$$D\left(\int_\Omega S : \delta E\right)[\Delta u]$$

$$\int_\Omega \underline{D(S)[\Delta u] : \delta E} + \underline{\int_\Omega S : \underline{D(E)[\Delta u]}}$$

$$S : D(\delta F^T F + F^T \delta F)[\Delta u]$$

$$= S : \left(\delta F^T \nabla_X u + (\nabla_X u)^T \delta F\right)$$

$$\delta F = \nabla_X \delta u$$

$$D(\underline{E})[\Delta \underline{u}]$$

$$\underline{E} = \frac{1}{2}(\underline{C} - \underline{I})$$

$$F^T F - \underline{I}$$

$$= \frac{1}{2}\left(D(\underline{C})[\Delta \underline{u}] - \cancel{D(\underline{I})(\Delta \underline{u})}^{20} \right)$$

$$= \frac{1}{2}\left(D(\underline{F}^T)(\Delta \underline{u}) F + F^T D[F](\Delta \underline{u}] \right)$$

$$= \frac{1}{2}\left(\left(F^T (\nabla_X \underline{u}) \right)^T F + F^T (\nabla_X \underline{u}) \cdot F \right)$$

$$\delta \underline{E} = D \underline{E}[\delta \underline{u}] = (\nabla_X \underline{u})^T F + F^T (\nabla_X \underline{u})$$

$$\underline{\int_\Omega D(S)[\Delta u] : \delta E} \quad + \int_\Omega S : D(\delta E)[\underline{u}]$$

$$= \qquad + \underline{\int S : \tfrac{1}{2}\left(\nabla_{\underline{X}} \underline{u}^T F + F^T \nabla_{\underline{X}} \underline{u}\right)}$$

$$\underline{S} = S_{eos} + S_{el} \rightarrow S_{eos} = -J\, p_{eos}\, C_e^{-1}$$

$$S_{el} = \mathbb{C} : \left(E_e - \alpha\,\underline{(T - T_0)}\right)$$

$$\underline{E}_e = \frac{1}{2}\left(F_e^T F_e - \underline{I}\right)$$

$$D(S_{el})[\Delta \underline{u}]$$

$$= \cancel{D(\mathbb{C})[\Delta u]}^{0} + \mathbb{C} : D[\underline{E_e} - \cancel{\alpha(T - T_0)}[\Delta \underline{u}]]$$

$$\mathbb{C} : \frac{1}{2}\left(D(F_e^T)[\Delta \underline{u}] F_e + F_e^T D(F_e)[\Delta \underline{u}]\right)$$

$$\boxed{D(S_{el})[\Delta \underline{u}] = \mathbb{C} : \frac{1}{2}\left(\nabla_X \underline{u}^T F_e + F_e^T \nabla_X \underline{u}\right)}$$

$$S_{eos} = -\underline{J}\, p_{eos}\, \underline{C_e}^{-1}$$

$$D(S_{eos})[\Delta \underline{u}] = -p_{eos}\left(D[\underline{J}][\Delta \underline{u}] + D(C_e^{-1})[\Delta \underline{u}]\right)$$

$$\left.\frac{\partial C_e^{-1}}{\partial C_e} : D(C_e)[\Delta \underline{u}]\right) \qquad \frac{\partial C_e^{-1}}{\partial \underline{C}_e} = -C^{-1} \otimes C^{-1}$$

$$-p_{eos}\left(\underline{J F^{-T} : \nabla_X \underline{u}} + \underline{\frac{\partial C_e^{-1}}{\partial C_e}}\right.$$

$$-p_{eos}\left(J F^{-T} : \nabla_{X} \underline{u} + \left(-\underline{C}^{-1} \otimes \underline{C}^{-1}\right) : D_i(C)[\Delta \underline{u}]\right) = D_i(S_{e})[\Delta \underline{v}]$$

$$\int_\Omega S : \delta E$$

$$S = S_{el} + S_{eos} :$$

$$\rightarrow \int_\Omega S_{el} : \delta E + \int_\Omega S_{eos} : \delta E$$

$$S_{el} = \mathbb{C} : (E_{el} - \alpha(T - T_0))$$

$$D\left(\int_\Omega S_{el} : \delta E\right) = \int_\Omega D(S_{el}) : \delta E + \int_\Omega S_{el} : D(\delta E)$$

$$= \int_\Omega D(E_{el}) : \mathbb{C} : \delta E + \int_\Omega S_{el} : D(\delta E)$$

$$= \int \left((\nabla_X (\Delta u))^T \underline{F} + \underline{F}^T (\nabla_X (\Delta u)) \right) : \underline{\mathbb{C}} : \underline{\delta E} + \int_\Omega \underline{S_{el}} : \left(\delta F^T \nabla_X u + (\nabla_X u)^T \delta F \right)$$

$$+ \int_\Omega S_{el} : \left(\nabla_X \delta v^T \nabla_X u + (\nabla_X u)^T \nabla_X \delta v \right)$$

$$\int_\Omega S_{eos} : \delta E = \int_\Omega -J p_{eos} C_{el}^{-1} : \delta E$$

$$D \int_\Omega S_{eos} : \delta E = \int_\Omega D(S_{eos}) : \delta E + \int_\Omega S_{eos} : D(\delta E)$$

$$\Delta \delta E$$

$$D(S_{eos}) = D(-J p_{eos} C_e^{-1})$$

$$= -D(J) p_{eos} C_e^{-1} - J p_{eos} D(C_e)^{-1}$$

$$= -p_{eos} \left(D(J) C_e^{-1} + J D(C_e^{-1}) \right) : \delta E$$

$$(JF^T : \nabla_{\underline{x}} u) C_e^{-1} + J \left(\frac{\partial C_e^{-1}}{\partial C_e} : D(C_e) \right)$$