Tutorial 3

1. Show that Cauchy Riemann equation take the form:

$$u_r = \frac{1}{r} v_\theta \; and \; v_r = -\frac{1}{r} u_\theta$$

in polar coordinates.

2. Prove Cauchy's theorem assuming Cauchy integral formula.

3. Let γ be the boundary of the triangle $\{0 < y < 1 - x;\; 0 \leq x \leq 1\}$ taken with the anticlockwise orientation. Evaluate:
a) $\int_\gamma Re(z)dz$ b) $\int_\gamma z^2 dz$

4. Compute $\int_{|z-1|=1} \frac{2z-1}{z^2-1} dz$

5. Show that if γ is a simple closed curve traced counter clockwise, the integral $\int_\gamma \bar{z} dz$ equals $2\imath Area(\gamma)$. Evaluate $\int_\gamma \bar{z}^m dz$ over a circle γ centered at the origin.

6. Let $\mathbb{H} = \{z \in \mathbb{C} | Re(z) > 0\}$ be the (strict) open right half plane. Construct a function f which is holomorphic on $\mathbb{H}$ and such that $f(\frac{1}{n}) = 0$ for $n \in \mathbb{N}$.

7. Let f be a holomorphic function on $\mathbb{C}$ such that $f(\frac{1}{n}) = 0$ for $n \in \mathbb{N}$. Show that f is a constant.

8. Expand $\frac{1+z}{1+2z^2}$ into a power series around 0. Find the radius of convergence.