Electrical Engineering
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EE 722, Spring 2023
Assignment-2
Total Points = 50
Due: February 22, 2023

1. (Total = 4 points) A market operator receives the sell bids in the form: $SB_i(q_i) = \beta_i + \gamma_i * q_i$ where $SB_i(q_i)$ is the asking price for selling q_i MW in \$. The seller also submits a $\bar{q}_i$ which is the maximum MW he can sell. The sell bid information for an hour k is given below:

Index i	β_i in \$/MWh	γ_i in $\$^2/MWh$	$\bar{q}_i$ in MW
1	3.5	0.002	1000
2	5	0.005	800
3	4.5	0.003	900
4	3.8	0.004	850
5	3.8	0.004	850

If the price discovered is 6 \$/MWh, answer the following:

(a) (1 point) State true or false: Seller 1 is selling 1000 MW.

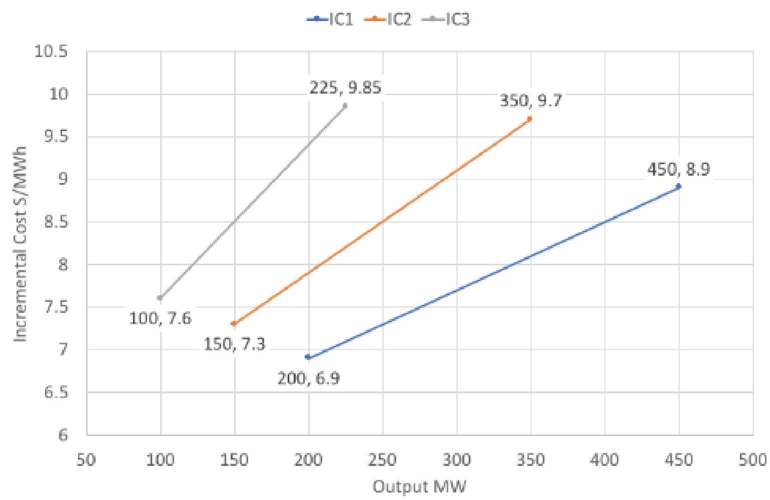
(b) (2 points) How much MW is the market clearing volume?

2. (Total = 2 points) The purchase bid received of buyer i is $PB_i(q_i) = \alpha_i - \delta_i * q_i$ where $PB_i(q_i)$ is the maximum price he is willing to pay in \$ to buy q_i MW. The buyer also submits a $\bar{q}_i$ which is the maximum MW he can buy. If the buyer is asked to purchase 800 MW for when the buying price is 8 \$/MWh, is he satisfied with the market outcome? State yes or no and provide a short justification for your answer.

index i	α_i	δ_i	$\bar{q}_i$
1	20	0.015	1000
2	21	0.018	1000

3. (Total = 4 points) What are the options available for electricity trading in India? Read reports from CEA/MOP/POSOCO and find out the volume of trading through each of these options over the past few years. Cite your sources.

4. (Total = 3 points) The incremental costs $\delta Ci/\delta Pi$ for generators i = 1,2,3 are plotted below with the labeled points indicating the incremental costs at the limits. Suppose the dispatch problem is solved with the Lambda iteration algorithm.



If the final λ is 7 $/MWh, then

(a) (1 point) How much MW is the power output for generator 2 is MW?

(b) (2 points) How much MW is the total demand served?

5. (Total = 12 points) Adapted from Kirschen and Strbac, problem 3.3
The following six companies participate, along with others, in the Southern Antarctica electrical energy market:

- Red: A generating company owning a portfolio of plants with a maximum capacity of 1000 MW.
- Green: Another generating company with a portfolio of plants with a maximum capacity of 800 MW.
- Blue: A retailer of electrical energy.
- Yellow: Another retailer of electrical energy.
- Magenta: A trading company with no generating assets and no demand.
- Purple: Another trading company with no physical assets.

The following information pertains to the operation of this market for Monday 29 February 2016 between 1 : 00 and 2 : 00 P.M.

Load forecasts
Blue and Yellow forecast that their customers will consume 1200 MW and 900 MW respectively during that hour.

Long-term contracts
June 2015: Red signs a contract for the supply of 600 MW at 15 $/MWh for all hours between 1 January 2015 and 31 December 2020
July 2015: Blue signs a contract for the purchase of 700 MW for all hours between 1 February 2016 and 31 December 2016. The price is set at 12 $/MWh for off-peak hours and at 15.50 $/MWh for peak hours.
August 2015: Green signs a contract for the supply of 500 MW at 16 $/MWh for peak hours in February 2016.

