Basic Terminology

1. Random Experiment:

   An expieriment is called random when it satisfies 2 conditions. a. it has more then 1 outcome b. not possible to predict outcome in advance ex: tossing a coin, It have 2 outcome & cant be predicted

   A random experiment is like a test or an activity where the outcome is uncertain or not predictable with certainty. Imagine flipping a coin, rolling a die, or picking a card from a deck. These are all examples of random experiments because the result can vary each time you do them. The outcome of a random experiment is usually influenced by chance, luck, or randomness, rather than being controlled or predetermined.

2. Trial:

   It refers to single execution on random experiment, each trial produces an outcome. for example: when we rolled a dice once.. rolling a dice is a random experiment we might be rolling it multiple times, rolling it once is 1st trial, rolling it again is 2nd trial & so on

3. Outcome:

   Every tria will have an outcome.. like when rolling a dice it can have any value b/w 1 to 6.. whatever value comes when rolling a dice once is an outcome of the trial

4. Sample Space:

   Its a set of all possible outcome that can occur in a random experiment.. like when tossnig a coin there can be only 2 outcomes Head or Tail so the sample space of that random experiment will be {Head,Tail}

5. Event:

   Its the unique combination of outcome which can happen in the random experiment, It can be single or multiple like when tossing a coin you might get head or tail, or you might head times in a row etc. So, an event is just a specific thing that you're looking out for or interested in when you're doing a random experiment like tossing a coin or rolling a dice.

   Event depends on us, on the problem we are trying to solve, depends on scenario.

In [17]:
```
#### Random Experiment: Drawing a card from a standard deck of 52 cards.

# Trial: Drawing one card from the deck.
# Outcome: whatever card that came when drawn.
# Sample Space: The set of all possible outcomes, which is the entire deck of 52 cards.
# Event: Drawing a red card.


#### Random Experiment: Flipping a fair coin twice.

# Trial: Flipping the coin twice in a row.
# Outcome: Getting a heads tails
# Sample Space: {HH, HT, TH, TT} (where H represents heads and T represents tails)
# Event: Getting at least one head in the two flips.

#### Random Experiment: Rolling a six-sided die three times.

# Trial: Rolling the die three times consecutively.
# Outcome: getting any number each time b/w 1 to 6
# Sample Space: The set of all possible outcomes, which is {1, 2, 3, 4, 5, 6} repeated three times.
# Event: Getting a total sum of 10 or more from the three rolls.

#### Random Experiment: Selecting a random student from a class of 20 students.

# Trial: Choosing one student randomly.
# Outcome: whichever student being selected.
# Sample Space: The set of all possible outcomes, which is the list of 20 students in the class.
# Event: Choosing a student whose name starts with the letter 'A'.
```

Types of events

1. Simple event: which can have only 1 possible outcome like getting a head in tossing a coin, getting a 3 in rolling a dice
2. Compound event: which can have more then 1 outcomes like getting odd numbers in rolling a dice, getting ace in card flusing
3. Independent event: Independent events are events where the occurrence of one event does not affect the probability of the occurrence of another event. In simpler terms, the outcome of one event has no influence on the outcome of the other event. like tossing a fair coin twice: whether the first toss results in heads or tails does not affect the outcome of the second toss. The two events (each toss) are independent of each other.
4. Dependend Events: Dependent events are events where the outcome of one event does affect the probability of the occurrence of another event. In other words, the outcome of one event changes the probability of the outcome of the second event like drawing 2 cards from a deck without replacement & got Ace in the 1st card now the probability of getting an ace for the 2nd card will reduce.
5. Mutually Exclusive Events: Two events are exclusive if they both can not occur at the same time like getting 2 & 6 at the same time when rolling a die.. thats impossible
6. Exhaustive event: A set of event is exhaustive when at least one of the event will definetly occur like when tossing a coin you'll get either head or tail.. or when rolling a dice you will get either an odd num or even num.
7. Impossible event: An event which will never happen like getting 7 when rolling a dice.
8. certain event: An event of surity like getting a number b/w 1 to 6 in dice rolling or getting head or tail in tossing a coin

