University of Rajshahi

Department of Computer Science and Engineering

B.Sc. Engg.(CSE) 1st Year 2016

Course: MATH1111(Algebra, Trigonometry and Vector)

Time: 3 Hrs. Full Marks: 52.5

[N.B. Answer SIXquestions taking at least THREE from each part.]

Ryadh

Part A

1.a) Define null set, subset, power set, union and intersection of two sets with example. 3

b) Define one-one and onto functions. Can a constant function be one-one? Justify your answer. Show that if a relation R is transitive, then its inverse relation R^{-1} is also transitive. 3

c) Use Cramer's rule to solve the system of linear equations: $x+y+z=3,\ x+2y+3z=6,\ 5x+8y+11z=24$. 2.75

2.a) Show that in an equation with real coefficients imaginary roots occurs in pairs. 3

b) Solve the equation $54x^3-39x^2-26x+16=0$, the roots being in geometrical progression. 3

c) In the equation $x^4-x^3-7x^2+x+6=0$, find the value of S_4. 2.75

3.a) If a,b,c are the roots of $x^3+qx+r=0$,find the equation whose roots are $bc+\frac{1}{a}, ca+\frac{1}{b}, ab+\frac{1}{c}$. 2

b) Solve the cubic equation $x^3-15x^2-33x+847=0$ by Cardan's method. 3.75

c) If $x=cos\theta+isin\theta$ and $1+\sqrt{1-a^2}=na$ then prove that $1+acos\theta=\frac{a}{2n}(1+nx)(1+\frac{n}{x})$ 3

4.a) If $x=\frac{2}{1!}-\frac{4}{3!}+\frac{6}{5!}-\frac{8}{7!}+\cdots \ldots\ldots\ldots\ldots to\ \infty$ and $y=1+\frac{2}{1!}-\frac{2^3}{3!}+\frac{2^5}{5!}-\cdots \ldots\ldots\ldots\ldots to\ \infty$ then show that $x^2=y$. 3

b) If$i^{i^{i \ldots\ldots adinf}}=A+iB$, then prove that $tan\frac{\pi A}{2}=\frac{B}{A}$ and $A^2+B^2=e^{-\pi B}$. 2.75

c) Find the sum to infinity of the series $sin\theta.sin\theta-\frac{1}{2}sin2\theta.sin^2\theta+\frac{1}{2}sin3\theta.sin^3\theta \ldots\ldots\ldots\ldots\ldots\ldots$ 3

Part B

5.a) Define dot product of two vectors $\vec{A}$ and $\vec{B}$. Prove that the area of a parallelogram with sides $\vec{A}$ and $\vec{B}$ is $|\vec{A}\times\vec{B}|$. 3

b) Determine the unit vector perpendicular to the plane of $\vec{A}=2\vec{i}-6\vec{j}-3\vec{k}$ and $\vec{B}=4\vec{i}+3\vec{j}-\vec{k}$. 2.75

c) Show that the vectors $\vec{A}=3\vec{i}-2\vec{j}+\vec{k}, \vec{B}=\vec{i}-3\vec{j}+5\vec{k}$and $\vec{C}=2\vec{i}+\vec{j}-4\vec{k}$ form a right-angled triangle. 3

6.a) Find a unit tangent vector to any point on the curve $x=t^2+1,\ y=4t-3,\ z=2t^2-6t$. Also determine it where t=2. 2.75

b) A particle moves on a curve so that its position vector is given by $\vec{r}=coswt\vec{i}+\sin wt\vec{j}$, where w is a constant. Show that the velocity of the particle is perpendicular to $\vec{r}$and the acceleration is directed towards the origin. 3

c) What is the physical significance of the curl of a vector field? Determine the value of λ so that the vector field $\vec{v}(x,y,z)=(x+3y)\vec{i}+(y-2z)\vec{j}+(x+\lambda z)\vec{k}$is solenoidal. 3

7.a) What is meant by $\vec{\nabla}\varphi$, where φ is a scalar field? Find a unit normal to the surface $x^2y+2xyz=4$ at the point (2, -2, 3). 3

b) Find angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2, -1, 2). 2.75

c) Find the volume of the region common to the intersecting cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$. 3

8.a) State the divergence theorem of Gauss. Verify Green's theorem in the plane for $\oint_C (x^2-2xy)dx+(x^2y+3)dy$ around the boundary of the region defined by $y^2=8x$ and x=2. 4.75

b) Verify Stroke's theorem for $\vec{A}=(y-z+2)\vec{i}+(yz+4)\vec{j}-xz\vec{k}$, where S is the surface of the cube $x=0,y=0,z=0,x=2,y=2,z=2$ above the xy plane. 4

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University of Rajshahi
Department of Computer Science and Engineering
B.Sc. (Engg.) Part-I (Odd Semester) Examination-2015
Course: MATH-1111 (Algebra, Trigonometry and Vector)

Marks: 52.5 **Time:** 03 Hours.