September 2015: Yellow signs a contract for the purchase of electrical energy. The contract specifies a profile of daily and weekly volumes and a profile for daily and weekly prices. In particular, on weekdays between 1 : 00 and 2 : 00 P.M., the volume purchased is 550 MW at 16.25 $/MWh.

Futures contracts

All contracts are for delivery on 29 February 2016 between 1 : 00 and 2 : 00 P.M.

Date	Company	Type	Amount	Price
10/9/15	Magenta	Buy	50	14.50
20/9/15	Purple	Sell	100	14.75
30/9/15	Yellow	Buy	200	15.00
10/10/15	Magenta	Buy	100	15.00
20/10/15	Red	Sell	200	14.75
30/10/15	Green	Sell	250	15.75
30/10/15	Blue	Buy	250	15.75
10/11/15	Purple	Buy	50	15.00
15/11/15	Magenta	Sell	100	15.25
20/11/15	Yellow	Buy	200	14.75
30/11/15	Blue	Buy	300	15.00
10/12/15	Red	Sell	200	16.00
15/12/15	Red	Sell	200	15.50
20/12/15	Blue	Sell	50	15.50
15/1/16	Purple	Sell	200	14.50
20/1/16	Magenta	Buy	50	14.25
10/2/16	Yellow	Buy	50	14.50
20/2/16	Red	Buy	200	16.00
25/2/16	Magenta	Sell	100	17.00
28/2/16	Purple	Buy	250	14.00
28/2/16	Yellow	Sell	100	14.00

Options contracts

In November 2015, Red bought a put option for 200 MWh at 14.75 $/MWh. The option fee was $50.

In December 2015, Yellow bought a call option for 100 MWh at 15.50 $/MWh. The option fee was $25.

Outcome

- The spot price on the Southern Antarctica electricity market was set at 15.75 \$/MWh for 29 February 2016 between 1 : 00 and 2 : 00 P.M.
- Owing to the difficulties at one of its major plants, Red was able to generate only 800 MW. Its average cost of production was 14.00 \$/MWh.
- Green generated 770 MW at an average cost of 14.25 \$/MWh.
- Blue's demand turned out to be 1250 MW. Its average retail price was 16.50 \$/MWh.
- Yellow's demand turned out to be 850 MW. Its average retail price was 16.40 \$/MWh.

Assuming that all imbalances are settled at the spot market price, calculate the profit or loss made by each of these participants.

6. (Total = 7 points) Adapted from Kirschen and Strbac, problem 3.4
The operator of a centralized market for electrical energy has received the bids shown in the table below for the supply of electrical energy during a given period.

Company	**Amount (MWh)**	**Price (\$/MWh)**
Red	100	12.5
Red	100	14.0
Red	50	18.0
Blue	200	10.5
Blue	200	13.0
Blue	100	15.0
Green	50	13.5
Green	50	14.5
Green	50	15.5

(a) (1 point) Build the supply curve.

(b) (3 points) Assume that this market operates unilaterally, that is, that the demand does not bid and is represented by a forecast. Calculate the market price, the quantity produced by each company and the revenue of each company for each of the following loads: 400 MW, 600 MW, 875 MW.

(c) (2 points) Suppose that instead of being treated as constant, the load is represented by its inverse demand curve, which is assumed to have the following form:

$$D = L - 4 * \pi$$

where D is the demand, L is the forecasted load and π is the price. Calculate the effect that this price sensitivity of demand has on the market price and the quantity traded.

7. (Total = 7 points) Adapted from Kirschen and Strbac, problem 3.5
The Syldavian Power and Light Company owns one generating plant and serves some load. It has been actively trading in the electricity market and has established the following position for 11 June between 10 : 00 and 11 : 00 A.M.:

- Long-term contract for the purchase of 600 MW during peak hours at a price of 20.00 \$/MWh
- Long-term contract for the purchase of 400 MW during off-peak hours at a price of 16.00 \$/MWh
- Long-term contract with a major industrial user for the sale of 50 MW at a flat rate of 19.00 \$/MWh
- The remaining customers purchase their electricity at a tariff of 21.75 \$/MWh
- Future contract for the sale of 200 MWh at 21.00 \$/MWh
- Future contract for the purchase of 100 MWh at 22.00 \$/MWh
- Call option for 150 MWh at an exercise price of 20.50 \$/MWh
- Put option for 200 MWh at an exercise price of 23.50 \$/MWh
- Call option for 300 MWh at an exercise price of 24.00 \$/MWh

The option fee for all the options is 1.00 \$/MWh. The peak hours are defined as being the hours between 8 : 00 A.M. and 8 : 00 P.M.

The outcome for 11 June between 10 : 00 and 11 : 00 is as follows:

- The spot price is set at 21.50 \$/MWh.
- The total load of the Syldavian Power and Light Company is 1200 MW, including the large industrial customer.
- The power plant produces 300 MWh at an average cost of 21.25 \$/MWh.

(a) (3 points) Assuming that all imbalances are settled at the spot market price, calculate the profit or loss made by the company during that hour.

(b) (4 points) What value of the spot market would reduce the profit or loss of the company to zero? Would this change in spot price affect any of the option contracts?

8. (Total = 4 points) Adapted from Kirschen and Strbac, problem 4.1
Cheapo Electrons is an electricity retailer. The table below shows the load that it forecast its consumers would use over a 6-h period. Cheapo Electrons purchased in the forward market and the power exchange exactly enough energy to cover this forecast. The table shows the average price that it paid for this energy for each hour. As one might expect, the actual consumption of its customers did not exactly match the load forecast and it had to purchase or sell the difference on the spot market at the prices indicated. Assuming that Cheapo Electrons sells energy to its customers at a flat rate of 24.00 \$/MWh, calculate the profit or loss that it made during this 6-h period. What would be the rate that it should have charged its customers to break even?

Period	**1**	**2**	**3**	**4**	**5**	**6**
Load Forecast (MWh)	120	230	310	240	135	110
Average cost (\$/MWh)	22.5	24.5	29.3	25.2	23.1	21.9
Actual load (MWh)	110	225	330	250	125	105
Spot price (\$/MWh)	21.6	25.1	32	25.9	22.5	21.5

9. (Total = 4 points) Adapted from Kirschen and Strbac, problem 4.7
Borduria Generation owns three generating units that have the following cost functions:

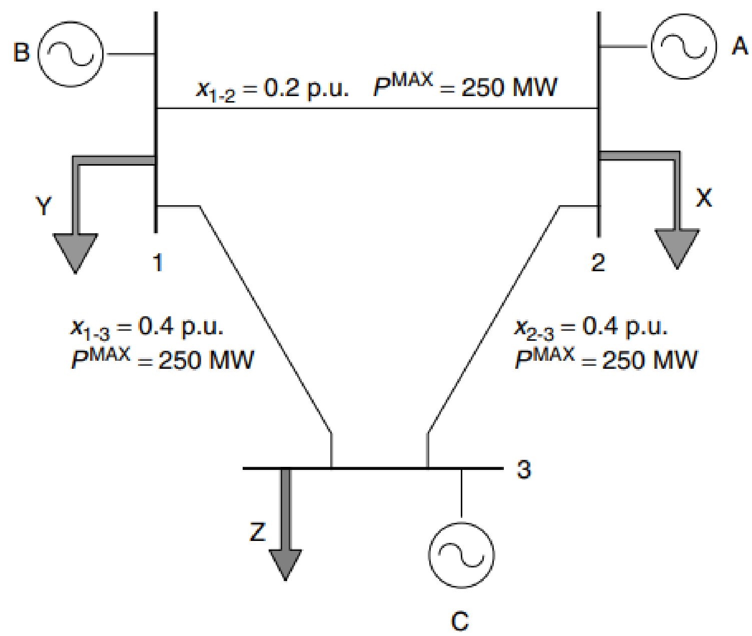
Unit A: $15 + 1.4\,P_A + 0.04\,P_A^2$ \$/h

Unit B: $25 + 1.6\,P_B + 0.05\,P_B^2$ \$/h

Unit C: $20 + 1.8\,P_C + 0.02\,P_C^2$ \$/h

Borduria Generation had the opportunity to buy some of this energy on the spot market at a price of 8.20 \$/MWh. How should these units be dispatched if Borduria Generation must supply a load of 350 MW at minimum cost?

10. (Total = 3 points) Adapted from Kirschen and Strbac, problem 6.1
Consider the power system shown in the Figure below. Assuming that the only limitations imposed by the network are imposed by the thermal capacity of the transmission lines and that the reactive power flows are negligible, check that the following sets of transactions are simultaneously feasible.



	Seller	Buyer	Amount
Set 1	B	X	200
	A	Z	400
	C	Y	300
Set 2	B	Z	600
	A	X	300
	A	Y	200
	A	Z	200
Set 3	C	X	1000
	X	Y	400
	B	C	300
	A	C	200
	A	Z	100