Q) let the one-dimensional image f be of length n

$f = [I_1, I_2, I_3 \ldots\ldots\ldots I_n]$

Where I_i is the intensity of the i^{th} pixel of the image

and $w = [w_0, w_1, w_2 \ldots\ldots w_6]$ is the 1D convolution mask

The convolution operation $(w * f)(x)$ is done by:

(i) Padding the image with $7-1=6$ zeros on both sides, slide the 180° rotated filter and then compute the sum of products at each location

↓ (slide till the last zero)

0 0 0 0 0 0 I_1 I_2 ..... I_n 0 0 0 0 0 0

w_6 w_5 w_4 w_3 w_2 w_1 w_0

↓ (last zero) ↓

0 0 0 0 0 0 I_1 I_2 ---- I_n 0 0 0 0 0 0

w_6 w_5 w_4 w_3 w_2 w_1 w_0

But then we have to truncate $(7-1)/2 = 3$ values from both sides of the convolved value

This means that the center of the mask (w_3) is sliding from the first pixel to the last pixel

↓

I_1 I_2 ..... I_n

w_3 w_2 w_1 w_0

After sliding till the last pixel

I_1 I_2 ..... I_n

w_6 w_5 w_4 w_3

The above operation can be realised by Wf^T where W is a $n \times n$ matrix

$$W = \begin{bmatrix} w_3 & w_2 & w_1 & w_0 & 0 & 0 & 0 \cdots 0 & \cdots & 0 \\ w_4 & w_3 & w_2 & w_1 & w_0 & 0 & 0 & 0 & \cdots & 0 \\ w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & 0 & 0 & \cdots & 0 \\ w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & 0 & \cdots & 0 \\ 0 & w_6 & w_5 & w_4 & w_3 & w_2 & w_1 & w_0 & \cdots & 0 \\ \vdots & & & & & & & & & \vdots \\ 0 & 0 & 0 & \cdots & \cdots & \cdots & 0 & w_6 & w_5 & w_4 & w_3 \end{bmatrix}$$

The convolved values are $(Wf^T)^T$

The above matrix is a circular matrix (each row is a linear shift of the previous one by one unit)

Applications:

i) In numerical analysis, circulant matrices are important because they are diagonalized by a discrete Fourier transform and hence linear equation that contain them may be quickly solved using a Fast Fourier Transform (FFT)

(ii) In cryptography, a circulant matrix is used in the Mix columns step of the Advanced Encryption standard