Module-02

1 marks Questions

1. Define Reflexive Relation with suitable example.

2. Define Equivalence Relation with suitable example.

3. Define Reflexive closure with suitable example.

4. Define symmetric closure with suitable example.

5. Define transitive closure with suitable example.

6. Define injective function with an example.

7. Define Bi-jective function with an example

8. Define surjective function with an example.

9. Let $f: R \to R$, $g: R \to R$ be defined by $f(a) = 4a-1$, $g(b) = b^3$, find $f(5)$, $g(3)$.

10. Let $f: R \to R$, $g: R \to R$ be defined by $f(x) = x^2+2$, $g(x) = 3x$, find $f(3)$, $g(2)$.

Relations & function

Module 02

Q1). Let $A = \{1,2,3,4\}$ and $B = \{a,b,c\}$
Let $R = \{(1,a), (3,a), (3,c)\}$
find
i) R^c
ii) $D(R^c)$
iii) $Rn(R^c)$

Solution:
$R = \{(1,a), (3,a), (3,c)\}$
i) $R^c = \{(a,1), (a,3), (c,3)\}$

ii) $D(R^c) = \{a,c\} = Rn(R)$

iii) $Rn(R^c) = \{1,3\} = D(R)$

Q2). Let $A = \{2,3,4,6\}$. Let R and S be relations on A such that.

$R = \{(a,b) \mid a = b+1 \text{ or } b = 2a\}$ and

$S = \{(a,b) \mid a \text{ divides } b\}$. Find $(R \cap S)^c$.

Solution:
$A = \{2,3,4,6\}$
$R = \{(a,b) \mid a = b+1 \text{ or } b = 2a\}$

$S = \{(a,b) \mid a \text{ divides } b\}$

$R = \{(3,2), (4,3), (2,4), (3,6)\}$

$S = \{(2,2), (2,4), (2,6), (3,3), (3,6)$
$(4,4), (6,6)\}$

$\therefore R \cap S = \{(2,4),(3,6)\}$

$\therefore (R \cap S)^c = \{(4,2),(6,3)\}$

Q3). Let $A = \{a,b,c,d\}$ and $B = \{1,2,3\}$
Let $R = \{(a,1),(a,2),(b,1),(c,2),(d,1)\}$
find the relation matrix.

Solution:

M_R will have 4 rows & 3 columns

(1, 2)
row column

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

Numbers of Rows (A, B) Number of Columns

Q4) Let $A = \{1,2,3,4,8\}$, $B = \{1,4,6,9\}$
Let aRb iff $a|b$ (a divides b).
Find the relation matrix.

Solution:

$R = \{(1,1), (1,4), (1,6), (1,9), (2,4), (2,6), (3,6), (3,9), (4,4)\}$

M_R will have
5 rows &
4 columns

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 8 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 4 & 6 & 9 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Q5). Let $A = \{1, 2, 3, 4, 8\}$, $B = \{1, 2, 3, 4, 8\}$
aRb iff $a+b \leq 9$ find it's relation matrix.

Solution:

$R = \{(1,1), (1,2), (1,3), (1,4), (1,8)$
$, (2,1), (2,2), (2,4), (2,3), (3,1)$
$, (3,2), (3,3), (3,4), (4,1), (4,2)$
$, (4,3), (4,4), (8,1)\}$

M_R will have
5 rows &
5 columns.

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 8 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 8 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Q6) Let $A = \{a, b, c, d\}$ and

Let
$$M_R = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

find R.

Solution:

$$M_R = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

$R = \{ (a,a), (a,b), (b,c), (b,d), (c,c), (c,d), (d,a), (d,c) \}$

1 mark

Q1) Define Reflexive Relation with suitable example.

Solution:

Definition:

R is Reflexive relation if for every element $a \in A$, aRa.

i.e $(a,a) \in R$.

e.g. $A = \{1,2,3\}$

$R_1 = \{(1,1), (2,2), (3,3)\}$

$R_2 = \{(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)\}$

1 marks

Q2) Define symmetric Relation with suitable example.

Solution:

Definition:

R is said to be symmetric Relation if whenever aRb then bRa.

i.e $a,b \in A$, if $(a,b) \in R$ then $(b,a) \in R$.

e.g. $A = \{1,2,3\}$

$R_1 = \{(1,2), (2,1), (1,3), (3,1), (2,3), (3,2)\}$.

$R_2 = \{ (1,1), (1,2), (1,3)$
$(2,1), (2,2), (2,3)$
$(3,1), (3,2), (3,3) \}$.

1 marks

Q3) Define Transitive Relation with suitable example.

Solution:

Definition:

R is said to be transitive if whenever aRb and bRc then aRc.

i.e. $a, b, c \in A$.

if (a,b) & $(b,c) \in R$ then $(a,c) \in R$.

e.g. $A = \{1,2,3\}$

$R_1 = \{ (1,2), (2,1), (1,1), (2,2) \}$

$R_2 = \{ (1,1), (1,2), (1,3),$
$(2,1), (2,2), (2,3),$
$(3,1), (3,2), (3,3) \}$

1 marks

Q4) Define Equivalence relation with suitable example.

Solution:

Definition:

A binary relation R on a set A is called as equivalence relation if it is Reflexive, Symmetric & transitive.

e.g. $A = \{1, 2, 3\}$

$R_1 = \{(1,1), (2,2), (3,3), (1,2), (2,1), (3,2), (1,3) (2,3), (3,1)\}$.

1 marks

Q5) Define Reflexive closure with suitable example.

Solution:

Definition:

The reflexive closure of a relation R on a set A is the smallest reflexive relation that contains R.

e.g. ① $A = \{1,2,3\}$ and $R = \{(1,2), (2,3)\}$ then reflexive closure of R

$r(R) = \{(1,2), (2,3), (1,1), (2,2), (3,3)\}$

1 marks
Q6) Define symmetric closure with suitable example.

Solution:

Definition:

The symmetric closure of a relation R is the smallest symmetric relation that contains R.

e.g. $A = \{1,2,3\}$ and $R = \{(1,2),(2,3)\}$ then symmetric closure of R

$$r(R) = \{(1,2),(2,1),(2,3),(3,2)\}.$$

1 marks
Q7) Define transitive closure with suitable example.

Solution:

Definition:

The transitive closure of a relation R is the smallest transitive relation that contains R.

e.g. ① Let $A = \{1,2,3,4\}$ and $R = \{(1,2),(2,3),(3,4)\}$ then transitive closure of R,

$$r(R) = \{(1,2),(2,3),(3,4),(1,3),(1,4),(2,4)\}$$

1 marks

Q8) Let $f: R \to R$, $g: R \to R$ be defined by $f(a) = 4a - 1$, $g(b) = b^3$ find $f(5)$, $g(3)$.

