Electrical Engineering
IIT Bombay, Powai
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EE 722, Spring 2023
Assignment-1
Total Points = 50
Due: January 30, 2023

1. (Total = 13 points) Adapted from Kirschen and Strbac, problem 2.2
   The inverse demand function of a group of consumers for a given type of widgets is given by the following expression:
   $$\pi = -10q + 2000 \ \$$
   where q is the demand and π is the unit price for this product.
   (a) (2 points) Draw the demand curve for the group of consumers and mark on it the (i) their maximum consumption, and, (ii) the price that no consumer is prepared to pay for this product.
   (b) (2 points) Determine the maximum consumers' surplus. Explain why the consumers will not be able to realize this surplus.
   (c) (3 points) For a price π of 1000 \$/unit, calculate the quantity bought, consumer surplus and the revenue collected by the producers.
   (d) (3 points) If the price π increases by 20%, calculate the changes in quantify bought, consumer surplus and the revenue collected by the producers.
   (e) (1 point) What is the price elasticity of demand for this product and this group of consumers when the price π is 1000 \$/unit.
   (f) (2 points) Derive an expression for the gross consumers' surplus and the net consumers' surplus as a function of the demand. Check these expressions using the results of part (d).

2. (Total = 8 points) Adapted from Kirschen and Strbac, problem 2.3
   Economists estimate that the supply function for the widget market is given by the following expression:
   $$q = 0.2\pi - 40$$
   (a) (1 point) Draw the supply curve for the widget market.
   (b) (1 point) What is the price when $q = 0$? What does this imply?
   (c) (2 points) Calculate the demand and price at the market equilibrium if the demand is as defined in Problem 2.2 (*problem 1 above*).
   (d) (4 points) For this equilibrium, calculate the consumer surplus, the producers' revenue, the producer surplus and the social welfare.

3. (Total = 12 points) Adapted from Kirschen and Strbac, problem 2.4
   Calculate the effect on the market equilibrium of Problem 2.2/2.3 (*above two problems*) of the following interventions:
   (a) (4 points) A minimum price of \$ 900 per widget.
   (b) (4 points) A maximum price of \$ 600 per widget.
   (c) (4 points) A sales tax of \$ 450 per widget.

Note: Specifically, calculate the new market outcome (clearing price and quantity), changes in the consumer surplus/producer surplus/social welfare and mention how the intervention should be considered in the analysis.

4. (Total = 4 points) Adapted from Kirschen and Strbac, problem 2.6
Vertically integrated utilities often offer two-part tariffs to encourage their consumers to shift demand from on-peak load periods to off-peak periods. Consumption of electrical energy during on-peak and off-peak periods can be viewed as substitute products. The table below summarizes the results of experiments that the Southern Antarctica Power and Light Company has conducted with its two-part tariff. Use these results to estimate the elasticities and cross-elasticities of the demand for electrical energy during peak and off-peak periods.

	On-peak price	**Off-peak price**	**Average on-peak demand**	**Average off-peak demand**
	π_1	π_2	D_1	D_2
	($/MWh)	**($/MWh)**	**(MWh)**	**(MWh)**
Base case	0.08	0.06	1000	500
Experiment 1	0.08	0.05	992	509
Experiment 2	0.09	0.06	985	510

5. (Total = 6 points) Recall dead weight loss – it is the reduction in social welfare due to market interventions. In class we saw how restrictions on price or volume can lead to dead weight loss. Using simple linear supply-demand curves, illustrate the dead weight loss incurred due to
   (a) (2 points) Introduction of a price floor (minimum allowable MCP)
   (b) (2 points) Introduction of a price ceiling (maximum allowable MCP)
   (c) (2 points) Imposition of a curtailment in quantity cleared (decrease in MCQ)

   Likewise, a price floor can also result in dead weight losses.

6. (Total = 4 points) A power plant has two generating units. The cost of generation is given by the function $F = 0.6P_1 + 0.4P_2$ Rs/hr. The generation limits are $0 \leq P_1 \leq 400$ and $100 \leq P_2 \leq 300$.
   (a) (2 points) If both units are always on, what is the range of demand this power plant can supply?
   (b) (2 points) If the plant supplies a 500 MW load, what is the least cost dispatch solution? Specify P_1^* and P_2^* in MW.

7. (Total = 3 points) Consider two generators submitting linear bids to a market operator that take the form:

$$B_1(P_1) = 5000 + 1.005P_1 \quad \text{and} \quad B_2(P_2) = 4800 + 1.195P_2$$

which can be interpreted as follows: $B(P)$ is the asking price of the generator in INR/MWh for producing P MW of power for an hour. If an inelastic demand (willing to pay infinite price) of 600 MW is to be met in this market, determine

(a) (2 points) Quantities sold by both generators in MW.

(b) (1 point) Market clearing price in INR/MWh