What is Heisenberg's Uncertainty principle?

Ans. It is impossible to measure simultaneously the exact position and exact velocity (or momentum) of a sub atomic particle like electron and proton.

Mathematical expression for the principle:

If Δx represents the error (or uncertainty) in the measurement of the position and Δp represents the uncertainty in the measurement of the momentum of a sub atomic particle like electron and neutron, then according to the principle, these two quantities are related as:

$$\Delta x \cdot \Delta p \geq \frac{h}{2\pi} \quad \text{(uncertainty relation)}$$

Where h is the plank's constant and the sign $\geq$ indicates equal to or greater than: This relation given above is called Uncertainty relation.

From this relation it is evident that, if Δx is small (i.e if the position of a particle is measured accurately or with accuracy), Δp will be large (i.e the momentum will be measured less accurately or with less accuracy) and vice versa. Thus, if one quantity is measured accurately the other quantity is measured less accurately.

Math – 01 : Calculate the Uncertainty in the position (Δx) of an electron, if Δv is 0.1 percent. Take velocity of the electron $= 2.2 \times 10^{6}\ ms^{-1}$, mass of electron $= 9.1 \times 10^{-31}$ kg and $h = 6.627 \times 10^{-34}$ J s
Coment (comment) on the result.

Soln. Here, $\Delta v = 0.1\%$ of the velocity of the electron

$$= \frac{0.1 \times 2.2 \times 10^{6}}{100}\ ms^{-1}$$

$$= 2.2 \times 10^{3}\ ms^{-1}$$

Now
$$\Delta x = \frac{h}{2\pi m . \Delta v}$$

$$= \frac{6.627 \times 10^{-34}\ Js}{2 \times (3.1416)(9.1 \times 10^{-31} kg)(2.2 \times 10^{3}\ ms^{-1})}$$

$$= \frac{6.627 \times 10^{-6}}{2 \times 3.1416 \times 9.1 \times 2.2}\ m$$

$$= 0.0524953 \times 10^{-6}\ m$$

$$= 524.953 \times 10^{-10}\ m$$

Since the calculated value of Δx is much longer than the atomic diameter ($\simeq 10^{-10}$ m), Uncertainty principle is applicable to electron.

$= 0.2911 \times 10^{-10}$ m

Math 03 Calculate the wave length of matter-wave associated with an electron moving with a velocity of 1.20×10^{7} cm s^{-1}
(mass of electron $= 9.11 \times 10^{-31}$ kg, $h = 6.63 \times 10^{-27}$ erg s)
Answer in C.G.S Unit.

Soln. Here, $v = 1.20 \times 10^{7}$ cm s^{-1}

$m = 9.11 \times 10^{-}$ gm

$h = 6.63 \times 10^{-27}$ erg. s

We know that,

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-27} \text{ erg.s}}{(9.11 \times 10^{-28} g) \times (1.20 \times 10^{7} \text{ cm } s^{-1})}$$

$= 0.6057 \times 10^{-6}$ cm. Ans.

Numericals Problems

S.I. Unit:

Math : 01 Calculate the wave length of a body of mass moving with a velocity of $3 ms^{-1}$

Soln: We know that according to the de Broglie's equat

$$\lambda = \frac{h}{mv}$$

Substituting $h = 6.624 \times 10^{-24}$ Js

$m = 1$ mg

$= 1 \times 10^{-6}$ kg and

$v = 3 ms^{-1}$ in the above equation,

We get,
$$\lambda = \frac{6.624 \times 10^{-34} \text{ Js}}{(1 \times 10^{-6} \times kg) \times (3 ms^{-1})}$$

$$= \frac{6.624 \times 10^{-34} \text{ Js}}{3 \times 10^{-6} \text{ kg ms}^{-1}}$$

$$= \frac{2.20 \times 10^{-28} \text{ Js}}{\text{kg ms}^{-1}}$$

$$= \frac{2.20 \times 10^{-28} \text{ kg.m}^2 s^{-2} s}{\text{kg ms}^{-1}}$$

$$= 2.20 \times 10^{-28} m$$ (Ans.

S.I Unit:

Math : 02 Calculate the wave length of an electron moving with a velocity of $2.5 \times 10^7 ms^{-1}$

($h = 6.624 \times 10^{-34}$ J s, mass of the electron $= 9.11 \times 10^{-31}$ kg)

Soln. Applying the de-Broglies equation

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31}) \times 2.5 \times 10^7} m$$

Q. Deduce the Schrodinger's Wave Equation.

Ans. The wave motion of a wave of a vibrating string can be described by the equation:

$$\psi = A \sin \frac{2\pi x}{\lambda} \longrightarrow (1)$$

where ψ represents the amplitude of the wave and is called the wave function; A is a constant; x is the displacement of the wave in a given direction and λ is the wave length.

On differentiating equation (1) with respect to x, we get

$$\frac{d\psi}{dx} = \left(A \cos \frac{2\pi x}{\lambda}\right)\left(\frac{2\pi}{\lambda}\right)$$

$$\text{or, } \frac{\partial\psi}{\partial x} = \frac{2\pi A}{\lambda} \cdot \cos \frac{2\pi x}{\lambda} \longrightarrow (II)$$

By differentiating again equation (II), we get.

$$\frac{\partial\psi}{dx} = \left(A \cos \frac{2\pi x}{\lambda}\right)\left(\frac{2\pi}{\lambda}\right)$$

$$\text{or, } \frac{d^2\psi}{dx^2} = \frac{2\pi A}{\lambda}\left(-\sin \frac{2\pi x}{\lambda}\right)\left(\frac{2\pi}{\lambda}\right)$$

$$\text{or, } \frac{\partial^2\psi}{dx^2} = -\frac{4\pi^2 A}{\lambda} \cdot \sin \frac{2\pi x}{\lambda}$$

$$= -\frac{4\pi^2}{\lambda^2}\left(A \cdot \sin \frac{2\pi x}{\lambda}\right)$$

$$= -\frac{4\pi^2}{\lambda^2} \cdot \psi \quad \left[\because \psi = A \sin \frac{2\pi x}{\lambda}\right]$$

$$\text{or, } \frac{d^2\psi}{dx^2} + \frac{4\pi^2}{\lambda^2}\psi = 0 \longrightarrow (III)$$

This is the classical wave equation which describes the wave motion of any particle vibrating along x - axis.

with the confirmation of the idea that an electron in atom is described as a standing wave round the nucleus, the classical wave equation (III) which describes the wave motion of any particle vibrating along X-axis should be applicable to the standing wave of an electron.

We have been seen in Bohr's theory that the total energy (E) of an electron is the sum of its kinetic energy (K.E) and the potential energy (V), i.e.

$$E = K.E + V$$

$$\text{or, } E = \frac{1}{2}mv^2 + V \longrightarrow \text{(IV)}$$

In equation (IV) m is the mass of the electron and v is velocity with which the electron is moving.

Now from the de Broglie's equation, we have

$$\lambda = \frac{h}{mv}$$

$$\text{or, } mv = \frac{h}{\lambda}$$

$$\text{or, } m^2v^2 = \frac{h^2}{\lambda^2}$$

$$\text{or, } \frac{1}{2}m^2v^2 = \frac{1}{2}\frac{h^2}{\lambda^2}$$

$$\text{or, } \frac{1}{2}m^2v^2 = \frac{h^2}{2\lambda^2}$$

$$\text{or, } \frac{1}{2}mv^2 = \frac{h^2}{2\lambda^2 m} \longrightarrow \text{(V)}$$

putting the value of $\frac{1}{2}mv^2$ from equation (V) in equat (IV), we get: $E = \frac{h^2}{2\lambda^2 m} + V$

$$\text{or, } \frac{1}{\lambda^2} = \frac{2m}{h^2}(E-V) \longrightarrow \text{(VI)}$$

putting the value of $\frac{1}{\lambda^2}$ from equation (VI) in equation (

we get ——

$$\frac{d^2\psi}{dx^2} + \frac{4\pi^2\psi \times 2m}{h^2}(E-V) = 0$$

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0 \longrightarrow \text{(VII)}$$

Equation (VII) is the schrodinger's wave equation of a wave moving in one dimension x, i.e the wave is moving in one direction x. If an electron wave is moving in any of three axis x, y, z its wave motion can be described by the wave equation :

$$\frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2} + \frac{8\pi^2 m}{h^2}(E-V)\psi = 0$$

$$\longrightarrow \text{(VIII)}$$

Equation (viii) is called schrodinger's wave equatio

Q. What is Eigen value and Eigen function ?

amplitude, ψ.

(iii) $\frac{d\psi}{dx}$, $\frac{\partial\psi}{\partial y}$ and $\frac{\partial\psi}{\partial z}$ must be continuous functions of x, y and z respectively

(iv) The Solutions must be normalised i.e they must satisfy the relation

$$\int_0^{+\alpha} \psi^2 d\gamma = 0$$

Where $d\gamma$ is a small volume element. The above condition clearly show that, the wave function, ψ is always finite, Single valued and continuous. It has zero value at finite distance.

Q. Describe the physical Significance of ψ and ψ^2

Ans. In Schrodinger's wave equation the wave equation, ψ represents the amplitude of the wave. From this equation the value of ψ for 1s electron situated at a distance of r from the proton in the ground state of Hydrogen atom is given by $\psi_{1s} = C_1 . e^{-C_2 r}$

Where C_1 and C_2 are constants.

(i) By ψ^2 finding the probability of electron in any place surrounding the nucleous.

