Theory of Equation

Examples

Ex. If $f(x) = x^4 + 10x^3 + 39x^2 + 76x + 65$ find the value of $f(x-4)$

Solution: Given that,

$$f(x) = x^4 + 10x^3 + 39x^2 + 76x + 65.$$

By Horner's method,

1	10	39	76	65	-4
	-4	-24	-60	-64	
1	6	15	16	1	
	-4	-8	-28		
1	2	7	-12		
	-4	8			
1	-2	15			
	-4				
1	-6				
1					

Hence the value of $f(x-4) = x^4 - 6x^3 + 15x^2 - 12x + 1$.

Ex-5: If $f(x) = ax^8 + bx^5 + cx + d$ find the value of $f(x+h) - f(x-h)$.

Solution: Given that,

$$f(x) = ax^8 + bx^5 + cx + d$$

$$f(x+h) = a(x+h)^8 + b(x+h)^5 + c(x+h) + d$$

$$= a\left(x^8 + {}^8c_1 x^7 h + {}^8c_2 x^6 h^2 + {}^8c_3 x^5 h^3 + {}^8c_4 x^4 h^4 + {}^8c_5 x^3 h^5 + {}^8c_6 x^2 h^6 + {}^8c_7 x h^7 + h^8\right) + b\left(x^5 + {}^5c_1 x^4 h + {}^5c_2 x^3 h^2 + {}^5c_3 x^2 h^3 + {}^5c_4 x h^4 + h^5\right) + cx + ch + d \longrightarrow (1)$$

and

$$f(x-h) = a(x-h)^8 + b(x-h)^5 + c(x-h) + d$$

$$= a\left(x^8 - {}^8c_1 x^7 h + {}^8c_2 x^6 h^2 - {}^8c_3 x^5 h^3 + {}^8c_4 x^4 h^4 - {}^8c_5 x^3 h^5 + {}^8c_6 x^2 h^6 - {}^8c_7 x h^7 + h^8\right) + b\left(x^5 - {}^5c_1 x^4 h + {}^5c_2 x^3 h^2 - {}^5c_3 x^2 h^3 + {}^5c_4 x h^4 - h^5\right) + cx - ch + d \longrightarrow (2)$$

(1) − (2) ⇒

$$f(x+h) - f(x-h) = 2a\,{}^8C_1 x^7 h + 2a\,{}^8C_3 x^3 h^3 + 2a\,{}^8C_5 x^3 h^5$$

$$+ 2a\,{}^8C_7 x h^7 + 2b\,{}^5C_1 x^4 h + 2b\,{}^5C_3 x^2 h^3 + 2b h^5 + 2c$$

$$= 2a(8x^7h + 56x^5h^3 + 56x^3h^5 + 8xh^7)$$

$$+ 2b(5x^4h + 10x^2h^3 + h^5) + 2ch$$

$$= 2a \cdot 8xh(x^6 + 7x^4h^2 + 7x^2h^4 + h^6) + 2bh(5x^4 \cdot$$

$$+ h^4) + 2ch$$

$$\therefore f(x+h) - f(x-h) = 16axh(x^6 + 7x^4h^2 + 7x^2h^4 + h^6)$$

$$+ 2bh(5x^4 + 5x^2h^2 + h^4) + 2ch$$

Answer.

H.K class

Ex. If $f(a)$ and $f(b)$ are of contrary signs then one root of the equation $f(x)=0$ must lie between a and b.

EX-6. Show that the equation $10x^3 - 17x^2 + x + 6 = 0$ has a root between 0 and -1.

solution:- Let, $f(x) = 10x^3 - 17x^2 + x + 6$

Then $f(0) = 0 - 0 + 0 + 6 = 6$

and $f(-1) = -10 - 17 - 1 + 6 = -22$

Since $f(0)$ and $f(-1)$ has contrary sign. [বিপরীত চিহ্ন]

Hence the given equation has a root between 0 and -1.

H.K class

EX-7. Show that the equation $x^4 - 5x^3 + 3x^2 + 35x - 70 = 0$ has a root between 2 and 3 and one between -2 and -3.

solution: Let, $f(x) = x^4 - 5x^3 + 3x^2 + 35x - 70$

$f(2) = 2^4 - 5.2^3 + 3.2^2 + 35.2 - 70$

$= 16 - 40 + 12 + 70 - 70$

$= -12$

and $f(3) = 3^4 - 5.3^3 + 3.3^2 + 35.3 - 70$

$= 81 - 135 + 27 + 105 - 70$

$= 8$

Since $f(2)$ and $f(3)$ has contrary sign. Hence the given equation has a root between 2 and 3.

