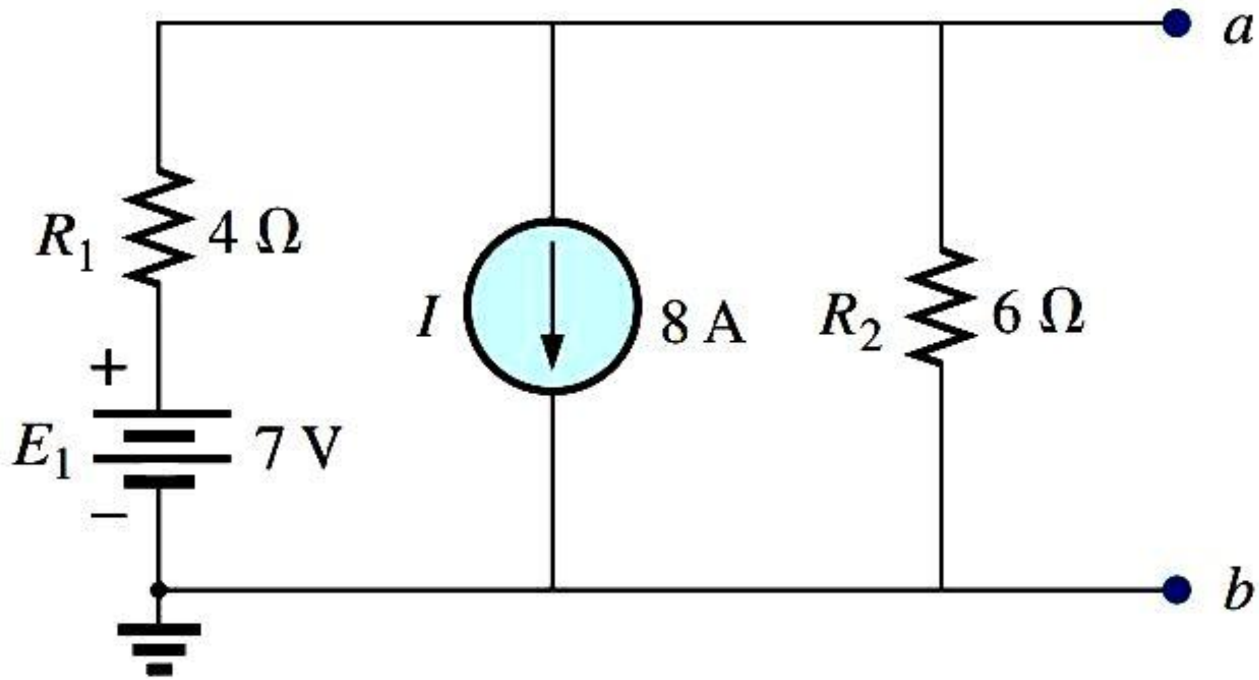
Converting between Thévenin and Norton equivalent circuits

Example of Norton's Theorem

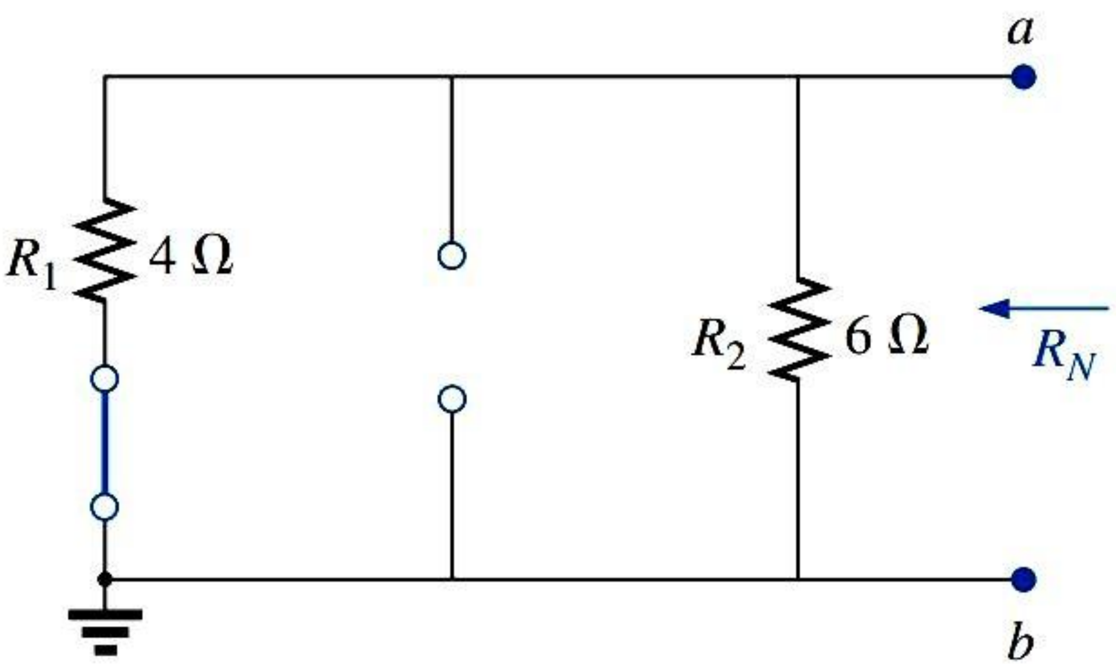
Find the Norton equivalent circuit for the portion of the network to the left of *a*-*b*



Example of Norton's Theorem



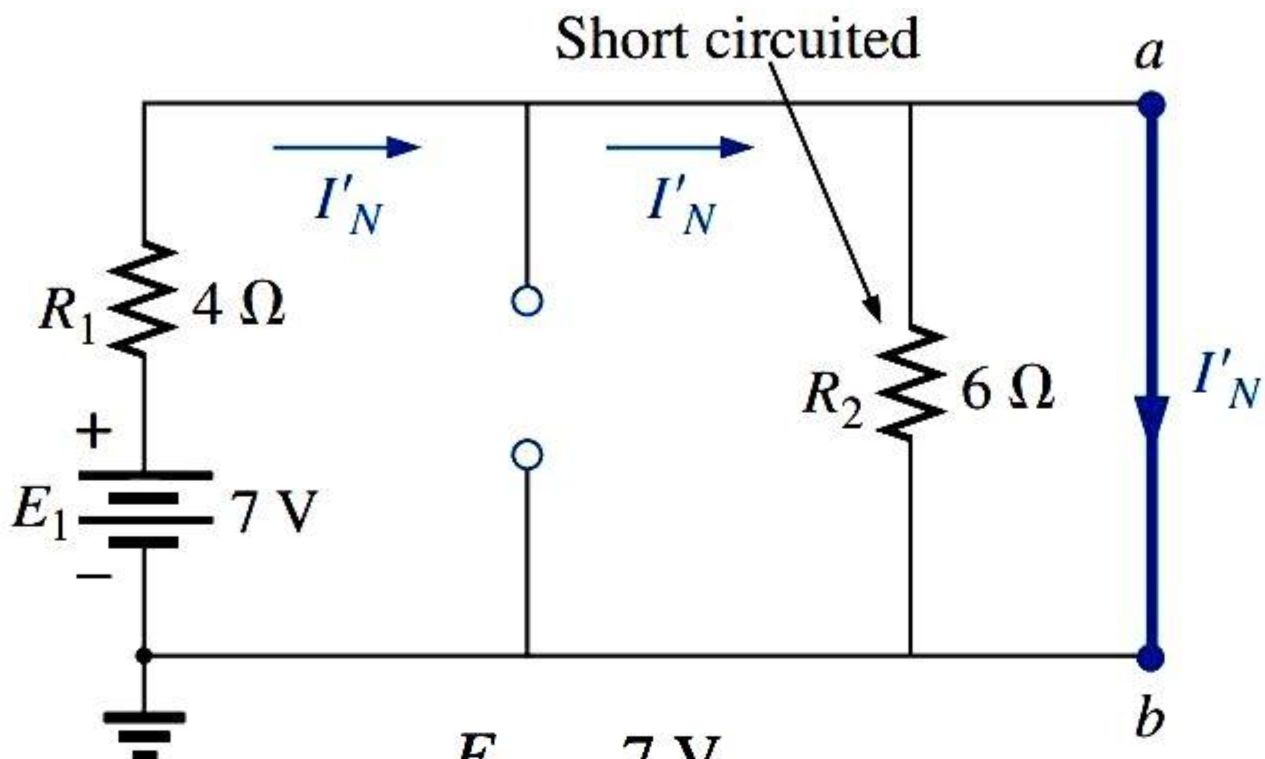
Example of Norton's Theorem



$$R_N = R_1 \parallel R_2 = 4\ \Omega \parallel 6\ \Omega = \frac{(4\ \Omega)(6\ \Omega)}{4\ \Omega + 6\ \Omega} = \frac{24\ \Omega}{10} = \mathbf{2.4\ \Omega}$$

