University of Rajshahi

Department of Computer Science and Engineering

B.Sc. Engg. Part-1 Even Semester, Examination-2016

Course: MATH-1211 (Differential and Integral Calculus)

Time: 3 Hours | Full Marks: 52.5

Answer any **six** questions taking **three** from each group

PART: A

1 (a) Define domain and range of a function. Find the domain and range of the function f defined by $f(x) = \frac{x-3}{2x+1}$. 2.75

(b) State L' Hospital's rule. Evaluate $\lim_{x\to 0}(cosx)^{cot^2x}$. 3

(c) Define continuity of functions. Show that $f(x) = |x|$ is continuous at x=0 but $f'(x)$ does not exist. 3

2 (a) If siny=xsin(a+y), prove that $dy/dx = \frac{sin^2(a+y)}{sina}$. 3

(b) If $y = e^{a\sin^{-1}x}$, show that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$ 2.75

(c) State and prove Mean Value theorem. 3

3 (a) Show that the maximum value of x+1/x is less than its minimum value. 2.75

(b) If $u=F(x^2+y^2+z^2)f(xy+yz+zx)$, then show that $(y-z)\frac{\partial u}{\partial x} + (z-x)\frac{\partial u}{\partial y} + (x-y)\frac{\partial u}{\partial z} = 0$. 3

(c) Define homogeneous functions. If $u = \cos^{-1}(\frac{x+y}{\sqrt{x}+\sqrt{y}})$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$. 3

4 (a) If $x\cos\alpha+y\sin\alpha=p$ touches the curve $\frac{x^m}{a^m} + \frac{y^m}{b^m} = 1$, show that $(a\cos\alpha)^{\frac{m}{m-1}} + (b\sin\alpha)^{\frac{m}{m-1}} = p^{\frac{m}{m-1}}$. 3

(b) If lx+my=1 is normal to the parabola $y^2=4ax$, then prove that $al^3+2alm^2=m^2$. 2.75

(c) Define asymptotes. Find the asymptotes of $x^2y^2-4(x-y)^2+2y-3=0$. 3

PART: B

5 (a) Evaluate any three of the followings: 8.75

(i) $\int\sqrt{\frac{a+x}{x}}dx$ (ii) $\int\frac{xe^x}{(1+x)^2}dx$

(iii) $\int\frac{x+sinx}{1+cosx}dx$ (iv) $\int\sqrt{2ax-x^2}dx$

6 (a) Evaluate $\int_0^{\pi/2}\frac{dx}{3+5cosx}$. 3

(b) Show that $\int_0^{\pi} x\log sinx\, dx = \frac{\pi^2}{2}log\frac{1}{2}$. 3

(c) Evaluate $lt_{n\to\infty}[\frac{1^2}{n^3+1^3} + \frac{2^2}{n^3+2^3} + \cdots + \frac{n^2}{n^3+n^3}]$ 2.75

7 (a) Show that $\int_0^{\pi/2} sin^n x dx = \frac{(n-1)}{n}\frac{(n-3)}{n-2}\cdots\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{\pi}{2}$, where n is a positive integer. 3

(b) Evaluate $\int_0^{\pi/2}\frac{sin^2x}{sinx+cosx}dx$. 2.75

(c) If $I_n = \int_0^{\pi/4} tan^n\theta d\theta$, show that $I_n = \frac{1}{n-1} - I_{n-2}$. 3

8 (a) Find the area bounded by the curves $y^2=4ax$ and $x^2=4ay$. 2.75

(b) If s is the arc length of $3ay^2=x(x-a)$ measured from the origin to the point (x, y), show that $3s^2=4x^2+3y^2$. 3

(c) Show that the volume of the solid produced by the revolution of the loop of the curve $y^2(a+x)=x^2(a-x)$ about x-axis is $2\Pi a^3(\log2-2/3)$. 3

