Dept. of Computer Science and Engineering
CSE2131 (Discrete Mathematics)-2018
Class Test 1 Time: 1 (One) Hour
NB: Answer any three Questions. (3x10=30 Marks)

1. Use the truth table functions to determine which of the following formulas are tautologies:

I. $(P \lor Q) \rightarrow (Q \lor P)$

II. $((P \lor Q) \land (P \lor R)) \leftrightarrow (P \land (Q \lor R))$

2. Verify that the expression is a WFF by analyzing its constituents at all levels down to the atomic primitives. A WFF is said to be satisfiable if it is True for some values of its propositional variables. All WFFs fall into exactly one of three categories: tautology, contradiction, or satisfiable but not tautology. Based on the results in the truth table, place the WFF in one of these categories.

I. $((P \land \neg Q) \lor (Q \land \neg P)) \rightarrow \neg(P \leftrightarrow Q)$

Ans:

- P and Q are WFFs.
- Since P and Q are WFFs, $\neg P$ and $\neg Q$ are WFFs.
- Since P , Q, $\neg P$ and $\neg Q$ are WFFs, $P \land \neg Q$ and $Q \land \neg P$ are WFFs.
- Since P and Q are WFFs, $P \leftrightarrow Q$ is a WFF.
- Since $P \leftrightarrow Q$ is a WFF, $\neg(P \leftrightarrow Q)$ is a WFF.
- Since $P \land \neg Q$ and $Q \land \neg P$ are WFFs, $(P \land \neg Q) \lor (Q \land \neg P)$ is a WFF.
- Since $(P \land \neg Q) \lor (Q \land \neg P)$ and $\neg(P \leftrightarrow Q)$ are WFFs, $((P \land \neg Q) \lor (Q \land \neg P)) \rightarrow \neg(P \leftrightarrow Q)$ is a WFF.

After truth table verification the proposition is a tautology.

II. $(P \rightarrow Q) \land (\neg P \rightarrow Q)$

Ans:

- P and Q are WFFs.
- Since P is a WFF, $\neg P$ is a WFF.
- Since P , Q, and $\neg P$ are WFFs, $P \rightarrow Q$ and $\neg P \rightarrow Q$ are WFFs.
- Since $P \rightarrow Q$ and $\neg P \rightarrow Q$ are WFFs, $(P \rightarrow Q) \land (\neg P \rightarrow Q)$ is a WFF.

After truth table verification the proposition is satisfiable but not a tautology.

3. Prove by equational reasoning: $(P \land ((Q \lor R) \lor Q)) \land S = S \land ((R \lor Q) \land P)$.

Ans:

$(P \land ((Q \lor R) \lor Q)) \land S$
$= S \land (P \land ((Q \lor R) \lor Q))$ {$\land$ commutative}
$= S \land (((Q \lor R) \lor Q) \land P)$ {$\land$ commutative}

$= S \wedge ((Q \vee (R \vee Q)) \wedge P)$ {$\vee$ associative}
$= S \wedge ((Q \vee (Q \vee R)) \wedge P)$ {$\vee$ commutative}
$= S \wedge (((Q \vee Q) \vee R) \wedge P)$ {$\vee$ associative}
$= S \wedge ((Q \vee R) \wedge P)$ {$\vee$ idempotent}
$= S \wedge ((R \vee Q) \wedge P)$ {$\vee$ commutative}

4.

I. Let the universe be the set of integers. Expand the following expression:
$\forall x \in \{1, 2, 3, 4\}.\ \exists y \in \{5, 6\}.\ F(x, y)$.

Ans: $(F(1, 5) \vee F(1, 6))$
$\wedge (F(2, 5) \vee F(2, 6))$
$\wedge (F(3, 5) \vee F(3, 6))$
$\wedge (F(4, 5) \vee F(4, 6))$

II. Let $S = \{0, 2, 4, 6\}$ and $R = \{0, 1, 2, 3\}$. Expand the following expressions into propositional term (i.e., remove the quantifiers): $\forall x \in S.\ \exists y \in R.\ x = 2 \times y$.

Ans: Let $S = \{0, 2, 4, 6\}$ and $R = \{0, 1, 2, 3\}$. Then we can state that every element of S is twice some element of R as follows:
$\forall x \in S.\ \exists y \in R.\ x = 2 \times y$
This can be expanded into a quantifier-free expression in two steps. The first step is to expand the outer quantifier:

$(\exists y \in R. 0 = 2 \times y)$
$\wedge (\exists y \in R. 2 = 2 \times y)$
$\wedge (\exists y \in R. 4 = 2 \times y)$
$\wedge (\exists y \in R. 6 = 2 \times y)$
The second step is to expand all four of the remaining quantifiers:
$((0 = 2 \times 0) \vee (0 = 2 \times 1) \vee (0 = 2 \times 2) \vee (0 = 2 \times 3))$
$\wedge ((2 = 2 \times 0) \vee (2 = 2 \times 1) \vee (2 = 2 \times 2) \vee (2 = 2 \times 3))$
$\wedge ((4 = 2 \times 0) \vee (4 = 2 \times 1) \vee (4 = 2 \times 2) \vee (4 = 2 \times 3))$
$\wedge ((6 = 2 \times 0) \vee (6 = 2 \times 1) \vee (6 = 2 \times 2) \vee (6 = 2 \times 3))$