EE324 - Control Systems Lab

PROBLEM SHEET 4

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Question 1:

a. Here is the code:

```
--> s = poly(0, 's');
--> G = (1/s^2)*(50*s/(s^2+s+100))*(s-2);
--> C = G/(1+G)
 C  =
        -100 +50s
   -------------------
   -100 +150s +1s² +s³
```

b. Here is the code:

```
--> s = poly(0, 's');
--> G1 = s^2 + (1/s);
--> C1 = G1/(1+G1);
--> G2 = C1/s;
--> C2 = G2/(1+s*G2)
 C2  =
     1 +s³
   ---------
   2s +s² +2s⁴
```

c. Here is the code:

```
--> s = poly(0, 's');
--> G1 = (3*s^2)/(1+s); G2 = 1/(s+2);
--> C = (G1*G2 + 2*s*G2)/(1+G2*(G1+2*s+4))
 C  =
   0.3333333s +0.8333333s²
   -----------------------
         1 +1.5s +s²
       2s +5s²
 =  --------------
      6 +9s +6s²
```

Question 2:

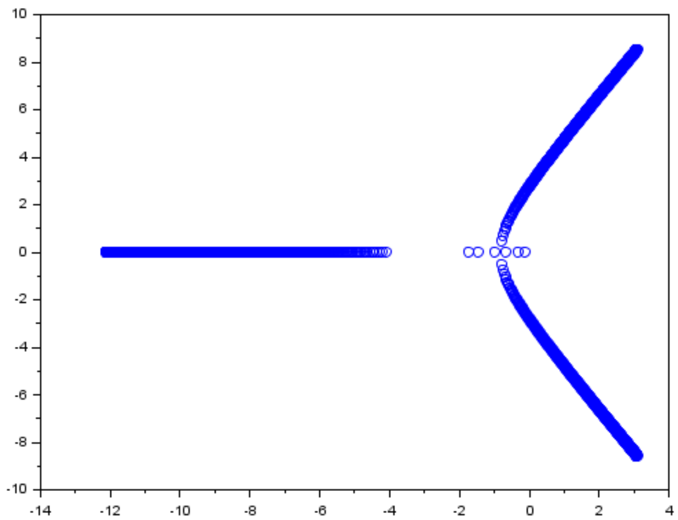
a. Here is the code of my function:

```
--> s = poly(0, 's');

--> function transfer_fn = TF(K)
  > G = 10/(s*(s+2)*(s+4));
  > transfer_fn = (K*G)/(1+(K*G));
  > endfunction
```

b. Code to plot loci as k varies from 0 to 100 in steps of 0.1

```
--> for k = 0:0.1:100
  > tf = TF(k);
  > poles = roots(tf.den);
  > scatter(real(poles), imag(poles));
  > end
```



c. My estimation for when the system crosses the stability line is at k=4.8
(This was found by manually constructing the routh table and noticing imaginary roots at k=4.8)

d. Verification of part (c) via Routh Table:

```
--> tf = TF(4.8);
--> disp(routh_t(tf.den))

   1.           8.
   6.           48.
  -8.882D-15    0.
   48.          0.
```

Here the -8.882D-15 is of the order of 10^-15 implying that at 4.8 the routh table row is entirely zero, which for a 3 degree polynomial can only mean imaginary roots (bordering on instability)

Question 3:

a. Routh Table is as shown below:

```
--> s = poly(0, 's');

--> Ga = s^5 + 3*s^4 + 5*s^3 + 4*s^2 + s + 3;
--> disp(routh_t(Ga))

   1.          5.   1.
   3.          4.   3.
   3.6666667   0.   0.
   4.          3.   0.
  -2.75        0.   0.
   3.          0.   0.
```

b. Routh Table is as shown below:

```
--> Gb = s^5 + 6*s^3 + 5*s^2 + 8*s + 20;
--> disp(routh_t(Gb))
               1                   6      8
               -                   -      -
               1                   1      1

              eps                  5      20
              ---                  -      --
               1                   1      1

           -5 +6eps            -20 +8eps  0
           --------            ---------  -
              eps                 eps     1

      -25 +50eps -8eps²           20      0
      -----------------           --      -
          -5 +6eps               1        1

   -2.274D-13 -160eps -64eps²      0      0
   --------------------------      -      -
      -25 +50eps -8eps²            1      1

               20                  0      0
               --                  -      -
               1                   1      1
```

c. Routh Table is as shown below:

```
--> Gc =  s^5 - 2*s^4 + 3*s^3- 6*s^2 + 2*s - 4;
--> disp(routh_t(Gc))
   1.          3.    2.
  -2.         -6.   -4.
  -8.         -12.   0.
  -3.         -4.    0.
  -1.3333333   0.    0.
  -4.          0.    0.
```

d. Routh Table is as shown below:

```
--> Gd = s^6 + s^5 - 6*s^4 + s^2 + s - 6;
--> disp(routh_t(Gd));
     1    -6   1   -6
     -    --   -   --
     1     1   1    1

     1     0   1    0
     -     -   -    -
     1     1   1    1

    -6     0  -6    0
    --     -  --    -
     1     1   1    1

   -24     0   0    0
   ---     -   -    -
     1     1   1    1

   eps    -6   0    0
   ---    --   -    -
     1     1   1    1

   -144    0   0    0
   ----    -   -    -
   eps     1   1    1

   864     0   0    0
   ----    -   -    -
   -144    1   1    1
```