[Answer any six (06) questions taking three (03) from each section]

Section-A

1. a) Define null set and subset. State and prove De Morgan's rule. 3
   b) Define function. Find the domain and range of the function $f(x) = \frac{x-3}{2x+1}$. 2.75
   c) Using Cramer's rule solve the following system: 3
   $x + y + z = 1;\ x + 2y + 3z = 2;\ x + 4y + 9z = 4.$

2. a) Evaluate the determinant: $\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}$ 3
   b) Prove that in an equation with real coefficients imaginary roots occur in pairs. 3
   c) If a, b, c are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the value of $\sum \frac{b^2+c^2}{bc}$. 2.75

3. a) Prove that every equation of n^{th} degree has exactly n roots and no more. 3
   b) Solve the cubic equation: $28x^3 - 9x^2 + 1 = 0$. 3
   c) State Demoiver's theorem and prove it when n is fractional either positive or negative. 2.75

4. a) If $x_r = \cos\frac{\pi}{2r} + i\sin\frac{\pi}{2r}$, then prove that, $x_1, x_2, x_3, \ldots\ldots\ to\ infinity = -1$. 3
   b) If $(1+x)^n = P_0 + P_1x + P_2x^2 + \cdots\cdots$ then show that, $P_1 - P_3 + P_5 - \cdots\cdots = 2^{\frac{n}{2}}\sin\frac{n\pi}{4}$ 3
   c) Solve $x^4 - 4x^2 + 16 = 0$ using Demoiver's theorem. 2.75

Section-B

5. a) If $A + iB = log(x + iy)$, then show that $B = \tan^{-1}\frac{y}{x}$ and $A = \frac{1}{2}log(x^2 + y^2)$. 2.75
   b) Using Gregory's series prove that $\frac{\pi}{8} = \frac{1}{1.3} + \frac{1}{5.7} + \frac{1}{9.11} + \frac{1}{13.15} + \cdots\cdots$ 3
   c) Find the sum of the series $\operatorname{cosec}\alpha + \operatorname{cosec}2\alpha + \operatorname{cosec}2^2\alpha + \cdots + \operatorname{cosec}2^{n-1}\alpha$ 3

6. a) Find the projection of the vector $\bar{A} = \hat{\imath} - 2\hat{\jmath} + \hat{k}$ on the vector $\bar{B} = 4\hat{\imath} - 4\hat{\jmath} + 7\hat{k}$. 2.75
   b) Find principal normal and binormal at point $t = \pi$ to the curve 3
   $x = 3\cos t,\quad y = 3\sin t,\quad z = 4t.$
   c) Find the total work done in moving a particle in a force field given by 3
   $\bar{F} = 3xy\hat{\imath} - 5z\hat{\jmath} + 10x\hat{k}$ along the curve $x = 1 + t^2, y = 2t^2, z = t^2$ from $t = 1$ to $t = 2$.

7. a) Define gradient and divergence. What is the physical significance of gradient? 3
   b) Find directional derivative of $\varphi = x^2yz + 4xz^2$ at $(1, -2, -1)$ in the direction $2\hat{\imath} - \hat{\jmath} - 2\hat{k}$. 2.75
   c) Find the constants a, b, c so that $\bar{F} = (x + 2y + az)\hat{\imath} + (bx - 3y - z)\hat{\jmath} + (4x + cy + 2z)\hat{k}$ is irrotational. 3

8. a) If $\bar{F} = 4xz\hat{\imath} - y^2\hat{\jmath} + yz\hat{k}$, evaluate $\iint_s \bar{F}.\bar{n}\, ds$ where s is the surface of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. 4
   b) State Green's theorem in the plane. Verify Green's theorem in the plane for $\varphi\{(xy + y^2)dx + x^2dy\}$, where c is the closed curve of the region bounded by $y = x$ and $y = x^2$. 4.75

University of Rajshahi

Department of Computer Science and Engineering

B. Sc. (Engg.) Part-I Odd Semester Exam - 2014

Course: MATH-1111 (Algebra, Trigonometry and Vector ~~Analysis~~)

Full Marks: 52.5 **Time:** 3 Hours

[N.B.: Answer any SIX questions taking THREE from each section. Marks for each question are shown on the right side margin.]

