Design of Quadrature Mirror Filter Bank using Particle Swarm Optimization (PSO)

A. Kumar, G. K. Singh, and R. S. Anand
Department of Electrical Engineering
Indian Institute of Technology
Roorkee, Uttrakhand – 247667, India
Email: gksngfee@iitr.ernet.in, anandfee@iitr.ernet.in

***Abstract*—In this paper, the particle swarm optimization technique is used for the design of a two channel quadrature mirror filter (QMF) bank. A new method is developed to optimize the prototype filter response in passband, stopband and overall filter bank response. The design problem is formulated as nonlinear unconstrained optimization of an objective function, which is weighted sum of square of error in passband, stop band and overall filter bank response at frequency (ω=0.5π). For solving the given optimization problem, the particle swarm optimization (PSO) technique is used. As compared to the conventional design techniques, the proposed method gives better performance in terms of reconstruction error, mean square error in passband, stopband, and computational time. Various design examples are presented to illustrate the benefits provided by the proposed method.**

***Index Terms*—Filter banks, quadrature mirror filter, subband coding.**

I. Introduction

Over the past two decades, the research on efficient design of filter banks has received considerable attention as improved design can have significant impact on numerous fields such as speech coding, scrambling, image processing, and transmission of several signals through same channel [1]. Among the various filter banks, two-channel QMF bank was the first type of filter bank used in signal processing applications for separating signals into subbands and reconstructing them from individual subbands [2, 3]. Subsequently, a substantial progress has been made in other fields like antenna systems [4], analog to digital (A/D) convertor [5], and design of wavelet base [6] due to advances in QMF bank. In two channel QMF bank, input signal (x[n]) is divided into two equal subbands with the help of analysis filters ($H_0(z);H_1(z)$) and then, these subbands are decimated by a factor two. Finally, these are up sampled before being recombined into the reconstruction output y[n] with the help of synthesis filters ($G_0(z);G_1(z)$) as shown in Fig. 1. Ideally, the reconstructed output signal is an exact replica of the input signal with some delay, called perfect reconstruction. But, it is not possible due to aliasing distortion, phase distortion, and amplitude distortion [1].

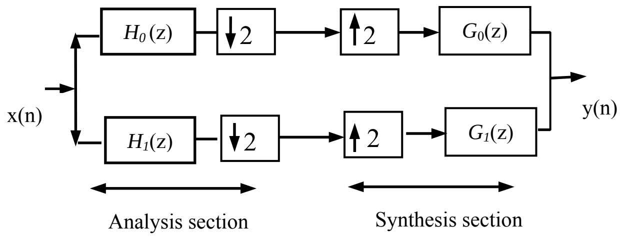

Fig. 1. Quadrature mirror filter bank

Various design techniques including optimization based, and non optimization based techniques have been reported in literature for the design of QMF bank. In optimization based technique, the design problem is formulated either as multi-objective or single objective nonlinear optimization problem, which is solved by various existing methods such as genetic algorithm [7-9], least square technique [10-12], and weighted least square (WLS) technique [13-21]. In early stage of research, the design methods developed were based on direct minimization of error function in frequency domain [10, 11]. But due to high degree of nonlinearity and complex optimization technique, these methods were not suitable for the filter with larger taps. Therefore, Jain and Crochiere [12] have introduced the concept of iterative algorithm and formulated the design problem in quadratic form in time domain. Thereafter, several new iterative algorithms [13, 17-22] have been developed either in time domain or frequency domain. In general, reconstruction error in above mentioned methods [10, 12] is not equiripple. Therefore, Chen and Lee [13] have proposed an iterative technique that results in equiripple reconstruction error, and the generalization of this method was carried out in [14-16] to obtain equiripple behaviors in stopband. Unfortunately, these techniques are complicated, and are only applicable to the two-band QMF banks that have low orders. Xu *et al* [17-19] has proposed some iterative methods in which, the perfect reconstruction condition is formulated in time domain for reducing computational complexity in the design. For some application, it is required that the reconstruction error shows equiripple behavior, and the stop band energies of filters are to be kept at minimum value. To solve these problems, a two-step approach for the design of two-channel filter banks was developed [23, 24]. But this approach results in nonlinear phase, and is not suitable for the wideband audio signal. Therefore, a new algorithm for the design of QMF banks using linear optimization has developed in which either the passband