Example of Norton's Theorem

Using Superposition for the Voltage Source

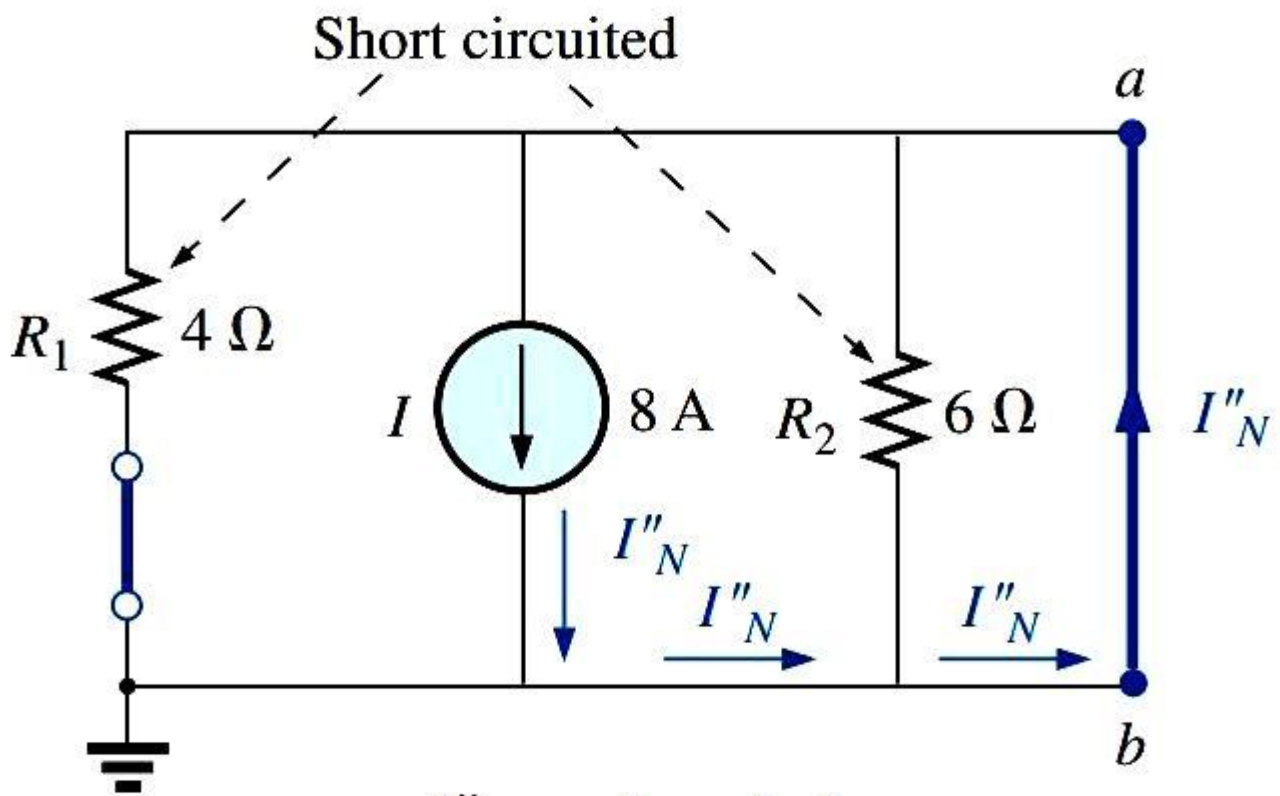


$$I'_N = \frac{E_1}{R_1} = \frac{7\text{ V}}{4\,\Omega} = 1.75\text{ A}$$

Ref: Boylestad: Example 9.13: Page 366

Example of Norton's Theorem

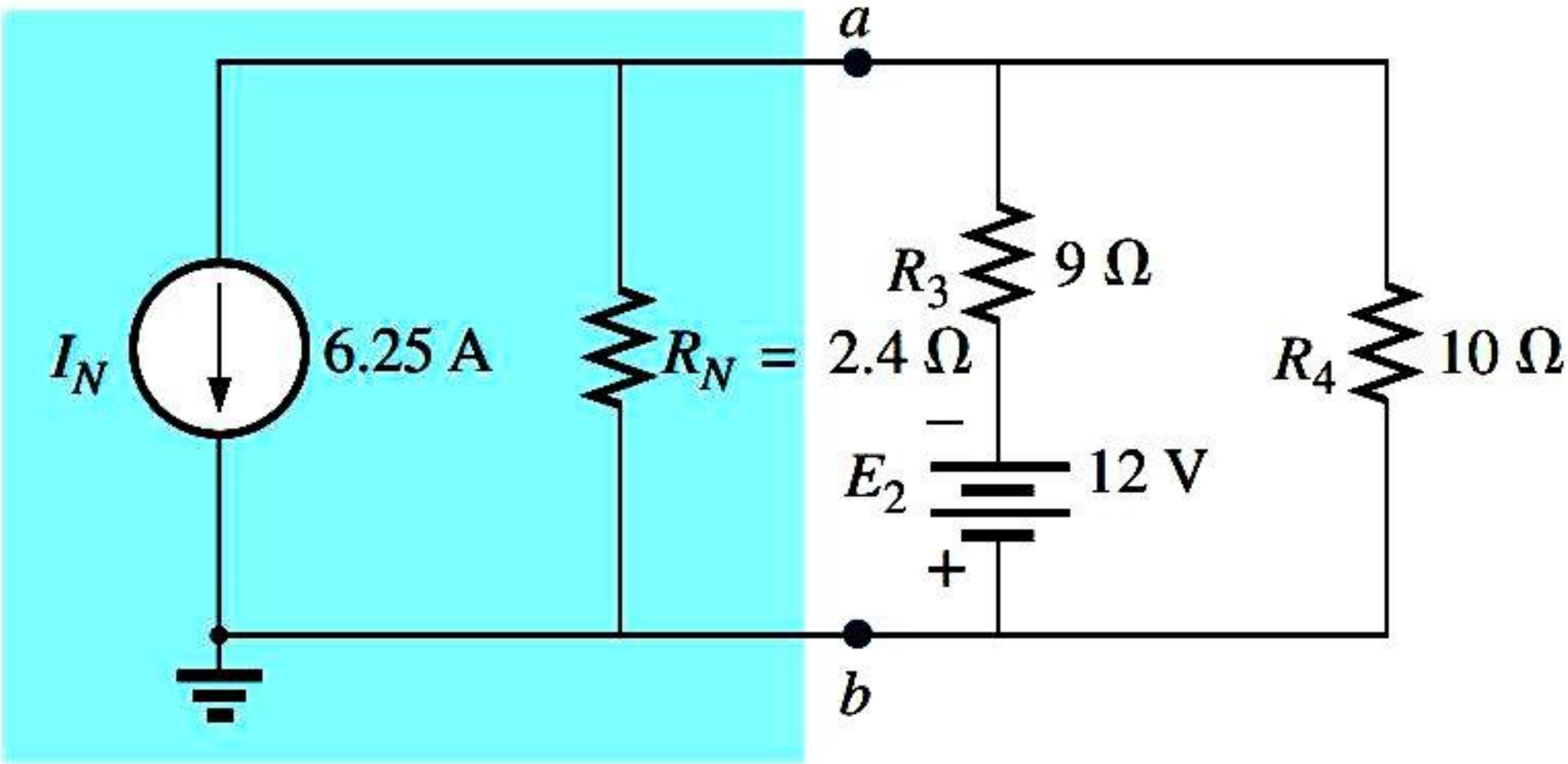
Using Superposition for the Voltage Source



$$I''_N = I = 8\text{ A}$$

Example of Norton's Theorem

$$I_N = I''_N - I'_N = 8\ \text{A} - 1.75\ \text{A} = \mathbf{6.25\ A}$$



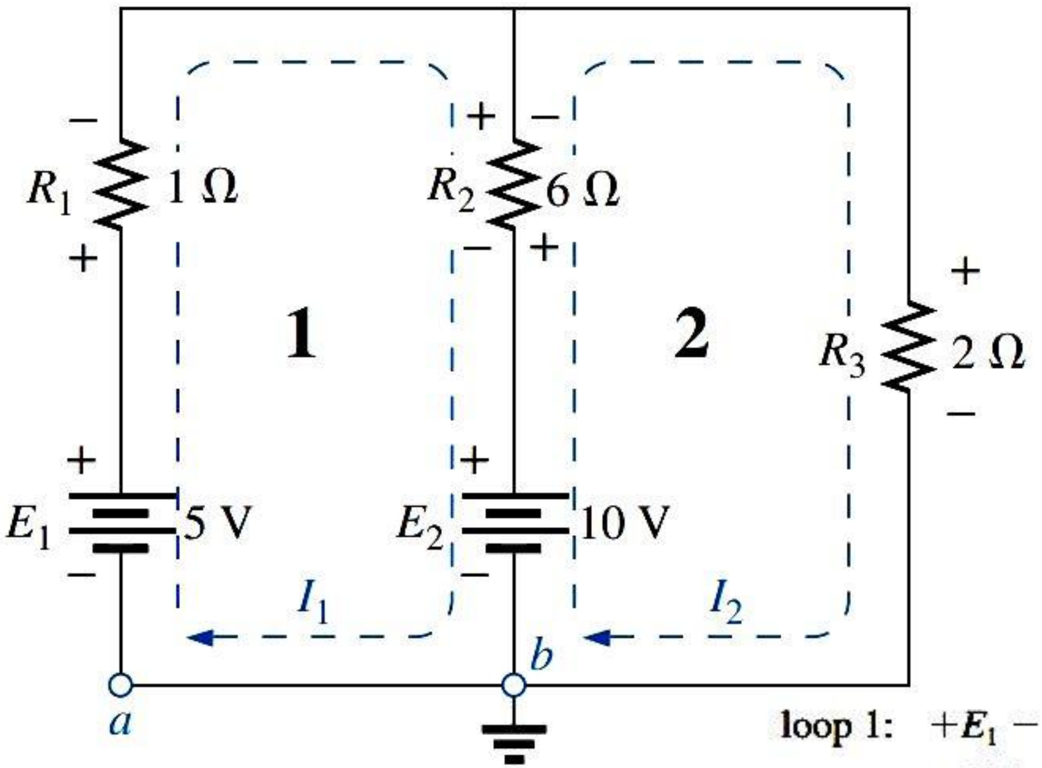
Maximum Power Transfer Theorem

Book – Boylestad

Article: 9.5

Page: 367

Mesh Analysis



loop 1: $+E_1 - V_1 - V_2 - E_2 = 0$ (clockwise starting at point a)

$$+5\text{ V} - (1\ \Omega)I_1 - (6\ \Omega)(I_1 - I_2) - 10\text{ V} = 0$$

I_2 flows through the 6 Ω resistor in the direction opposite to I_1.

loop 2: $E_2 - V_2 - V_3 = 0$ (clockwise starting at point b)