Section A

1. a) Explain the operations of union, intersection and difference of sets with the aid of Venn-Euler diagrams. 3

b) Is there any difference between mappings and operators? Explain your answer. Give an example of a relation which is not symmetric. 2.75

c) Using Cramer's rule solve the following system: 3

$$3x - y + 2z = 7$$
$$2x + y + z = 7$$
$$x + y - 2z = -3$$

2. a) Solve the cubic equation $3x^3 - 26x^2 + 52x - 24 = 0$, the roots being in geometrical progression. 2.75

b) What is Descartes' rule of signs? Use the rule to find the nature of the roots of the quintic equation $x^5 + 5x^4 - 20x^2 - 19x - 2 = 0$. Show that the equation has a real root between 2 and 3. 3

c) Obtain the value of S_6 in equation $x^3 - x - 1 = 0$. 3

3. a) Test the equation $2x^4 + x^3 - 6x^2 + x + 2 = 0$ whether it is reciprocal. If a, b and c are roots of the equation $x^3 + qx + r = 0$, form the equation whose roots are $\frac{b+c}{a^2}$, $\frac{c+a}{b^2}$, $\frac{a+b}{c^2}$. 2.75

b) Solve the quartic equation $x^4 - 6x^3 + 12x^2 - 10x + 3 = 0$ which has equal roots. 2

c) Use Cardan's method to solve the cubic equation $x^3 + 21x + 342 = 0$. 4

4. a) Mention some applications of Demoivre's theorem. With the aid of Demoivre's theorem solve the polynomial equation $x^7 + x^4 + x^3 + 1 = 0$. 3.75

b) If $x_r = \cos\frac{\Pi}{2^r} + i\sin\frac{\Pi}{2^r}$, prove that $x_1 x_2 x_3$.............to infinity = -1. 3

c) If $\frac{Sinx}{x} = \frac{5045}{5046}$ then show that x is nearly $1^0 58'$. 2

Section B

5. a) Explain a technique to find the numerical value of Π using Gregory's series. 2.75

b) Show that $i^i = e^{-(4n+1)\frac{\Pi}{2}}$. 3

c) Find the sum of the following series up to n terms 3

$$\tan^{-1}\left(\frac{1}{2.1^2}\right)+\tan^{-1}\left(\frac{1}{2.2^2}\right)+\tan^{-1}\left(\frac{1}{2.3^2}\right)+\dots\dots\dots$$

6. a) Show graphically that $-(\vec{A}-\vec{B})=-\vec{A}+\vec{B}$. Graph the vector field defined by $\vec{V}(x,y)=x\hat{i}+y\hat{j}$. 3

b) Show that commutative law for dot products is valid. Find the projection of the vector $2\hat{i}-3\hat{j}+6\hat{k}$ on the vector $\hat{i}+2\hat{j}+3\hat{k}$. Draw a rough sketch of it. 3.75

c)Show that $\vec{A}.\left(\vec{B}\times\vec{C}\right)=\vec{B}.\left(\vec{C}\times\vec{A}\right)=\vec{C}.\left(\vec{A}\times\vec{B}\right)$. 2

7. a) A particle moves so that its position vector is given by $\vec{r}=\cos\omega t\,\hat{i}+\sin\omega t\,\hat{j}$, where ω is a constant. Show that 3

(i) the velocity $\vec{v}$ of the particle is perpendicular to $\vec{r}$.

(ii) $\vec{r}\times\vec{v}$ is a constant vector.

b) Show that $div.curl\,\vec{A}=0$. 2.75

c) If $\vec{\nabla}.\vec{E}=0$, $\vec{\nabla}.\vec{H}=0$, $\vec{\nabla}\times\vec{E}=-\frac{\partial\vec{H}}{\partial t}$, $\vec{\nabla}\times\vec{H}=\frac{\partial\vec{E}}{\partial t}$, show that $\vec{E}$ and $\vec{H}$ satisfy 3

$$\nabla^2\vec{u}=\frac{\partial^2\vec{u}}{\partial t^2}.$$

8. a) The acceleration of a particle at any time t given by 3

$$\vec{a}=\frac{d\vec{v}}{dt}=12\cos 2t\,\hat{i}-8\sin 2t\,\hat{j}+16t\,\hat{k}.$$

If the velocity $\vec{v}$ and displacement $\vec{r}$ are zero at $t=0$, find $\vec{v}$ and $\vec{r}$ at any time.

b) Find the value of $\int_{-3}^{3}\int_{0}^{4}\int_{2}^{5}(x+y+z)dzdydx$. 2.75

c) State Green's theorem. Verify Green's theorem in the plane for $\oint_C (xy+y^2)dx+x^2dy$, where C is the closed curve of the region bounded by $y=x$ and $y=x^2$. 3

