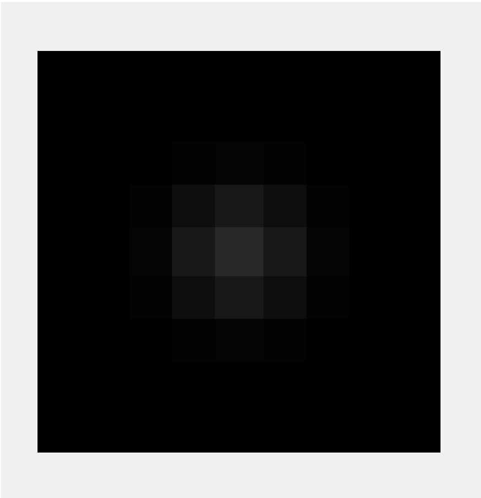
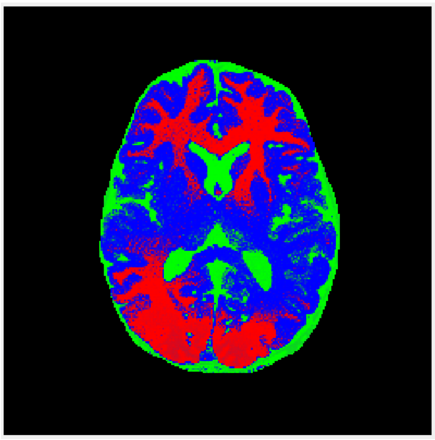
Q1)

a) q = 2.48 was chosen
I iterated in steps of 0.02 from q=1.6 to q=3.6 and checked J values for all those cases. J converged to a global optimum at q=2.48 and elsewhere there were local optima only

b) The neighborhood mask was chosen arbitrarily on discussion with peers. I took a 9x9 filter with mean the central pixel and std = 1
As an image:

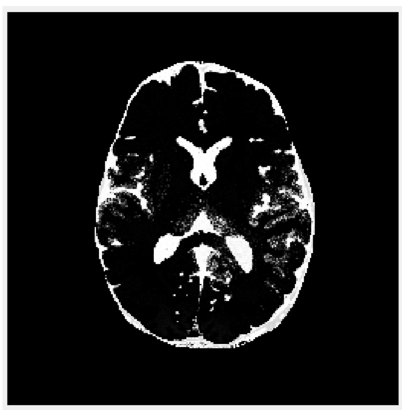


c) Initial estimate for U_jk was by doing the fuzzy c means update for U given C:
Shown as images:



RGB channels for different classes

Class 1



Class 2

Class 3

I chose this initialization as I felt it was a quick initialisation and would lead to faster convergence as the U values would be (in theory) closer to the ideal U values

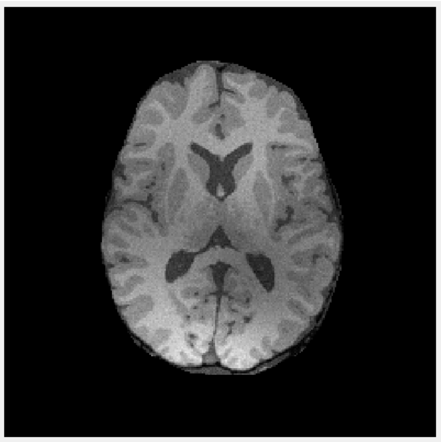
d) Initial class mean estimate I did by k means on the image with 4 classes. Discarding one class which corresponded to the background. This C was used to initialize U.

e) The objective function :- (Varies for different initialisations)

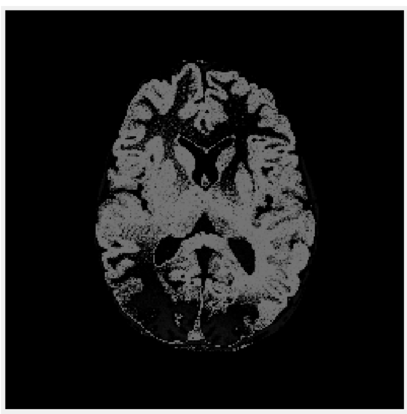
```
J_set =

  1×4 single row vector

   29.9957   22.5349   20.8842   20.7514
```

