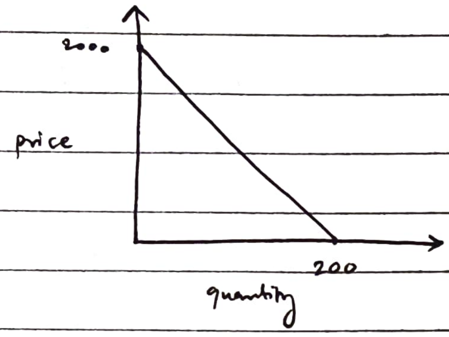
Prasann Viswanathan Iyer
190070047
EE722
Assignment 1

Q1.) $\pi = -10q + 2000$ \$ q = demand, π = unit price

a.)



i) Maximum consumption = 200
ii) Maximum price = 2000 \$

b.) The maximum consumer surplus is the area under the demand curve

$= \frac{1}{2}(2000)(200) = 200,000$ \$

This consumer surplus is not realised because to obtain this, the market price of the product must be 0 \$ which is infeasible

c.) π = \$ 1000/unit

$\Rightarrow q = \frac{2000 - 1000}{10} = 100$

Consumer surplus $= \frac{1}{2}(2000-1000)(100) = 50,000$ \$

revenue collected $= 100 \times 1000 = 100,000$ \$

d.) π increases by 20% $\Rightarrow \pi' =$ \$ 1200/unit

$\Rightarrow q = \frac{2000 - 1200}{10} = 80$

Consumer surplus $= \frac{1}{2}(2000-1200)(80) = 32000$ \$

revenue collected $= 80 \times 1200 = 96,000$ \$

quantity drops by 20 $(100-80)$
consumer surplus drops by \$ 18,000
revenue collected drops by \$ 4,000

e.) $\pi = 1000\,\$ \Rightarrow q = 100$

price elasticity of demand $= \frac{dQ}{dp} \times \frac{P}{Q}$

$= -\frac{1}{10} \times \frac{1000}{100} = -1$

f.) $\pi = 2000 - 10q\ \$$

gross consumer surplus = area under curve

$= \pi q + \frac{1}{2}(2000 - \pi) q$

$= (2000 - 10q) q + \frac{1}{2}(10q) q$

$= 2000q - 5q^2$

net consumer surplus $= \frac{1}{2}(2000 - \pi) q$

$= 5q^2$

Comparing to results of part d:

gross cons. surp = revenue + net cons. surplus

$= 128{,}000$

$= 2000(80) - 5(80)^2$

$= 2000q - 5q^2$ ☐

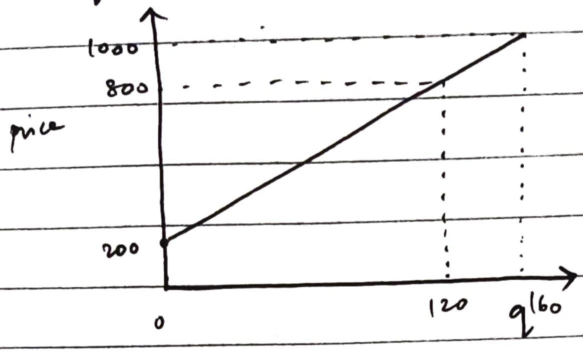
net cons. surp $= 32000$

$= 5(80)^2$

$= 5q^2$ ☐ (The expressions match)

Q2.) $q = 0.2\pi - 40$

a.)



b.) When $q = 0$ price is 200\$. This implies that no producer is willing to sell the product for anything less than 200 \$.

c.) Supply $\Rightarrow$ $q = 0.2\pi - 40$

demand $\Rightarrow$ $\pi = -10q + 2000$

At market equilibrium, the curves intersect

$$\pi = -10(0.2\pi - 40) + 2000$$

$$\pi = -2\pi + 400 + 2000$$

$$3\pi = 2400$$

$$\pi = 800 \Rightarrow q = 120$$

d.) Consumer surplus $= \frac{1}{2}(2000 - 800)(120)$

$= \$\ 72{,}000$

producer revenue $= 800 \times 120 = \$\ 96{,}000$

producer surplus $= \frac{1}{2}(800 - 200)(120) = \$\ 36000$

Social welfare = producer surplus + consumer surplus

$= 72000 + 36000$

$= \$\ 108{,}000$

Q3.) demand curve : $\pi = -10q + 2000$ \$

supply curve : $q = 0.2\pi - 40$

a.) $\pi \geq 900$

Without this intervention the equilibrium price would be \$800

Since $\pi \geq 900$ and $\pi_{eq} = 800$ implies

$\pi = 900$ is the new resulting equilibrium

(By lagrange multipliers, either the equilibrium price is attained or the boundary condition is satisfied)

In this case, $q = \frac{2000 - 900}{10} = 110$

as dictated by the demand curve.