Solution:

$f(a) = 4a - 1$ put $a = 5$

$f(5) = 4(5) - 1$

$= 20 - 1$

$= 19$

$\therefore \boxed{f(5) = 19}$

$g(b) = b^3$ put $b = 3$

$g(3) = 3^3$

$= 27$

$\therefore \boxed{g(3) = 27}$

1 marks

Q9) Let $f: R \to R$, $g: R \to R$ be defined by $f(x) = x^2 + 2$, $g(x) = 3x$. find $f(3)$, $g(2)$

Solution:

$f(x) = x^2 + 2$ put $x = 3$

$f(3) = 3^2 + 2$

$= 9 + 2$

$= 11$

$\boxed{f(3) = 11}$

$g(x) = 3x$ put $x = 2$

$g(2) = 3(2)$

$= 6$

$\boxed{g(2) = 6}$

1 marks

Q10) Define injective function with an example.

Solution :

Definition:

An injective function, (one-one) is a function where each element in the domain maps to a unique element in the co-domain.

or//

A function $f: A \rightarrow B$ is injective if for all x, y in A, whenever $f(x) = f(y)$ then $x = y$.

e.g.

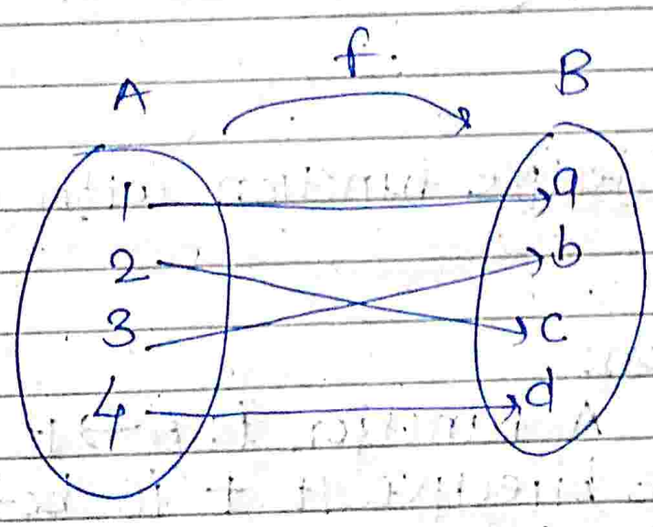

fig. Injective function.

1 marks

Q1) Define surjective function with an example

Solution:

Definition:

A surjective function (onto) is a function whose image is equal to it's co-domain.

eg.

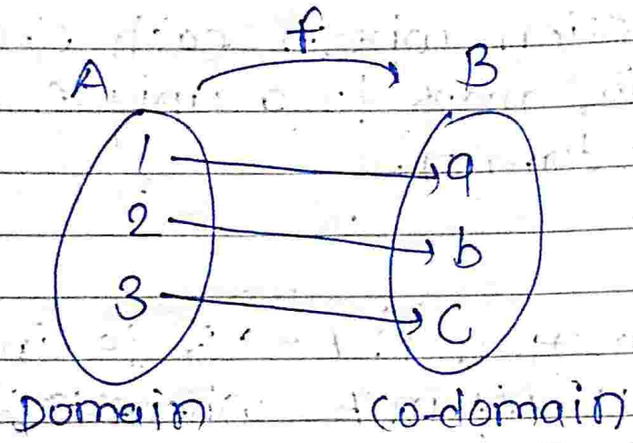

fig: surjective function.

1 marks

Q2) Define Bi-jective function with an example

Solution:

Definition:

A function $f: A \to B$ is said to be bijective if f is both injective and surjective.

eg.

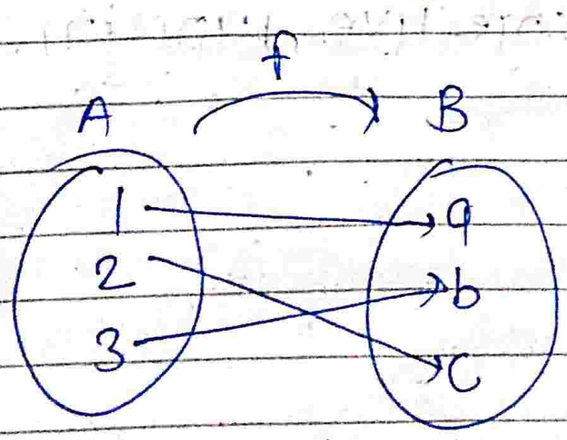

fig Bi-jective function.

Important information.

Q. Define reflexive, irreflexive, symmetric, asymmetric, antisymmetric, Transitive with example.

Solution:

① Reflexive Relation:

Definition:

A relation R on a set A is said to be reflexive if

$(x,x) \in R \ \forall x \in A$.

e.g. $A = \{1,2,3\}$

$R_1 = \{(1,1), (2,2), (3,3)\}$

$R_2 = \{(1,1), (1,2), (1,3)$
$(2,1), (2,2), (2,3)$
$(3,1), (3,2), (3,3)\}$

② irreflexive Relation:

Definition:

A Relation R on a set A is said to be irreflexive if $x \not R x$, $\forall x \in A$.

i.e. $(x,x) \notin R$.

e.g. $A = \{1,2,3\}$

$R_1 = \{(1,2), (1,3), (2,4), (3,2)\}$

$R_2 = \{(3,1), (2,1), (4,3)\}$

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③ Symmetric Relation:

Definition:

A Relation R on a set 'A' is Said to be symmetric if xRy then yRx. $x, y \in A$

e.g. $A = \{1, 2, 3, 4\}$

$R_1 = \{(2,3), (3,2), (4,1), (1,4)\}$

$R_2 = \{(1,2), (1,1), (2,1)\}$

④ Asymmetric relation

Definition:

A Relation R on a set A is Said to be asymmetric if whenever xRy then $y \not R x$, $x, y \in A$

eg. $A = \{1, 2, 3, 4\}$

$R_1 = \{(1,2), (1,3), (2,3)\}$

$R_2 = \{(1,3), (1,2), (1,4)\}$

$R_3 = \{(2,3), (2,4), (1,4), (1,2)\}$

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⑤. Antisymmetric Relation:

Definition:

A Relation R on a set A is Said to be antisymmetric if whenever aRb and bRa then $a=b$. , $a,b \in A$.

e.g. $A = \{1,2,3,4\}$

$R_1 = \{(1,1), (1,2), (1,3), (1,4)\}$

$R_2 = \{(2,2), (2,1), (2,3), (2,4)\}$

$R_3 = \{(3,1), (3,2), (3,3), (3,4)\}$.

⑥ Transitive Relation:

Definition:

A Relation R on a set A is Said to be transitive if whenever aRb and bRc then aRc , $a,b,c \in A$.

e.g. $A = \{1,2,3\}$

$R_1 = \{(1,2), (2,3), (1,3)\}$

$R_2 = \{(3,4), (4,2), (3,2)\}$.