Question 4:

a. For the s^3 row to be entirely zero, we need to ensure that a factor of our polynomial has terms of the form (as^4 + bs^2 + c). One such arbitrary factor is (s^4 + 11s^2 + 13) and this into (s^2 + 6s + 69) gives a possible polynomial as (**$s^6 + 6s^5 + 80s^4 + 66s^3 + 772s^2 + 78s^1 + 897s^0$**)

characteristic equation:

$1.0s^6 + 6.0s^5 + 80.0s^4 + 66.0s^3 + 772.0s^2 + 78.0s^1 + 897.0s^0$

1.0	80.0	772.0	897.0	0.0	0.0
6.0	66.0	78.0	0.0	0.0	0.0
69.0	759.0	897.0	1e-100	1e-100	1e-100
1e-100	1e-100	-8.696e-102	-8.696e-102	-8.696e-102	1e-100
690.0	903.0	6.0	6.0	-69.0	1e-100
-3.087e-101	-9.565e-102	-9.565e-102	1.304e-102	1e-100	1e-100
689.2	-207.8	35.15	2166.0	2235.0	1e-100

b. By similar logic as part (a) we can choose an arbitrary factor := (s^4 + 3s^2 + 7) and a quotient polynomial of degree 4 := (s^4 + 2s^3 + 3s^2 + 4s + 5)
This gives our polynomial as (**$1s^8 + 2s^7 + 6s^6 + 10s^5 + 21s^4 + 26s^3 + 36s^2 + 28s^1 + 35s^0$**)

characteristic equation:

$1.0s^8 + 2.0s^7 + 6.0s^6 + 10.0s^5 + 21.0s^4 + 26.0s^3 + 36.0s^2 + 28.0s^1 + 35.0s^0$

1.0	6.0	21.0	36.0	35.0	0.0	0.0	0.0
2.0	10.0	26.0	28.0	0.0	0.0	0.0	0.0
1.0	8.0	22.0	35.0	1e-100	1e-100	1e-100	1e-100
-6.0	-18.0	-42.0	-2e-100	-2e-100	-2e-100	-2e-100	1e-100
5.0	15.0	35.0	6.667e-101	6.667e-101	6.667e-101	1.167e-100	1e-100
1e-100	1e-100	-1.2e-100	-1.2e-100	-1.2e-100	-6e-101	2.2e-100	1e-100
10.0	41.0	6.0	6.0	3.0	-11.0	-5.0	1e-100
-3.1e-100	-1.8e-100	-1.8e-100	-1.5e-100	5e-101	2.7e-100	1e-100	1e-100
35.19	0.1935	1.161	4.613	-2.29	-1.774	3.226	1e-100

c. Let our s^3 row entry be [0 1 0 0]
Then let our s^5 row be (a b c) and s^4 row be (d e f)
=> db - ae = 0 and c - (af/d) = 1
=> We can choose (a, b, c, d, e, f) = (8, 2, 5, 12, 3, 6) to satisfy the same
=> Then to fill row s^6 consider it to be (x, y, z, w)
=> Equations to satisfy are: y-(x/4)=12; z-(5x/8)=3; w=6;
=> (x, y, z, w) can be chosen to be (4, 13, 5.5, 6)

Sanity check for resultant polynomial : p(s) = **$4s^6 + 8s^5 + 13s^4 + 2s^3 + 5.5s^2 + 5s^1 + 6s^0$**

characteristic equation:

$4.0s^6 + 8.0s^5 + 13.0s^4 + 2.0s^3 + 5.5s^2 + 5.0s^1 + 6.0s^0$

4.0	13.0	5.5	6.0	0.0	0.0
8.0	2.0	5.0	0.0	0.0	0.0
12.0	3.0	6.0	1e-100	1e-100	1e-100
1e-100	1.0	-6.667e-101	-6.667e-101	-6.667e-101	1e-100
-1.2e+101	14.0	8.0	8.0	-12.0	1e-100
1.0	-6.667e-101	-6.667e-101	-6.667e-101	1e-100	1e-100
6.0	1e-100	1e-100	1e-100	12.0	1e-100