Assignment 1: CS 663, Fall 2021

Question 4

All the images are of dimensions 460x532. Casting them as double arrays transforms them into arrays of size 1x244720.

(a) J3 = **imrotate**(J3, 28.5, 'crop')
(b)

1. Normal Correlation function/Correlation coefficient between J1 and J4 calculated as follows:
$$NCC = \frac{\sum_{(x,y)\in\Omega}(J1(x,y) - \bar{J}1)(J4(x,y) - \bar{J}4)}{\sqrt{\sum_{(x,y)\in\Omega}(J1(x,y) - \bar{J}1)^2 \sum_{(x,y)\in\Omega}(J4(x,y) - \bar{J}4)^2}}$$

2. Joint entropy:
$$JE = \sum_{i1}\sum_{i2} p(J1 = i1, J4 = i2) \log_2 p(J1 = i1, J4 = i2)$$

3. Quadratic Mutual Information(QMI):
$$QMI = \sum_{i1}\sum_{i2}(p(J1 = i1, J4 = i2) - p(J1 = i1)p(J4 = i2))^2$$
Marginal Histograms built by integrating the joint histogram as follows:
$$p(J1 = i1) = \sum_{i2}(p(J1 = i1, J4 = i2)$$
$$p(J4 = i2) = \sum_{i1}(p(J1 = i1, J4 = i2)$$

   i1 and i2 used in calculations of JE and QMI are bins of size 10 ranging from 0 - 260

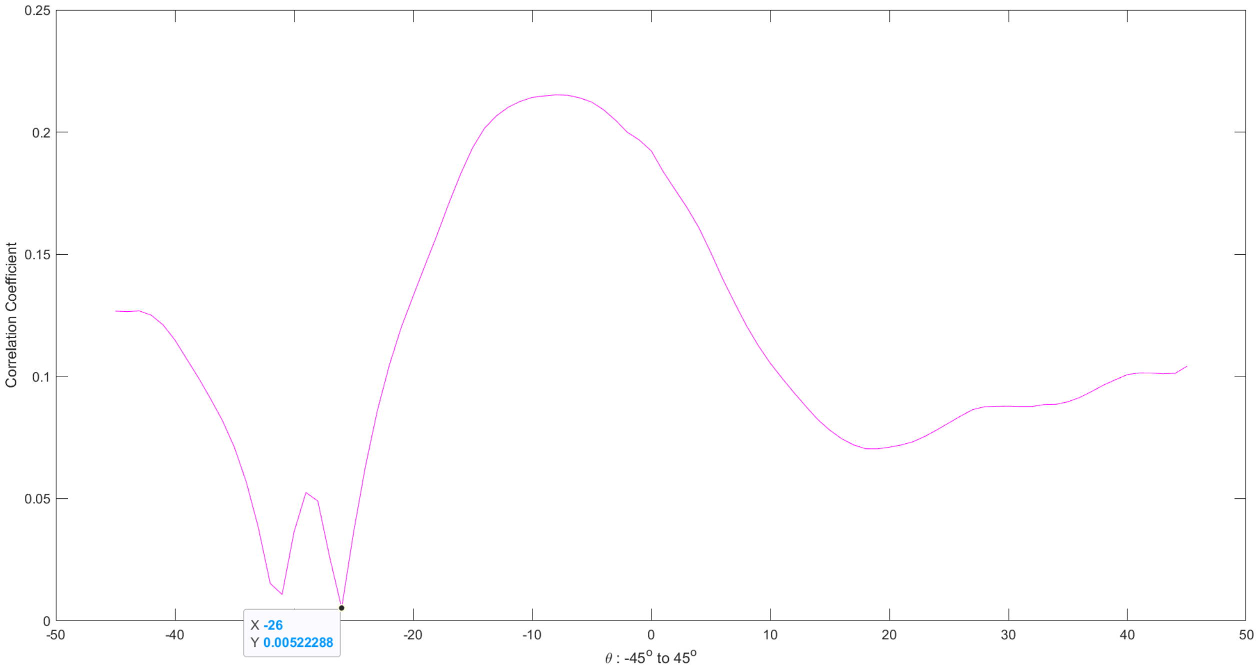

Figure 1: Correlation coefficient v/s θ

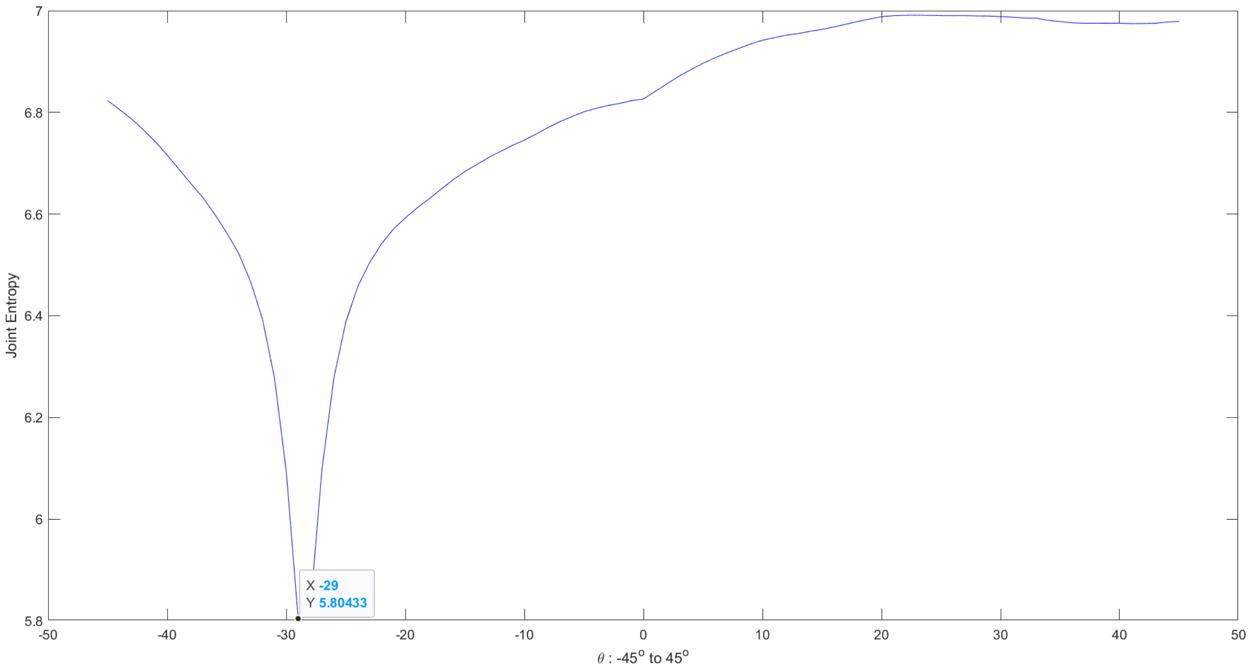

Figure 2: Joint Entropy v/s θ

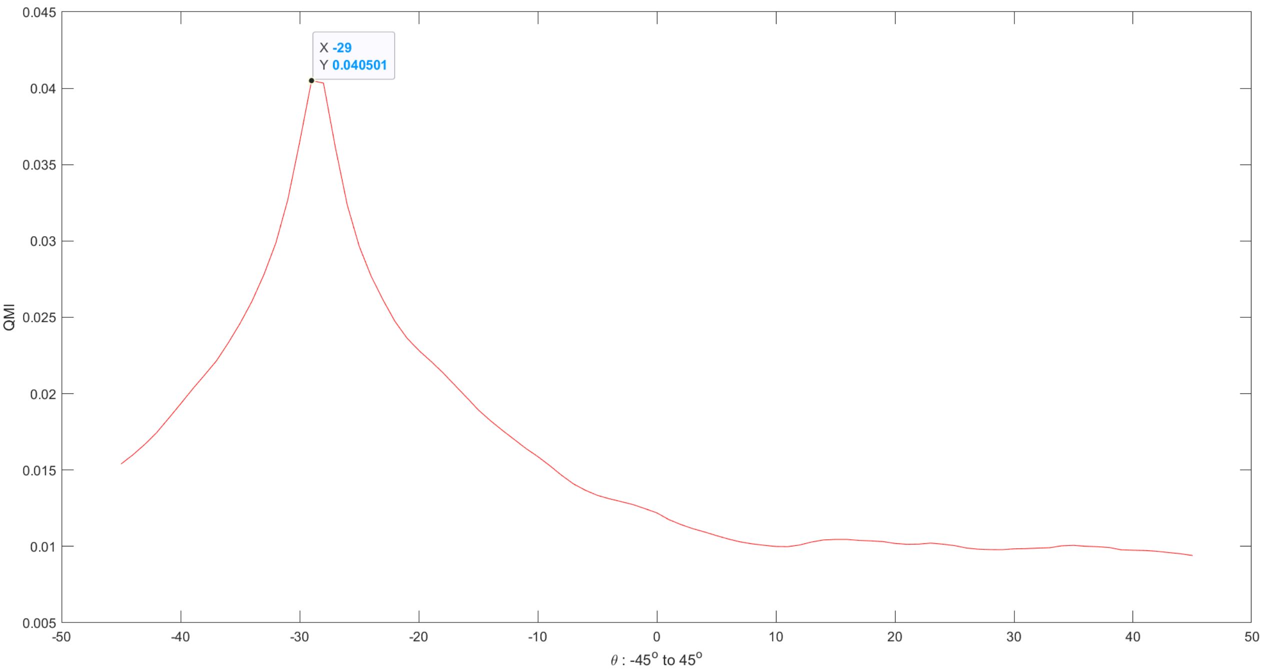

Figure 3: Quadratic MI v/s θ

(d) From the plots, we can infer that the optimal rotation occurs at angles at which, the plot has a very steep peak or a trough.

1. In the NCC plot, the best alignment occurs at the minimum value. In the plot, the NCC is mimimum at $\theta = -26^o$ (i.e. 26^o clockwise). Also can be observed is that the plot has a local peak at -29^o.
2. In the Joint Entropy plot, minimum is at -29^o and the entropy at -28^o is just negligibly higher.

3. The QMI at $\theta = -29^o$ and $\theta = -28^o$ is almost the same. -29^o has the highest QMI and is, hence, the optimal angle in the QMI plot.

The three measures do a pretty good job at estimating the rotation which is 28.5^o. The NCC plot's estimation is a bit off compared to others because it is only good at estimating alignment when the images' intesities are linearly related and the images we used likely have a functional relationship that isn't linear. Joint Entropy does a better job in this case.

(e)
Optimal rotation in **JE vs** $\theta = -29$ degrees.
Defined a variable **'JH'** to store the Joint Histogram between **J1** and **optimal J4**. The code to plot this is as follows:
imagesc(5:10:255, 5:10:255, JH)
colorbar

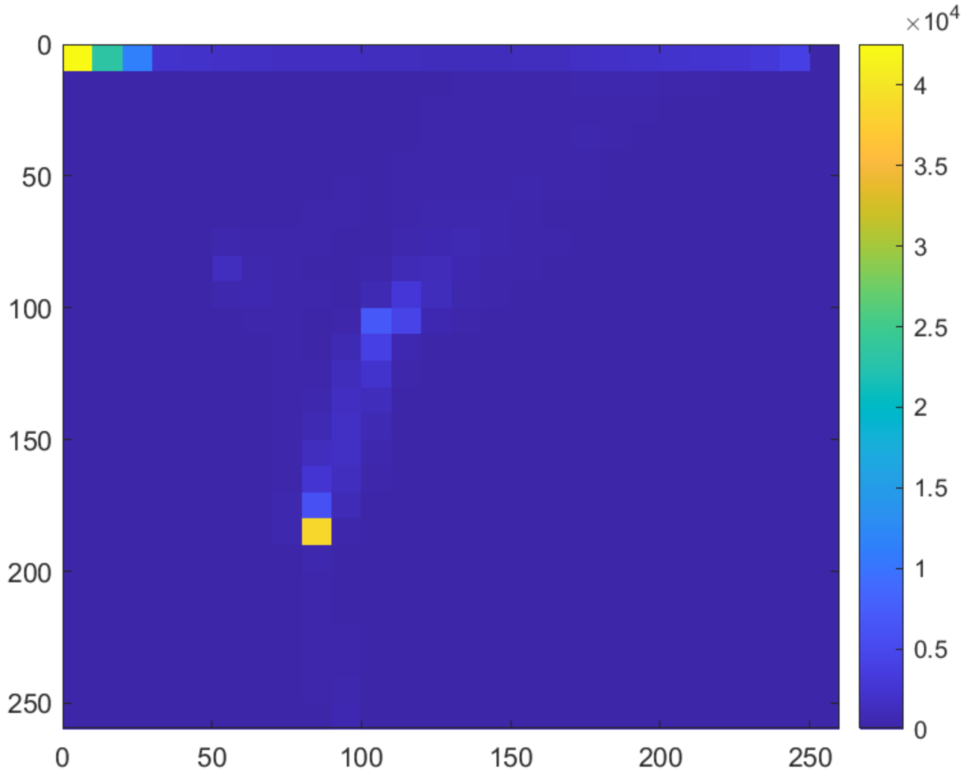

Figure 4: Joint Histogram between J1 and best J4

(f) We can say that the random variable I_1 and I_2 are statistically independent if

$$p(I_1 = i_1, I_2 = i_2) = p(I_1 = i_1)p(I_2 = i_2) \forall i_1, i_2 \in \Omega$$

$$QMI = \sum_{i1} \sum_{i2} (p(J1 = i1, J4 = i2) - p(J1 = i1)p(J4 = i2))^2$$

- We can observe that the above expression for QMI is always ≥ 0 and would equate to 0 if J_1 and J_4 satisy the above condition of being statistically independent.
- We can say the statistical independence between intensity random variables of two images directly corresponds to the amount of misalignment between them.
- Hence, QMI would be very less or close to 0 for images which have no physical correspondance or are misaligned. And, this measure achieives its maximum when images are aligned. This can be observed from the plot.