Module - 02

Q1) Let $A = \{2, 3, 4, 6, 8\}$ Let the relation R be defined on set A by if x divides y then xRy find R it's matrix and it's digraph

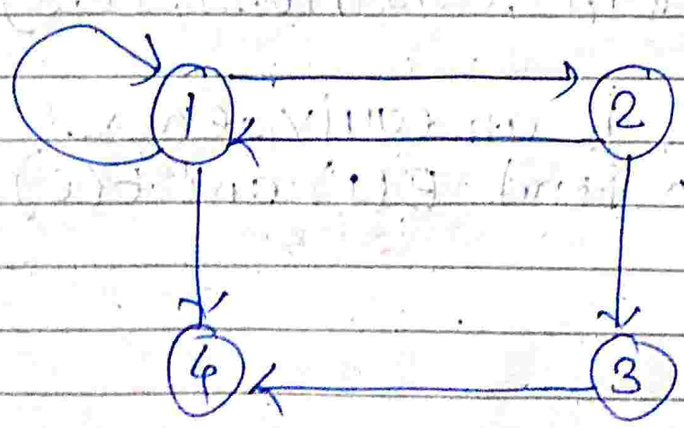
Q2) Determine whether the relation R whose matrix

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is reflexive, irreflexive, symmetric ~~and~~ asymmetric and Transitive where $A = \{1, 2, 3, 4\}$

Q3) Determine wether the relation R on the set A is reflexive, irreflexive, symmetric antisymmetric, transitive where,

$A = \{1, 2, 3, 4\}$



fig

Q4) Determine wether the relation R whose matrix

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

it is reflexive, irreflexive, symmetric, asymmetric, and transitive where $A = \{1,2,3,4\}$

Q5) Let $A = \{1,2,3,4\}$ and
$R = \{(1,1), (2,2), (3,3), (4,4), (2,1), (1,2), (2,3), (3,2), (3,1), (1,3)\}$
prove that R is an equivalence relation and also find matrix and draw it digraph

Q6) Let $A = \{1,2,3,4,5,6\}$ and $R = \{(1,1), (1,5), (2,2), (2,3), (2,6), (3,2), (3,3), (3,6), (4,4), (5,1), (5,5), (6,2), (6,3) (6,6),\}$
Show that R is an equivloence relation. Also find $E(1)$ and $E(6)$.

Q7) Show that the mapping $f: R \to R$ given by
i) $f(x) = 4x - 3$
ii) $f(x) = 4x + 7$
is bijective. find it's inverse.

Q8) Is the mapping $f: R \sim \{2\} \to R \sim \{0\}$,
$f(x) = \left(\frac{1}{x-2}\right)$ bijective & if so find the inverse of f.

Q9) A function $h: R \to R$, $g: R \to R$, $f: R \to R$ are defined as
$f(x) = 2x + 5$
$g(x) = x^2 + 1$
$h(x) = 2x + 5$
find
i) fog
ii) goh
iii) foh
iv) hoh
v) gog

Q10) A function $h: R \to R$ is defined by $h(x) = x^2 - 1$. Test the function whether it is
i) injective
ii) Surjective
iii) Bijective.

Q11). Let $A=B=C=R$ and Let $f: A \to B$, $g: B \to C$, be defined by

$$f(a) = a-1$$
$$g(b) = b^2$$

find,

$(fog)(2)$
$(gof)(x)$
$(fof)(y)$
$(gof)(2)$
$(gog)(y)$

Q12). is the mapping

$$f: R \sim \{\tfrac{2}{5}\} \to R \sim \{\tfrac{4}{5}\}$$

is a function defined by

$$f(x) = \left(\frac{4x+3}{5x-2}\right)$$

prove that f is a bijective function and find f^{-1}.

5 marks

Q1) Let $A = \{2, 3, 4, 6, 8\}$
Let the relation R be defined on set A by it x divides y. then xRy
Find R, it's matrix and it's digraph

Solution:

$$R = \{(2,2), (3,3), (4,4), (6,6), (8,8), (2,4), (2,6), (2,8), (3,6), (4,8)\}$$

$$M_R = \begin{array}{c} \\ 2 \\ 3 \\ 4 \\ 6 \\ 8 \end{array} \begin{array}{c} \begin{array}{ccccc} 2 & 3 & 4 & 6 & 8 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

it is matrix representation of R.

Now Digraph of R is

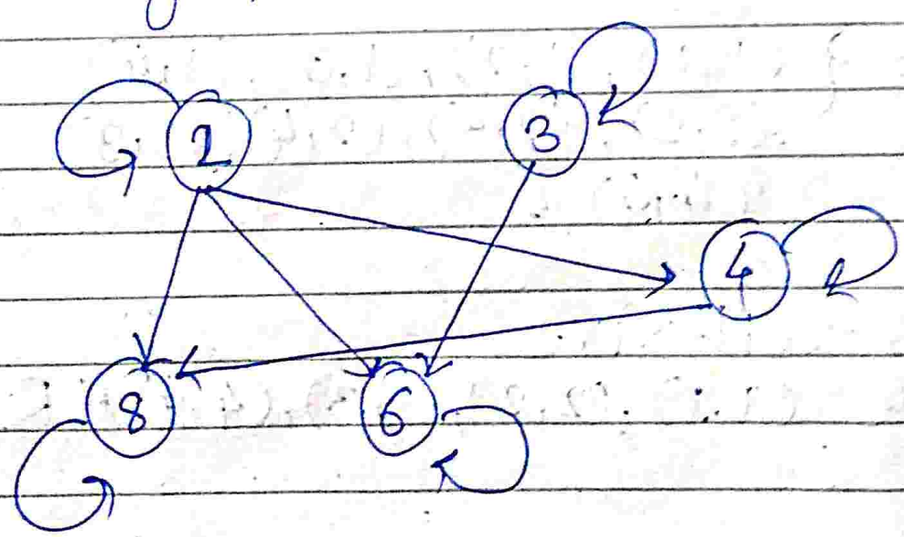


fig. Digraph of R.

5 marks

Q2) Determine whether the relation R whose matrix

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

is reflexive, irreflexive, symmetric, asymmetric, and Transitive where $A = \{1, 2, 3, 4\}$

Solution:

$A = \{1, 2, 3, 4\}$

$$M_R = \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array}$$

$R = \{(1,1), (1,2), (1,3), (1,4)$
$(2,2), (2,3), (2,4), (3,3)$
$(4,4)\}$

① R is reflexive
since $(1,1), (2,2), (3,3), (4,4) \in R$

② $(1,2) \in R$ but $(2,1) \notin R$
$\therefore$ R is not symmetric
$\Rightarrow$ ~~Asymmetric~~.

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(3) For transitive.

