Full Name: ID Number:

Instructions for the Examination

- There are four questions with equal weight. You should answer all the questions.
- Standard type calculator may be used. You are not allowed to have any electronics device except calculator on you.
- The exam is closed book and you may use the attached formula sheet.
- Available time is 2 hours.

Note 1:

You should submit the exam booklet and this question paper with attached formula sheets. Do not separate any sheet from the booklet or this question paper.

Note 2:

The grades of this exam and other grades of the course will be announced based on your ID number. If you want your grades not to be announced based your ID number, you may provide a nick name to replace the ID number in the announcement.

Optional Nickname:

Question 1:

A discrete system is defined by following difference equation where $x[n]$ and $y[n]$ are input and output of the system, respectively.

$$y[n] = x[2n]$$

a- (15%) Is this system linear? Is this system time-invariant? Justify your answers.
b- (10%) Is this system causal? Is this system memory-less? Justify your answers.

Question 2:

A discrete-time LTI system is defined by its impulse response

$$h[n] = (0.8)^n u[n] + (0.8)^{n-2} u[n-2]$$

a- (15%) Find the difference equation defining this system if $x[n]$ and $y[n]$ are input and output of the system, respectively.
b- (10%) Is this system stable? Justify your answer.

Question 3:

Consider following continuous time system with band-limited input of $x(t)$ limited to $\Omega_M = 8\pi \times 10^3\, rad/\sec$. The sampling period is $T = \frac{1}{24} \times 10^{-3} \sec$ and the frequency of the cosine function is $\Omega_0 = 2\pi f_0 = 32\pi \times 10^3\, rad/\sec$. In this system, $H(j\Omega)$ is an ideal lowpass filter with cut-off frequency of $\frac{\pi}{T}$ and gain of T.

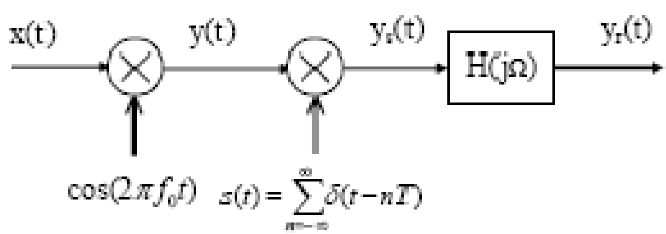
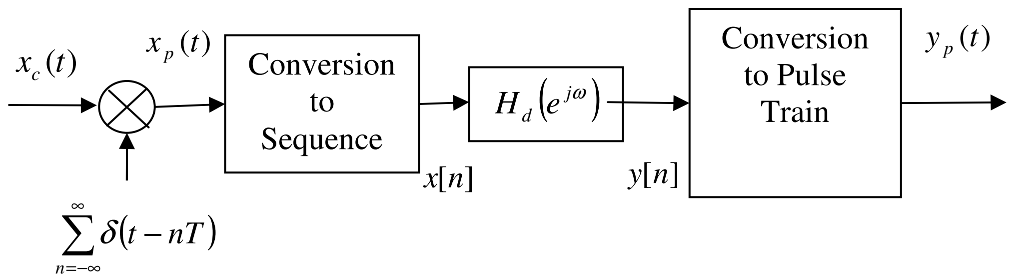


Fig. 2

a- (15%) Write expressions for $Y(j\Omega)$, $Y_s(j\Omega)$ and $Y_r(j\Omega)$ and sketch them.
b- (10%) Find the relation between $y_r(t)$ and $x(t)$.

Question 4:

The following figure shows the discrete time implementation of a continuous-time system. The band-limited signal $x_c(t)$ is input to the system which is sampled by an ideal periodic sampler each T seconds. The maximum frequency Ω_M of $x_c(t)$ is equal to $\frac{\pi}{2T}$. The output of the sampler $x_p(t)$ is converted to discrete sequence $x[n]$ and passed through the discrete system $H_d(e^{j\omega})$. The output of the digital system $y[n]$ is converted to a pulse train $y_p(t) = \sum_{n=-\infty}^{+\infty} y[n]h(t-nT)$ where $h(t) = \begin{cases} 1 & 0 < t < T \\ 0 & otherwise \end{cases}$.



a- (15%) Assume that $H_d(e^{j\omega}) = 1$ and design a non-ideal reconstruction filter to recover $x_c(t)$ from $y_p(t)$. Determine the pass band, stop band and transition band of the non-ideal reconstruction filter. Sketch the transfer function $H_r(j\Omega)$ of the reconstruction filter.

b- (10%) Assume that the discrete system is a single sample delay circuitry which means that $y[n] = x[n-1]$. Evaluate and sketch the frequency responses of $x_c(t)$, $x_p(t)$, $x[n]$, $y[n]$ and $y_p(t)$.