*corresponding author: A. Kumar, Phone: +91-9411755329, Email: anilkdee@iitr.ernet.in

frequency (ω_p) or cutoff frequency (ω_c) is optimized to minimize the reconstruction error [25, 26]. In all such method discussed above, the filter response in transition band has not been taken into account. The design problem was constructed using either linear or nonlinear combination of reconstruction error and stop band residual energy or passband energy, which was solved by some classical optimization techniques which might suffer from slow convergence. Particle swarm optimization (PSO), a computational intelligence based technique has emerged as a powerful and robust tool for solving the real valued optimization problem [27-30].

In the above context, this paper presents a new algorithm for the design of two channels QMF bank using PSO technique. The paper is organized as fellows. A brief introduction has been provided in this section on the existing design techniques of QMF banks. Section II contains the formulation of the design problem. In section III, the basic concept of PSO technique is explained. Finally, the comparison of results is described in section IV, followed by conclusion in section V.

II. Formulation of Design Problem

The basic structure of a QMF bank to be considered is shown in Fig. 1, the reconstructed output signal is defined as [1]:

$$\begin{aligned} Y(z) &= 1/2\left[H_0(z)G_0(z)+H_1(z)G_1(z)\right]X(z) \\ &\quad +1/2\left[H_0(-z)G_0(z)+H_1(-z)G_1(z)\right]X(-z) \\ &= T(z)X(z)+A(z)X(-z) \end{aligned} \quad (1)$$

where, Y(z) is the reconstructed signal and X(z) is the original signal. From Eqn. (1), the output of the system contains two terms. First term, T(z) is the desired translation of the input to the output, called distortion transfer function, while second term, A(z) is aliasing distortion, which can be completely eliminated with use of the condition given by Eqn. (2).

$$\begin{aligned} H_1(z) = H_0(-z), G_0(z) = H_1(-z) \\ \text{and } G_1(z) = -H_0(-z) \end{aligned} \quad (2)$$

Therefore, Eqn. (1) becomes

$$Y(z) = 1/2\left[H_0(z)H_0(z) - H_0(-z)H_0(-z)\right]X(z) \quad (3)$$

It implies that the overall design problem of filter bank reduces to determination of the filter taps coefficients of a low pass filter $H_0(z)$, called a prototype filter. Let $H_0(z)$ be a linear phase finite impulse response (FIR) filter with even length (N):

$$H_0(e^{j\omega}) = \left|H_0(e^{j\omega})\right| e^{-j\omega(N-1)/2} \quad (4)$$

If all above mentioned conditions are put together, the overall transfer function of QMF bank is reduced to Eqn. (5) [1].

$$T(e^{j\omega}) = \frac{e^{-j\omega(N-1)}}{2}\left\{\left|H_0(e^{j\omega})\right|^2 + \left|H_0(e^{j(\omega-\pi)})\right|^2\right\} \quad (5)$$

If $\left|H_0(e^{j\omega})\right|^2 + \left|H_0(e^{j(\omega-\pi)})\right|^2 = 1$, it results in perfect reconstruction; the output signal is exact replica of the input signal. If it is evaluated at frequency ($\omega=0.5\pi$), the perfect reconstruction condition reduces to Eqn. (6).

$$\left|H_0(e^{j0.5\pi})\right| = 0.707 \quad (6)$$

Therefore, perfect reconstruction condition is changed in to simpler form, in term of the prototype filter response at frequency ($\omega=0.5\pi$). This is the condition of filter bank response in transition band.