$$+10\text{ V} - (6\ \Omega)(I_2 - I_1) - (2\ \Omega)I_2 = 0$$

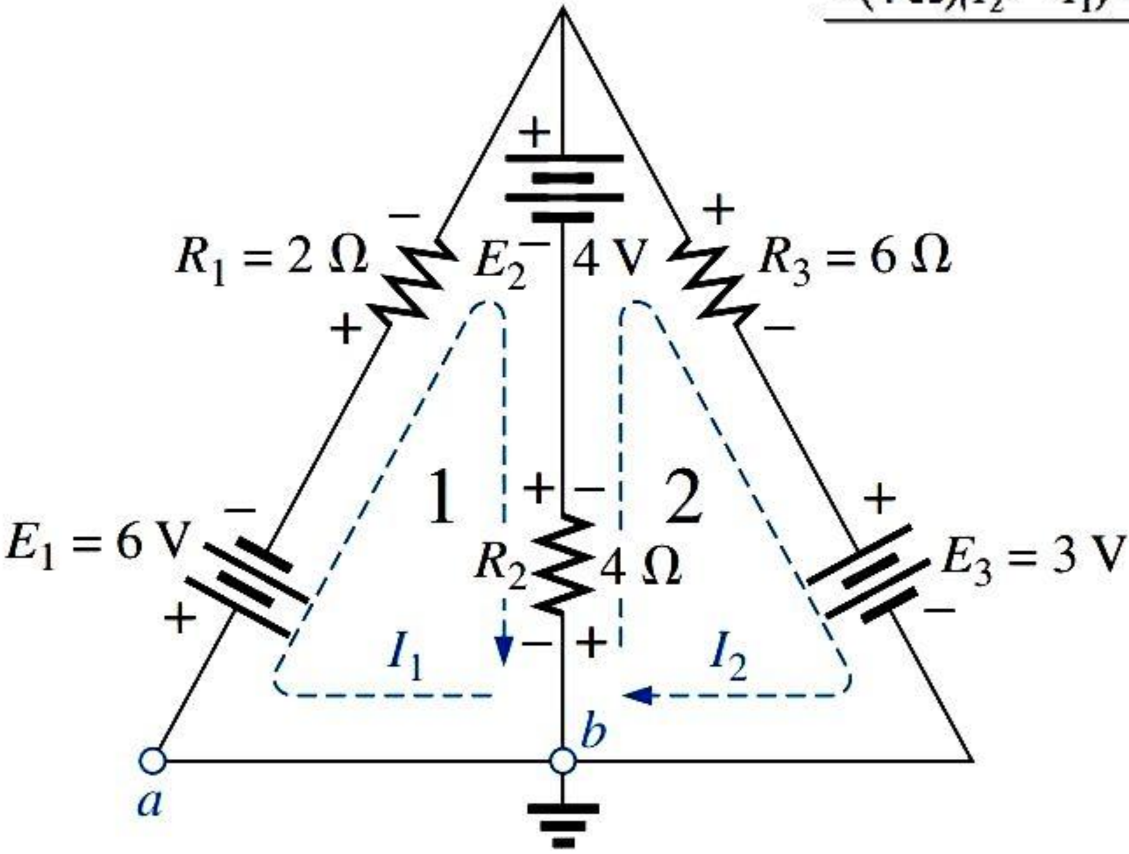
Mesh Analysis

loop 1: $-E_1 - I_1R_1 - E_2 - V_2 = 0$ (clockwise from point a)

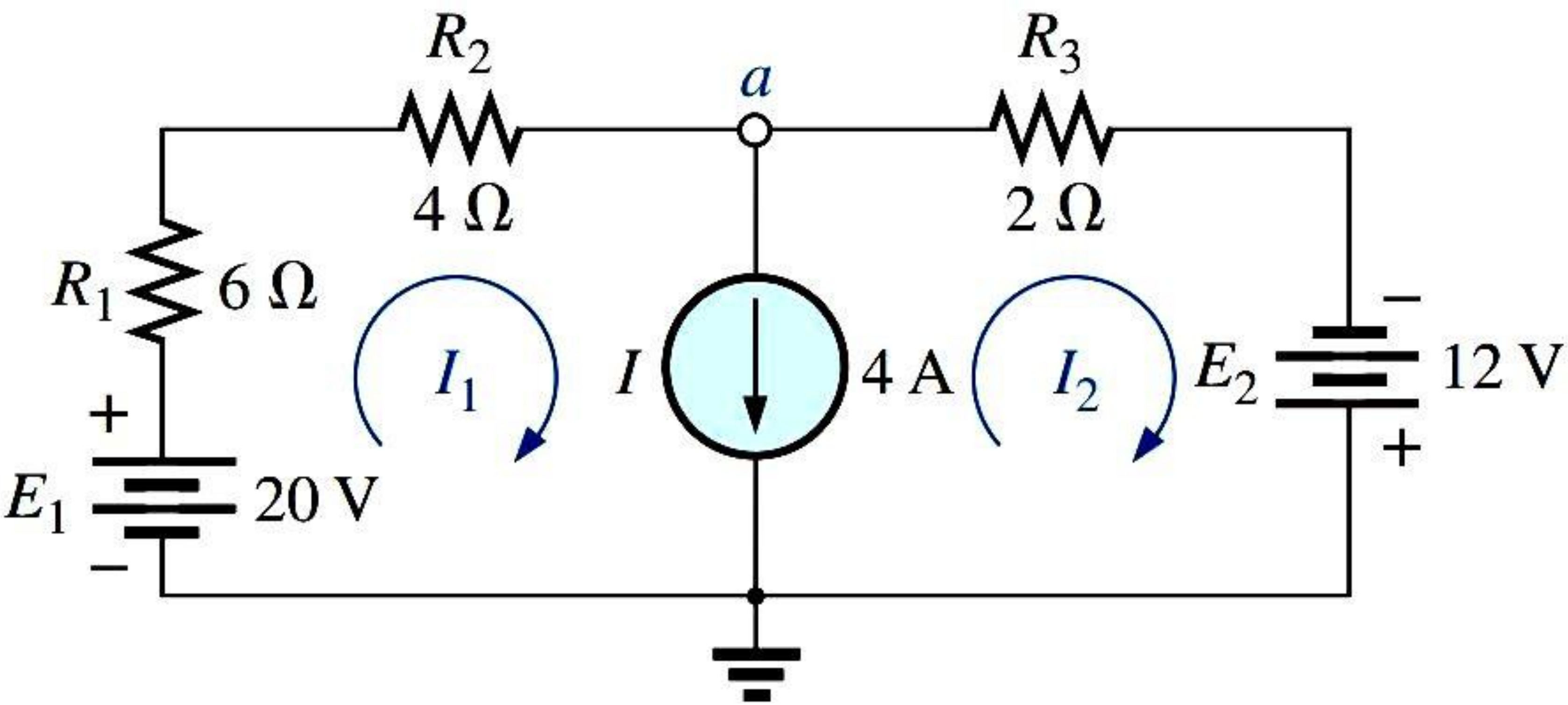
$- 6\text{ V} - (2\ \Omega)I_1 - 4\text{ V} - (4\ \Omega)(I_1 - I_2) = 0$

loop 2: $-V_2 + E_2 - V_3 - E_3 = 0$ (clockwise from point b)

$-(4\ \Omega)(I_2 - I_1) + 4\text{ V} - (6\ \Omega)(I_2) - 3\text{ V} = 0$

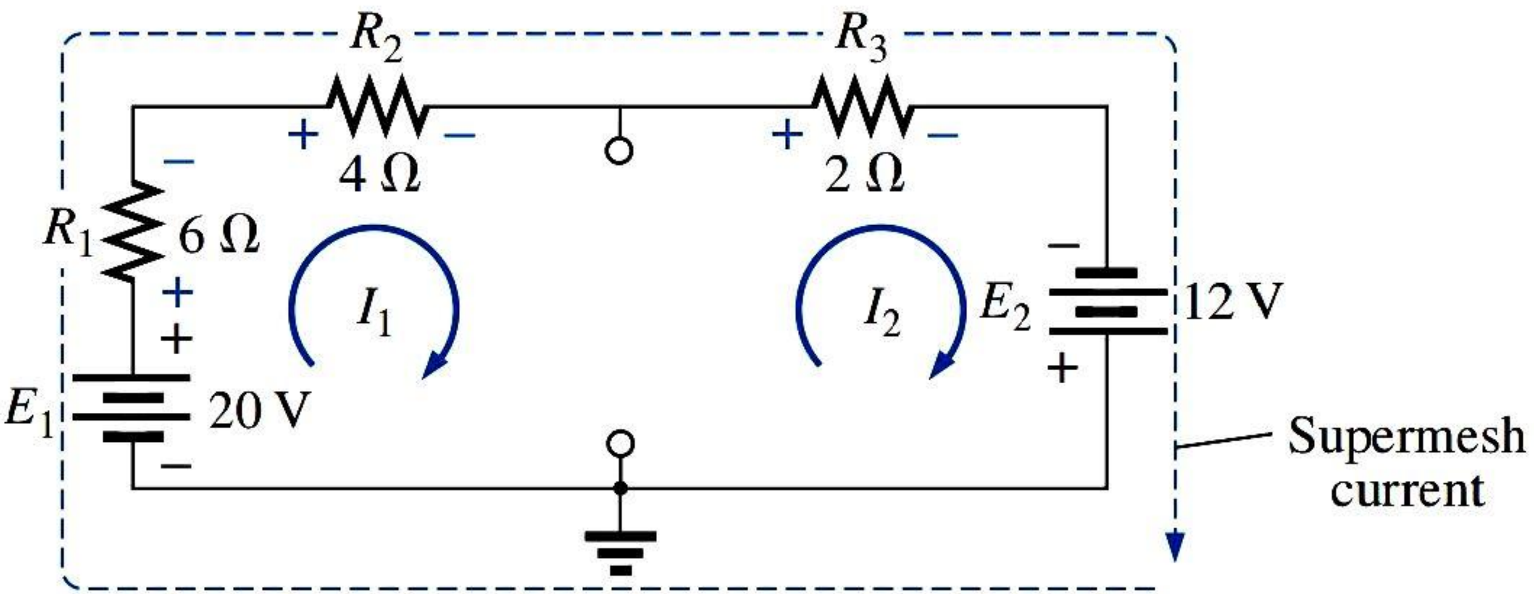


Mesh Analysis



Ref: Boylestad: Example 8.14: Page 299

Mesh Analysis

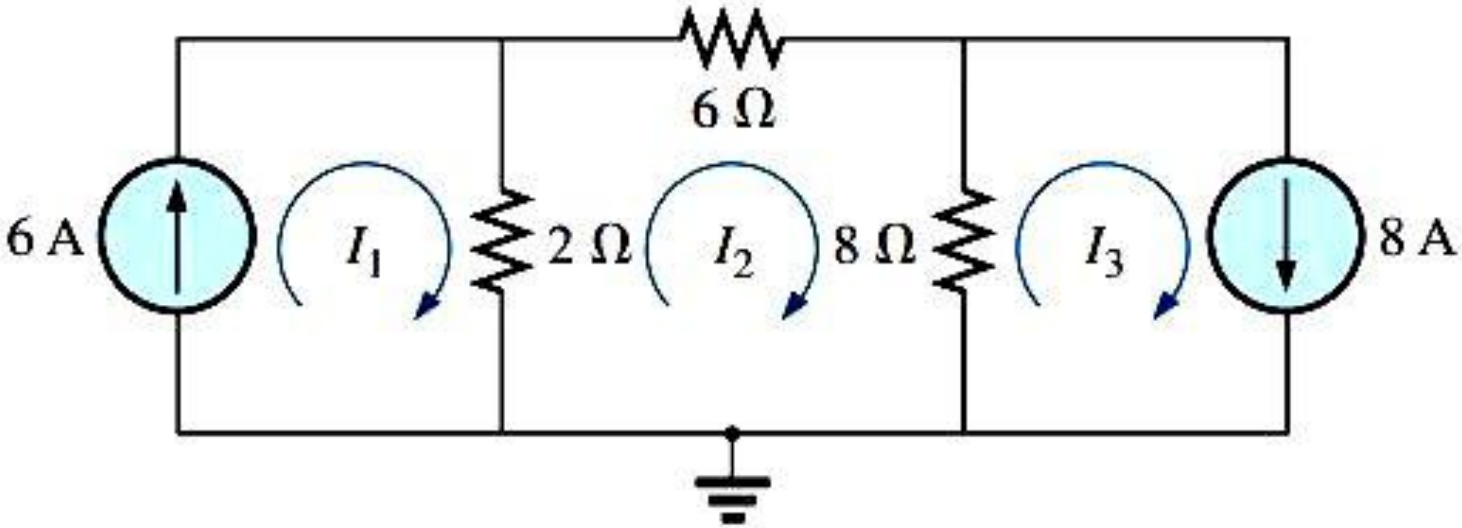


$$20\ \text{V} - I_1(6\ \Omega) - I_1(4\ \Omega) - I_2(2\ \Omega) + 12\ \text{V} = 0$$