Minimum at J = 20.7514

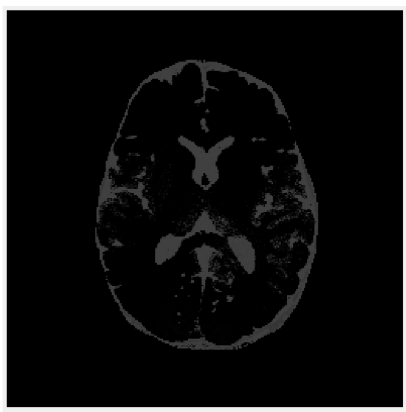
f) Corrupted image provided:-



Optimal class estimates:- (Via Fuzzy Cmeans)



Class 1

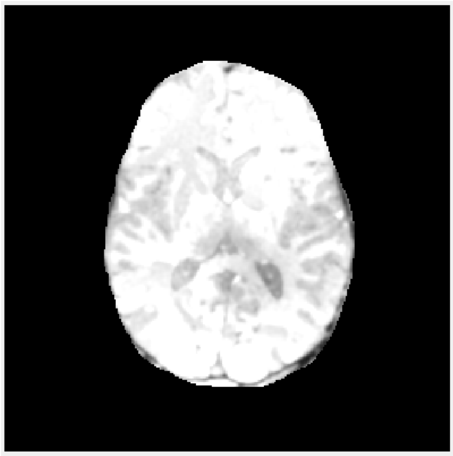


Class 2

Class 3

Bias field:-

Bias removed image:-

Residual image:-

(Increase brightness to view subtle features)

g) The optimal mean values are:

```
C =

  3×1 single column vector

    0.4690
    0.2737
    0.6133
```

Note that they keep varying each time I run the same code.

I feel this does not lead to a unique solution as whenever I run it there are small differences in final results

Q.3) We know probability dist of θ : $P(\theta)$

$$\arg\max_{\theta} P(\theta|y)$$

$$= \arg\max_{\theta} \frac{P(y|\theta)\,P(\theta)}{P(y)} \quad \ldots \text{ (Using Bayes' Rule)}$$

$$\therefore \log\left(\frac{P(y|\theta)\,p(\theta)}{P(y)}\right) = \left[\log\left(P(y|\theta)\right) + \log\left(P(\theta)\right) - \log(P(y))\right]$$

Let $G(q,\theta) = \log\left(P(\theta|y)\right) - \lambda L\left(q \,||\, p(x|y,\theta)\right)$

E step:

Design $q(\cdot)$ to maximize $G(q, \theta_i)$

$$G(q,\theta) = \log\left(P(\theta|y)\right) - \lambda L\left(q \,||\, p(x|y,\theta)\right)$$

$$\therefore q(x) = P(x|y, \theta_i)$$

$G(\cdot)$ is lower bound of the log posterior function and:

$$G(q,\theta_i) = \log\left(P(y|\theta_i)\right) \quad \text{at } \theta = \theta_i$$

M step:

Choose θ to max $G(q,\theta)$

We know

$$\log\left(P(y|\theta)\right) - \lambda L\left(q \,||\, p(x|y,\theta)\right) = E_{q(\cdot)}\left[\log\left(P(x|y,\theta)\right]\right. + H(q)$$

where $H(q) = E(-\log q)$

$$\therefore G(q,\theta) = E_{q(\cdot)}\left[\log\left(P(x|y,\theta)\right]\right. + H(q) + \log P(\theta) - \log P(y)$$

Let $Q(\theta;\theta_i) = E_{q(\cdot)}\left[\log(P(x|y,\theta)\right]$

We maximise $Q(\theta;\theta_i) + \log(P(\theta))$