Tutorial 2

1. If $u(X, Y)$ and $v(X, Y)$ are harmonic conjugates of each other, show that they are constant functions.

2. Show that $u = XY + 3X^2Y - Y^3$ is harmonic and find its harmonic conjugate.

3. Find the radius of convergence of the following power series :

a) $\sum_{n=0}^{\infty} nz^n$
b) $\sum\limits_{p \text{ prime}} z^p$
c) $\sum \frac{n!z^n}{n^n}$

4. Show that $L > 1$ in the ratio test (Lecture 3 slides) does not neccessarily imply that the series is divergent.

5. Construct a infinitely differentiable function $f : \mathbb{R} \to \mathbb{R}$ which is non-zero but vanishes outside a bounded set. Show that there are no holomorphic functions which satisfy this property.

6. Show that $\exp : \mathbb{C} \to \mathbb{C}^\times$ is onto.

7. Show that $\sin, \cos : \mathbb{C} \to \mathbb{C}$ are surjective. (In particular, note the difference with real sine and cosine which were bounded by 1).

8. Show that for any complex number z, $sin^2(z) + cos^2(z) = 1$.