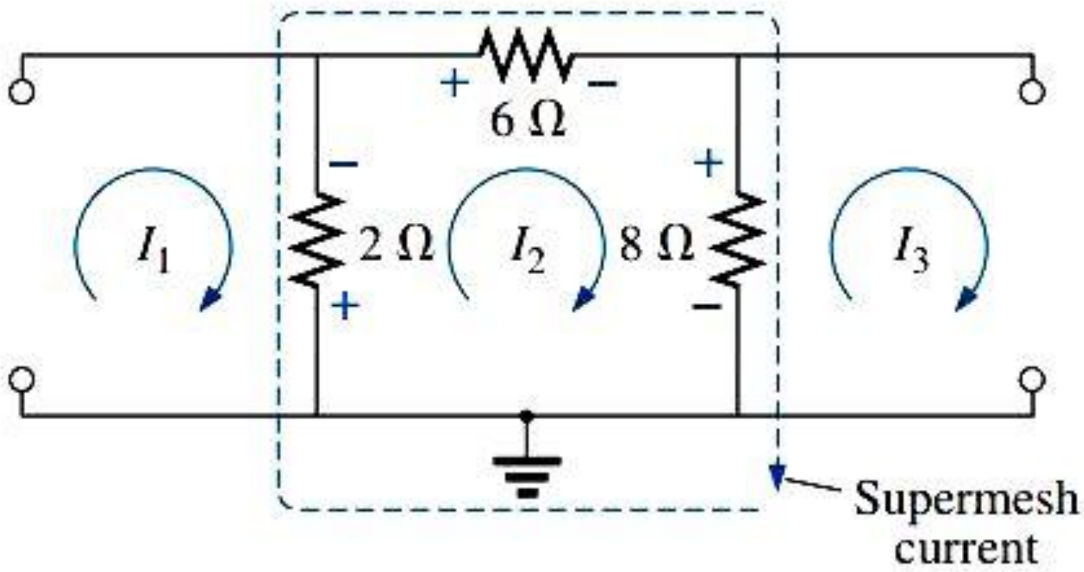
$$10I_1 + 2I_2 = 32$$

$$I_1 = I + I_2$$

Mesh Analysis



Mesh Analysis



$$-V_{2\Omega} - V_{6\Omega} - V_{8\Omega} = 0$$

$$-(I_2 - I_1)2\ \Omega - I_2(6\ \Omega) - (I_2 - I_3)8\ \Omega = 0$$

$$-2I_2 + 2I_1 - 6I_2 - 8I_2 + 8I_3 = 0$$

$$2I_1 - 16I_2 + 8I_3 = 0$$

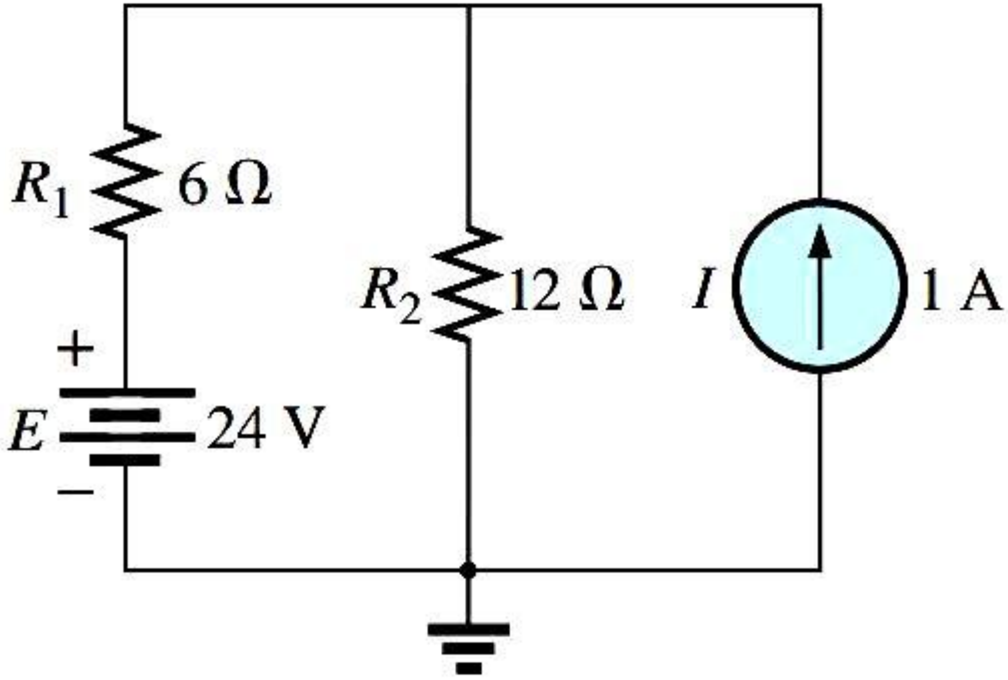
$$I_1 = 6\ \text{A}$$

$$I_3 = 8\ \text{A}$$

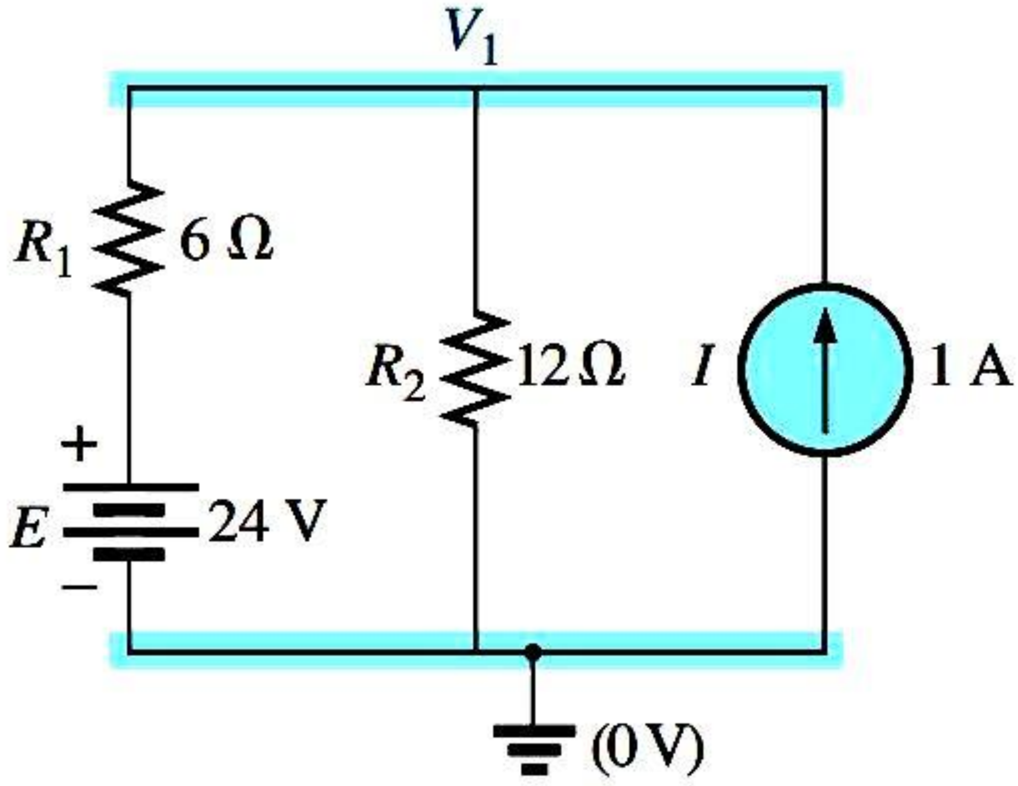
Node Analysis

- The number of nodes for which the voltage must be determined using nodal analysis is ONE less than the total number of nodes.
- The number of equations required to solve for all the nodal voltages of a network is ONE less than the total number of independent nodes.