Probability

- Probability is all about how likely an event will occur like whats the probability of getting head when tossing a coin is 0.5 or the probability of getting 3 when rolling a die is 1/6.
- Its expressed in a number b/w 0 to 1
- 0 means it will not happen, 1 means it will happen & 0.5 means it will happen half the time

Emperical Probability

- Empirical probability, also known as experimental probability, is a type of probability that is based on observations or experiments. Instead of using theoretical calculations or mathematical formulas, empirical probability is determined by actually conducting experiments or observations and recording the outcomes.
- when tossing a coin 100 times, we got head 55 times so emperical probability of getting a head will be 55/100

Theoritical Probability

- Theoretical probability, also known as classical probability, is a type of probability when each outcome in a sample space is equally likely to occur
- like when tossing a coin, possibility of both outcomes in sample space {H,T} is equal or when rolling a dice, probability of getting any number is 1/6
- formula: no of favorable outcomes/ total number of outcomes in sample space
- getting 3 when rolling a dice is : 1/6

In [3]:
```
# Emperical probability will be very close to theoritical probability when no of trials increases
```

Random Variable

(Its not a "variable", Its a function)

- random variable is a function which takes as input, runs a logic & provides an ouput
- input will be complete sample space for example: in tossing a coin experiment, input will be {H,T} or in rolling a dice experiment input will be {1,2,3,4,5,6}
- output will always be a real numberthat we asign to each possible outcome
- random variable will convert sample space into real number based on logic which comes from the events you want to study so that we can study them with the help of probability distributions

How do we decide the logic

- logic comes from event --> for whatever reason we're conducting the experiment
- for example: rolling 2 dice & we want a probability of "getting a sum of 7"
- so the sample space will be sum of 1-1 number {(1,1)(1,2)(1,3)... (2,1)(2,2).. (3,1)... (6,6)}
- sample space will have 6 x 6 = 36 items --> X = {2,3,4,5,6,7,8,9... 36}
- probability will be: 1/36

There are 2 types of random variables:

1. discrete
2. continous

Probability distribution of a random variable

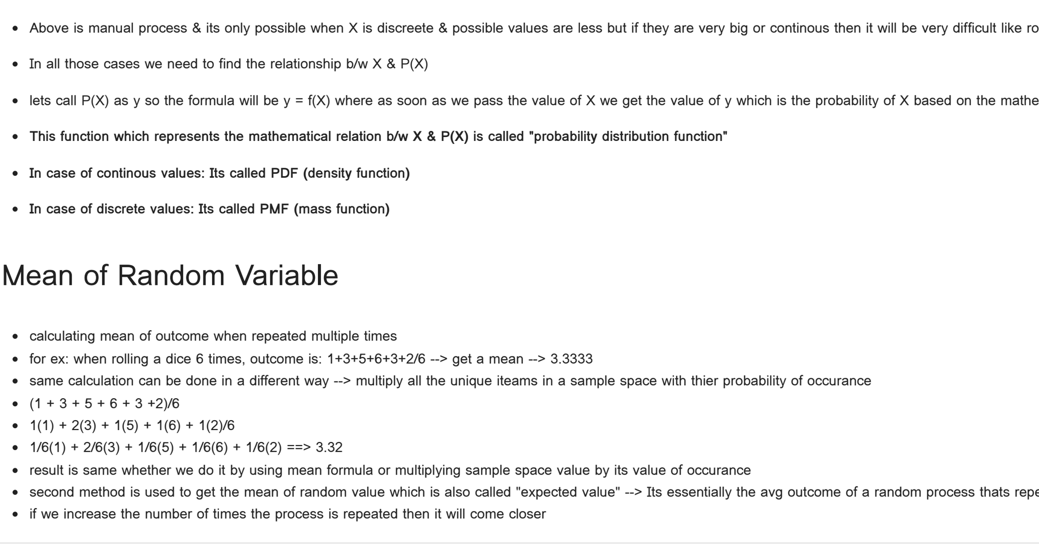
- probability distribution is a list of all the possible outcomes of a random variable along with thier corresponding probability values
- Its a table which contains all possible outcomes & its probability
- for tossing a coin experiment:

X	1	0
P(X)	1/2	1/2

- for rolling a dice experiment:

X	1	2	3	4	5	6
P(X)	1/6	1/6	1/6	1/6	1/6	1/6

- for rolling 2 dice experiment:



- Above is manual process & its only possible when X is discreete & possible values are less but if they are very big or continous then it will be very difficult like rolling 1000 dice altogether
- In all those cases we need to find the relationship b/w X & P(X)
- lets call P(X) as y so the formula will be y = f(X) where as soon as we pass the value of X we get the value of y which is the probability of X based on the mathematical relationship function "f"
- **This function which represents the mathematical relation b/w X & P(X) is called "probability distribution function"**
- **In case of continous values: Its called PDF (density function)**
- **In case of discrete values: Its called PMF (mass function)**

Mean of Random Variable

- calculating mean of outcome when repeated multiple times
- for ex: when rolling a dice 6 times, outcome is: 1+3+5+6+3+2/6 --> get a mean --> 3.3333
- same calculation can be done in a different way --> multiply all the unique iteams in a sample space with thier probability of occurance
- (1 + 3 + 5 + 6 + 3 +2)/6
- 1(1) + 2(3) + 1(5) + 1(6) + 1(2)/6
- 1/6(1) + 2/6(3) + 1/6(5) + 1/6(6) + 1/6(2) ==> 3.32
- result is same whether we do it by using mean formula or multiplying sample space value by its value of occurance
- second method is used to get the mean of random value which is also called "expected value" --> Its essentially the avg outcome of a random process thats repeated many times.
- if we increase the number of times the process is repeated then it will come closer

In [13]:
```python
import random
import numpy as np
```

In [15]:
```python
outcome = []

for i in range(10000000):
    outcome.append(random.randint(1,6))

np.array(outcome).mean()
```

Out[15]: 3.4999741

$$E[X] = \sum_{i=1}^{n} x_i P(x_i)$$

- Expected value of X = sigma i = 1 to n where n is the number of unique values, we multiply that value Xi wqith its probability P(Xi)

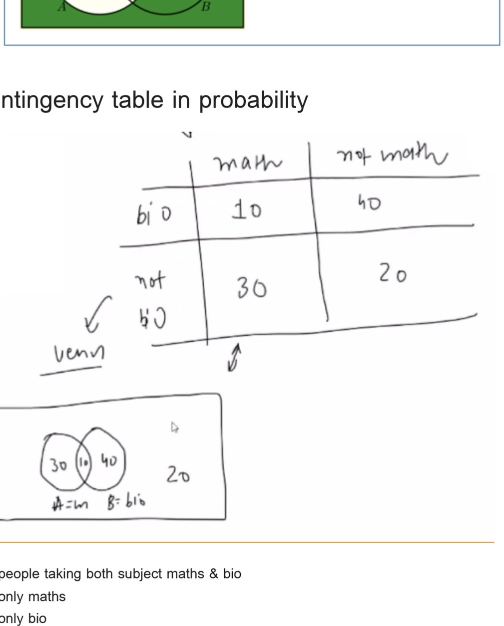
Variance of Random Variable

- variance of a random variable describes how much individual observation in a group differs from its mean
- this can be calculated from variance formula
- $$\text{var} = \sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}$$
- finding avg of sum of squared distance from the mean
- formula to calculate variance of random variable:
- $$Var(X) = E[(X - E[X])^2]$$
- when E[X] is mean of random variable

