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MSc(MATH),BED,GATE,MH SET

Module-03
posets and Lattice

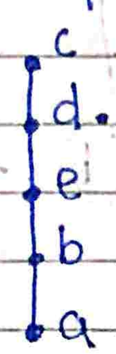
partial order Relations
poset
supremum and Infimum
Hasse Diagram
Lattice
Types of Sub-lattice
Distributive lattice.

1 mark
Q1) Define partial order with an example.

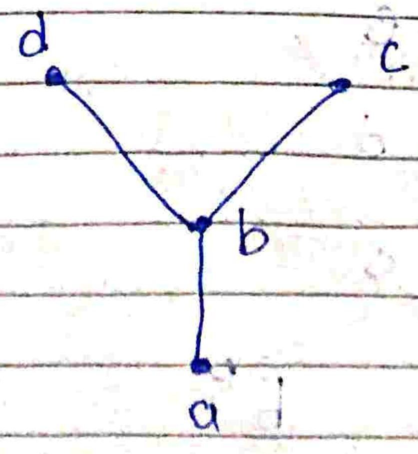
1 mark
Q2) Define Distributive lattice

1 marks
Q3) Define lattice

Q4) find the maximal and minimal elements in the given poset if exist.
1 mark



1 mark
Q5) find the maximal and minimal elements in the given poset, if exist.



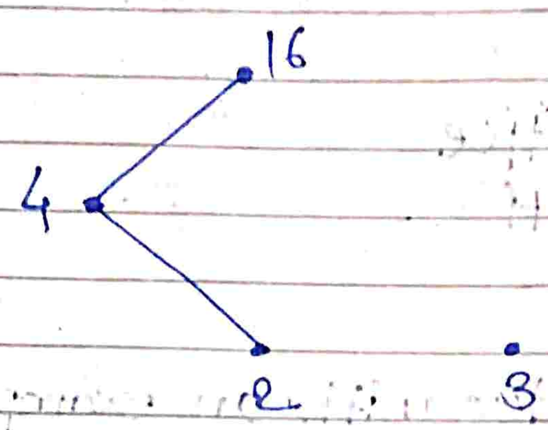
Page No.
Date

1 mark

Q6) Define sub-lattice.

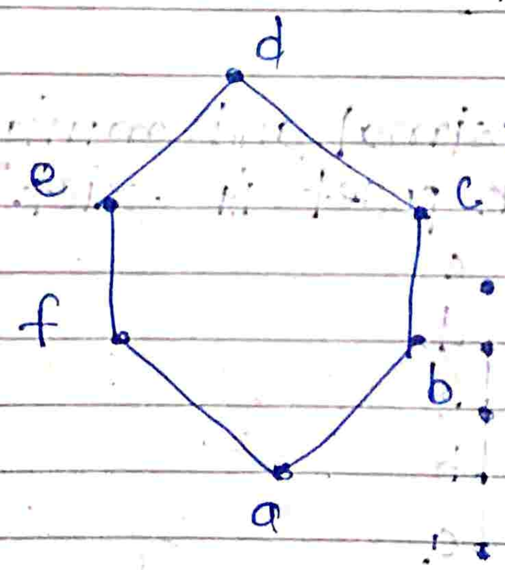
Q7) find the maximal and minimal elements in the given poset, if exist.

1 mark



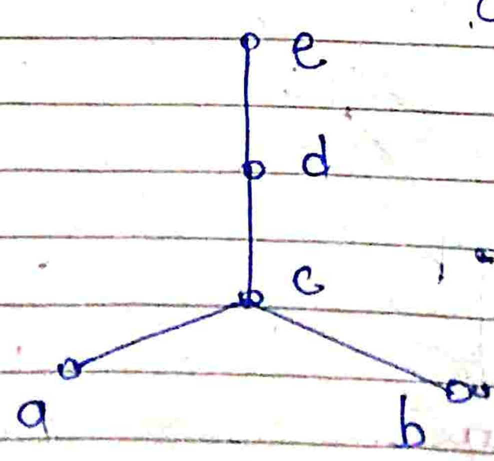
1 mark

Q8) find infimum and supremum for the poset whose Hasse digram is given Below.



1 mark

Q9) find infimum and Supremum for the poset whose Hasse Digram is given Below.



1 mark

Q10) what is the supremum of a set in a poset ?

1 mark.

Q11) What is the infimum of a set in a poset ?

1 mark

Q12) state whether the following Hasse Digram represent lattice.

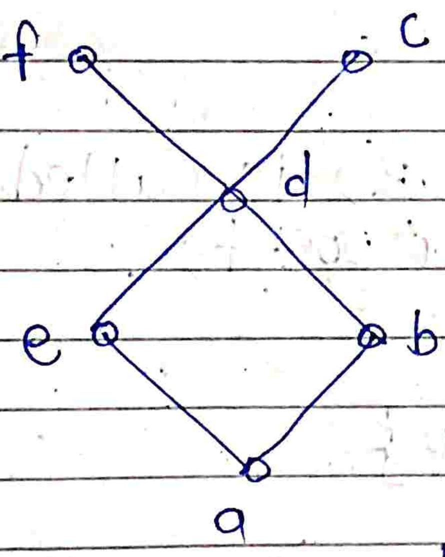


fig 1

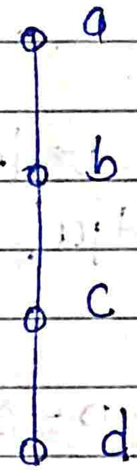


fig 2

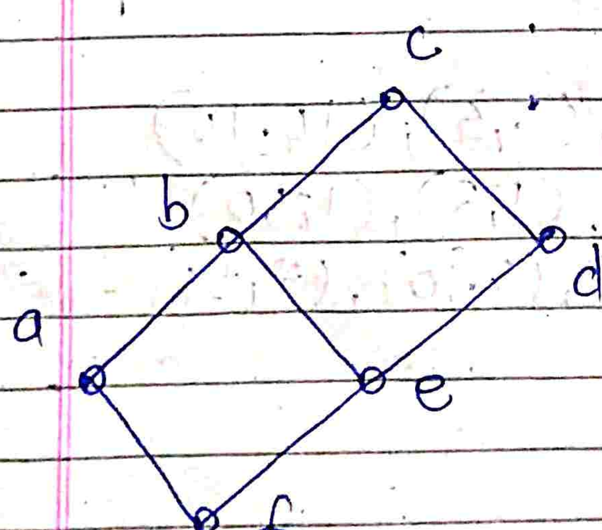


fig 3

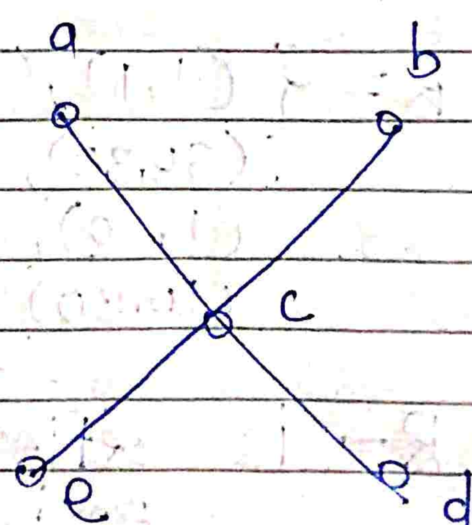


fig 4.

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1 mark

Q1) Define partial order relation with an example.

Solution:

partial order:

Definition:

A binary relation R on a non-empty set A is a partial order if

R is reflexive

antisymmetric and

Transitive

Note:

The ordered pair (A, R) is called a partial ordered set (poset).

e.g. ① $P = \{1, 3, 5, 10, 30\}$

$\leq$ - division

$(P, \leq)$ — partial order Relation

$R = \{(1,1), (3,3), (5,5), (10,10)$
$(30,30), (1,3), (1,5), (1,30)$
$(1,10), (3,30), (5,10), (5,30)$
$(10,30)\}$

R — is reflexive

Antisymetric

Transitive.

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Date

Q2) Define Distributive lattice.

Solution:

Distributive lattice:

Definition:
A lattice is said to be distributive if it satisfies distributive property.

i) $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$
ii) $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

or

if every element has maximum 1 complement then lattice is distributive.

e.g.

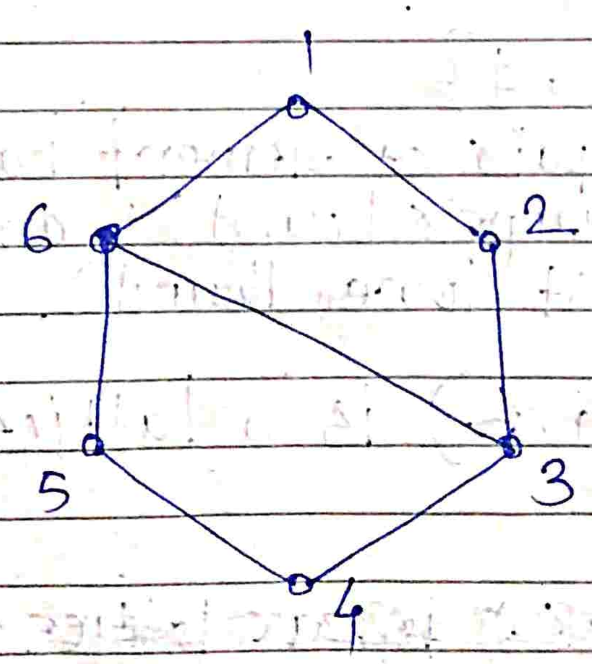


fig. Distributive Lattice.

Q3) Define lattice.

Solution:

lattice:

Definition:

A lattice is a poset $(L, \leq)$ in which every subset $\{a, b\}$ of L has a least upper bound and a greatest lower bound.

e.g.

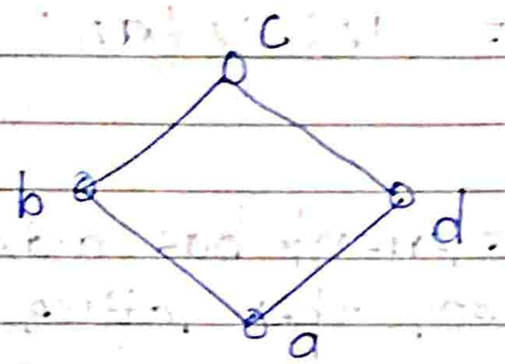


fig Lattice

$A = \{a, b, c, d\}$

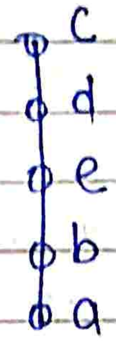
Here every pair of element has a lub (Least upper bound) and glb (greatest lower bound).

Hence $(A, \leq)$ is a lattice.

Q4.

