Electrical Circuit and Electronics

Filters

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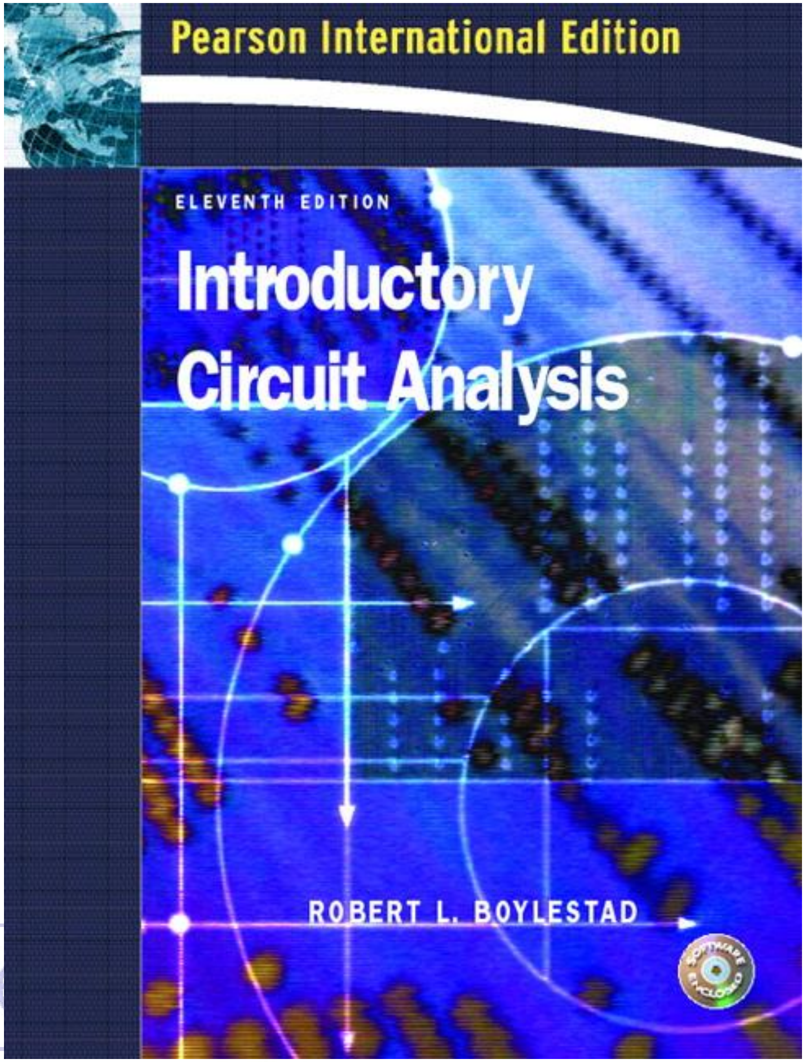
Reference Books Recommended

- **Introductory Circuit Analysis**

  - Robert L. Boylestad



Filter

Any combination of passive (*R, L,* and *C*) and/or active (transistors or operational amplifiers) elements designed to select or reject a band of frequencies is called a filter.

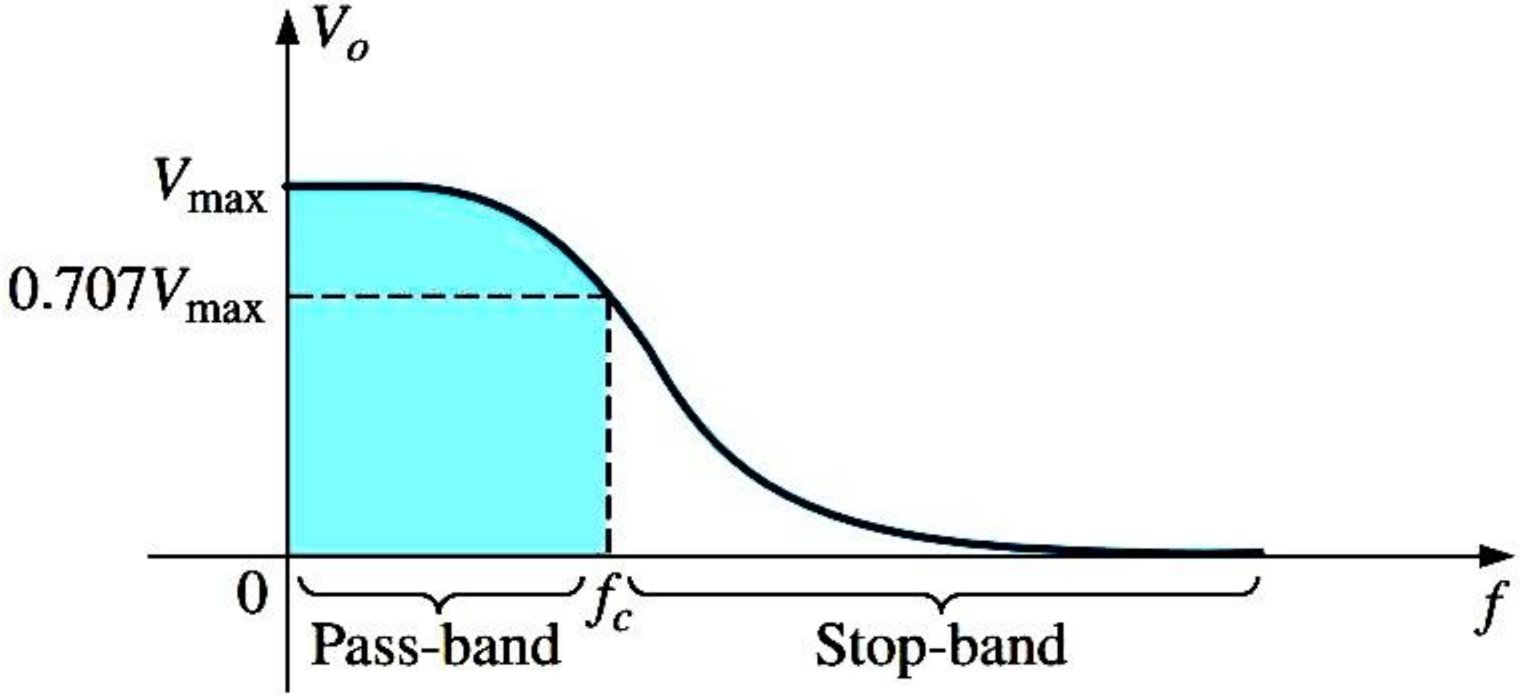
- **Passive filters** are those filters composed of series or parallel combinations of *R, L,* and *C* elements
- **Active filters** are filters that employ active devices such as transistors and operational amplifiers in combination with *R, L,* and *C* elements

Filter

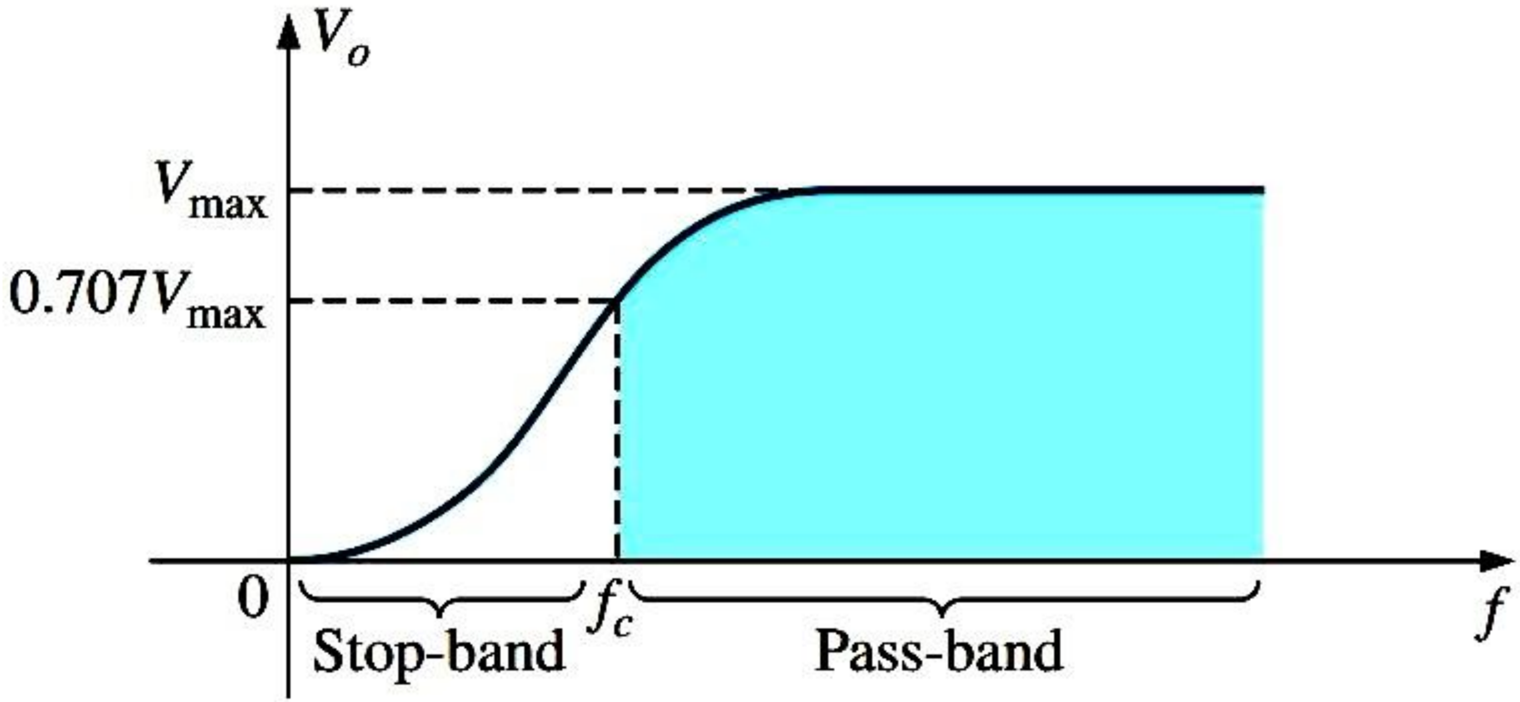
All filters belong to the four broad categories of:

- **low-pass**
- **high-pass**
- **pass-band**
- **stop-band**

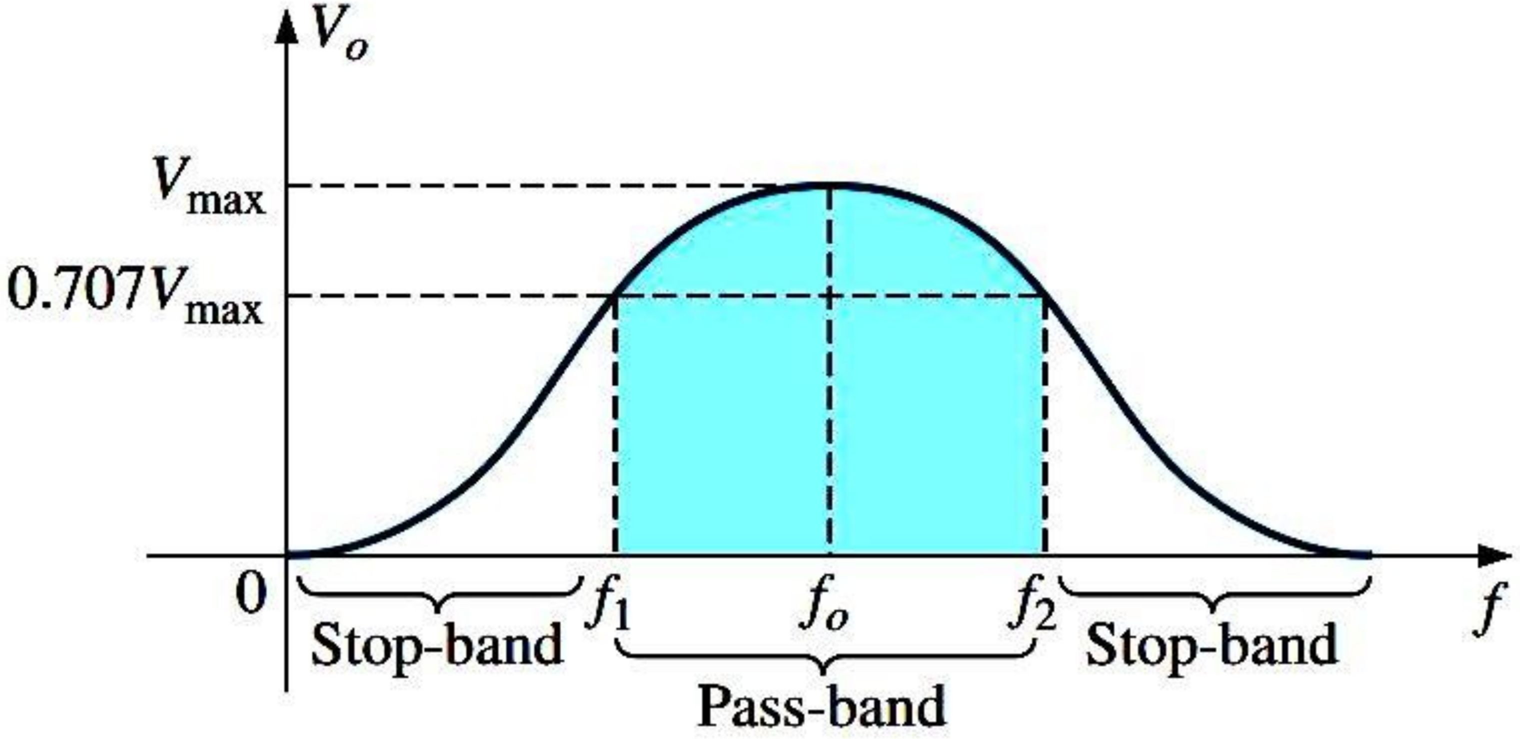
Low Pass Filter



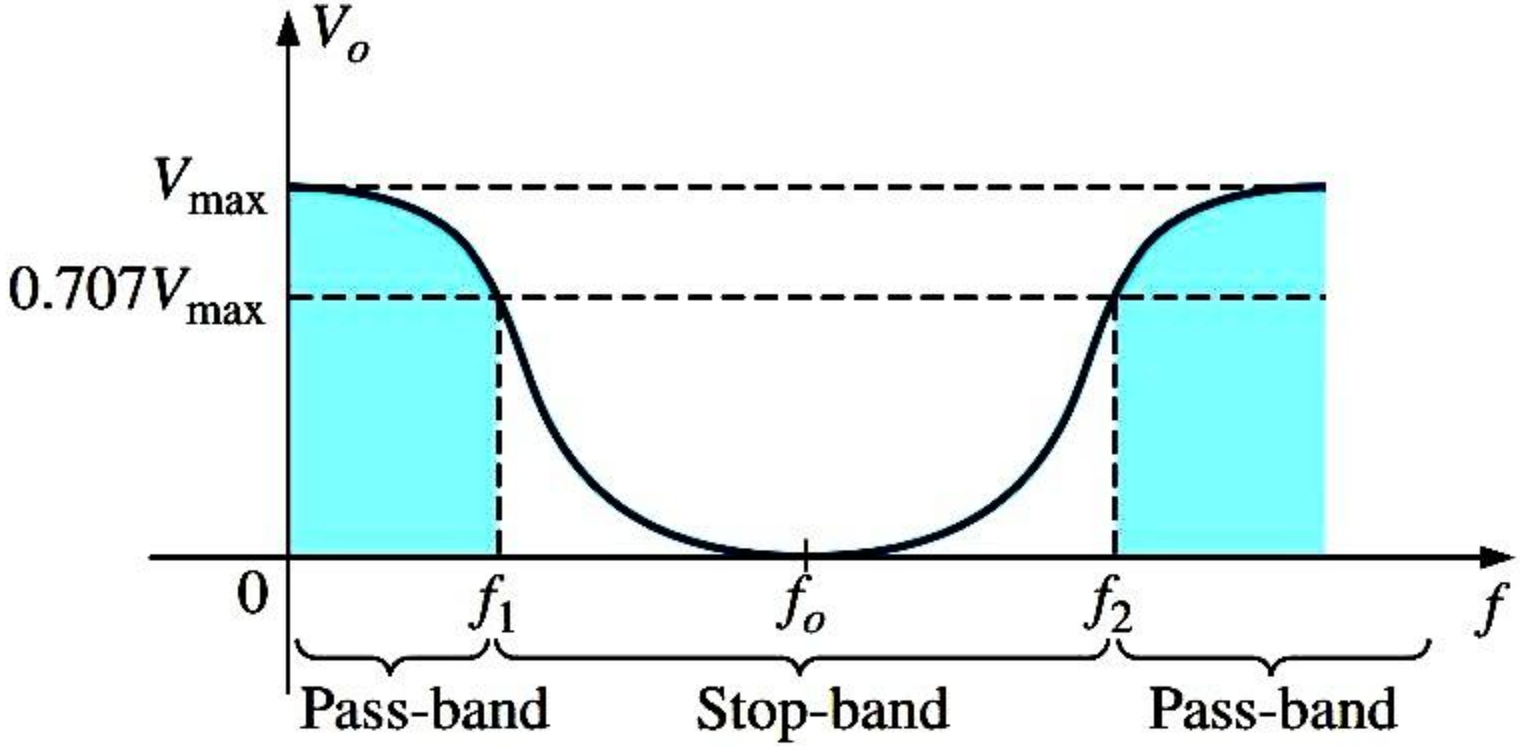
High Pass Filter



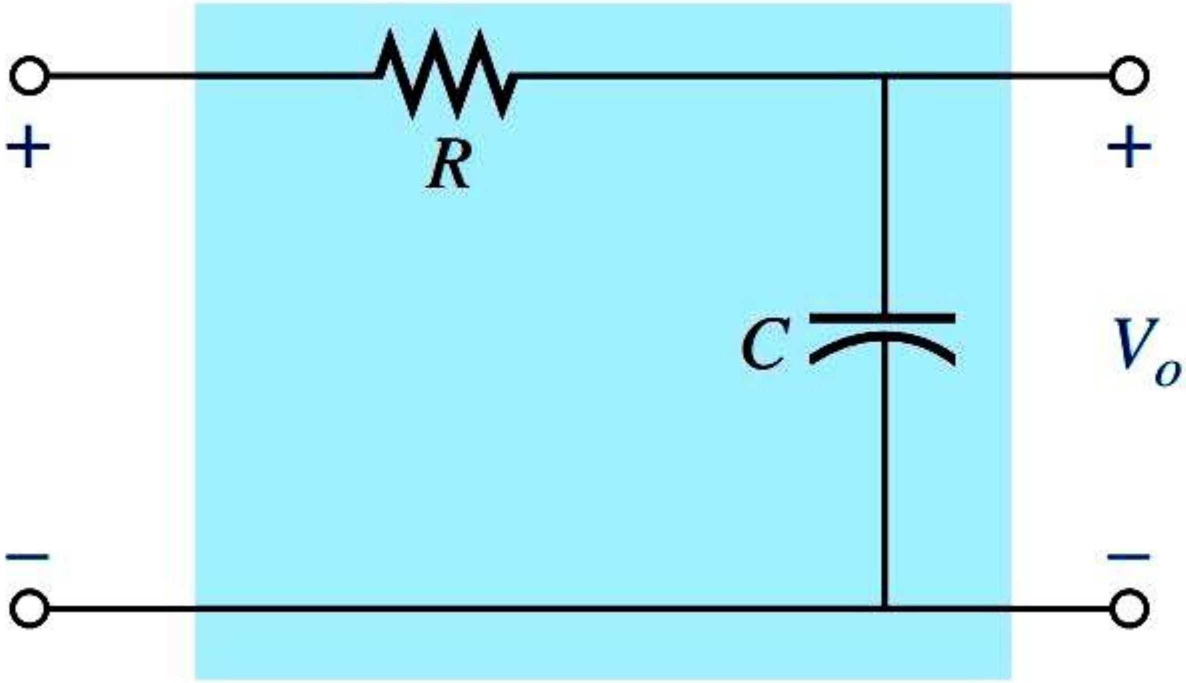
Band Pass Filter



Stop Band Filter



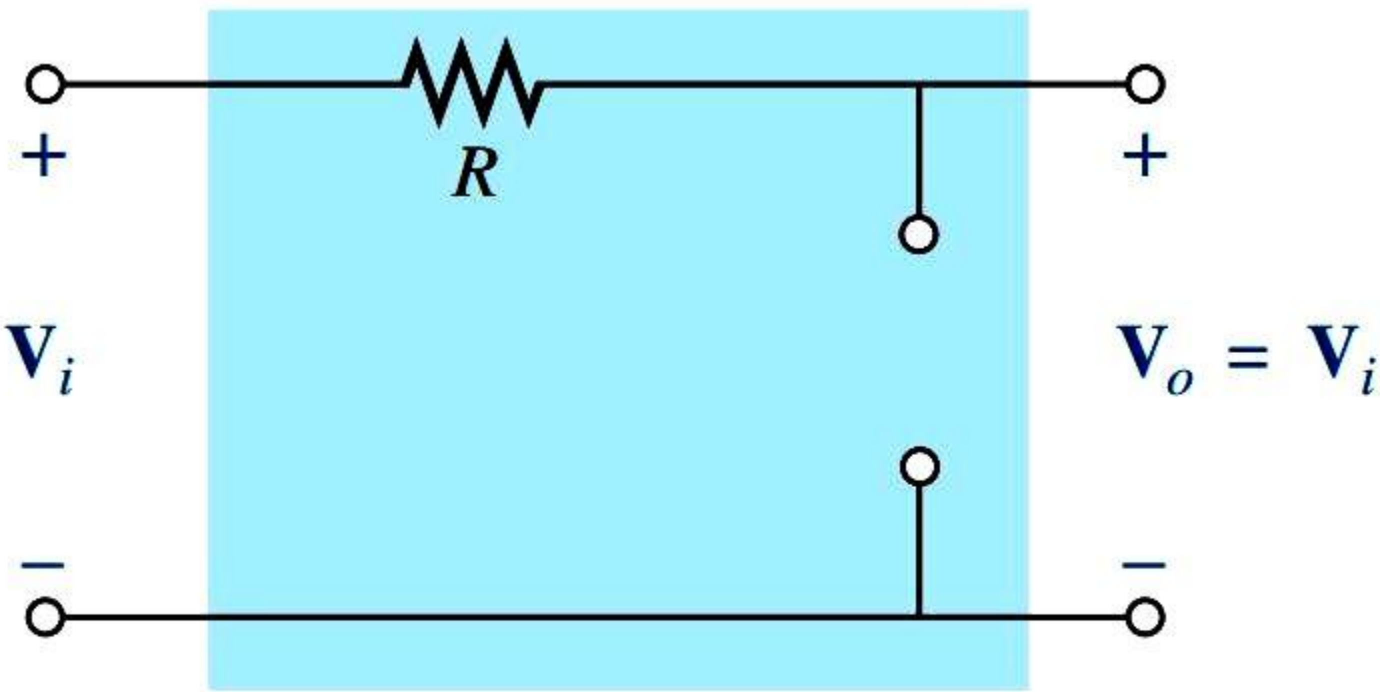
R-C Low Pass Filter



R-C Low Pass Filter

At $f = 0$ Hz,

$$X_C = \frac{1}{2\pi f C} = \infty\ \Omega$$



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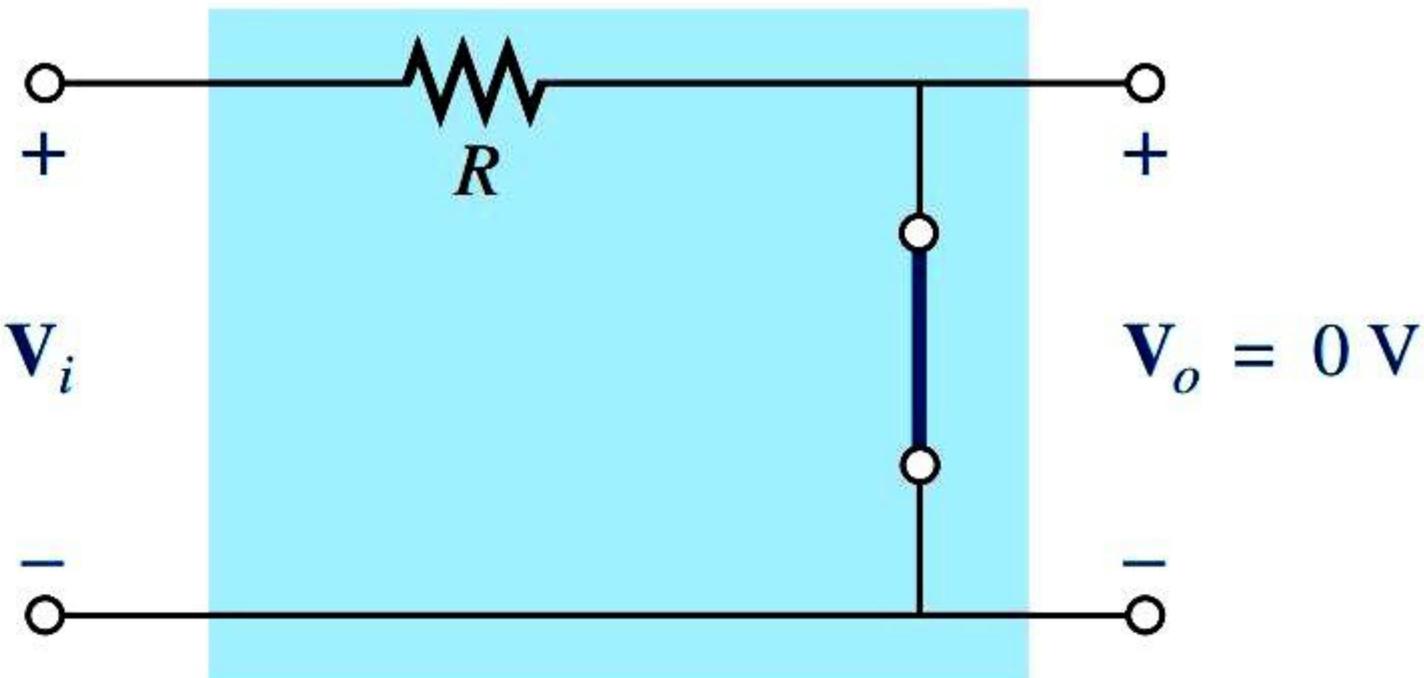
R-C Low Pass Filter

