The tikz package

This is a general purpose graphics package. To load it for this document, I used:

```
\usepackage{tikz}
\usetikzlibrary{matrix,arrows,decorations.pathmorphing}
```

There are now three ways to enter commutative diagrams using tikz: with the package tikz-cd, with matrix, and directly with tikz (listed roughly in order of decreasing ease but increasing flexibility).

$$A \stackrel{a}{\longrightarrow} B \qquad \qquad A \stackrel{a}{\longrightarrow} B \qquad \qquad b$$

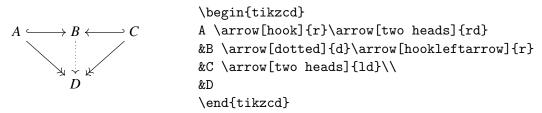
This is part of: Guide to Commutative Diagrams, www.jmilne.org/not/CDGuide.html Last revised October 30, 2012

Using tikz-cd

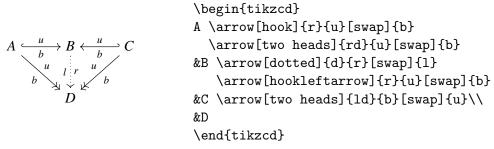
Load¹ this with \usepackage{tikz-cd}. As the code on p.1 illustrates, the syntax for tikz-cd is similar to that of array. Note that tikz-cd handles large objects and tall labels better than amscd:

$$\begin{array}{cccc}
A \times A \times A \times A \times A \times A & \xrightarrow{a} & B & & A \xrightarrow{a} & B \\
\downarrow b & & \downarrow c & & \downarrow b & \downarrow c \\
C & \xrightarrow{d} & D & & C \xrightarrow{\overline{AA^A}} & D
\end{array}$$

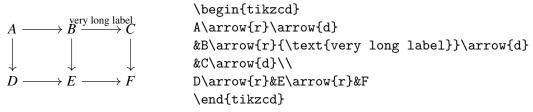
The next example illustrates the use of different arrows in a commutative diagram:



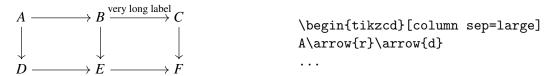
Now an example with labels on the arrows:



Long labels may cause problems:



However, this can be fixed as follows:



tikz-cd does not have a problem with objects of different heights.

$$\hat{A} \longrightarrow \prod_{n \in \mathbb{Z}} A_n \longrightarrow \prod_{n \in \mathbb{Z}} A_n.$$

 $^{^{1}}$ Before using tikz-cd, check that your $T_{E}X$ installation is using version 2.10 of pgf — you can do this by running $T_{E}X$ on a file containing pgfversion.

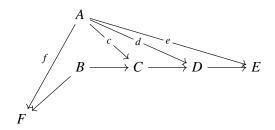
Curving arrows is easy.

$$A \begin{tikzcd} \\ A \arrow[bend left]{r}\arrow[bend right]{r}&B \\ \end{tikzcd} \\$$

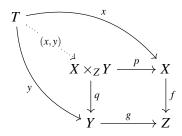
When a diagram is numbered, this is placed correctly:

$$\begin{array}{ccc}
A & \xrightarrow{a} & B \\
\downarrow_{b} & \downarrow_{c} \\
C & \xrightarrow{d} & D
\end{array} \tag{1}$$

Two more examples:



\begin{tikzcd}
&A\arrow{ldd}[swap]{f}\arrow{rd}[description]{c}
\arrow{rrd}[description]{d}
\arrow{rrrd}[description]{e}\\
&B\arrow{ld}\arrow{r}&C\arrow{r}&D\arrow{r}&E\\
F
\end{tikzcd}

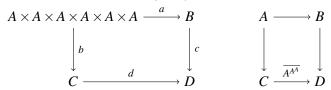


```
\begin{tikzcd}
T\arrow[bend left]{drr}{x}
\arrow[bend right]{ddr}[swap]{y}
\arrow[dotted]{dr}[description]{(x,y)} & & \\
& X \times_Z Y \arrow{r}{p} \arrow{d}{q} & X \arrow{d}{f} \\
& Y \arrow{r}{g} & Z
\end{tikzcd}
```

Using matrix

The code on p.1 sets up a matrix named m with some options, and then places A, B, C, and D at the four positions of a 2×2 matrix. The next line specifies normal arrows with labels in scriptsize and a nondefault arrow head, and the following line specifies an arrow from the (1,1) position of the matrix m to the (1,2) position with a label a in the default position.