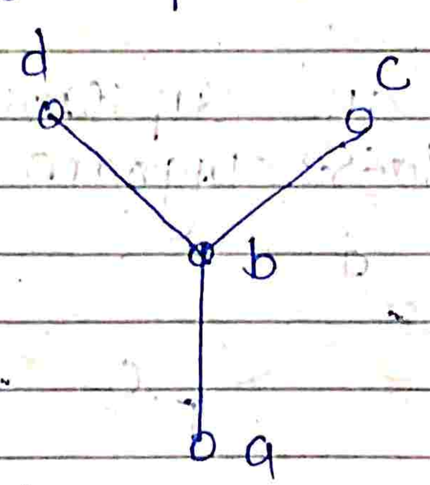
~~e.g. poset (P, ≤) is a lattice iff~~

Q4) find the maximal and minimal elements in the given poset if exist



Solution: maximal element – c
minimal element – a

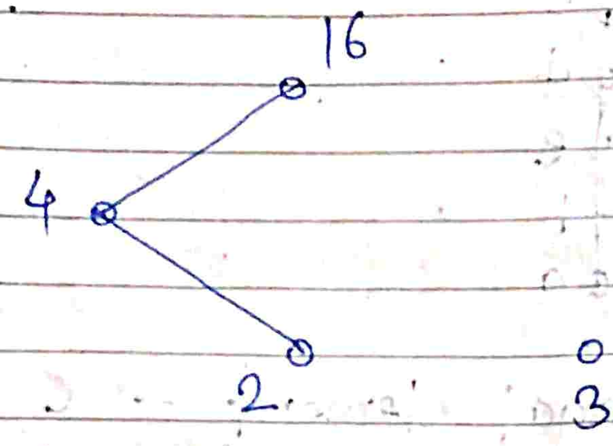
Q5) find the maximal and minimal elements in the given poset if exist.



Solution:

maximal element – d, c
minimal element – a

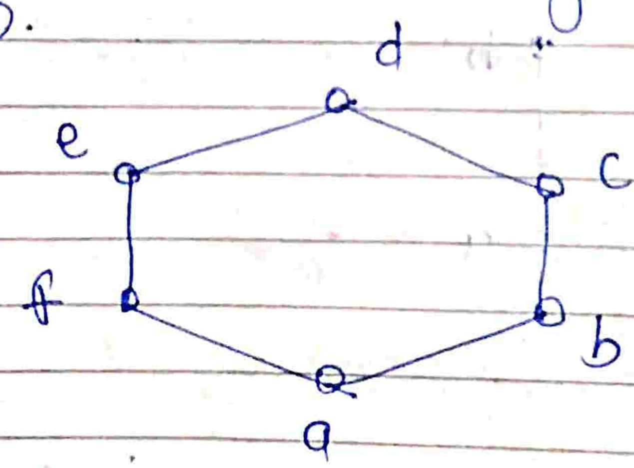
Q6). find the maximal and minimal elements in the given poset if exist.



solution:

maximal element — 16, 3
minimal element — 2, 3

Q7) find infimum and supremum for the poset whose Hasse Digram is given Below.

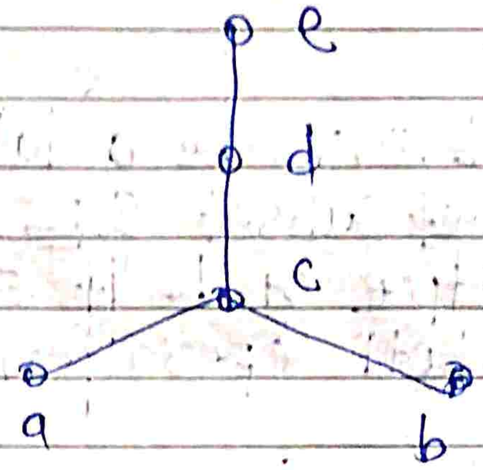


solution:

maximal element — d
minimal element — a

supremum — d
infimum — a

Q8) find infimum and supremum for the poset whose Hasse Digram is given below.



solution:

maximal element — e
minimal element — a, b

Supremum — e
infimum — not exist.

Q9) Define Sublattice:

Solution:

Sublattice:

Definition:

Let $(L, \wedge, \vee)$ be a lattice. A non empty ~~set~~ subset $S \sqsubseteq L$, is called Sublattice of L if S is closed under both operation $\wedge$ and $\vee$.

or In otherwords:

S is called sublattice of L iff

$a, b \in S$

$\Rightarrow a \wedge b, a \vee b \in S$.

e.g. ① Let $A = \{a, b, c\}$

$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$

$(P(A), \sqsubseteq)$ — Lattice

$S_1 = \{\{a\}, \{b\}, \{a,b\}, \phi\}$

i.e. $S_1 \sqsubseteq P(A)$

$\therefore S_1$ is Sublattice of the lattice $(P(A), \sqsubseteq)$.

Q10) what is the supremum of a set in a poset?

Solution:

In partially ordered set (poset):

Supremum (Least upper Bound)

The supremum of a set A, denoted as sup(A) or $\vee A$ that is the least element is greater than or equal to every element of A.

eg ① For the poset $A = \{1, 2, 3, 4, 6, 12\}$ with the partial order relation division

Supremum:

The supremum of A is the least element that is divisible by all elements of A.

$\therefore$ $Sup(A) = 12$

$\vee A = 12$

Q11) what is infimum of a set in a poset.

Solution:

In partially ordered set (poset):
Infimum (Greatest lower Bound)
the infimum of a set A, denoted as inf(A) or $\wedge A$, is the greatest element that is less than or equal to every element of A.

e.g. ① For the poset $A = \{2, 4, 6, 8, 10\}$ with the relation division

Infimum
The infimum of A is the greatest element that divides all elements of A.

$\therefore \quad inf(A) = 2$
$\wedge A = 2$

1 mark

Q12) state whether the following Hasse Digram represent lattice

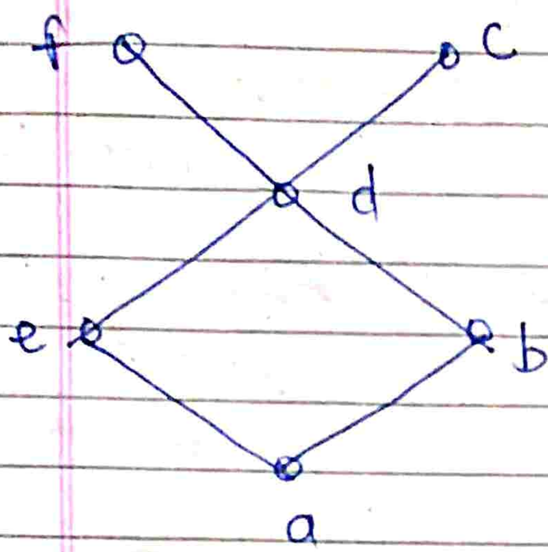


fig:1

Ans: not lattice

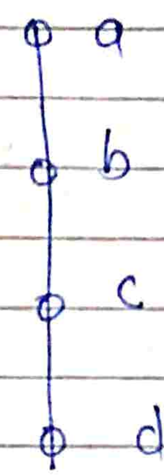


fig:2

Ans: Lattice

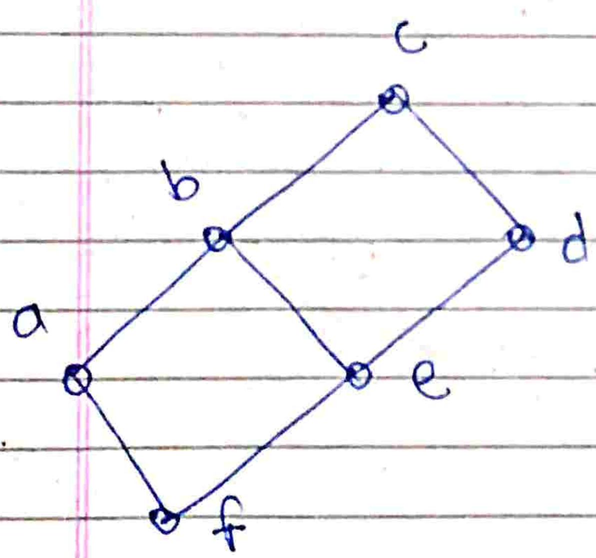


fig:3

Ans: Lattice

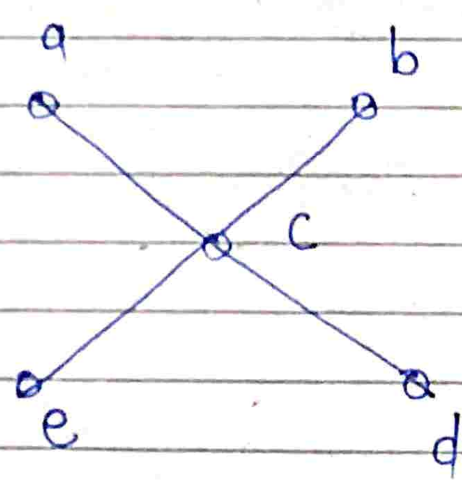


fig:4

Ans: ~~non~~ not Lattice

5marks

Q1) consider a set $A = \{a, b, c, d\}$ and R be the relation on A whose matrix is

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

show that (A, R) is a poset. Also contruct digraph of poset.

5marks

Q2) consider a set $A = \{1, 2, 3, 6, 8, 12\}$ and R be the relation of 'a divides b' show that (A, R) is a poset. Also Construct digraph of the poset.

5marks

Q3) consider a set $A = \{1, 2, 3, 4\}$ and $R = \{(1,1), (1,2), (2,2), (2,4), (1,3), (3,3), (1,4), (4,4)\}$ Show that (A, R) is a poset. Also construct digraph of the poset.

5marks

Q4) consider a set $A = \{1, 2, 3, 4\}$ and $R = \{(1,1), (2,2), (3,3), (4,4), (1,3), (1,4), (2,3), (2,4), (3,4)\}$ show that (A, R) is a poset. Also construct digraph of the poset.

5 marks

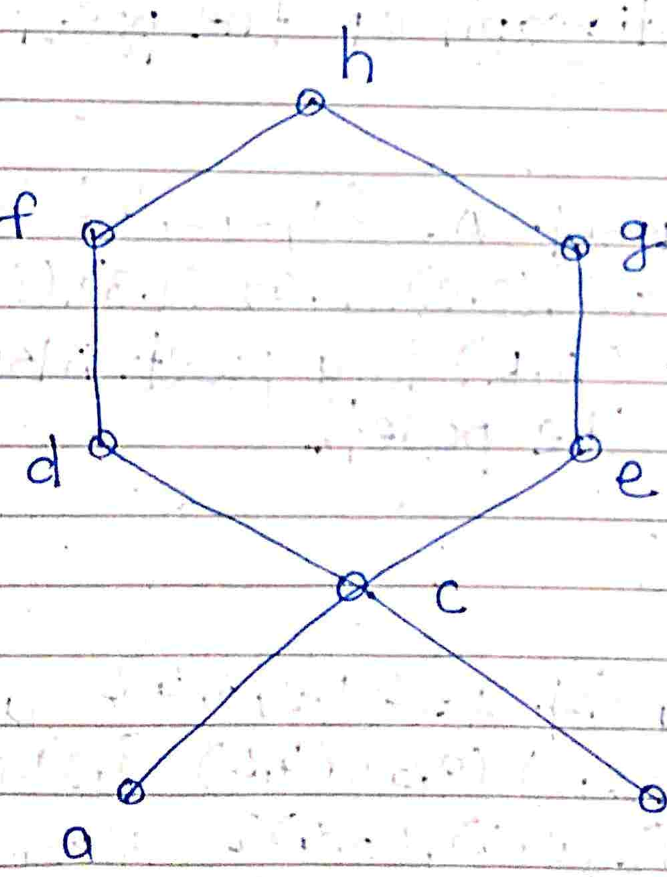
Q5) Let $A = \{3, 9, 12\}$ and
$R = \{(3,3), (3,9), (3,12), (9,9), (12,12)\}$
Construct the Hasse diagram of R.