At very high frequencies, the reactance is

$$X_C = \frac{1}{2\pi f C} \cong 0\ \Omega$$



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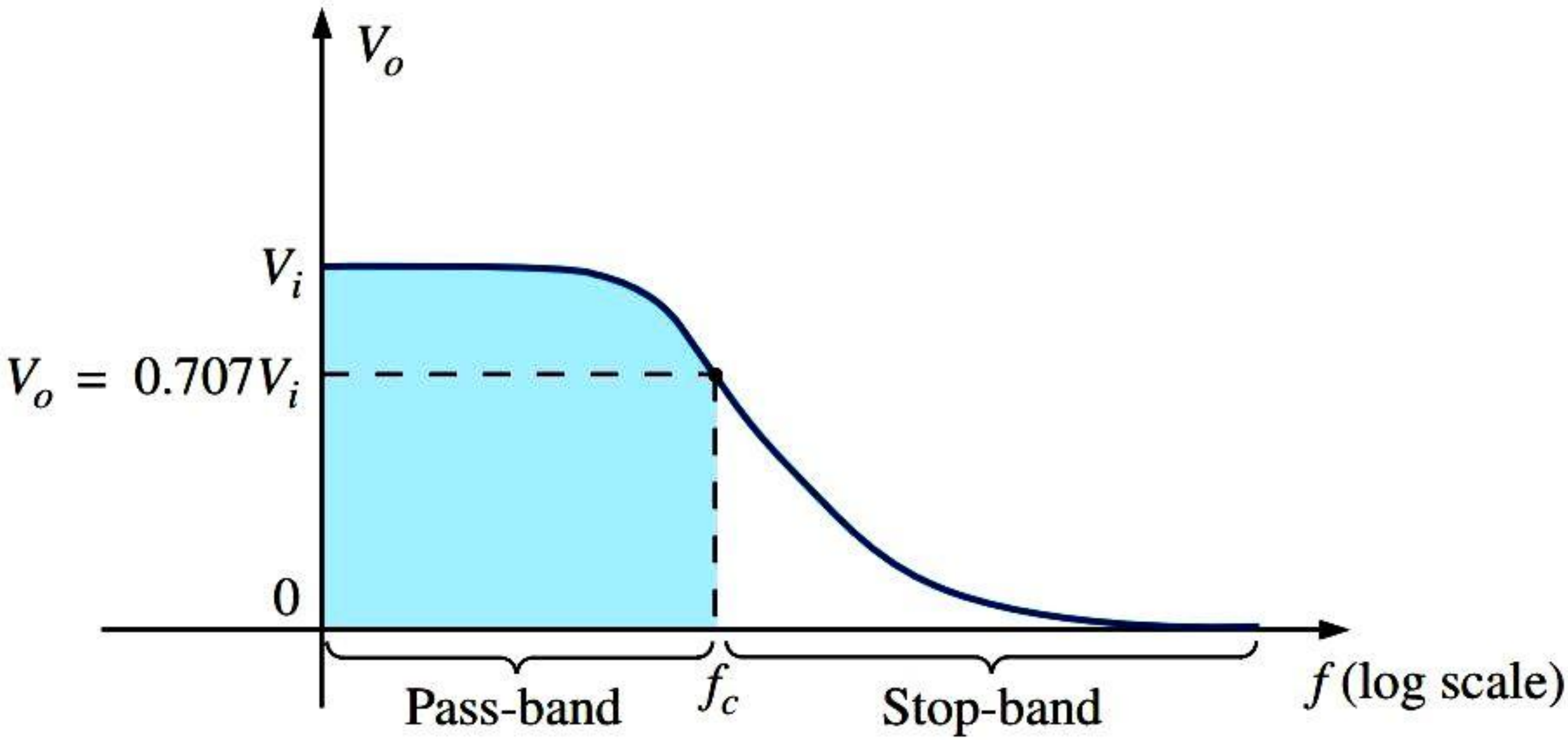
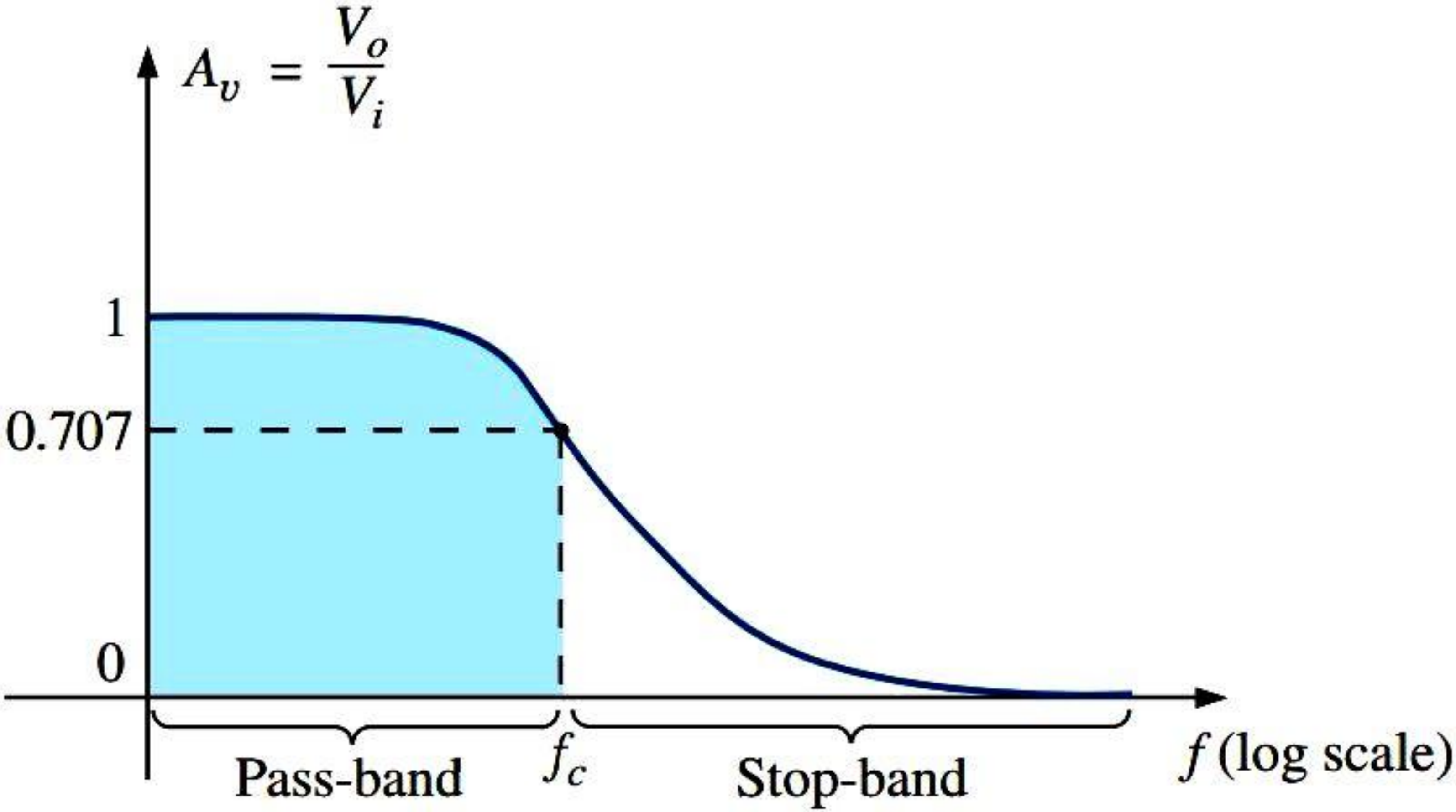
R-C Low Pass Filter



R-C Low Pass Filter



R-C Low Pass Filter

$$\mathbf{V}_o = \frac{X_C \angle -90° \mathbf{V}_i}{R - j X_C}$$

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{X_C \angle -90°}{R - j X_C} = \frac{X_C \angle -90°}{\sqrt{R^2 + X_C^2} \angle -\tan^{-1}(X_C/R)}$$

$$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{X_C}{\sqrt{R^2 + X_C^2}} \angle -90° + \tan^{-1}\left(\frac{X_C}{R}\right)$$

R-C Low Pass Filter

$$A_v = \frac{V_o}{V_i} = \frac{X_C}{\sqrt{R^2 + X_C^2}} = \frac{1}{\sqrt{\left(\frac{R}{X_C}\right)^2 + 1}}$$

$$\theta = -90° + \tan^{-1}\frac{X_C}{R} = -\tan^{-1}\frac{R}{X_C}$$

R-C Low Pass Filter

For the special frequency at which $X_C = R$
The frequency at which $X_C = R$ is the critical or cutoff frequency

$$A_v = \frac{V_o}{V_i} = \frac{1}{\sqrt{\left(\frac{R}{X_C}\right)^2 + 1}} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$$

$$\frac{1}{2\pi f_c C} = R$$

$$\boxed{f_c = \frac{1}{2\pi RC}}$$

R-C Low Pass Filter

$$\mathbf{V}_o = \left[\frac{X_C}{\sqrt{R^2 + X_C^2}} \angle\theta\right]\mathbf{V}_i = \left[\frac{X_C}{\sqrt{R^2 + X_C^2}} \angle\theta\right] V_i \angle 0^\circ$$

$$\mathbf{V}_o = \frac{X_C V_i}{\sqrt{R^2 + X_C^2}} \angle\theta$$

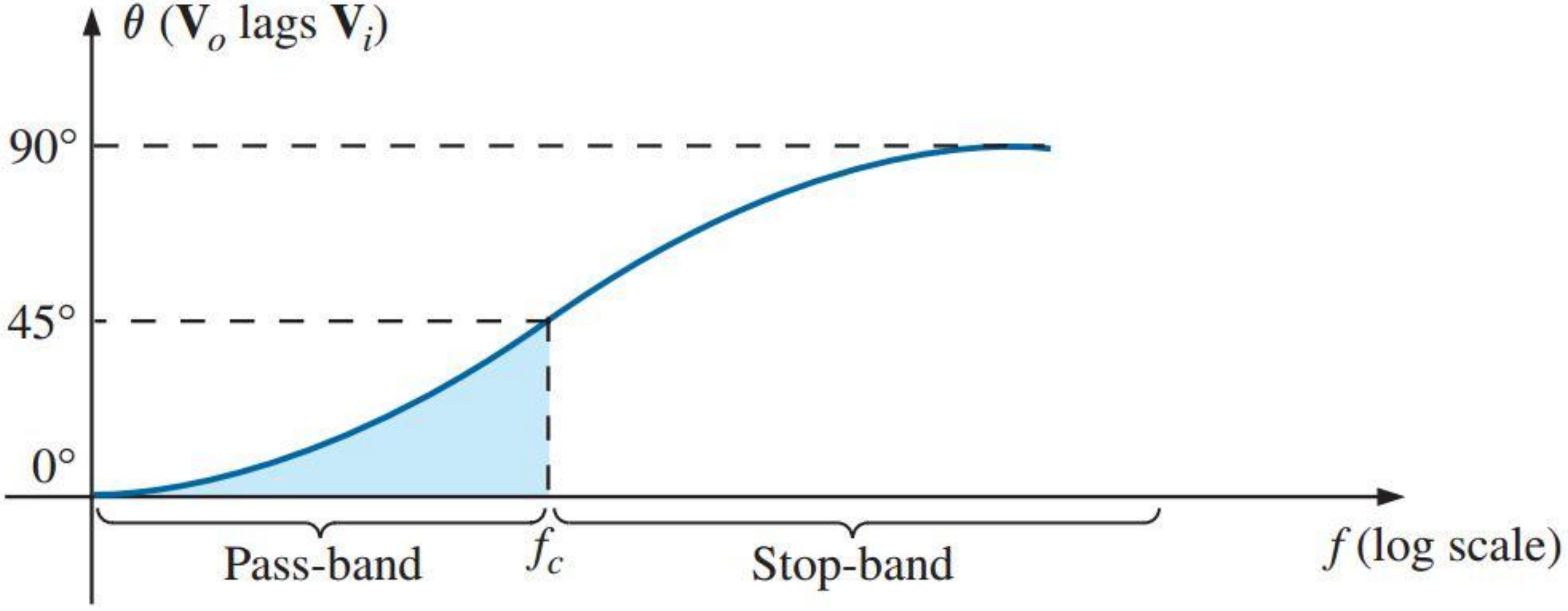
$$f_c = \frac{1}{2\pi RC}$$

For $f < f_c$, $V_o > 0.707V_i$

whereas for $f > f_c$, $V_o < 0.707V_i$

At f_c, $\mathbf{V}_o$ lags $\mathbf{V}_i$ by 45°

R-C Low Pass Filter



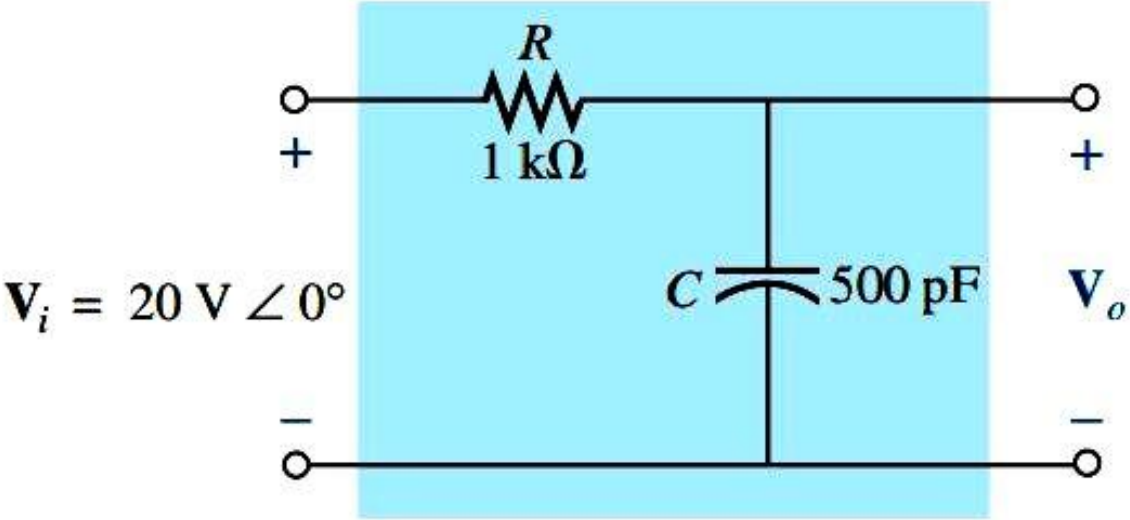
R-C Low Pass Filter

EXAMPLE

a. Sketch the output voltage V_o versus frequency for the low-pass R-C filter in Fig.

b. Determine the voltage V_o at $f = 100$ kHz and 1 MHz, and compare the results to the results obtained from the curve in part (a).

c. Sketch the normalized gain $A_v = V_o/V_i$.



R-C Low Pass Filter

a.

$$f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi(1\ \text{k}\Omega)(500\ \text{pF})} = \mathbf{318.31\ kHz}$$

At f_c, $V_o = 0.707(20\ \text{V}) = 14.14\ \text{V}$. See Fig.