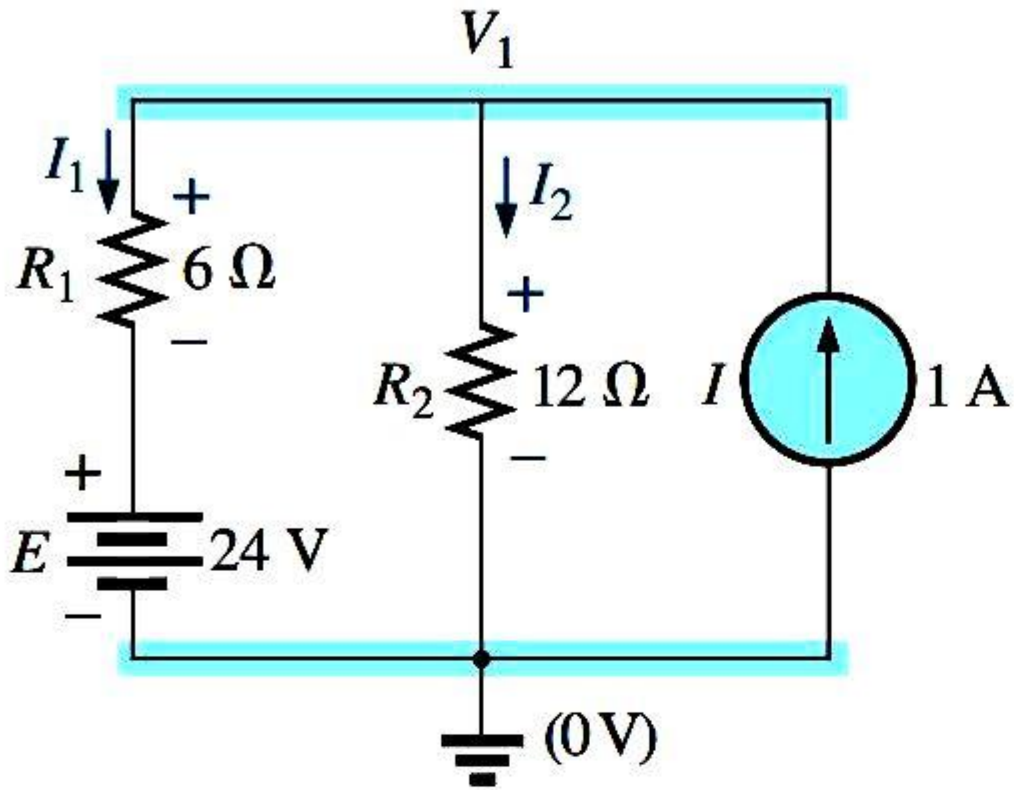
Node Analysis



Node Analysis

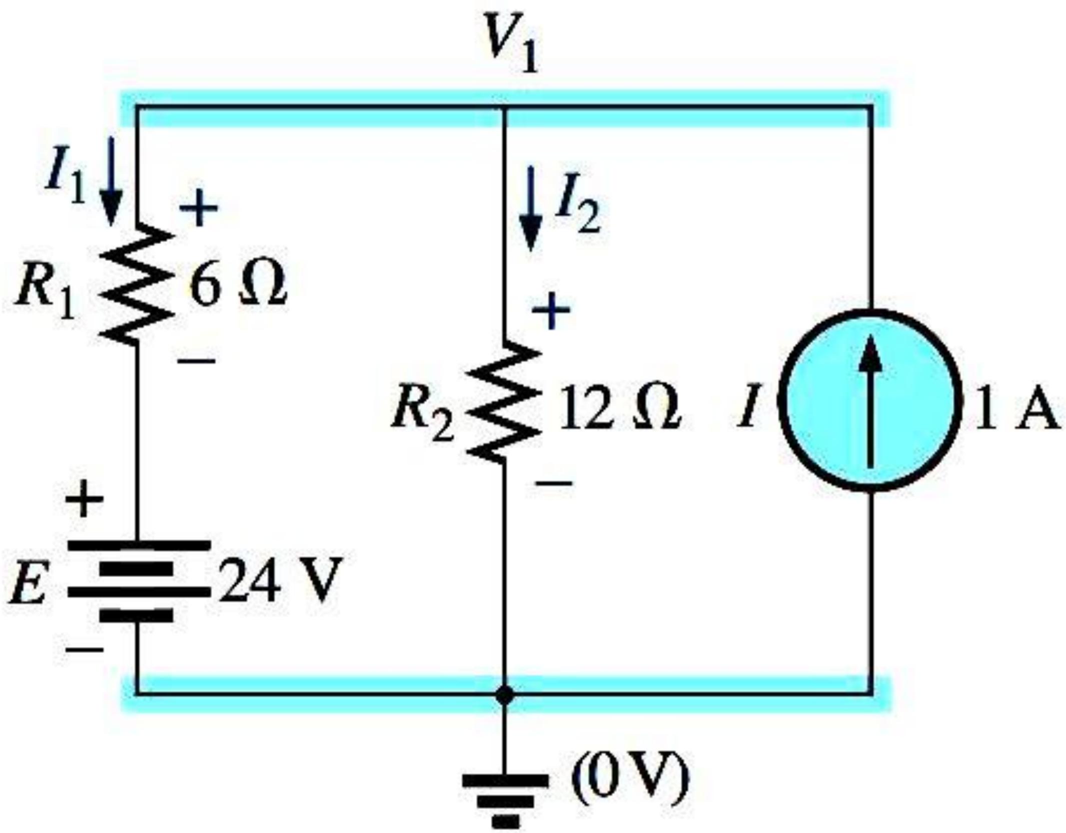


Node Analysis



$$I = I_1 + I_2$$

Node Analysis



$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1}{R_2}$$

$$I_1 = \frac{V_{R_1}}{R_1}$$

$$V_{R_1} = V_1 - E$$

Node Analysis

$$I = \frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} = V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - \frac{E}{R_1}$$

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{E}{R_1} + 1$$

$$V_1\left(\frac{1}{6\ \Omega} + \frac{1}{12\ \Omega}\right) = \frac{24\ \text{V}}{6\ \Omega} + 1\ \text{A} = 4\ \text{A} + 1\ \text{A}$$

$$V_1\left(\frac{1}{4\ \Omega}\right) = 5\ \text{A}$$

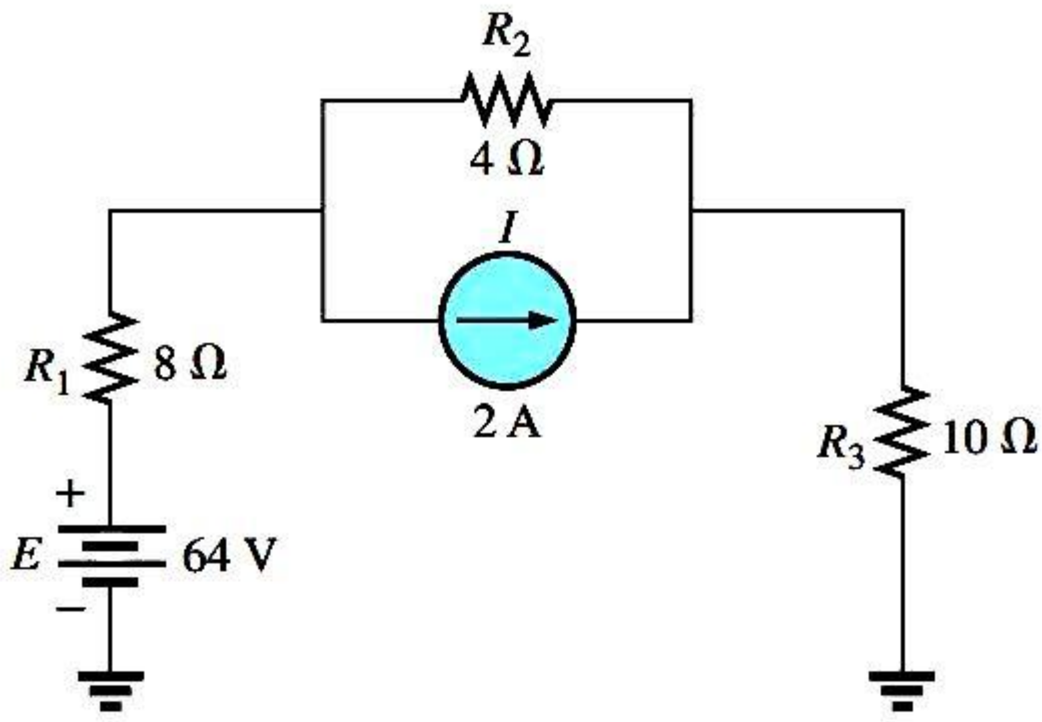
$$V_1 = \mathbf{20\ V}$$

Node Analysis

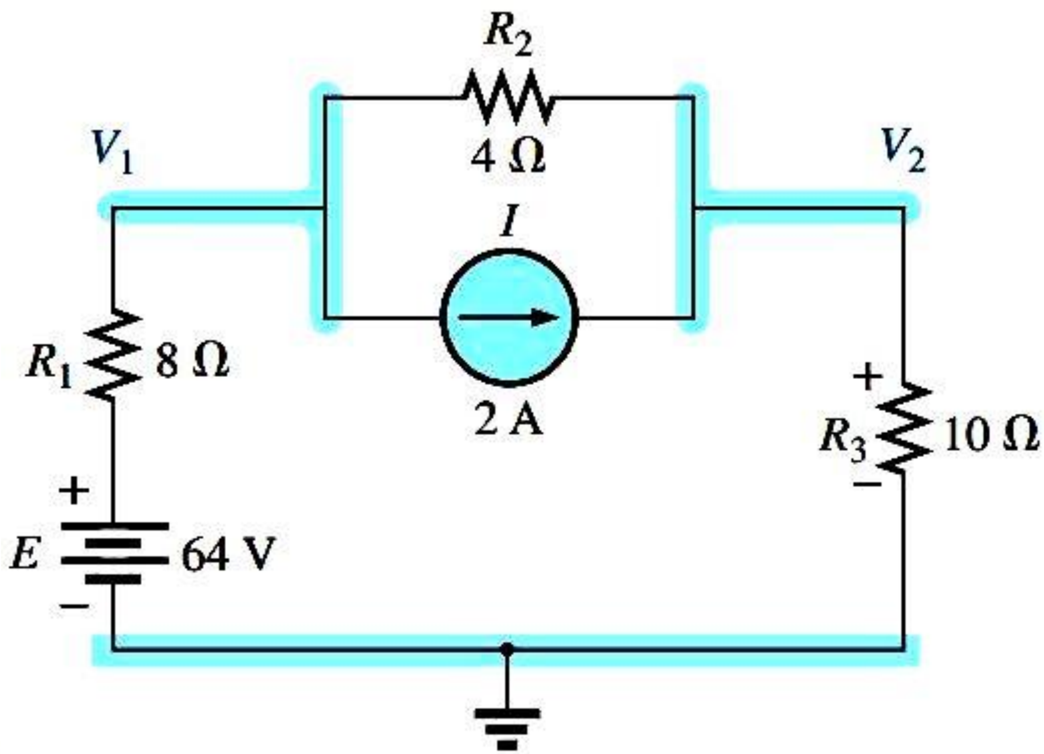
$$I_1 = \frac{V_1 - E}{R_1} = \frac{20\text{ V} - 24\text{ V}}{6\ \Omega} = \frac{-4\text{ V}}{6\ \Omega}$$
$$= \mathbf{-0.67\ A}$$