5 marks

Q6) Consider the poset $A = \{a, b, c, d, e, f, g, h\}$ whose Hasse diagram is shown below. Find all upper and lower bounds of the following subsets of A. Also find GLB & LUB.

i) $B_1 = \{a, b\}$

ii) $B_2 = \{d, e\}$



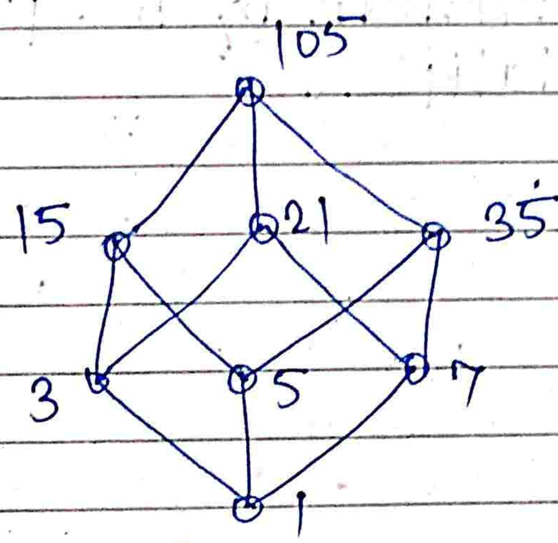
fig

5 marks

Q7) Consider the poset $A = \{1, 3, 5, 7, 15, 21, 35, 105\}$ whose Hasse diagram is given below. find all upper and lower bound of the following subset of A. Also find GLB & LUB.

i) $B_1 = \{3, 7\}$

ii) $B_2 = \{3, 5\}$



fig

5 marks

Q8) Let $A = \{1, 2, 3, 4, 5\}$ and
$R = \{(1,1), (1,2), (1,3), (1,4), (1,5), (2,2), (2,4), (3,3), (3,5), (4,4), (5,5)\}$
Construct the Hasse diagram of R.

5 marks

Q9) Let $L = \{1, 2, 3, 6\}$ and R be the relation 'a divides b'. prove that L is Lattice.

5marks

Q10) Draw the Hasse diagram of D_4.
Check if it is a lattice.

5marks

Q11) Draw the Hasse diagram of D_8.
Check if it is a lattice.

5marks

Q12) Let $L = \{1, 2, 3, 5, 30\}$ and R be the relation 'a divisible by b'. prove that L is a lattice.

5 marks

Q1) Consider a set $A = \{a, b, c, d\}$ and R be the relation on A whose matrix is

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

show that (A, R) is a poset. Also construct digraph of poset.

Solution: $A = \{a, b, c, d\}$

$$M_R = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 0 & 1 & 1 \\ b & 0 & 1 & 1 & 1 \\ c & 0 & 0 & 1 & 1 \\ d & 0 & 0 & 0 & 1 \end{array}$$

$$R = \{(a,a), (b,b), (c,c), (d,d), (a,c), (b,c), (a,d), (b,d), (c,d)\}$$

first we want to show that R is partial order Relation that mean

R is Reflexive

R is Antisymmetric and

R is Transitive

i) since R is Reflexive

$(a,a), (b,b), (c,c), (d,d) \in R$.

ii) For Antisymmetric

$(a,c) \in R \longrightarrow (c,a) \notin R$
$(b,c) \in R \longrightarrow (c,b) \notin R$
$(a,d) \in R \longrightarrow (d,a) \notin R$
$(b,d) \in R \longrightarrow (d,b) \notin R$
$(c,d) \in R \longrightarrow (d,c) \notin R$

$\therefore$ R is Antisymmetric

iii) For Transitive:

(a,a) and $(a,c) \in R \rightarrow (a,c) \in R$
(a,a) and $(a,d) \in R \rightarrow (a,d) \in R$
(b,b) and $(b,c) \in R \rightarrow (b,c) \in R$
(b,b) and $(b,d) \in R \rightarrow (b,d) \in R$
(c,c) and $(c,d) \in R \rightarrow (c,d) \in R$
(a,c) and $(c,c) \in R \rightarrow (a,c) \in R$
(a,c) and $(c,d) \in R \rightarrow (a,d) \in R$
(b,c) and $(c,c) \in R \rightarrow (b,c) \in R$
(b,c) and $(c,d) \in R \rightarrow (b,d) \in R$
(a,d) and $(d,d) \in R \rightarrow (a,d) \in R$
(b,d) and $(d,d) \in R \rightarrow (b,d) \in R$
(c,d) and $(d,d) \in R \rightarrow (c,d) \in R$

$\therefore$ R is Transitive.

R is Reflexive
Antisymmetric
Transitive
$\Rightarrow$ R is partial order Relation

Hence (A, R) is poset.
proved.

Digraph:

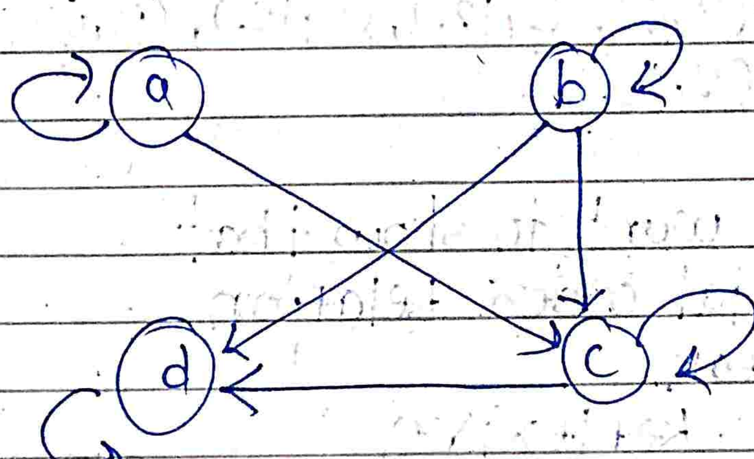


fig: Digraph of poset (A, R).

5 marks

Q2) Consider a set $A = \{1, 2, 3, 6, 8, 12\}$ and R be the relation of 'a divides b'. Show that (A, R) is poset. Also Construct Digraph of the poset.

Solution:

$A = \{1, 2, 3, 6, 8, 12\}$

R — Relation 'a divides b'

$R = \{(1,1), (2,2), (3,3), (6,6), (8,8), (12,12), (1,2), (1,3), (1,6), (1,8), (1,12), (3,6), (3,12), (6,12), (2,6), (2,8), (2,12)\}$

first we want to show that R is partial order Relation that mean

R is Reflexive
R is Antisymmetric
R is Transitive

i) Since

$(1,1), (2,2), (3,3), (6,6), (8,8), (12,12) \in R$

$\therefore$ R is Reflexive

ii) For antisymmetric

$(1,2) \in R \longrightarrow (2,1) \notin R$

$(1,3) \in R \longrightarrow (3,1) \notin R$

$(1,6) \in R \longrightarrow (6,1) \notin R$

$(1,8) \in R \longrightarrow (8,1) \notin R$

$(1,12) \in R \longrightarrow (12,1) \notin R$

$(3,6) \in R \longrightarrow (6,3) \notin R$

$(3,12) \in R \longrightarrow (12,3) \notin R$

$(6,12) \in R \longrightarrow (12,6) \notin R$
$(2,6) \in R \longrightarrow (6,2) \notin R$
$(2,4) \in R \longrightarrow (4,2) \notin R$
$(2,12) \in R \longrightarrow (12,2) \notin R$

$\therefore$ R is Antisymmetric

iii) For transitive
$(1,2)$ and $(2,6) \in R \longrightarrow (1,6) \in R$
Similarly,
$(1,2)$ and $(2,8) \in R \rightarrow (1,8) \in R$
$(1,2)$ and $(2,12) \in R \rightarrow (1,12) \in R$
$(1,3)$ and $(3,6) \in R \longrightarrow (1,6) \in R$
$(1,3)$ and $(3,12) \in R \longrightarrow (1,12) \in R$
$(1,6)$ and $(6,12) \in R \longrightarrow (1,12) \in R$
$(3,6)$ and $(6,12) \in R \rightarrow (3,12) \in R$
$(2,6)$ and $(6,12) \in R \rightarrow (2,12) \in R$

$\therefore$ R is Transitive.

R is Reflexive
Antisymmetric
Transitive

$\Rightarrow$ R is partial order Relation
$\therefore$ (A,R) is poset.

Hence proved.

Digraph :

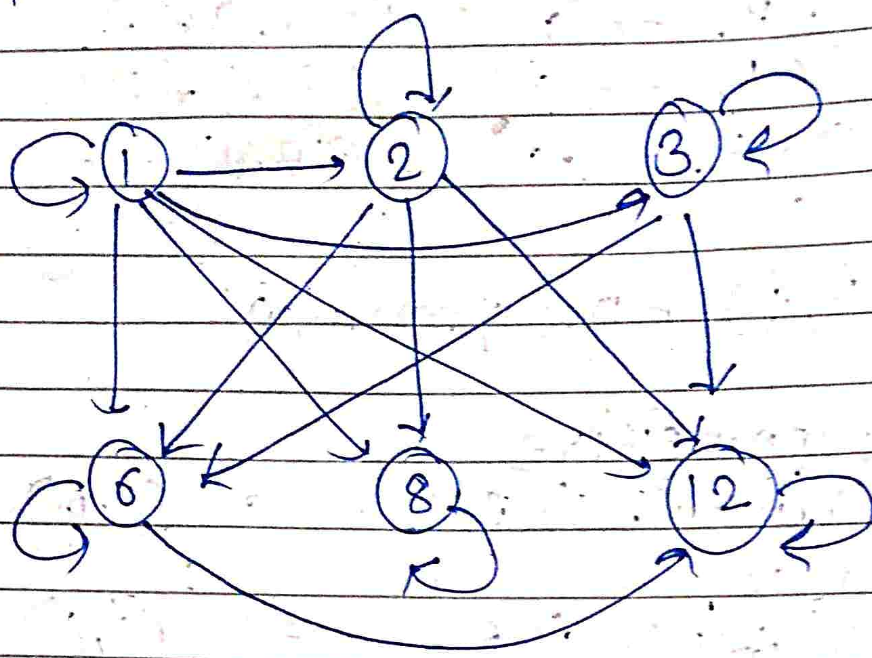


fig. Digraph of poset (A, R).