Since, the QMF banks design problem is a prototype filter design problem. If the prototype filter is assumed ideal in passband and stopband, then there is reconstruction error in transition region, which can be minimized for improving the performance of the filter bank. Therefore, the proposed algorithm is based on the prototype filter response in passband, stopband and overall response of filter bank at frequency ($\omega=0.5\pi$). The objective function (E) is constructed using weighted sum of mean square of errors in these responses.

$$E = E_{FB} + \alpha E_p + (1-\alpha)E_s \quad 0 < \alpha \le 1 \quad (7)$$

where, E_{FB} is the square of error in filter bank response at frequency ($\omega=0.5\pi$).

Since, the amplitude responses of a low pass filter in passband and stopband are one and zero respectively. Therefore, E_p and E_s are the mean square error in passband and stop band given by Eqns. (8) and (9).

$$E_P = \frac{1}{\pi}\int_0^{\omega_p}(1-H(\omega))d\omega \quad (8)$$

$$E_s = \frac{1}{\pi}\int_{\omega_s}^{\pi}(H(\omega))^2 d\omega \quad (9)$$

As $H_0(z)$ is a low pass filter with linear phase response ($h_0(n)=h_0(N-n-1)$), whose frequency response is given Eqn. (4), which can be rewritten in Eqn. (10) [1, 24].

$$\begin{aligned} H(e^{j\omega}) &= e^{-j\omega(N-1)/2}\sum_{n=0}^{N/2} 2h_0(n)\cos(\omega((N-1/2)-n)) \\ &= e^{-j\omega(N-1)/2}H(\omega) \end{aligned} \quad (10)$$

The amplitude response, $H(\omega)$ is defined as

$$H(\omega) = \sum_{n=0}^{N/2} b_n \cos(\omega n) \quad (11)$$

where, $b=[b_0\ b_1\ b_2 \ldots b_{N/2}]$, and

$c(\omega)=[\cos(\omega(N-1)/2)\ \ \cos(\omega((N-1)/2-1)) \ldots \cos(\omega/2)]$

Using Eqns. (10) and (11), previous Eqns. (6), (8) and (9) can be implemented as follows:

$$b^T c(0.5\pi) = 0.707 \text{ at } \omega = 0.5\pi \quad (12)$$

$$E_{FB} = (b^T c(0.5\pi) - 0.707)^2 \quad (13)$$

$$\begin{aligned} E_s &= \frac{1}{\pi}\int_{\omega_s}^{\pi} b^T c(\omega) b c(\omega)^T d\omega \\ &= b^T C b \end{aligned} \quad (14)$$

where, C is matrix, whose elements $C(m,n)$ is given by

$$C(m,n) = \frac{1}{\pi}\int_{\omega_s}^{\pi}\cos(A\omega)\cos(B\omega)d\omega \quad (15)$$

where, A and B are [(N-1)/2-m] and [(N-1)/2-n] respectively.

$$E_p = \frac{1}{\pi}\int_0^{\omega_p}(1-b^T c(\omega))(1-b^T c(\omega))d\omega$$
$$= (\omega_p/\pi) - 2b^T P + b^T Qb \quad (16)$$

where, Q and P are respectively a matrix and a vector, and whose elements are given by Eqns. (17 and 18).

$$Q(m,n) = \frac{1}{\pi}\int_0^{\omega_p}\cos(A\omega)\cos(B\omega)d\omega \quad (17)$$

$$P = \frac{1}{\pi}\int_0^{\omega_p}\cos(A\omega)d\omega \quad (18)$$

Finally, the objective function constructed in Eqn. (7) can be rewritten as:

$$\phi = (b^T c(0.5\pi) - 0.707)^2 + (1-\alpha)b^T Cb$$
$$+\alpha\left[(\omega_p/\pi) - 2b^T P + b^T Qb\right], \quad 0 < \alpha \le 1 \quad (19)$$

It is in quadratic form without any constraint. Therefore, the design problem is reduced to unconstrained minimization of the objective function as given in Eqn. (19).

