Q1 Bilinear interpolation function of x, y $v(x,y)$

$$v(x,y) = ax + by + cxy + d$$

For $v(x,y)$ to be a linear function of x keeping y constant

Assume 2 points (x_1, y_1) and $\cancel{(x_2, y_1)}$ (x_2, y_1)

$$v(\alpha x_1 + x_2, y_1) = \alpha v(x_1, y_1) + v(x_2, y_1)$$

But $v(\alpha x_1 + x_2, y_1) = a(\alpha x_1 + x_2) + by_1 + c(\alpha x_1 + x_2)y_1 + d$ — (1)

$\alpha v(x_1, y_1) + v(x_2, y_1) = a(\alpha x_1 + x_2) + (b\alpha + b)y_1 + c(\alpha x_1 + x_2)y_1 + d(\alpha+1)$ (2)

LHS $\neq$ RHS

(1) $\neq$ (2), Hence $v(x,y)$ is not a linear function of x keeping y constant

For $v(x,y)$ to be a linear function of y keeping x constant

Assume 2 points (x_1, y_1) and (x_1, y_2)

$$v(x_1, \alpha y_1 + y_2) = \alpha v(x_1, y_1) + v(x_1, y_2)$$

But $v(x_1, \alpha y_1 + y_2) = ax_1 + b(\alpha y_1 + y_2) + cx_1(\alpha y_1 + y_2) + d$

and,

$\alpha v(x_1, y_1) + v(x_1, y_2) = \cancel{ax+} (a\alpha + a)x_1 + b(\alpha y_1 + y_2) + cx_1(\alpha y_1 + y_2) + d(\alpha+1)$

LHS $\neq$ RHS

Hence $v(x,y)$ is not a linear function of y keeping x constant

For $v(x,y)$ to be a linear function of $z \triangleq (x,y)$
Take 2 points $z_1 \triangleq (x_1, y_1)$ and $z_2 \triangleq (x_2, y_2)$

$$v(\alpha z_1 + z_2) = \alpha v(z_1) + v(z_2) \quad - (3)$$

But $v(\alpha z_1 + z) = v(\alpha x_1 + x_2, \alpha y_1 + y_2)$

$$= a(\alpha x_1 + x_2) + b(\alpha y_1 + y_2) + c(\alpha x_1 + x_2)(\alpha y_1 + y_2) + d$$

$$\alpha v(z_1) + v(z_2) = \alpha v(x_1, y_1) + v(x_2, y_2)$$

$$= a(\alpha x_1 + x_2) + b(\alpha y_1 + y_2) + c(\alpha x_1 y_1 + \alpha x_2 y_2) + d(\alpha + 1)$$

LHS $\neq$ RHS in (3)

Hence $v(x,y)$ is not a linear function of $z \triangleq (x,y)$