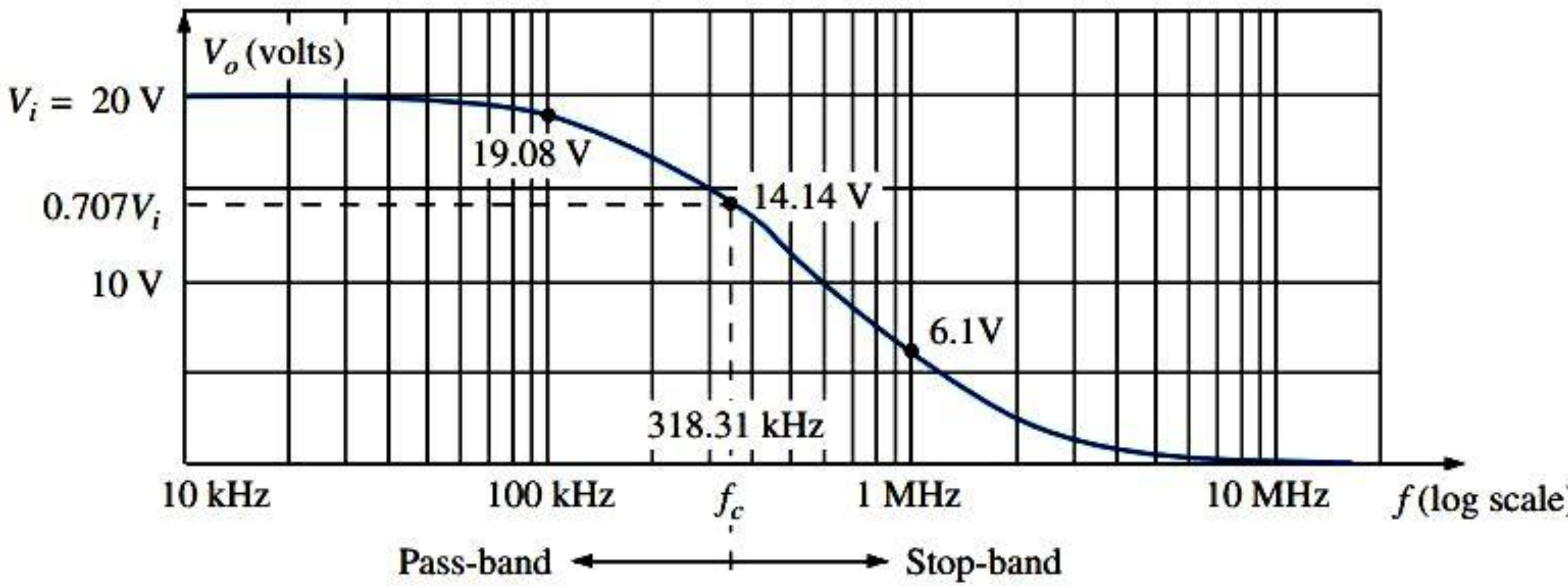
b.

$$V_o = \frac{V_i}{\sqrt{\left(\frac{R}{X_C}\right)^2 + 1}}$$

At $f = 100\ \text{kHz}$:

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(100\ \text{kHz})(500\ \text{pF})} = 3.18\ \text{k}\Omega$$

R-C Low Pass Filter



R-C Low Pass Filter

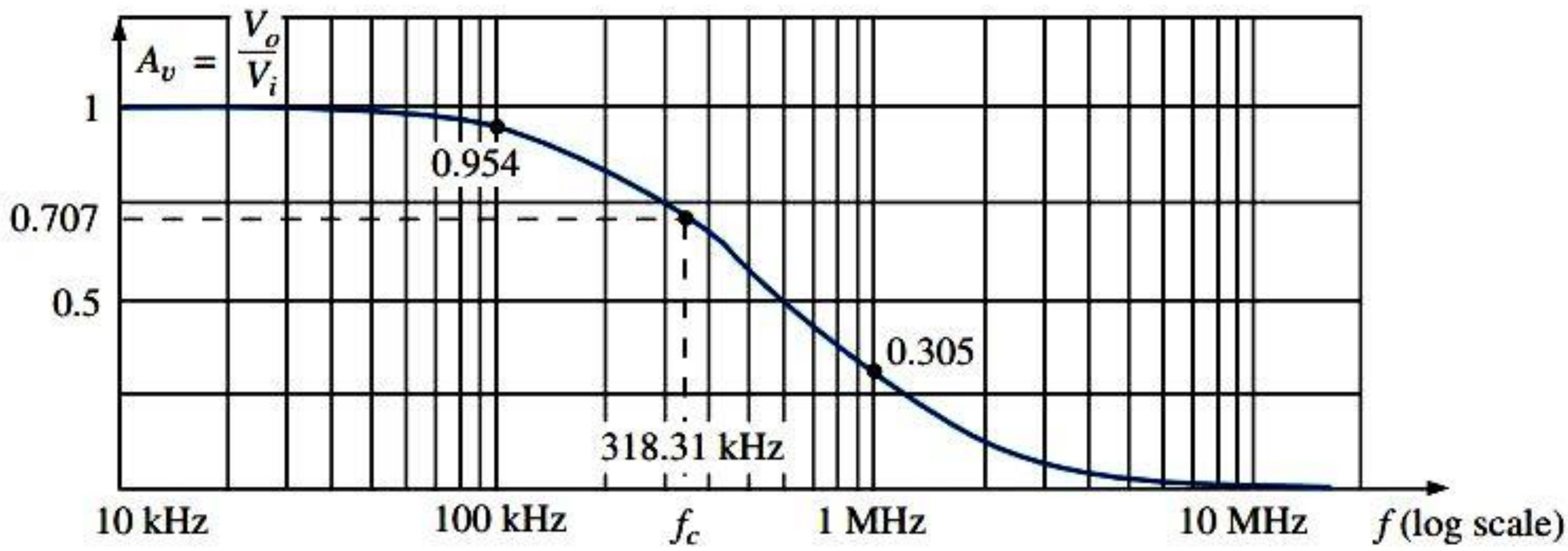
and $$V_o = \frac{20\text{ V}}{\sqrt{\left(\frac{1\text{ k}\Omega}{3.18\text{ k}\Omega}\right)^2 + 1}} = \mathbf{19.08\text{ V}}$$

At $f = 1$ MHz:

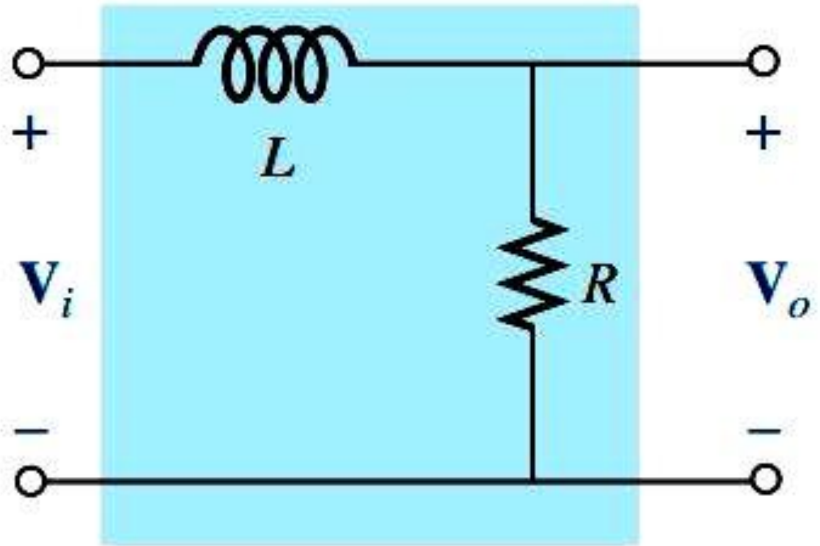
$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(1\text{ MHz})(500\text{ pF})} = 0.32\text{ k}\Omega$$

and $$V_o = \frac{20\text{ V}}{\sqrt{\left(\frac{1\text{ k}\Omega}{0.32\text{ k}\Omega}\right)^2 + 1}} = \mathbf{6.1\text{ V}}$$

R-C Low Pass Filter

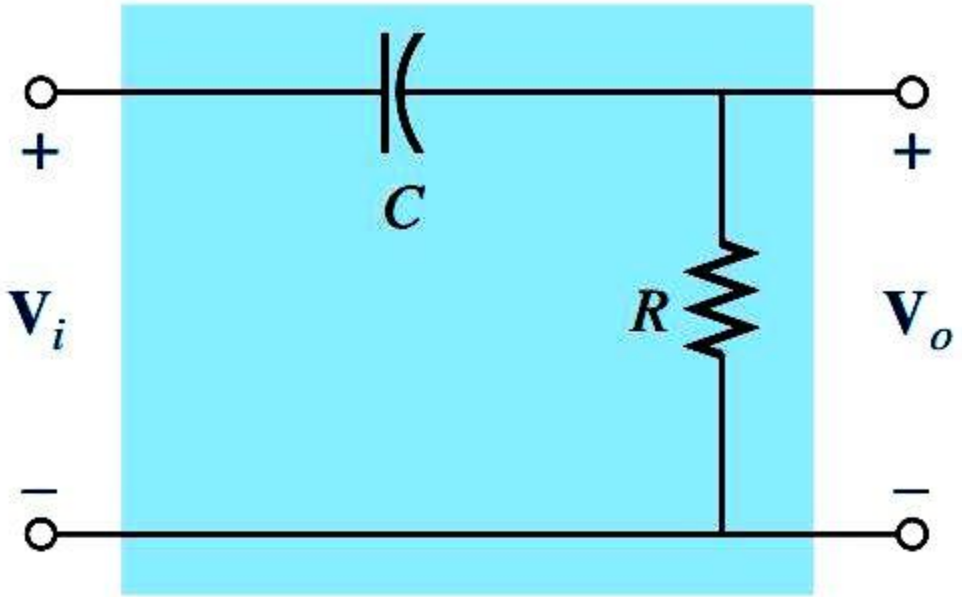


R-L Low Pass Filter

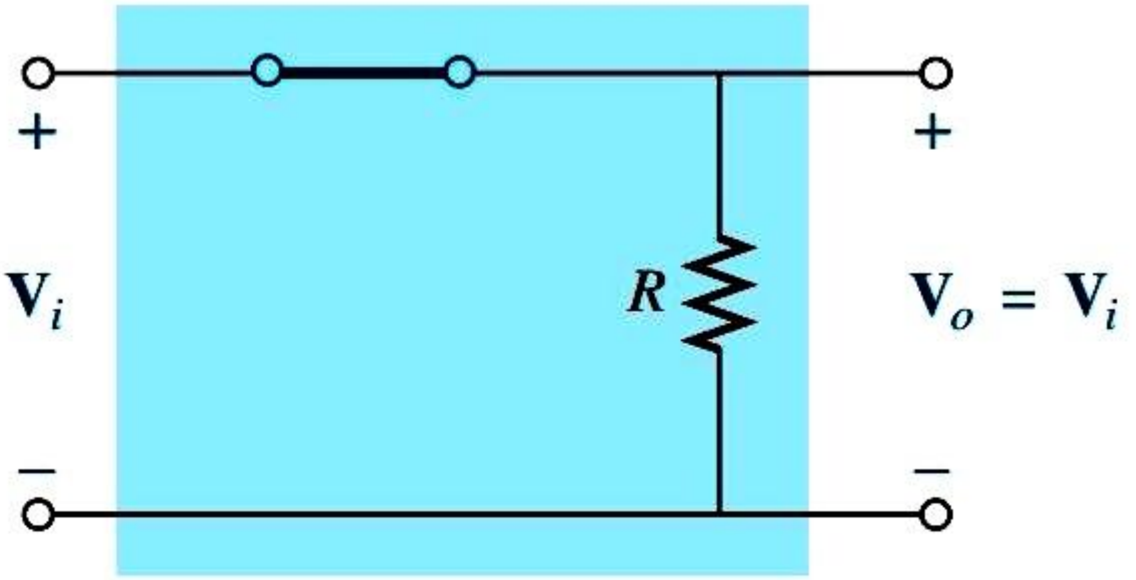


$$f_c = \frac{R}{2\pi L}$$

R-C High Pass Filter

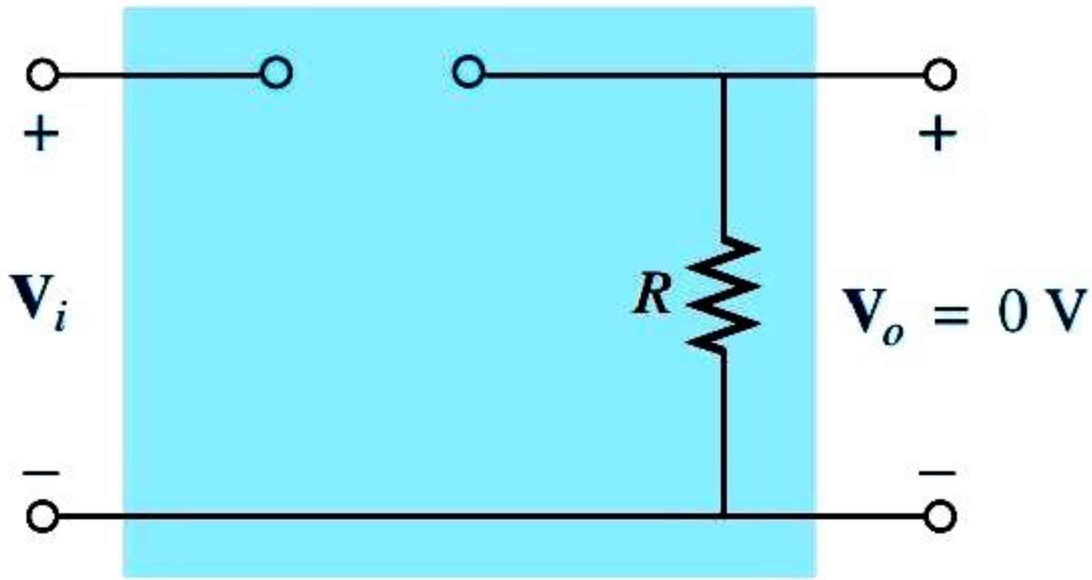


R-C High Pass Filter



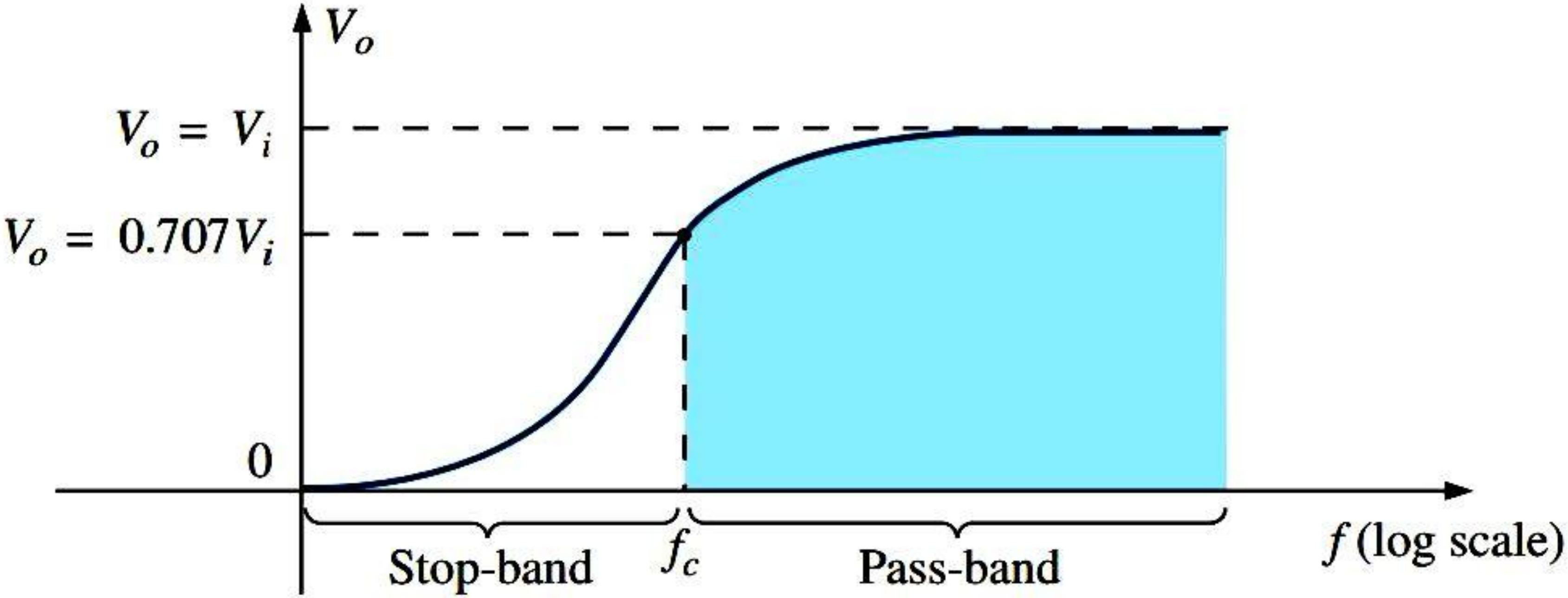
R-C high-pass filter at very high frequencies