III. Particle Swarm Optimization Technique

PSO is a population based stochastic optimization technique developed in 1995, inspired by social behavior of bird flocking or fish schooling [27, 28]. It utilizes a population of particles called swarms, which fly in the given problem space. In every iteration, each particle is updated by two values: the best solution or fitness that has achieved so far termed pbest. Another value, called gbest obtained so far by any particle in the population. After finding these two values, the particles adjust its velocity and position. Like genetic algorithm (GA), PSO is also computational based technique, share many similarities such as both algorithms start with a randomly generated population, and have fitness values to evaluate the population. Compared to the other optimization techniques, PSO has following several advantages [29]:

- It is not affected by the size and nonlinearity of the problems and it converse to solution when other analytical methods fail.
- In PSO, few parameters are required to be adjusted, therefore, it is easy to implement and there are few parameters to adjust.
- As compared to Genetic algorithm, in PSO, information sharing is one way and every evolution only looks for the best solution. Therefore, all the particles tend to converge to the best solution quickly even in the local version in most cases.

Several research papers on development of PSO algorithm are available in literature [27-29]. In this paper, PSO program developed in Matlab [30] is used for optimizing the objective function given by Eqn. (19).

IV. Result and Discussions

In this section, a new algorithm based PSO technique has been developed in Matlab for the design of two-channel QMF bank. Several design examples are given to illustrate the proposed algorithm. The performances of the designs are evaluated in terms of: peak reconstruction error (PRE), computational time (CPU), stopband edge attenuation, and mean square errors in passband and stopband. These performance parameters are evaluated using following Eqns. (14), (16), (20) and (21).

$$PRE = \max\left\{10\log\left(\left|H_0(e^{j\omega})\right|^2 + \left|H_0(e^{j(\omega-\pi)})\right|^2\right)\right\} \quad (20)$$

$$A_s = Min[-20\log_{10}|H(\omega)|] \text{ for } \omega_s \le \omega \le \pi \quad (21)$$

Example-I: A two channel QMF bank is designed using following design specifications: Length of filter (N+1) =16, stopband edge frequency (ω_s=0.6π), passband edge frequency (ω_p=0.4π), and $\alpha = 0.94$. The results obtained are:

PRE = 0.037dB, E_p = 2.06 x 10^{-6}, E_s = 5.06 x 10^{-4}, A_s = 16.71dB and CPU = 27s.

Fig. 2 shows the amplitude response of the analysis filters of designed QMF bank. Variation of reconstruction error is depicted in Figs. 3.

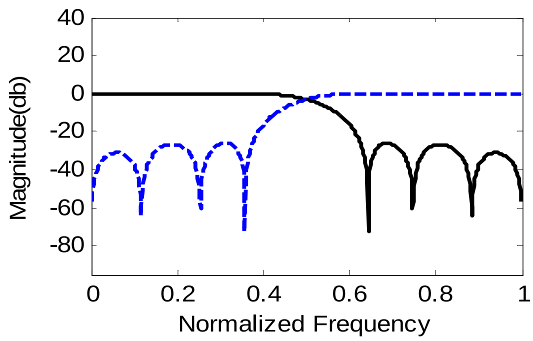

Fig. 2. Amplitude response of the analysis filters of QMF bank

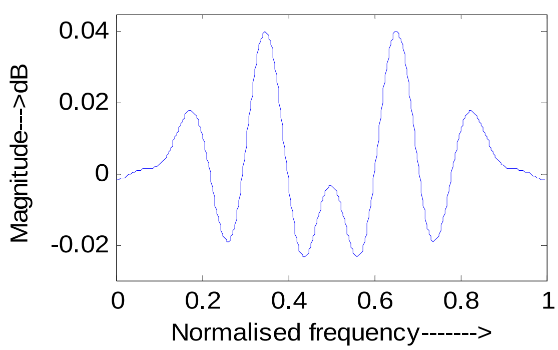

Fig. 3. Variation of reconstruction error in dB

Example-II: A two channel QMF bank is designed using following specifications: (N+1) =32, ω_s=0.6π, ω_p=0.4π and α=0.92. In this case, the performance measuring parameters obtained are:

PRE =0.0131dB, E_p = 6.02x10^{-8}, E_s = 4.80 x 10^{-6}, A_s = 36.98dB, CPU = 37s.