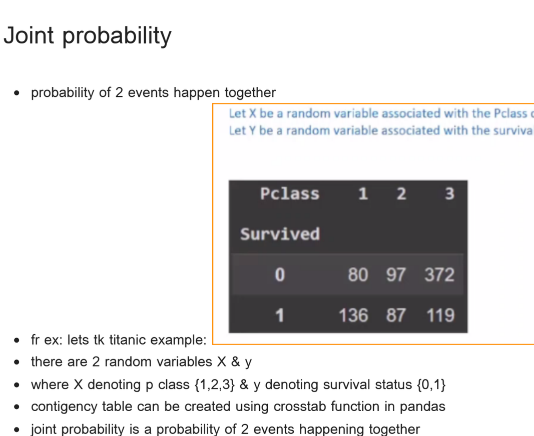
Types of Probability

Venn Diagram

- A Venn diagram is a way to show the relationship between different groups or sets of things. It uses circles to represent each group or set, and where the circles overlap, it shows the things that are common to both sets.



Contingency table in probability



- people taking both subject maths & bio
- only maths
- only bio
- none
- we can easily create venn diagram using contingency table

Joint probability

- probability of 2 events happen together

Let X be a random variable associated with the Pclass of a passenger
Let Y be a random variable associated with the survival status of a passenger

Pclass Survived	1	2	3
0	80	97	372
1	136	87	119

- fr ex: lets tk titanic example:
- there are 2 random variables X & y
- where X denoting p class {1,2,3} & y denoting survival status {0,1}
- contigency table can be created using crosstab function in pandas
- joint probability is a probability of 2 events happening together
- we want to know probability of X = 0 & Y = 1 (probability of people diying in pclass 1)
- as per contingency table, count is 80.

Pclass Survived	1	2	3
0	0.089787	0.108866	0.417508
1	0.152637	0.097643	0.133558

- so probability is 80/891 (891 is total no of pax)
- when you add all probabilities, it will be equal to 1 & this table of probabilities is "Joint Probability Distribution"

- Joint probability refers to the probability of two (or more) events happening together. In your example, you're interested in the joint probability of being in Pclass 1 (X = 0) and surviving (Y = 1), which is calculated as the count of individuals in both categories divided by the total number of individuals.
- The concept of joint probability distributions is about the probabilities of all possible combinations of events occurring together. In this case, the contingency table summarizes these probabilities for the given variables X and Y. The sum of all probabilities in a joint probability distribution is always 1, as it covers all possible outcomes.

marginal probability

- marginal probability refers to the probability of event occurring irrespective of other possibilities

Pclass Survived	1	2	3	All
0	80	97	372	549
1	136	87	119	342
All	216	184	491	891

- like in titanic data, it will just look at the no of people survived or died irrespective of pclass or gender
- 549 is the total no of people died irrespective of pclass
- 342 is the total people survived irrespective of pclass
- 891 is total pax travelling
- 216 is total pax in pclass 1 irrespective of survived or dead
- 184 is total pax in pclass 2 irrespective of survived or dead
- 491 is total pax in pclass 3 irrespective of survived or dead

Pclass Survived	1	2	3	All
0	0.089787	0.108866	0.417508	0.616162
1	0.152637	0.097643	0.133558	0.383838
All	0.242424	0.206510	0.551066	1.000000

- so if we divided all counts by total no of pax which is 891, we'll get probabilities

