(5marks)

Module - 01

Q1) Construct truth tables to check whether the statement $[\sim(p \wedge q)] \rightarrow [\sim p \vee \sim q]$ is tautology, contradictory or contingent.

Solution:

p	q	$\sim p$	$\sim q$	$\sim(p \wedge q)$ (1)	$\sim p \vee \sim q$ (2)	(1) $\rightarrow$ (2)
T	T	F	F	F	F	T
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	T	T	T

from last column, all values are True.

$\therefore$ given statement $[\sim(p \wedge q)] \rightarrow [\sim p \vee \sim q]$ is tautology

Remember:

Tautology — Always True
Contradiction — Always False
Contingency — mix T & F

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(5 marks)

Q2) Construct truth tables to check whether statement $(\sim p \land q) \land (q \to p)$ is tautology, contradictory or contingent.

Solution:

p	q	$\sim p$	$\sim p \land q$	$q \to p$	$(\sim p \land q) \land (q \to p)$
T	T	F	F	T	F
T	F	F	F	T	F
F	T	T	T	F	F
F	F	T	F	T	F

From last column, All values are False

$\therefore$ given statement $(\sim p \land q) \land (q \to p)$ is contradictory (Contradiction)

(5 marks)

Q3) Construct truth table to check whether the statement
$[(p \to q) \wedge \sim q] \to p$ is tautology, contradictory or contingent.

solution:

p	q	$\sim q$	$p \to q$	$(p \to q) \wedge \sim q$	$[(p \to q) \wedge \sim q] \to p$
T	T	F	T	F	T
T	F	T	F	F	T
F	T	F	T	F	T
F	F	T	T	T	F

From last column, True and False both values are exist.

$\therefore$ given statement
$[(p \to q) \wedge \sim q] \to p$ is contingent.

5 marks

Q4). construct the truth table to check whether the statement $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \wedge q) \rightarrow r]$ is tautology, contradictory or contingent.

Solution:

p	q	r	$p \wedge q$	$q \rightarrow r$	① $p \rightarrow (q \rightarrow r)$	② $(p \wedge q) \rightarrow r$	① $\rightarrow$ ②
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	F	T	T	T	T
F	T	F	F	F	T	T	T
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

From last column, All values are True

$\therefore$ given statement $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \wedge q) \rightarrow r]$ is Tautology.

(5 marks)

Q5) prove equivalance of $(p \to q) \equiv \sim p \vee q$ using Truth Table.

Solution:

p	q	$\sim p$	$p \to q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

From last two column, which give the same value

$\therefore (p \to q) \equiv \sim p \vee q$

it is True. (logical equivalance is true)

Hence proved.

Q6) prove equivalance of
$(p \to q) \equiv (\sim q \to \sim p)$
using truth table.

Solution:

p	q	$\sim p$	$\sim q$	$p \to q$	$\sim q \to \sim p$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

From last two column, which give the same value

$\therefore (p \to q) \equiv (\sim q \to \sim p)$ it is true.

Hence proved.

(5 marks)

Q7) prove by Mathematical Induction

$$P(n) = 1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

Solution:

① Base case:

prove the formula for $n=1$

$$P(1): 1 = \frac{1(1+1)}{2}$$

$$1 = \frac{1(2)}{2}$$

$$1 = 1 \quad \text{True}$$

$P(1)$ is True ——— ①

② Inductive step

Assume the formula is true for some positive integers. K i.e $P(K)$ is True.

$$P(K): 1+2+3+\cdots+K = \frac{K(K+1)}{2} \quad \text{——— ②}$$

③ Now we want to prove ~~for~~ $P(n)$ is True for $K+1$.

$$P(K+1) = \underbrace{1+2+3+\cdots+K}+K+1$$

$$= \frac{K(K+1)}{2} + K+1$$

$$= \frac{K(K+1)+2(K+1)}{2}$$

$$= \frac{(K+1)(K+2)}{2}$$

$$P(K+1) = \frac{(K+1)\cdot(K+1+1)}{2}$$

Hence the formula is True for $K+1$
i.e $P(K+1)$ it is True.

Conclusion:
By Mathematical Induction the formula

$$P(n) : 1+2+3+\cdots+n = \frac{n(n+1)}{2}$$

is True for all natural Numbers.

Hence proved.

(5 marks)

Q8) prove by mathematical Induction

$$p(n): 1^3 + 2^3 + 3^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

Solution: using mathematical Induction we want to prove formula

$$1^3 + 2^3 + \cdots + n^3 = \frac{n^2(n+1)^2}{4}$$

① Base case:

prove the formula for $n=1$

$$P(1) = 1^3 = \frac{1^2(1+1)^2}{4}$$

$$1 = \frac{4}{4}$$

$$1 = 1 \quad \text{True}$$

P(1) is True.

