Q3 let I be a random variable for the intensities of Image I with PMF $P_I(i)$

let J be a random variable for the intensities of Image J with PMF $P_J(j)$

let the intensities of both images $\in [0, L-1]$

Consider $Z = I + J$

let $j = k$

$i = z - k$ $\quad k \in [z-(L-1), z]$

The PMF of Z is $P_Z(z) = \sum_{k=z-(L-1)}^{z} P_{IJ}(z-k, k)$

Since I and J are independent variable

$P_{IJ}(z-k, k) = P_I(z-k) \times P_J(k)$

$P_Z(z) = \sum_{k=z-(L-1)}^{z} P_I(z-k) \times P_J(k)$

Since $P_I(i)$ and $P_J(j)$ are only non zero when $i, j \in [0, L-1]$

$P_Z(z) = \sum_{k=0}^{L-1} P_I(z-k) \times P_J(k)$

$$P_Z(z) = P_I(z) * P_J(z)$$

This is the convolution operation that we are current studying in class