University of Rajshahi
Department of Computer Science and Engineering
B. Sc. (Engg.) Part-I, Even Semester, Examination 2015
Course: MATH1211 (Differential and Integral Calculus)
Full Marks: 52.5 **Time:** 3 Hours

[Answer any Six questions taking three from each part]

Part A

1. a) Find the domain and range of the function $\frac{x-3}{2x+1}$ and also find its inverse function, if exists. 3

   b) $f(x)$ is defined as follows: 3

   $$f(x) = 0, \; x=0$$
   $$= x, \; x>0$$
   $$= -x, \; x<0$$

   Draw the graph of the function. Does $f'(x)$ exist at $x=0$? Justify your answer.

   c) Examine the continuity of the function $f(x)$ at $x=3/2$ where 2.75

   $$f(x) = \begin{cases} 3-2x, 0 \le x < 3/2 \\ -3-2x, \; x \ge 3/2 \end{cases}$$

2. a) Define differentiability of a function. Prove that every finitely derivable function is continuous. 3

   b) If $y = e^{ax} sinbx$, prove that $y_2 - 2ay_1 + (a^2 + b^2)y = 0$ 2.75

   c) If $y = \sin(asin^{-1}x)$, with the help of Leibnitz's theorem prove that 3

   $$y_{n+2}(1-x^2) - (2n+1)xy_{n+1} - (n^2 - a^2)y_n = 0$$

3. a) State and prove the Rolle's theorem. 3

   b) Verify the mean value theorem for $f(x)=2x^2-7x+10$, $a=2$, $b=5$. 2.75

   c) Expand $e^x cosx$ in a finite series in power of x with Lagrange's remainder using Maclaurin series. 3

4. a) Define maxima and minima of a function. Find the maximum and minimum values of $2x^3-9x^2+12x-3$. 3

   b) State Euler's theorem. Verify Euler's theorem for the function $u = sin\frac{x^2+y^2}{xy}$. 3

   c) Find the asymptotes of $x^3+2x^2y-xy^2-2y^3+xy+y^2-1=0$. 2.75

Part B

5. a) Integrate the following with respect to x (any two) 3

i). $\int \frac{dx}{(1+x)\sqrt{(1+2x+x^2)}}$. ii). $\int \frac{dx}{a+bsinx}$ iii). $\int \sqrt{\{\frac{\sin(x-\alpha)}{\sin(x+\alpha)}\}}dx$

b) Integrate $\int \frac{\log(logx)}{x} dx$.

c) Evaluate $\int \frac{dx}{cosx\ (5+3cosx)}$. 3

2.75

6. a) Evaluate $\int_0^{\pi/2} \frac{sinx}{sinx+cosx} dx$. 2.75

b) Prove that $\int_0^1 \frac{dx}{(1+x^2)\sqrt{(1-x^2)}} = \frac{\pi}{2\sqrt{2}}$. 3

c) Evaluate $\lim_{n\to\infty} \sum_{r=0}^{n} \frac{n}{n^2+r^2}$. 3

7. a) Prove that $\int_0^{\pi/4} \log(1+tan\theta)\, d\theta = \frac{\pi}{8} log2$. 3

b) Prove that $\int_0^{\pi} \frac{xtanx}{secx+tanx} dx = \frac{1}{2}\pi(\pi-2)$. 2.75

c) Obtain the reduction formula for $\int \frac{dx}{(x^2+a^2)^{n/2}}$. 3

Hence, find the value of $\int \frac{dx}{(x^2+a^2)^{7/2}}$.

