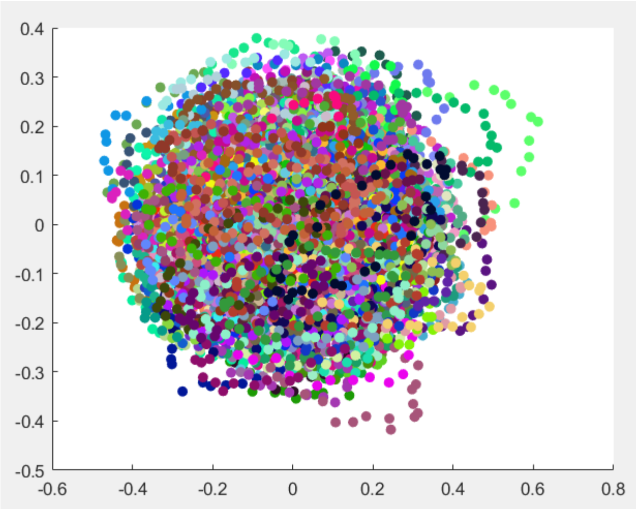
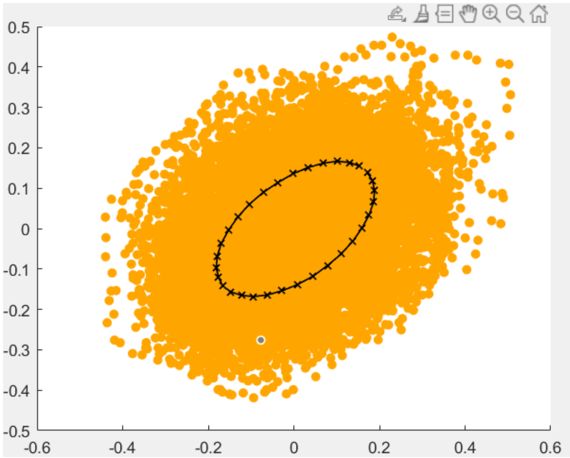
1) ELLIPSE SHAPE

d)

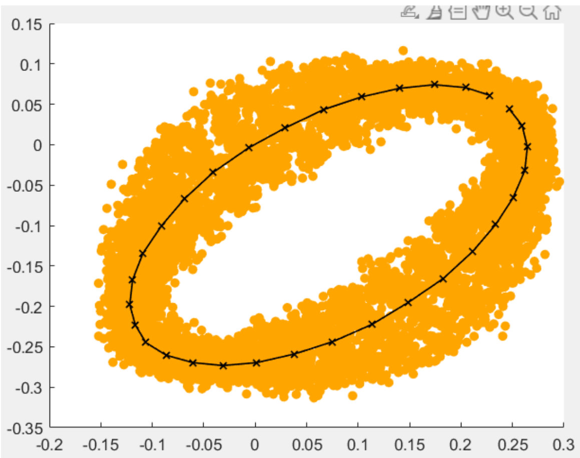


All initial pointsets

e)