(ii) the value of ψ^2 is proportional to frequency of the wave. where The value of ψ^2 more there must be more electron density.

(iii) In the total place the summation of finding electron is 1. So $\int_{-\alpha}^{+\alpha} \psi^2 dv = 1$

(iv) The value of ψ positive or Negative but the value of ψ^2 must be positive.

Math: Calculate the wavelength of an electron moving with a velocity of $2.5 \times 10^7 ms^{-1}$ ($h = 6.63 \times 10^{-34} Js$, $m = 9.11 \times 10^{-31} kg$)

Soln: Applying de Broglie's equation,

$$\lambda = \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34}}{9.11 \times 10^{-34} \times 2.5 \times 10^7} m$$

$$= 0.2911 \times 10^{-10} m$$

Ans.

** Q. Quantum Theory of Radiation ?

Ans. The general theory of Radiation or Electromagnetic radiation in its present form may stated as —

(i) When atoms or molecules absorb or emit radiant energy, they donot separate units of waves called quanta or photons.

(ii) The energy of E of a quantum or photon is given by the relation,

$$E = h\nu \longrightarrow (1)$$

where ν is the frequency of the emitted radiation and h is the planck's constant The value of

$h = 6.63 \times 10^{-34}$ Js

$= 6.63 \times 10^{-27}$ erg sec

we know that c, the velocity of radiation is gi[ven] by the equation

$$c = h\nu \longrightarrow (11)$$

Substituting the value of ν from (11) in (1)

We can write,

$$E = \frac{hc}{\lambda}$$

Thus the magnetude of a quantum or photon of energ[y] is directly proportional to the frequency of the radia[tion] energy or is inversely proportional to its wavelength λ.

(iii) An atom or molecule can emit (or absorb) either one quantum of energy ($h\nu$) or any whole Number multiple of this unit.

Quantum theory provided admireably a basis for explaning the photoelectric effect, atomic spectra and also helped in understanding the modern concepts of atomic and molecular structure.

Application of Planck's Quantum Theory of Radiation.

(i) In 1905 Einstein applied this theory to explain the photo electric effect.

(ii) In 1913 Bohr used this theory to explain the Structure of the atom and the origin of Spectral lines seen in Hydrogen atom.

(iii) In 1922 Compton applied this theory to explain the phenomena of scattering of X-rays (Compton eff

* Note:

$$E_n = -\frac{2\pi^2 m e^4}{n^2 h^2} \quad \text{(C.G.S Unit)}$$

$$E_n = \frac{-me^4}{8\epsilon_0^2 n^2 h^2} \quad \text{(S.I Unit)}$$

$$r_n = \frac{\epsilon_0 n^2 h^2}{\pi m e^2} \quad \text{(SI Unit)}$$

$$r_n = \frac{n^2 h^2}{4\pi^2 m e^2} \quad \text{(C.G.S Unit)}$$

* Problem: 01 (C.G. Unit) Calculate the excitation energy for the electronic transition from state $n=1$ to state $n=2$ in a hydrogen atom ($h = 6.627\times10^{-27}$ erg sec)

$e = 4.803\times10^{-10}$ esu

$m = 9.11\times10^{-28}$ gm.

Soln. Let, the energy associated with the orbit having $n=1$ be E_1 and that associated with the orbit with $n=2$ be E_2

and h is the planck's constant The value of

$h = 6.63 \times 10^{-34}$ JS
$= 6.63 \times 10^{-27}$ erg sec

we know that c, the velocity of radiation is given by the equation

$c = h\nu$ ⟶ (II)

Substituting the value of ν from (II) in (I)
We can write,

$$E = \frac{hc}{\lambda}$$

Thus the magnetude of a quantum or photon of energy is directly proportional to the frequency of the radiation energy or is inversely proportional to its wavelength λ.

(III) An atom or molecule can emit (or absorb) either one quantum of energy ($h\nu$) or any whole Number multiple of this unit.

Quantum theory provided admireably a basis for explaning the photoelectric effect, atomic spectra and also helped in understanding the modern concept of atomic and molecular structure.

sub-shell are as follows.

Electron Number

		Quantum Numbers		
	n	l	m	s
1st electron:	3	0	0	$+\frac{1}{2}$
2nd electron:	3	0	0	$+\frac{1}{2}$

12Q. Deduce the de Broglie's Equation.

