Q6 Given 1D ramp image $I(x) = cx+d$

When I is filtered by a zero mean Gaussian with standard deviation σ, the output is J

$$J = \frac{\sum_{j=-\infty}^{\infty} I(x+j)\, e^{-\frac{j^2}{2\sigma^2}}}{\sum_{j=-\infty}^{\infty} e^{-\frac{j^2}{2\sigma^2}}} = \frac{\sum_{j=-\infty}^{\infty} (c(x+j)+d)\, e^{-\frac{j^2}{2\sigma^2}}}{\sum_{j=-\infty}^{\infty} e^{-\frac{j^2}{2\sigma^2}}}$$

$$J = cx+d + \frac{\sum_{j=-\infty}^{\infty} cj\, e^{-\frac{j^2}{2\sigma^2}}}{\sum_{j=-\infty}^{\infty} e^{-\frac{j^2}{2\sigma^2}}}$$

$$\left.\begin{array}{l} cj \rightarrow \text{odd function} \;;\; e^{-j^2/2\sigma^2} \rightarrow \text{even function} \\ \text{odd} \times \text{even} \rightarrow \text{odd} \\ \text{Sum of an odd function on the real axis is zero} \end{array}\right\} - (1)$$

Hence $J = cx+d+0 = I$

When I is filtered using a Bilateral filter with parameters σ_s, σ_r, the output is $B_F I(x)$

let x be the target pixel and j be the pixel in the neighbourhood of x $(q = j + x)$

$$B_F I(x) = \frac{1}{W_x} \sum_{j=-\infty}^{\infty} I(x+j)\, e^{\frac{-j^2}{2\sigma_s^2}} \times e^{\frac{-(I_p - I_q)^2}{2\sigma_r^2}}$$

$$I_p - I_q = cj$$

$$W_x = \sum_{j=-\infty}^{\infty} e^{\frac{-j^2}{2\sigma^2}}\, e^{\frac{-(cj)^2}{2\sigma_r^2}}$$

$$B_F I(x) = cx + d + \frac{\overbrace{\sum_{j=-\infty}^{\infty} \underbrace{cj}_{\text{odd}}\, \underbrace{e^{-\frac{j^2}{2\sigma_s^2}}}_{\text{even}}\, \underbrace{e^{\frac{-(cj)^2}{2\sigma_r^2}}}_{\text{even}}}^{\text{odd}}}{\sum_{j=-\infty}^{\infty} e^{-\frac{j^2}{2\sigma_s^2}}\, e^{-\frac{(cj)^2}{2\sigma_r^2}}}$$

Sum of odd function over infinty $= 0$

$$B_F I(x) = cx + d + 0 = cx + d = I(x)$$