Q3 Let I be the random variable for the intensities of the clean image $I(x,y)$ with pmf $P_I(i)$

Let N be the random variable for the additive noise N which is a zero mean Gaussian distribution

PMF of N $P_N(n) = \frac{1}{\sigma\sqrt{2\pi}} e^{-n^2/\sigma^2}$

The new image with noise has the random variable Z for the intensities

Since both the image and gaussian noise are continous RV's

$Z = I + N \qquad (i = z - n)$

PMF of Z is $P_Z(z) = \int_{n=-\infty}^{\infty} P_{IN}(z-n, n)\, dn$

Since I and N are independent Random Variable $(P_{IN} = P_I \times P_N)$

$P_Z(z) = \int_{n=-\infty}^{\infty} P_I(z-n) \times P_N(n)\, dn$

$P_Z(z) = (P_I * P_N)(z)$

$$P_Z(z) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} P_I(z-n) e^{-n^2/2\sigma^2} \, dn$$