$(1,1)$ and $(1,2) \in R \rightarrow (1,2) \in R$
$(1,1)$ and $(1,3) \in R \rightarrow (1,3) \in R$
$(1,1)$ and $(1,4) \in R \rightarrow (1,4) \in R$
$(2,2)$ and $(2,3) \in R \rightarrow (2,3) \in R$
$(2,2)$ and $(2,4) \in R \rightarrow (2,4) \in R$
$(1,2)$ and $(2,2) \in R \rightarrow (1,2) \in R$
$(1,2)$ and $(2,3) \in R \rightarrow (1,3) \in R$
$(1,2)$ and $(2,4) \in R \rightarrow (1,4) \in R$
$(1,3)$ and $(3,3) \in R \rightarrow (1,3) \in R$
$(1,4)$ and $(4,4) \in R \rightarrow (1,4) \in R$
$(2,3)$ and $(3,3) \in R \rightarrow (2,3) \in R$
$(2,4)$ and $(4,4) \in R \rightarrow (2,4) \in R$

Since R is transitive.

(4) $(1,1) \in R \Rightarrow$ R is not irreflexive.

(5) $(1,1), (2,2), (3,3), (4,4) \in R$
$\Rightarrow$ R is not Asymmetric

5 marks

Q3). Determine wether the relation R on the set A is reflexive, irreflexive, symmetric, antisymmetric, transitive where

$A = \{1, 2, 3, 4\}$

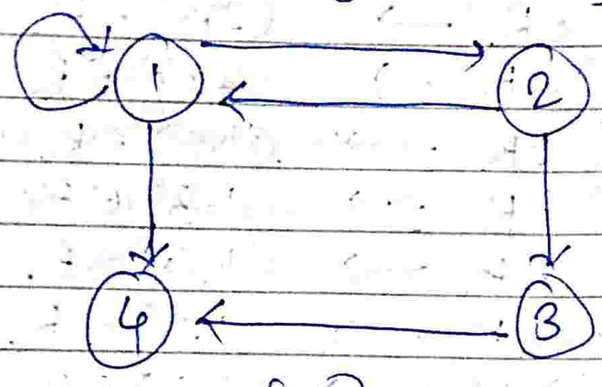

fig.

Solution: $A = \{1, 2, 3, 4\}$

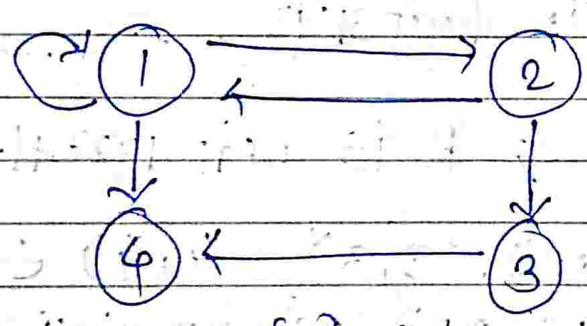

fig.

$R = \{(1,1), (1,2), (1,4), (2,1), (2,3), (3,4)\}$

(1) $(2,2), (3,3), (4,4) \notin R$
$\therefore$ R is not reflexive

(2) $(1,1) \in R$
$\therefore$ R is not irreflexive

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③ $(2,3) \in R$ but $(3,2) \notin R$
$\therefore$ R is not symmetric

④ $(1,2) \in R$, and $(2,1) \in R$
$\therefore$ R is not antisymmetric

⑤ For transitive
$(1,1)$ and $(1,2) \in R \rightarrow (1,2) \in R$
$(1,1)$ and $(1,4) \in R \rightarrow (1,4) \in R$
$(1,2)$ and $(2,1) \in R \rightarrow (1,1) \in R$
$(1,2)$ and $(2,3) \in R \rightarrow (1,3) \notin R$
$\therefore$
R is not transitive.

5 marks

Q4) Determine the wether relation R whose matrix.

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

it is reflexive, irreflexive, symmetric, asymmetric and transitive where

$A = \{1, 2, 3, 4\}$

Solution: $A = \{1, 2, 3, 4\}$

$$M_R = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$R = \{(1,1), (1,2), (1,4), (2,1), (2,2), (3,3), (4,1), (4,4)\}$

① since

$(1,1), (2,2), (3,3), (4,4) \in R$

$\therefore$ R is reflexive

② $(1,1) \in R \Rightarrow$ R is irreflexive

(3) $(1,2) \in R \longrightarrow (2,1) \in R$
$(4,1) \in R \longrightarrow (1,4) \in R$
$\therefore$ R is symmetric

(4) $(1,1) \in R$
$\therefore$ R is not asymmetric.

(5) For Transitive

$(1,1)$ and $(1,2) \in R \longrightarrow (1,2) \in R$
$(1,1)$ and $(1,4) \in R \longrightarrow (1,4) \in R$
$(1,2)$ and $(2,1) \in R \longrightarrow (1,1) \in R$
$(1,2)$ and $(2,2) \in R \longrightarrow (1,2) \in R$
$(1,4)$ and $(4,1) \in R \longrightarrow (1,1) \in R$
$(1,4)$ and $(4,4) \in R \longrightarrow (1,4) \in R$
$(2,1)$ and $(1,1) \in R \longrightarrow (2,1) \in R$
$(2,1)$ and $(1,2) \in R \longrightarrow (2,2) \in R$
$(2,1)$ and $(1,4) \in R \longrightarrow (2,4) \notin R$

$\therefore$ R is not transitive.

5 marks

Q5) Let $A = \{1,2,3,4\}$ and
$R = \{(1,1), (2,2), (3,3), (4,4), (2,1), (1,2)$
$(2,3), (3,2), (3,1), (1,3)\}$
prove that R is an equivalence relation and also find matrix and draw it digraph.

Solution: $A = \{1,2,3,4\}$
$R = \{(1,1), (2,2), (3,3), (4,4), (2,1), (1,2),$
$(2,3), (3,2), (3,1), (1,3)\}$.

(I)

(1) Since
$(1,1), (2,2), (3,3), (4,4) \in R$
$\therefore$ R is reflexive.