Note that tikz handles large objects and tall labels better than amscd:



To my eyes, the arrow heads are too small.² This can be fixed by adding >=angle 90, as an option to the path or to the whole picture:

```
______ \path[->](1,1) edge (2,1);
______ \path[->,>=angle 90](1,1) edge (2,1);
```

Here is the code for some arrows.

```
path[->](1,1) edge (2,1);

path[->](1,1) edge (2,1);

path[-](1,1) edge (2,1);

path[right hook->](1,1) edge (2,1);

path[->>](1,1) edge (2,1);

path[dotted,->](1,1) edge (2,1);

path[dashed,->](1,1) edge (2,1);

path[*->](1,1) edge (2,1);

and the path are part of the path are path ar
```

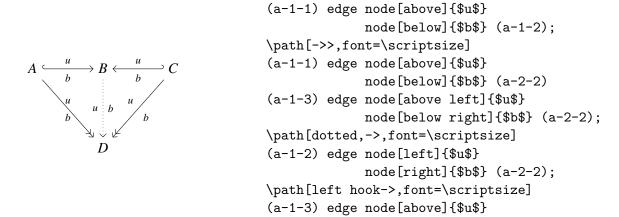
The next example illustrates the use of the different arrows in a commutative diagram

```
\label{eq:localization} $\operatorname{\mathtt{matrix}}(a) [\operatorname{\mathtt{matrix}} \text{ of math nodes}, \\ \operatorname{\mathtt{row}} \text{ sep=3em, column sep=2.5em,} \\ \operatorname{\mathtt{text}} \text{ height=1.5ex, text depth=0.25ex}] \\ \{A\&B\&C\setminus \\ \&D\setminus \}; \\ \operatorname{\mathtt{path}}[\operatorname{\mathtt{right}} \text{ hook->}] (a-1-1) \text{ edge } (a-1-2); \\ \operatorname{\mathtt{path}}[\operatorname{\mathtt{lotted}}, ->] (a-1-1) \text{ edge } (a-2-2); \\ \operatorname{\mathtt{path}}[\operatorname{\mathtt{left}} \text{ hook->}] (a-1-3) \text{ edge } (a-1-2); \\ \operatorname{\mathtt{path}}[->>] (a-1-3) \text{ edge } (a-2-2); \\ \operatorname{\mathtt{lend}}\{\text{tikzpicture}\} $
```

\begin{tikzpicture}[>=angle 90]

 $^{^2}$ See http://tex.stackexchange.com/questions/37320/ for an erudite discussion of this problem, with solutions.

Now an example with labels on the arrows:



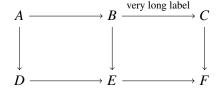
\path[right hook->,font=\scriptsize]

 $node[below]{$b$} (a-1-2);$

Long labels may cause a problem:

```
\begin{tikzpicture}
                                \matrix(m)[matrix of math nodes,
                                row sep=3em, column sep=2.5em,
                                 text height=1.5ex, text depth=0.25ex]
                                 {A&B&C\\
A \longrightarrow B \xrightarrow{\text{very long label}} C
\downarrow \qquad \qquad \downarrow \qquad \qquad \downarrow
                                D&E&F\\};
                                \path[->,font=\scriptsize]
                                (m-1-1) edge (m-1-2)
                                           edge (m-2-1)
                                 (m-1-2) edge node[auto] {very long label} (m-1-3)
                                           edge (m-2-2)
                                 (m-1-3) edge (m-2-3)
                                 (m-2-1) edge (m-2-2)
                                 (m-2-2) edge (m-2-3);
                                 \end{tikzpicture}
```

However, this can be fixed by setting column sep=5.0em.



tikz does not have a problem with objects of different heights.

$$\hat{A} \longrightarrow \prod_{n \in Z} A_n \longrightarrow \prod_{n \in Z} A_n.$$

But that is because of the options text height=1.5ex, text depth=0.25ex. When you omit

them, you get:

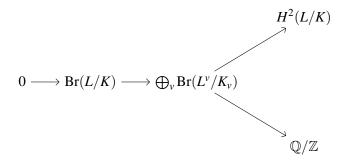
$$\hat{A} \longrightarrow \prod_{n \in \mathbb{Z}} A_n \longrightarrow \prod_{n \in \mathbb{Z}} A_n.$$

Curving arrows is easy.

\begin{tikzpicture}
\matrix(m) [matrix of math nodes,
row sep=3em, column sep=2.8em,
text height=1.5ex, text depth=0.25ex]
{A&B\\};
\path[->]
(m-1-1) edge [bend left] (m-1-2)
 edge [bend left=40] (m-1-2)
 edge [bend left=60] (m-1-2)
 edge [bend left=80] (m-1-2)
 edge [bend right] (m-1-2);



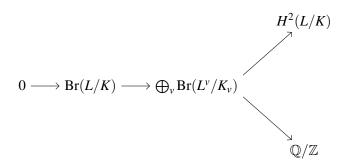
Arrows may not attach themselves correctly to the nodes:



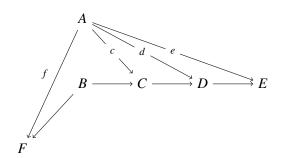
\end{tikzpicture}

To fix this, use

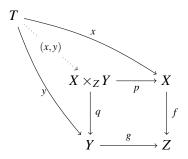
$$(m-2-3.north east) edge $(m-1-4)$
 $(m-2-3.south east) edge $(m-3-4)$;$$$



Two more examples:



```
1/
\begin{tikzpicture}[descr/.style={fill=white}]
\matrix(m)[matrix of math nodes, row sep=3em, column sep=2.8em,
text height=1.5ex, text depth=0.25ex]
\{&A\\\&B\&C\&D\&E\\F\\\};
\path[->,font=\scriptsize]
(m-1-2) edge node[above left] {$f$} (m-3-1)
        edge node[descr] {$c$} (m-2-3)
        edge node[descr] {$d$} (m-2-4)
        edge node[descr] \{\$e\$\}\ (m-2-5);
\path[->]
(m-2-2) edge (m-3-1)
        edge (m-2-3);
\path[->]
(m-2-3) edge (m-2-4);
\path[->]
(m-2-4) edge (m-2-5);
\end{tikzpicture}
\]
```



```
\[
\begin{tikzpicture}[descr/.style={fill=white}]
\matrix(m)[matrix of math nodes, row sep=3em, column sep=2.8em,
text height=1.5ex, text depth=0.25ex]
{T\\&X\\times_Z Y&X\\&Y\Z\\};
```

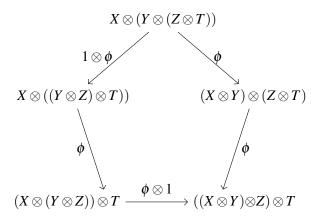
```
\path[->,font=\scriptsize]
(m-1-1) edge [bend left=10] node[above] {$x$} (m-2-3)
(m-1-1) edge [bend right=10] node[below] {$y$} (m-3-2);
\path[->,dotted,font=\scriptsize]
(m-1-1) edge node[descr] {$(x,y)$} (m-2-2);
\path[->,font=\scriptsize]
(m-2-2) edge node[below] {$p$} (m-2-3)
(m-2-2) edge node[right] {$q$} (m-3-2);
\path[->,font=\scriptsize]
(m-2-3) edge node[right] {$f$} (m-3-3);
\path[->,font=\scriptsize]
(m-3-2) edge node[above] {$g$} (m-3-3);
\end{tikzpicture}
\]
```

Using tikz directly

Instead of using a matrix grid, you can use tikzpicture directly to construct a diagram.

```
\\ \text{begin{tikzpicture}} \\ \text{node (A) at (-1,0) {$A$};} \\ \text{node (B) at (1,0) {$B$};} \\ \text{node at (0,0) {\rotatebox{270}{$\Rightarrow$}};} \\ \text{path[->,font=\scriptsize,>=angle 90]} \\ (A) \text{edge [bend left] node[above] {$\alpha$} (B) \\ \text{edge [bend right] node[below] {$\alpha$} (B);} \\ \text{end{tikzpicture}} \end{tikzpicture}
```

(rotatebox requires graphicx.)



Pentagon Axiom

```
\begin{tikzpicture}
\node (P0) at (90:2.8cm) {$X\otimes (Y\otimes (Z\otimes T))$};
\node (P1) at (90+72:2.5cm) {$X\otimes ((Y\otimes Z)\otimes T))$};
\node (P2) at (90+2*72:2.5cm) {$\mathllap{(X\otimes (Y\otimes Z))}\otimes T$};
\node (P3) at (90+3*72:2.5cm) {$((X\otimes Y)\mathrlap{\otimes Z)\otimes T}$};
\node (P4) at (90+4*72:2.5cm) {$(X\otimes Y)\otimes (Z\otimes T)$};
\draw
(P0) edge[->,>=angle 90] node[left] {$1\otimes\phi$} (P1)
(P1) edge[->,>=angle 90] node[left] {$\phi$} (P2)
(P2) edge[->,>=angle 90] node[above] {$\phi\otimes 1$} (P3)
(P4) edge[->,>=angle 90] node[right] {$\phi$} (P3)
(P0) edge[->,>=angle 90] node[right] {$\phi$} (P4);
\end{tikzpicture}
```

Here I used \mathlap and \mathrlap to adjust the positions of the nodes. They require the package mathtools.

When you number a displayed commutative diagram

```
\begin{equation}
\begin{tikzpicture}
.....
```

\end{tikzpicture}
\end{equation}

$$\begin{array}{ccc}
A & \xrightarrow{a} & B & \xrightarrow{b} & C \\
\downarrow^{c} & \downarrow^{d} & \downarrow^{e} \\
D & \xrightarrow{f} & E & \xrightarrow{g} & F
\end{array}$$
(2)

the number appears below the level of the diagram. To centre the number, use:

\begin{equation}
\begin{tikzpicture}[baseline=(current bounding box.center)]
.....
\end{tikzpicture}
\end{equation}

$$\begin{array}{ccc}
A & \xrightarrow{a} & B & \xrightarrow{b} & C \\
\downarrow^{c} & \downarrow^{d} & \downarrow^{e} \\
D & \xrightarrow{f} & E & \xrightarrow{g} & F
\end{array} \tag{3}$$

Here are two examples with multiple arrows:

$$Y \times_X Y \xrightarrow{p_1} Y \longrightarrow X$$

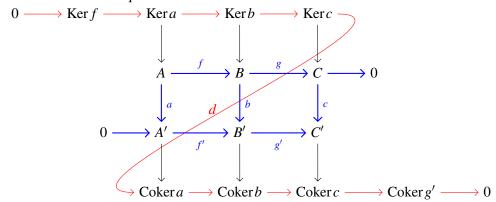
\begin{tikzpicture}
\node (a) at (0,0) {\$Y\times_X Y\$};
\node (b) at (2,0) {\$Y\$};
\node (c) at (3.5,0) {\$X\$};
\path[->,font=\scriptsize,>=angle 90]
([yshift= 2pt]a.east) edge node[above] {\$p_1\$} ([yshift= 2pt]b.west)
([yshift= -2pt]a.east) edge node[below] {\$p_2\$} ([yshift= -2pt]b.west)
(b) edge (c);
\end{tikzpicture}