$$I_2 = \frac{V_1}{R_2} = \frac{20\text{ V}}{12\ \Omega} = \mathbf{1.67\ A}$$

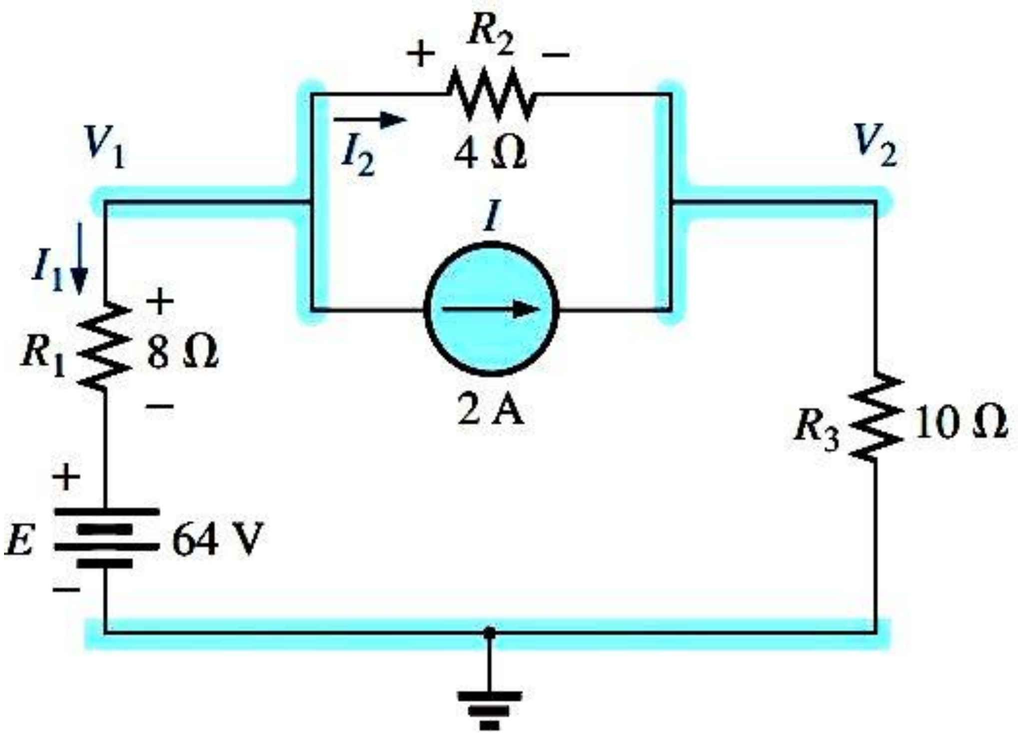
Node Analysis



Node Analysis



Node Analysis



$$0 = I_1 + I_2 + I$$

$$I_1 = \frac{V_1 - E}{R_1}$$

$$I_2 = \frac{V_{R_2}}{R_2} = \frac{V_1 - V_2}{R_2}$$

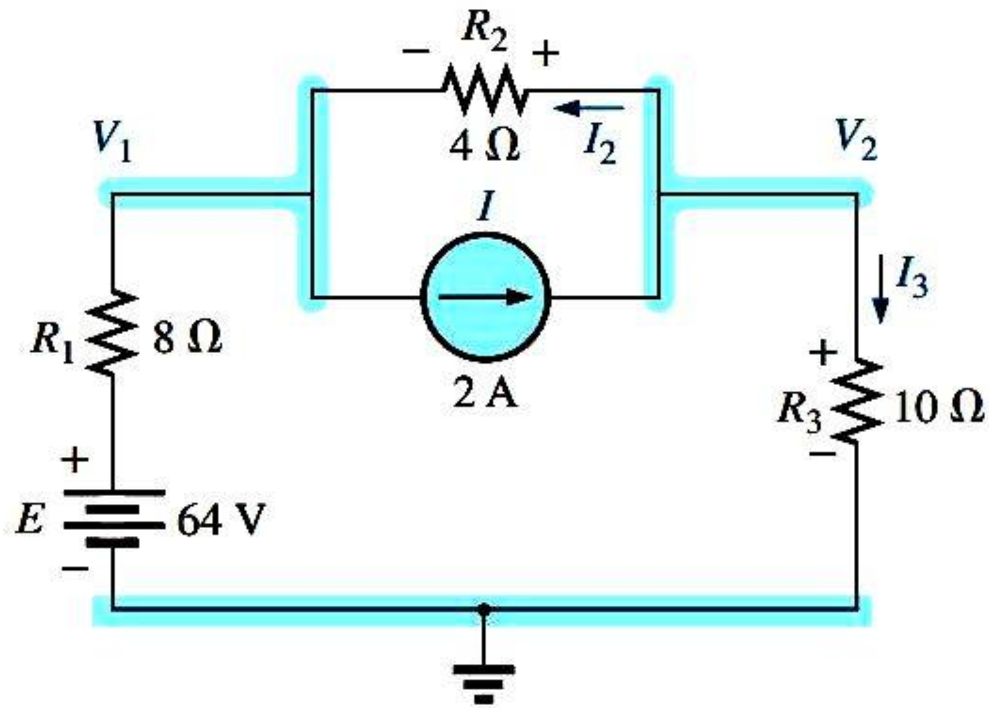
$$\frac{V_1 - E}{R_1} + \frac{V_1 - V_2}{R_2} + I = 0$$

$$\frac{V_1}{R_1} - \frac{E}{R_1} + \frac{V_1}{R_2} - \frac{V_2}{R_2} + I = 0$$

$$V_1\left(\frac{1}{R_1} + \frac{1}{R_2}\right) - V_2\left(\frac{1}{R_2}\right) = -I + \frac{E}{R_1}$$

$$V_1\left(\frac{1}{8\,\Omega} + \frac{1}{4\,\Omega}\right) - V_2\left(\frac{1}{4\,\Omega}\right) = -2\text{ A} + \frac{64\text{ V}}{8\,\Omega} = 6\text{ A}$$

Node Analysis



$$I = I_2 + I_3$$

$$I = \frac{V_2 - V_1}{R_2} + \frac{V_2}{R_3}$$

$$I = \frac{V_2}{R_2} - \frac{V_1}{R_2} + \frac{V_2}{R_3}$$

$$V_2\left(\frac{1}{R_2} + \frac{1}{R_3}\right) - V_1\left(\frac{1}{R_2}\right) = I$$

$$V_2\left(\frac{1}{4\ \Omega} + \frac{1}{10\ \Omega}\right) - V_1\left(\frac{1}{4\ \Omega}\right) = 2\ \text{A}$$

Node Analysis

$$V_1\left(\frac{1}{8\ \Omega}+\frac{1}{4\ \Omega}\right)-V_2\left(\frac{1}{4\ \Omega}\right)=6\ \text{A}$$

$$-V_1\left(\frac{1}{4\ \Omega}\right)+V_2\left(\frac{1}{4\ \Omega}+\frac{1}{10\ \Omega}\right)=2\ \text{A}$$

$$V_1 = \mathbf{37.82\ V}$$

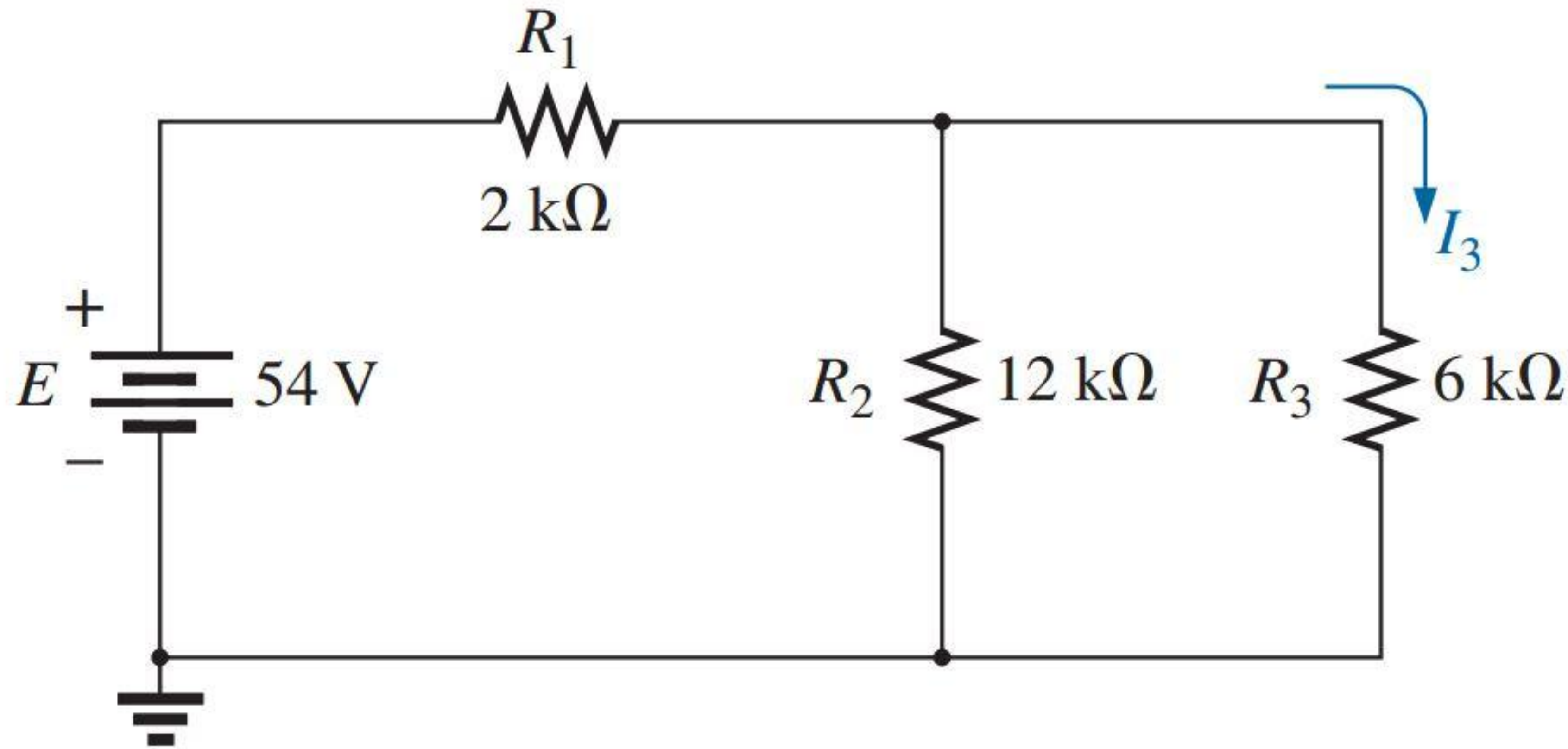
$$V_2 = \mathbf{32.73\ V}$$

$$I_{R_1}=\frac{E-V_1}{R_1}=\frac{64\ \text{V}-37.82\ \text{V}}{8\ \Omega}=\mathbf{3.27\ A}$$