② Inductive Step:

Assume the formula is true for some positive integer K.

i.e P(K) is True.

$$P(K): 1^3 + 2^3 + \cdots + K^3 = \frac{K^2(K+1)^2}{4}$$

③ Now we want to prove p(n) is true for K+1.

$$P(K+1) = \underbrace{1^3+2^3+\cdots+K^3} + (K+1)^3$$

$$= \frac{K^2(K+1)^2}{4} + (K+1)^3$$

$$= \frac{K^2(K+1)^2 + 4(K+1)^3}{4}$$

$$= \frac{\{K^2+4(K+1)\}(K+1)^2}{4}$$

$$= \frac{(K^2+4K+4)(K+1)^2}{4}$$

$$= \frac{(K+2)^2(K+1)^2}{4}$$

$$P(K+1) = \frac{(K+1)(K+1+1)^2}{4}$$

Hence the formula is True for $K+1$.
i.e $P(K+1)$ is True.

Conclusion:
By mathematical Induction the Formula
$$P(n): 1^3+2^3+\cdots+n^3 = \frac{n^2(n+1)^2}{4}$$
is True for all natural Numbers.

Hence proved.

(5marks)

Q9) prove by mathematical induction
$3^n - 2n - 1$ is divisible by 4.

Solution: using mathematical induction we want to prove
$P(n): 3^n - 2n - 1$ is divisible by 4

① Base case:
prove that is true for $n=1$

$P(n): 3^n - 2n - 1$
$= 3^1 - 2(1) - 1 \quad n=1$
$= 3 - 3$
$= 0$

0 is divisible by 4

$\therefore$ it is true for $n=1$

② Inductive step:
Assume that it is true for some positive integer K

i.e $(3^K - 2K - 1)$ is divisible by 4
$P(K)$ is true
$\Rightarrow 3^K - 2K - 1 = 4m \Rightarrow \boxed{3^K = 4m + 2K + 1}$

③ Now we want to prove that
$P(K+1)$ is true.
mean $(3^{K+1} - 2(K+1) - 1)$ is divisible by 4.

$$P(K+1) = 3^{K+1} - 2(K+1) - 1$$

$$= 3^K \cdot 3^1 - 2K - 2 - 1$$

$$= 3(4m + 2K + 1) - 2K - 3$$

$$= 12m + 6K + \cancel{3} - 2K - \cancel{3}$$

$$= 12m + 4K$$

$$= 4(3m + K)$$

where $n = 3m + K \in N$

$$P(K+1) = 4n$$

$\therefore$ $P(K+1)$ is divisible by 4.

Conclusion:

From all above steps and by the principle of Mathematical Induction $P(n)$ is true for all Natural Numbers.

i.e. $(3^n - 2n - 1)$ is divisible by 4, for all Natural Numbers.

Hence proved.

(5 marks)

Q10) prove by mathematical Induction that $7^n - 1$ is divisible by 6 for $n \in N$.

solution: using mathematical Induction we want to prove:
$7^n - 1$ is divisible by 6.

① Base case:
prove that it is True for $n = 1$

$P(n) = 7^n - 1$

$P(1) = 7^1 - 1$
$= 6$

6 is divisible by 6
$\therefore$ it is True for $n=1$

② Inductive step:
Assume that it is True for some positive integer K.

i.e $(7^K - 1)$ is divisible by 6
P(K) is True

$\Rightarrow 7^K - 1 = 6m$
$\Rightarrow \boxed{7^K = 6m + 1}$

③ Now we want to prove that $p(k+1)$ is True, mean

$(7^{k+1}-1)$ is divisible by 6

$$p(k+1) = 7^{k+1}-1$$
$$= 7 \cdot 7^{k} - 1$$
$$= 7(6m+1) - 1$$
$$= 42m + 7 - 1$$
$$= 42m + 6$$
$$= 6(7m+1)$$
$$p(k+1) = 6n$$

where $n = 7m+1 \in N$

$\therefore$ $p(k+1)$ is divisible by 6

Conclusion:

From all above steps and by the principle of Mathematical Induction $p(n)$ is true for all Natural Numbers.

i.e (7^n-1) is divisible by 6 for all Natural Numbers.

Hence proved.

1 marks questions

Module – 01

Q1) State De morgan's law

Solution: Statement:

The negation of a conjunction is equivalent to the disjunction of the negation

Symbolic representation:

$$\sim(p \wedge q) \equiv \sim p \vee \sim q$$
$$\sim(p \vee q) \equiv \sim p \wedge \sim q$$

Q2) Explain the principle of Mathematical Induction

Solution:

principle of Mathematical induction

Step 1 : prove $p(1)$ is True

↓

Step 2 : Assume $p(K)$ is True for some $n=K$

↓

Step 3 : prove $p(K+1)$ is True

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Q3) what are the types of quantifiers?

Solution:
In logic, there are two main types of quantifiers
① universal quantifiers
$\forall$ for all

②. Existential quantifier
$\exists$ there exists

Q4) what is Associative law for conjunction?

solution:
The associative Law of conjunction state that the grouping of statements connected by the logical '$\wedge$' AND (conjunction) does not affect the truth value of the overall statement.

e.g. let p, q, r propositions

$$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$$

Q5) what is the associative Law for disjunction ?

solution:

The associative Law for disjunction state that the grouping of propositions in a disjunction (using the 'or' operator) does not affect the truth Value of the overall statement:

e.g. let p, q, r propositions

$(p \vee q) \vee r \equiv p \vee (q \vee r)$

Q6) what is distributive law of disjunction over conjunction ?