University of Rajshahi
Department of Computer Science and Engineering
B. Sc. (Engg.) Examination-2013, Part-1, Odd Semester
Course: MATH-1111 (Algebra, Trigonometry, Vector Analysis)
Full Marks-52.5 Time: 4 hours

[N. B: Answer any six questions taking three from each group]

PART-A

1. a) State and prove De-Morgan's Laws. 3
   b) If ω is a root of $x^4-1=0$, show that $a+b\omega+c\omega^2+d\omega^3$ is a factor of 3

$$\begin{vmatrix} a & b & c & d \\ b & c & d & a \\ c & d & a & b \\ d & a & b & c \end{vmatrix}$$

Hence show that the determinant is equal to $-(a+b+c+d)(a-b+c-d)[(a-c)^2+(b-d)^2]$.

   c) Solve the following system of linear equations (By Cramer's rule) 2.75

$$x-2y+3z=11$$
$$2x+y+2z=10$$
$$3x+2y+z=9$$

2. a) Define the complement of a set. Prove that $B-A^C=B\cap A$. 3
   b) Let S be the set of all integers. Given $a, b \in S$ define aRb if a-b is divisible by 2. Show that the relation R defines an equivalence relation. 3
   c) What is the difference between an into function and an onto function? 2.75

3. a) If a, b, c are the roots of $x^3-3qx+s=0$, show that the equation whose roots are a-b, b-c, c-a is $x^3+9qx\pm 3k=0$, where $k^2=3(4q^2-s^2)$. 3
   b) Solve $x^3-12x^2-6x-10=0$ by Cardan's method. 3
   c) If the roots of $x^n-1=0$ are $1, \alpha, \beta, \gamma, \ldots$ show that $(1-\alpha)(1-\beta)(1-\gamma)\ldots\ldots=n$. 2.75

4. a) State De-Moiver's theorem and find the value of (i) $(-1)^{1/4}$, (ii) $(-i)^{1/6}$. 3
   b) If $x=\frac{2}{1!}-\frac{4}{3!}+\frac{6}{5!}-\frac{8}{7!}+\cdots\infty$ and $y=1+\frac{2}{1!}-\frac{2^3}{3!}+\frac{2^5}{5!}-\cdots$ *to* ∞, show that $x^2=y$. 3
   c) If $x+\frac{1}{x}=2\cos\theta$, then show that If $x^n+\frac{1}{x^n}=2\cos n\theta$. 2.75

PART-B

5. a) Evaluate Log(a+ib), where a and b are real. 3
   b) Express Log {Log $(\cos\theta+i\sin\theta)$} in the form A + iB. 3
   c) If tan Log(x+iy) = a+ib, where $a^2+b^2\neq 1$, prove that $\tan\log(x^2+y^2)=\frac{2a}{1-(a^2+b^2)}$. 2.75

6. a) Find the numerical value of π to 4 places of decimals by Dase's Series. 3
   b) Find the summation of the following series to n terms: $\tan\alpha+2\tan2\alpha+2^2\tan2^2\alpha+\ldots\ldots$ 3
   c) Show that $\cosh\theta$ is periodic. 2.75

7. a) Define the dot product of two vectors. Find the angles which the vector $\underset{A}{\rightarrow}=3i-6j+2k$ makes with the coordinate axes. 3
   b) Prove that $\underset{A}{\rightarrow}\cdot\left(\underset{A}{\rightarrow}\times\underset{C}{\rightarrow}\right)=0$. 2.75
   c) Find the unit tangent vector to any point on the curve $x=t^2+1$, $y=4t-3$, $z=2t^2-6t$. 3

8. a) Define the curl of a differential vector field. Show that $\nabla\times(\nabla\varphi)=0$. 3
   b) Find the volume of the region common to the intersecting cylinders $x^2+y^2=a^2$ and $x^2+z^2=a^2$. 2.75
   c) State and prove the Divergence Theorem of Gauss. 3

University of Rajshahi
Department of Computer Science and Engineering
B.Sc. Engg. (CSE) 1st Year Odd Semester 2012
Course: MATH 1111(Algebra, Trigonometry and Vector Analysis)
Time: 4 Hrs Full Marks: 52.5
[N.B. Answer any SIX questions taking at least THREE from each part]

Section A

1. a) Define function. Find the domain and range of f given by $y = f(x) = \frac{x-1}{2x-3}$. $x, y \in \mathbb{R}$ 2.75
   b) Define difference of two sets. Prove $A\cup(B\cap C)=(A\cup B)\cap(B\cup C)$. 3
   c) Solve the system of equations using Cramer's rule $x+2y-z=4$, $x+4y-2z=-6$, $2x+3y+z=3$ 3

2. a) State and prove De Moivre's theorem for positive integer n. 3
   b) Prove that $\sin\alpha = \alpha - \frac{\alpha^3}{3!} + \frac{\alpha^5}{5!} + \cdots$ ……… to infinity term 3
   c) Expand $\tan x$ in power of x as far as the term involving x^6 2.75

3. a) Prove that if $\alpha_1, \alpha_2 \ldots\ldots \alpha_n$ are the roots of the equation. 3
   $f(x)=c_0x^n+c_1x^{n-1}+\ldots\ldots+c_n=0$, *then the sum of the roots is* $-\frac{c_1}{c_0}$, the sum of the products of roots taken two at a time is $\frac{c_2}{c_0}$, the sum of products of the roots taken tree at a time is $-\frac{c_3}{c_0}$, etc, finally the product of all the roots is equal to $(-1)^n\frac{c_n}{c_0}$
   b) If a, b, c are roots of the equation $x^3+p_1x^2+p_2x+p_3=0$, form the equation whose roots are a^2, b^2, c^2. 3
   c) State Descartes' rule of signs, Find the nature of the roots of the equation $x^9+5x^8-x^3+7x+2=0$ 2.75

4. a) Prove that in an equation with real coefficients imaginary roots occur in pairs. 2.75
   b) Solve the cubic equation $x^3-15x-126=0$ 3
   C Solve the equation $3x^3-26x^2+52x-24=0$, the roots being in geometrical progression. 3

Section B

5. a) Separate $\log(a+ib)$ into real and imaginary parts. 3
   b) Show that $i^i = e^{-(4n+1)\pi/2}$ 3
   c) Using Gregory's series show that $\pi = \frac{8}{1.3} + \frac{8}{5.7} + \frac{8}{9.11} + \cdots$ 2.75

6 a) Sum to n terms the series $\cot^{-1}(2.1^2)+\cot^{-1}(2.2^2)+\cot^{-1}(2.3^2)+\ldots\ldots\ldots\ldots$ 2.75
   b) If $\sin^{-1}(u+iv)=\alpha+i\beta$ prove that $\sin^2\alpha$ and $\cosh^2\beta$ are roots of the equation $x^2-x(1+u^2+v^2)+u^2=0$ 3
   c) Sum to n terms the series $\sqrt{1+\sin\alpha}+\sqrt{1+\sin2\alpha}+\sqrt{1+\sin3\alpha}\ldots\ldots\ldots\ldots$ 3

7. a) Determine a unit vector perpendicular to the plane of $\vec{A}=2\hat{\imath}-6\hat{\jmath}-3\hat{k}$ and $\vec{B}=4\hat{\imath}+3\hat{\jmath}-\hat{k}$. 3
   b) Find the projection of $\vec{A}=6\hat{\imath}-2\hat{\jmath}+3\hat{k}$ on $\vec{B}=4\hat{\imath}+\hat{\jmath}-8\hat{k}$ 2.75
   c) A particle moves along the curve $\vec{r}=3t^2\hat{\imath}+2t\hat{\jmath}+3t\hat{k}$ at time t, find its velocity $\vec{V}$ and acceleration $\vec{a}$. 3

8. a) If $\vec{A}=x^2yz\hat{\imath}-2xz^3\hat{\jmath}+xz^2\hat{k}$ and $\vec{B}=2z\hat{\imath}+y\hat{\jmath}-x^2\hat{k}$, find $\frac{\delta^2}{\delta x\delta y}(\vec{A}\times\vec{B})$ at $(1,0,-2)$. 2.75
   b) If $\vec{\nabla}.\vec{E}=0, \vec{\nabla}.\vec{H}=0, \vec{\nabla}\times\vec{E}=-\frac{\delta\vec{H}}{\delta t}, \vec{\nabla}\times\vec{H}=\frac{\delta\vec{E}}{\delta t}$, show that $\vec{E}$ *and* $\vec{H}$ satisfy $\nabla^2\vec{u}=\frac{\delta^2\vec{u}}{\delta t^2}$ 3
   c) The acceleration of a particle at any time $t\geq0$ is given by $\vec{a}=\frac{d\vec{v}}{dt}=12\cos2t\hat{\imath}-8\sin2t\hat{\jmath}+16t\hat{k}$, if the velocity $\vec{V}$ and displacement $\vec{r}$ are zero at t=0, find $\vec{V}$ and $\vec{r}$ at any time 3