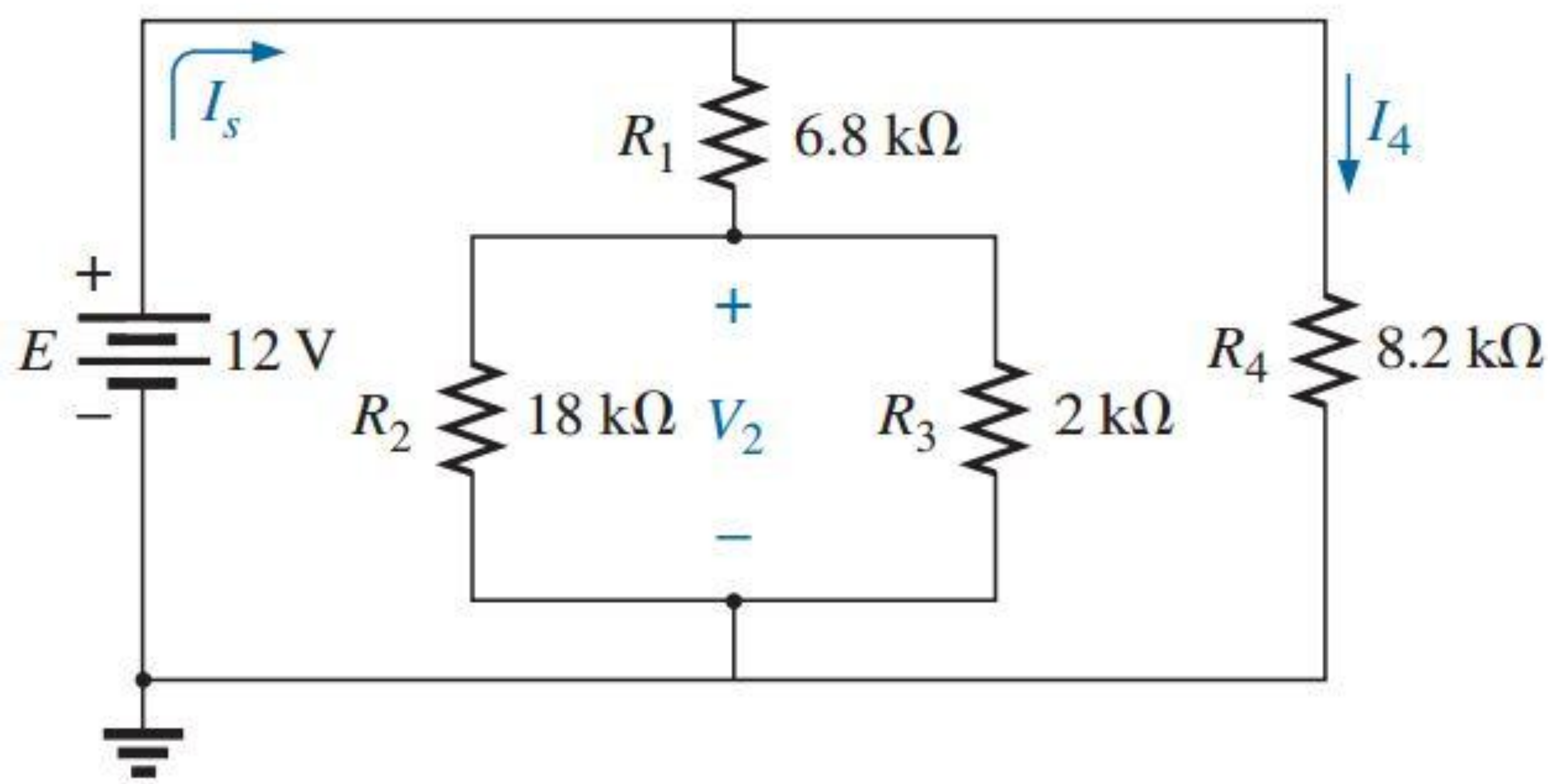
$$I_{R_2}=\frac{V_1-V_2}{R_2}=\frac{37.82\ \text{V}-32.73\ \text{V}}{4\ \Omega}=\mathbf{1.27\ A}$$

$$I_{R_3}=\frac{V_{R_3}}{R_3}=\frac{V_2}{R_3}=\frac{32.73\ \text{V}}{10\ \Omega}=\mathbf{3.27\ A}$$

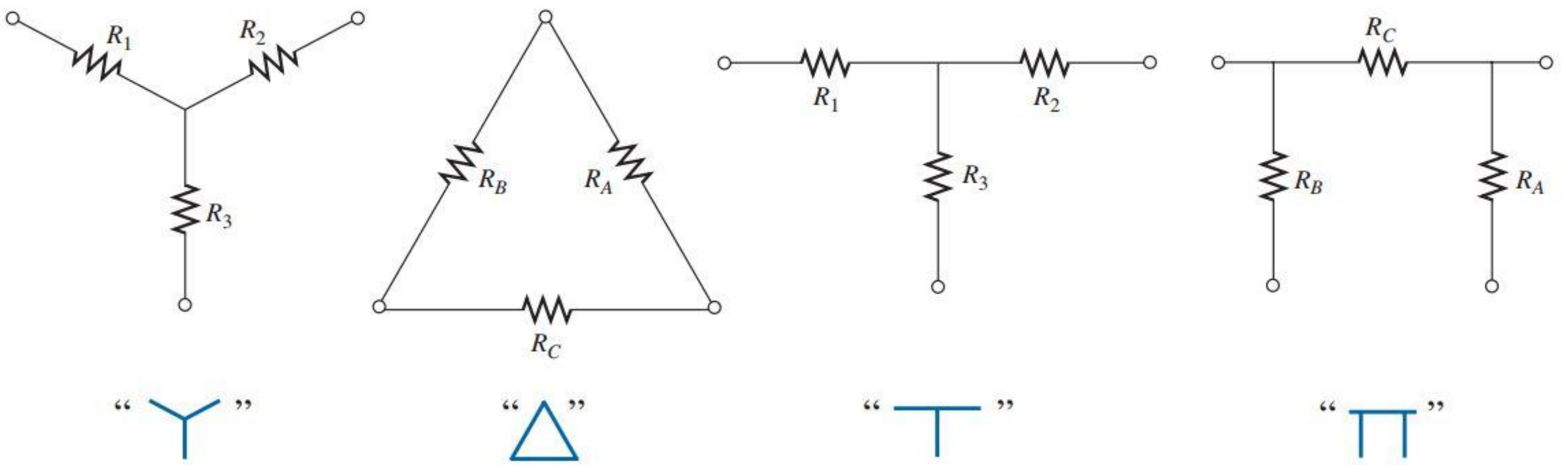
Network Reduction



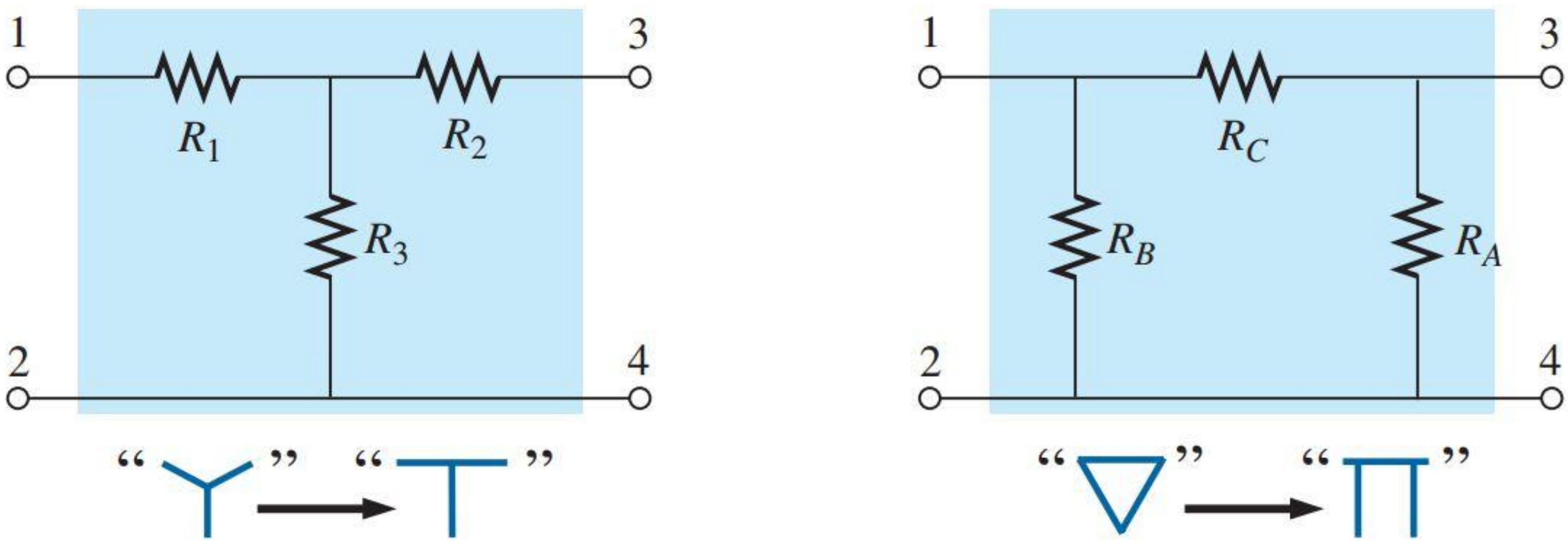
Network Reduction



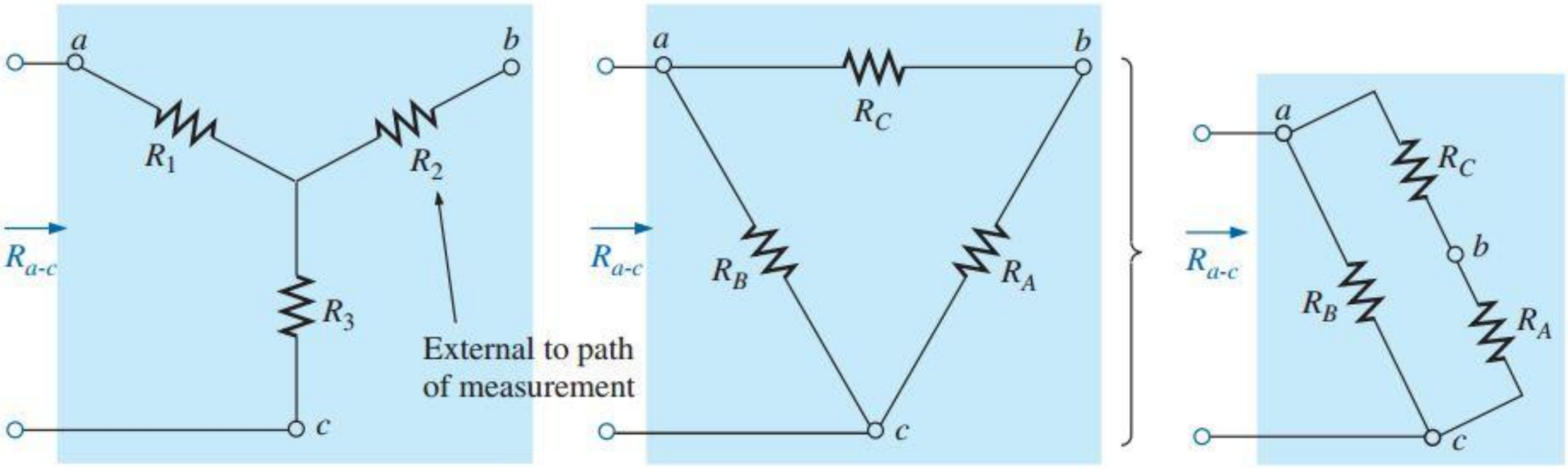
Y-Δ (T-P) And Δ -Y (P-T) Conversions



Y-Δ (T-P) And Δ -Y (P-T) Conversions



Y-Δ (T-P) And Δ -Y (P-T) Conversions



Y-Δ (T-P) And Δ -Y (P-T) Conversions

$$R_{a\text{-}c}\,(\mathrm{Y}) = R_{a\text{-}c}\,(\Delta)$$

$$\boxed{R_{a\text{-}c} = R_1 + R_3 = \frac{R_B(R_A + R_C)}{R_B + (R_A + R_C)}}$$

$$\boxed{R_{a\text{-}b} = R_1 + R_2 = \frac{R_C(R_A + R_B)}{R_C + (R_A + R_B)}}$$

$$\boxed{R_{b\text{-}c} = R_2 + R_3 = \frac{R_A(R_B + R_C)}{R_A + (R_B + R_C)}}$$