$(\pi, q) = (900, 110)$

consumer surplus $= \frac{1}{2}(2000 - 900)(110) = \$ 60,500$

producer revenue

$= 900 \times 110 = \$ 99,000$

producer surplus $= \frac{1}{2}(150 + 900)(110)$

$= \cancel{\$ 57,750}$ \$ 46,750

Social welfare $=$ \$107,250

b.) $\pi \leq 600$

Proceeding from the previous argument, again the price will be its limiting case, i.e. $\pi = 600$

This time quantity will be determined by the supply curve

$q = 0.2(600) - 40 = 80$

$(\pi, q) = (600, 80)$

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Consumer surplus $= \frac{1}{2}(2000-600 + 1200-600)\ 80$

$= 80,000\ \$$

producer revenue $= 600 \times 80 = 48000\ \$$

producer surplus $= \frac{1}{2}(600-200)\ 80 = 16,000\ \$$

social welfare $= 96,000\ \$$

c.) Sales tax \$ 450 per midget

Sales tax drops the demand curve by the amount described by tax

$\therefore$ The new demand curve will be

$$\pi + 450 = -10q + 2000\ \$$$

or $$\pi = -10q + 1550$$

For this new demand curve, the equilibrium is at

$$\pi = -10q + 1550$$

$$\pi = -10(0.2\pi - 40) + 1550$$

$$\pi = -2\pi + 400 + 1550$$

$$3\pi = 1950$$

$$\pi = 650 \quad \& \quad q = 0.2(650) - 40 = 90$$

$$(\pi, q) = (650, 90)$$

Consumer surplus $= \frac{1}{2}(1550-650)\ 90 = 40,500\ \$$

producer revenue $= 650 \times 90 = 58,500\ \$$

producer surplus $= \frac{1}{2}(650-200) \times 90 = 20,250\ \$$

social welfare $= 60,750\ \$$

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Q4.)

	π_1 On peak price (π_1) (\$/MWh)	π_2 Off peak price (π_2) (\$/MWh)	D_1 Avg on pk dem (MWh)	D_2 Avg off pk dem (MWh)
Base case	0.08	0.06	1000	500
Exp. 1	0.08	0.05	992	509
Exp. 2	0.09	0.06	985	510

a.) On peak elasticity $= \dfrac{\Delta D_1}{\Delta \pi_1} \cdot \dfrac{\pi_1}{D_1}$

$$= \left(\frac{D_{12} - D_{1a}}{\pi_{12} - \pi_{1a}}\right) \frac{\pi_{1a}}{D_{1a}}$$ ($\because$ price changes at Exp. 2)

$$= \left(\frac{985 - 1000}{0.09 - 0.08}\right) \frac{0.08}{1000}$$

$$= \underline{\underline{-0.12}}$$

b.) Off peak elasticity $= \dfrac{\Delta D_2}{\Delta \pi_2} \cdot \dfrac{\pi_2}{D_2}$

$$= \left(\frac{D_{21} - D_{2a}}{\pi_{21} - \pi_{2a}}\right) \frac{\pi_{2a}}{D_{2a}}$$ ($\because$ price changes at exp. 1)

$$= \left(\frac{509 - 500}{0.05 - 0.06}\right) \frac{0.06}{500}$$

$$= \underline{\underline{-0.108}}$$

c.) cross elasticity on peak to off

$$= \frac{\Delta D_1}{D_1} \times \frac{\pi_2}{\Delta \pi_2}$$

$$= \left(\frac{1000 - 992}{1000}\right) \times \left(\frac{0.06}{0.01}\right)$$ (Exp 1)

$$= \underline{\underline{0.048}}$$

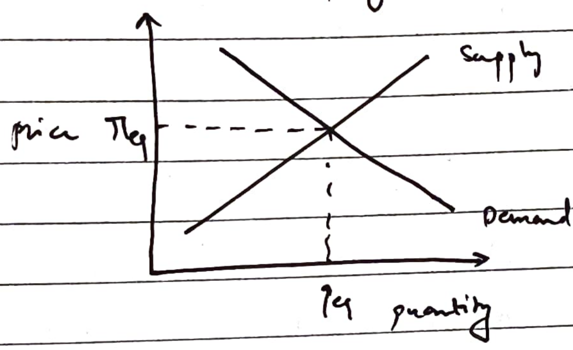
d.) Cross elasticity off peak to on

$$= \frac{\Delta D_2}{D_2} \times \frac{\pi_1}{\Delta \pi_1}$$ (Exp 2)