$$\mathsf{S}(Z) \xleftarrow{i^*} \underbrace{i^*}_{i_!} \longrightarrow \mathsf{S}(X) \xleftarrow{j_!} \underbrace{j^*}_{j_*} \longrightarrow \mathsf{S}(U).$$

\begin{tikzpicture}[descr/.style={fill=white},text height=1.5ex, text depth=0.25ex]
\node (a) at (0,0) {\$\mathsf{S}(Z)\$};
\node (b) at (2.5,0) {\$\mathsf{S}(X)\$};
\node (c) at (5,0) {\$\mathsf{S}(U).\$};
\path[->,font=\scriptsize,>=angle 90]
([yshift= 9pt]b.west) edge node[above] {\$i^{\ast}\$} ([yshift= 9pt]a.east)

```
(a.east) edge node[descr] {$i_{\ast}$} (b.west)
([yshift= -9pt]b.west) edge node[below] {$i^!$} ([yshift= -9pt]a.east)
([yshift= 9pt]c.west) edge node[above] {$j_!$} ([yshift= 9pt]b.east)
(b.east) edge node[descr] {$j^{\ast}$} (c.west)
([yshift= -9pt]c.west) edge node[below] {$j_*$} ([yshift= -9pt]b.east);
\end{tikzpicture}
```

One final example: the extended snake lemma says that the exact commutative diagram in blue gives rise to the exact sequence in red.



```
\begin{tikzpicture}[>=angle 90,scale=2.2,text height=1.5ex, text depth=0.25ex]
%%First place the nodes
\node (k-1) at (0,3) \{$0$\};
\node (k0) [right=of k-1] {$\Ker f$};
\node (k1) [right=of k0] {$\Ker a$};
\node (k2) [right=of k1] {$\Ker b$};
\node (k3) [right=of k2] {$\Ker c$};
\node (a1) [below=of k1] {$A$};
\node (a2) [below=of k2] {$B$};
\node (a3) [below=of k3] \{C\};
\node (a4) [right=of a3] {$0$};
\node (b1) [below=of a1] {$A'$};
\node (b0) [left=of b1] {$0$};
\node (b2) [below=of a2] {$B'$};
\node (b3) [below=of a3] {$C'$};
\node (c1) [below=of b1] {$\Coker a$};
\node (c2) [below=of b2] {\color{b}};
\node (c3) [below=of b3] {$\Coker c$};
\node (c4) [right=of c3] {$\Coker g'$};
\node (c5) [right=of c4] {$0$};
%%Draw the red arrows
\draw[->,red,font=\scriptsize]
(k-1) edge (k0)
(k0) edge (k1)
(k1) edge (k2)
(k2) edge (k3)
```

```
(c1) edge (c2)
(c2) edge (c3)
(c3) edge (c4)
(c4) edge (c5);
%%Draw the curvy red arrow
\draw[->,red]
(k3) edge[out=0,in=180,red] node[pos=0.55,yshift=5pt] {$d$} (c1);
%%Draw the black arrows
\draw[->]
(k1) edge (a1)
(k2) edge (a2)
(k3) edge (a3)
(b1) edge (c1)
(b2) edge (c2)
(b3) edge (c3);
%%Draw the thick blue arrows
\draw[->,font=\scriptsize,blue,thick]
(a1) edge node[auto] {$f$} (a2)
(a2) edge node[auto] {$g$} (a3)
(a3) edge (a4)
(a1) edge node[auto] {$a$} (b1)
(a2) edge node[auto] {$b$} (b2)
(a3) edge node[auto] {$c$} (b3)
(b0) edge (b1)
(b1) edge node[below] {$f'$} (b2)
(b2) edge node[below] {$g'$} (b3);
\end{tikzpicture}
   For the last diagram, I added the following lines to the preamble
\usepackage{amsmath}
\DeclareMathOperator{\Coker}{Coker}
\DeclareMathOperator{\Ker}{Ker}
```

\usetikzlibrary{positioning}