[illegible]

[illegible]

Again, $f(-2) = (-2)^4 - 5(-2)^3 + 3(-2)^2 + 35(-2) - 70$

$= 16 + 40 + 12 - 70 - 70 = -72$

and $f(-3) = (-3)^4 - 5(-3)^3 + 3(-3)^2 + 35(-3) - 70$

$= 81 + 135 + 27 - 105 - 70 = 68$

Since $f(-2)$ and $f(-3)$ has contrary sign. Hence the given equation has a root between -2 and -3.

Ex-10. Solve the following equation which have equal roots.

$$x^4 - 9x^2 + 4x + 12 = 0$$

Solution: Let, $f(x) = x^4 - 9x^2 + 4x + 12 = 0 \longrightarrow (1)$

$f'(x) = 4x^3 - 18x + 4 = 0$

$\Rightarrow f'(x) \Rightarrow 2x^3 - 9x + 2 = 0 \longrightarrow (2)$ [2 common]

put, $x = 2$ then $f(x)$ and $f'(x)$ are satisfied.

$\therefore$ $(x-2)$ is a hight common factor (H.C.F) of $f(x)$ and $f'(x)$.

Hence $(x-2)^2$ or (x^2-4x+4) is a factor of $f(x)$

[illegible]
$\therefore x = a + a$
$(x-a)(x-a) = (x-a)$

Now. $x^4 - 9x^2 + 4x + 12 = 0$

$\Rightarrow x^4 - 4x^3 + 4x^2 + 4x^3 - 16x^2 + 16x + 3x^2 - 12x + 12 = 0$

$\Rightarrow x^2(x^2-4x+4) + 4x(x^2-4x+4) + 3(x^2-4x+4) = 0$

$\Rightarrow (x^2-4x+4)(x^2+4x+3) = 0$

$x^2-4x+4=0$

$\Rightarrow (x-2)^2 = 0$

$\therefore x = 2, 2$

$x^2+4x+3=0$

$\Rightarrow x^2+3x+x+3=0$

$\Rightarrow x(x+3)+1(x+3)=0$

$\Rightarrow (x+3)(x+1)=0$

$x = -3, -1$

Hence the roots are $2, 2, -3, -1$.

Ex-11. Solve the following equation which have equal roots

$$x^4 - 6x^3 + 12x^2 - 10x + 3 = 0$$

Solution:- Let, $f(x) = x^4 - 6x^3 + 12x^2 - 10x + 3 = 0$

$f'(x) = 4x^3 - 18x^2 + 24x - 10 = 0$

put, $x = 1$ then $f(x)$ and $f'(x)$ are satisfied.

$\therefore$ $(x-1)$ is a hight common factor of $f(x)$ and $f'(x)$

Hence $(x-1)^2$ is a factor of $f(x)$.

Now, $x^4-6x^3+12x^2-10x+3=0$

$\Rightarrow x^2(x^2-2x+1)-4x(x^2-2x+1)+3(x^2-2x+1)=0$

$\Rightarrow (x^2-2x+1)(x^2-4x+3)=0$

$\Rightarrow (x^2-2x+1)=0$

$\Rightarrow (x-1)^2=0$

$\therefore x=1,1$

$x^2-4x+3=0$

$\Rightarrow x^2-3x-x+3=0$

$\Rightarrow (x-1)(x-3)=0$

$\therefore x=1,3$

Hence the roots are $1, 1, 1, 3$.

Ex-12. ~~solve the~~ following equation which have equal roots

$$x^5-13x^4+67x^3-171x^2+216x-108=0$$

Solution :- Let, $f(x)=x^5-13x^4+67x^3-171x^2+216x-108=0$

$f'(x)=5x^4-52x^3+201x^2-342x+216=0$

put, $x=3$ then $f(x)$ and $f'(x)$ are satisfied.

$\therefore (x-3)$ is a H.C.F of $f(x)$ and $f'(x)$

Hence $(x-3)^2$ is a factor of $f(x)$

Now, $x^5-13x^4+67x^3-171x^2+216x-108=0$

$\Rightarrow x^3(x^2-6x+9)-7x^2(x^2-6x+9)+16x(x^2-6x+9)-12(x^2-6x+9)=0$

$\Rightarrow (x^2-6x+9)(x^3-7x^2+16x-12)=0$

$x^2-6x+9=0$

$\Rightarrow (x-3)^2=0$

$\therefore x=3,3$

$x^3-7x^2+16x-12=0$

$\Rightarrow x^3-3x^2-4x^2+12x+4x-12=0$

$\Rightarrow x^2(x-3)-4x(x-3)+4(x-3)=0$

$\Rightarrow (x-3)(x^2-4x+4)=0$

$\Rightarrow (x-3)(x-2)^2=0$

$\therefore x=3,2,2$

Hence the roots are $2, 2, 3, 3, 3$.

