Tutorial 6

1. Evaluate $\int_0^{2\pi} \frac{cos^2(3x)dx}{5-4cos(2x)}$

2. Evaluate $\int_{|z-2|=4} \frac{2z^3+z^2+4\ dz}{z^4+4z^2}$

3. Show with and without using open mapping theorem that if $f(z)$ is a holomorphic function on a domain such that $|f(z)|$ is constant, then $f(z)$ is constant.

4. Show that $\int_{-\infty}^{\infty} \frac{x}{(x^2+2x+2)(x^2+4)} dx = -\pi/10$

5. Compute the number of zeros of the polynomial $z^5 + z^2 - 6z + 3$ in the annulus $1/3 < |z| < 1$ using Rouche's theorem.

6. Show that the function $u(x, y) = log(x^2 + y^2)$ is harmonic on the annulus $1 < |z| < 2$. Does $u(x, y)$ have a harmonic conjugate ?

7. Show that if $f(z)$ is a non-zero polynomial, then $g(z) = e^z f(z)$ has an essential singularity at ∞.