R-C High Pass Filter

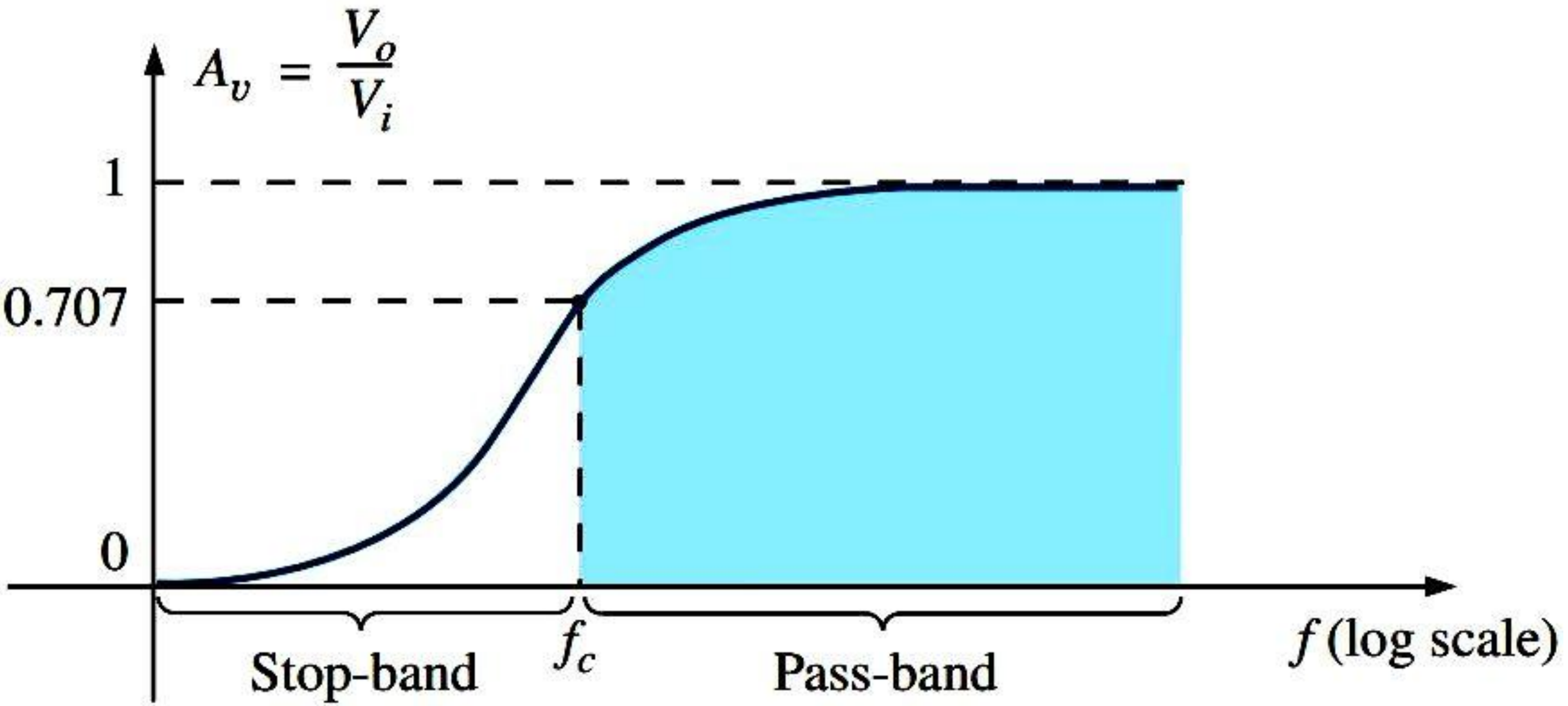


R-C high-pass filter at $f = 0$ Hz

R-C High Pass Filter



R-C High Pass Filter



R-C High Pass Filter

$$\mathbf{V}_o = \frac{R\angle 0^\circ\, \mathbf{V}_i}{R - jX_C}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R\angle 0^\circ}{R - jX_C} = \frac{R\angle 0^\circ}{\sqrt{R^2 + X_C^2}\,\angle -\tan^{-1}(X_C/R)}$$

$$\frac{\mathbf{V}_o}{\mathbf{V}_i} = \frac{R}{\sqrt{R^2 + X_C^2}}\angle \tan^{-1}(X_C/R)$$

R-C High Pass Filter

$$A_v = \frac{V_o}{V_i} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{1}{\sqrt{1 + \left(\frac{X_C}{R}\right)^2}}$$

$$\theta = \tan^{-1} \frac{X_C}{R}$$

R-C High Pass Filter

For the frequency at which $X_C = R$, the magnitude becomes

$$\frac{V_o}{V_i} = \frac{1}{\sqrt{1+\left(\frac{X_C}{R}\right)^2}} = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$$

The frequency at which $X_C = R$ is determined by

$$X_C = \frac{1}{2\pi f_c C} = R$$

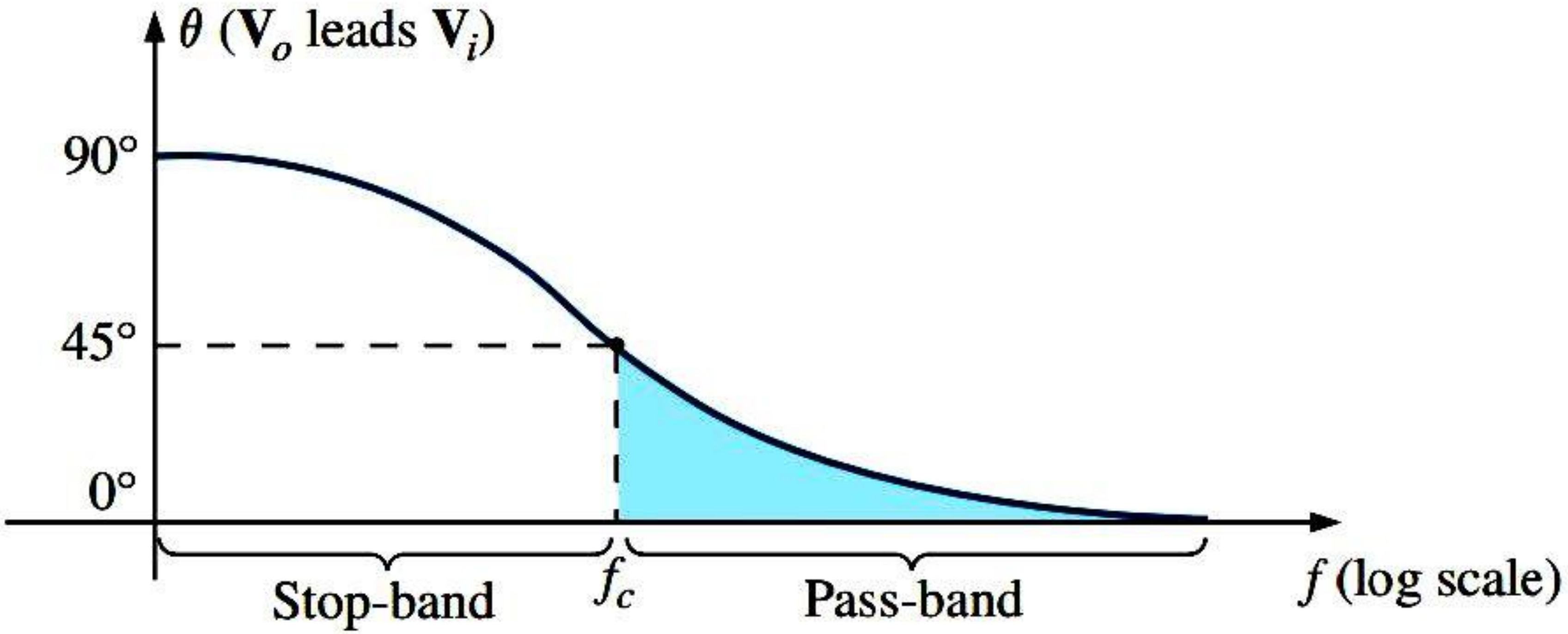
$$\boxed{f_c = \frac{1}{2\pi RC}}$$

R-C High Pass Filter

$$f_c = \frac{1}{2\pi RC}$$

For	$f < f_c$,	$V_o < 0.707V_i$
whereas for	$f > f_c$,	$V_o > 0.707V_i$
At f_c,	$\mathbf{V}_o$ leads $\mathbf{V}_i$ by 45°	

R-C High Pass Filter



R-C High Pass Filter

EXAMPLE Given $R = 20\ \text{k}\Omega$ and $C = 1200\ \text{pF}$:

a. Sketch the normalized plot if the filter is used as both a high-pass and a low-pass filter.
b. Sketch the phase plot for both filters in part (a).
c. Determine the magnitude and phase of $\mathbf{A}_v = \mathbf{V}_o/\mathbf{V}_i$ at $f = \frac{1}{2}f_c$ for the high-pass filter.

R-C High Pass Filter

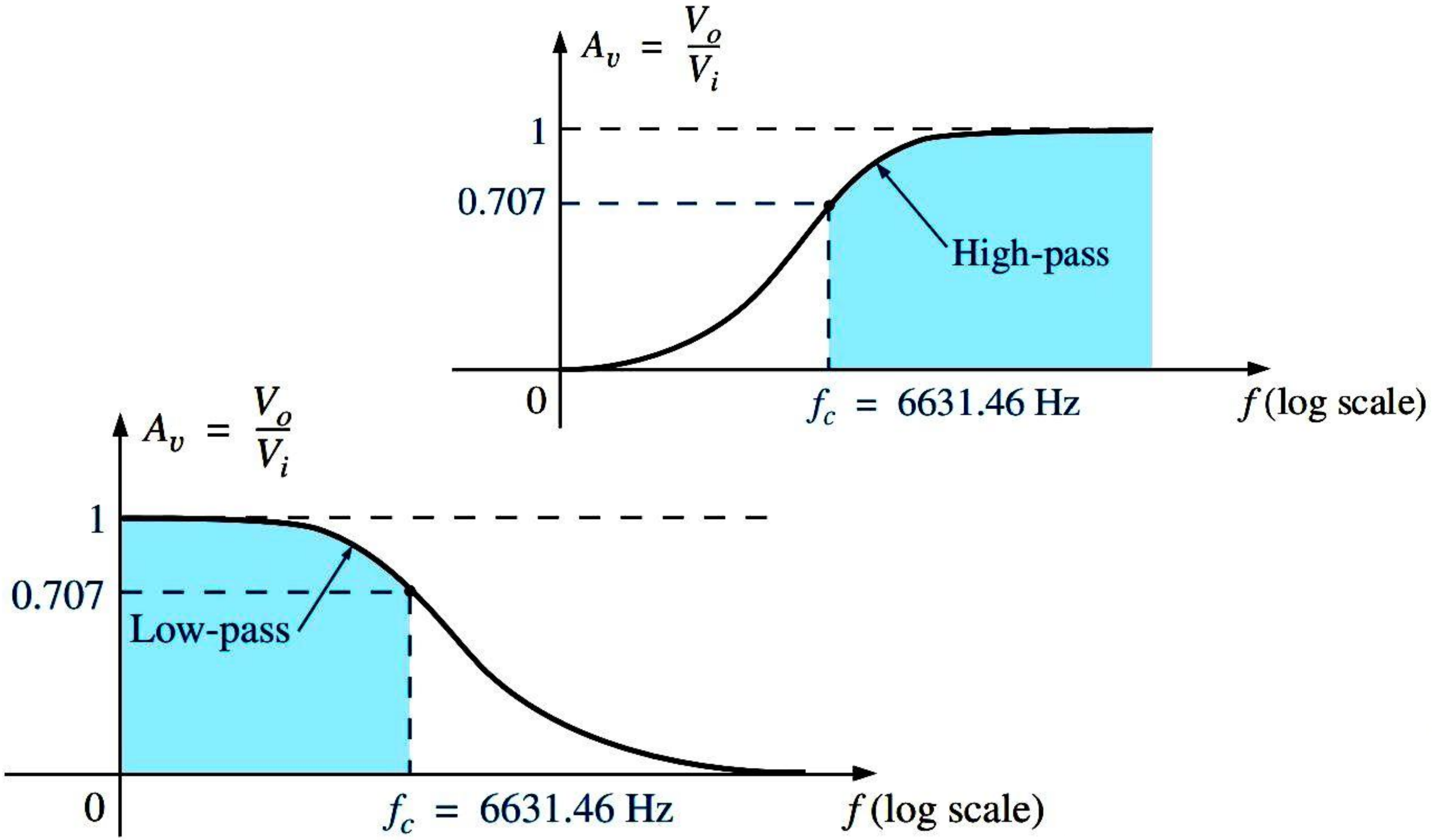
a. $f_c = \dfrac{1}{2\pi RC} = \dfrac{1}{(2\pi)(20\text{ k}\Omega)(1200\text{ pF})}$