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Ex-16. solve the following equation which have equal roots.

$$x^6-3x^5+6x^3-3x^2-3x+2=0$$

Solution:- Let, $f(x)=x^6-3x^5+6x^3-3x^2-3x+2=0$

$\therefore f'(x)=6x^5-15x^4+18x^2-6x-3=0$

put, $x=1$ then $f(x)$ and $f'(x)$ are satisfied

$\therefore (x-1)$ is a H.C.F of $f(x)$ and $f'(x)$

Hence $(x-1)^2$ is a factor of $f(x)$

Now, $x^6-3x^5+6x^3-3x^2-3x+2=0$

$\Rightarrow x^4(x^2-2x+1)-x^3(x^2-2x+1)-3x^2(x^2-2x+1)$
$+x(x^2-2x+1)+2(x^2-2x+1)=0$

$\Rightarrow (x^2-2x+1)(x^4-x^3-3x^2+x+2)=0$

$\Rightarrow (x^2-2x+1)\{x^3(x-1)-2x(x-1)-2(x-1)\}=0$

$\Rightarrow (x^2-2x+1)\ (x-1)\ (x^3-2x-2)=0$

$\Rightarrow (x-1)^2(x-1)\ \{x^2(x+1)-x(x+1)-2(x+1)\}=0$

$\Rightarrow (x-1)^3\ (x+1)\ (x^2-x-2)=0$

$\Rightarrow (x-1)^3\ (x+1)\ (x^2-2x+x-2)=0$

$\Rightarrow (x-1)^3\ (x+1)\ (x-2)(x+1)=0$

$\therefore x=1,1,1,-1,-1,2$

Hence the roots are $1,1,1,-1,-1,2$

Ex-17. Solve the following equation which have equal roots

$$x^4-(a+b)x^3-a(a-b)x^2+a^2(a+b)x-a^3b=0$$

Solution: Let, $f(x)=x^4-(a+b)x^3-a(a-b)x^2+a^2(a+b)x-a^3b=0$

$f'(x)=4x^3-3(a+b)x^2-2a(a-b)x+a^2(a+b)=0$

put, $x=a$ then $f(x)$ and $f'(x)$ are satisfied

$\therefore (x-a)$ is a H.C.f of $f(x)$ and $f'(x)$

Hence $(x-a)^2$ is a factor of $f(x)$

Now, $x^4-(a+b)x^3-a(a-b)x^2+a^2(a+b)x-a^3b=0$

$\Rightarrow x^2(x^2-2ax+a^2)+ax(x^2-2ax+a^2)-bx(x^2-2ax+a^2)$
$-ab(x^2-2ax+a^2)=0$

$\Rightarrow (x^2-2ax+a^2)(x^2+ax-bx-ab)=0$

$\Rightarrow (x-a)^2\ (x+a)(x-b)=0$

$\therefore x=a,a,-a,b$

Hence the roots are $a,a,-a,b$

EX-18. ~~Find~~ the solution of the following equation which have common ~~roots~~

$2x^4-2x^3+x^2+3x-6=0,\ 4x^4-2x^3+3x-9=0$

Solution :- Given that,

$2x^4-2x^3+x^2+3x-6=0 \longrightarrow (1)$

$4x^4-2x^3+3x-9=0 \longrightarrow (2)$

$(1)-(2) \Rightarrow$
$-2x^4+x^2+3=0$

$\Rightarrow 2x^4-x^2-3=0$

$\Rightarrow 2x^4-3x^2+2x^2-3=0$

$\Rightarrow x^2(2x^2-3)+1(2x^2-3)=0$

$\Rightarrow (2x^2-3)(x^2+1)=0$

$\Rightarrow 2x^2-3=0$ | $x^2+1=0$
$\Rightarrow x^2=-1$ [Not Accetabl

$\therefore x^2=-1$, | $2x^2-3=0$
$\Rightarrow x=\pm\sqrt{3}/2$

$\therefore (2x^2-3)$ is a common factor of the equ^n (1) and (2)

Now, $2x^4-2x^3+x^2+3x-6=0$

$\Rightarrow x^2(2x^2-3)-x(2x^2-3)+2(2x^2-3)=0$

$\Rightarrow (2x^2-3)(x^2-x+2)=0$

$\Rightarrow 2x^2-3=0$, $x^2-x+2=0$

$x=\pm\sqrt{3}/2$, $x=\frac{1\pm\sqrt{-7}}{2}$

Again, $4x^4 - 2x^3 + 3x - 9 = 0$

$\Rightarrow 2x^2(2x^2-3) - x(2x^2-3) + 3(2x^2-3) = 0$

$\Rightarrow (2x^2-3)(2x^2-x+3) = 0$

$2x^2 - 3 = 0$, $2x^2 - x + 3 = 0$

$x = \pm\sqrt{3}/2$ $\qquad x = \dfrac{1 \pm \sqrt{-23}}{2\cdot 2} = \dfrac{1 \pm \sqrt{-23}}{4}$

