Q2 The image intensity values in bicubic interpolation is

$$V(x,y) = \sum_{i=0}^{3}\sum_{j=0}^{3} a_{ij}\, x^i y^j$$

$$V(x,y) = \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \overbrace{\begin{bmatrix} a_{00} & a_{01} & a_{02} & a_{03} \\ a_{10} & a_{11} & a_{12} & a_{13} \\ a_{20} & a_{21} & a_{22} & a_{23} \\ a_{30} & a_{31} & a_{32} & a_{33} \end{bmatrix}}^{A} \begin{bmatrix} 1 \\ y \\ y^2 \\ y^3 \end{bmatrix}$$

We are given 16 intensity values for 16 points for (x_i, y_j)

$\equiv$ $i = 1, 2, 3, 4$ and $j = 1, 2, 3, 4$

We have 16 equations for 16 points of intensity values

$$V = XAY$$

$$A = X^{-1} V Y^{-1}$$

$$X = \begin{bmatrix} 1 & x_1 & x_1^2 & x_1^3 \\ 1 & x_2 & x_2^2 & x_2^3 \\ 1 & x_3 & x_3^2 & x_3^3 \\ 1 & x_4 & x_4^2 & x_4^3 \end{bmatrix} \qquad Y = \begin{bmatrix} 1 & y_1 & y_1^2 & y_1^3 \\ 1 & y_2 & y_2^2 & y_2^3 \\ 1 & y_3 & y_3^2 & y_3^3 \\ 1 & y_4 & y_4^2 & y_4^3 \end{bmatrix}$$

$$V = \begin{bmatrix} V(x_1,y_1) & V(x_1,y_2) & V(x_1,y_3) & V(x_1,y_4) \\ V(x_2,y_1) & V(x_2,y_2) & V(x_2,y_3) & V(x_2,y_4) \\ V(x_3,y_1) & V(x_3,y_2) & V(x_3,y_3) & V(x_3,y_4) \\ V(x_4,y_1) & V(x_4,y_2) & V(x_4,y_3) & V(x_4,y_4) \end{bmatrix}$$

Using $A = X^{-1} V Y^{-1}$, we can find the values of all 16 coefficients (X and Y should be invertible)
(Since all points are distinct X and Y should be invertible)
We require 16 nearest neighbours because we need 16 equations to find to values of 16 coefficients