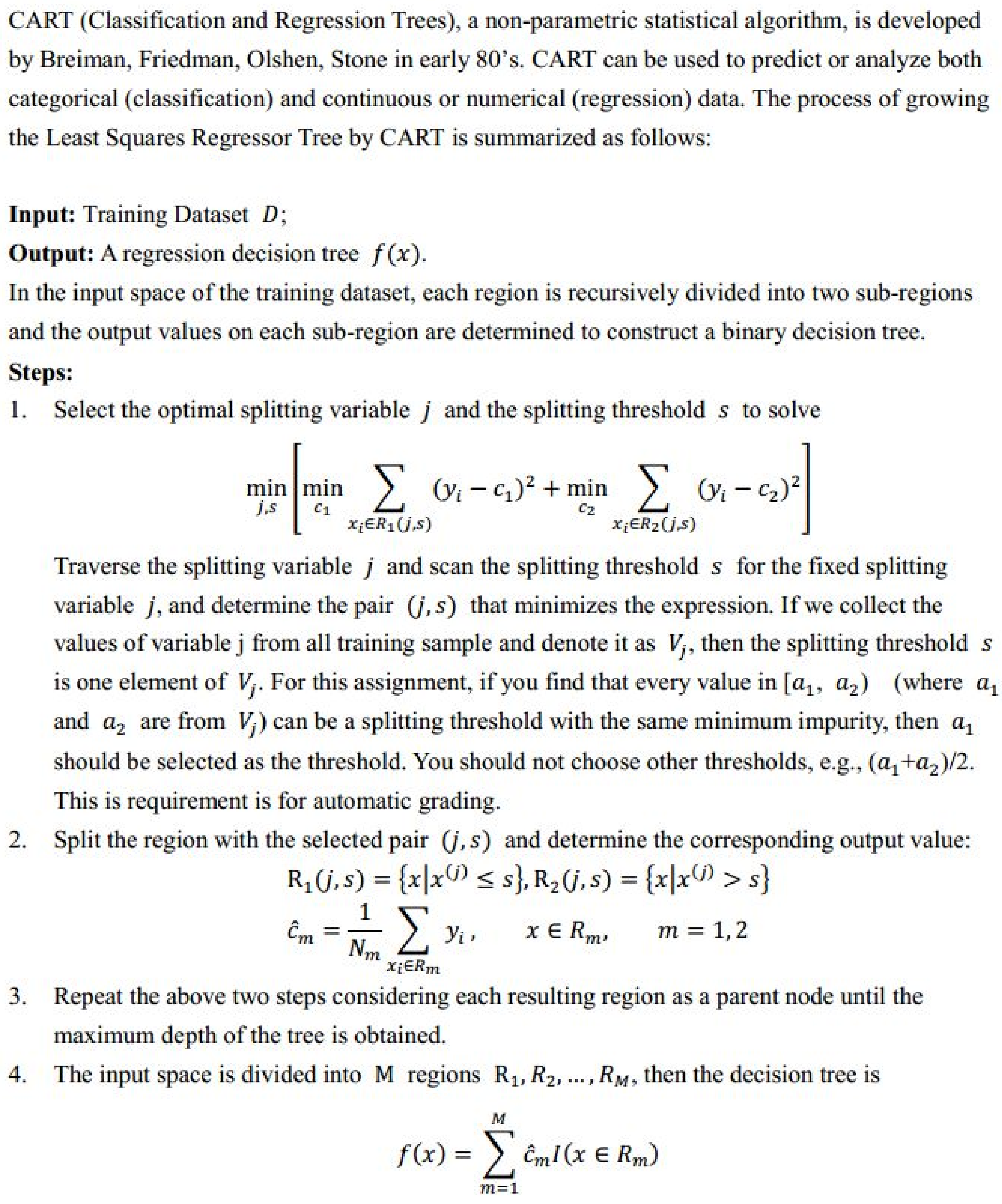
Reference: CS5228: Assignment



CART (Classification and Regression Trees), a non-parametric statistical algorithm, is developed by Breiman, Friedman, Olshen, Stone in early 80's. CART can be used to predict or analyze both categorical (classification) and continuous or numerical (regression) data. The process of growing the Least Squares Regressor Tree by CART is summarized as follows:

Input: Training Dataset D;

Output: A regression decision tree $f(x)$.