Hence the roots are $\pm\sqrt{3}/2$, $\dfrac{1\pm\sqrt{-7}}{2}$,

$\pm\sqrt{3}/2$, $\dfrac{1\pm\sqrt{-23}}{4}$

EX-19: Find the solution of the following equations which have common roots $4x^4 + 12x^3 - x^2 - 15x = 0$, $6x^4 + 13x^3 - 4x^2 - 15x = 0$

Solution: - Given that,

$4x^4 + 12x^3 - x^2 - 15x = 0 \longrightarrow (1)$

$6x^4 + 13x^3 - 4x^2 - 15x = 0 \longrightarrow (2)$

$(1) - (2) \Rightarrow -2x^4 - x^3 + 3x^2 = 0$

$\Rightarrow 2x^4 + x^3 - 3x^2 = 0$

$\Rightarrow x^2(2x^2 + x - 3) = 0$

$\Rightarrow x^2 = 0$, $2x^2 + x - 3 = 0$ is a common factor of the eqn (1) and (2).

$\Rightarrow x = 0, 0$

Now, $4x^4 + 12x^3 - x^2 - 15x = 0$

$\Rightarrow 2x^2(2x^2 + x - 3) + 5x(2x^2 + x - 3) = 0$

$\Rightarrow (2x^2 + x - 3)(2x^2 + 5x) = 0$

$\Rightarrow x(2x+5)(2x^2 + 3x - 2x - 3) = 0$

$\Rightarrow x(2x+5)\{x(2x+3) - 1(2x+3)\} = 0$

$\Rightarrow x(2x+5)(2x+3)(x-1) = 0$

$\therefore x = 0, 1, -5/2, -3/2$

Again,

$$6x^4+13x^3-4x^2-15x=0$$

$$\Rightarrow 3x^2(2x^2+x-3)+5x(2x^2+x-3)=0$$

$$\Rightarrow (2x^2+x-3)(3x^2+5x)=0$$

$$\Rightarrow x(3x+5)(2x^2+3x-2x-3)=0$$

$$\Rightarrow x(3x+5)\{x(2x+3)-1(2x+3)\}=0$$

$$\Rightarrow x(3x+5)(2x+3)(x-1)=0$$

$$\therefore x=0,1,-5/3,-3/2$$

Hence the roots are

$$0,1,-5/2,-3/2$$

$$0,1,-5/3,-3/2.$$

Class

EX-20. Find the condition that $x^n-px^2+r=0$ may have equal roots.

solution: Let, $f(x)=x^n-px^2+r=0 \longrightarrow (1)$

$$f'(x)=nx^{n-1}-2px=0 \longrightarrow (2)$$

$\therefore (1)\times n-(2)\times x \Rightarrow$

$$nx^n-pnx^2+rn=0$$
$$nx^n-2px^2=0$$
$$-\quad\quad +$$

$$2px^2-pnx^2+rn=0$$

$$\Rightarrow x^2(2p-pn)=-rn$$

$$\Rightarrow px^2(2-n)=-rn$$

$$\Rightarrow px^2(n-2)=rn$$

$$\Rightarrow x^2=\frac{rn}{p(n-2)}$$

$$\therefore x=\left\{\frac{nr}{p(n-2)}\right\}^{1/2}$$

putting the value of x in (1), we get

$$\left\{\frac{nr}{p(n-2)}\right\}^{n/2}-p\left\{\frac{nr}{p(n-2)}\right\}^{1/2\cdot 2}+r=0$$

$$\Rightarrow \left\{\frac{nr}{p(n-2)}\right\}^{n/2}-\left\{\frac{nr}{(n-2)}\right\}+r=0$$

$$\Rightarrow \left(\frac{nr}{p(n-2)}\right)^{n/2} = \frac{nr}{n-2} - r$$

$$= \frac{nr - nr + 2nr}{n-2} = \frac{2r}{n-2}$$

$$\Rightarrow \left\{\frac{nr}{p(n-2)}\right\}^{n/2} = \frac{2r}{n-2}.$$

squaring on both sides, we get

$$\left\{\frac{nr}{p(n-2)}\right\}^{n} = \frac{2^2 \cdot r^2}{(n-2)^2}$$

$$\Rightarrow \frac{n^n \cdot r^n}{p^n (n-2)^n} = \frac{4r^2}{(n-2)^2}$$

$$\Rightarrow \frac{n^n \cdot r^{n-2}}{p^n (n-2)^{n-2}} = 4$$

$\Rightarrow n^n \cdot r^{n-2} = 4p^n (n-2)^{n-2}$ which is the required condition.

Ex-21. Show that $x^4 + qx^2 + s = 0$ can not have three equal roots.

solution: Let, $f(x) = x^4 + qx^2 + s = 0$

$$f'(x) = 4x^3 + 2qx = 0$$

$$f''(x) = 12x^2 + 2q = 0$$

If $f(x) = 0$ have three equal roots then we have a common roots between $f'(x) = 0$ and $f''(x) = 0$

Here, $4x^3 + 2qx = 0 \longrightarrow (1)$

$12x^2 + 2q = 0 \longrightarrow (2)$

$(1) \times 3 - (2) \times x \Rightarrow$

$$12x^3 + 6qx = 0$$

$$12x^3 + 2qx = 0$$

$$- \quad -$$

$$4qx = 0$$

$$\therefore x = 0$$

But $x=0$ does not satisfied the equation $f(x)=0$.

Hence we do not get a common root. So $f(x)$ have not three equal roots.

Ex-22. Find the ratio of b to a in order that the equation $ax^2+bx+a=0$ and $x^3-2x^2+2x-1=0$ may have

(i) one

(ii) two roots in common.

solution: Let, $f(x) = ax^2+bx+a=0 \longrightarrow (A)$

$\Rightarrow f(x) = x^2+b/a\,x+1=0 \longrightarrow (1)$ [dividing 'a']

Again, $f(x)=x^3-2x^2+2x-1=0 \longrightarrow (B)$

$f'(x) = 3x^2-4x+2=0$

$\Rightarrow f'(x) = x^2-4/3\,x+2/3=0 \longrightarrow (2)$ [dividing 3]

(i) If one root common then (1) and (2) must be identical

$$\frac{1}{1} = \frac{b/a}{-4/3} = \frac{1}{2/3}.$$

$$\Rightarrow b/a = {-4/3}/{2/3} = -2$$

$$\Rightarrow b/a = -2$$

(ii) If two roots common

Let, α and β be the roots of (A)

$\therefore \alpha+\beta = -b/a$, $\alpha\beta = 1$

Again, α, β, γ be the roots of (B)

$\therefore \alpha+\beta+\gamma = 2 \longrightarrow (iii)$

$\alpha\beta\gamma = 1$ $\quad$ $[\because \alpha\beta = 1]$

$\Rightarrow \gamma = 1$

From (iii), $\alpha+\beta = 2-\gamma = 2-1 = 1.$

$\Rightarrow \alpha+\beta = 1$

$\Rightarrow -b/a = 1$

$\therefore b/a = -1$

Hence the required ratio of b to a is $b/a = -2$ when one root common and $b/a = -1$ when two roots common.

Ex-23. Show that the equation $x^n + nx^{n-1} + n(n-1)x^{n-2} + \cdots + n! = 0$ can not have equal roots.

Solution: Let,

$$f(x) = x^n + nx^{n-1} + n(n-1)x^{n-2} + \cdots + n! = 0$$

$$= x^n + \frac{n(n-1)!}{(n-1)!}x^{n-1} + \frac{n(n-1)(n-1)!}{(n-2)!}x^{n-2} + \cdots + \frac{n!}{1!}x + n! = 0$$

$$= x^n + \frac{n!}{(n-1)!}x^{n-1} + \frac{n!}{(n-2)!}x^{n-2} + \cdots + \frac{n!}{1!}x + n! = 0 \longrightarrow (1)$$

$$f'(x) = nx^{n-1} + n(n-1)x^{n-2} + n(n-1)(n-2)x^{n-3} + \cdots + n! = 0$$

$$= \frac{n!}{(n-1)!}x^{n-1} + \frac{n!}{(n-2)!}x^{n-2} + \frac{n!}{(n-3)!}x^{n-3} + \cdots + n! = 0 \longrightarrow (2)$$

Now, $(2) - (1) \Rightarrow$

$$x^n = 0$$

$$\Rightarrow x = 0$$

putting the value of $x = 0$ in (1), we get $n! = 0$ which is impossible. Therefore the equation can not have equal roots.

Ex-24. If the equation $x^5 - 10a^3x^2 + b^4x + c^5 = 0$ has three equal roots show that $ab^4 - 9a^5 + c^5 = 0$.

Solution: Let, $f(x) = x^5 - 10a^3x^2 + b^4x + c^5 = 0$

$$f'(x) = 5x^4 - 20a^3x + b^4 = 0$$

$$f''(x) = 20x^3 - 20a^3 = 0$$

Now, $f''(x) = 0$

$$\Rightarrow 20x^3 - 20a^3 = 0$$

$$\therefore x = a$$

putting the value of $x=a$ in the original equation we get

$$a^5 - 10a^3 \cdot a^2 + b^4 a + c^5 = 0$$

$$\Rightarrow ab^4 - 9a^5 + c^5 = 0$$

Proved.

close (08)

Ex-25. If the equation $x^4 + ax^3 + bx^2 + cx + d = 0$ has three equal roots show that each of them is equal to $\dfrac{6c - ab}{3a^2 - 8b}$. ✓

solution: Let, $f(x) = x^4 + ax^3 + bx^2 + cx + d = 0$

$$f'(x) = 4x^3 + 3ax^2 + 2bx + c = 0$$

$$f''(x) = 12x^2 + 6ax + 2b = 0$$

Since $f(x) = 0$ has three equal roots so $f'(x) = 0$ and $f''(x) = 0$ have a root common.

$$4x^3 + 3ax^2 + 2bx + c = 0 \longrightarrow (1)$$

$$12x^2 + 6ax + 2b = 0 \longrightarrow (2)$$

$(1) \times 3 - (2) \times x \Rightarrow$

$$\begin{array}{r} 12x^3 + 9ax^2 + 6bx + 3c = 0 \\ 12x^3 + 6ax^2 + 2bx \quad\quad = 0 \\ \hline 3ax^2 + 4bx + 3c = 0 \end{array} \longrightarrow (3)$$

From (2) and (3),

$$12x^2 + 6ax + 2b = 0$$

$$3ax^2 + 4bx + 3c = 0$$

By cross multiplication rule,

$$\frac{x^2}{18ca - 8b^2} = \frac{x}{6ab - 36c} = \frac{1}{48b - 18a^2}$$

Last two ratio, $\dfrac{x}{6(ab - 6c)} = \dfrac{1}{6(8b - 3a^2)}$

$$\therefore x = \frac{6c - ab}{3a^2 - 8b}$$ which is common roots. (showed)

Ex-26. If $x^5+qx^3+rx^2+t=0$ has two equal roots prove that one of them will be a root of the quadratic

$$15rx^2-6q^2x+25t-4qr=0$$

Solution: ~~The given equation.~~

Let, $f(x)=x^5+qx^3+rx^2+t=0 \longrightarrow (1)$

$f'(x)=5x^4+3qx^2+2rx=0 \longrightarrow (2)$

$(1)\times 5-(2)\times x \Rightarrow$

$$
\begin{array}{l}
5x^5+5qx^3+5rx^2+5t=0\\
5x^5+3qx^3+2rx^2=0\\
-\qquad -\qquad -\\
\hline
2qx^3+3rx^2+5t=0
\end{array}
$$

$\longrightarrow (3)$

$(2)\times 2q-(3)\times 5x \Rightarrow$

$$
\begin{array}{l}
10qx^4+6q^2x^2+4qrx=0\\
10qx^4+15rx^3+25tx=0\\
-\qquad -\qquad -\\
\hline
-15rx^3+6q^2x^2-25tx+4qrx=0
\end{array}
$$

$\Rightarrow 15rx^2-6q^2x+25t-4qr=0$

Proved.

close

Ex-27 In the equation $x^3-x-1=0$ find the value of S_6.

Solution: Let, $f(x)=x^3-x-1=0$

$f'(x)=3x^2-1=0$

By the method of synthetic division, we get

1	3	0	-1							
0		0	3	3						
1			0	0	0					
1				0	2	2				
					0	3	3			
						0	2	2		
							0	5	5	
								0	5	5

$$\underset{}{3} + \underset{S_1}{0} + \underset{S_2}{2} + \underset{S_3}{3} + \underset{S_4}{2} + \underset{S_5}{5} + \underset{S_6}{5} + \cdots$$

Hence the quotient is $\frac{3'(x)}{x} = \frac{3}{x} + \frac{0}{x^2} + \frac{2}{x^3} + \frac{3}{x^4} + \frac{2}{x^5} + \frac{5}{x^6} + \frac{5}{x^7} + \cdots$

Hence the value of S_6 is 5.

Newton method

Given that,

$x^3 - x - 1 = 0$

$\Rightarrow x^3 - 0.x^2 - x - 1 = 0$

Here, $P_1 = 0,\ P_2 = -1,\ P_3 = -1,\ P_4 = P_5 = P_6 = 0$

$S_1 + P_1 = 0$

$\Rightarrow S_1 = 0$

$S_2 + S_1P_1 + 2P_2 = 0$

$\Rightarrow S_2 + 0 + 2(-1) = 0$

$\Rightarrow S_2 = 2$

$S_3 + S_2P_1 + S_1P_2 + 3P_3 = 0$

$\Rightarrow S_3 + 0 + 0 + 3(-1) = 0$

$\Rightarrow S_3 = 3$

$S_4 + S_3P_1 + S_2P_2 + S_1P_3 + 4P_4 = 0$

$\Rightarrow S_4 + 0 + 2.(-1) + 0 + 4.0 = 0$

$\Rightarrow S_4 = 2$

$S_5 + S_4P_1 + S_3P_2 + S_2P_3 + S_1P_4 + 5P_5 = 0$

$\Rightarrow S_5 + 0 + 3.(-1) + 2.(-1) + 0 + 0 = 0$

$\Rightarrow S_5 = 5$;

$$s_6 + s_5 p_1 + s_4 p_2 + s_3 p_3 + s_2 p_4 + s_1 p_5 + 6 p_6 = 0$$

$$\Rightarrow s_6 + 0 + 2(-1) + 3\cdot(-1) + 0 + 0 + 0 = 0$$

$$\Rightarrow s_6 - 2 - 3 = 0$$

$$\Rightarrow s_6 = 5$$

Hence the value of s_6 is 5.

Close

Ex-28. Find the values of s_4 and s_6 of the following equation

$$x^4 - x^3 - 7x^2 + x + 6 = 0$$

Solution: Let, $f(x) = x^4 - x^3 - 7x^2 + x + 6 = 0$

$$f'(x) = 4x^3 - 3x^2 - 14x + 1 = 0$$

By the method of synthetic division, we get —

1	4	−3	−14	1					
1		4	28	−4	−24				
7			1	7	−1	−6			
−1				15	105	−15	−90		
−6					19	133	−19	−114	
						99	693	−99	−594
						211	1477	−211	−1266

$4 + 1 + 15 + 19 + 99 + 211 + 795 + ---$

Hence the quotient is $\frac{f'(x)}{f(x)} = \frac{4}{x} + \frac{1}{x^2} + \frac{15}{x^3} + \frac{19}{x^4} + \frac{99}{x^5} + \frac{211}{x^6} + \frac{795}{x^7} + ---$

Hence the values of s_4 and s_6 is 99 and 795.

Newton method Given that

$$x^4 - x^3 - 7x^2 + x + 6 = 0$$

Here, $p_1 = -1,\ p_2 = -7,\ p_3 = 1,\ p_4 = 6$

$$p_5 = p_6 = 0$$

$s_1 + p_1 = 0$

$\Rightarrow s_1 - 1 = 0$

$\Rightarrow s_1 = 1$

$s_2 + s_1 p_1 + 2p_2 = 0$

$\Rightarrow s_2 + 1\cdot(-1) + 2(-7) = 0$

$\Rightarrow \underline{s_2 = 15}$

$s_3 + s_2 p_1 + s_1 p_2 + 3p_3 = 0$

$\Rightarrow s_3 + 15(-1) + 1(-7) + 3\cdot 1 = 0$

$\Rightarrow s_3 = 19$

$s_4 + s_3 p_1 + s_2 p_2 + s_1 p_3 + 4p_4 = 0$

$\Rightarrow s_4 + 19(-1) + 15(-7) + 1\cdot 1 + 4\cdot 6 = 0$

$\Rightarrow s_4 - 19 - 105 + 1 + 24 = 0$

$\Rightarrow s_4 = 99$

$s_5 + s_4 p_1 + s_3 p_2 + s_2 p_3 + s_1 p_4 + 5p_5 = 0$

$\Rightarrow s_5 + 99\cdot(-1) + 19(-7) + 15\cdot 1 + 1\cdot 6 + 0 = 0$

$\Rightarrow s_5 - 99 - 133 + 15 + 6 = 0$

$\Rightarrow s_5 = 211$

$s_6 + s_5 p_1 + s_4 p_2 + s_3 p_3 + s_2 p_4 + s_1 p_5 + 6p_6 = 0$

$\Rightarrow s_6 + 211\cdot(-1) + 99(-7) + 19\cdot 1 + 15\cdot 6 + 0 + 0 = 0$

$\Rightarrow s_6 - 211 - 693 + 19 + 90 = 0$

$\Rightarrow s_6 = 795$

Hence the values of s_4 and s_6 is 99 and 795.