$= \mathbf{6631.46\ Hz}$

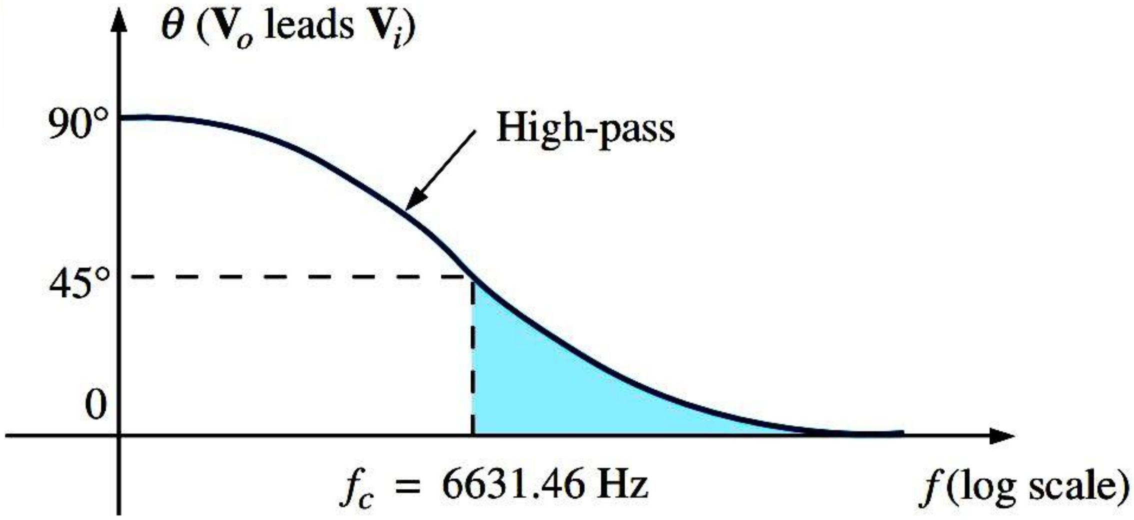
The normalized plots appear in Fig. 1

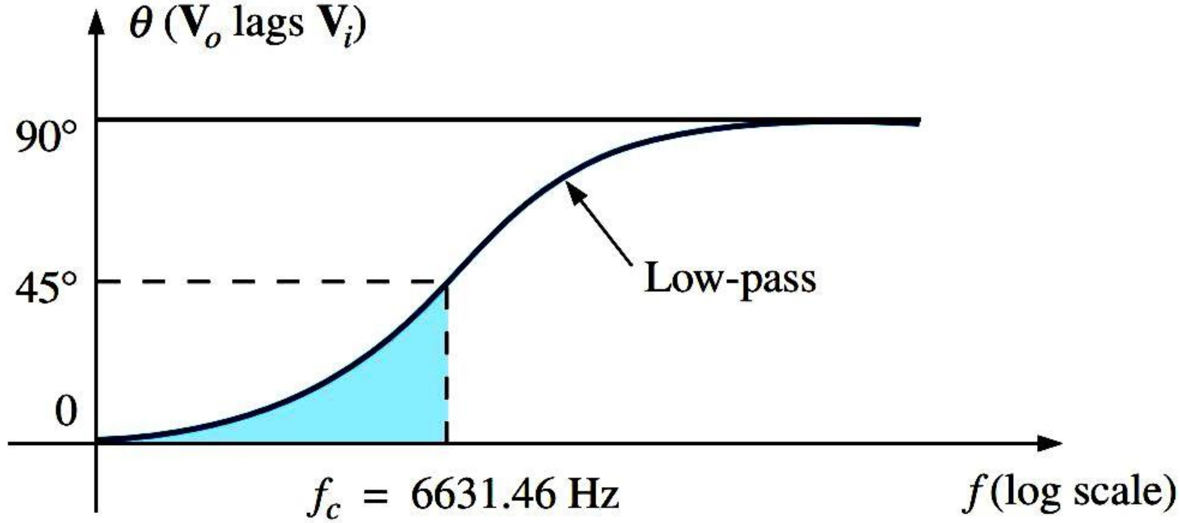
b. The phase plots appear in Fig. 2

R-C High Pass Filter



R-C High Pass Filter





R-C High Pass Filter

c. $f = \frac{1}{2} f_c = \frac{1}{2}(6631.46\text{ Hz}) = 3315.73\text{ Hz}$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{(2\pi)(3315.73\text{ Hz})(1200\text{ pF})}$$

$$\cong 40\text{ k}\Omega$$

R-C High Pass Filter

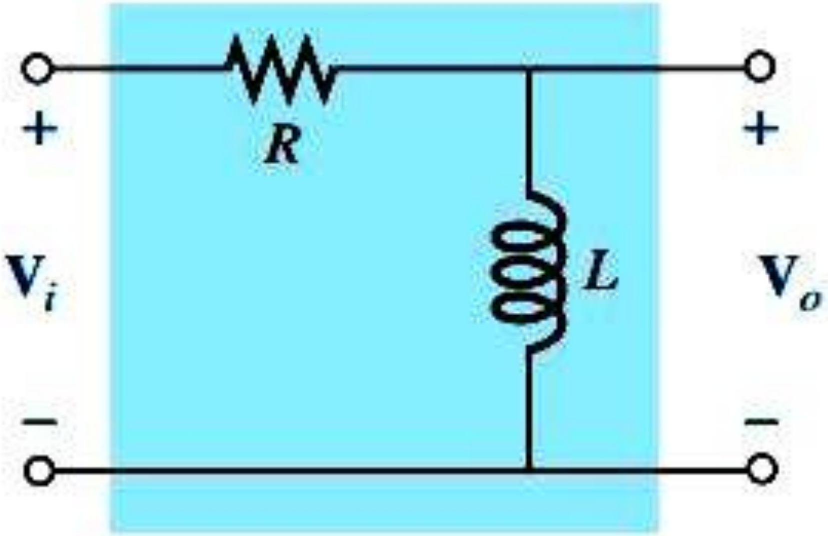
$$A_v = \frac{V_o}{V_i} = \frac{1}{\sqrt{1 + \left(\frac{X_C}{R}\right)^2}} = \frac{1}{\sqrt{1 + \left(\frac{40\ \text{k}\Omega}{20\ \text{k}\Omega}\right)^2}} = \frac{1}{\sqrt{1 + (2)^2}}$$

$$= \frac{1}{\sqrt{5}} = 0.4472$$

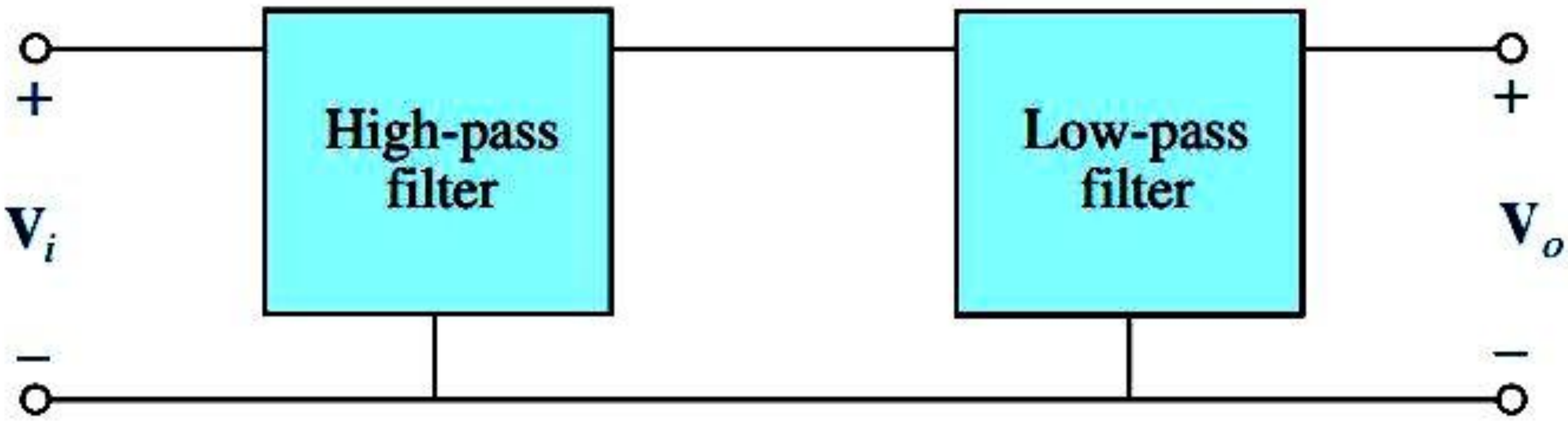
$$\theta = \tan^{-1}\frac{X_C}{R} = \tan^{-1}\frac{40\ \text{k}\Omega}{20\ \text{k}\Omega} = \tan^{-1} 2 = 63.43^\circ$$

and $$\mathbf{A}_v = \frac{\mathbf{V}_o}{\mathbf{V}_i} = \mathbf{0.447\ \angle 63.43^\circ}$$

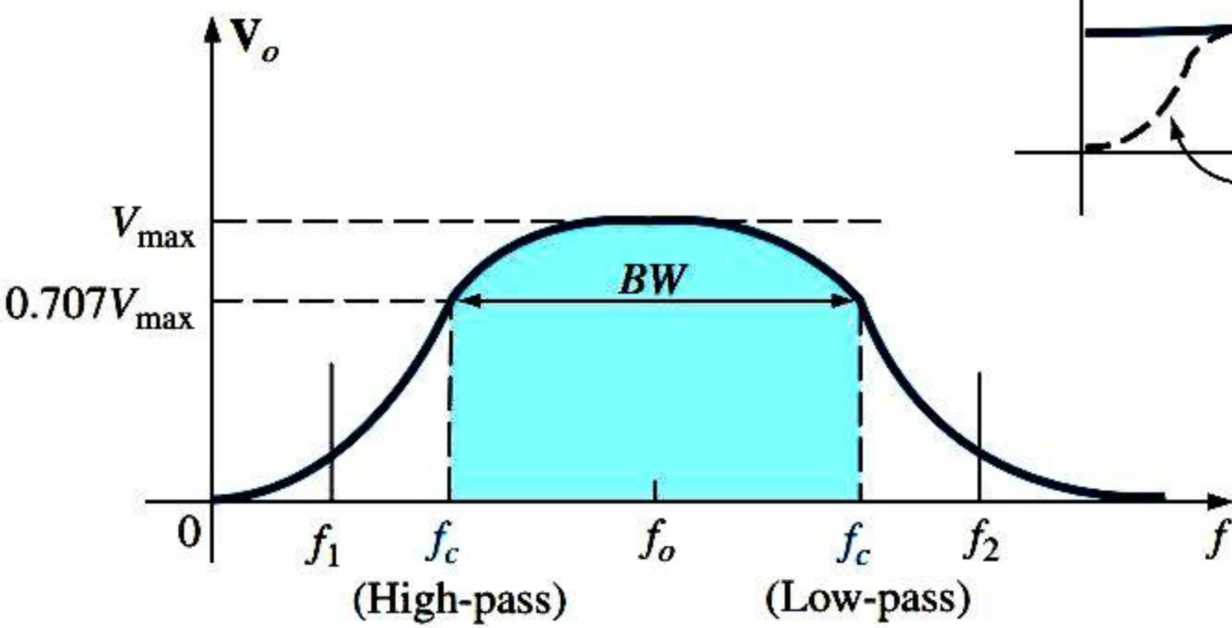
R-L High Pass Filter

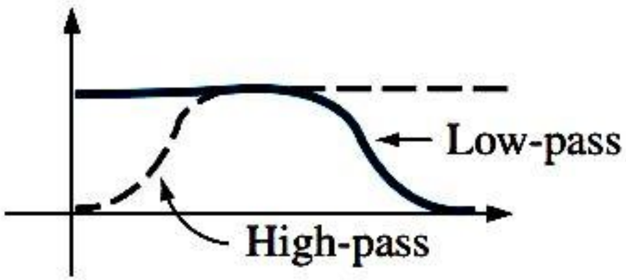


Pass Band Filter

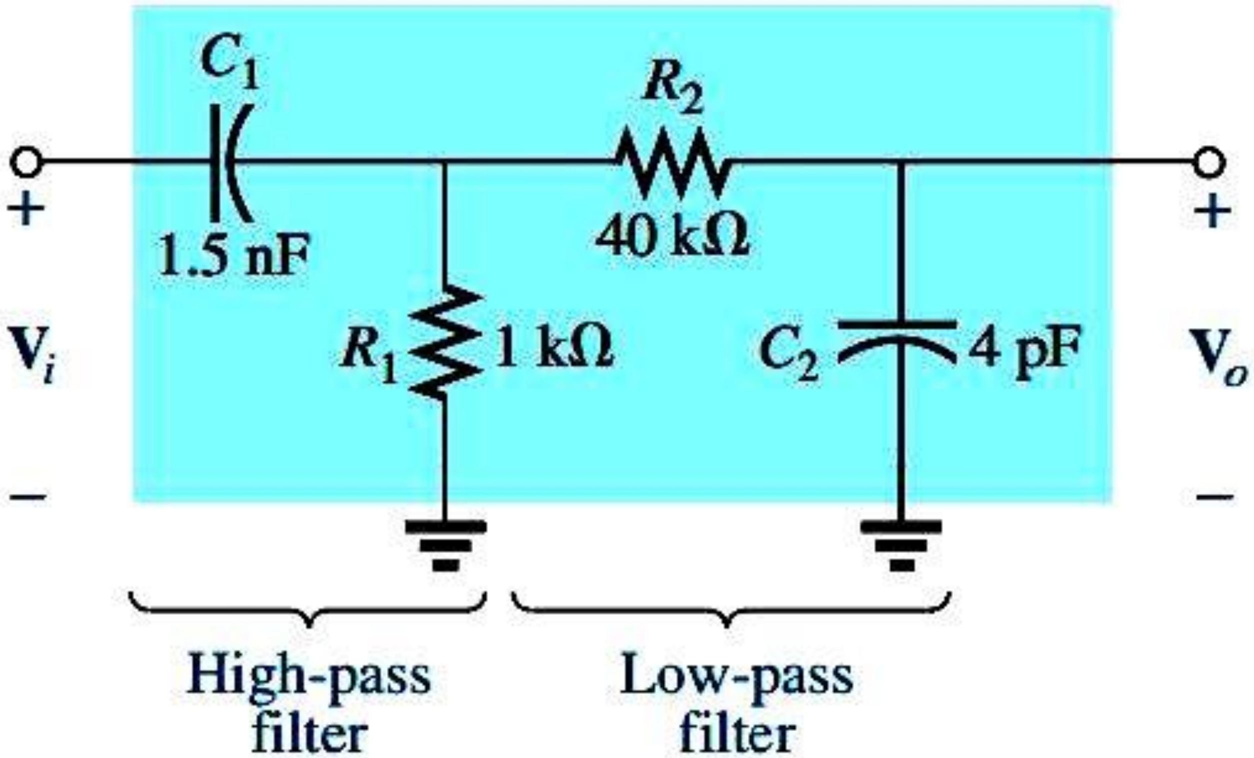


Pass Band Filter





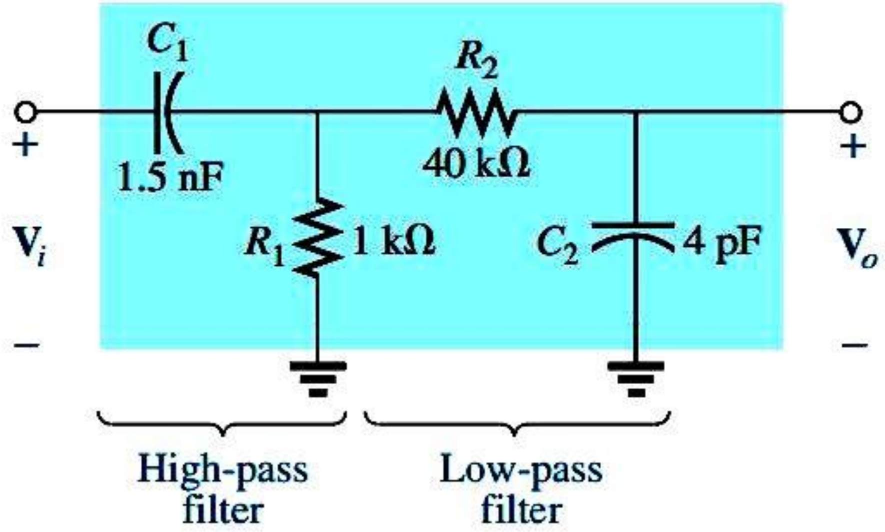
Pass Band Filter



Pass Band Filter

EXAMPLE For the pass-band filter in Fig.