In the input space of the training dataset, each region is recursively divided into two sub-regions and the output values on each sub-region are determined to construct a binary decision tree.

Steps:

1. Select the optimal splitting variable j and the splitting threshold s to solve

$$\min_{j,s}\left[\min_{c_1}\sum_{x_i\in R_1(j,s)}(y_i-c_1)^2+\min_{c_2}\sum_{x_i\in R_2(j,s)}(y_i-c_2)^2\right]$$

   Traverse the splitting variable j and scan the splitting threshold s for the fixed splitting variable j, and determine the pair (j,s) that minimizes the expression. If we collect the values of variable j from all training sample and denote it as V_j, then the splitting threshold s is one element of V_j. For this assignment, if you find that every value in $[a_1,\ a_2)$ (where a_1 and a_2 are from V_j) can be a splitting threshold with the same minimum impurity, then a_1 should be selected as the threshold. You should not choose other thresholds, e.g., $(a_1+a_2)/2$. This is requirement is for automatic grading.

2. Split the region with the selected pair (j,s) and determine the corresponding output value:

$$\mathrm{R}_1(j,s)=\{x|x^{(j)}\le s\},\mathrm{R}_2(j,s)=\{x|x^{(j)}>s\}$$

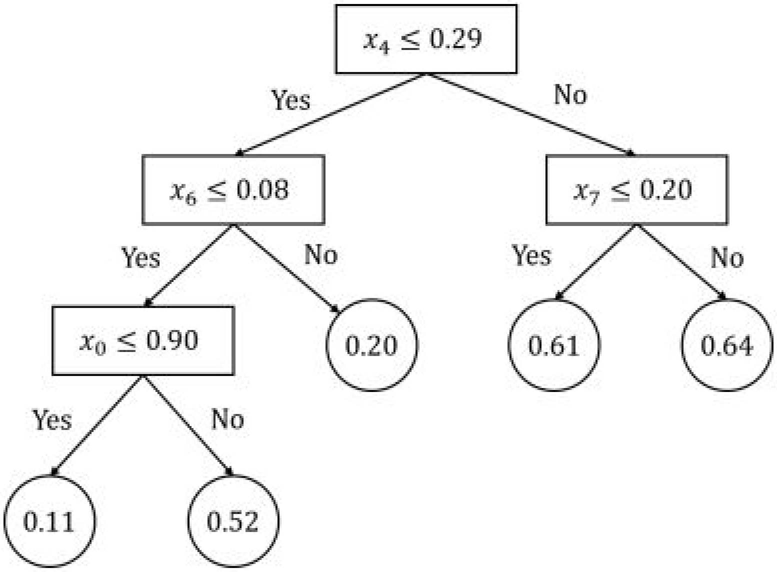
$$\hat{c}_m=\frac{1}{N_m}\sum_{x_i\in R_m}y_i\,,\qquad x\in R_m,\qquad m=1,2$$

3. Repeat the above two steps considering each resulting region as a parent node until the maximum depth of the tree is obtained.

4. The input space is divided into M regions $\mathrm{R}_1,R_2,\ldots,R_M$, then the decision tree is

$$f(x)=\sum_{m=1}^{M}\hat{c}_m I(x\in R_m)$$

For example,



the above decision tree can be represented by

```
self.root = {"splitting_variable": 4,
             "splitting_threshold": 0.29,
             "left": {"splitting_variable": 6,
                      "splitting_threshold": 0.08,
                      "left": {"splitting_variable": 0,
                               "splitting_threshold": 0.90,
                               "left": 0.11,
                               "right": 0.52},
                      "right":0.20}
             "right": {"splitting_variable": 7,
                       "splitting_threshold": 0.20,
                       "left": 0.61,
                       "right": 0.64}
             }
```