(2) $(2,1) \in R \rightarrow (1,2) \in R$
$(2,3) \in R \rightarrow (3,2) \in R$
$(3,1) \in R \rightarrow (1,3) \in R$

$\therefore$ R is symmetric

(3) For transitive

$(1,1)$ and $(1,2) \in R \rightarrow (1,2) \in R$
$(2,2)$ and $(2,1) \in R \rightarrow (2,1) \in R$
$(2,2)$ and $(2,3) \in R \rightarrow (2,3) \in R$
$(3,3)$ and $(3,2) \in R \rightarrow (3,2) \in R$
$(3,3)$ and $(3,1) \in R \rightarrow (3,1) \in R$
$(2,1)$ and $(1,1) \in R \rightarrow (2,1) \in R$
$(2,1)$ and $(1,2) \in R \rightarrow (2,2) \in R$
$(2,1)$ and $(1,3) \in R \rightarrow (2,3) \in R$

$(1,2)$ and $(2,2) \in R \rightarrow (1,2) \in R$
$(1,2)$ and $(2,1) \in R \rightarrow (1,1) \in R$
$(1,2)$ and $(2,3) \in R \rightarrow (1,3) \in R$
$(2,3)$ and $(3,3) \in R \rightarrow (2,3) \in R$
$(2,3)$ and $(3,2) \in R \rightarrow (2,2) \in R$
$(2,3)$ and $(3,1) \in R \rightarrow (2,1) \in R$
$(3,2)$ and $(2,2) \in R \rightarrow (3,2) \in R$
$(3,2)$ and $(2,1) \in R \rightarrow (3,1) \in R$
$(3,2)$ and $(2,3) \in R \rightarrow (3,3) \in R$
$(3,1)$ and $(1,1) \in R \rightarrow (3,1) \in R$
$(3,1)$ and $(1,2) \in R \rightarrow (3,2) \in R$
$(3,1)$ and $(1,3) \in R \rightarrow (3,3) \in R$
$(1,3)$ and $(3,3) \in R \rightarrow (1,3) \in R$
$(1,3)$ and $(3,2) \in R \rightarrow (1,2) \in R$
$(1,3)$ and $(3,1) \in R \rightarrow (1,1) \in R$

Hence R is Transitive

From ①, ② and ③ The Relation R is Reflexive
Symmetric
Transitive
(implies) therefore R is Equivalence Relation.

Hence proved.

(II)

$$M_R = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 1 & 1 & 1 & 0 \\ 4 & 0 & 0 & 0 & 1 \end{array}$$

M_R is the matrix Representation of Relation R.

(III)

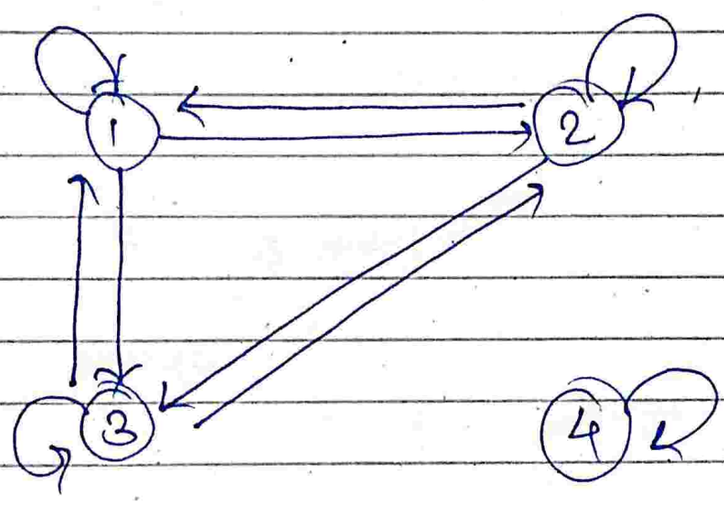

fig. Digraph of R.

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MSc (Math),BEd,GATE SET

5 marks

Q6) Let $A = \{1,2,3,4,5,6\}$ and
$R = \{(1,1),(1,5),(2,2),(2,3),(2,6),(3,2),(3,3),(3,6),(4,4),(5,1),(5,5),(6,2),(6,3),(6,6)\}$
Show that R is an equivalence relation
Also find $E(1)$ and $E(2)$ and $E(6)$.

Solution:

$A = \{1,2,3,4,5,6\}$
$R = \{(1,1),(1,5),(2,2),(2,3),(2,6),(3,2),(3,3),(3,6),(4,4),(5,1),(5,5),(6,2),(6,3),(6,6)\}$

① Since
$(1,1),(2,2),(3,3),(4,4),(5,5),(6,6) \in R$
$\therefore$ R is reflexive

② $(1,5) \in R \rightarrow (5,1) \in R$
$(2,3) \in R \rightarrow (3,2) \in R$
$(3,6) \in R \rightarrow (6,3) \in R$
$(5,1) \in R \rightarrow (1,5) \in R$
$\therefore$ R is symmetric

③ For Transitive
- $(1,1)$ and $(1,5) \in R \rightarrow (1,5) \in R$
- $(1,5)$ and $(5,1) \in R \rightarrow (1,1) \in R$
- $(1,5)$ and $(5,5) \in R \rightarrow (1,5) \in R$
- $(2,2)$ and $(2,3) \in R \rightarrow (2,3) \in R$
- $(2,2)$ and $(2,6) \in R \rightarrow (2,6) \in R$
- $(2,3)$ and $(3,2) \in R \rightarrow (2,2) \in R$
- $(2,3)$ and $(3,3) \in R \rightarrow (2,3) \in R$
- $(2,3)$ and $(3,6) \in R \rightarrow (2,6) \in R$

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$(2,6)$ and $(6,2) \in R \rightarrow (2,2) \in R$
$(2,6)$ and $(6,3) \in R \rightarrow (2,3) \in R$
$(2,6)$ and $(6,6) \in R \rightarrow (2,6) \in R$
$(3,2)$ and $(2,2) \in R \rightarrow (3,2) \in R$
$(3,2)$ and $(2,3) \in R \rightarrow (3,3) \in R$
$(3,2)$ and $(2,6) \in R \rightarrow (3,6) \in R$
$(3,3)$ and $(3,2) \in R \rightarrow (3,2) \in R$
$(3,3)$ and $(3,6) \in R \rightarrow (3,6) \in R$
$(3,6)$ and $(6,2) \in R \rightarrow (3,2) \in R$
$(3,6)$ and $(6,3) \in R \rightarrow (3,3) \in R$
$(3,6)$ and $(6,6) \in R \rightarrow (3,6) \in R$
$(5,1)$ and $(1,1) \in R \rightarrow (5,1) \in R$
$(5,1)$ and $(1,5) \in R \rightarrow (5,5) \in R$
$(5,5)$ and $(5,1) \in R \rightarrow (5,1) \in R$
$(6,2)$ and $(2,2) \in R \rightarrow (6,2) \in R$
$(6,2)$ and $(2,3) \in R \rightarrow (6,3) \in R$
$(6,2)$ and $(2,6) \in R \rightarrow (6,6) \in R$
$(6,3)$ and $(3,2) \in R \rightarrow (6,2) \in R$
$(6,3)$ and $(3,3) \in R \rightarrow (6,3) \in R$
$(6,3)$ and $(3,6) \in R \rightarrow (6,6) \in R$
$(6,6)$ and $(6,2) \in R \rightarrow (6,2) \in R$
$(6,6)$ and $(6,3) \in R \rightarrow (6,3) \in R$

Hence R is Transitive.

From ①, ② and ③ the Relation R is Reflexive
Symmetric
Transitive
therefore R is Equivalence Relation.

Hence proved.

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$E(1)$ is Equivalence class of 1.

$E(1) = \{1, 5\}$

$E(2)$ is Equivalence class of 2.

