

$$D \left(\int_{\Omega} S : SE \right) [u]$$

$$\int_{\Omega} \underline{D(S[u]) : SE} + \int_{\Omega} \underline{S : D(E)[u]}$$

$$S : D(\delta F^T(F) + F^T \delta F) [u]$$

$$\Sigma S : (\delta F^T \underline{\nabla_x u} + (\underline{\nabla_x u})^T \delta F)$$

$$\delta F = \underline{\nabla_x S u}$$

$$D(E)[\Delta u]$$

$$E = \frac{1}{2} (C - I)$$

$$= \frac{1}{2} (D(C)[\Delta u] - \cancel{D(E)[\Delta u]}) \quad F^T F = I$$

$$= \frac{1}{2} (D(F^T C \Delta u) F + F^T D(F \Delta u))$$

$$= \frac{1}{2} (F^T (\nabla_x u) F + F^T (\nabla_x u) F)$$

$$\delta E = D E[\delta u] = (\nabla_x u)^T F + F^T (\nabla_x u)$$

$$\int_{\Omega} D(\beta) [\Delta u] : \delta E$$

$$+ \int_{\Omega} S : D(\delta E) [u]$$

$$+ \int_{\Omega} S : \frac{1}{2} (\nabla_{\underline{x}} u^T F + F^T \nabla_{\underline{x}} u)$$

11

$$\underline{S} = S_{eos} + S_{el} \rightarrow S_{eos} = -J p_{eos} \underline{C}_e^{-1}$$

$$S_{el} = Q : (\underline{E}_e - \alpha (\underline{T} - T_0))$$

$$\underline{E}_e = \frac{1}{2} (\underline{F}_e^T \underline{F}_e - \underline{I})$$

$$D(S_{el})[\Delta \underline{u}]$$

$$= \cancel{D(Q)[\Delta \underline{u}]} + Q : D(\underline{E}_e - \alpha (\underline{T} - T_0))[\Delta \underline{u}]$$

$$Q : \frac{1}{2} (D(\underline{F}_e^T) \underline{C}_{\underline{u}} \underline{F}_e + \underline{F}_e^T D(\underline{F}_e) \underline{C}_{\underline{u}})$$

$$D(S_{el})[\Delta \underline{u}] = Q : \frac{1}{2} (\nabla_{\underline{x}} \underline{u}^T \underline{F}_e + \underline{F}_e^T \nabla_{\underline{x}} \underline{u})$$

$$S_{eos} = -J p_{eos} \underline{C}_e^{-1}$$

$$D(S_{eos})[\Delta \underline{u}] = -p_{eos} (D[J][\Delta \underline{u}] + D(\underline{C}_e^{-1})[\Delta \underline{u}])$$

$$-p_{eos} (J F^{-T} : \nabla_{\underline{x}} \underline{u} +$$

$$\frac{\partial \underline{C}_e^{-1}}{\partial \underline{C}_e} : D(\underline{C}_e)[\Delta \underline{u}])$$

$$\frac{\partial \underline{C}_e^{-1}}{\partial \underline{C}_e} = -\underline{C}_e^{-1} \otimes \underline{C}_e^{-1}$$

$$- \text{Rees } (JF^{-1} : \nabla_{\mathbb{A}^1}^U +$$

$$(-\mathbb{C}^1 \otimes \mathbb{C}^1) : D_1(\mathbb{C}) \cong \mathbb{C} \Rightarrow (Se_1) \cup \mathbb{C}^1$$