8. a) Find the area included between the ellipses 3

$x^2+2y^2=a^2$ and $2x^2+y^2=a^2$

b) Show that the length of the area of the evolute $27ay^2 = 4(x-2a)^3$ of the parabola $y^2=4ax$, from the cusp to one of the points where the evolute meets the parabola is $2a(3\sqrt{3}-1)$. 3

c) Find the volume and the surface area of the solid generated by revolving the cardioids $r=a(1-cos\theta)$ about the initial line. 2.75

University of Rajshahi

Department of Computer Science and Engineering

B. Sc. (Engg) Part-I Even Semester Examination 2014

Course: MATH-1211 (Differential and Integral Calculus)

Full Marks: 52.5 Duration: 3(Three) Hours

Answer 06(Six) questions taking any 03(Three) questions from each section in separate answer script

Section - A

1. a) Draw the graph of $y = x - [x]$, where $[x]$ denotes the greatest integer not greater than x. 2.75
   b) Define the differentiability of a function at $x = a$. Let $f(x)$ be defined by 3
   $$f(x) = \begin{cases} x\sin\frac{1}{x}, & when\ x \neq 0 \\ 0, & when\ x = 0 \end{cases}$$
   Examine the differentiability of $f(x)$ at $x = 0$.
   c) Evaluate $\lim_{x\to 0}(sinx)^{2tanx}$. 3

2. a) If $y = \sin(10\sin^{-1}x)$, then use Leibnitz's theorem to show that $(1-x^2)_{12} - 21xy_{11} = 0$. 3
   b) Differentiate $\tan^{-1}\frac{\sqrt{(1+x^2)}-1}{x}$ with respect to $\tan^{-1}x$. 2.75
   c) If $y = x^{2n}$, where n is a positive integer, show that $y_n = 2^n\{1.3.5..........(2n-1)\}x^n$. 3

3. a) State Mean Value theorem. In the Mean Value theorem $f(h) = f(0) + hf'(\theta h), 0 < \theta < 1$, show that the limiting value of θ as $h \to 0$ is $1/2$, according as $f(x) = \cos x$. 3
   b) If $f(x)$ be a maximum at $x = c$ and if $f'(c)$ exists, then show that $f'(c) = 0$. 2.75
   c) Given $x/2 + y/3 = 1$, find the maximum value of xy and minimum value of $x^2 + y^2$. 3

4. a) Define homogeneous function for n variables. Verify Euler's theorem for $u = \tan^{-1}\frac{x^3+y^3}{x-y}$. 3
   b) Define subtangent and subnormal. Show that for the curve $by^2 = (x+a)^3$, the square of the subtangent varies as the subnormal. 3
   c) Prove that the asymptotes of the cubic $(x^2 - y^2)y - 2ay^2 + 5x - 7 = 0$ form a triangle of area a^2. 2.75

Section– B

5. a) Integrate the following: 3
   i. $\int \frac{e^x-1}{e^x+1}dx$.
   ii. $\int \sqrt{\frac{x}{a-x}}dx$.
   b) Evaluate the integral $\int \frac{e^x}{x}(1 + x\log x)\,dx$. 2.75
   c) Integrate $\int \frac{dx}{5+4\sin x}$ 3

6. a) State the Fundamental Theorem of Integral Calculus. Evaluate $\int_0^a \sqrt{a^2 - x^2}\,dx$. 3
   b) Prove that $\int_2^e \left\{\frac{1}{\log x} - \frac{1}{(\log x)^2}\right\}dx = e - \frac{2}{\log 2}$. 3
   c) Evaluate $\lim_{n\to\infty}\left[\frac{n!}{n^n}\right]^{1/n}$. 2.75

7. a) If $I_n = \int_0^{\pi/4} tan^n\theta\, d\theta$, show that $I_n = \frac{1}{n-1} - I_{n-2}$. Hence find the value of $\int_0^{\pi/4} tan^6 x\, dx$. 3
   b) Obtain a reduction formula for $\int \frac{dx}{(a+b\sin x)^n}$. 3
   c) If $u_n = \int_0^{\pi/2} x^n \sin x\, dx\ (n > 0)$. Prove that $u_n + n(n-1)u_{n-2} = n(\pi/2)^{n-1}$. 2.75

8. a) Show that the area bounded by the parabola $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. 3
   b) Find the whole length of the loop of the curve $3ay^2 = x(x-a)^2$. 2.75
   c) Find the volume of the solid generated by revolving the cardioide $\pi = a(1 - \cos\theta)$ about the initial line. 3

University of Rajshahi

Department of Computer Science and Engineering

B.Sc. Engg. Part-1 Even Semester, Examination-2013

Course: MATH-1211 (Differential and Integral Calculus)

Full Marks: 52.5 Time: 3 Hours

Answer six questions taking any three from each Section

Section: A

1 (a) Find the domain and range of the function $\frac{x^2-1}{x-1}$. Also sketch the graph. 3

(b) Define continuity at a point. If the function $f(x) = \begin{cases} -\frac{x^2-16}{x-4} & if\ x \neq 4 \\ a & if\ x = 4 \end{cases}$ is continuous at point 4, what is the value of a? 2.75

(c) Evaluate $\lim_{x\to 0}\left(\frac{tanx}{x}\right)^{\frac{1}{x}}$. 3

2 (a) Show that f(x) =|x| is not differentiable at x=0. 2.75

(b) State Leibnitz's theorem. If $y = (\sin^{-1} x)^2$, then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-n^2y_n=0$. 3

(c) On a dark night, a thin man 6 feet tall walks away from a lamp post 24 feet high at the rate of 5 mph. How fast is the end of his shadow moving? How fast is the shadow lengthening? 3

3 (a) State Rolle 's Theorem. Verify Roll's theorem for $f(x) = x^2-2x-3$ on [-1, 3] and give geometrical interpretation. 4

(b) Suppose that $f(0) = -3$ and $f'(x) \leq 5$ for all values of x. How large can f(2) be possible? 3

(c) Find two positive numbers whose sum is 100 and the sum of whose square is minimum. 1.75

4 (a) Find the shortest distance from the point P (1, 0) to the parabola $x=y^2$. 3

(b) State Euler's Theorem. If $u = x\emptyset({}^{y}/_{x}) + \emptyset({}^{y}/_{x})$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = x\emptyset({}^{y}/_{x})$. 3

(c) Find the condition that the conics $ax^2+by^2=1$ and $a_1x^2+b_1y^2=1$ shall cut orthogonally. 2.75

Section: B

5 (a) Find the antidevivative of the following: 6

(i) $f(x) = \cos(2cot^{-1}(\sqrt{\frac{1-x}{1+x}}))$.

(ii) $f(x) = log(x + \sqrt{x^2 + a^2})$.

(b) A rocket shot straight up from the ground hits the ground 8 seconds later. Find its maximum height using integration. 2.75

6 (a) Using the definition of definite integral Evalute. 4

(i) $\int_a^b e^x\, dx$.

(ii) $\lim_{n\to\alpha}[\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \cdots + \frac{1}{2n}]$.

(b) Evaluate $\int_a^b \frac{logx}{x} dx$. 3

(c) Prove that $\int_0^{\pi/2} \frac{\sqrt{sinx}}{\sqrt{sinx}+\sqrt{cosx}} dx = \frac{\pi}{4}$. 1.75

7 a) Evaluate $\int cos^m x\, dx$ and hence $\int_0^{\pi/2} cos^6 x\, dx$. 3

b) Obtain the reduction formula for $\int x^m (logx)^n dx$. 2.75

c) If $u_n = \int_0^{\pi/2} \theta sin^n\theta\, d\theta$ and $n > 1$, then prove that $u_n = \frac{n-1}{n} u_{n-2} + \frac{1}{n^2}$. 3

8 a) Find the area of the region bounded by the x-axis and one of the arc of y=sinx. 2.75

b) If s be the length of an arc of $3ay^2=x(x-a)^2$ measured from the origin to the point (x, y), then show that $3s^2=4x^2+3y^2$. 3

c) Find the volume of a paraboloid of revolution formed by revolving the parabola $y^2=4ax$ about x-axis and bounded by the section $x=x_1$. 3

University of Rajshahi

Dept. Of Computer Science and Engineering

B. Sc. Engg.(CSE) 1st Year Even Semester Examination 2012

Course: MATH 1211 (Differential and Integral calculus)

Full Mark: 52.5 Duration: 4 hours

Answer 6 questions taking at least 3 from each part

Part -A

1. a) Find the domain and range of 3

$$f(x) = \frac{x^2+1}{x^2-5x+6}$$

b) A function defined as 3

$$f(x) = x \qquad 0 \le x < \tfrac{1}{2}$$
$$= 1-x \qquad \tfrac{1}{2} \le x < 1$$

Discuss the continuity and differentiability of $f(x)$ at $x = \frac{1}{2}$

c) Evaluate $\underset{x\to 0}{Lt}(\cos x)^{Cot^2 x}$ 2.75

2. a) If $\sin y = x\sin(a+y)$, prove that 3

$$\frac{dy}{dx} = \frac{\sin^2(a+y)}{\sin a}$$

b) If $y = \frac{x}{x^2+a^2}$, find y_n 3

c) If $\sin^{-1} y = m \sin^{-1} x$, prove that 2.75

$$(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (m^2-n^2)y_n = 0$$

3. a) State and prove mean value theorem. 3

b) Expand $Sin\, x$ in a finite series in power of x, with the reminder in Lagrange's form. 3

c) Examine whether $x^{\frac{1}{x}}$ possesses a maximum or a minimum, and determine the same. 2.75

4. a) If $u = \cos^{-1}\{(x+y)/(\sqrt{x}+\sqrt{y})\}$, then show that 2.75

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0.$$

b) If the tangent at (x_1, y_1) to the curve $x^3 + y^3 = a^3$ meets the curve again in (x_2, y_2), show that 3

$$\frac{x_2}{x_1} + \frac{y_2}{y_1} + 1 = 0.$$

c) Define asymptotes. Find the asymptotes of the curve $x^3 + y^3 = 3axy$. 3

Part -B

5. Integrate any three of the following: 8.75

i) $\int\sqrt{\frac{x}{a-x}}dx$ ii) $\int\frac{dx}{(2x+1)\sqrt{4x+3}}$

iii) $\int\frac{xe^x}{(x+1)^2}dx$ iv) $\int\frac{\cos x\, dx}{5-3\cos x}$

6. a) Evaluate $\int_0^1 x^3 dx$ and $\underset{n\to\infty}{Lt}\left\{\frac{1}{n+1}+\frac{1}{n+2}+.........+\frac{1}{2n}\right\}$ using the definition of definite integral. 3

b) Show that $\int_0^{\frac{\pi}{2}}\frac{x\sin x\cos x}{(a^2\cos^2 x+b^2\sin^2 x)^2}dx=\frac{\pi}{4ab^2(a+b)}$, $[a,b>0]$. 3

c) Evaluate the definite integral $\int_0^1\frac{\log(1+x)}{1+x^2}dx$. 2.75

7. a) Find the reduction formula for $\int\sin^n x dx$, and hence show that 3

$\int_0^{\frac{\pi}{2}}\sin^n x\,dx=\frac{(n-1)}{n}\frac{(n-3)}{(n-2)}.........\frac{3}{4}.\frac{1}{2}.\frac{\pi}{2}$ where n is a positive integer.

b) If $I_n=\int e^{ax}\cos^n x dx$, prove that 3

$I_n=\int\frac{e^{ax}\cos^{n-1}x(a\cos x+n\sin x)}{n^2+a^2}+\frac{n(n-1)}{n^2+a^2}I_{n-2}$

c) If $I_n=\int_0^{\frac{\pi}{4}}\tan^n\theta\, d\theta$, Show that $I_n=\frac{1}{n-1}-I_{n-2}$. Hence find the value of $\int_0^{\frac{\pi}{4}}\tan^6 x\,dx$. 2.75

8. a) Find the area of the quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ between the major and minor axes. 3

b) If s be the length of an arc of $3ay^2=x(x-a)^2$ measured from the origin to the point (x,y), show that $3s^2=4x^2+3y^2$. 3

c) Evaluate the integral $\int_0^1\int_0^{1-y^2}[(x-1)^2+y^2]\,dxdy$. 2.75

University of Rajshahi
Department of Computer Science and Engineering
B.Sc. Engg.(CSE) 1st Year 2nd Semester 2011
Course: MATH 1211 (Differential and Integral Calculus)
Time: 4 Hrs. Full Marks: 52.5
[N.B. Answer SIX questions taking at least THREE from each part.]