$E(2) = \{2, 3, 6\}$

$E(6)$ is Equivalence class of 6

$E(6) = \{2, 3, 6\}$

5 marks

Q7) A function $h: R \to R$, $g: R \to R$, $f: R \to R$ are defined as

$f(x) = 2x+5$

$g(x) = x^2+1$

$h(x) = 2x-5$

find

i) fog

ii) goh

iii) foh

iv) hoh

v) gog

Solution: $h: R \to R$

$g: R \to R$

$f: R \to R$

$f(x) = 2x+5$

$g(x) = x^2+1$

$h(x) = 2x-5$

i) $fog(x) = f(x^2+1)$

$= 2(x^2+1)+5$

$= 2x^2+2+5$

$= 2x^2+7$

$\therefore \boxed{fog = fog(x) = 2x^2+7}$

ii) $goh(x) = g(2x-5)$

$= (2x-5)^2+1$

$= (2x)^2 - 2(2x)(5) + (5)^2 + 1$

$= 4x^2 - 20x + 25 + 1$

$goh(x) = 4x^2 - 20x + 26$

$\therefore \boxed{goh = goh(x) = 4x^2 - 20x + 26}$

iii) $foh(x) = f(2x-5)$

$= 2(2x-5)+5$

$= 4x - 10 + 5$

$foh(x) = 4x - 5$

$\therefore \boxed{foh = foh(x) = 4x - 5}$

iv) $hoh(x) = h(2x-5)$

$= 2(2x-5) - 5$

$= 4x - 10 - 5$

$hoh(x) = 4x - 15$

$\therefore \boxed{hoh = hoh(x) = 4x - 15}$

v) $gog(x) = g(x^2+1)$

$= (x^2+1)^2 + 1$

$= x^4 + 2x^2 + 1 + 1$

$gog(x) = x^4 + 2x^2 + 2$

$\therefore \boxed{gog = gog(x) = x^4 + 2x^2 + 2}$

5 marks

Q8) Let $A = B = C = R$ and let $f : A \to B$
$g : B \to C$ be defined by

$f(a) = a - 1$
$g(b) = b^2$

find
- $(f \circ g)(2)$
- $(g \circ f)(x)$
- $(f \circ f)(y)$
- $(g \circ f)(2)$
- $(g \circ g)(y)$

Solution: $f(a) = a - 1$
$g(b) = b^2$

i) $(f \circ g)(2) = f(4)$
$= 4 - 1$
$= 3$

$\boxed{(f \circ g)(2) = 3}$

ii) $(g \circ f)(2) = g(2-1)$
$= 1$

$\boxed{(g \circ f)(2) = 1}$

iii) $(g \circ f)(x) = g(x-1)$
$= (x-1)^2$
$= x^2 - 2x + 1$

$\boxed{(g \circ f)(x) = x^2 - 2x + 1}$

iv) $(f \circ f)(y) = f(y-1)$

$= (y-1)-1$

$= y-2$

$$\boxed{(f \circ f)(y) = y-2}$$

v). $(g \circ g)(y) = g(y^2)$

$= (y^2)^2$

$= y^4$

$$\boxed{(g \circ g)(y) = y^4}$$

5 marks

Q1) Let $A = \{1, 2, 3, 4\}$ and
$R = \{(1,2), (2,4), (1,3), (3,2)\}$
find the transitive closure of Relation R using Warshall's Algorithm.

Solution: $A = \{1, 2, 3, 4\}$
$R = \{(1,2), (2,4), (1,3), (3,2)\}$

$$M_R = W_0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

1st column: —
1st Row : 2, 3

$$W_1 = W_0 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

2nd column: 1, 3
2nd Row : 4

1 will be inserted at
(1,4), (3,4)

$$W_2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

3rd column : 1
3rd row : 2, 4
1 will be inserted at
(1,2), (1,4)

$$W_3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

4th column : 1, 2, 3
4th Row : ——

$W_3 = W_4$.

$$W_4 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Hence

$$M_{R^*} = W_4$$

∴ we obtain the Transitive closure

$R^* = \{(1,2), (1,3), (1,4), (2,4), (3,2)$
$(3,4)\}$

5 marks

Q2) Let $A = \{1,2,3,4\}$

$R = \{(1,1), (1,2), (1,4), (2,4), (3,1), (3,2), (4,2), (4,3), (4,4)\}$

Find the transitive closure of R using Warshall's Algorithm.

Solution: $A = \{1,2,3,4\}$

$R = \{(1,1), (1,2), (1,4), (2,4), (3,1), (3,2), (4,2), (4,3), (4,4)\}$

$$M_R = W_0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 1 & 1 & 1 \end{array}$$

1st Column: 1, 3

1st Row : 1, 2, 4

1 will be inserted at

(1,1), (1,2), (1,4), (3,1), (3,2), (3,4).

$$W_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 0 & 1 & 1 & 1 \end{array}$$

2nd Column: 1, 3, 4

2nd Row: 4

1 will be inserted at

(1,4), (3,4), (4,4).

$$W_2 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 0 & 1 & 1 & 1 \end{array}$$

3rd Column: 4
3rd Row: 1, 2, 4
1 will be inserted at
(4,1), (4,2), (4,4).

$$W_3 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 1 \\ 2 & 0 & 0 & 0 & 1 \\ 3 & 1 & 1 & 0 & 1 \\ 4 & 1 & 1 & 1 & 1 \end{array}$$

4th column: 1, 2, 3, 4
4th Row: 1, 2, 3, 4

1 will be inserted at
(1,1), (1,2), (1,3), (1,4)
(2,1), (2,2), (2,3), (2,4)
(3,1), (3,2), (3,3), (3,4)
(4,1), (4,2), (4,3), (4,4)

$$W_4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 \\ 3 & 1 & 1 & 1 & 1 \\ 4 & 1 & 1 & 1 & 1 \end{array}$$

$M_{R^*} = W_4$

$\therefore$ we obtain the transitive closure

$$R^* = \{ (1,1), (1,2), (1,3), (1,4)$$
$$(2,1), (2,2), (2,3), (2,4)$$
$$(3,1), (3,2), (3,3), (3,4)$$
$$(4,1), (4,2), (4,3), (4,4) \}$$

5 marks

Q9) Show that the mapping $f: R \to R$ given by

i) $f(x) = 4x - 3$

ii) $f(x) = 4x + 7$

is Bijective. Find it's Inverse.

Solution:

i) $f(x) = 4x - 3$

• For Injective or 1-1 function

Let

$$f(x_1) = f(x_2)$$

$$4x_1 - \cancel{3} = 4x_2 - \cancel{3}$$

$$\cancel{4}x_1 = \cancel{4}x_2$$

$$\boxed{x_1 = x_2}$$

$\therefore$ $f(x)$ is Injective function
1-1 function

ii) For onto or Surjective function

Let

$$y = f(x)$$

$$y = 4x - 3$$

$$4x = y + 3$$

$$x = \left(\frac{y+3}{4}\right)$$

Here

$\forall y \in R, \exists x \in R$

$\therefore$ function is onto.