Y-Δ (T-P) And Δ -Y (P-T) Conversions

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C}$$

Y-Δ (T-P) And Δ -Y (P-T) Conversions

$$\boxed{R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_1}}$$

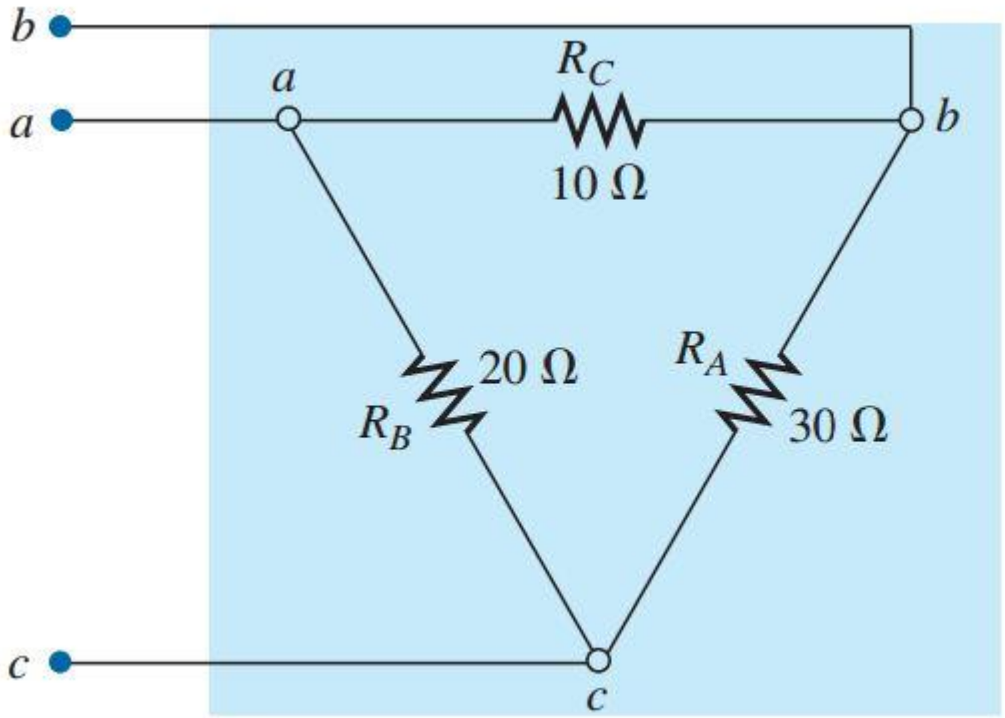
$$\boxed{R_Y = \frac{R_\Delta}{3}}$$

$$\boxed{R_B = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}}$$

$$\boxed{R_\Delta = 3R_Y}$$

$$\boxed{R_C = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}}$$

Y-Δ (T-P) And Δ -Y (P-T) Conversions



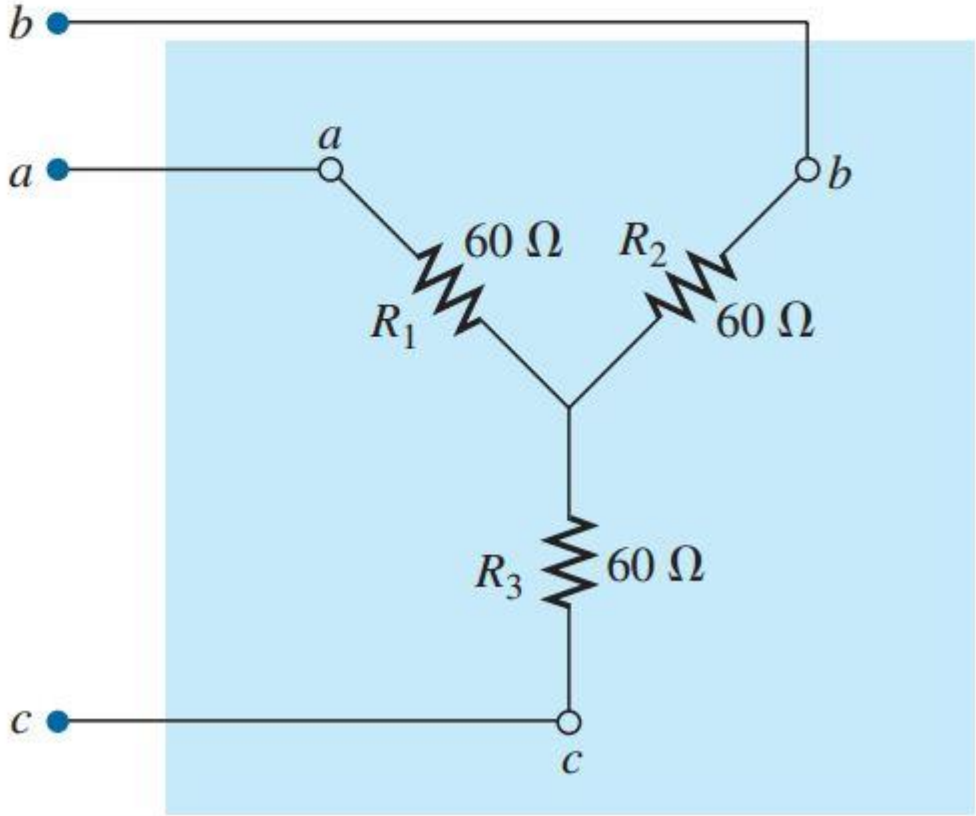
Y-Δ (T-P) And Δ -Y (P-T) Conversions

$$R_1 = \frac{R_B R_C}{R_A + R_B + R_C} = \frac{(20\ \Omega)(10\ \Omega)}{30\ \Omega + 20\ \Omega + 10\ \Omega} = \frac{200\ \Omega}{60} = \mathbf{3\tfrac{1}{3}\ \Omega}$$

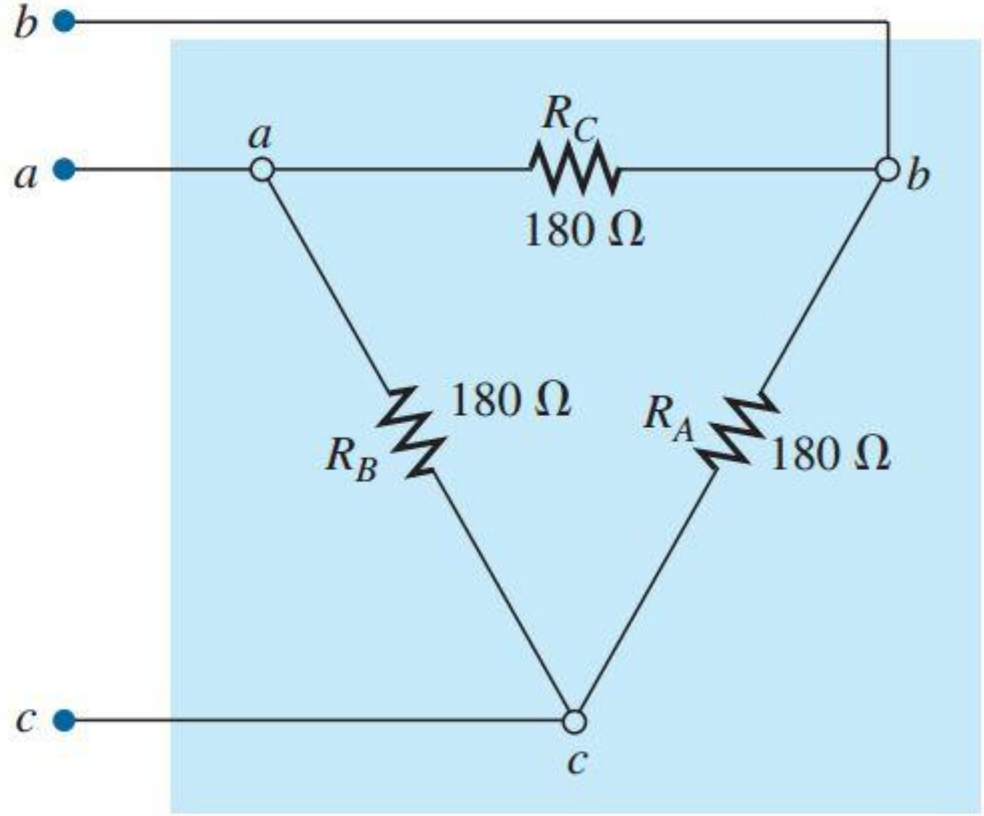
$$R_2 = \frac{R_A R_C}{R_A + R_B + R_C} = \frac{(30\ \Omega)(10\ \Omega)}{60\ \Omega} = \frac{300\ \Omega}{60} = \mathbf{5\ \Omega}$$

$$R_3 = \frac{R_A R_B}{R_A + R_B + R_C} = \frac{(20\ \Omega)(30\ \Omega)}{60\ \Omega} = \frac{600\ \Omega}{60} = \mathbf{10\ \Omega}$$

Y-Δ (T-P) And Δ -Y (P-T) Conversions



Y-Δ (T-P) And Δ -Y (P-T) Conversions



Y-Δ (T-P) And Δ -Y (P-T) Conversions

Practice Problems

Book: Boylestad

Example: 8.29 – 8.30

Page: 326

ASSIGNMENT - 1

Write the following theorems:

- **Superposition Theorem**
- **Millman's Theorem**
- **Reciprocity Theorem**
- **Thévenin's Theorem**
- **Norton's Theorem**
- **Maximum Power Transfer Theorem**