$$= \frac{(510 - 500)}{500} \times \left(\frac{0.08}{0.09 - 0.08} \right)$$

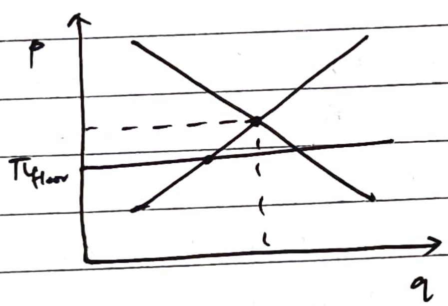
$$= \underline{\underline{0.160}}$$

Q5.) Consider a supply demand curve graph as below



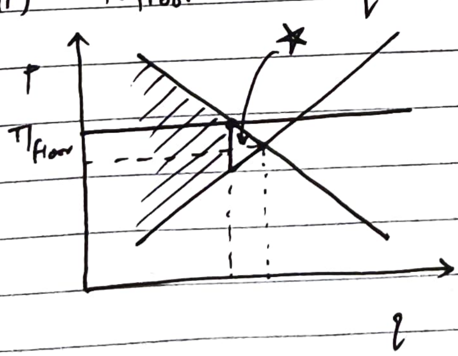
a.) When we have a minimum price floor there are 2 cases

i) $\Pi_{floor} \le \Pi_{eq}$



In this case, market forces drive the price to Π_{eq} and there is no loss in social welfare

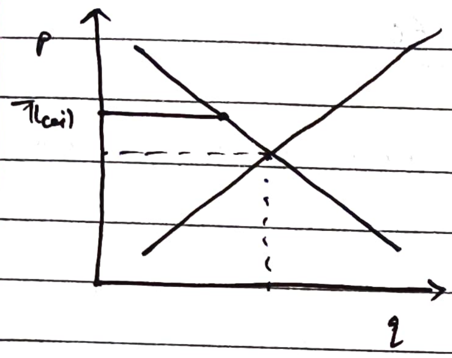
ii) $\Pi_{floor} > \Pi_{eq}$



In this case, the floor price becomes the market price and dead weight loss is illustrated by the ★ marked area.

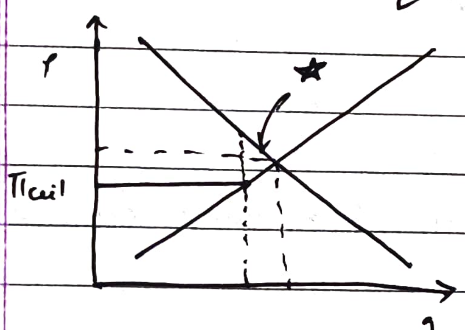
b.) Similarly, when we have a price ceiling, there are 2 cases

i) $\Pi_{ceil} \geq \Pi_{eq}$



In this case market forces drive the market to equilibrium and there is no social welfare loss.

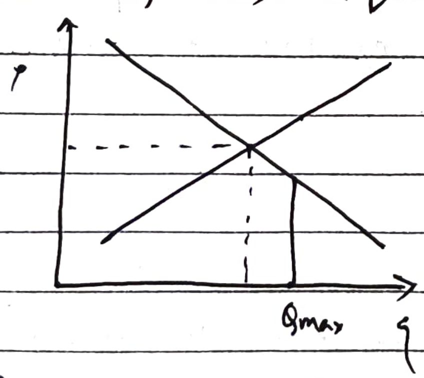
ii) $\Pi_{ceil} < \Pi_{eq}$



Here, the ceil price becomes the market price and the area marked by ☆ acts as the deadweight loss.