Part A

1.a) Define function, domain and range of a function. Determine the domain and range of the function $f(x) = \frac{|x|}{x}$. 3

b) A function $f(x)$ is defined as follows: 2.75

$$f(x) = 3 + 2x \quad for\ -\tfrac{3}{2} \le x < 0$$
$$= 3 - 2x \quad for\ 0 \le x < \tfrac{3}{2}$$
$$= -3 - 2x \quad for\ x \ge \tfrac{3}{2}$$

Show that $f(x)$ is continuous at $x = 0$ and discontinuous at $= \frac{3}{2}$.

c) State L' Hospital theorem. Evaluate $\lim_{x\to 0}(sinx)^x$ 3

2.a) Differentiate $x^{\sin^{-1}x}$ with respect to $\sin^{-1}x$. 3

b) State Leibnitz's theorem. If $y = e^{a\sin^{-1}x}$, then show that $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - (n^2+a^2)y_n = 0$. 3

c) Suppose that $f(0) = -3$ and $f'(x) \le 5$ for all values of x. How large can $f(2)$ possible be? 2.75

3.a) Define the differentiability of a function $f(x)$ at $x = a$. Give the geometrical interpretation of $\frac{dy}{dx}$. 3

b) Explain $log(1-x)$ in a finite series in powers of x with remainder of Lagrange's form. 2.75

c) Define maxima and minima of a function at a point $x=c$. Find the maximum and minimum values of u where $u = \frac{4}{x} + \frac{36}{y}$ and $x+y=2$. 3

4.a) Define homogeneous function. If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$. 3

b) Find the asymptotes of the curve $xy=1$. 2.75

c) Find the condition that the conics $ax^2 + by^2 = 1\ and\ a_1x^2 + b_1y^2 = 1$ shall cut orthogonally. 3

Part B

5. Evaluate **any three** of the following: 8.75

(i) $\int \frac{e^{m\tan^{-1}x}}{(1+x^2)^2}dx$ (ii) $\int \frac{dx}{13+3cosx+4sinx}$ (iii) $\int \frac{11cosx-16sinx}{2cosx+5sinx}dx$ (iv) $\int \frac{dx}{cosx(5+3cosx)}$

6. Evaluate the following: 8.75

(a) $\int_0^{\pi/2} log\ sinx\ dx$ (b) $\lim_{n\to\infty}\left[\frac{1}{\sqrt{n^2-1^2}} + \frac{1}{\sqrt{n^2-2^2}} + \cdots\ \ldots\ldots + \frac{1}{\sqrt{n^2-(n-1)^2}}\right]$ (c) $\int_0^1 x^3(1-x)^3 dx$

7.a) Show that $\int_0^1 \frac{logx}{\sqrt{(1-x^2)}}dx = \frac{\pi}{2}log\frac{1}{2}$. 2.75

b) If $I_n = \int_0^{\frac{1}{4}\pi} tan^n\theta d\theta$, show that $I_n = \frac{1}{n-1} - I_{n-2}$. Hence find the value of $\int_0^{\frac{1}{4}\pi} tan^6x dx$. 3

c) Define Gamma and Beta function. Find the relation between Gamma and Beta function. 3

8.a) Find the area of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$. 3

b) Show that the volume of a right circular cone of height h and base of radius a is $\frac{1}{3}\pi a^2 h$. 3

c) Find the perimeter of the circle $x^2 + y^2 = a^2$. 2.75