If function $f(x) = 4x-3$ is 1-1 (Injective) and onto (Surjective) function then it is Bijective function.

function is Bijective therefore inverse are exist.

$$y = f(x)$$
$$x = f^{-1}(y)$$
$$x = \left(\frac{y+3}{4}\right)$$

ii) $f(x) = 4x+7$

- For injective or 1-1 function

Let

$$f(x_1) = f(x_2)$$
$$4x_1 + \cancel{7} = 4x_2 + \cancel{7}$$
$$\cancel{4}x_1 = \cancel{4}x_2$$
$$\boxed{x_1 = x_2}$$

$\therefore$ $f(x)$ is injective function
1-1 function

•) For onto or Surjective function

Let

$$y = f(x)$$

$$y = 4x + 7$$

$$4x = y - 7$$

$$x = \left(\frac{y-7}{4}\right)$$

Here

$$\forall y \in R; \ \exists x \in R$$

$\therefore$ function is onto.

If function $f(x) = 4x + 7$ is 1-1 (Injective) and onto (Surjective) function then it is Bijective function...

function is Bijective therefore inverse are exist

$$y = f(x)$$

$$x = f^{-1}(y)$$

$$\boxed{x = \left(\frac{y-7}{4}\right)}$$

Equivalence classes:

Q1) Let $\mathbb{Z}$ denote the set of integers. Let n be a positive integer and define a relation R on $\mathbb{Z}$ by setting aRb iff $n|(a-b)$. Show that R is an Equivalence relation and determine the Equivalence classes.

Solution:

① R is reflexive
since $n|(a-a)$
i.e. n divides zero

② R is symmetric
since $aRb \longrightarrow n|(a-b)$
which implies $n|(b-a)$
i.e. bRa

③ Let aRb and bRc Then $n|(a-b)$ and $n|(b-c)$ implies

$$n|[(a-b)+(b-c)]$$

$$n|(a-c)$$

i.e. aRc
Hence R is transitive.

$\therefore$ Therefore R is an Equivalence Relation.

The Equivalence classes are

$[0], [1], [2], \ldots [n-1]$

We denote the set of these Equivalence classes by Z_n

$Z_1 = \{[0]\} = \{\ldots -2, -1, 0, 1, 2, \ldots\} = Z$

$Z_2 = \{[0], [1]\}$

$Z_3 = \{[0], [1], [2]\}$

$Z_4 = \{[0], [1], [2], [3]\}$

and so on.

The relation R is known as the Congruence relation.

Q2) Let $\mathbb{Z}$ be the set of integers. Define a relation R on $\mathbb{Z}$ as aRb iff $6 \mid (a-b)$
Show that R is an Equivalence relation and find $\mathbb{Z}/R$

solution:

① Since $6 \mid (a-a)$,
aRa. Hence that R is reflexive.

② If $6 \mid (a-b)$ then $6 \mid (b-a)$
which show that R is symmetric

③ If $6 \mid (a-b)$ and $6 \mid (b-c)$ then
$6 \mid [(a-b)+(b-c)]$

i.e $6 \mid (a-c) \Rightarrow aRc$

Hence R is also transitive.

$\therefore$ R is an Equivalence Relation.

$$(\mathbb{Z}/R) = \{[0],[1],[2],[3],[4],[5]\}$$

where,

$$[0] = \{\ldots,-12,-6,0,6,12,18,\ldots\}$$

$$[1] = \{\ldots,-11,-5,1,7,13,\ldots\}$$

$[4] = \{\ldots, -2, 4, 10, 16, \ldots\}$

$[2] = \{\ldots, -10, -4, 2, 8, 14, \ldots\}$

$[3] = \{\ldots, -3, 3, 9, 15, \ldots\}$

$[5] = \{\ldots, -7, -1, 5, 11, 17, \ldots\}$

The Quotient set (Z/R) is denoted by Z_6 and is called as the set of congruence classes modulo 6. R is also called as a congruence relation.

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MSc(Math),BED,SET,GATE

5marks

Q10) A function $h: R \to R$ is defined by
$h(x) = x^2 - 1$
Test function whether it is
i) Injective
ii) Surjective
iii) Bijective.

Solution:

i) Let $h(x_1) = h(x_2)$

$$x_1^2 - \cancel{1} = x_2^2 - \cancel{1}$$

$$x_1^2 = x_2^2$$

$$\pm x_1 = \pm x_2$$

$+x_1 \begin{cases} +x_2 \\ -x_2 \end{cases}$ $\boxed{OR}$ $-x_1 \begin{cases} +x_2 \\ -x_2 \end{cases}$

$\therefore$ $h(x)$ is not Injective function

ii) Let $y = h(x)$

$$y = x^2 - 1$$

$$x^2 = y - 1$$

$$x = \pm\sqrt{y-1}$$

$\forall y \in R, \exists x \in R$

$\therefore$ $h(x)$ is not Surjective function

iii) Hence from (i) & (ii) function $h(x)$ is not Injective and $h(x)$ is not Surjective.

Therefore $h(x)$ is not Bijective function.

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Warshall's Algorithm

(10 marks)

Q1) Let $A = \{1, 2, 3, 4\}$ and
$R = \{(4,3), (2,2), (2,1), (3,1), (1,2)\}$
i) show that R is not transitive
ii) find the Transitive closure of R by using Warshall's Algorithm.

(10 marks)

Q2) Let $A = \{1, 4, 7, 13\}$ and
$R = \{(1,4), (4,7), (7,4), (1,13)\}$
i) show that R is not transitive
ii) find the Transitive closure of R by using Warshall's Algorithm.

(10 marks)

Q3) Let $A = \{1, 2, 3, 4\}$ and
$R = \{(1,2), (2,3), (3,4), (2,1)\}$
i) show that R is not transitive
ii) find the Transitive closure of R by using Warshall's Algorithm

(10 marks)

Q4) Let $A = \{11, 12, 13, 14\}$ and
$R = \{(11,12), (12,13), (13,14), (12,11)\}$
i) show that R is not transitive
ii) find the Transitive closure of R by using Warshall's Algorithm.

(10 marks)

Q1) Let $A = \{1,2,3,4\}$ and
$R = \{(4,3),(2,2),(2,1),(3,1),(1,2)\}$

i) Show that R is not transitive

ii) Find the transitive closure of R by using Warshall's Algorithm.