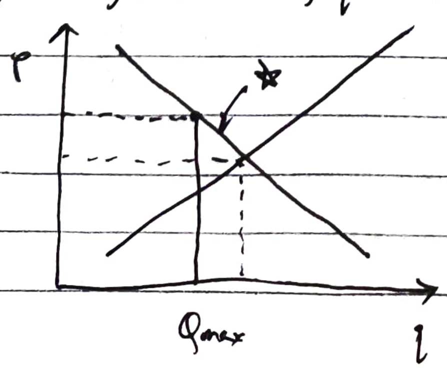
c.) Maximum permissible clearing quantity Q_{max} has 2 cases

i) $Q_{max} \geq Q_{eq}$



The market reaches equilibrium and there is no social welfare loss

ii) $Q_{max} < Q_{eq}$



The Q_{max} restriction forces the price to rise and results again in loss marked by ☆ area

Q6.) $F = 0.6P_1 + 0.4P_2$

$0 \le P_1 \le 400$

$100 \le P_2 \le 300$

a.) If both units are on,

the maximum demand generated $= P_{1max} + P_{2max}$

$= 400 + 300$

$= 700$ MW

the minimum demand $= P_{1min} + P_{2min}$

$= 0 + 100$

$= 100$ MW

$\therefore$ The range $= [100, 700]$ MW

b) Load supplied $= 500$ MW

λ : $P_1 + P_2 - 500 = 0$

$\bar{\eta}_1$: $P_1 - 400 \le 0$

$\underline{\eta}_1$: $-P_1 \le 0$

$\bar{\eta}_2$: $P_2 - 300 \le 0$

$\bar{\eta}_2$ $100 - P_2 \le 0$

f_{min} : $0.6P_1 + 0.4P_2$

$\mathcal{L}(P_1, P_2, \lambda, \bar{\eta}_1, \underline{\eta}_1, \bar{\eta}_2, \underline{\eta}_2)$

$= 0.6P_1 + 0.4P_2 + \lambda(P_1 + P_2 - 500) +$
$+ \bar{\eta}_1(P_1 - 400) + \underline{\eta}_1(-P_1) +$
$+ \bar{\eta}_2(P_2 - 300) + \underline{\eta}_2(100 - P_2)$

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By the KKT conditions, we have:

$$\left.\frac{\partial \mathcal{L}}{\partial P}\right|_* = 0 \Rightarrow \left(0.6 + \lambda + \bar{\mu}_1 - \underline{\mu}_1 ,\ 0.4 + \lambda + \bar{\mu}_2 - \underline{\mu}_2\right) = \bar{0}$$

$$\left.\frac{\partial \mathcal{L}}{\partial \lambda}\right|_* = 0 \Rightarrow P_1 + P_2 - 500 = 0$$

$$\left.\frac{\partial \mathcal{L}}{\partial \mu}\right|_* \leq 0 \Rightarrow (P_1 - 400, -P_1, P_2 - 300, 100 - P_2) \leq 0$$

On solving these conditions, we get

$P_2^* = 300MW$, $P_1^* = 200MW$

$\bar{\mu}_1 = \underline{\mu}_1 = \underline{\mu}_2 = 0$

$\bar{\mu}_2 = 0.2$

$\lambda = -0.6$ as the feasible & optimal solution

Minimum cost is $F = 0.6 P_1^* + 0.4 P_2^*$

$= 0.6 (200) + 0.4 (300)$

$= 120 + 120$

$= 240$ Rs/hr

Q7.) $B_1(P_1) = 5000 + 1.005 P_1$

$B_2(P_2) = 4800 + 1.195 P_2$

Demand is inelastic at 600MW

a.) We have $P_1 + P_2 = 600$ (λ)

We wish to minimize total cost of production

$B_1(P_1) + B_2(P_2)$

also, $P_1 \geq 0$ & $P_2 \geq 0$

$-P_1 \leq 0$ & $-P_2 \leq 0$

$$\mathcal{L}(P_1, P_2, \lambda, y_1, y_2) = 5000 + 1.005 P_1 + 4800 + 1.195 P_2 + \lambda(P_1 + P_2 - 600) + y_1(-P_1) + y_2(-P_2)$$

$$\left.\frac{\partial \mathcal{L}}{\partial P}\right|_* = 0 \Rightarrow \left(1.005 + \lambda - y_1,\ 1.195 + \lambda - y_2\right) = 0$$

$$\left.\frac{\partial \mathcal{L}}{\partial \lambda}\right|_* = 0 \Rightarrow P_1 + P_2 - 600 = 0$$

$$\left.\frac{\partial \mathcal{L}}{\partial y}\right|_* \leq 0 \Rightarrow (-P_1, -P_2) \leq 0$$

On solving, we get

$\lambda = -1.005$ $\quad y_1 = 0$ $\quad y_2 = 0.19$

$P_1^* = 600$ $\quad P_2^* = 00$

b.) The clearing price then is

$B_1(P_1^*) + B_2(P_2^*)$

$= 5000 + 1.005(600) + 4800 + 0$

$= 10,403$ Rs/MWh