a. Determine the critical frequencies for the low- and high-pass filters.
b. Using only the critical frequencies, sketch the response characteristics.
c. Determine the actual value of V_o at the high-pass critical frequency calculated in part (a), and compare it to the level that defines the upper frequency for the pass-band.



Pass Band Filter

a. High-pass filter:

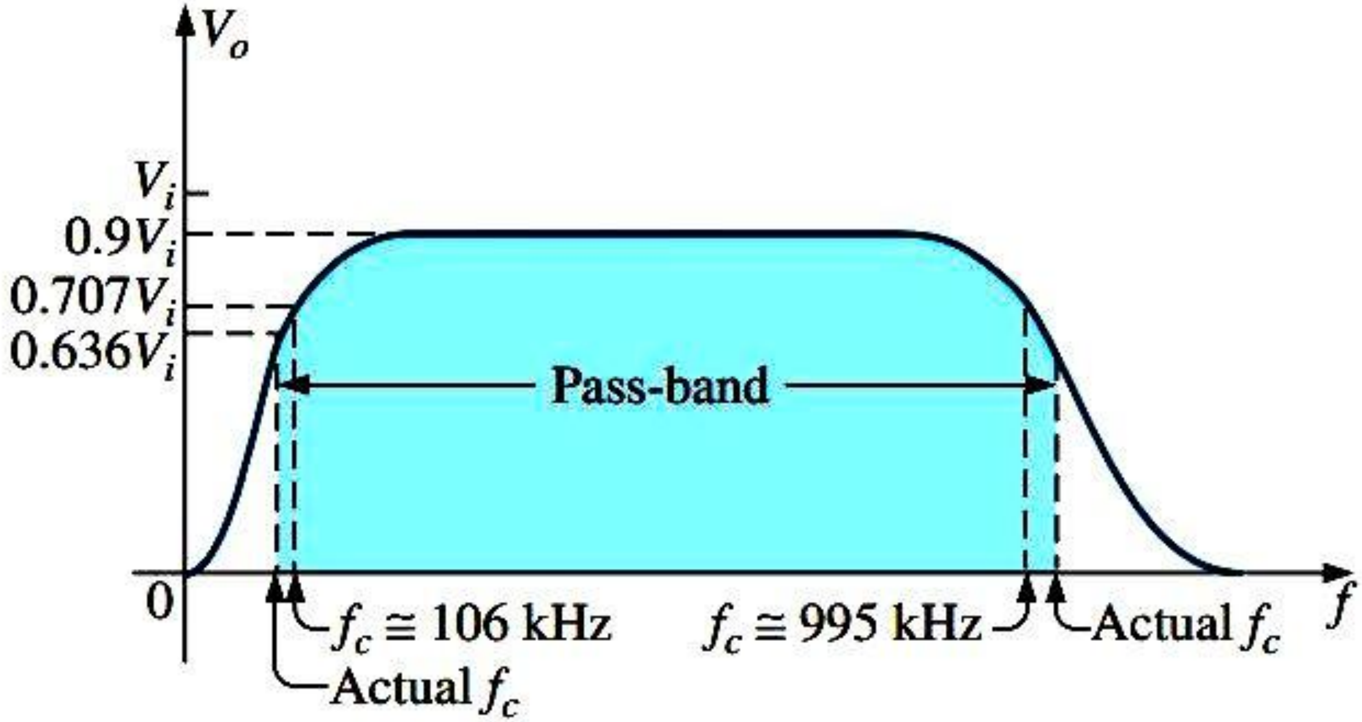
$$f_c = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi(1\text{ k}\Omega)(1.5\text{ nF})} = \mathbf{106.1\ kHz}$$

Low-pass filter:

$$f_c = \frac{1}{2\pi R_2 C_2} = \frac{1}{2\pi(40\text{ k}\Omega)(4\text{ pF})} = \mathbf{994.72\ kHz}$$

Pass Band Filter

b. In the mid-region of the pass-band at about 500 kHz, an analysis of the network reveals that $V_o \cong 0.9V_i$ as shown in Fig. bandwidth is therefore defined at a level of $0.707(0.9V_i) = 0.636V_i$ as also shown in Fig.



c. At $f = 994.72$ kHz,

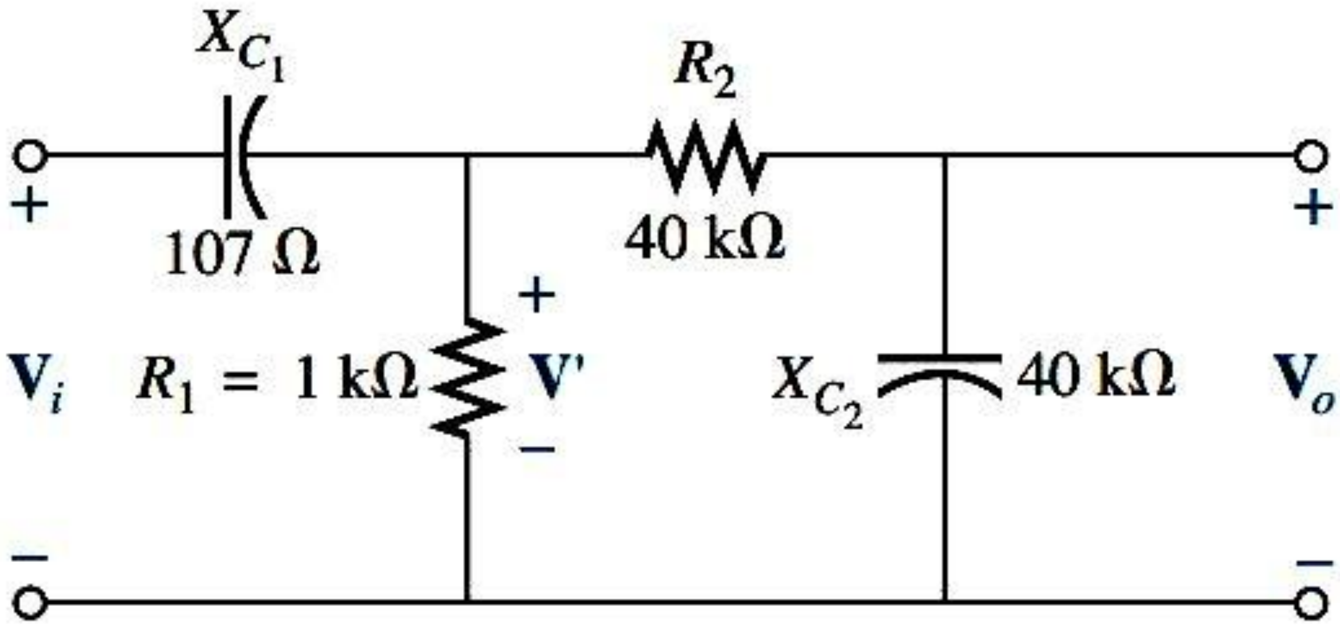
$$X_{C_1} = \frac{1}{2\pi f C_1} \cong 107\ \Omega$$

and $$X_{C_2} = \frac{1}{2\pi f C_2} = R_2 = 40\ \text{k}\Omega$$

resulting in the network in Fig.

The parallel combination $R_1 \| (R_2 - j X_{C_2})$ is essentially 0.976 kΩ $\angle 0°$ because the $R_2 - X_{C_2}$ combination is so large compared to the parallel resistor R_1.

Pass Band Filter



Pass Band Filter

$$\mathbf{V}' = \frac{0.976\text{ k}\Omega\ \angle 0°(\mathbf{V}_i)}{0.976\text{ k}\Omega - j0.107\text{ k}\Omega} \cong 0.994\mathbf{V}_i\ \angle 6.26°$$

with

$$\mathbf{V}_o = \frac{(40\text{ k}\Omega\ \angle -90°)(0.994\mathbf{V}_i\ \angle 6.26°)}{40\text{ k}\Omega - j40\text{ k}\Omega}$$

$$\mathbf{V}_o \cong 0.703\mathbf{V}_i\ \angle -39°$$

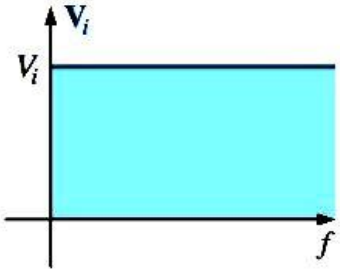
so that

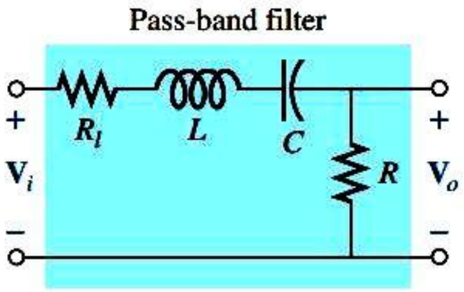
$$V_o \cong \mathbf{0.703V}_i \quad \text{at } f = 994.72\text{ kHz}$$

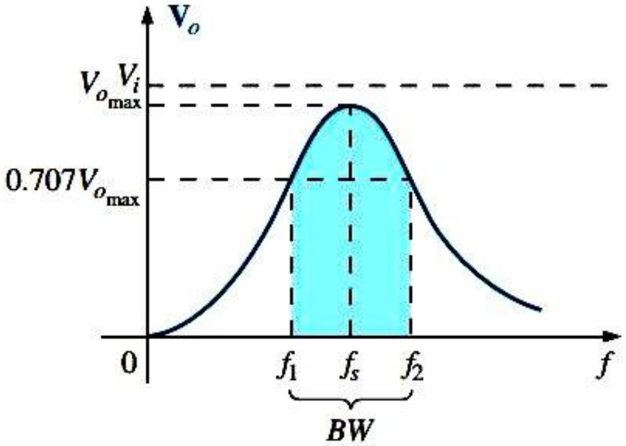
Since the bandwidth is defined at $0.636V_i$ the upper cutoff frequency will be higher than 994.72 kHz as shown in Fig.

