TUTORIAL 1

1. Show that a real polynomial that is irreducible has degree at most two. i.e., if

$$f(x) = a_0 + a_1x + \ldots + a_nx^n, \ a_i \in \mathbb{R},$$

then there are non-constant real polynomials g and h such that $f(x) = g(x)h(x)$ if $n \geq 3$.
(Assume FTA)

2. Show that a non-constant polynomial $f(z_1, z_2)$ in complex variables z_1 and z_2 and with complex coefficients has infinitely many roots (in $\mathbb{C}^2$).
(Assume FTA)

3. Show that the complex plane minus a countable set is path-connected.

4. Check for real differentiability and holomorphicity:

   1. $f(z) = c$
   2. $f(z) = z$
   3. $f(z) = z^n, n \in \mathbb{Z}$
   4. $f(z) = \mathrm{Re}(z)$
   5. $f(z) = |z|$
   6. $f(z) = |z|^2$
   7. $f(z) = \bar{z}$
   8. $f(z) = \begin{cases} \frac{z}{\bar{z}} & \text{if } z \neq 0 \\ 0 & \text{if } z = 0. \end{cases}$