Ans. We have seen that energy of a photon, E is related to its frequency, ν by plank's equation as:

$$E = h\nu \quad \text{(Plank equation)}$$

$$\text{or, } E = h \cdot \frac{c}{\lambda} \quad \left(\because \nu = \frac{c}{\lambda}\right) \quad \cdots\cdots \longrightarrow (i)$$

Here, λ is the wavelength of radiation (photon)

Again, the energy of a photon, E with mass, m and Velocity, c is also given by Einstein mass energy relationship as:

$$E = mc^2 \quad \text{(Einstein mass Energy relationship)} \quad \cdots\cdots\cdots \longrightarrow (ii)$$

Equating equations (i) and (ii), we get,

$$h \frac{c}{\lambda} = mc^2$$

$$\text{or, } \frac{h}{\lambda} = mc$$

$$\text{or } \lambda = \frac{h}{mc}$$

If the above equation is applied to a moving elec of mass m and moving with a velocity v; then the above equation reduces to

$$\lambda = \frac{h}{mv}$$ (de Broglie's equation)

This is de Broglie's equation which give the value of wavelength (λ) of a moving electron in terms of its mass (m) and velocity (v)

Q. What are Isotopes ?

Soln) Different kinds of atoms of the same element which have the same atomic number but different mass numbers or atomic masses are called Isotopes of the elements. For example. Hydrogen has 3 Isotopes.

$^{1}_{1}H \longrightarrow$ Hydrogen or Protium

$^{2}_{1}H \longrightarrow$ Deuterium

$^{3}_{1}H \longrightarrow$ Tritium

Isotopes of oxygen are: $^{16}_{8}O$, $^{17}_{8}O$, $^{18}_{8}O$

Isotopes of Neon are : $^{20}_{10}Ne$, $^{21}_{10}Ne$, $^{22}_{10}Ne$

Isotopes of chlorine : $^{35}_{17}Cl$, $^{37}_{17}Cl$.

Q. What are Isobars ?

The atoms of different elements which have the same mass number but different atomic number are called Isobars.

For example: $^{40}_{18}Ar$, $^{40}_{19}K$, $^{40}_{20}Ca$ are the examples of isobars. since each of them has the same mass number. i.e. the sum of protons and neutrons is the same.

$^{235}_{92}U$; $^{235}_{93}Np$ and $^{235}_{94}Pu$ are Isobars.

11

Q/ What are Isotones?

Atoms which have different atomic number and different atomic masses but the same number of neutrons are called Isotones.

For example: ${}^{14}_{6}C$, ${}^{15}_{7}N$ and ${}^{16}_{8}O$ are Isotones

since each contain eight neutrons.

${}^{30}_{14}Si$, ${}^{31}_{15}P$, ${}^{32}_{16}S$ are Isotones since each contain sixteen Neutrons.

${}^{5}_{3}Li$, and ${}^{6}_{4}Be$,

${}^{5}_{2}He$ and ${}^{6}_{3}Li$

Q. What are magic numbers? How is the stability of Nuclei related with magic Numbers?

Ans The nuclei which contain 2, 8, 20, 50, 82 or 126 protons (P) or neutrons (n) have been found to be extra stable, have large number of isotopes and hav their nuclear shells completely-filled (i.e closed shel The numbers, 2, 8, 20, 50, 82 and 126 are called magic numbers for the nuclear shells.

For example of stable nuclei:

① The nuclei ${}_{18}Ar^{38}$ $(n = 38 - 18 = 20)$,

${}_{40}Zr^{90}$ $(n = 90 - 40 = 50)$ and

${}^{138}_{56}Ba$ $(n = 138 - 56 = 82)$ are stable since the number of Neutrons in these nuclei are magic numbers.

(ii) ${}_{2}He^{4}$ ($p=2, n=4-2=2$)

${}^{16}_{8}O$ ($p=8, n=16-8=8$)

${}_{20}Ca^{40}$ ($p=20, n=40-20=20$) and

${}_{82}Pb^{208}$ ($p=82, n=208-82=126$) nuclei are extra stable. Since both the numbers of protons (p) and neutrons (n) in these nuclei are magic numbers

(iii) The end products in thorium (4n), uranium [(4n+2)] and actinium [(4n+3)] series are ${}_{82}Pb^{208}$, ${}_{82}Pb^{206}$ and ${}_{82}Pb^{207}$ respectively. All these isotopes are stable nuclei of ${}_{82}Pb$. Since the number of Proton ($p=82$) in each is a magic Number.

Q. Which of the following nuclei would you expect to be radioactive and why?

${}^{4}_{2}He$, ${}^{39}_{20}Ca$, ${}^{210}_{85}At$

Ans. (i) ${}^{4}_{2}He$ has protons $=2$; Neutrons $=2$ i.e both are magic numbers

Hence, it is expected to be stable (non-radioactive)

(ii) ${}^{39}_{20}Ca$ has protons $=20$ (even Number) and Neutrons $=19$ (odd Number). 20 is one of the magic numbers

Nevertheless, $\frac{n}{p}$ ratio is less than 1. Hence it would lie below the stability belt. Hence it is expected to be radioactive.

(iii) $^{210}_{85}At$ is radioactive as there is no stable nuclei with atomic number > 83