The frequency response of QMF bank is shown in Fig. 4 while that of the reconstruction error is depicted in Fig. 5.

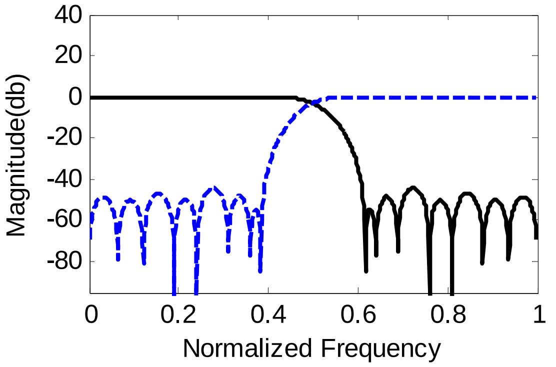

Fig. 4. Frequency response of QMF bank

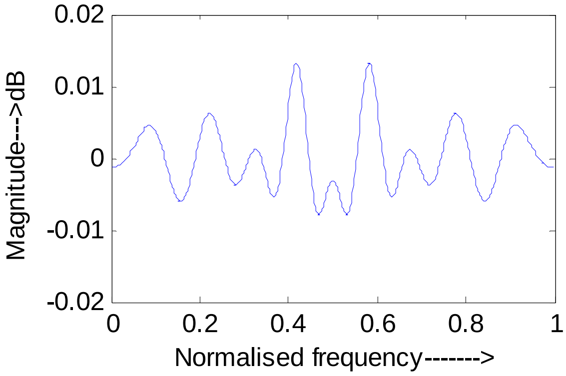

Fig. 5. Variation of reconstruction error in dB

Several other examples have been also designed using the proposed method and the results obtained are summarized in Table I. The Respective filter coefficients obtained in each example are listed in Table II.

TABLE- I
SUMMARY OF RESULTS

Taps (N+1)	A_s (dB)	PRE (dB)	E_p	E_s	CPU (s)
16	16.71	0.037	2.06×10^{-6}	5.09×10^{-4}	26
22	18.62	0.0197	6.54×10^{-7}	2.41×10^{-4}	29
24	28.86	0.0165	1.28×10^{-7}	4.69×10^{-5}	31
32	35.98	0.0131	6.08×10^{-8}	4.80×10^{-6}	36

As it can be observed from simulation results in Table I, the proposed method gives better performance in terms of mean square error in passband, stopband and peak reconstruction error. Computation time required for this method is very small. Therefore, the proposed algorithm can be effectively used for the design of QMF banks.

V. CONCLUSINON

In this paper, a new PSO based technique has been proposed for the design of QMF bank. The proposed method optimizes the prototype filter response characteristics in passband, stopband and also the overall filter bank response. Simulation results included in this paper clearly show that the proposed method leads to filter banks with improved performance in terms of peak reconstruction error, mean square error in stopband, passband and computation time required.

TABLE- II
COEFFICIENTS OF THE PROTOTYPE IN DIFFERENT EXAMPLES

N	Filter taps (16)	Filter taps (22)	Filter taps (24)	Filter taps (32)
	h(n)	h (n)	h(n)	h(n)
0	0.009095	-0.000656	0.002964	0.001655
1	-0.024191	-0.003535	-0.006208	-0.002954
2	0.003421	0.008654	-0.002532	-0.001490
3	0.049317	0.003001	0.015046	0.006213
4	-0.029376	-0.024836	-0.001278	0.000771
5	-0.100047	0.006987	-0.028997	-0.011798
6	0.120719	0.048472	0.011369	0.001252
7	0.470753	-0.031247	0.051950	0.020213
8		-0.098388	-0.036576	-0.006163
9		0.120155	-0.099724	-0.032966
10		0.470865	0.126108	0.016933
11			0.467839	0.053913
12				-0.041756
13				-0.099830
14				0.130903
15				0.465027

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