Pass Band Filter

Series Resonant Circuit, $X_L = X_C$







Pass Band Filter

Series Resonant Circuit,

$X_L = X_C$

$$V_{o_{\max}} = \frac{R}{R + R_l} V_i \qquad f = f_s$$

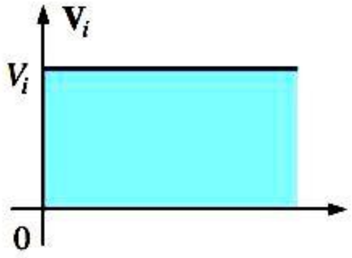
$$f_s = \frac{1}{2\pi\sqrt{LC}}$$

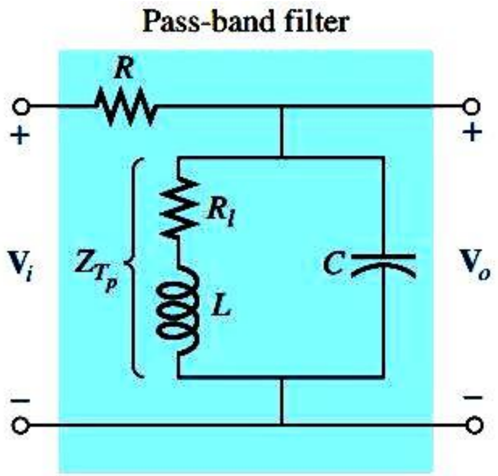
$$Q_s = \frac{X_L}{R + R_l}$$

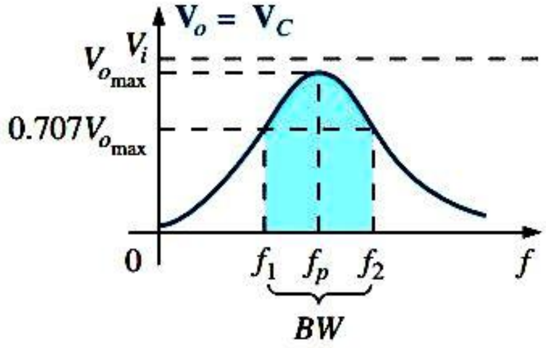
$$BW = \frac{f_s}{Q_s}$$

Pass Band Filter

Parallel Resonant Circuit, $X_L = X_C$







Parallel Resonant Circuit, $X_L = X_C$

$$\boxed{V_{o_{\max}} = \frac{Z_{T_p} V_i}{Z_{T_p} + R}} \quad f = f_p$$

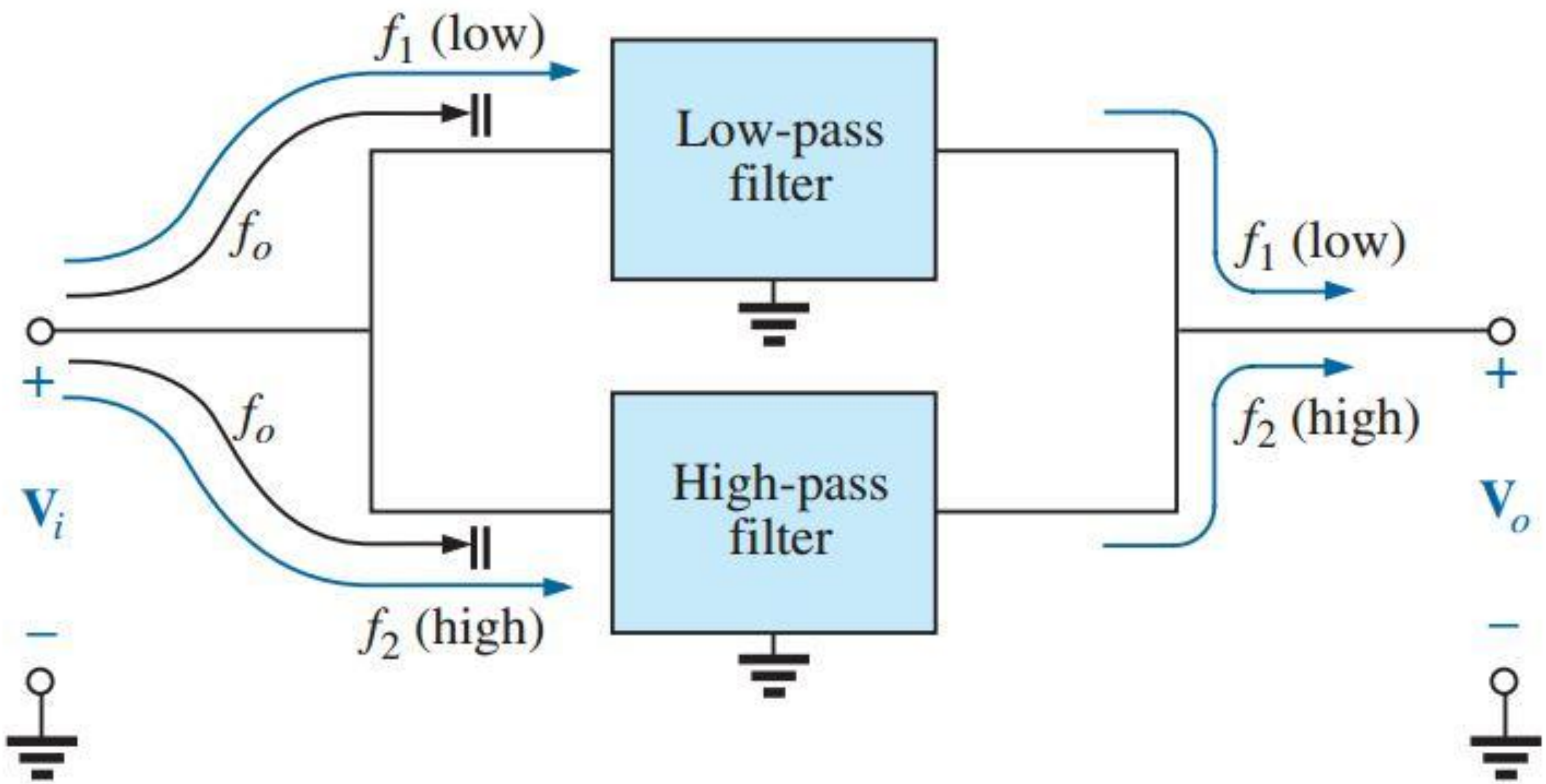
$$\boxed{Q_p = \frac{X_L}{R_l}}$$

$$\boxed{Z_{T_p} = Q_l^2 R_l} \quad Q_l \geq 10$$

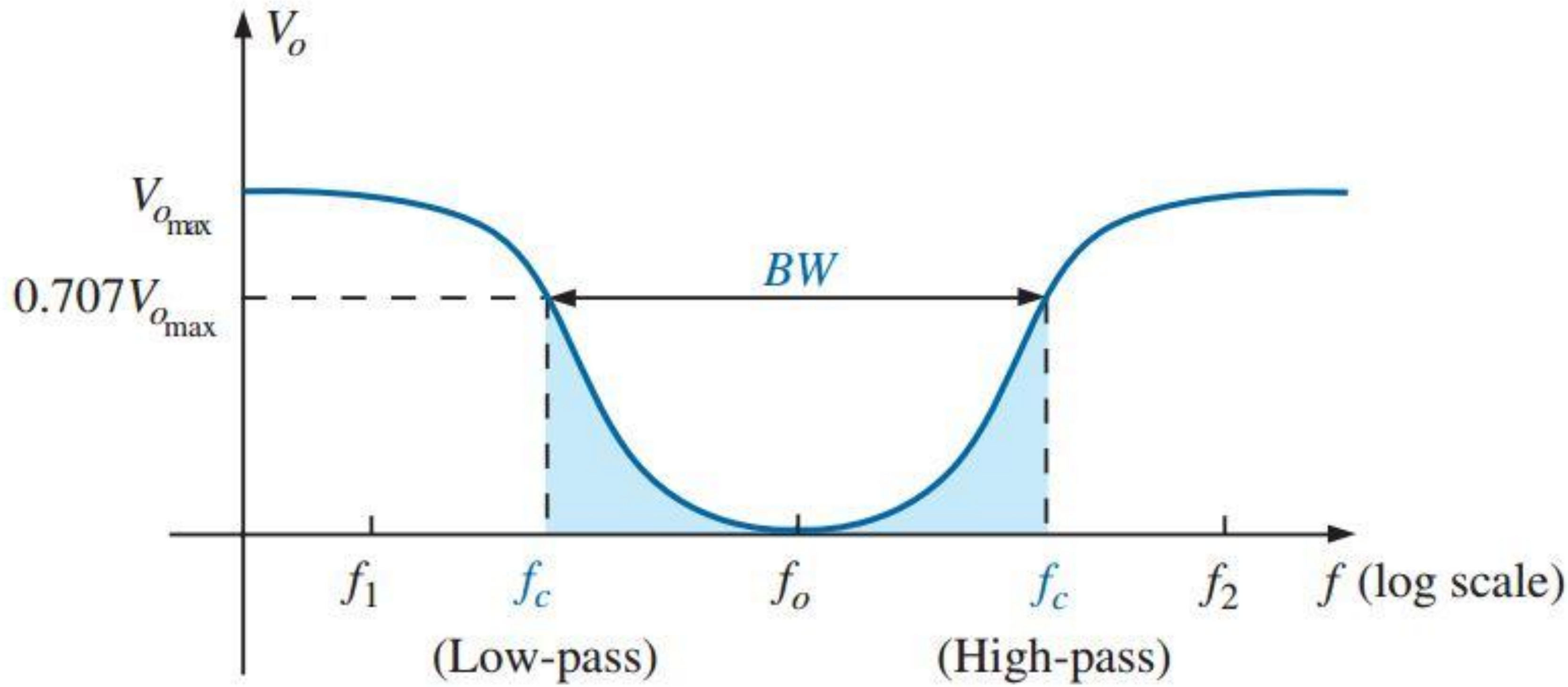
$$\boxed{BW = \frac{f_p}{Q_p}}$$

$$\boxed{f_p = \frac{1}{2\pi\sqrt{LC}}} \quad Q_l \geq 10$$

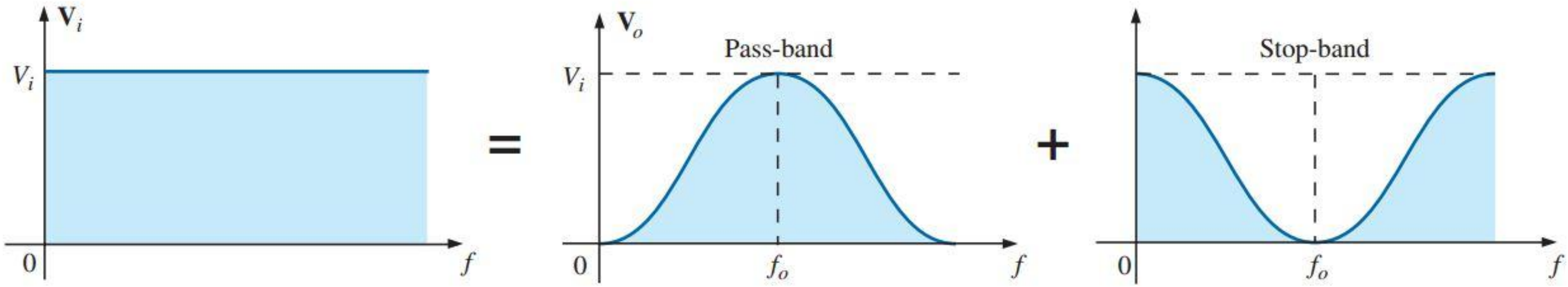
Stop Band Filter



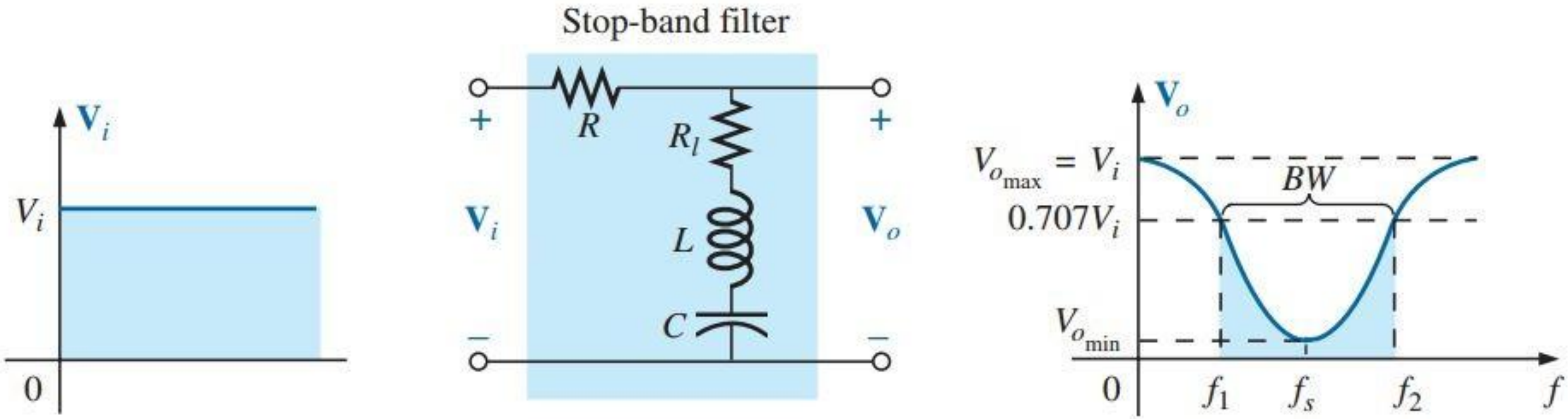
Stop Band Filter



Stop Band Filter



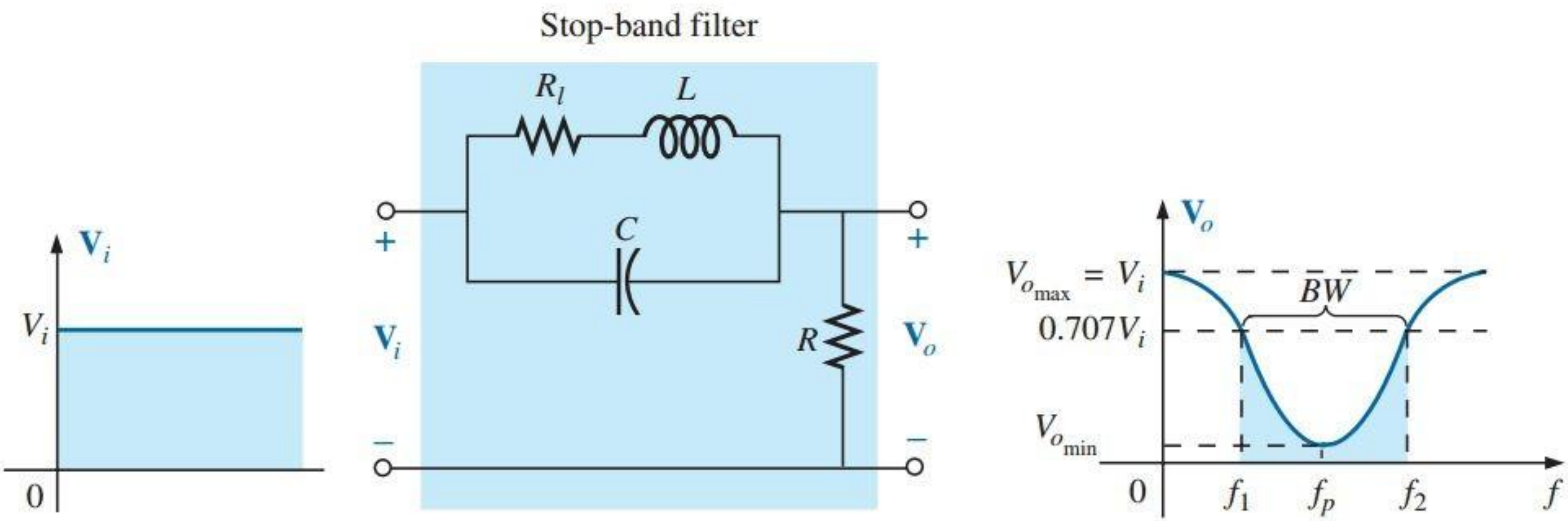
Stop Band Filter



Stop-band filter using a series resonant circuit.

$$V_{o_{\min}} = \frac{R_l V_i}{R_l + R}$$

Stop Band Filter



Stop-band filter using a parallel resonant network.

$$\boxed{V_{o_{\text{min}}} = \frac{RV_i}{R + Z_{T_p}}}$$

ASSIGNMENT – 2

EXAMPLE 21.8

a. Determine the frequency response for the voltage V_o for the series circuit in Fig. 21.35.

b. Plot the normalized response $A_v = V_o/V_i$.

c. Plot a normalized response defined by $A'_v = A_v/A_{v_{max}}$.