Theorem:- If $a, b, c, \ldots, k$ are the roots of the equation $f(x)=0$, to prove that $f'(x) = \frac{f(x)}{x-a} + \frac{f(x)}{x-b} + \frac{f(x)}{x-c} + \cdots + \frac{f(x)}{x-k}$

proof:- since $f(x)=0$ be the equation whose roots are $a, b, c, \ldots, k$.

then we can write

$$f(x) = (x-a)(x-b)(x-c) \cdots (x-k)$$

$$= \prod_{r=1}^{n} (x-\alpha_r)$$

$$= (x-\alpha_1)(x-\alpha_2)(x-\alpha_3) \cdots (x-\alpha_n)$$

Let, $a=\alpha_1$, $b=\alpha_2$, $c=\alpha_3$, $\vdots$ $k=\alpha_n$

$$\Rightarrow f(x) = \prod_{r=1}^{n} (x-\alpha_r) \longrightarrow (1)$$

Taking 'ln' on both sides we get

$$\ln f(x) = \sum_{r=1}^{n} \ln (x-\alpha_r)$$

Differentiation on both sides w.r. to x

$$\frac{f'(x)}{f(x)} = \sum_{r=1}^{n} \frac{1}{(x-\alpha_r)}$$

$$\Rightarrow f'(x) = \sum_{r=1}^{n} \frac{f(x)}{(x-\alpha_r)}$$

$$\Rightarrow f'(x) = \frac{f(x)}{(x-\alpha_1)} + \frac{f(x)}{(x-\alpha_2)} + \cdots + \frac{f(x)}{(x-\alpha_n)}$$

$$\therefore f'(x) = \frac{f(x)}{(x-a)} + \frac{f(x)}{(x-b)} + \frac{f(x)}{(x-c)} + \cdots + \frac{f(x)}{x-k}$$

proved.

OR

since $f(x)=0$ be the equation whose roots are $a, b, c, \ldots, k$

then we can write

$$f(x) = (x-a)(x-b)(x-c) \cdots (x-k) \longrightarrow (1)$$

put, $x = x+h$

$$f(x+h) = (x+h-a)(x+h-b)(x+h-c) \cdots (x+h-k)$$

$\Rightarrow f(x+h) = (x-a+h)(x-b+h)(x-c+h) \cdots (x-k+h)$

We know that, $\longrightarrow (2)$

$f(x+h) = f(x) + h' f'(x) + \frac{h^2}{2!} f''(x) + \cdots$, [Taylor's theorem]

$\longrightarrow (3)$

From (2) and (3), we get—

$$f(x) + h f'(x) + \frac{h^2}{2!} f''(x) + \cdots = (x-a+h)(x-b+h)(x-c+h) \cdots (x-k+h)$$

Equaling the coefficient of h on both sides, we get

$$f'(x) = (x-b)(x-c) \cdots (x-k) + (x-a)(x-c) \cdots (x-k) + \cdots \cdots$$

$$\Rightarrow f'(x) = \frac{(x-a)(x-b)(x-c) \cdots (x-k)}{(x-a)} + \frac{(x-a)(x-b)(x-c) \cdots (x-k)}{(x-b)} + \cdots + \frac{(x-a)(x-b) \cdots (x-k)}{x-k}$$

$$\Rightarrow f'(x) = \frac{f(x)}{x-a} + \frac{f(x)}{x-b} + \cdots + \frac{f(x)}{x-k}$$

proved.

Theorem: If $\alpha_1, \alpha_2, \cdots, \alpha_n$ are the roots of the equation $f(x) = 0$ then show that

$$\frac{f'(x)}{f(x)} = \frac{n}{S} + \frac{S_1}{x^2} + \frac{S_2}{x^3} + \cdots + \frac{S^n}{x^{n+1}} + \cdots$$

where, $S_1 = \sum \alpha_1$, $S_2 = \sum \alpha_1^2$, $\cdots S_n = \sum \alpha_1^n$ etc.

proof: We know that, (see- pg0) +

$$\frac{f'(x)}{f(x)} = \sum_{r=1}^{n} \frac{1}{x - \alpha_r}$$

$$= \sum_{r=1}^{n} \frac{1}{x} \left(1 - \frac{\alpha_r}{x}\right)^{-1}$$

$$= \sum_{r=1}^{n} \frac{1}{x} \left\{ 1 + \frac{\alpha_r}{x} + \frac{\alpha_r^2}{x^2} + \cdots + \frac{\alpha_r^n}{x^n} + \cdots \right\}$$

$$= \frac{n}{x} + \frac{s_1}{x^2} + \frac{s_2}{x^3} + \cdots + \frac{s_n}{x^{n+1}} + \cdots$$

where, $s_1 = \sum \alpha_1 = \sum_{r=1}^{n} \alpha_r$

$s_2 = \sum \alpha_1^2 = \sum_{r=1}^{n} \alpha_r^2$

— — — — — —

$s_n = \sum \alpha_1^n = \sum_{r=1}^{n} \alpha_r^n$ etc.

~~1) What is function~~

~~2) " " limiting point~~

~~3) " " continuous function~~

~~4) What is domai~~