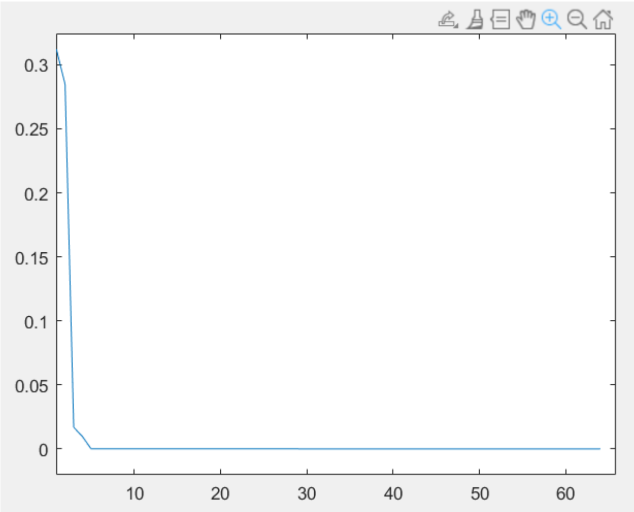


Computed shape mean using code11



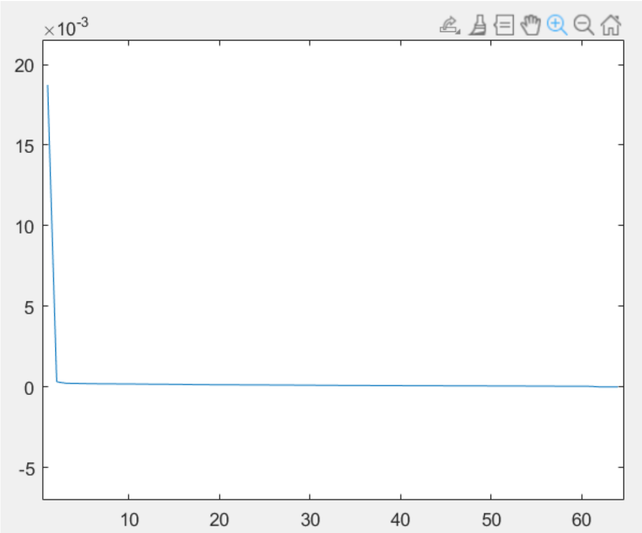
Computed shape mean using code22

f)



Eigen values relying on Code11

Eigen Values for top 3 principal modes = 0.3122, 0.2848, 0.0169



Eigen values relying on Code22

Eigen Values for top 3 principal modes = 0.0187, 3.1820e-04, 2.2382e-04

g) In all the plots below,

Green plot: Computed shape mean

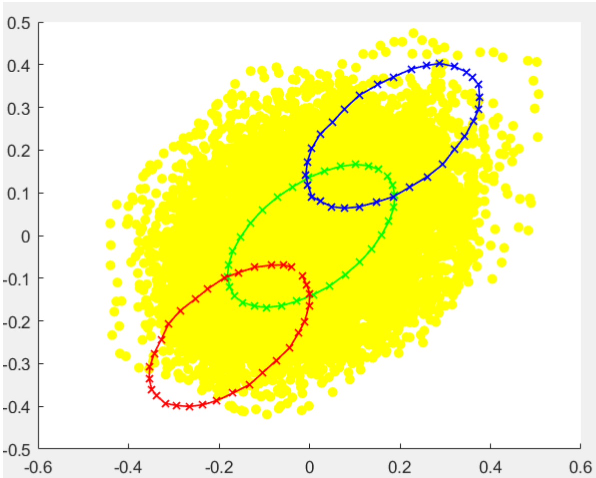
Yellow Points: All aligned pointsets

Shape Variation:

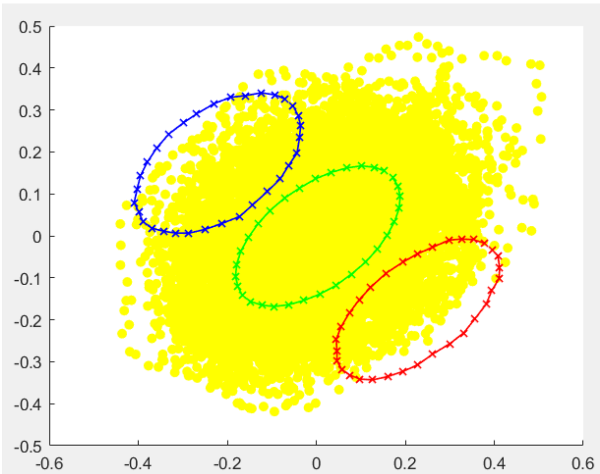
Red plot: b= $+3\sqrt{\lambda}$

Blue Plot: b= $-3\sqrt{\lambda}$

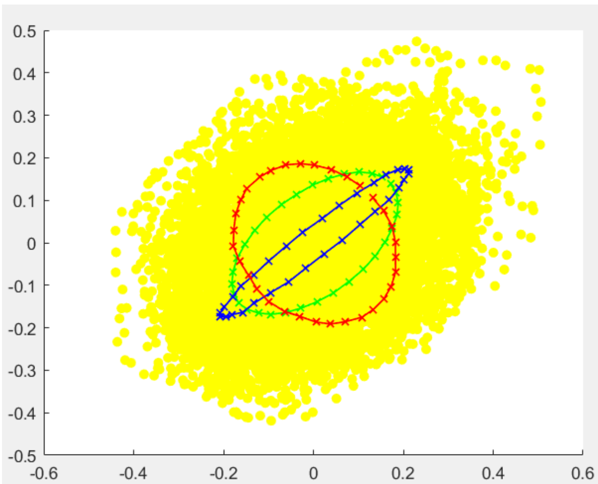
Results relying on Code11:



Principal Mode 1

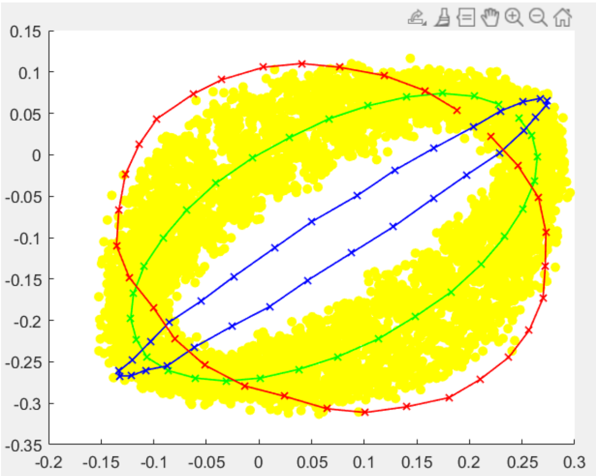


Principal Mode 2

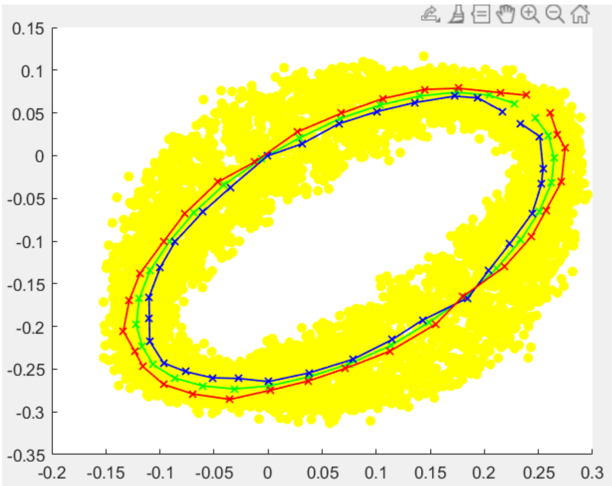


Principal Mode 3

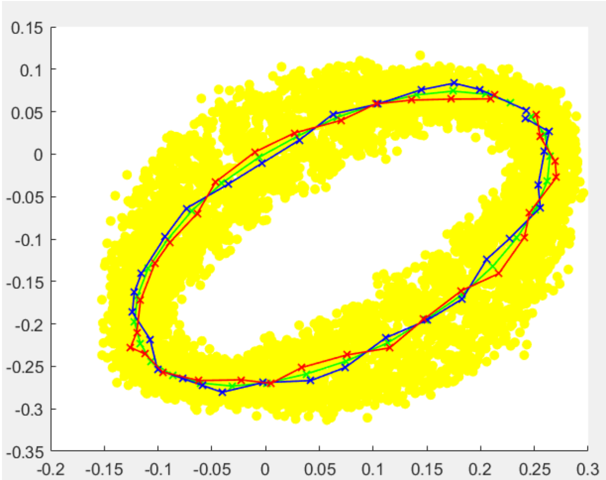
Result relying on Code22:



Principal Mode 1



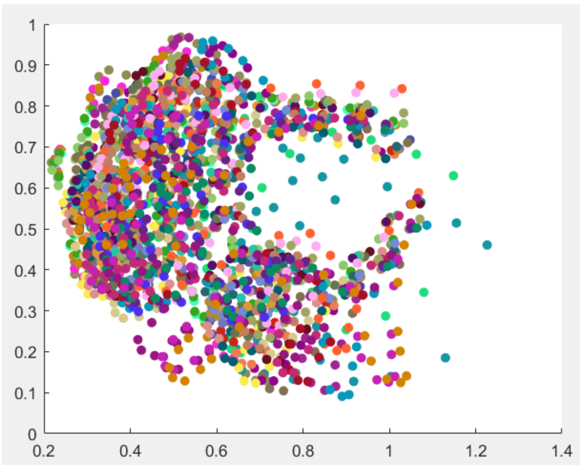
Principal Mode 2



Principal Mode 3

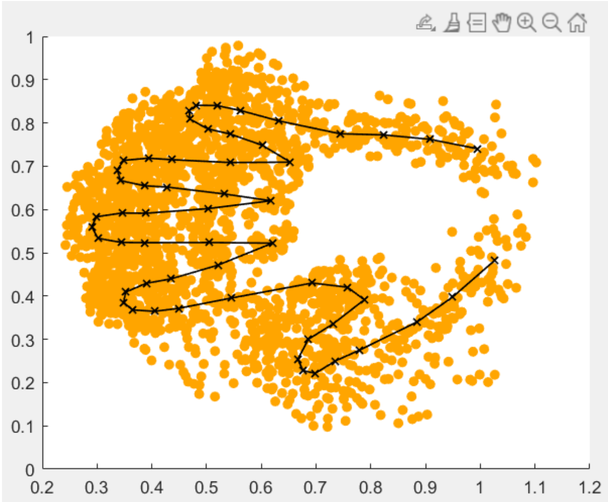
2) HAND SHAPE

d)

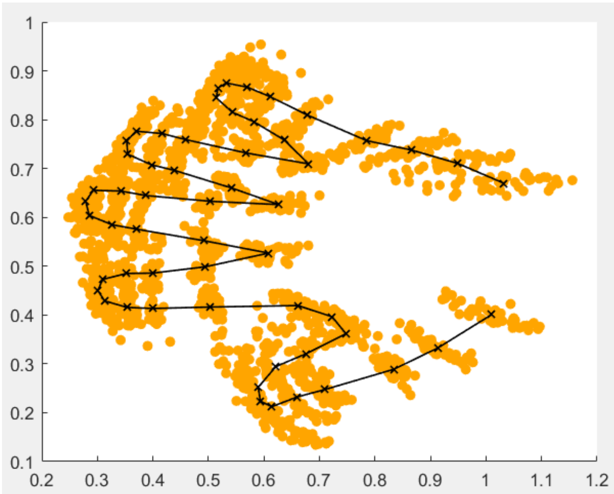


All initial pointsets

e)

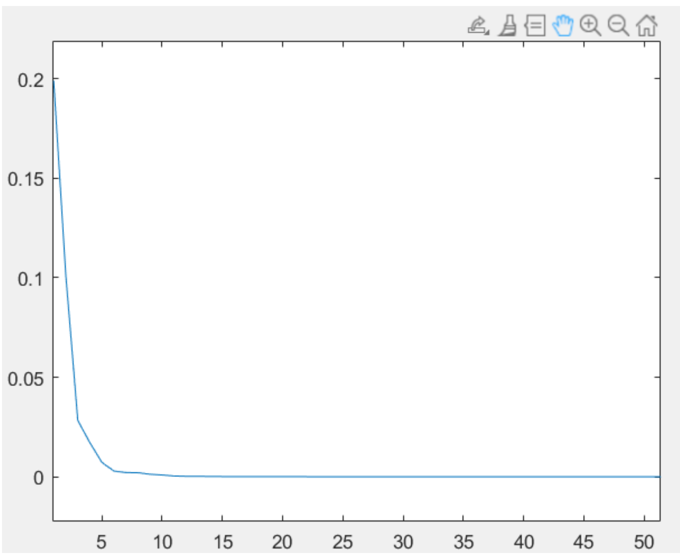


Computed shape mean using code11



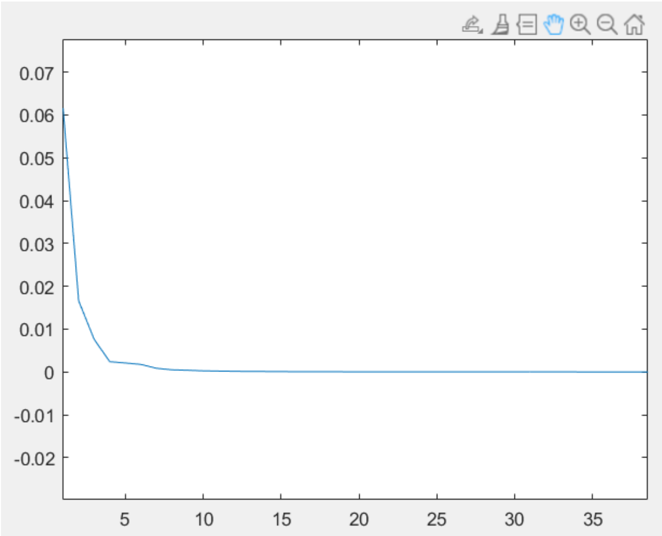
Computed shape mean using code22

f)



Eigen values relying on Code11

Eigen Values for top 3 principal modes = 0.1990, 0.1016, 0.0283



Eigen values relying on Code22

Eigen Values for top 3 principal modes = 0.0616, 0.0166, 0.0076

g) In all the plots below,

Green plot: Computed shape mean

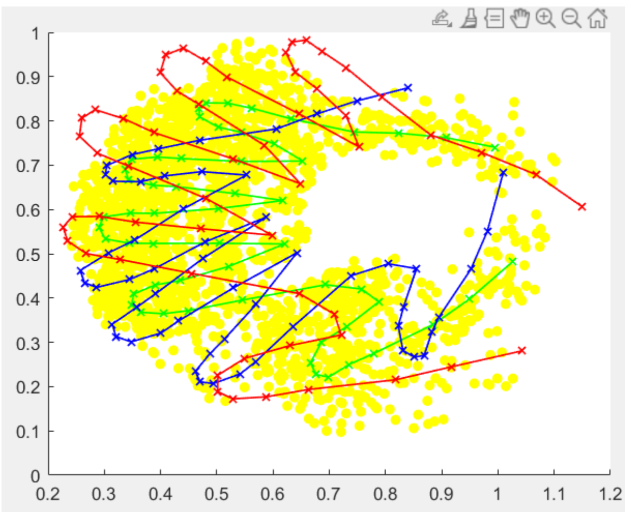
Yellow Points: All aligned pointsets

Shape Variation:

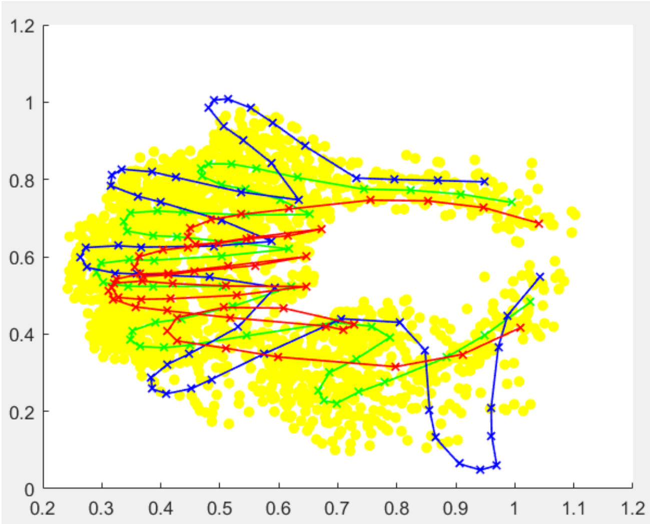
Red plot: b= $+3\sqrt{\lambda}$

Blue Plot: b= $-3\sqrt{\lambda}$

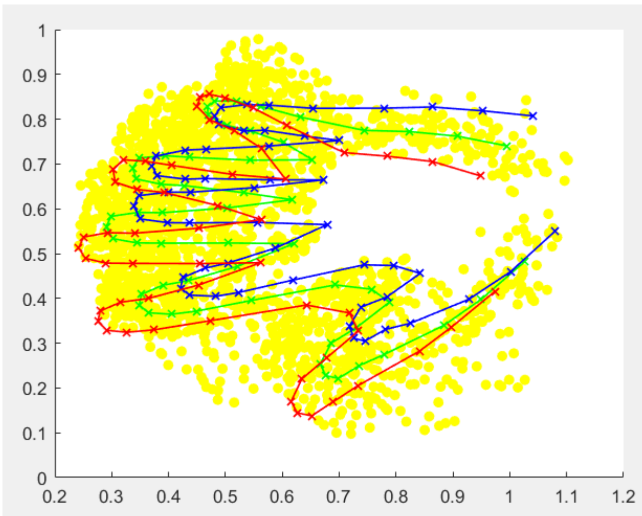
Results relying on Code11:



Principal Mode 1

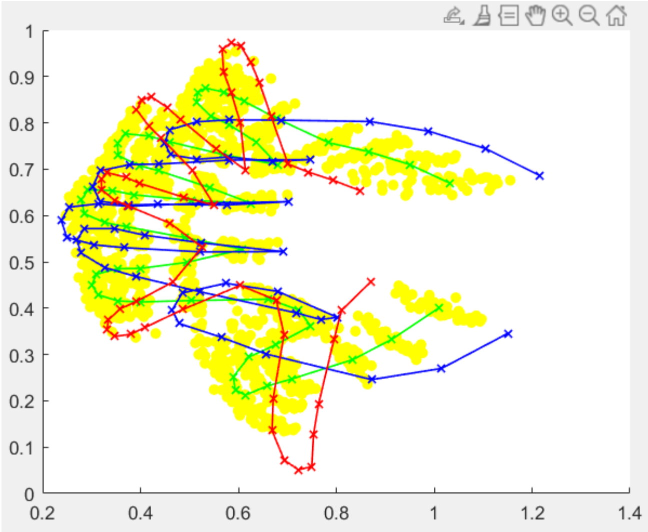


Principal Mode 2

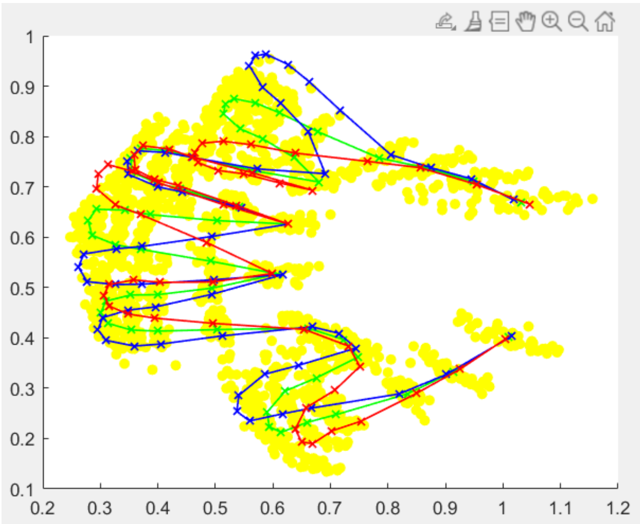


Principal Mode 3

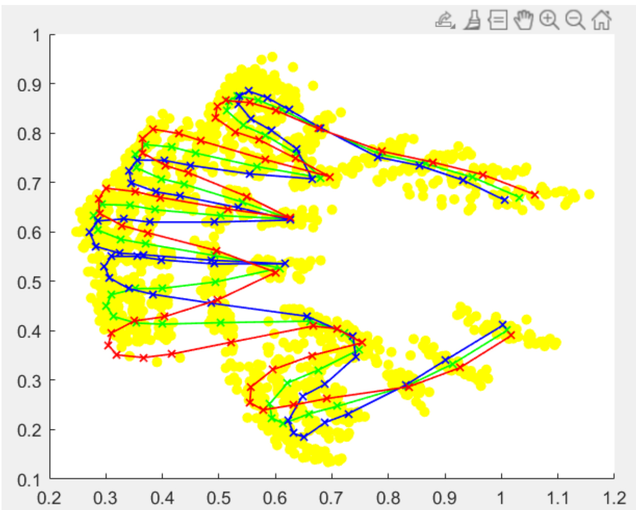
Results relying on Code22:



Principal Mode 1



Principal Mode 2



Principal Mode 3

Q3.) Suppose we have T shapes belonging to K classes each of N points.

a.)

i) We perform K means ++ on the T shapes to get good estimates of the different types of shape clusters in our data set.

Then, within one shape type, procrustes distance is given by

$$\sum_{n=1}^{N} \| p_{in} - sR\,p_{jn} - T \|_2^2$$

Where p_i, p_j are two shapes of the same cluster

s - scaling term

T - Translation matrix in 3D

R - Rotation matrix in 3D.

b.) For our purpose, the objective function can be designed as :-

Suppose the cluster a shape belongs to is given by super script (k) and no. of shapes in cluster is $F(K)$

Objective function $:= \displaystyle\sum_{K=1}^{K} \sum_{M=1}^{F(K)} \sum_{n=1}^{N} \| p_n^{(k)} - s_m^{(k)} R_m^{(k)} p_{mn}^{(k)} - T_m \|_2^2$

(Across all clusters)

Where $p_m^{(k)}$ - m^{th} shape in the cluster (k)

and $p^{(k)}$ - Mean shape of k^{th} cluster

(Outliers shall be dealt with in part (c)) c.) The Algorithm will be a 3 step optimization algorithm. We will optimize

⟶ The K clusterings

⟶ R, s, T w.r.t each shape within a class

⟶ Mean of a class

Iteratively.

i) Given T shapes we perform K means ++ on the entire shape as a data entry to classify the shapes into K types.

ii) Initialise the 'mean' of each shape cluster $p^{(k)}$ by choosing one of the shapes $p_m^{(k)}$ uniformly at random.

⟶ Minimise $R_m^{(k)}$, $s_m^{(k)}$, $T_m^{(k)}$ using the kabsch algorithm.

(1.- Translate all points to origin, eliminating T)

2. - Find rotation matrix R using kabsch

3. - Optimise $s_m^{(k)}$ to minimize cost function.

iii) Optimise $p^{(k)}$ by setting it to

$$p^{(k)} = \frac{\sum_{m=1}^{F(k)} \left(s_m^{(k)} R_m^{(k)} z_m^{(k)} + T_m^{(k)} \right)}{\left\| \sum_{m=1}^{F(k)} \left(s_m^{(k)} R_m^{(k)} z_m^{(k)} + T_m^{(k)} \right) \right\|}$$

Then repeat.

Stopping criterion :-

The objective function (part (b)) should steadily decrease across iterations.

We set a limiting threshold (10^{-3} %) - example

Change in the value will serve as a stopping criterion.

Outliers :-

Once the K shape types along with their means have been optimized, we can decompose each shape within a cluster to its Principal component equivalent.

Checking projections of the 1st two components (2D graph) we can find the shapes that are outliers.

Eliminate these shapes from the dataset for each k^{th} cluster and re-run the optimization algorithm.

Then we will have gotten rid of the negative effect of outliers in our analysis.

Source :-

Mentioned in document.

Reference:
https://graphics.stanford.edu/courses/cs164-09-spring/Handouts/paper_shape_spaces_imm403.pdf