Solution:

$A = \{1,2,3,4\}$

i) $R = \{(4,3),(2,2),(2,1),(3,1),(1,2)\}$

$(4,3)$ and $(3,1) \in R \rightarrow (4,1) \notin R$

$\therefore$ R is not transitive.

ii)

$$M_R = W_0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

1st Column : 2, 3

1st Row : 2

1 will be inserted at $(2,2), (3,2)$

$$W_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 0 & 0 & 1 & 0 \end{array}$$

2nd column: 1, 2, 3
2nd Row: 1, 2
1 will be inserted at
(1,1), (1,2), (2,1), (2,2), (3,1), (3,2)

$$W_2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

3rd column: 4
3rd Row: 1, 2
1 will be inserted at
(4,1), (4,2)

$$W_3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

4th column: ——
4th Row: 1, 2, 3

$W_4 = W_3$

$$M_{R^*} = W_4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 0 & 0 \\ 2 & 1 & 1 & 0 & 0 \\ 3 & 1 & 1 & 0 & 0 \\ 4 & 1 & 1 & 1 & 0 \end{array}$$

$\therefore$ we obtain the Transitive Closure

$R^* = \{ (1,1), (1,2), (2,1), (2,2), (3,1)$
$(3,2), (4,1), (4,2), (4,3) \}$

(10 marks)

Q2) Let $A = \{1, 4, 7, 13\}$ and
$R = \{(1,4), (4,7), (7,4), (1,13)\}$

i) Show that R is not transitive

ii) find the transitive closure of R by using Warshall's Algorithm.

Solution:

i) $A = \{1, 4, 7, 13\}$
$R = \{(1,4), (4,7), (7,4), (1,13)\}$

$(1,4)$ and $(4,7) \in R \rightarrow (1,7) \notin R$

$\therefore$ R is not transitive.

ii)

$$M_R = W_0 = \begin{array}{c|cccc} & 1 & 4 & 7 & 13 \\ \hline 1 & 0 & 1 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 \\ 7 & 0 & 1 & 0 & 0 \\ 13 & 0 & 0 & 0 & 0 \end{array}$$

1st column : ——
1st Row : 4, 13

$W_1 = W_0$

$$W_1 = \begin{array}{c|cccc} & 1 & 4 & 7 & 13 \\ \hline 1 & 0 & 1 & 0 & 1 \\ 4 & 0 & 0 & 1 & 0 \\ 7 & 0 & 1 & 0 & 0 \\ 13 & 0 & 0 & 0 & 0 \end{array}$$

2nd column: 1, 7
2nd Row: 7
1 will be inserted at
(1,7), (7,7)

$$W_2 = \begin{array}{c} \\ 1 \\ 4 \\ 7 \\ 13 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 4 & 7 & 13 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

3rd column: 1, 4, 7
3rd Row: 4, 7
1 will be inserted at
(1,4), (1,7), (4,4), (4,7)
(7,4), (7,7)

$$W_3 = \begin{array}{c} \\ 1 \\ 4 \\ 7 \\ 13 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 4 & 7 & 13 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

4th column: 1
4th Row: ———

$W_4 = W_3$

$$M_{R^*} = W_4 = \begin{array}{c} \\ 1 \\ 4 \\ 7 \\ 13 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 4 & 7 & 13 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$\therefore$ We obtain the transitive closure

$R^* = \{(1,4), (1,7), (1,13), (4,4), (4,7), (7,4), (7,7)\}$

(10 marks)

Q3) Let $A = \{1,2,3,4\}$ and
$R = \{(1,2),(2,3),(3,4),(2,1)\}$

i) show that R is not transitive

ii) find the transitive closure of R by using Warshall's Algorithm.

Solution:

i)
$$A = \{1,2,3,4\}$$
$$R = \{(1,2),(2,3),(3,4),(2,1)\}$$

$(1,2)$ and $(2,1) \in R \longrightarrow (1,1) \notin R$

$\therefore$ R is not transitive

ii)

$$M_R = W_0 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

1st column: 2

1st Row: 2

1 will be inserted at (2,2)

$$W_1 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1 & 0 & 0 \\ 2 & 1 & 1 & 1 & 0 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

2nd column : 1, 2
2nd Row : 1, 2, 3
1 will be inserted at
(1,1), (1,2), (1,3)
(2,1), (2,2), (2,3)

$$W_2 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

3rd column: 1, 2
3rd Row : 4
1 will be inserted at
(1,4), (2,4).

$$W_3 = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

4th column : 1, 2, 3
4th Row : ———

$W_4 = W_3$

$$M_{R^*} = W_4 = \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 1 & 1 & 1 & 1 \\ 2 & 1 & 1 & 1 & 1 \\ 3 & 0 & 0 & 0 & 1 \\ 4 & 0 & 0 & 0 & 0 \end{array}$$

$\therefore$ We obtain the transitive closure

$$R^* = \{ (1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4), (3,4) \}$$

(10 marks)

Q4) Let $A = \{11, 12, 13, 14\}$ and
$R = \{(11,12), (12,13), (13,14), (12,11)\}$

i) Show that R is not transitive.

ii) Find the transitive closure of R by using Warshall's Algorithm.

Solution:

i) $A = \{11, 12, 13, 14\}$
$R = \{(11,12), (12,13), (13,14), (12,11)\}$

$(11,12)$ and $(12,11) \in R \longrightarrow (11,11) \notin R$

$\therefore$ R is not Transitive.

ii)

$$M_R = W_0 = \begin{array}{c} \\ 11 \\ 12 \\ 13 \\ 14 \end{array} \begin{array}{c} \begin{array}{cccc} 11 & 12 & 13 & 14 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

1st column : 12
1st Row : 12

1 will be inserted at (12,12)

$$W_1 = \begin{array}{c} \\ 11 \\ 12 \\ 13 \\ 14 \end{array} \begin{array}{c} \begin{array}{cccc} 11 & 12 & 13 & 14 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

2nd Column : 11, 12
2nd Row : 11, 12, 13
1 will be inserted at
(11, 11), (11, 12), (11, 13)
(12, 11), (12, 12), (12, 13).

$$W_2 = \begin{array}{c} \\ 11 \\ 12 \\ 13 \\ 14 \end{array} \begin{array}{c} \begin{array}{cccc} 11 & 12 & 13 & 14 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

3rd Column : 11, 12
3rd Row : 14
1 will be inserted at
(11, 14), (12, 14)

$$W_3 = \begin{array}{c} \\ 11 \\ 12 \\ 13 \\ 14 \end{array} \begin{array}{c} \begin{array}{cccc} 11 & 12 & 13 & 14 \end{array} \\ \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

4th Column : 11, 12, 13
4th Row : ——

$$W_4 = W_3$$

$$M_{R^*} = W_4 = \begin{array}{c|cccc} & 11 & 12 & 13 & 14 \\ \hline 11 & 1 & 1 & 1 & 1 \\ 12 & 1 & 1 & 1 & 1 \\ 13 & 0 & 0 & 0 & 1 \\ 14 & 0 & 0 & 0 & 0 \end{array}$$

$\therefore$ We obtain the transitive closure

$$R^* = \{(11,12), (11,11), (11,13), (11,14), (12,11), (12,12), (12,13), (12,14), (13,14)\}$$

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