Solution:

The Distributive Law of disjunction over conjunction is expressed as

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Q7). what is the distributive law of Conjunction over disjunction ?

solution :

The distributive law of conjunction over disjunction is expressed as

$$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$$

Q8). what is the idempotent law for conjunction ?

Solution:

The idempotent law in conjunction ('∧' AND) in propositional logic state that

$$p \wedge p \equiv p$$

Q9) what is the idempotent law for disjunction ?

solution:

The Idempotent law in disjunction ('∨' OR) in propositional logic state that.

$$p \vee p \equiv p$$

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Q10) what is law of Identity ?

Solution:

The law of identity is a fundamental principle in logic state that something is identical to itself.

e.g. $P \wedge T \equiv P$, $P \wedge F \equiv F$
& $P \vee T \equiv T$, $P \vee F \equiv P$
it is law of identity

Q12) If A and B are true statements and X is false statement find $A \vee (B \wedge X)$.

Solution

$$A \vee (B \wedge X) \equiv T \vee (T \wedge F)$$
$$\equiv T \vee F$$
$$\equiv T$$

$\therefore A \vee (B \wedge X)$ is true.

10 marks

Q1) using the Law of logic simplyfy $(p\wedge \sim q)\vee q\vee(\sim p\wedge q)$

Solution:

$$(p\wedge\sim q)\vee q\vee(\sim p\wedge q)$$
$\equiv (p\vee q)\wedge(\sim q\vee q)\vee(\sim p\wedge q)$ Distributive law
$\equiv (p\vee q)\wedge T\vee(\sim p\wedge q)$ complement law
$\equiv (p\vee q)\vee(\sim p\wedge q)$ identity law
$\equiv p\vee(q\vee(\sim p\wedge q))$ Distributive law
$\equiv p\vee((q\vee\sim p)\wedge(q\vee q))$ idempotent law
$\equiv p\vee((q\vee\sim p)\wedge q)$
$\equiv p\vee(q\wedge(q\vee\sim p))$ Absorption law
$\equiv p\vee q$

$\therefore (p\wedge\sim q)\vee q\vee(\sim p\wedge q)\equiv p\vee q$

10 marks

Q2) using law of logic prove that
$(\sim q \land (p \to q)) \to \sim p$

Solution:

$(\sim q \land (p \to q)) \to \sim p$

$\equiv (\sim q \land (\sim p \lor q)) \to \sim p$ Conditional law

$\equiv ((\sim q \land \sim p) \lor (\sim q \lor q)) \to \sim p$ Distributive law

$\equiv ((\sim q \land \sim p) \lor F) \to \sim p$ Complement law

$\equiv (\sim q \land \sim p) \to \sim p$ Identity law.

$\equiv \sim(\sim q \land \sim p) \lor \sim p$ Conditional law

$\equiv (q \lor p) \lor \sim p$ Demorgan's law.

$\equiv q \lor (p \lor \sim p)$ Associative law

$\equiv q \lor T$ Complement law

$\equiv T$ identity law.

True.

$\therefore (\sim q \land (p \to q)) \to \sim p$ it is True.

Hence proved.

10marks

Q3) using law of logic simplify.

$$(p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q))$$

Solution:

$(p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q))$

$\equiv \sim (p \wedge q) \vee (\sim p \vee (\sim p \vee q))$ Conditional law

$\equiv \sim p \vee \sim q \vee (\sim p \vee (\sim p \vee q))$ Demorgan's law

$\equiv \sim p \vee \sim q \vee \sim p \vee \sim p \vee q$ Associative law

$\equiv \sim p \vee \sim p \vee \sim p \vee \sim q \vee q$ Commutative law

$\equiv \sim p \vee \sim q \vee q$ Idempotent law

$\equiv \sim p \vee (\sim q \vee q)$ Associative law

$\equiv \sim p \vee T$ complement law

$\equiv T$ identity law.

True.

$\therefore (p \wedge q) \rightarrow (\sim p \vee (\sim p \vee q)) \equiv$ True.

Important laws :

1) Idempotent law. $p \wedge p \equiv p$ $p \vee p \equiv p$
2) Commutative law $p \vee q \equiv q \vee p$, $p \wedge q \equiv q \wedge p$
3) Associative law $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$
$p \vee (q \vee r) \equiv (p \vee q) \vee r$
4) Distributive law
$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

5) De Morgan's law
$\sim(p \wedge q) \equiv \sim p \vee \sim q$
$\sim(p \vee q) \equiv \sim p \wedge \sim q$

6) Identity law $p \wedge T \equiv p$, $p \wedge F \equiv F$
& $p \vee T \equiv T$, $p \vee F \equiv p$

7) Complement law $p \wedge \sim p \equiv F$, $p \vee \sim p \equiv T$

8) Absorption law $p \vee (p \wedge q) \equiv p$
$p \wedge (p \vee q) \equiv p$

9) Conditional law $p \rightarrow q \equiv \sim p \vee q$

10) Bi-conditional law

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$
$p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$