OR

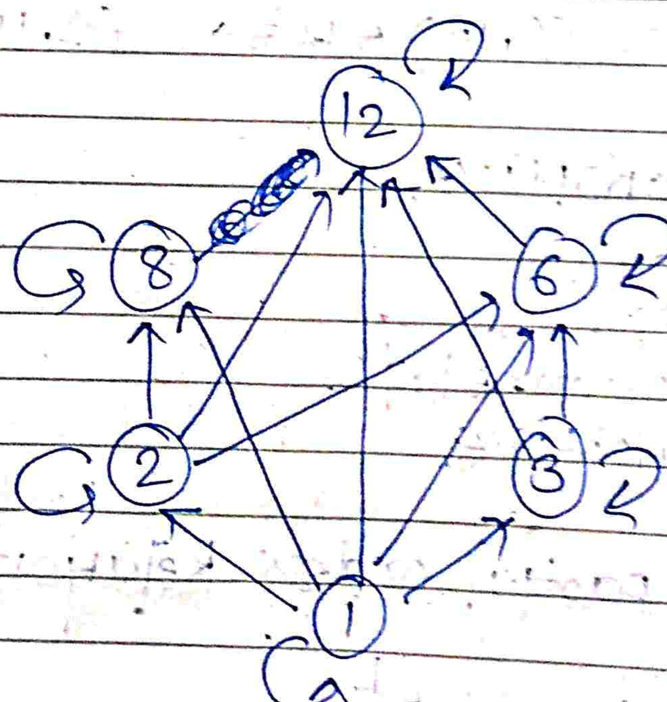


fig. Digraph of poset (A, R).

5marks

Q3) Consider a set $A = \{1,2,3,4\}$ and
$R = \{(1,1),(2,2),(3,3),(4,4),(1,3),(1,4),$
$(2,3),(2,4),(3,4)\}$
Show that (A,R) is poset. Also
Construct the digraph of poset.

Solution: $A = \{1,2,3,4\}$
$R = \{(1,1),(2,2),(3,3),(4,4),(1,3),(1,4),$
$(2,3),(2,4),(3,4)\}$

First we want to show that
R is partial order Relation
that mean
R is Reflexive
R is Antisymmetric
R is Transitive

i) since
$(1,1),(2,2),(3,3),(4,4) \in R$
$\therefore$ R is Reflexive

ii) For Antisymmetric
$(1,3) \in R \rightarrow (3,1) \notin R$
$(1,4) \in R \rightarrow (4,1) \notin R$
$(2,3) \in R \rightarrow (3,2) \notin R$
$(2,4) \in R \rightarrow (4,2) \notin R$
$(3,4) \in R \rightarrow (4,3) \notin R$

$\therefore$ R is Antisymmetric

iii) For Transitive:

$(1,3)$ and $(3,4) \in R \rightarrow (1,4) \in R$

similarly

$(2,3)$ and $(3,4) \in R \rightarrow (2,4) \in R$

$\therefore$ R is Transitive.

$\therefore$ R is Reflexive

R is Symmetric

R is Transitive

$\Rightarrow$ R is partial order Relation

$\therefore$ (A, R) is poset

Hence proved.

Digraph.

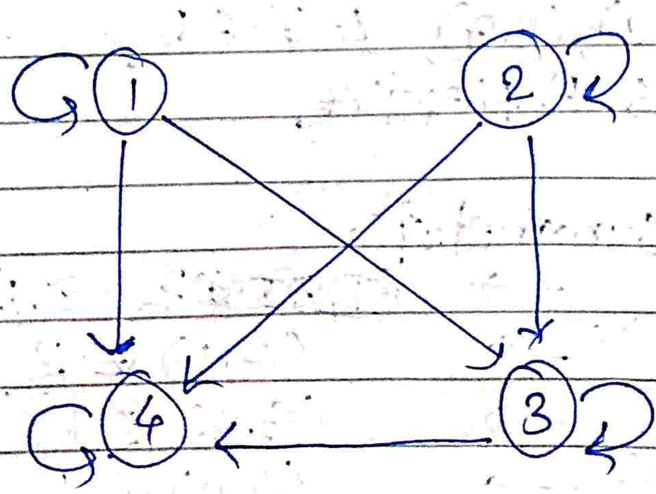


fig: Digraph of poset (A, R).

5marks

Q4). Consider a set $A=\{1,2,3,4\}$ and
$R=\{(1,1),(1,2),(2,2),(2,4),(1,3),(3,3),(1,4),(4,4)\}$
Show that (A,R) is poset.
Also construct digraph of the poset.

Solution:
$A=\{1,2,3,4\}$
$R=\{(1,1),(1,2),(2,2),(2,4),(1,3),(3,3),(1,4),(4,4)\}$

first we want to show that
R is partial order Relation
that mean
R is Reflexive
R is Antisymmetric
R is Transitive

i) since
$(1,1),(2,2),(3,3),(4,4) \in R$
$\therefore$ R is Reflexive

ii). For Antisymmetric
$(1,2) \in R \longrightarrow (2,1) \notin R$
$(2,4) \in R \longrightarrow (4,2) \notin R$
$(1,3) \in R \longrightarrow (3,1) \notin R$
$(1,4) \in R \longrightarrow (4,1) \notin R$

$\therefore$ R is Antisymmetric

iii) For Transitive

(1,2) and (2,4) $\in R \rightarrow (1,4) \in R$

Similarly.

(2,4) and (4,4) $\in R \rightarrow (2,4) \in R$

(1,3) and (3,3) $\in R \rightarrow (1,3) \in R$

(1,2) and (2,2) $\in R \rightarrow (1,2) \in R$

(1,1) and (1,2) $\in R \rightarrow (1,2) \in R$

$\therefore$ R is Transitive.

$\therefore$ R is Reflexive
R is Antisymmetric
R is Transitive

$\Rightarrow$ R is partial order Relation

$\therefore$ (A, R) is poset

Hence proved.

Digraph:

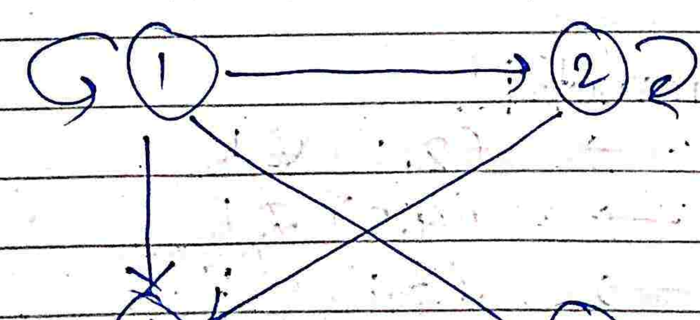


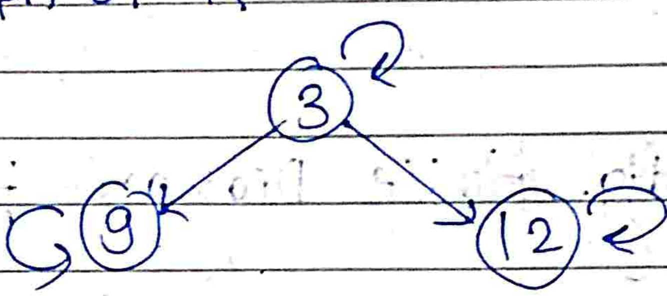
fig. Digraph of poset (A, R).

5 marks

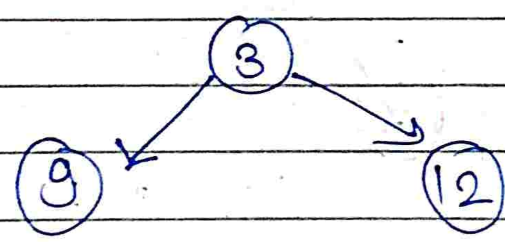
Q5) Let $A = \{3, 9, 12\}$ and
$R = \{(3,3), (3,9), (3,12), (9,9), (12,12)\}$
construct the Hasse Digram of R

Solution: $A = \{3, 9, 12\}$
$R = \{(3,3), (3,9), (3,12), (9,9), (12,12)\}$

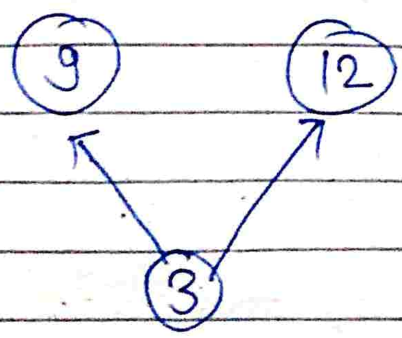
i) Digraph of R



ii) Remove Reflexive & Transitive Arrow.



iii) Arrange all arrow to uppword direction



iv) Remove Direction and Repraesented Number by dot.

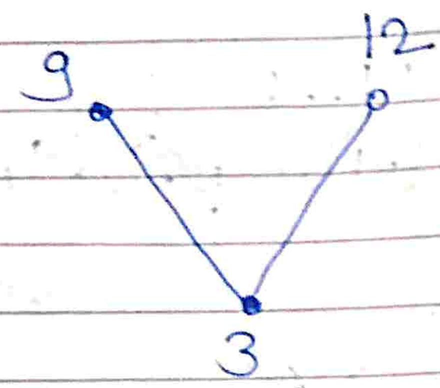
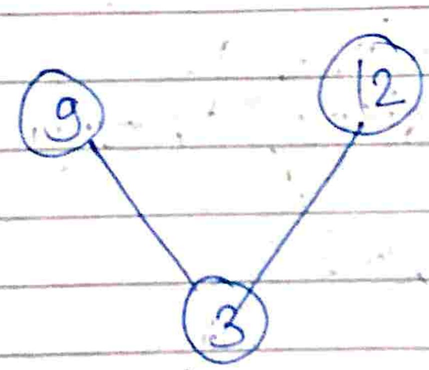


fig. Hasse Digram R

OR



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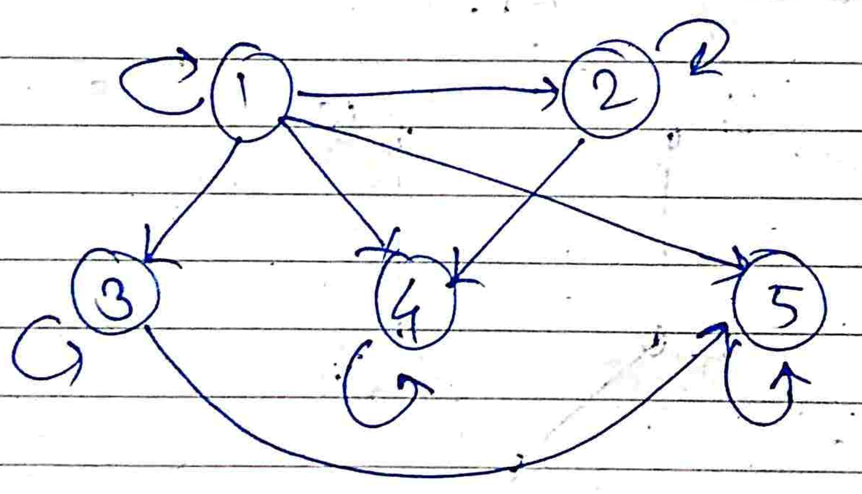
6Q8) Let $A = \{1,2,3,4,5\}$ and
$R = \{(1,1),(1,2),(1,3),(1,4),(1,5),$
$(2,2),(2,4),(3,3),(3,5),(4,4)$
$(5,5)\}$

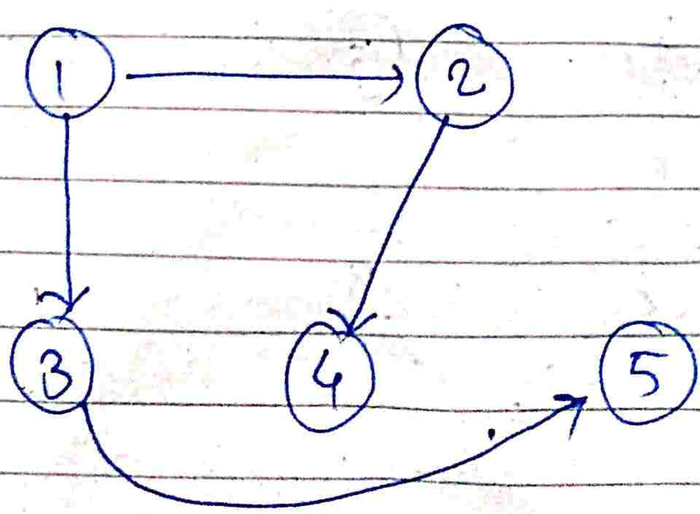
Construct the Hasse Digram of R

Solution:
$A = \{1,2,3,4,5\}$
$R = \{(1,1),(1,2),(1,3),(1,4),(1,5),(2,2)$
$(2,4),(3,3),(3,5),(4,4),(5,5)\}$

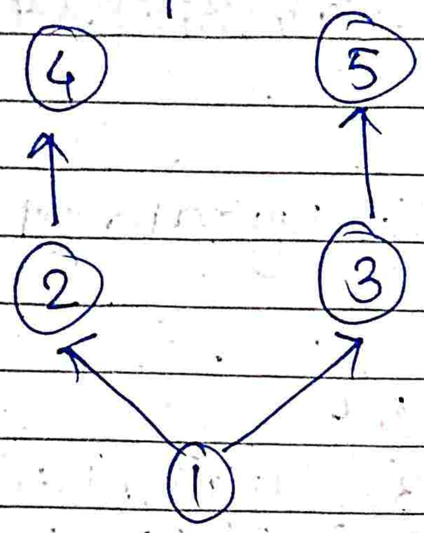
i) Draw Directed graph of R



ii) Remove Reflexive & Transitive Arrow



iii) Arrange All ~~Arrow~~ Arrow to upword direction



iv) Remove Direction and Reprasented Number by dot.

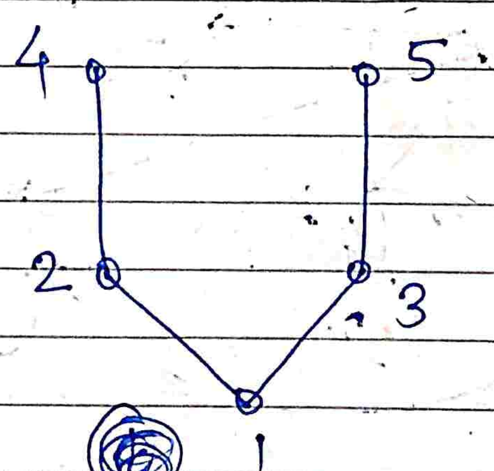
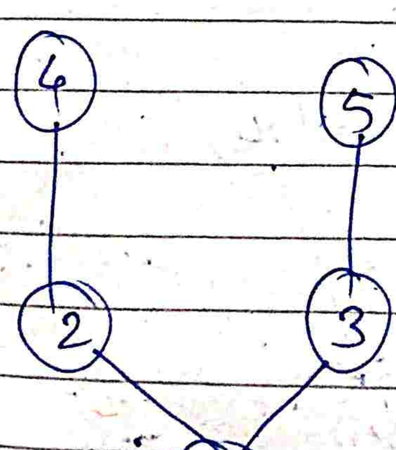


fig. Hasse Digram

OR



5 marks

Q7) Let $L = \{1, 2, 3, 6\}$ and R be the Relation 'a divides b' prove that L is Lattice.

Solution: $L = \{1, 2, 3, 6\}$

R – Relation 'a divides b'

$R = \{(1,1), (2,2), (3,3), (6,6), (1,2), (1,3), (1,6), (2,6), (3,6)\}$

first we want to prove that R – is partial order Relation

it mean

R is Reflexive

Antisymmetric

Transitive

After that By using Definition of Lattice prove (L, R) is Lattice.

i) Since

$(1,1), (2,2), (3,3), (6,6) \in R$

$\therefore$ R is Reflexive

ii) for Antisymmetric

$(1,2) \in R \longrightarrow (2,1) \notin R$

$(1,3) \in R \longrightarrow (3,1) \notin R$

$(1,6) \in R \longrightarrow (6,1) \notin R$

$(2,6) \in R \longrightarrow (6,2) \notin R$

$(3,6) \in R \longrightarrow (6,3) \notin R$

$\therefore$ R is Antisymmetric.

iii) For Transitive

(1,2) and (2,6) ∈ R ⟶ (1,6) ∈ R

Similarly

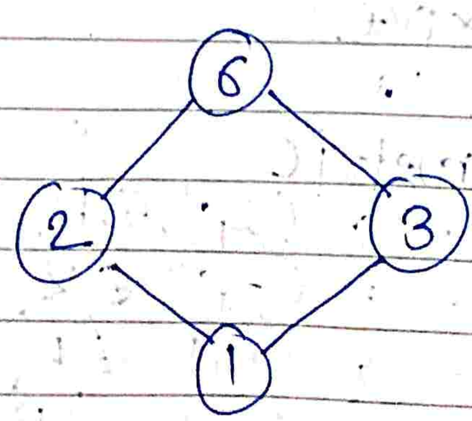
(1,3) and (3,6) ∈ R ⟶ (1,6) ∈ R

∴ R is Transitive

∴ R is Reflexive
R is Antisymmetric
R is Transitive

(L, R) —— poset.

pair	glb ∧ meet	lub join ∨
1, 2	1	2
1, 3	1	3
1, 6	1	6
2, 6	2	6
3, 6	3	6



Hasse Digram.

In poset
(L, R) each pair glb & lub are exist
Therefore (L, R) is Lattice.

$\therefore$ L — Lattice.

Hence prooved.

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Q8). Let $L = \{1, 2, 3, 5, 30\}$ and R be the Relation 'a divisible by b' prove that L is Lattice.

Solution:

$L = \{1, 2, 3, 5, 30\}$

R- Relation
a divisible by b.

$R = \{(1,1), (2,2), (3,3), (5,5), (30,30)$
$(30,5), (30,3), (30,2), (30,1),$
$(5,1), (3,1), (2,1)\}$

first we want to prove that R is partial order Relation
it mean.
R is Reflexive
Antisymmetric
Transitive

After that By using Definition of Lattice prove that (L, R) is lattice

i) since
$(1,1), (2,2), (3,3), (5,5), (30,30) \in R$
$\therefore$ R is Reflexive

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ii) For Antisymmetric
(30,5) ∈ R ⟶ (5,30) ∉ R
(30,3) ∈ R ⟶ (3,30) ∉ R
(30,2) ∈ R ⟶ (2,30) ∉ R
(30,1) ∈ R ⟶ (1,30) ∉ R
(5,1) ∈ R ⟶ (1,5) ∉ R
(3,1) ∈ R ⟶ (1,3) ∉ R
(2,1) ∈ R ⟶ (1,2) ∉ R

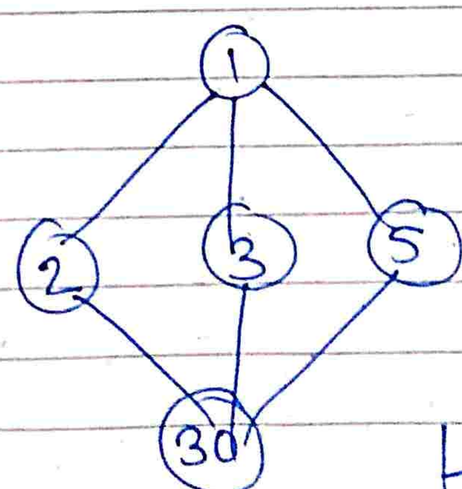
∴ R is Antisymmetric

iii) For Transitive
(30,5) and (5,1) ∈ R ⟶ (30,1) ∈ R
Similarly
(30,2) and (2,1) ∈ R ⟶ (30,1) ∈ R
(30,3) and (3,1) ∈ R ⟶ (30,1) ∈ R

∴ R is Transitive.

R is Reflexive
Antisymmetric
Transitive

⇒ (L,R) is poset.



Hasse Digram.

pair	glb meet ∧	lub join ∨
1,2	2	1
1,3	3	1
1,5	5	1
2,30	30	2
3,30	30	3
5,30	30	5

In poset (L, R) each pair, glb & lub are exist

Therefore (L, R) is Lattice.

∴ L – Lattice.

Hence proved.

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Imp.
5 marks

Q8) Let $A = \{3, 1, 2, 5, 30\}$ and R be the Relation 'a divides b' check if is a Lattice.

solution: $A = \{1, 2, 3, 5, 30\}$
R – Relation:
a divides b

$R = \{(1,1), (2,2), (3,3), (5,5), (30,30)$
$(1,2), (1,3), (1,5), (1,30), (2,30)$
$(3,30), (5,30)\}$

first we want to prove that
R is partial order Relation
it mean
R is Reflexive
is Antisymmetric
is Transitive

After By using Definition of Lattice prove that (A, R) is Lattice

i) since
$(1,1), (2,2), (3,3), (5,5), (30,30) \in R$
$\therefore$ R is Reflexive

ii) For Antisymmetric
$(1,2) \in R \longrightarrow (2,1) \notin R$
$(1,3) \in R \longrightarrow (3,1) \notin R$
$(1,5) \in R \longrightarrow (5,1) \notin R$
$(1,30) \in R \longrightarrow (30,1) \notin R$
$(2,30) \in R \longrightarrow (30,2) \notin R$
$(3,30) \in R \longrightarrow (30,3) \notin R$
$(5,30) \in R \longrightarrow (30,5) \notin R$
$\therefore$ R is Antisymmetric.

iii) For Transitive
(1,2) and (2,30) ∈ R → (1,30) ∈ R
similarly,
(1,3) and (3,30) ∈ R → (1,30) ∈ R
(1,5) and (5,30) ∈ R → (1,30) ∈ R

∴ R is Transitive.

R is Reflexive
is Antisymmetric
is Transitive

R is partial order Relation

⇒ (A, R) is poset

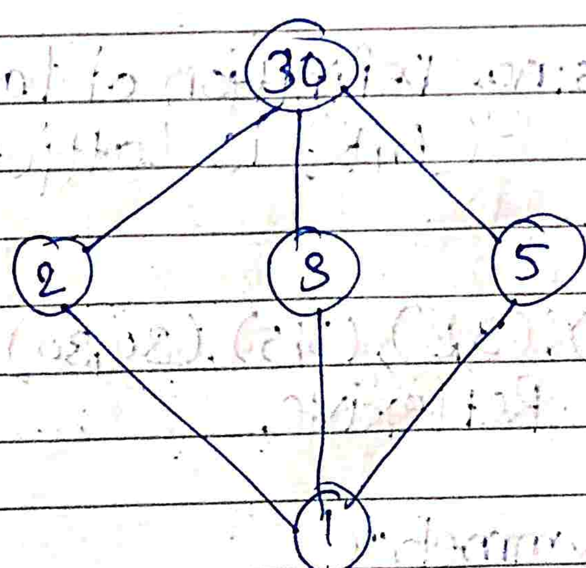


fig: Hasse Digram.

pair	glb meet ∧	lub join ∨
1,1	1	1
2,2	2	2
3,3	3	3
5,5	5	5
30,30	30	30
1,2	1	2
1,3	1	3
1,5	1	5
1,30	1	30
2,30	2	30
3,30	3	30
5,30	5	30

In poset (A,R) each pair glb & lub are exist
Therefore (A,R) is Lattice.

Hence proved.

5 marks

Q9) Draw the Hasse Digram of D_4
Check if it is a Lattice.

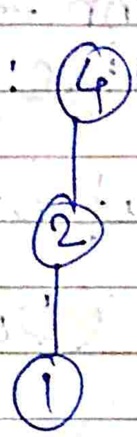
Solution:

D_4 — Divisor of 4

$D_4 = \{1, 2, 4\}$

$R = \{(1,1), (2,2), (4,4)$
$(1,2), (1,4), (2,4)\}$

Hasse Digram:



first we want to show that R is partial order Relation. it mean
R — is Reflexive
is Antisymmetric
is Transitive

i) Since $(1,1), (2,2), (4,4) \in R$
$\therefore$ R is Reflexive

ii) For Antisymmetric
$(1,2) \in R \longrightarrow (2,1) \notin R$
$(1,4) \in R \longrightarrow (4,1) \notin R$
$(2,4) \in R \longrightarrow (2,4) \notin R$

$\therefore$ R is Antisymmetric

iii) For Transitive

$(1,2)$ and $(2,4) \in R \Rightarrow (1,4) \in R$

similarly

$(1,2)$ and $(2,2) \in R \rightarrow (1,2) \in R$

$(1,4)$ and $(4,4) \in R \rightarrow (1,4) \in R$

$(2,4)$ and $(4,4) \in R \rightarrow (2,4) \in R$

$\therefore$ R is Transitive

R - is Reflexive
is Antisymmetric
is Transitive

$\therefore$ R - is partial order Relation

$(D_4, |)$ OR (D_4, R) is poset.

pair	glb $\wedge$ meet	lub join $\vee$
1,1	1	1
2,2	2	2
4,4	4	4
1,2	1	2
1,4	1	4
2,4	2	4

In poset $(D_4, |)$ OR (D_4, R), each pair glb & lub are exist.

$\therefore (D_4, |)$ OR (D_4, R) — is lattice

$\therefore D_4$ — is Lattice

5 marks

Q10) Draw the Hasse diagram of D_8 check if it is a Lattice.

Solution:

$D8$ — Divisor of 8

$D_8 = \{1, 2, 4, 8\}$

$R = \{(1,1), (2,2), (4,4), (8,8), (1,2), (1,4), (1,8), (2,4), (2,8), (4,8)\}$

Hasse Digram:

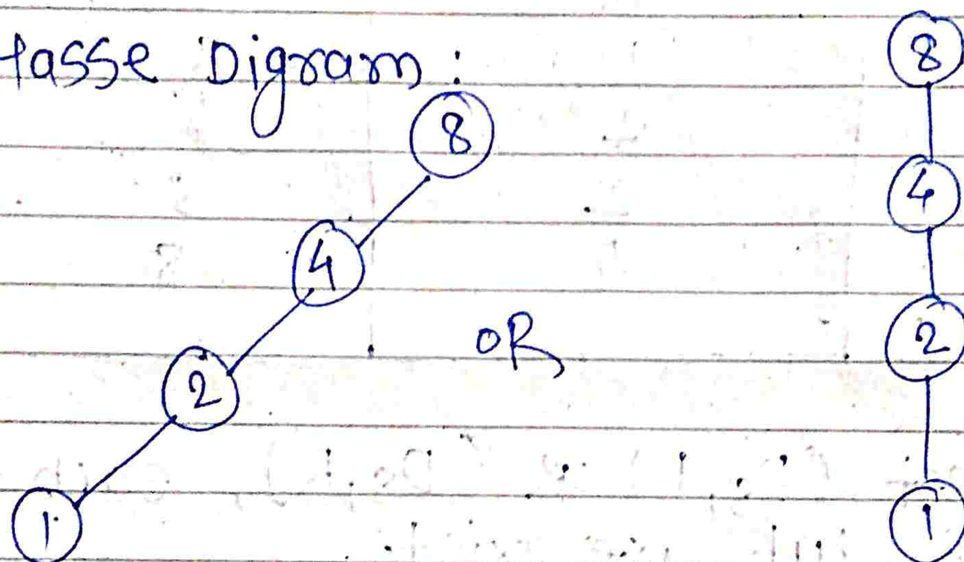


fig. Hasse Digram of D_8

first we want to show that R is partial order Relation it mean

R – is Reflexive
is Antisymmetric
is Transitive

i) since

$(1,1), (2,2), (4,4), (8,8) \in R$

$\therefore$ R is Reflexive

ii) For Antisymmetric

$(1,2) \in R \rightarrow (2,1) \notin R$

$(1,4) \in R \rightarrow (4,1) \notin R$

$(1,8) \in R \rightarrow (8,1) \notin R$

$(2,4) \in R \rightarrow (4,2) \notin R$
$(2,8) \in R \rightarrow (8,2) \notin R$
$(4,8) \in R \rightarrow (8,4) \notin R$

$\therefore$ R is Antisymmetric

iii) For transitive
$(1,2)$ and $(2,4) \in R \rightarrow (1,4) \in R$
$(1,2)$ and $(2,8) \in R \rightarrow (1,8) \in R$
Similarly.
$(1,4)$ and $(4,8) \in R \rightarrow (1,8) \in R$
$(2,4)$ and $(4,8) \in R \rightarrow (2,8) \in R$

$\therefore$ R is transitive.

R — is Reflexive
is Antisymmetric
is transitive

$\therefore$ R — is partial order Relation

$(D_8, |)$ OR (D_8, R) is poset.

pair	glb ∧ meet	lub join ∨
1,1	1	1
2,2	2	2
4,4	4	4
8,8	8	8
1,2	1	2
1,4	1	4
1,8	1	8
2,4	2	4
2,8	2	8
4,8	4	8

In poset $(D_8, |)$ OR (D_8, R), each pair glb & lub are exist.

$\therefore$ $(D_8, |)$, OR (D_8, R) – is Lattice

$\therefore$ D_8 – is Lattice

5marks

Q(i). Consider the poset $A = \{a, b, c, d, e, f, g, h\}$ whose Hasse Digram is shown below find all upper and lower bounds of the following subset of A. Also find GLB & LUB.

i) $B_1 = \{a, b\}$
ii) $B_2 = \{d, e\}$

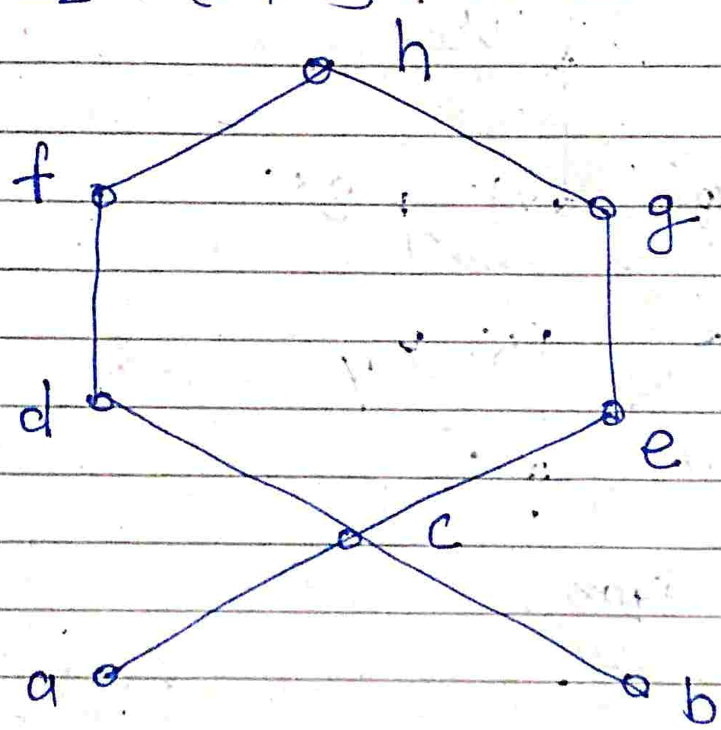


fig.

solution:

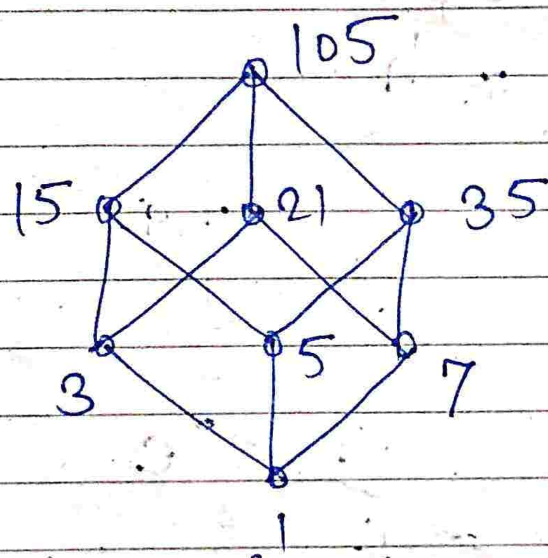
Sub set	upper Bounds	LUB	lower Bounds	GLB
$B_1 = \{a, b\}$	c d e f g h	c	—	—
$B_2 = \{d, e\}$	h	h	c b a	c

5 marks

Q12) consider the poset $A=\{1,3,5,7,15,21,35,105\}$ whose Hasse Digram is given below find all upper and lower bound of the following subset of A. Also find GLB & LUB.

i) $B_1=\{3,7\}$

ii) $B_2=\{3,5\}$



fig

solution:

Subset	upper Bounds	LUB	lower Bounds	GLB
$B_1=\{3,7\}$	21 105	21	1	1
$B_2=\{3,5\}$	15 105	15	1	1

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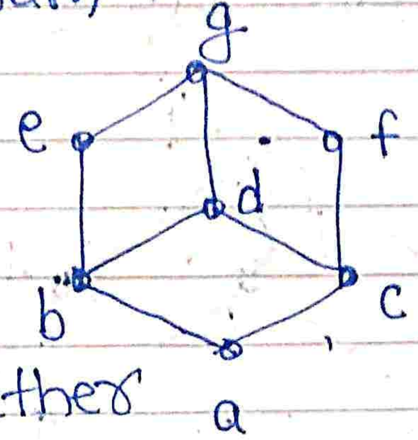
10 mark

Q1) Given lattice L where
$L = \{a, b, c, d, e, f, g\}$ shown in the Hasse Diagram



examine whether

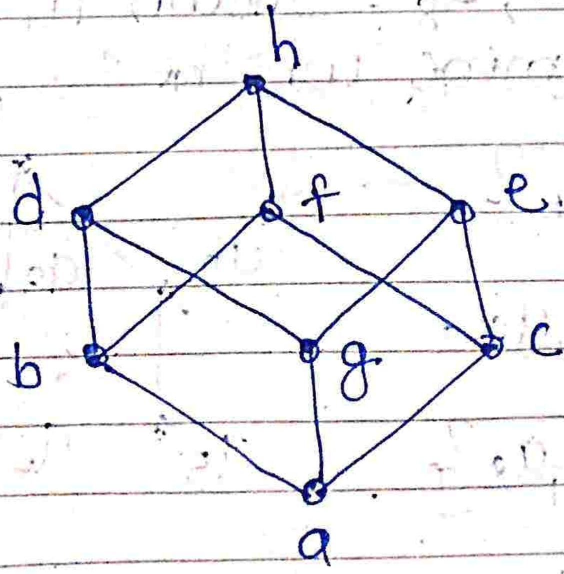
$S_1 = \{b, c, e, f, g\}$

$S_2 = \{a, b, c, d, e, f, g\}$

$S_3 = \{a, b, c, d\}$ are sublattice.

10 marks

Q2) Given the lattice L where
$L = \{a, b, c, d, e, f, g, h\}$ shown in the Hasse Digram



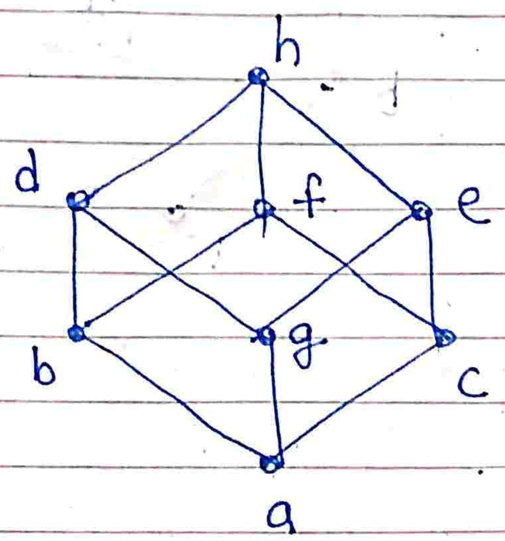
examine whether
$S_1 = \{a, b, d, f\}$
$S_2 = \{c, e, h, g\}$
$S_3 = \{a, b, h, d\}$ are sublattice.

10 marks

Q1) Given Lattice L where

$L = \{a, b, c, d, e, f, g, h\}$

Shown in Hasse Diagram.



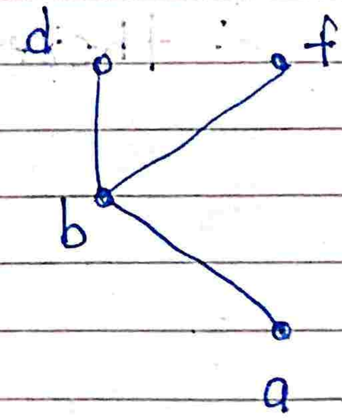
examine whether

1). $S_1 = \{a, b, d, f\}$

2). $S_2 = \{c, e, h, g\}$

3). $S_3 = \{a, b, h, d\}$ are sublattice.

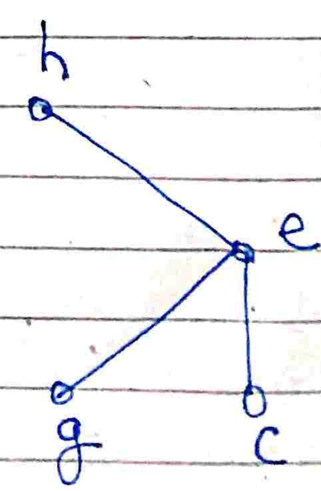
i) For $S_1 = \{a, b, d, f\}$



$d \vee f = h \notin S_1$

$\therefore$ S_1 is not Sublattice.

ii) For $S_2 = \{c, e, h, g\}$



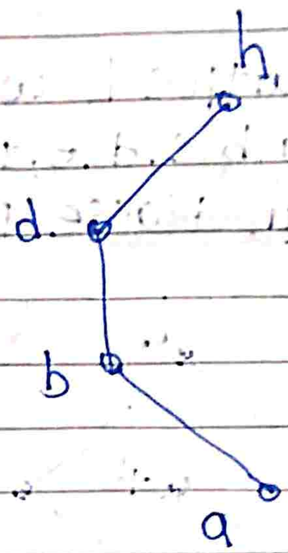
$g \wedge c = a \notin S_2$

$\therefore$ S_2 is not Sublattice.

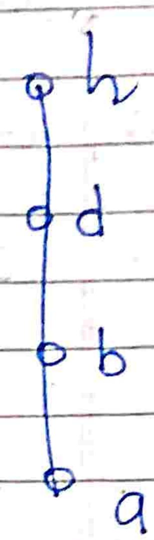
iii) For $S_3 = \{a, b, h, d\}$

$\wedge$ — meet GLB

$\wedge$	a	b	h	d
a	a	a	a	a
b	a	b	b	b
h	a	b	h	d
d	a	b	d	d



OR



$\vee$ — join LUB

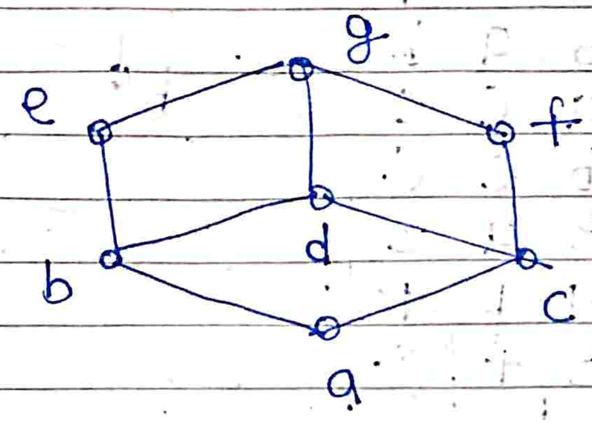
$\vee$	a	b	h	d
a	a	b	h	d
b	b	b	h	d
h	h	h	h	h
d	d	d	h	d

Here S_3 are closed with $\wedge$ meet & $\vee$ join

$\therefore$ therefore S_3 is Sublattice.

10 marks

Q2) Given Lattice L where
$L = \{a, b, c, d, e, f, g\}$
shown in the Hasse Diagram.



examine whether

1. $S_1 = \{b, c, e, f, g\}$
2. $S_2 = \{a, b, c, d, e, f, g\}$
3. $S_3 = \{a, b, c, d\}$

solution:

i) For $S_1 = \{b, c, e, f, g\}$

$b \cap c = a \notin S_1$

$\therefore$ S_1 is not sublattice.

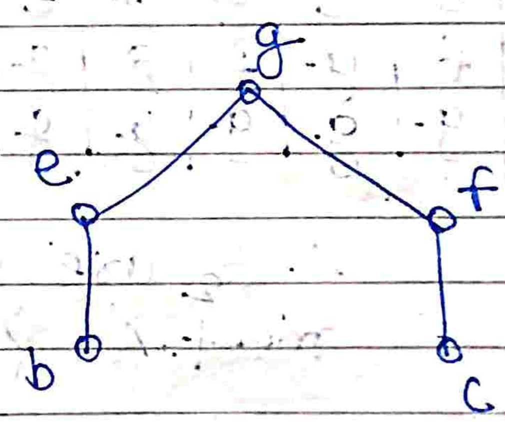
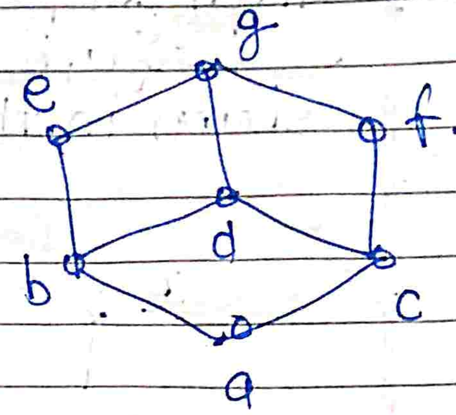


fig S_1

2.) For $S_2 = \{a, b, c, d, e, f, g\}$

$\wedge$	a	b	c	d	e	f	g
a	a	a	a	a	a	a	a
b	a	b	a	b	b	a	b
c	a	a	c	c	a	c	c
d	a	b	c	d	b	c	d
e	a	b	a	b	e	a	e
f	a	a	c	c	a	f	f
g	a	b	c	d	e	f	g

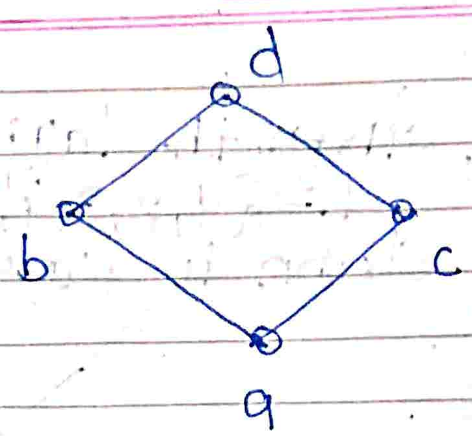


$\vee$	a	b	c	d	e	f	g
a	a	b	c	d	e	f	g
b	b	b	d	d	e	g	g
c	c	d	c	d	g	f	g
d	d	d	d	d	g	g	g
e	e	e	g	g	e	g	g
f	f	g	f	g	g	f	g
g	g	g	g	g	g	g	g

$\therefore$ S_2 are closed with meet $\wedge$ & join $\vee$.

$\therefore$ S_2 is sublattice.

iii) For $S_3 = \{a, b, c, d\}$



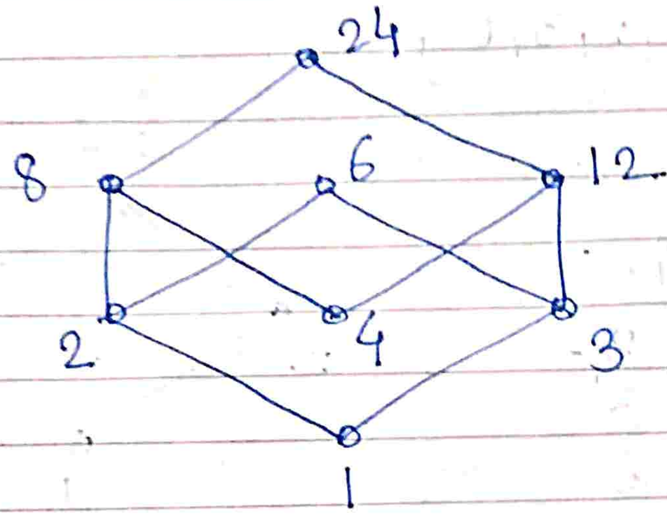
$\wedge$	a	b	c	d
a	a	a	a	a
b	a	b	a	b
c	a	a	c	c
d	a	b	c	d

$\vee$	a	b	c	d
a	a	b	c	d
b	b	b	d	d
c	c	d	c	d
d	d	d	d	d

S_3 are closed with
meet $\wedge$ & join $\vee$
$\therefore S_3$ is sublattice.

10 marks

Q3) Given the lattice L where
$L = \{1,2,3,4,6,8,12,24\}$
shown in Hasse Diagram



examine whether

1) $S_1 = \{4,8,12,24\}$

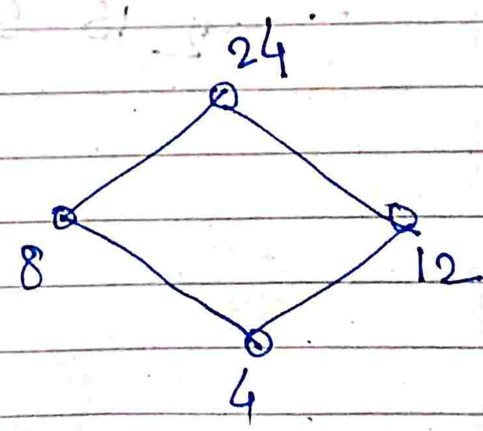
2). $S_2 = \{1,2,3,6\}$

3). $S_3 = \{1,8,12,24\}$

Solution

i) For $S_1 = \{4,8,12,24\}$

$\wedge$	4	8	12	24
4	4	4	4	4
8	4	8	4	8
12	4	4	12	12
24	4	8	12	24

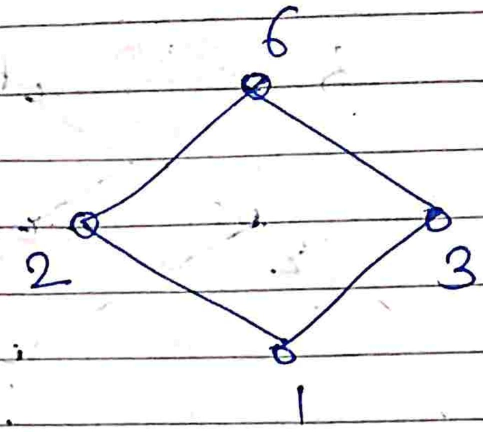


$\vee$	4	8	12	24
4	4	8	12	24
8	8	8	24	24
12	12	24	12	24
24	24	24	24	24

S_1 are closed with meet $\wedge$ & join $\vee$
$\therefore$ S_1 is sublattice.

2). For $S_2 = \{1, 2, 3, 6\}$

$\wedge$	1	2	3	6
1	1	1	1	1
2	1	2	1	2
3	1	1	3	3
6	1	2	3	6



$\vee$	1	2	3	6
1	1	2	3	6
2	2	2	6	6
3	3	6	3	6
6	6	6	6	6

$\therefore$ S_2 are closed with meet $\wedge$ & join $\vee$
$\therefore$ S_2 is Sublattice.

3). $S_3 = \{1, 8, 12, 24\}$

$8 \wedge 12 = 4 \notin S_3$

$\therefore S_3$ is not sublattice.

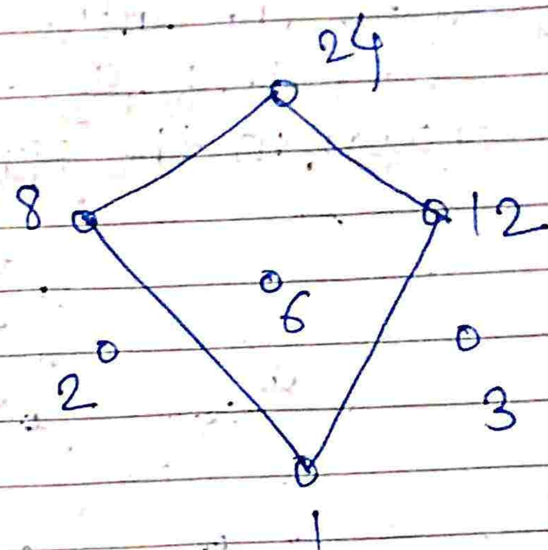
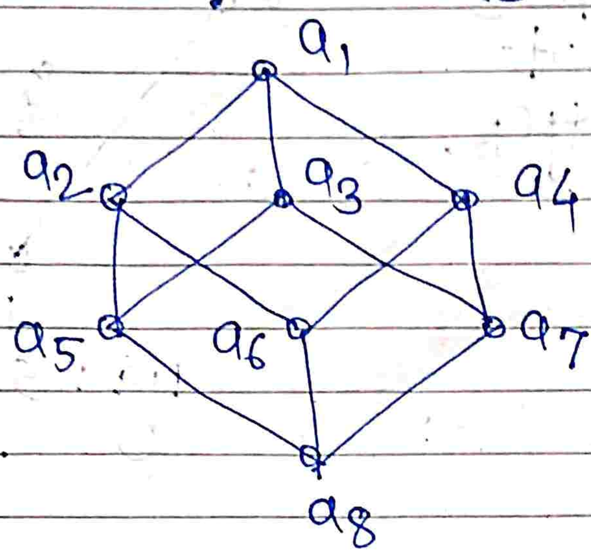


fig : S_3

10 marks

Q4) Given the lattice L where $L = \{a_1, a_2, a_3, \ldots, a_8\}$ Shown in the Hasse Diagram

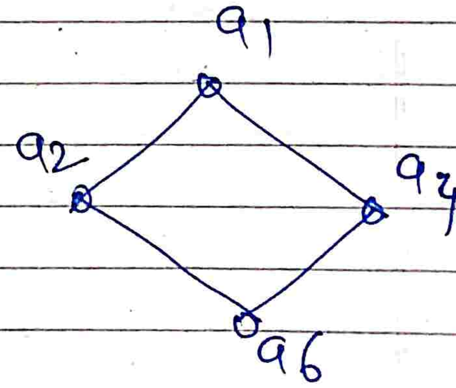


examine whether

1) $S_1 = \{a_1, a_2, a_4, a_6\}$.
2) $S_2 = \{a_3, a_5, a_7, a_8\}$
3). $S_3 = \{a_1, a_2, a_4, a_8\}$

Solution:

i) For $S_1 = \{a_1, a_2, a_4, a_6\}$



$\vee$	a_1	a_2	a_4	a_6
a_1	a_1	a_1	a_1	a_1
a_2	a_1	a_2	a_1	a_2
a_4	a_1	a_1	a_4	a_4
a_6	a_1	a_2	a_4	a_6

$\wedge$	a_1	a_2	a_4	a_6
a_1	a_1	a_2	a_4	a_6
a_2	a_2	a_2	a_6	a_6
a_4	a_4	a_6	a_4	a_6
a_6	a_6	a_6	a_6	a_6

$\therefore S_1$ are closed with meet $\wedge$ & join $\vee$
$\therefore S_1$ is Sublattice.

ii) For $S_2 = \{a_3, a_5, a_7, a_8\}$

$\wedge$	a_3	a_5	a_7	a_8
a_3	a_3	a_5	a_7	a_8
a_5	a_5	a_5	a_8	a_8
a_7	a_7	a_8	a_7	a_8
a_8	a_8	a_8	a_8	a_8

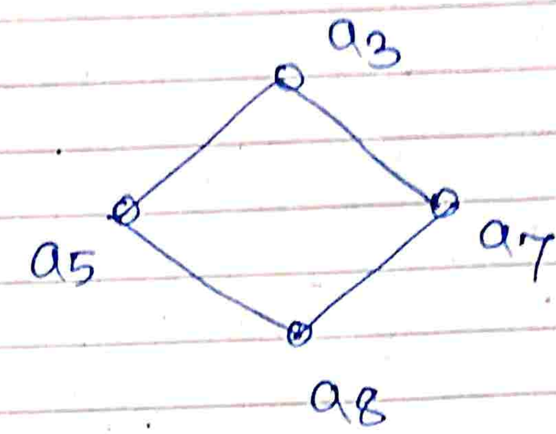


fig: S_2

$\vee$	a_3	a_5	a_7	a_8
a_3	a_3	a_3	a_3	a_3
a_5	a_3	a_5	a_3	a_5
a_7	a_3	a_3	a_7	a_7
a_8	a_3	a_5	a_7	a_8

$\therefore S_2$ are closed with meet $\wedge$ & join $\vee$
$\therefore S_2$ is Sublattice.

iii) For $S_3 = \{a_1, a_2, a_4, a_8\}$

$a_2 \wedge a_4 = a_6 \notin S_3$

$\therefore S_3$ is not Sublattice.

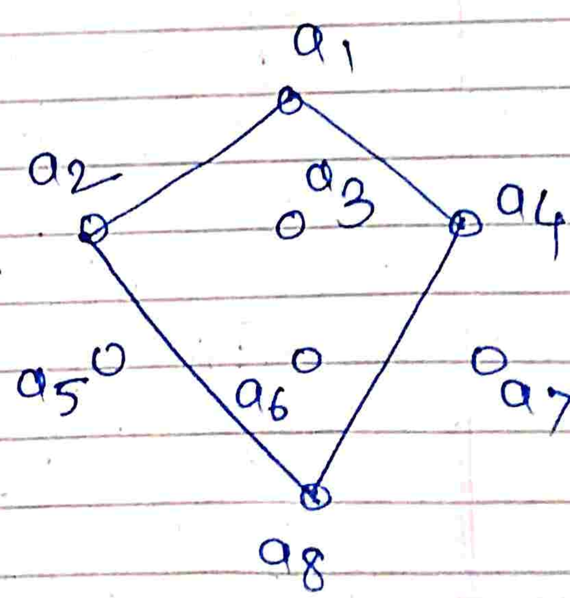


fig. S_3