conditional probability

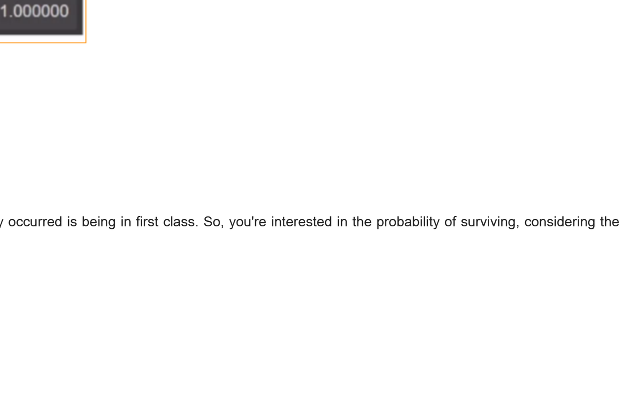
- Conditional probability is indeed about figuring out the probability of one event happening, given that another event has already occurred.
- In simpler terms, it's like asking: "What's the chance of something happening, knowing that something else has already happened?"
- let's say you want to find the probability that a passenger survived, given that they were in first class. In this case, the event that has already occurred is being in first class. So, you're interested in the probability of surviving, considering the fact that the person is already known to be in first class.
- That's the essence of conditional probability – it's about adjusting probabilities based on what we already know has happened.
- P(A | B) (probability of A given B)

Examples

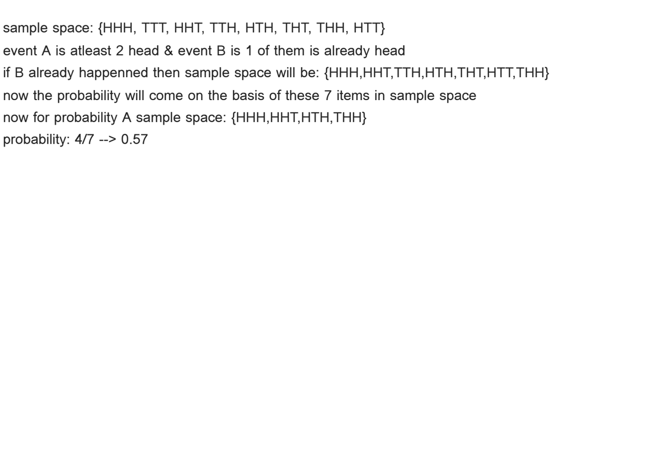
1. 3 unbiased coins are tossed, whats the probability that 2 coins will show head given that 1 already showed head

- sample space: {HHH, TTT, HHT, TTH, HTH, THT, THH, HTT}
- event A is atleast 2 head & event B is 1 of them is already head
- if B already happenned then sample space will be: {HHH,HHT,TTH,HTH,THT,HTT,THH}
- now the probability will come on the basis of these 7 items in sample space
- now for probability A sample space: {HHH,HHT,HTH,THH}
- probability: 4/7 --> 0.57

2. 2 fair six sided dice are rolled, whats the conditional probability that the sum of the numbers rolled is 7, given first dice shows odd number?



3.



Formula to find out conditional probability

Formula for Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

joint prob A and B

marginal prob of B

In [4]:
```
#In python
#pd.crosstab
#joint -- normalize = True
#marginal -- margins = True
#conditional -- normalize = 'columns/index'
```

Intution behind conditional probability

- Its based no reducing sample space
- If bas has already occurred then sample space will be reduced to events where B is present
- Now AuB which is joint probability shows in the universe of B, how many times A occurred

Types of events

1. Independent events: events where occurance of 1 event doesnt affect occurance of another like tossing a coin & rolling a dice.. doesnt matter coin gets head or tail, probability of gettting any number in dice will remain be 1/6
2. dependent events: events where occurance of 1 event affect another like drawing a card wihtout replacement, earlier it was 1/52.. after drawing without replacement it will be 1/51
3. mutually exclusive events: where 2 events cant occur at the same time.. like tossing a coin can not get head & tail both.. it will be either head or tail

Bayes Theorem

- Bayes' Theorem is a way of updating our beliefs or predictions based on new evidence. It helps us figure out the probability of something happening, given what we already know.

conditional Prob

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad A \cap B = B \cap A$$

$$P(B|A) = \frac{P(B \cap A)}{P(A)} \Rightarrow P(B \cap A) = P(A \cap B) = P(B|A)\,P(A)$$

$$\boxed{P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}} \rightarrow \text{Bayes Theorem}$$

In [ ]: